



On INM's Use of Corrected Net Thrust for the Prediction of Jet Aircraft Noise

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Abstract

The Federal Aviation Administration's (FAA) Integrated Noise Model¹ (INM) employs a prediction methodology that relies on corrected net thrust (F_n/δ_a) as the sole correlating parameter between aircraft and engine operating states and aircraft noise. Thus aircraft noise measured for one set of atmospheric and aircraft operating conditions is assumed to be applicable to all other conditions as long as the corrected net thrust remains constant. This hypothesis is investigated under two primary assumptions: (1) the sound field generated by the aircraft is dominated by jet noise, and (2) the sound field generated by the jet flow is adequately described by Lighthill's theory of noise generated by turbulence.

The analysis shows that the correlating parameter F_n/δ_a employed by the INM prediction methodology can be reconciled with relative jet velocity, the independent variable used in Lighthill's jet noise theory. The ratio of INM predictions to those made using Lighthill's theory can be separated into two factors, one dependent on atmospheric conditions and the other dependent on aircraft performance parameters. The factor that depends on atmospheric conditions is generally very near unity. Thus prediction errors will be small, even under the most extreme temperature and altitude conditions. The one that depends on aircraft performance parameters indicates that small deviation of the Mach number of the prediction flight from the Mach number of the aircraft during the data collection flight will yield relatively large prediction errors. The INM prediction will under predict the acoustic field if the prediction flight Mach number is less than the data collection flight Mach number, and over predict the acoustic field if the prediction flight's Mach number is greater than the data collection flight's Mach number.

Introduction

The Federal Aviation Administration's (FAA) Integrated Noise Model¹ (INM) employs a prediction methodology based on the procedures developed by the Society of Automotive Engineers (SAE) and presented in SAE's Aerospace Information Report (AIR) 1845, "Procedure for the Calculation of Airplane Noise in the Vicinity of Airports²." The procedures presented in that report were developed for the prediction of aircraft noise near airports, primarily airports located near sea level and particularly noise generated by aircraft flying into or out of those airports.

There is some indication^{3,4} that INM predictions are in error both for locations more distant from airports and for aircraft taking off from airports located at a considerable altitude above sea level. These differences between measured and predicted levels could be due to any of several factors including incorrect modeling of: (1) aircraft flight performance, (2) sound propagation through the atmosphere, or (3) the noise source itself. One hypothesis, perhaps viewed most correctly as falling within the third category, is that this error in the prediction of the noise levels by INM is due to INM's use of corrected net thrust as the sole correlating parameter between engine state and aircraft noise. In the following this hypothesis is investigated under two primary assumptions: (1) the sound field generated by the aircraft is dominated by jet noise, and (2) the acoustic field generated by the jet flow is adequately described by Lighthill's theory of noise generated by turbulence. The assumption that the atmospheric properties depend only on the altitude is also employed. Further assumptions, which do not constrain the conclusions of this analysis in any essential way, will be introduced as required.

Airplanes and Flights (Real and Imaginary) A Note on Notation

In the following it is necessary to differentiate between two quite different aircraft "flights." The first requires flying a real airplane and deploying real microphones to measure the acoustic field generated by that airplane. The atmospheric properties, aircraft performance parameters, and data collected from these flights are indicated by a subscript "d" in all that follows, and these flights are referred to as data collection flights. The second requires only imaginary airplanes generating imaginary acoustic fields that are sampled by imaginary microphones. For these "flights," the data previously collected is used to predict the acoustic field that would be generated by a particular airplane flying a given trajectory through a prescribed atmosphere. The atmospheric properties, aircraft performance parameters, and predicted acoustic field variables for these "flights" though generally displayed without a subscript will be subscripted with the letter "a" when required for clarity. These flights are referred to as prediction flights.

The Fundamental Assumption of INM

Consider an aircraft flying at an altitude $z = z_a$ above sea level with its engine developing net thrust F_n . Let $P(z)$ be the atmospheric pressure, P_{SLST} be the sea level atmospheric pressure on a standard day, and $\delta(z) = P(z)/P_{SLST}$. Then, $P(z_a) = P_a$ is the atmospheric pressure at the aircraft's altitude, $\delta(z_a) = P_a/P_{SLST} = \delta_a$, and the corrected net thrust is defined as

$$F_c(P_a, F_n; P_{SLST}) = \frac{F_n}{\delta_a} \quad (1)$$

The INM noise data is presented in so-called Noise-Power-Distance (NPD) tables in which noise levels are tabulated for a range of engine "power" settings and a range of aircraft to observer distances. Significantly, INM expresses the engine "power" setting as F_n/δ_a , i.e., as corrected net thrust. Therefore, the engine's corrected net thrust determines entirely the INM prediction of the noise level at a given distance from the aircraft. Notably there is no further adjustment for airport altitude even though altitude variation affects both atmospheric density and temperature, and both of these may reasonably be expected to affect the sound field as, for example, the ambient density appears explicitly and the ambient temperature implicitly (through the sound speed) in Lighthill's jet noise equation (see Equation (2) below). Further, apart from an adjustment for the flyover duration, there is no accounting for aircraft speed in the INM noise-tables even though the acoustic source strength may depend on aircraft speed.

Therefore, the fundamental assumption of the method employed by INM for the prediction of aircraft noise is: The acoustic source strength of a given aircraft is a function of corrected net thrust alone.

A Relationship Between Jet Noise and Corrected Net Thrust

Whereas the methodology employed by INM predicts the acoustic field of an aircraft, Lighthill's jet noise theory predicts the acoustic field of a turbulent jet flow. However, when the sound field generated by an aircraft is dominated by jet noise it follows that the INM prediction of the acoustic field caused by that aircraft is in essence a prediction of the sound field produced by the turbulent jet flow generated by that aircraft's engine and therefore that the two predictions are in some sense comparable. One further incongruity remains however: The methodology employed by INM uses F_n/δ_a as the sole correlating parameter for its prediction of the acoustic field; Lighthill's jet noise theory expresses that field as a function of the relative jet velocity, $U_j = v_j - U_a$, where v_j is the jet

exhaust speed and U_a is the aircraft speed. Therefore in addition to the assumption that jet noise dominates the acoustic field, the comparison of a prediction by INM with the prediction of the same acoustic field by Lighthill's jet noise theory requires the reconciliation of the correlating parameter F_n/δ_a used by INM with the independent variable U_j used by Lighthill's theory.

In the following, this reconciliation is accomplished by algebraic manipulation of the four equations: (1) Lighthill's jet noise equation; (2) the equation governing the thrust of a jet engine (Newton's second law); (3) the mass conservation equation applied to the jet flow; and (4) the definition of corrected net thrust, Equation (1), to produce a form of Lighthill's equation in which the primary independent variable U_j has been eliminated in favor of INM's correlation parameter F_n/δ_a . If the two primary assumptions of this study are valid, evaluation of this equation for the conditions under which the data in the INM tables was acquired will reproduce that data. Then enforcing INM's fundamental assumption will reproduce the INM NPD tables. The analysis that follows carries out this procedure thereby providing the analytical representation of the INM NPD tables. That analytical representation is then compared directly with the analytical prediction of the acoustic field given by Lighthill's theory of jet noise thus determining the fidelity of the INM prediction.

Let

A_i be the jet engine inlet cross-sectional area

A_j be the jet engine exhaust cross-sectional area

c_a be the sound speed at the altitude at which the aircraft is flying

c_o be the sound speed at the altitude of the observer

$h(|\bar{x}_s - \bar{x}_o|)$ be a proportionality function

\bar{x}_o be the observer location

\bar{x}_s be the source location

$|\bar{x}_s - \bar{x}_o|$ be the distance between the source and the observer

ρ_a be the mass density of the atmosphere at the altitude at which the aircraft is flying

ρ_j be the mass density of the jet exhaust flow

and ρ_o be the mass density of the atmosphere at the altitude of the observer, then the mean-squared acoustic pressure, \bar{p}^2 , generated by a jet flow at a given location as predicted by Lighthill's jet noise theory is⁵

$$\bar{p}_{\text{Lighthill}}^2 = h(|\bar{x}_s - \bar{x}_o|) \left\{ \left[\frac{A_j}{\rho_a c_a^5} \right] \rho_j^2 U_j^8 \right\} (\rho_o c_o) \quad (2)$$

Assuming one-dimensional flow through the engine, application of Newton's second law to that flow provides the equation

$$F_n = \rho_a U_a A_i (v_j - U_a) = \rho_a U_a A_i U_j \quad (3)$$

and neglecting fuel flow which is generally a small percentage of the total mass flow conservation of mass requires that

$$\rho_a U_a A_i = \rho_j v_j A_j \quad (4)$$

Eliminating v_j between Equations (3) and (4), and solving the resulting equation for ρ_j , yields

$$\rho_j = \frac{\rho_a A_i}{(1 + \frac{F_n}{R_D}) A_j} \quad (5)$$

where

$$R_D = \rho_a U_a^2 A_i \quad (6)$$

is the ram drag. Solving Equation (3) for the relative jet velocity, $U_j = v_j - U_a$, yields

$$U_j = \frac{F_n}{\rho_a U_a A_i} \quad (7)$$

Finally, employing Equations (5) and (7) to eliminate ρ_j and U_j from Equation (2) provides the equation

$$\bar{p}_{\text{Lighthill}}^2 = h(|\bar{x}_s - \bar{x}_o|) \left\{ \left[\frac{1}{\rho_a^3 c_a^5 A_i^2 A_j} \right] \frac{F_n^6}{R_D^2 (1 + \frac{R_D}{F_n})^2} \right\} (\rho_o c_o) \quad (8)$$

Given the net thrust, F_n , being developed by a jet engine, Equation (8) provides the estimate of the mean-squared acoustic pressure, \bar{p}^2 , in the acoustic field generated by that engine as obtained by application of Lighthill's jet noise theory. However, INM uses corrected net thrust rather than net thrust as the independent variable. Therefore Equation (1) is employed to write Equation (8) in the form

$$\bar{p}_{\text{Lighthill}}^2 = h(|\bar{x}_s - \bar{x}_o|) \left\{ \left(\frac{\delta_a^4}{\rho_a^3 c_a^5 A_i^2 A_j} \right) \frac{1}{(\frac{R_D}{\delta_a})^2 [1 + \frac{R_D}{\delta_a} (\frac{F_n}{\delta_a})^{-1}]^2} \left(\frac{F_n}{\delta_a} \right)^6 \right\} (\rho_o c_o) \quad (9)$$

This is the form of Lighthill's theory of sound generated by turbulence that is most convenient for the current study. Note, however, that except for the use of a different primary independent variable: U_j in Equation (2), F_n in Equation (8), and F_n/δ_a in Equation (9), there is no fundamental difference between these three equations; they are simply different forms of Lighthill's equation, and Equation (9) is as general as Equation (2). That this cannot be true for the INM prediction method will become clear before the current analysis is completed.

The Analytical Representation of the INM Noise Tables

The next step in the quest for an analytical representation of the INM noise tables is to realize that those tables represent a severely limited subset of the set of all predictions possible by application of Equation (9). That subset is, in fact, precisely the subset represented by realizations for which acoustic data have been obtained and that are then presented in the INM tables. Denoting the members of that set by subscripting the physical parameters with the letter "d," for data, we obtain

$$\bar{p}_d^2 = h(|\bar{x}_s - \bar{x}_o|) \left\{ \left(\frac{\delta_d^4}{\rho_d^3 c_d^5 A_i^2 A_j} \right) \frac{1}{\left(\frac{R_{Dd}}{\delta_d} \right)^2 \left[1 + \frac{R_{Dd}}{\delta_d} \left(\frac{F_{nd}}{\delta_d} \right)^{-1} \right]^2} \left(\frac{F_{nd}}{\delta_d} \right)^6 \right\} (\rho_{od} c_{od}) \quad (10)$$

Application of Equation (10) is restricted to combinations of aircraft, aircraft flights, weather conditions, and source and receiver locations for which acoustic data have been acquired; this equation takes the place of the raw data from which the INM NPD tables are produced, and as currently employed Equation (10) can replicate *any* of the data used to construct the INM NPD tables but it *cannot* predict any realization not *directly* represented in that data set. The set of raw data used to construct the INM NPD noise tables and Equation (10) both lack predictive capability. The INM NPD tables are not so restricted; they do in fact provide a predictive capability. That capability is obtained, however, *only* by invoking the fundamental assumption of INM introduced earlier: the acoustic source strength for the NPD noise table entry with corrected net thrust F_{nd}/δ_d is *the* acoustic source strength for *any* prediction flight of the given aircraft for which $F_n/\delta_a = F_{nd}/\delta_d$. Invoking this assumption not only gives the INM NPD tables whatever predictive capability they possess, it also places Equation (10) in the form

$$\bar{p}_{INM'}^2 = h(|\bar{x}_s - \bar{x}_o|) \left\{ \left(\frac{\delta_d^4}{\rho_d^3 c_d^5 A_i^2 A_j} \right) \frac{1}{\left(\frac{R_{Dd}}{\delta_d} \right)^2 \left[1 + \frac{R_{Dd}}{\delta_d} \left(\frac{F_n}{\delta_a} \right)^{-1} \right]^2} \left(\frac{F_n}{\delta_a} \right)^6 \right\} (\rho_o c_o) \quad (11)$$

(where a pc correction has been applied) and endows this final equation with the same predictive capability as the INM NPD tables.

Under the assumptions adopted for the current study, that jet noise dominates the acoustic field and that Lighthill's jet noise theory adequately models jet noise, Equation (11) is the desired result, the analytical representation of the INM NPD tables, that is Equation (11) provides the mean-squared acoustic pressure, $\bar{p}_{INM'}^2$, as a function of the corrected net thrust, F_n/δ_a , as would be predicted by INM. Note that the state of the aircraft and its engine is represented solely by the quantity F_n/δ_a , and that all other relevant quantities in Equation (11): δ_d when not part of the expression F_{nd}/δ_d ($= F_n/\delta_a$), ρ_d , c_d , and R_{Dd} , in all cases, are unchanged from their values for the data collection flight as represented in Equation (10) even if, as is generally true, other values of these parameters are more appropriate for a given prediction flight.

Comparing the Noise Estimates of INM and Lighthill

Equations (9) and (11) both claim to predict the acoustic field for a turbulent jet flow or, equivalently, for an aircraft for which the acoustic field is dominated by jet noise. If these two models are to provide the same estimate of the acoustic field the ratio

$$\frac{\bar{p}_{INM'}^2}{\bar{p}_{Lighthill}^2} = \left\{ \left(\frac{\delta_d^4 \rho_a^3 c_a^5}{\delta_a^4 \rho_d^3 c_d^5} \right) \left(\frac{\delta_d}{R_{Dd}} \frac{R_D}{\delta_a} \right)^2 \frac{\left[1 + \frac{R_D}{\delta_a} \left(\frac{F_n}{\delta_a} \right)^{-1} \right]^2}{\left[1 + \frac{R_{Dd}}{\delta_d} \left(\frac{F_n}{\delta_a} \right)^{-1} \right]^2} \right\} \quad (12)$$

must be unity, an equality that does not hold in most cases thereby implying that Equation (11) and hence the prediction methodology used in INM is not as general as Lighthill's theory, Equation (9). Although the right-hand-side of Equation (12) cannot be reduced to unity it can be simplified considerably. Define the parameters

$$\xi \equiv \frac{P_d R_D}{P_a R_{Dd}} \quad (13)$$

and

$$\eta \equiv \frac{P_a R_{Dd}}{P_d F_n} \quad (14)$$

Then, minor algebraic manipulation employing the ideal gas law, $P = \rho RT$, the isentropic relation $c^2 = \gamma RT$, the relations $\delta_a = P_a/P_{SLST}$, and $\delta_d = P_d/P_{SLST}$, the parameters ξ and η and the notation $T_a = T(z_a)$, places Equation (12) in the form

$$\frac{\bar{P}_{INM}^2}{\bar{P}_{Lighthill}^2} = \left[\left(\frac{T_d}{T_a} \right)^{\frac{1}{2}} \left(\frac{P_d}{P_a} \right) \right] \left[\frac{\xi(1 + \xi\eta)}{(1 + \eta)} \right]^2 \quad (15)$$

Alternate Forms For ξ and η

Employing the ideal gas law, the definition of ram drag, and the isentropic relation, $c^2 = \gamma RT$, we have

$$\xi = \frac{M_a^2}{M_d^2} \quad (16)$$

where $M_a = U_a/c_a$, and $M_d = U_d/c_d$, are the Mach number of the aircraft for the prediction and data collection flights, respectively. Another alias for ξ is obtained by employing INM's fundamental assumption, $F_{nd}/\delta_d = F_n/\delta_a$. The definitions of δ_a and δ_d may be used to write this equation as $P_d/P_a = F_{nd}/F_n$, thereby showing that $\xi = F_{nd}R_D/(F_nR_{Dd})$. Now, for most INM data collection flights the aircraft is in constant speed level flight, consequently the equation $F_{nd} = \rho_d U_d^2 A_w C_{Dd}/2$, where A_w is the aircraft's wing area, and C_{Dd} is the aircraft's drag coefficient during the data collection flight, must apply. If for the prediction flight the aircraft is also to be in steady level flight, the equation $F_n = \rho_a U_a^2 A_w C_{Da}/2$, where C_{Da} is the drag coefficient for the aircraft during the given segment of the prediction flight, also applies. Utilizing these equations and the equation for ram drag we may write $\xi = C_{Dd}/C_{Da}$.

A similar analysis may be employed to find two alternate aliases for η . The definition of δ implies that $P_a/P_d = \delta_a/\delta_d$. Therefore, we may write $\eta = \delta_a R_{Dd}/(\delta_d F_n) = R_{Dd}/[\delta_d (F_n/\delta_a)]$, and INM's fundamental assumption implies that this may be written as $\eta = R_{Dd}/[\delta_d (F_{nd}/\delta_d)]$ or more simply as $\eta = R_{Dd}/F_{nd}$. Employing the definitions of ram drag and net thrust, this may be written as $\eta = U_d/(v_{jd} - U_d)$, where U_d and v_{jd} are the aircraft speed and the jet exhaust speed for the data

collection flight, respectively. This is the first alternate form and shows that η is fully determined by the conditions of the data collection flight.

The second alternate form is obtained by assuming that the data collection flight was a constant speed flight at a constant altitude. Thus, $F_{nd} = \rho_d U_d^2 A_w C_{Dd}/2$, which, coupled with the definition of ram drag in the form $R_{Dd} = \rho_d U_d^2 A_i$ required for the present purposes, and the equation $\eta = R_{Dd}/F_{nd}$ shows that $\eta = 2A_i/(A_w C_{Dd})$. The previous alias for η implied that η depends only on conditions of the data collection flight. This representation shows that, for constant speed flight at a fixed altitude, except for fixed aircraft parameters, i.e., wing area and engine inlet area, η depends only on the drag coefficient of the aircraft during the data collection flight, and that η decreases as the drag coefficient increases.

These results are summarized in Table I.

Table I. Parameter Aliases				
Parameter	Definition	No Further Assumptions Required*	Fundamental Assumption of INM $F_n/\delta_a = F_{nd}/\delta_d$	Fundamental Assumption Plus Constant Velocity Level Flight
ξ	$P_d R_D / (P_a R_{Dd})$	M_a^2 / M_d^2		C_{Dd} / C_{Da}
η	$P_a R_{Dd} / (P_d F_n)$		$U_d / (v_{jd} - U_d)$	$2A_i / (A_w C_{Dd})$
*Derived using only the ideal gas law, the isentropic relation, and defined quantities.				

The form $\xi = M_a^2 / M_d^2$ shows that $\xi > 1$ if the Mach number of the aircraft for the prediction flight is greater than the Mach number of the aircraft during the data collection flight, while the form $\xi = C_{Dd} / C_{Da}$ implies that $\xi \geq 1$, for a fixed aircraft weight, if the aircraft configuration is “cleaner” (e.g. retracted landing gear and flaps) for the prediction flight that it was for the data collection flight. Neither of these conclusions is easily obtained from the definition $\xi \equiv P_d R_D / (P_a R_{Dd})$. Also since both ξ and η are positive the conditions presented in Table II must hold.

Table II. Simple Relations Between Mach Numbers, Drag Coefficients, ξ and $[\xi(1 + \xi\eta)/(1 + \eta)]^2$			
General Conditions	Steady Level Flight	ξ	$[\frac{\xi(1 + \xi\eta)}{(1 + \eta)}]^2$
$M_a < M_d$	$C_{Dd} < C_{Da}$	< 1	< 1
$M_a = M_d$	$C_{Dd} = C_{Da}$	$= 1$	$= 1$
$M_a > M_d$	$C_{Dd} > C_{Da}$	> 1	> 1

Table III provides the wing area, A_w , and the number of engines and fan diameter, hence an *estimate* of the total engine inlet area, for four typical aircraft. From this data it is possible to obtain an estimate of the quantity $C_{d\eta}$, which is also presented in the table. These estimates are remarkably consistent.

Table III. Typical Values of $2A_i/A_w = C_d\eta$						
Aircraft	Wing Area ⁶ (M ²)	Typical Engine ⁷	Number of Engines	Fan Diameter ⁷ (M)	Total Inlet Area* (M ²)	$C_d\eta = \frac{2A_i}{A_w}$
747	511	PW4152	4	2.36	17.5	0.07
757	185.25	PW2037	2	1.99	6.2	0.07
767	283.3	JT9D	2	2.36	8.7	0.06
MD-80	118	JT8D	2	1.17	2.2	0.04
* Total inlet area $\approx \frac{\pi D^2 N_e}{4}$, where N_e is the number of engines and D is the fan diameter.						

If it is assumed that the drag coefficient of a typical aircraft is in the range $0.02 < C_D \leq 0.1$, it will appear that $0.4 \leq \eta \leq 3.5$, for the aircraft listed above. The parameter η will decrease if the drag coefficient is increased, by, for example, increasing the aircraft angle of attack, deploying slats and/or the landing gear, or increasing aircraft weight. Assuming that the quantity $2A_i/A_w$ may be as large as 0.1 for some aircraft suggests that it is not unreasonable to assume that $0.01 \leq \eta \leq 5$ for a reasonable range of transport aircraft under most flight conditions.

Although ξ and η are defined in terms that involve atmospheric properties, the analysis presented here shows that in the case of constant velocity flight at a fixed altitude they actually depend on aircraft performance parameters alone.

The value of the function $\bar{p}_{\text{INM}}^2 / \bar{p}_{\text{Lighthill}}^2$ (Equation (15)) depends on the two factors

$$G = \left(\frac{T_d}{T_a}\right)^{\frac{1}{2}} \left(\frac{P_d}{P_a}\right) \quad (17)$$

and

$$H = \left[\frac{\xi(1 + \xi\eta)}{(1 + \eta)}\right]^2 \quad (18)$$

the first of which depends on atmospheric conditions alone, and the second of which depends on aircraft performance parameters alone. These two factors will now be investigated in turn.

Effect of Atmospheric Conditions

For the purposes of the current study we consider atmospheric temperature variation of the form

$$T(z) = T_{sl} - \ell z \quad (19)$$

Here, T_{sl} is the sea level temperature, and $\ell = -dT/dz$ is the temperature lapse rate. For this temperature profile, it may be shown that⁸

$$\frac{P(z)}{P_{sl}} = \left(\frac{T_{sl} - \ell_a z}{T_{sl}} \right)^{\frac{g}{R\ell_a}} \quad (20)$$

where P_{sl} is the sea level pressure, and therefore that

$$G = \left(\frac{T_d}{T_{sla}} \right)^{\frac{1}{2}} \left(\frac{P_d}{P_{sla}} \right) \left(\frac{T_{sla}}{T_{sla} - \ell_a z_a} \right)^{\frac{2g + R\ell_a}{2R\ell_a}} \quad (21)$$

Here, T_{sla} , P_{sla} , are the sea level atmospheric temperature, pressure, respectively, and ℓ_a is the lapse rate for the prediction flight, $g = 9.8$ meter/second², is the acceleration due to gravity, and $R = 287$ meter²/(° Kelvin second²), is the gas constant for air. Employing Equation (21) places Equation (15) in the form

$$\frac{\bar{P}_{INM'}^2}{\bar{P}_{Lighthill'}^2} = \left(\frac{T_d}{T_{sla}} \right)^{\frac{1}{2}} \left(\frac{P_d}{P_{sla}} \right) \left(\frac{T_{sla}}{T_{sla} - \ell_a z_a} \right)^{\frac{2g + R\ell_a}{2R\ell_a}} \left[\frac{\xi(1 + \xi\eta)}{(1 + \eta)} \right]^2 \quad (22)$$

which, for the following discussion, is written in the form

$$10 * \text{Log}_{10} \left| \frac{\bar{P}_{INM'}^2}{\bar{P}_{Lighthill'}^2} \right| = 10 * \text{Log}_{10}(G) + 10 * \text{Log}_{10}(H) \quad (23)$$

where Equations (17), (18) and (21) have been employed. In all cases it will be assumed that $P_{sla} = P_{SLST}$ as the variation in sea level pressure is a very small percentage of P_{SLST} , and is assuredly negligible for the purposes of the current discussion.

Data collection flights are generally flown at an altitude of approximately 1,000 feet, and the sea level temperature for these flights is, or is corrected to, $77^\circ\text{F} = 298.15^\circ\text{K}$. If it is assumed that the lapse rate at the time of the flight was the standard lapse rate, $1.98^\circ\text{K}/1,000$ feet, and that the data collection flight was at an altitude of 1,000 feet, the temperature at the aircraft's altitude is $T_d = 296.17^\circ\text{K}$, the value used for all of the following. Also, for prediction flights with a standard lapse rate, $(2g + R\ell_a)/(2R\ell_a) \approx 5.75$; the value used for the following calculations.

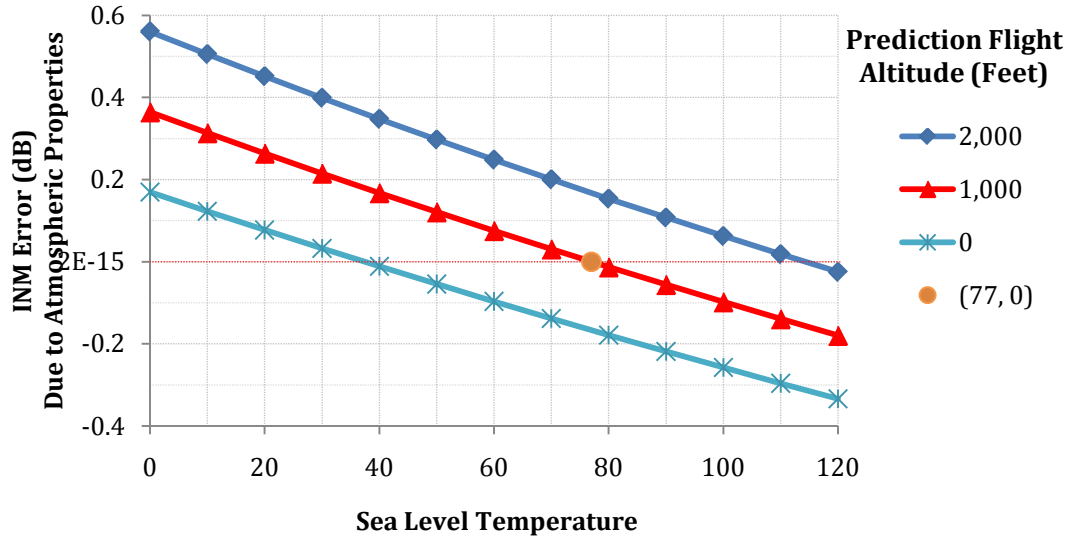


Figure 1. Effect of sea level temperature on INM prediction error, $10 * \text{Log}_{10}(G)$. Prediction flight altitude is between sea level and 2,000 feet.

Figure 1 presents the quantity $10 * \text{Log}_{10}(G)$ as a function of sea level temperature for prediction flights at altitudes of 0, 1,000 and 2,000 feet above sea level. As expected, the point $T_{\text{sla}} = 77^{\circ}\text{F}$ and 0 dB falls on the curve for 1,000 feet, indicating that there is no error at the point at which the INM data is obtained. Clearly, for prediction flights within 1,000 feet of the altitude of the data collection flight the error due to this factor is small, its extremes for the temperature range considered, $0^{\circ}\text{F} \leq T_{\text{sla}} \leq 120^{\circ}\text{F}$, being 0.6 dB at $T_{\text{sla}} = 0^{\circ}\text{F}$ and $z_a = 2,000$ ft., and minus 0.4 dB at $T_{\text{sla}} = 120^{\circ}\text{F}$ and $z_a = 0$ ft. Note that for a fixed sea level temperature the error increases with increasing altitude of the prediction flight, and that the error decreases for a constant prediction flight altitude as the sea level temperature increases. Thus INM predictions are expected to under-predict measured levels for low altitude and hot conditions, i.e. negative values in Figure 1.

Figure 2 presents the quantity $10 * \text{Log}_{10}(G)$ as a function of sea level temperature, again for $0^{\circ}\text{F} \leq T_{\text{sla}} \leq 120^{\circ}\text{F}$, for prediction flights at altitudes of 3,000, 4,000 and 5,000 feet above sea level. Again the error is reasonably small, being less than 1.2 dB over-prediction throughout the region $0^{\circ}\text{F} \leq T_{\text{sla}} \leq 120^{\circ}\text{F}$, and 3,000 feet $\leq z_a \leq 5,000$ feet. Again, for a fixed sea level temperature the error increases with increasing altitude of the prediction flight, and decreases for a constant prediction flight altitude as the sea level temperature increases.

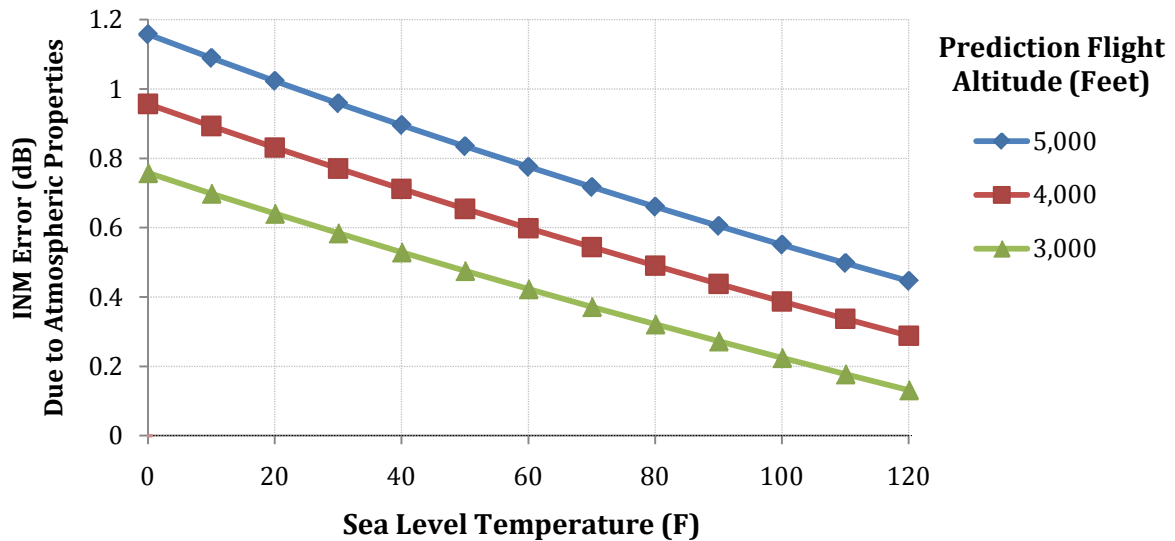


Figure 2. Effect of sea level temperature on INM prediction of flyover noise, $10 \cdot \text{Log}_{10}(G)$. Prediction flight altitude is between 3,000 feet and 5,000 feet.

On occasion, INM is employed to predict noise levels for aircraft at altitudes greater than 5,000 feet. Therefore, Figure 3 presents the quantity $10 \cdot \text{Log}_{10}(G)$ as a function of sea level temperature, again for $0^\circ\text{F} \leq T_{\text{sla}} \leq 120^\circ\text{F}$, for prediction flights at altitudes of 10,000, 20,000 and 30,000 feet above sea level. Clearly, for prediction flights at an altitude below 10,000 feet the error is reasonably small, being less than 3 dB throughout the region. Figure 3 suggests that this factor is 5 - 7 dB at $z_a = 30,000$ feet, implying that measured levels are expected to be significantly below predicted ones for cruise altitudes.

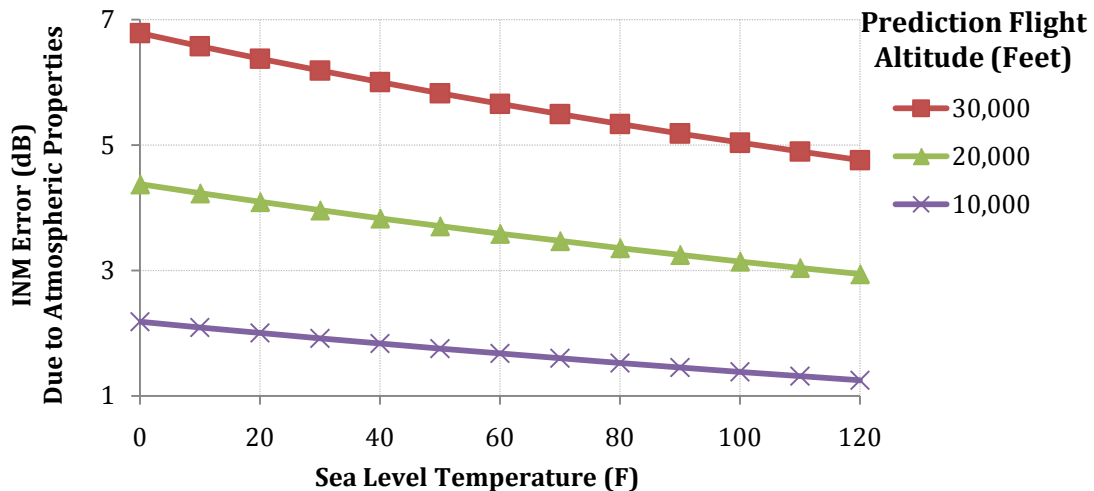


Figure 3. Effect of sea level temperature on INM prediction of flyover noise, $10 \cdot \text{Log}_{10}(G)$. Prediction flight altitude is between 10,000 feet and 30,000 feet.

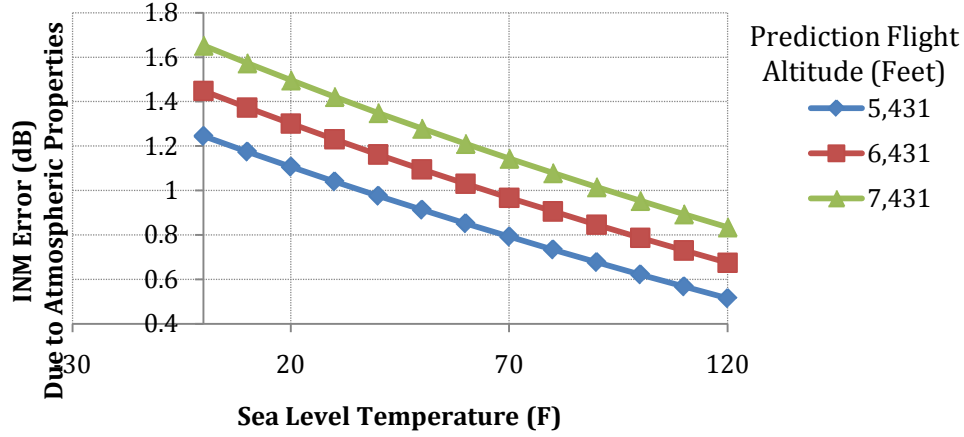


Figure 4. Effect of sea level temperature on INM prediction of flyover noise, $10 * \text{Log}_{10}(G)$. Prediction flight altitude is between 5,431 feet and 7,431 feet.

Figure 4 presents the quantity $10 * \text{Log}_{10}(G)$ as a function of sea level temperature for prediction flight altitudes of 5,431, 6,431, and 7,431 feet, as would be applicable for the prediction of an aircraft flying into or out of Denver International, which is at an altitude of 5,431 feet. Again, the error is relatively small being less than 1.8 dB for the entire range of sea level temperature and prediction flight altitude considered.

Potential errors in the INM prediction due to performance parameters captured in the function $H(\xi, \eta)$ is considered in the next section.

Effect of Aircraft Performance Parameters

Figure 5 through 7 display the function $10 * \text{Log}_{10}(H(\xi, \eta))$ for $\eta = 0.04, 0.25, 0.5, 1.5$ and 5 . Remember that η is determined by the data flight parameters ($\eta = R_{\text{Dd}}/F_{\text{nd}}$). Since $\xi = (M_a/M_d)^2$, it is convenient to use $\sqrt{\xi} = M_a/M_d$ as the independent variable.

Most prediction flights for aircraft near an airport will have $M_a/M_d \approx 1$. Figure 5 presents $10 * \text{Log}_{10}(H)$ for $0.95 \leq M_a/M_d \leq 1$. As shown, for this range of M_a/M_d INM will under-predict measured levels by less than 2 dB for all reasonable values of η . Note that the magnitude of the error increases with increasing η for all $M_a/M_d < 1$, and that it increases as M_a/M_d decreases if $M_a/M_d < 1$.

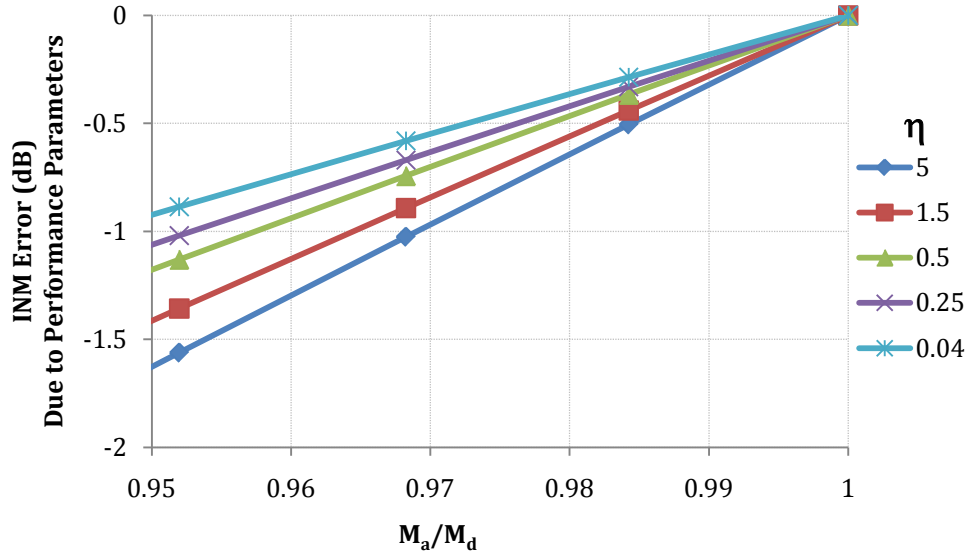


Figure 5. The function $10 \cdot \text{Log}_{10}(H)$ for $0.95 \leq M_a/M_d \leq 1$.

Figure 6 presents $10 \cdot \text{Log}_{10}(H)$ for $1 \leq M_a/M_d \leq 1.05$. In this region INM will over-predict measured levels by no more than 1.6 dB. The magnitude of the error increases with increasing η and M_a/M_d . The results presented in Figures 5 and 6 suggest that in general the magnitude of the error increases as η increases and as $|M_a/M_d|$ increases. However, for realistic values of η , the error in the INM prediction due to performance parameters is less than ± 1.6 dB if the prediction flight Mach number differs from the data collection flight Mach number by $\pm 5\%$ or less, as is likely for low altitude flights, i.e., operations into or out of airports located near sea level.

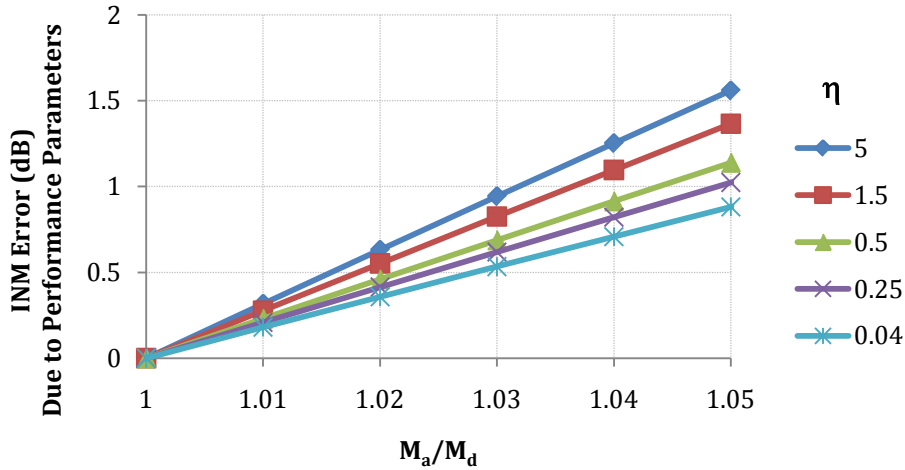
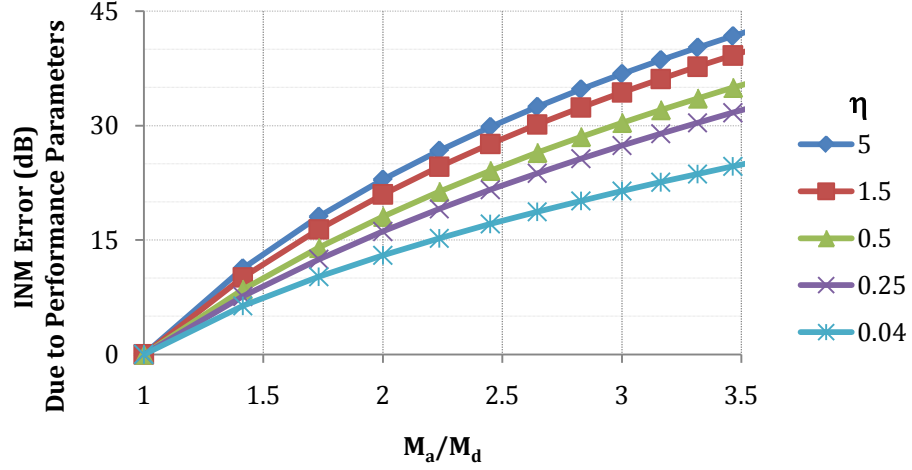


Figure 6. The function $10 \cdot \text{Log}_{10}(H)$ for $1 \leq M_a/M_d \leq 1.05$.

For aircraft at much higher altitudes than the data collection flights, the Mach number will typically be considerably greater than the Mach number of the data collection flight. Thus, Figure 7 presents

$10 * \text{Log}_{10}(H)$ for $1 \leq M_a/M_d \leq 3.5$. It is clear that the error increases rather rapidly with increasing M_a/M_d if $M_a/M_d > 1$. Hence INM is likely to significantly over predict noise from high



altitude flights.

Figure 7. The function $10 * \text{Log}_{10}(H)$ for $1 \leq M_a/M_d \leq 3.5$.

Conclusions

The current analysis shows:

1. The correlating parameter F_n/δ_a employed by the INM prediction methodology can be reconciled with relative jet velocity, $v_j - U_a$ the independent variable used in Lighthill's jet noise theory.
2. Errors in the INM prediction can be separated into two factors, one that depends on atmospheric conditions and one that depends on aircraft performance parameters.
3. The factor that depends on atmospheric conditions is generally very near unity. Thus prediction errors will be small, even under the most extreme temperature and altitude conditions. The one that depends on aircraft performance parameters indicates that small deviations of the Mach number for the prediction flight from the Mach number of the aircraft during the data collection flight will yield relatively large prediction errors. The INM prediction will under predict the acoustic field if the prediction flight Mach number is less than the data collection flight Mach number, and over predict the acoustic field if the prediction flight's Mach number is greater than the data collection flight's Mach number. Thus, INM is expected to significantly over predict noise levels from high altitude cruise conditions.

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