NASA/TM—2012–217455

Theoretical Consolidation of Acoustic Dissipation

M.J. Casiano and T.F. Zoladz
Marshall Space Flight Center, Huntsville, Alabama

March 2012

The NASA STI Program…in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and mission, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results…even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI program home page at <http://www.sti.nasa.gov>

- E-mail your question via the Internet to <help@sti.nasa.gov>

- Fax your question to the NASA STI Help Desk at 443–757–5803

- Phone the NASA STI Help Desk at 443–757–5802

- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7115 Standard Drive
  Hanover, MD 21076–1320

NASA/TM—2012–217455

Theoretical Consolidation of Acoustic Dissipation

M.J. Casiano and T.F. Zoladz
Marshall Space Flight Center, Huntsville, Alabama

National Aeronautics and
Space Administration

Marshall Space Flight Center • Huntsville, Alabama 35812

March 2012

Available from:

NASA Center for AeroSpace Information
7115 Standard Drive
Hanover, MD 21076–1320
443–757–5802

This report is also available in electronic form at
<https://www2.sti.nasa.gov/login/wt/>

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

Symbols

A	area (in^2)
a_0	sound speed (in/s)
a_{ph}	phase speed (in/s)
a_∞	frozen sound speed (in/s)
b	damping-related coefficient (s/in^2)
c_p	specific heat at constant pressure ($lb_f\,in/lb_m$-°R)
D_H	hydraulic diameter (in)
E_0	energy amplitude ($lb_f\,in$)
f	Darcy friction factor (-); frequency (Hz)
f_M	molecular relaxation frequency (Hz)
g_c	constant of proportionality; for EE: (386.087 $lb_m in/lb_f$-s^2), for SI: (1 kg m/N-s^2)
h	head (in)
i	imaginary unit, $-1 = i^2$ (-)
k	wave number (rad/in); spring constant-related coefficient ($1/in^2$)
L	length (in)
m	mass-related coefficient (s^2/in^2)
$\dot{m}$	mass flow rate (lb_m/s)
n	number of end walls (-)
P	perimeter (in)
P_0	pressure amplitude (lb_f/in^2)
Pr	Prandtl number (-)
p	pressure (lb_f/in^2)
q	flow rate (in^3/s)

NOMENCLATURE (Continued)

R	radius (in)
R_L	linearized resistance (using pressure and mass flow rate) (lb_f s/lb_m-in^2)
$R_{L,W}$	linearized resistance per unit length (using head and flow) (s/in^3)
T	period (s)
t	time (s)
u	velocity (in/s)
z	axial location (in)

Greek Symbols

α	total absorption coefficient (Np/in)
α_M	intrinsic molecular relaxation absorption coefficient (Np/in)
$\alpha_{w\kappa}$	wall thermal absorption coefficient (Np/in)
$\alpha_{w\mu}$	wall viscous absorption coefficient (Np/in)
α_κ	intrinsic thermal absorption coefficient (Np/in)
α_μ	intrinsic viscous absorption coefficient (Np/in)
γ	propagation constant (1/in); specific heat ratio (-)
δ_κ	thermal acoustic boundary layer thickness (in)
δ_μ	viscous acoustic boundary layer thickness (in)
ε	acoustic energy density (lb_f/in^2)
ζ	damping ratio (-)
κ	thermal conductivity (lb_f/s-°R)
λ	wavelength (in)
μ	dynamic viscosity (lb_m/in-s)
$\mu_{\max}$	maximum value of absorption per wavelength (Np)
μ^V	volume viscosity (lb_m/in-s)

NOMENCLATURE (Continued)

ρ	density (lb_m/in^3)
σ_E	energy damping rate (Np/s)
σ_p	pressure damping rate (Np/s)
τ	shear stress (lb_f/in^2)
ω	angular frequency (rad/s)

Other Symbols

O	order of magnitude
$\bar{p}$	mean component
p'	oscillatory component
Re	real part
Δ	change in
$\langle \; \rangle$	time average over an acoustic period, $\frac{1}{T}\int_0^T (\;)\,dt$

TECHNICAL MEMORANDUM

THEORETICAL CONSOLIDATION OF ACOUSTIC DISSIPATION

1. INTRODUCTION

In many engineering problems, the effects of dissipation can be extremely important. Dissipation can be represented by several parameters depending on the context and the models that are used. Some examples of dissipation-related parameters are damping ratio, viscosity, resistance, absorption coefficients, pressure drop, or damping rate. This Technical Memorandum (TM) describes the theoretical consolidation of the classic absorption coefficients with several other dissipation parameters including linearized resistance. The primary goal of this TM is to theoretically consolidate the linearized resistance with the absorption coefficient. As a secondary goal, other dissipation relationships are presented. Table 2 in section 6 presents all the dissipation relationships in this TM.

Reducing the linearized resistance parameter as a function of acoustic absorption coefficient is a primary goal because of its importance in impedance or lumped parameter modeling of the dynamics in pipe systems without flow. Examples of its use would be for dead-headed cavities, closed pipes, or sensor ports when the flow resistance is reduced to zero. It is well known that pressure drop is a function of flow rate, and linearized resistance follows directly from a Taylor series expansion of $\Delta p(\dot{m})$ as shown in equation (1):

$$\Delta p(\dot{m}) = \Delta p\left(\bar{\dot{m}}\right) + \left.\frac{\partial \Delta p}{\partial \dot{m}}\right|_{\bar{\dot{m}}} \left(\dot{m} - \bar{\dot{m}}\right) + O\left(\dot{m}'^2\right) \ . \tag{1}$$

The dependent variables in this TM are made up of a steady part and an oscillatory part; e.g., $p = \bar{p} + p'$, so in oscillatory form, equation (1) can be written as equation (2) after dropping the higher order terms, $O\left(\dot{m}'^2\right)$:

$$\Delta p(\dot{m})' = \left.\frac{\partial \Delta p}{\partial \dot{m}}\right|_{\bar{\dot{m}}} \dot{m}' \ . \tag{2}$$

Linearized resistance can be defined as the linear relationship between oscillatory pressure drop and mass flow rate and is shown in equation (3):

$$R_L \equiv \left.\frac{\partial \Delta p}{\partial \dot{m}}\right|_{\bar{\dot{m}}} \ . \tag{3}$$

In a system without flow, the usually dominant form of resistance due to steady flow viscous effects, orifice pressure drops, or other dissipative components, is equal to zero. To model the effects of dissipation in a fluid system without flow, acoustic absorption coefficients can be used. The absorption coefficients, described in further detail in section 4, are used classically as estimates of wave propagation dissipation in a fluid. They are useful because they can be estimated directly with known properties of the fluid and also can be related to a linearized resistance so they can be used in a dynamic system without flow.

A resistor, inductor, and capacitor (RLC) circuit is commonly used as an analogy in fluid systems. There are several RLC fluid analogies, however one of the most commonly used analogy considers the RL in series with a shunt C for each component or lump. Reference 1 (chp. 12), for example, in dynamic fluid systems analysis, uses this formulation. Reference 2 (pp. 270–275) also discusses the circuit analogy in terms of modeling the feed system components in the combustion stability problems. Reference 3 (pp. 18–22) describes this fluid analogy along with others including the forms of their equations. Recognizing the dissipation formulation and wave equation used is an important step in consolidating dissipation parameters.

2. FORMULATION DESCRIPTION

The key to consolidating the dissipation terms is to develop identical theoretical formulations while incorporating generic forms of the dissipation parameters. By casting each theoretical formulation as a damped wave equation, each dissipation parameter can be directly compared. Keeping the dissipation terms generic from the start of the theoretical development, rather than substituting in known parameters, provides a good approach. In this section, a damped wave equation is developed with linearized resistance and one is also provided with the acoustic absorption coefficient.

A force balance in a differential section of pipe can be simplified to equation (4). This is similar to that described in reference 1 (p. 21, equation (2.1)) except that the pipe slope is ignored. The significant observation in this formulation is that the dissipation is represented by shear stress rather than viscous terms as in the Navier-Stokes equation:

$$A\frac{\partial p}{\partial z}+\frac{\rho A}{g_c}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial z}\right)+\tau P=0 \quad . \tag{4}$$

As described in reference 1 (p. 21), the usage of shear stress, a *steady* flow resistive force, is an assumption typically used in transient flow calculations and forms the basis for the unsteady flow equations. Also described in reference 1 (p. 290) and reference 4 (p. 231), it is recognized that the wall shear stress is not in phase with the mean velocity in pulsatile flow and the inertia term must be modified. Additionally, using wall friction as a static function of the mean velocity underestimates the wave attenuation at moderate and high frequencies (ref. 1, p. 290). Considering that the advective acceleration term, $u\times\partial u/\partial z$, is small in low Mach number unsteady flows, it is eliminated from the equation; equation (4) becomes the following momentum equation:

$$A\frac{\partial p}{\partial z}+\frac{\rho A}{g_c}\frac{\partial u}{\partial t}+P\tau=0 \quad . \tag{5}$$

In steady flow, a force balance gives equation (6), except now it is assumed that this relationship also applies in transient flow, thus the dependent variables contain an oscillatory component:

$$\Delta p\,A=\tau L\,P \quad . \tag{6}$$

In steady state, equation (6) reduces to $\Delta\bar{p}A=\bar{\tau}PL.$ Therefore, equation (6) can be reduced to oscillatory form as equation (7):

$$\Delta p' A=\tau' L\,P \, . \tag{7}$$

In oscillatory form, equation (5) can be written as equation (8):

$$A\frac{\partial p'}{\partial z}+\frac{\rho A}{g_c}\frac{\partial u'}{\partial t}+P\tau'=0 \ . \tag{8}$$

The relationship, $\Delta p' = R_L \dot{m}'$, holds from equation (2). After substituting the oscillatory mass flow rate, $\dot{m}' = \rho A u'$, into this relationship, the oscillatory pressure drop, $\Delta p'$, can be substituted into equation (7). The result is shown as equation (9) and is the oscillatory shear stress. Recall the hydraulic diameter, $D_H = 4A/P$.

$$\tau' = \frac{\rho A D_H R_L}{4L}u' \ . \tag{9}$$

Equation (9) can be substituted directly into equation (8). This is the final form of the oscillatory momentum equation while keeping the linearized resistance as a generic parameter:

$$\frac{\partial p'}{\partial z}+\frac{\rho}{g_c}\frac{\partial u'}{\partial t}+\frac{\rho A R_L}{L}u'=0 \ . \tag{10}$$

The approach used by reference 1 is to obtain shear stress specifically from the Darcy-Weisbach equation:

$$\Delta p = f\frac{L}{D_H}\frac{\rho u^2}{2g_c} \ . \tag{11}$$

And then by substituting equation (11) into equation (6), the shear stress relationship in equation (12) is obtained. Since the shear stress will be used in oscillatory flow, the absolute value on the velocity term ensures that the shear stress will always oppose the direction of velocity.

$$\tau = \frac{\rho f u|u|}{8g_c} \ . \tag{12}$$

Equation (12) is then linearized to obtain equation (13):

$$\tau' = f\frac{\rho\bar{u}}{4g_c}u' \ . \tag{13}$$

At this point, equation (13) is used directly in equation (8) to obtain a final form of the oscillatory momentum equation, which does *not* contain the generic linearized resistance parameter as shown in equation (14). This form of the oscillatory momentum equation is not helpful for determining linearized resistance since it has already been eliminated in the equation.

$$\frac{\partial p'}{\partial z}+\frac{\rho}{g_c}\frac{\partial u'}{\partial t}+f\frac{\rho\bar{u}}{D_H g_c}u'=0\ . \tag{14}$$

As an aside, equation (11) with equation (3) can also be used to obtain the linear resistance in a pipe with flow shown as equation (15):

$$R_L=f\frac{L}{D_H}\frac{\bar{\dot{m}}}{\rho A^2 g_c}\ . \tag{15}$$

The next step in obtaining a damped wave equation with a general linearized resistance term is to bring in the continuity equation. Also considering that the advective term is small, the oscillatory continuity equation can be written simply as equation (16) and is also limited to low Mach number flows:

$$\frac{\partial u'}{\partial z}+\frac{g_c}{\rho a_0^2}\frac{\partial p'}{\partial t}=0\ . \tag{16}$$

By differentiating equation (10) with respect to z, differentiating equation (16) with respect to t, and combining to eliminate the velocity terms, the following wave equation is obtained. This is a damped wave equation that incorporates the linearized resistance:

$$\frac{\partial^2 p'}{\partial z^2}-\frac{1}{a_0^2}\frac{\partial^2 p'}{\partial t^2}-\frac{AR_L g_c}{a_0^2 L}\frac{\partial p'}{\partial t}=0\ . \tag{17}$$

Reference 5 provides the development of a damped wave equation for waves in a viscous stationary medium. The linear momentum equation is derived by incorporating the pressure drop per unit length due to visco-thermal friction and can be written as equation (18):

$$\frac{\partial p'}{\partial z}+\frac{\rho}{g_c}\frac{\partial u'}{\partial t}+\frac{2\rho\alpha a_0}{g_c}u'=0\ . \tag{18}$$

And similarly combining with continuity, the damped wave equation becomes equation (19):

$$\frac{\partial^2 p'}{\partial z^2}-\frac{1}{a_0^2}\frac{\partial^2 p'}{\partial t^2}-\frac{2\alpha}{a_0}\frac{\partial p'}{\partial t}=0\ . \tag{19}$$

By direct comparison of the damped wave equations, equations (17) and (19), the relationship between linear resistance and absorption coefficient can be deduced and is described in section 3.

3. DISSIPATION PARAMETER CONSOLIDATION

Several dissipation parameters are consolidated in this section. The linearized resistance (using pressure and mass flow rate), pressure damping rate, energy damping rate, damping ratio, and linearized resistance (using head and flow rate) are all related to the acoustic absorption coefficients. The relationship between each individual parameter can also be found by comparison of the parameters in table 2 in section 6.

3.1 Linearized Resistance (Pressure and Mass Flow Rate)

By comparing equations (17) and (19), an equality can be written as equation (20):

$$\frac{AR_L g_c}{a_0^2 L} = \frac{2\alpha}{a_0} \quad . \tag{20}$$

Solving for the linear resistance gives the following relationship:

$$R_L = \frac{2L\alpha a_0}{Ag_c} \quad . \tag{21}$$

Equation (21) gives the linearized resistance as a function of the absorption coefficient. Absorption coefficient is in the form of dissipation per unit length with units of neper per unit distance. The absorption coefficient can be evaluated as described in detail in the next section.

3.2 Pressure Damping Rate

Another common parameter used for dissipation is the damping rate. The damping rate can be in two forms and care must be taken when using this parameter. The energy damping rate is twice that of the pressure damping rate because of the relationship between total energy and pressure. This will be described further below.

Because this derivation is usually discussed only in part in literature,[4,6,7] it is shown here in completion. To represent damping, the frequency or the wavenumber can be regarded as complex. For damping rate, the frequency is regarded as complex where $\omega \rightarrow \omega - i\sigma_p$. In this form, $\sigma_p > 0$ indicates that the wave decays as is seen in equation (22):

$$p(t) = P_0 e^{-i\left(\omega t - i\sigma_p t - kz\right)} = P_0 e^{-i\omega t} e^{ikz} e^{-\sigma_p t} \quad . \tag{22}$$

The acoustic energy density of a plane wave can be represented as equation (23):

$$\varepsilon(t) = \frac{p(t)^2 g_c}{\rho a_0^2} \ . \tag{23}$$

Since the complex form of equation (22) is normally used as a convenience in representing a harmonic signal in acoustics, the complex conjugate must be incorporated in the evaluation of $p(t)^2$. Also assuming that the fractional change of amplitude is small in one cycle of oscillation, $|\sigma_p/\omega| \ll 1$, the quantity $e^{-2\sigma_p t}$ can be regarded as constant in time. The evaluation of $p(t)^2$ becomes equation (24):

$$p(t)^2 = \frac{1}{2}\mathrm{Re}\left(P_0^2 e^{-2i\left(\omega t - i\sigma_p t - kz\right)} + P_0^2 e^{-2\sigma_p t} \right). \tag{24}$$

And the energy density can be simplified to equation (25):

$$\varepsilon(t) = \frac{P_0^2 e^{-2\sigma_p t} g_c}{2\rho a_0^2}\left(1 + \cos(2\omega t - 2kz)\right). \tag{25}$$

Continuing with the small fractional change in amplitude assumption, the quantity $e^{-2\sigma_p t}$ can again be regarded as constant in time and factored out of the integral in equation (26). The period is over one cycle and can be written as $T = 2\pi/\omega$. Parseval's theorem can also be applied directly.

$$\langle \varepsilon \rangle = \frac{1}{T}\int_0^T \varepsilon(t)\, dt = \frac{1}{T}\int_0^T \left(\frac{p(t)^2 g_c}{\rho a_0^2} \right) dt = \frac{P_0^2 e^{-2\sigma_p t} g_c}{2\rho a_0^2} \ . \tag{26}$$

It can be shown that the following equality is satisfied by evaluating:

$$\sigma_p = \left| \frac{1}{2\langle \varepsilon \rangle} \frac{d\langle \varepsilon \rangle}{dt} \right| . \tag{27}$$

For the absorption coefficient, the relationship in equation (28) holds. In this case, the wave number is regarded as complex where $k \to k - i\alpha$. In this form, $\alpha > 0$ indicates that the waves decay as is seen in equation (28):

$$p(t) = P_0 e^{-i\omega t} e^{ikz} e^{-\alpha z} = P_0 e^{-i\omega t} e^{ikz} e^{-\alpha a_0 t} \ . \tag{28}$$

Using equation (28) and following the same procedure gives equation (29):

$$\alpha = \left| \frac{1}{2a_0 \langle \varepsilon \rangle} \frac{d\langle \varepsilon \rangle}{dt} \right| . \tag{29}$$

And so the following relationship is obtained by comparing equations (27) and (29):

$$\sigma_p = \alpha a_0 \quad . \tag{30}$$

3.3 Energy Damping Rate

Acoustic energy can be described as the energy density multiplied by volume. Assuming the oscillatory part of acoustic energy follows the form $E_0 e^{-\sigma_E t}$, it is seen from equation (25) that the energy damping rate is given by equation (31):

$$\sigma_E = 2\sigma_p \, . \tag{31}$$

Or in terms of absorption coefficient, it is given by equation (32):

$$\sigma_E = 2\alpha a_0 \quad . \tag{32}$$

3.4 Damping Ratio

The damping ratio is used at times and is commonly used in finite element modeling because of analytic features designed to couple acoustics and structures. It is also representative of the damping parameter used in a system with a single degree of freedom. Analogous to a mass-spring-damper system, the wave equation can be rearranged and written as equation (33). The parameters *m*, *b*, and *k* do not represent the actual mass, spring constant, and damping coefficient. For the parameters to be representative, the wave equation would be written as a force balance, however doing this would only introduce a linear factor that would be eliminated when rewriting in a form for an analogous equation comparison:

$$m\frac{\partial^2 p'}{\partial t^2} + b\frac{\partial p'}{\partial t} + kp' = 0 \, . \tag{33}$$

The definition of damping ratio of a damped harmonic oscillator can be represented by the damping coefficient, equation (34):

$$b = 2\sqrt{km}\,\zeta = 2m\omega\zeta \, . \tag{34}$$

To provide a direct comparison, the lossless wave equation parameters for *m* and *k*, in the form of equation (33), are taken from equation (19). Also, by incorporating equation (34) using the newly defined parameter for *m*, the damped wave equation with damping ratio can be written as equation (35):

$$\left(-\frac{1}{a_0^2}\right)\frac{\partial^2 p'}{\partial t^2} + \left(-\frac{2\omega}{a_0^2}\zeta\right)\frac{\partial p'}{\partial t} + \left(\frac{\partial^2}{\partial z^2}\right)p' = 0 \, . \tag{35}$$

By direct comparison of equations (19) and (35), the equality shown as equation (36) is obtained:

$$\frac{-2\alpha}{a_0} = \frac{-2\omega\zeta}{a_0^2} . \tag{36}$$

Damping ratio as a function of absorption coefficient can be related as equation (37):

$$\zeta = \frac{\alpha a_0}{\omega} . \tag{37}$$

3.5 Linearized Resistance (Head and Flow Rate)

Care must also be taken for resistance terms in various formulations. For example, it is common to normalize resistance to density or length. Additionally, the linearized resistance may be based on head and flow relationships. For example, in reference 1, the linear resistance is based on head and flow rate, rather than pressure and mass flow rate and is also normalized to length. Their momentum equation is given by equation (38):

$$\frac{\partial h'}{\partial z} + \frac{1}{gA}\frac{\partial q'}{\partial t} + R_{L,W} q' = 0 . \tag{38}$$

With $h' = \frac{g_c}{\rho g} p'$ and $q' = Au'$, equation (38) can be written as equation (39):

$$\frac{\partial p'}{\partial z} + \frac{\rho}{g_c}\frac{\partial u'}{\partial t} + \frac{\rho g A R_{L,W}}{g_c} u' = 0 . \tag{39}$$

And by comparing the momentum equations, equations (10) and (39), the equality given by equation (40) is obtained:

$$\frac{\rho A R_L}{L} = \frac{\rho g A R_{L,W}}{g_c} . \tag{40}$$

Thus, the relationship between the Wyle & Streeter[1] form of linearized resistance (using head and flow rate) and the linearized resistance (using pressure and mass flow rate) is described by equation (41):

$$R_{L,W} = \frac{R_L}{L}\frac{g_c}{g} . \tag{41}$$

It is also clear that the Wyle & Streeter[1] form of linearized resistance is normalized to length.

In terms of absorption coefficient, the Wyle & Streeter[1] linearized resistance relationship is given as equation (42):

$$R_{L,W} = \frac{2\alpha a_0}{gA} \quad . \tag{42}$$

4. ABSORPTION COEFFICIENTS

Sources of dissipation can be broken into two main categories—those intrinsic to the medium and those associated with the boundary of the medium. The losses intrinsic to the medium can be further divided into three basic types: (1) viscous losses, (2) heat conduction losses, and (3) losses associated with internal molecular processes. At the boundary there are viscous losses and losses to heat conduction between the fluid and the surface. Without derivation the absorption coefficients are provided in this section. Most acoustics textbooks discuss these absorption coefficients in further detail.[3,4] For the dissipation associated with the boundary, the absorption coefficients are usually derived for standing waves. However, reference 7 gives the formulation of damping rate that includes nonresonant conditions. The forms in reference 7 are provided in this TM to account for both resonant and nonresonant conditions.

The total absorption coefficient can be regarded as the sum of the absorption coefficients for the individual loss mechanisms. Superposition is typically justified in practice, and true when losses are small; however, in general, they do have interactions with each other. The unit for the absorption coefficient is neper per unit length where a neper is analogous to a bel, but represents the natural logarithmic scale rather than the decadic logarithmic scale.

$$\alpha = \sum_i \alpha_i = \alpha_\mu + \alpha_\kappa + \alpha_M + \alpha_{w\mu} + \alpha_{w\kappa} + \ldots \tag{43}$$

4.1 Intrinsic Dissipation

The losses intrinsic to the medium can be further divided into three basic types: (1) viscous losses, (2) heat conduction losses, and (3) losses associated with internal molecular processes. The complete absorption coefficient is defined as equation (43), but the combination of equations (44) and (45) under Stokes' assumption is regarded as the classical absorption coefficient.

4.1.1 Intrinsic Absorption Due to Viscosity

The intrinsic absorption in a fluid due to viscosity is given by the intrinsic viscous absorption coefficient, equation (44). In Stokes' assumption, the bulk volume viscosity is zero. Substantial theoretical development in the field has occurred since Stokes' pioneering work that has introduced the bulk volume viscosity parameter into the absorption formulation. The sound speed dispersion for intrinsic viscous effects is very small and usually too small to measure.

$$\alpha_\mu = \frac{\omega^2}{2\rho a_0^3}\left(\frac{4}{3}\mu + \mu^V\right). \tag{44}$$

4.1.2 Intrinsic Absorption Due to Thermal Conduction

The intrinsic absorption in a fluid due to thermal conduction is given by the intrinsic thermal absorption coefficient, equation (45). Variations in temperature occur between the slightly warmer compressed portion and slightly colder expanded portion of the wave. The variation in temperatures gives rise to heat flow as the fluid medium tries to reestablish temperature equilibrium. The heat flow reduces the amount of energy available for the sound wave and represents an acoustic loss. The wave due to this absorption mechanism is shown to be nondispersive. For most gases the absorption due to intrinsic viscosity and intrinsic thermal conduction is comparable; for most liquids the intrinsic absorption due to thermal conduction is usually negligible.

$$\alpha_\kappa = \frac{\omega^2}{2\rho a_0^3} \frac{(\gamma - 1)\kappa}{c_P} . \tag{45}$$

4.1.3 Intrinsic Absorption Due to Molecular Thermal Relaxation

The intrinsic absorption in a fluid due to molecular thermal relaxation takes into account the internal structure of the molecules and the interactions between them that lead to internal vibrations, rotations, ionizations, and short-range ordering. In this case, the pressure is not just dependent on the local instantaneous values of density and pressure; additionally, it has a dependence on the rate at which density and temperature are changing. The molecular thermal relaxation time is associated with the time needed for a new equilibrium to be established. Transformation of the mechanical energy into heat represents a loss to the sound wave. The absorption is most significant at the molecular relaxation frequencies of the fluid. The fluid may have several relaxation frequencies in which the total molecular thermal relaxation is the sum of all the absorption coefficients calculated separately for each of the relaxations. The sound speed transitions from the equilibrium sound speed, a_0, at low frequencies that are well below the relaxation frequency, to the frozen sound speed, a_∞, at high frequencies that are well above the relaxation frequency. Experimental data for a particular fluid are needed to obtain the parameters, $\mu_{\max}$ and f_M, which are the maximum value of absorption per wavelength and the relaxation frequency, respectively. These parameters are both functions of temperature. This form of absorption can be responsible for most of the intrinsic sound absorption in the audio and low frequency regions. The absorption coefficient due to molecular relaxation is given by equation (46) and the phase speed dispersion for molecular relaxation is given as equation (47):

$$\alpha_M = \frac{2\mu_{\max}}{a_0} \frac{f_M}{1 + \left(\frac{f_M}{f}\right)^2} \tag{46}$$

and

$$a_{ph} = a_0 \sqrt{\frac{1 + \left(\frac{a_\infty^2 f}{a_0^2 f_M}\right)^2}{1 + \left(\frac{a_\infty f}{a_0 f_M}\right)^2}} . \tag{47}$$

4.2 Boundary Dissipation

Boundary dissipation is an entirely separate mode of absorption due to the passage of a wave over a surface boundary. The surface exerts a frictional shear force on the overlying fluid while thermal conduction allows heat transfer to take place between the fluid and the surface. Both result in loss of energy from the wave.

Similar to a boundary layer thickness in steady flow, acoustic waves have an analogous acoustic boundary layer thickness. It is much thinner because the boundary layer only has a period of half a cycle to develop. For the formulas to hold, the acoustic wavelength must be much greater than the acoustic boundary layer thickness $\lambda \gg \delta$. The viscous and thermal acoustic boundary layer thicknesses are given as equations (48) and (49), respectively:

$$\delta_\mu = \sqrt{\frac{2\mu}{\rho\omega}} \tag{48}$$

and

$$\delta_\kappa = \sqrt{\frac{2\mu}{\rho\omega\,\mathrm{Pr}}}\;. \tag{49}$$

For the wavelength, $\lambda = 2\pi a_0/\omega$, to be much greater than the acoustic boundary layer thickness, equations (50) and (51) must be satisfied:

$$\frac{1}{\pi a_0}\sqrt{\frac{\mu\omega}{2\rho}} \ll 1 \tag{50}$$

and

$$\frac{1}{\pi a_0}\sqrt{\frac{\mu\omega}{2\rho\,\mathrm{Pr}}} \ll 1\,. \tag{51}$$

The combination of the wall viscous absorption coefficient, equation (52), and the wall thermal conduction absorption coefficient, equation (54), is regarded as the combined absorption coefficient for wall losses. Normally, the formulation of the following boundary layer absorption coefficients are derived for resonant conditions, however, reference 7 gives the formulation of damping rate that includes nonresonant conditions. The absorption coefficients at general conditions are provided so that both resonant and nonresonant conditions can be estimated. For other pipe shapes, the radius can be replaced with a hydraulic diameter term, $R=D_H/2$. Reference 8 compares experimental data, to a very fine degree, to the combined absorption coefficient theoretical formulation.

4.2.1 Viscous Losses at a Rigid Wall

The boundary layer absorption in a fluid due to viscosity for a cylindrical pipe is given by the wall viscous absorption coefficient, equation (52). This absorption coefficient accounts also for nonresonant conditions. The phase speed dispersion for wall viscous effects is given as equation (53). It shows that the sound speed is slightly slower.

$$\alpha_{w\mu} = \frac{1}{a_0 R}\sqrt{\frac{\mu\omega}{2\rho}}\left(1 - \frac{\sin(2kL)}{2kL}\right) \tag{52}$$

and

$$a_{ph} = a_0\left(1 - \frac{\alpha_{w\mu} a_0}{\omega}\right). \tag{53}$$

4.2.2 Thermal Conduction at an Isothermal Wall

The boundary layer absorption in a fluid due to thermal conduction for a cylindrical pipe is given by the wall thermal absorption coefficient, equation (54). This absorption coefficient accounts also for nonresonant conditions, but also end-wall effects that are important in thermal conduction. The sound speed dispersion for wall thermal effects is given as equation (55). It shows that the sound speed is slightly slower.

$$\alpha_{w\kappa} = \frac{1}{a_0 R}\sqrt{\frac{\mu\omega}{2\rho}}\left(\frac{\gamma - 1}{\sqrt{\mathrm{Pr}}}\right)\left(1 + \frac{\sin(2kL)}{2kL} + n\frac{R}{L}\right) \tag{54}$$

and

$$a_{ph} = a_0\left(1 - \frac{\alpha_{w\kappa} a_0}{\omega}\right). \tag{55}$$

5. EXAMPLE

As an example, a 5-in-long senseline with an inner diameter of 0.5 in is examined. A schematic is shown in figure 1. The contents are air at a temperature of 540 °R and pressure of 4 torr.

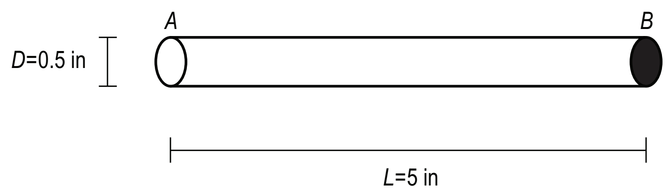

Figure 1. Sensor port example.

An expression for complex pressure ratio, p_B/p_A, is needed where A is the open end and B is the closed end. From reference 1, for a simple pipe closed at one end, this complex pressure ratio is described with the hyperbolic secant as equation (56):

$$\frac{p_B}{p_A} = \operatorname{sech}(\gamma L) \,. \tag{56}$$

And the propagation constant, γ, can be written as equation (57):

$$\gamma = \sqrt{-\left(\frac{\omega}{a_0}\right)^2 + i\left(\frac{\omega g A R_{L,W}}{a_0^2}\right)} \,. \tag{57}$$

In this case, the acoustic dissipation is dominated by the viscous and thermal losses at the senseline wall. Since the wall effects dominate in this problem, the acoustic absorption coefficient is found by substituting the fluid parameters in table 1 and the geometric parameters into equations (43), (52), and (54).

Table 1. Fluid parameters for air with a temperature of 540 ºR (80.33 ºF) and pressure of 4 torr (0.077347 psia).

Parameters	Variable	Value	Units
Viscosity (dynamic)	μ	1.0373×10^{-6}	lb_f/in-s
Density	ρ	2.2367×10^{-7}	lb_m/in^3
Sound speed	a_0	13,671.6	in/s
Specific heat ratio	γ	1.4	–
Prandtl number	Pr	0.70641	–

Additionally, substituting these parameters into equation (42) to obtain the linearized resistance term, using equation (57) to obtain the propagation constant, and then lastly combining into equation (56), the complex pressure transfer function can be obtained. Equation (42) allows access of the absorption coefficient in the form of linearized resistance and thus can be applied directly into the complex pressure ratio model. Figure 2 shows both acoustic absorption coefficient and damping ratio as a function of frequency. Figure 3 shows the gain and phase of the complex pressure transfer function between location *A* and *B* due to acoustic dissipation.

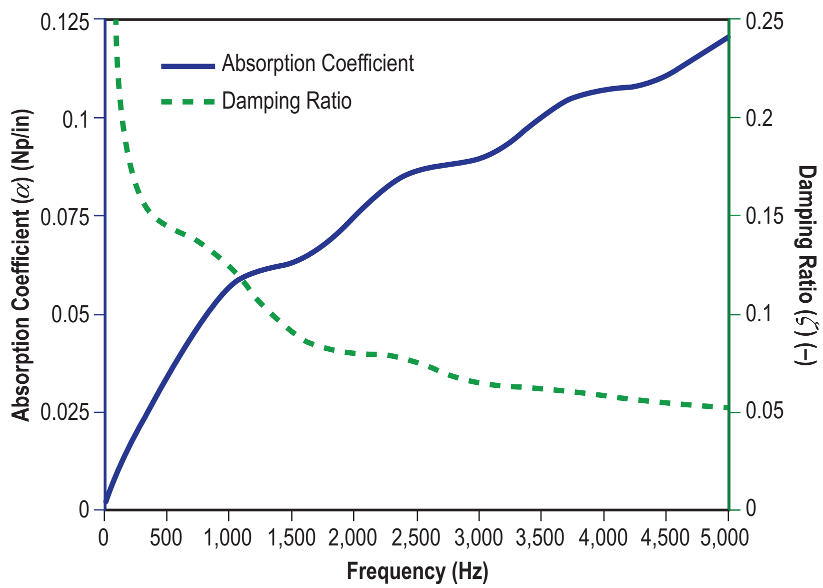

Figure 2. Acoustic dissipation parameters.

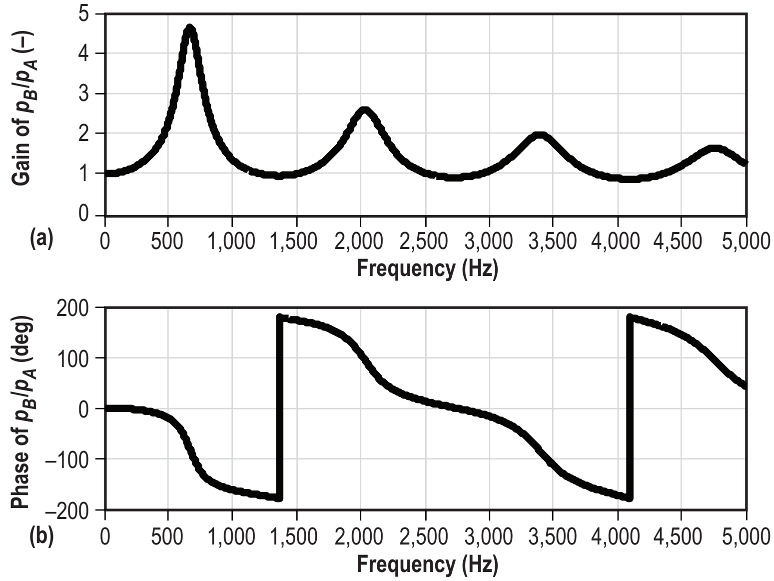

Figure 3. Pressure transfer function for a sensor port using acoustic absorption: (a) Gain of p_B/p_A and (b) phase of p_B/p_A.

6. SUMMARY

This TM describes the theoretical consolidation of the classic absorption coefficients with several dissipation parameters including linearized resistance. Table 2 provides the various relationships developed in this TM in terms of absorption coefficient. This allows engineering problems that use various dissipation parameters to account for acoustic dissipation by easily estimating and incorporating the needed parameters into their particular models. This TM summarizes the consolidation of several dissipation parameters and gives a simple example of a senseline problem.

Table 2. Acoustic dissipation relationship as a function of absorption coefficient.

Linearized resistance (pressure/mass flow rate) $R_L = \left.\frac{\partial \Delta p}{\partial \dot{m}}\right\|_{\dot{m}=\bar{\dot{m}}}$	$R_L = \frac{2L\alpha a_0}{Ag_c}$
Pressure damping rate	$\sigma_p = \alpha a_0$
Energy damping rate	$\sigma_E = 2\alpha a_0$
Damping ratio	$\varsigma = \frac{\alpha a_0}{\omega}$
Linearized resistance per unit length (head/flow rate) $R_{L,W} = \frac{1}{L}\left.\frac{\partial \Delta h}{\partial q}\right\|_{q=\bar{q}}$	$R_{L,W} = \frac{2\alpha a_0}{gA}$

REFERENCES

1. Wylie, E.B.; and Streeter, V.L.; Suo, L. (ed.): *Fluid Transients in Systems*, Prentice-Hall, Inc., Upper Saddle River, NJ, 1993.

2. Harrje, D.T.; and Reardon, F.H.: "Liquid Propellant Rocket Combustion Instability," NASA, *NASA SP-194*, Washington, DC, 1972.

3. Blackstock, D.T.: *Fundamentals of Physical Acoustics*, John Wiley & Sons, Inc., New York, 2000.

4. Kinsler, L.E.; Frey, A.R.; Coppens, A.B.; and Sanders, J.V.: *Fundamentals of Acoustics*, John Wiley & Sons, Inc., New York, 2000.

5. Munjal, M.L.: *Acoustics of Ducts and Mufflers*, John Wiley & Sons, New York, 1987.

6. Culick, F.E.C.: *Unsteady Motions in Combustion Chambers for Propulsion Systems*, NATO Research and Technology Organisation, RTO AGARDograph AG-AVT-039, 2006.

7. Searby, G.; Nicole, A.; Habiballah, M.; and Laroche, E.: "Prediction of the Efficiency of Acoustic Damping Cavities," *Journal of Propulsion and Power*, Vol. 24, No. 3, pp. 516–523, 2008.

8. Quinn, T.J.; Colclough, A.R.; and Chandler, T.R.D.: "A New Determination of the Gas Constant by an Acoustical Method," *Philosophical Transactions of the Royal Society*, Vol. 283, pp. 367–420, 1976.

REPORT DOCUMENTATION PAGE					Form Approved OMB No. 0704-0188
The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operation and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**					
1. REPORT DATE *(DD-MM-YYYY)* 01–03–2012			**2. REPORT TYPE** Technical Memorandum		**3. DATES COVERED** *(From - To)*
4. TITLE AND SUBTITLE Theoretical Consolidation of Acoustic Dissipation					**5a. CONTRACT NUMBER**
					5b. GRANT NUMBER
					5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) M.J. Casiano and T.F. Zoladz					**5d. PROJECT NUMBER**
					5e. TASK NUMBER
					5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) George C. Marshall Space Flight Center Huntsville, AL 35812					**8. PERFORMING ORGANIZATION REPORT NUMBER** M–1331
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546–0001					**10. SPONSORING/MONITOR'S ACRONYM(S)** NASA
					11. SPONSORING/MONITORING REPORT NUMBER NASA/TM—2012– 217455
12. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified-Unlimited Subject Category 71 Availability: NASA CASI (443–757–5802)					
13. SUPPLEMENTARY NOTES Prepared by the Propulsion, Structural, Thermal, and Fluid Analysis Division, Engineering Directorate					
14. ABSTRACT In many engineering problems, the effects of dissipation can be extremely important. Dissipation can be represented by several parameters depending on the context and the models that are used. Some examples of dissipation-related parameters are damping ratio, viscosity, resistance, absorption coefficients, pressure drop, or damping rate. This Technical Memorandum (TM) describes the theoretical consolidation of the classic absorption coefficients with several other dissipation parameters including linearized resistance. The primary goal of this TM is to theoretically consolidate the linearized resistance with the absorption coefficient. As a secondary goal, other dissipation relationships are presented.					
15. SUBJECT TERMS acoustic dissipation, absorption coefficient, linearized resistance, damping rate, unsteady fluid dynamic systems with no flow					
16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** STI Help Desk at email: help@sti.nasa.gov
a. REPORT U	**b. ABSTRACT** U	**c. THIS PAGE** U	UU	32	**19b. TELEPHONE NUMBER** *(Include area code)* STI Help Desk at: 443–757–5802

National Aeronautics and
Space Administration
IS20
George C. Marshall Space Flight Center
Huntsville, Alabama 35812