# Analytical Determination of Propeller Performance Degradation Due to Ice Accretion 

```
(NASA-CR-175092) ANALYTICAL DETERMINATION
N 86-23577
OF PROPELLER PERFORMANCE DEGRAEATION DUE TO
ICE ACCRETION Final Feport (Sverdrup
Technology. Inc.) 138 p HC AO7/MF AO1
                                    CSCL 01C G3/03-05976
```

Thomas L. Miller
Sverdrup Technology, Inc.
Lewis Research Center
Cleveland, Ohio

April 1986

Prepared for
Lewis Research Center
Under Contract NAS 3-24105

TABLE OF CONTENTS

## Page

iii
LIST OF SYMBOLS
1
CHAPTER 1. INTRODUCTION
CHAPTER 2. SUMMARY OF PREVIOUS WORK ..... 5
A. Early Work

1. Experimental Effort ..... 5
2. Analytical Effort ..... 7
B. Recent Work
3. Experimentel Effort ..... 10
4. Analytical Effort ..... 12
CHAPTER 3. CODES EMPLOYED ..... 17
A. Theodorsen Transformation ..... 17
B. Droplet Trajectory Code ..... 19
C. Aerodynamic Coefficient Correlationa ..... 22
5. Gray Correlation ..... 30
6. Bragg Correlation ..... 32
7. Flemming Correlations ..... 34
8. Miller Correlation ..... 36
D. Propeller Performance Code ..... 37
CHAPTER 4. PERFORMANCE DEGRADATION CODE ASSEMBLY ..... 39
CHAPTER 5. RESULTS ..... 44
A. Bragg ..... 48
B. Gray ..... 49
C. Flemming ..... 51
```SUMMARY AND CONCLUSIONS54
```

REFERENCES ..... 57
FIGURES ..... 62
TABLES OF EXPERIMENTAL \& ANALYTICAL DATA ..... 80
APPENDIX 1. PROGRAM FLOWCHART ..... 85
APPENDIX 2. USER'S GUIDE ..... 88
APPENDIX 3. SAMPLE CASE ..... 100

## LIST OF SYMBOLS



| $r$ | leading edge radius of curvature |
| :---: | :---: |
| $\mathrm{R}_{u}$ | droplet Reynolds number $=P 1 \bar{u}-\dot{\bar{j}} 148 / u$ |
| 5 | airfoil aurface arc length |
| $t / c$ | airfoil thickness to chord ratio |
| To | freeatream temperature |
| $\mathbf{u}$ | local air velocity |
| U, V | freestream velocity |
| W | liquid water content |
| $\mathbf{x}$ | propeller radial location in fraction of tip radiua |
| $Y_{0}$ | initial droplet $y$-coordinate |
| 0 | airfoil section angle of attack |
| $*_{1}$ | induced angle of attack |
| B | impingement efficiency $=$ dy $/ d x$ propeller twiat |
|  | angle |
| Amax | meximum local impingement efficiency |
| $\delta$ | droplet diameter |
| I) | $\text { propeller efficiency }=\frac{J_{T}}{C_{p}}$ |
| $\dot{\square}$ | local droplet velocity vector |
| $\mu$ | absolute air viscosity |
| $p$ | air density |
| Pice | ice density $\therefore$ |
| $\sigma$ | droplet density. |
| ${ }_{\underline{p}}$ | section solidity $=\underline{B b}$ |
| , |  |

## CHAPTER 1

## INTRODUCTION

The accretion of ice on the lifting or propulsive surfaces of an aircraft is a phenomenon which can heve aevere, if not disastrous, consequences if it is not dealt with properly and in a timely fashion. In many cases, this accretion may produce a serious reduction in lift and increage in drag, requiring more power than is available and reaulting in eserious departure from the miseion profile. Although measures and devices exist which generally prevent this worstcase scenario from happening, ice accretion on airfoila ia still a common occurrence in certain atmospheric conditions and as such merits further study because of the significant negative effect which it has on airfoil and aircreft performance. It is the objective of this thesis to provide a single computer code which will accurately predict the degree of performance degradation primarily with respect to drag increment, which will result from the exposure of an airfoil. to some set of atmospheric and flight conditions conducive to ice formation.

Two basic types of ice are found to occur as a result of exposing an airfoil in forward motion to supercooled water droplets in a sub-freezing environment (Fig. 1). The type of ice which will form may be determined by a variety of factora including freestream velocity, liquid water content of the cloud, droplet size distribution, and freestream temperature. Based upon an inveatigation of the data currently available,
the two latter factors appear to be the moat influential in determining the resulting ice type. The firat kind of ice, known as rime ice; occurs at relatively low velocities, low íquid water content values (typically 0.5 to 1.0 grams per Cubic meterj, and temperatures well below freezing. Due primariiy to the very cold temperatures associated with rime ice formation, the droplets tend to freeze on inpact to form a fairly smooth addition to the leading edge of the airfoil. At some point with respect to the combination of flight and atmospheric conditions present, ice type transitione from rime to glaze ice. Not only is the cryatalline atructure of these ice types different, but glaze ice is alao produced by higher freestream velocities, higher liquid water content values (on the order of 1.5 to $3: 0$ grams per cubic meter), and temperatures near, but below, freezing. Again, due chiefly to the warmer temperatures involved, the water droplets impacting the aurface tend not to freeze on impact but rather to strike the airfoil or existing ice formation and run back somewhat in a chordwise direction before freezing.

Both types of ice cause significant performence degradation of the airfoil. When compared to the aerodynamica of an airfoil in the clean, or non-iced, configuration, the iced airfoil wili virtually always exhibit a decrease in lift, an increase in drag, and a change in pitching moment which wili yield a detrimental effect on alrcraft stability. Examination of the available icing data base indicates that the degree of performance degradation, specifically with
reapect to drag increment, is generally more severe for the glaze ice condition. This may be attributed to the more drastic alteration of the leading edge of the airfoil as a result of glaze ice formation, as compared to the typically smoother and leas obtruaive rime ice formation. Increases in drag coefficient in excess of $100 \%$ or more are not unusual for either type of accretion.

Both rime and glaze ice act to negatively influence airfoil performance by physically altering the shape of the airfoil, thereby changing the flowfield around the airfoil. In computing this flowfield for the propeller case, the rotational and induced components of velocity, as well as the forward component, must be taken into account as exhibited in Figure 2. The reshaping of the airfoil surface by the ice accretion and the rough peaks and surfaces commonly preaent in ice accretions, especially in the glaze ice case, serve to induce premature transition and often separation of the flow around the airfoil, thus spoiling its deaigned aerodynamic characteristics. The associated ice roughness also acts as an energy loss device. For the propeller configuration dealt with in this thesis, these effects may occur along the entire apan of the propeller and have the same effect on propeller aerodynamics as they are found to have with respect to fixed wing aircraft. Ice accretion on a propeller may then translate for example into an increase in power required to maintain a given flight condition when ice is allowed to form on the propeller blades. As a pllot, passenger, or other

```
concerned party it would therefore be desirable before
undergoing an icing encounter to have some idea of just how
much power would be required to maintain a flight condition
with ice accreting on the propeller and other aircraft
components, so that some decision could be confidently made
regarding the safety of undergoing or avoiding such an
encounter. In addition, information provided by this code mey
be used" in establishing propeller ice protection system
specifications, as well as potentially in future icing
certification efforts.
```

CHAPTER 2
SUMMARY OF PREVIOUS WORK

Much work has already been done in attempting to better understand ice accretion and ita effect on eircraft performance. However, due to the very complicated phyaica which govern the ice accretion process, many questions remain unanswered in this area and it is still difficult with present technology to accurately predict the degree of performance degradation which will result from an arbitrary icing encounter. Several recent developmenta have made the prediction of performance degradation, especially in terms of drag increase, more reliable and have expanded the range of conditions over which relatively accurate predictions can now be made. Yet much of the groundwork for these developments was laid thirty to forty yeare ago. This initial work involved both experimental and analytical investigationa of ice accretion; and much of the icing data used in current correlation development work was actually collected during this early period, from roughly the late $1940^{\prime} s$ to the late 1950's.

## A. Early Work

1. Experimental Effort

Early propeller performance degradation data was gathered using both simulated ice and actual natural icing encounters. Corson and Maynard (1) in 1946 used simulated ice on propeller blades and measured average efficiency loseses on the order of three percent, with maximum losses of fifteen
percent noted. In 1948, Preston and Blackman (2) undertook a flight test program in which they also noted average decreases of roughly ten percent in propeller efficiency due to ice formation, as well as the associated increase in drag. This study was of special significance due to the fact that an attempt was made to examine the effects of ice accretion on all major components of the aircraft. Possibly the most complete and informative work relative to lee accretion on propellers was performed and documented by Neel and Bright (3) in 1950. In a flight test program in which efficiency lose was measured during actual natural icing encounters, they observed losses of roughly ten percent in most cases, with maximum loases on the order of twenty percent. The performance degradation data of Neel and Bright is still used by preaent reaearchers to develop or verify analytical propeller performance degradation codes.

Also of note with respect to experimental investigations of airfoil icing are a series of test programs undertaken in the Icing Research Tunnel at the NASA Lewis Research Center and documented by a variety of NACA personnel in the early and mid-1950's. Gray and von Glahn (4,5) looked at the effects of ice on the performance of NACA 65A004 and $65{ }_{1}-212$ airfoils, and lesser increases were noted for both airfoils with rime ice accretions. Brun, et. al. ( $6,7,8$ ) and Brun and Vogt (9) produced a series of reports dealing with experimental
measurements of performance degradation of NACA 65A004, 65, 208, and $65_{1}-212$ airfoils due to ice formation. These reports focused on the impingement characteristics of these airfoils, which in turn relate directly to drag increment and decreased performance. Included in their investigations were the effects of airfoil thickness and angle of attack on droplet impingement values, noting that thicknese tends to increase the volume of water collected and decrease the rearward limit of droplet impingement, and as expected total impingement increases with increasing angle of attack.

Gelder, Smyers, and von Glahn (10) in 1956 inveatigated droplet impingement properties on several airfoil sections of various thickness and concluded in part that the total and maximum local collection efficiencies were strong functions of $\ddot{f} i r a t$ of all the modified inertia parameter, a parameter which takes into account factors such as freestream velocity, droplet denaity and diameter, airfoil chord, abeolute air viscosity and air density, and also of thickness ratio and angle of attack.

These early experimental programs not only provided valuable insight into the process and effects of ice accretion, but also eatablished a data base from which analytical ice growth and performance degradation predictions could be developed or verified.
2. Analytical Effort

In the report by Neel and Bright (3), the authors attempted to predict analytically the degree of propeller
performance reduction to be expected as a reault of ice accretion. Using blade element theory as the basis for their analyaia, and by relating the change in airfoil drag/lift ratio to efficiency loss, they predicted efficiency losaes which agreed at least to the same order of magnitude with the experimentally obtained values.

Bergrun (11) in 1951 offered a method for determining droplet impingement characteriatics on an airfoil by solving a set of simulteneous differential equations which described the particle dynamics of a water droplet moving in an air stream. This same principle forms the basis for present droplet trajectory codes also. Along with Lewis in 1952 (12), Bergrun also investigated using probability analysis the atmospheric factors responsible for ice formation and identified with a quite limited data base the three parameters atill felt to be of primary importance in the ice accretion process: cloud liquid water content, water droplet aize, and ambient temperature.

In 1958, Gray undertook an icing study of the NACA 65A004 airfoil (13) and attempted to develop a drag coefficient correlation which would relate ice nature and droplet impingement rates to the associated aerodynamic penaltiea for thia airfoil. Uaing experimental icing data, he obtained a dimensional correlation which he found to be accurate at all but angles of attack greater than four degrees, presumably due to the flow separation occurring at the higher angles of attack. Gray then published in 1964 a
report (14) in which he examined available aerodynamic icing data for other airfoils and modified his correlation to make it applicable to these other airfoils as. well. He accomplished this through the introduction of the factor "r" Which represents the airfoil leading edge radius of curvature in percent of chord. This modified correlation represented the only available lcing drag coefficient correlation at that time, and saw widespread use for several years. Oniy recently has itts validity as a general icing correlation been questioned, and it will most probably remain in use until a suitable replacement 1 a offered.

The formulation of Gray's drag coefficient correlation represented the last major development with respect to ice accretion effects for several years. The advent and use of the jet engine and turbofan on the propulaion scene then deemphasized the use of the propeller as a propulsive device and subsequently drew much attention away from the problem of ice accretion, which was of lesser importance for the large commercial transporta and military aircraft of that time. However, with recent increased fuel costs and a renewed interest in commuter aircraft and all-weather helicoptera, new interest in ice protection systems and a need to better understand the phenomenon of ice accretion have again come about.
B. Recent Work

1. Experimental Effort


#### Abstract

Researchers involved in the renewed study of ice accretion recognized the need to expand the quite limited experimental data base, especially to include tests of lce accretion on the newer airfoils designed for use on current


 general aviation aircraft, propellera, and helicopter rotora. Shaw, Sotos, and Solano in a 1982 report (15) discussed the results of one such program in which aerodynamic performance degradation data was obtained for a NACA $63_{2}$-A415 airfoil. Glaze and rime ice formations were studied in both cruise and climb configurations and aignificant performance degradation was noted for all cases. Also evaluated was the effect of aft frost growth on airfoil performance, and this phonomenon was found to significantly increase section drag coefficient. This project was also of importance because it provided new data with which to test Gray's drag coefficient correlation. The authors found that the correlation was as accurate in moat cases for the new data as it was for the old data upon which the correlation was developed, but it became poorer in accuracy for higher liquid water content values. This conclugion again pointed out the need for a better, more accurate drag coefficient correlation.Also in 1982 at the Ohio State University, Bragg and Gregorek (16), along with Zaguli (17), used simulated ice shapes to investigate various aspects of the ice accretion problem. In one test a simulated rime ice shape was applied,

```
    with and without aurface roughness, to a NACA 65A413 airfoil
    and the ensuing performance degradation was measured. Surface
    roughnese was determined to be an important factor in
    modelling ice shapes, affecting drag coefficient and C C;
    In the second test, they also demonstrated the feasibility of
    using wood shapes to model ice accretions on a NACA 63.-A415
    airfoil in order to ascertain the effects of the ice shape on
    the aerodynamic flowfield.
        Flemming and Lednicer in 1983 (18) tested a series of
    scale models of helicopter airfoils to investigate the effecta
    of artificial ice accretion on airfoils at high speeds.
    Aerodynamic performance degradation of all the airfolls was
    noted, with the authors citing drag coefficient increases of
    up to roughly 300% in some cases.. Also accomplished in thia
    teat were better definitions of the boundaries for ice growth
    and ice type in terms of static temperature, Mach number, and
    liquid water content.
            Motivated by a desire to find a relatively simple,
    economical means of collecting performance degradation data
    relative to rotating syatems, Korkan, Crosa, and Cornell
    (19.20) and Korkan, Cross, and Miller, (21,22) in 1984
    undertook a test program utilizing a model helicopter with a
    53.375-inch diameter, 2.5-inch chord NACA 0012 main rotor with
    a simulated ice shape attached and measured main rotor
    performance degradation. Trends identical to those previously
    seen on larger or full-size airfoils and rotors were noted,
```


#### Abstract

with increases of up to $300 x$ in torque coefficient found to be required to maintain a given thruat coefficient after the addition of the aimulated lae to the rotor bladea. The sensitivity of the tip region of the rotor to the adverae effects of ice formation was also seen, with an increase in torque coefficient required to maintain a given thrust coefficient of as much as $150 \%$ measured when the simulated ice was extended from the 85x rotor radial location out to the 100\% location.

Juat prior to this model helicopter test, the iasue of Reynolds number effecta on aerodynamic data was also addreased by the same authors (20,22). In a 2-D airfoil test using a NACA 0012 airfoil section with a simulated ice formation geometrically identical to that used in the model helicopter test, the authors collected aerodynamic data for the airfoil in both clean and iced configurations over a Reynolds number range which included the operating range of the model helicopter rotor tip. Reynolds numbers effects appeared significant only at the highest Reynolds number tested, $3.4 \times 10^{6}$. 2. Analytical Effort

Much has been learned in recent years as a result of these experimental programs, and a major contribution of theae teata has been in the expansion of the icing data base to provide more current data with which to develop and evaluate analytical methods for prediction of icing-related performance degradation. As in the experimental area, many analytical


advances have also been recently made. These advances have taken the forms of both better methods of predicting droplet impingement characteristics as well as better aerodynamic performance degradation correlations.

Bragg et. al. recently developed a computer program to celculate water droplet trajectories and thereby determine airfoil impingement efficiencies and theoretical ice shapes (23). Development of this code represented a major step toward the goal of analytical prediction of airfoll performance degradation due to ice accretion and the code ia now of ten used for this purpose. The code itself will be discussed in more detail in a later chapter as it is one of the major componenta of the code development which la the subject of this report. Along with this droplet trajectory code, Bragg and Gregorek have also formulated a drag coefficient correlation for the rime ice condition (16). In this correlation, the change in drag due to ice is given as a function of several variables related to flight and atmospheric conditions, airfoil geometry, and duration of ice accretion. This correlation has seen widespread use since ita development and has only recently been studied in any detail to better define ite range of applicability. An unpublished investigation performed by this author shows the correlation to in several cases significantly overpredict drag increment for icing data which appears to be rime-oriented.

Cansdale and Gent in a 1983 report (24) detailed the development of another two-dimensional airfoil icing code.

Unlike the Bragg code, the effects of compreseibility, kinetic heating, and water runback are taken into account in this code, thus making it applicable to both rime and glaze ice conditions. Designed to be applied to helicopter configurations, the code employs a heat balance analyais to calculate the kinetic heating and runback effects. The authors have reported good agreement between predicted and experimentally obtained ice shepes, temperature distributions, and icing threshold conditions.

Flemming and Lednicer (25) have used experimentel icing data to formulate new rime and glaze ice drag coefficient correlations. In addition this data was also used to formulate separate lift and moment coefficient correlations. These correlations, recently published in final form (25), have shown some promise in preliminary evaluations and are discussed in more detail in a later section.

Using Bragg's droplet trajectory code, Korkan, Dadone. and Shaw (26) have developed a method for predicting performance degradation of rotating syatems under the influence of ice accretions. Limited currently to the rime ice condition, the effects of ice formetion on helicopter rotor or propeller thrust coefficient, torque coefficient, and efficiency may be calculated. Originally reatricted to the helicopter hover mode, extension of the method to include forward flight calculations has been performed and documented in a recent $A I A A$ paper by Korkan, Dadone, and Shaw (27). The method easentially involves first obtaining non-iced values of
angle of attack and Mach number as a function of radial location, then determining impingement efficiency and accumulation parameter for a given aet of icing conditiona for each radial location, and finally determining the reaulting drag increment at each radial location. An existing propeller. or helicopter performance code with the iced drag increment input then will produce values of iced performance which may be compared with known or calculated clean values to ascertain the degree of performance degradation which reaulte from the given icing encounter. Good agreement between theory and experiment has been obtained for both the propeller and helicopter rotor configurations.

Korkan et. al. have also developed a method of averaging (27) to compute rotor or propeller disk performance in which angle of attacic and Mach number are first averaged around the disk. From these averaged values, collection efficiency. accumulation parameter, and finally drag increment are calculated for given icing conditions. This procedure has been shown to be virtually as accurate as the previously described method but provides a significant reduction in the number of necessary calculations and hence computer run time.

Miller, Korkan, and Shaw in 1983 (28) attempted to develop a drag coefficient correlation for the glaze ice condition using atatistical analysis of icing data as a basia of the correlation development. Although a reaulting correlation was presented, it represented only the atate of the study at the time and showed a need for much further
modification and analysis. Feasibility of using such an analysis as the basis for correlation development was demonstrated, but the results of such an analysis must be interpreted correctly and in tandem with phyaical observationa of the associated phenomenon to make the method truly beneficial.

Bragg (29) has also recently documented a study in which he predicted airfoil performance with both aimulated rime and glaze ice shapes. Various codes were examined for possible use in calculating pressure distributions on an airfoil with glaze ice attached. The potential flow code of Briatow was determined to be suitable for this purpose and provided good agreement between predicted and experimental pressure distributions. Bragg also investigated the effects of surface roughness in the laminar boundary layer on drag increment and offered promising results for predicting drag increase by this method.

## CODES EMPLOYED

A. Theodorsen Transformation

The first atep in the performance degradation
determination process is computation of the flowfield about the airfoil section being investigated. The method used in this code is a Theodoreen technique of conformal mapping which translates the airfoil coordinates to a circle plane and from that plane determines the velocity diatribution about the airfoil. The calculations involved in this method make the assumption of potential flow, in which no viscous effects are considered. This has been found to be an acceptable assumption, allowing the method to yield accurate results, due to the fact that viscous effects existing in the boundery layer near the leading edge of the airfoil have little effect on droplet trajectory for all but the smallest of droplet sizes. The magnitude of the inertia force associated with the larger droplets greatly predominates over any viscous deflection or interaction phenomena in this region.

This particular flowfield calculation technique is limited to incompressible flow, which is acceptible aince compressibility effects on droplet trajectory have been found to be negligible up to the airfoil section critical Mach number for all but the smallest of drop sizes. In recent Joint atudies between NASA and the Royal Aircraft Establishment (RAE) (30), beck-to-back comparisons were made between the impingement efficiency values generated using the
incompresaible. Theodoresen method at NASA and those calculated using the compreasible flowfield code of Garabedian and Korn for supercritical airfoil sections at the RAE. Good agreement was observed between the two methods, thereby deemphasizing any compressibility effects and validating the use of the Theodorsen transformation in computing the flowfleld about airfoil sections exposed to icing.

Theodorsen's method makes use of the Karman-Trefftz transformation to relate the airfoll geometry to a near circle plane. This particular transformation is used because it has been found to yield a better near circle for airfoils with finite trailing edge angles by avoiding any cusp atthe trailing edge. It is given by the formula

$$
\begin{equation*}
\underline{x}=n b \frac{\left(z_{1}+b\right)^{n}+\left(z_{1}-b\right)^{n}}{\left(z_{1}+b\right)^{n}-\left(z_{1}-b\right)^{n}} \tag{1}
\end{equation*}
$$

where $z$ is a vector which may define some profile such as an airfoil section, and $z_{1}$ is a vector from the origin, 0 , to some arbitrary point on the circle. Here, b represents a vector directed along, the positive x-axis with length $O B^{\prime \prime}$. where $B^{\prime}$ is a point at which the circle and x-axis intersect. The value of $n$ is typically just less than two, and it may be observed that for $n=2$, the Karman-Trefftz transformation simplifies to the more familiar Joukowski profile, given by

$$
z=z_{1}+\frac{\underline{b}^{2}}{z_{1}}
$$

Finally, an iterative procedure calculates the Fourier coefficients in the Theodorsen mapping to the exact circle.

Using a Newton root-finding scheme, the point in the circle plane corresponding to a particular point on the airfoil may be determined and from this the flow velocity at that point may be obtained given the section angle of attack.
B. Droplet Trajectory Code

The droplet trajectory code employed in thia analyaia was developed by Bragg (31) in 1980. Using basic theory and equations developed several years earlier as its foundation, the code extends the earlier work to include the results of more recent studies as well as much improved computer techiology. Esaentially, the code is used to compute aingle droplet trajectories and points of impingement on a given airfoil, and from these calculations it is able to determine both total and maximum local collection efficiencies as well as a predicted lice shape if desired.
: Anain, the initial step in the trajectory calculation process requires a computation of the potential flowfield about the airfoil under investigation. This flowfield information, calculated using the Theodorsen transformation method described in the previous section, is passed to the trajectory calculation portion of the code in the form of the aforementioned Fourier coefficienta, which yield the flow velocity at any given point in the airfoil flowfield for a specific angle of attack. With this flowfield information available, droplet trajectories may then be computed. The trajectory calculation requirea solving a differential equation which results from the application of

Newton's Second Law to a water droplet moving in an air stream. This equation is given as

$$
\begin{equation*}
K \eta=\frac{C_{D} R}{24}(\bar{u}-\bar{\eta})+\frac{1}{F r^{2}} \bar{q} \tag{3}
\end{equation*}
$$

where $K$ and $F r$ are nondimensional parametera known reapectively as the inertia parameter and Froude number. They are given by

$$
K=\frac{\sigma \delta^{8} u}{18 c \mu}
$$

and

$$
\begin{equation*}
F r=\frac{U}{\sqrt{C g}} \tag{5}
\end{equation*}
$$

From Equation 3 a set of four simultaneous first order ordinary differential equations result, which when solved yield the droplet trajectory.

Once the droplet trajectoriea are known impingement efficiencies may then be calculated. Airfoil local impingement efficiency, B, physically repreaenta a ratio of airfoil surface droplet mass flux to freestream mass flux, and 18

$$
\begin{equation*}
B=\frac{d y_{0}}{d s} \tag{6}
\end{equation*}
$$

Here, $Y_{0}$ is the initial particle $y$-coordinate and $S$ ia the airfoil surface arc length from the leading edge to the point of impact. Figure 3 illustrates graphically the calculation of $B$, and Figure 4 shows a typical B-S curve. If Yo is ploted versus $S$, it may be seen that $B$ is aimply the slope of the yo-S curve at a given point. Total, or overall.
collection efficiency of an airfoil is also an important icing parameter and is defined by
$\because \because \because E=\frac{\Delta Y_{0}}{h}$
where $Y_{0}$ is the distance between the initial y-coordinate of the upper and lower droplet tangent trajectoriea and $h$ represents the maximum airfoil projected height. Total coliection efficiency is then a ratio of the amount of water masi collected by the airfoil to the amount in the freeatream sector which is swept out by the airfoll. Figure 5 exhibita typical variation of collection efficiencies with radial liocation a and Figure 6 is an example of the radial variation of local angle of attack and Mach number.

Other capabilities exist within the droplet trajectory code, perhaps most aignificantly the ability to develop a theoretical rime ice shape using a time-stepping routine, but this function is not necessary in the present atudy and will not be employed or further discussed here. Although it is true that the formation of an ice shape on an airfoil will indeed alter the flowfield about the airfoil, and hence the B curve for that airfoil, this phenomenon is not considered in this analysis because the values of $B$ and $E$ used in later calculations are taken for the clean airfoil. The importance of the collection efficiencies will be manifested in the following section dealing with aerodynamic coefficient correlations.
C. Aerodynamic Coefficient Correlations

Prior to the initiation of the present study, an effort was made to identify all of the dimensionless parametera relevant to an icing encounter using Buckingham'a $\mathrm{P}_{1}$ Theorem, with the goal of identifying and/or justifying the use of certain variables in empirical correlations. The $p i$ theom of Buckingham, presented first in 1914, provides a means of creating a dimenaionleas variable aet from an initial group of dimensional variablea. The theorem states that any function of $N$ variables, of the form

$$
f\left(P_{1}, P_{2}, P_{3}, \ldots P_{N}\right)=0
$$

may also be given in terms of ( $N-K$ ) dimensionless Pi producta, such thet

$$
f\left(P_{i_{1}}, P i_{2}, \ldots P i_{N-K}\right)=0
$$

Here $K$ represents the number of dimensionally independent quantities necessary to fully express the dimensions of the $N$ variables, and each $P i$ term is a dimensionless combination of an arbitrarily chosen set of $K$ dimensionally independent variablea. The Pi theorem will be used here to nondimensionalize parametera relevant to an airfoil icing encounter.

The procedure is initiated by first listing all of the suspected relevant dimensional parameters. In an icing encounter, these are taken to be as given in Table 1.

Table 1. Dimensional parameters considered in the ice accretion process.


```
performance lobses, namely leading edge radius of curvature,
alrfoll thickness, and camber, are not included in the above
variable set even though they are dimensional quantities.
This is because the only seemingly meaningful way to
nondimensionalize these terms is with respect to airfoil
chord, so that for the purposes of this analysis it will be
assumed that these parameters are all initially
nondimensionalized by chord and hence are not included in the
variable set. Instead they may be added to the list of
nondimensional variables at the conclusion of the application
of the theorem if desired.
It may be noted that the chord term makes its influence felt in the nondimensionalization of these airfoil geometry terms, and has also been included in the dimensional variable set to allow for the possible development of dimensionless terms which will be consistent with or similar to dimensionless quantities already in existence. For example:
\[
\begin{array}{ll}
A C=\frac{V_{O} W T}{P_{i c e}^{C}} & \text { (Accumulation Parameter) } \\
K_{\dot{O}}=f\left(K_{,} R_{u}\right) & \text { (Modified Inertia Parameter) }
\end{array}
\]
```

where

$$
\begin{aligned}
& \dot{K}=\frac{\rho_{i c e^{d}} V_{O}}{18 c \mu} \\
& R_{u}=\frac{v_{O}^{p}{ }_{\text {air }}{ }^{d}}{\mu}
\end{aligned}
$$

The next atep in the procedure is to formulate a $K$ set of four (equal to the number of fundamental dimensiona)
dimensionally independent variablea, from which the Pi products will be developed. To form the set, any four variables may be chosen from the dimenaional variable liat, with the restriction that they may not themselves combine to form a dimensionless combination. A $K$ set consisting of

$$
K=\left\langle T, \tau, \omega, V_{0}\right)
$$

is initially chosen. With this $K$ set, the six resulting Pi products are:

$$
\begin{aligned}
& P_{i_{1}}=f\left(T, T, w, V_{0}, T_{0}\right) \\
& P_{i_{2}}=f\left(T, T, w, V_{0}, d\right) \\
& P_{i_{3}}=f\left(T, T, w, V_{0}, P_{i c e}\right) \\
& P_{i_{4}}=f\left(T, T, w, V_{0,} P_{a i r}\right) \\
& P_{1}=f\left(T, T, w, V_{0}, u\right) \\
& P_{1_{1}}=f\left(T, T, w, V_{0}, C\right)
\end{aligned}
$$

To create nondimensional terms from these sets, the following procedure, as illustrated by an example using $\mathrm{p}_{1}$, is employed. The Pi product is first written as a product of all of ita terms, each raised to some power.

$$
P i_{2}=(T)(T)^{a}(\omega)^{b}\left(V_{0}\right)^{c}\left(T_{0}\right)^{d}
$$

It is then rewritten in terms of the fundamental dimensions involved:

$$
P_{i_{1}}=(t)(T)^{a}\left(M L^{-3}\right)^{b}\left(L T^{-1}\right)^{c}(t)^{d}
$$

For this product to be nondimensionalized, all of these dimensions must go to zero, i.e., the exponents on each dimension must sum to zero.

$$
\begin{aligned}
t: & 1+d=0 \\
T: & a-c=0 \\
M: & b=0 \\
L: & -3 b+c=0
\end{aligned}
$$

This set of equations is then solved for $a, b, c$, and $d$.
$d=-1$
$b=0$
$c=0$
$a=0$

The Pi product is then rewritten with these values included.

$$
\left.P i_{1}=(T)\left(T_{0}\right)^{-1}=\frac{(T}{T_{0}}\right)
$$

Which is the first dimensionless quantity.
Likewise, the other $P i$ products are found to be



$$
\mathrm{Pi}_{4}=\frac{W}{P_{\text {air }}}
$$

$$
\mathrm{Pi}_{5}=\omega \mathrm{Vg}^{2}
$$

$\mu$

$$
P i_{0}=\frac{V_{0}}{c}
$$

The resulting set of dimensionless variables as determined by the $P_{i}$ theorem is then

$$
f\left(t / T_{0}, V_{0} \tau / d, w / P_{i c e}, w V_{0}^{2} / \mu, V_{0} \tau / c\right)=0,
$$

or in the case of predicting drag coefficient change due to these variables,
$\mathbf{f}\left(T / T_{0}, \ldots . . . . . . . . . . . . . V_{0} T / C\right)=\Delta C_{d}$

At this point, any other dimensionless variables auch as collection efficiencies or airfoil geometry terms may be included in the equation of desired. The user may then apply some correlation development technique to determine the exact relationship between these dimensionless variablea and the associated dependent variable(a).

It may be demonstrated that the reaultant aet of dimensionless variables is dependent on both the set of dimensional variables which is used and on the $k$ aet which ia initially chosen. Since there are a finite number of dimensional variables and a finite number of $K$ sets which can betchosen, there will be a finite number of dimenaionlegs variables which will result for a given problem. However, not ali of these variablea will necessarily be produced by one $k$ set: For the icing encounter problem discussed here, this can be illustrated by choosing a second $K$ set:

$$
K=\left(T, W, C, V_{0}\right)
$$

This particular $K$ set is especially noteworthy for the icing encounter problem since it contains parameters which are thought to be of primary importance in the ice accretion process: temperature, liquid water content, freestream velocity, and airfoil chord length. The Pi products for this set are:

$$
\begin{aligned}
& P_{1}=f\left(T, W, C, V_{0}, T_{0}\right) \\
& P i_{2}=f\left(T, W, C, V_{0}, T\right) \\
& \mathrm{Pi}_{3}=f\left(T, \omega, C, V_{0}, d\right) \\
& \mathrm{PL}_{4}=\mathrm{f}\left(\mathrm{~T}, \mathrm{w}, \mathrm{C}, \mathrm{~V}_{\mathrm{O}}, \mathrm{P}_{\text {ice }}\right) \\
& P_{1_{g}}=f\left(T ; \omega, c, V_{0}, P_{a i r}\right) \\
& P_{i}=f\left(T, \omega, c, V_{0}, \mu\right)
\end{aligned}
$$

Following the procedure previously outlined for nondimensionlizing these products, the resultant set of dimensionless variables is:
 which differs somewhat from the set produced by the other $k$ set used. So for any given problem it is necessary to consider all of the possible $K$ sets to obtain a complete list of dimensionless variables which may be produced. Knowledge of the physics of the problem under consideration should then provide a starting point in the analysia of these variablea as they relate to the given dependent variable(s).

In the icing encounter problem, it was deaired to use the Pi theorem to provide some justification for the use of the terms $A C$ and $K o$, previously defined, in the development of a.glaze ice drag coefficient correlation. Although the pi theorem does not produce these terms directly, it does provide the following dimensionless terms:
(1) $V_{0} \tau / c$
(2) w/ pice
(3): $\frac{\rho_{i c e V_{0} d}^{\mu}}{\mu}$
(4) $d / c$
$(5): \frac{\rho a i r V_{0} d}{\mu}$
:-: " It may be noted that $A C$ is simply the product of terms
(1) sand (2). Ko is a function of $K$, the product of terms (3)
and (4), and $R_{u}$, which is identical to term (5), egg..
$3: 3$
$A C=\frac{V_{O T}}{C} \quad \frac{w}{P_{\text {ice }}}$
$\therefore$ (1) (2)

$\therefore$ Should the reader wish to verify that these terms are indeed obtainable from the original set of dimensional variables,
they can be found as follows:
(1) -from $K$ set ( $T, T, \omega, V_{0}$ )
(2) - from $K$ set ( $T, \tau, w, V_{0}$ )
(3) - from $K$ set ( $T, d, V_{0}, P_{1 c e}$ )
(4) - from $K$ set ( $T, w, c, V_{0}$ )
(5) - from $K$ set ( $T, d, V_{0}, P_{a i r}$ )

It can be aeen that the $P_{1}$ theorem does indeed provide some juatification for the use of the variables $A C$ and $K 0$ in developing a correlation, and any correlation which doea result will be in terms of dimensionless variables obtainable by Buckingham's Pi theorem.

At present, the only means available for analytically determining performance degradation of an airfoil in terma of its aerodynamic coefficients is through the use of empirical correlations. As previously indicated, several individuals have attempted development of one or more of these correlations and generally have met with only limited succeas in terma of accuracy and range of application. Incorporated into the present study are the two correlations currently most widely used to predict drag increment, as well as a series of new correlations. These empirical correlations represent only an interim solution to the problem of assesaing aerodynamic performance losses due to ice formation. A potentially more precise solution technique currently under development involves a Navier-Stokes solution for the flowfield around an iced airfoil (32).

1. Gray Correlation

The firat correlation, formulated by Gray (14) roughly twenty years ago is of the form $\Delta C_{D}=\left[8.7 \times 10^{-s}\left\{L_{\text {max }}\left(32-T_{0}\right)^{0.3}\right]\left(1+6\left(1+2.52 r^{0.1} \sin ^{4} 12 \alpha\right)\right.\right.$

$$
\begin{align*}
& \sin ^{2}\left[543 w\left(\frac{E}{32-T_{0}}\right) 1 / 3-81+65.3\left(\frac{1}{1.35^{\alpha i}}-\frac{1}{1+35^{\alpha}}\right)\right] \\
& \left.\left.\therefore \quad \frac{-0.17}{r} \sin ^{4} 11 \alpha\right]\right)  \tag{8}\\
& \therefore
\end{align*}
$$

where here $\Delta C_{d}$ represents the actual numerical value of drag coefficient increment; 1.e., $\Delta C_{d}=C_{d}(i c e d)-C_{d}(c l e a n) . ~ A a$ can be seen, this correlation involves combinations of various dimensional parameters, and the dimensionless total and lacal collection efficienciea calculated in the droplet trajectory code"previously discussed also appear. This correlation also has the capability of computing drag coefficient increment at angles of attack other than that at which the ice was formed, through the use of the $\alpha$ and $\alpha_{1}$ terms. Here $\alpha$ repreaents the angle of attack at which the experimental data was taken, or the angle at which the effecta of the ice accretion are to be evaluated, and $\alpha_{i}$ represents the angle of attack at which the ice was formed.

The dimensional variables have not in all cases been grouped so as to form dimensionless quantities, and this has evoked some criticism by various persons since the correlation's development. This criticism is based on the idea that any correlation which involves dimensional quantities will be to some extent a function of the data set from which it was formulated. Thus, if the correlation was applied to some other set of data in which the range of one or more of the relevent parametera differed from that of the original data aet, the variable affected may have an undue
effect on the prediction which was not accounted for in the original, limited data set. Indeed, as is seen in Figure 7 , Gray'a correlation has been found to predict quite poorly for much of the recently generated NASA glaze ice data, which ia typefied by aignificantly higher liquid water content valuea than are found in the data set used by Gray in developing hia correlation.: Nevertheleas, since no other glaze ice drag coefficient existed until quiter recently, Gray'a correlation still sees widespread use and has been included in this program as an option available to the user:

Because Gray's correlation accounta for only changes in drag coefficient, modification of $C_{h}$ has been added to take into account the decambering effect of ice accretion, reaulting in a shift in the $C_{k}-\alpha$ curve and a decrease in $C_{k}$ $\max$ due to flow separation on the airfoil upper surface. Trial values tested by the author have indicated that reducing $C_{A}$ to 95\% of its clean configuration value yields an acceptable approximation for iced $C_{A}$. This factor is incorporated into the present study for both Gray's and Bragg's correlations. 2. Bragg Correlation

A correlation has been developed by Bragg (16) for the rime ice condition. It has the form
$\Delta C_{d}=0.01\left(15.8 \ln (k / C)+28000 A_{c} E+I\right) \quad$ (9)
where Ac is accumulation parameter, given by

$$
A C=\frac{u W T}{P_{i c e} C}
$$

and I repreaenta a drag conatant which variea according to airfoil type as shown in Table 2.

Table 2. Constants in Bragg Drag Coefficient Correlation

| Alrioil Type | Orag Conatant, I |
| :---: | :---: |
| NACA 4 and 5 digit | 184 |
| NACA ES gerleg | 218 |
| NACA 64 series | 232 |
| NACA ES series | 252 |
| NACA 66 series | 290 | in, rather than an actual numerical value of, drag coefficient. Here $\Delta C_{d}$ may be ueed to find the iced airfoil drag coefficient using

$$
\begin{equation*}
c_{d_{\text {iced }}}=\left(1+\Delta C_{d}\right) c_{d_{\text {cleen }}} \tag{11}
\end{equation*}
$$

Previous studies of this correlation applied to NACA and NASA 2-D icing data have indicated a atrong tendency for the correlation to overpredict drag increment. Because the predictiona are commonly off by a relatively conatant percentage with reapect to the Neel and Bright propeller performance data, a factor was introduced which reduces the degree of drag coefficient increase calculated by the Bragg correlation. Replacement of the initial constant in the equation, 0.01 , by 0.0008 produced much more acceptable predictions and was therefore incorporated into the present analysia.

Bragg has also recently proposed a new, modified form of his correlation (33) which has also been included in the present study. This new form of the correlation given below, appears to work well for the cases examined to date, but it has not yet been applied to the $2-D$ NACA and NASA icing data with which the other correlations were tested.

$$
\Delta C_{d}=0.01(15.8 \ln (k / C)+1171 A C E+I) \quad \text { (12) }
$$

3. Flemming Correlations

The third correlation, or more preciaely set of
correlations, are those developed by Flemming (25). Whereas Gray and Bragg dealt atrictly with drag coefficient correlations, Flemming has also formulated correlations for lift and moment coefficients for both glaze and rime encounters. These correlations are of the forms:

For a glaze ice encounter.

$$
\begin{aligned}
& \Delta C_{d}=K D_{1}\left[.00686 K_{0}(t / C)^{1.5}(\alpha+6)-.0313(r / C)^{2}+K D(.006) M^{2.4}\right] \\
& {\left[\omega \operatorname{rc}\left(C^{c}\right)^{0.2} /(\underline{C})^{1.2}\right]\left[1-8 \Delta C_{d}^{\prime}\left(\underline{V_{h e l o}}\right)\right]} \\
& .1524 \quad .1524 \quad 278
\end{aligned}
$$

For a rime ice encounter.
$\Delta C_{d}=\left[.158 \ln (k / C)+175\left(\underset{i c e}{ } \underset{p_{1}}{v}\right) \omega \tau_{c} E+1.70\right]\left(\frac{\alpha+6)}{10}\right.$

$$
\begin{equation*}
\left[1-8 \Delta C_{d} \cdot \frac{\left.\left(v_{\text {helo }}\right)\right]}{278} C_{d_{c l e a n}}\right. \tag{14}
\end{equation*}
$$

and for both glaze and rime encounters,
$\Delta C_{i}=\left(-.01335 K_{0}(t / C)\left[\alpha+2+K L_{1}(.00555)(\alpha-\sigma)^{2}\right] K L\right]\left[\omega_{c}\right.$

$$
\begin{equation*}
\left(\frac{c}{.1524}\right)^{0.2} /\left(\frac{c}{1524}^{1.2}\right] \tag{15}
\end{equation*}
$$

and
$\Delta C m=\left[(.00179-.0045 M)(.00544) K_{0} \alpha /(t / C)^{2.7}+\right.$

$$
.00383 M(1-63.29 r / c) J
$$


where Ko is the modified inertia parameter, given by

$$
\begin{equation*}
K_{0}=K\left(1+.0967 R_{L} .6397\right) \tag{17}
\end{equation*}
$$

and $K L_{i}, K L_{i}$, and $K D$, and $K D_{2}$ are functions of temperature and angle of attack and are defined within the subroutine.

These correlationa were developed using data obtained in a series of tests conducted in the Canadian National Research Council's High Speed Icing Wind Tunnel in late 1982. The ten airfoil scale models tested, with typical chorda of roughly aix inches, were deaigned primarily for use in helicopter applications and cover a wide range of helicopter airfoil ahapes. For correlation development purpoaes this range should enable the resulting correlations to adequately account for airfoil geometry as a factor in aerodynamic coefficient increments due to icing.

The correlations are, however, dimensional in nature and again were developed using model data for high speed applications. The full effects of these two items on the range of applicability of the correlations remains to be aeen. The correlations have been found in a recent study to underpredict $\Delta C_{d}$ in many cases however. In spite of thia,

Flemming's correlations still hold promise and do in fact work well in many cases.
4. Miller Correlation

A fourth general drag coefficient correlation
development has been attempted by Miller, et. al. Deacribed in detail in Reference 28, this effort used as the basis for the correlation formulation the method of atatiatical analyaia as applied to a set of experimentally obtained icing data.

Although a correlation was presented in Reference 28, it represented only a status report and not a final result. Further study and modification of this correlation ia necessary before it may be deemed useable, and no modifications made to date have proven satisfactory. The method of statistical analysis is a powerful tool in correlation in development and should yield greater success at some future date, but because the present form of the correlation is not acceptable in terms of accuracy of prediction it will not be incorporated into the code.

So there are presently available three correlations or sets of correlations which are integrated into the code. They provided values of drag, lift, and/or moment coefficient increment for both glaze and rime ice encounters. These increments may then be passed to a propeller performance code, deacribed in the following section, to evaluate the ensuing propeller rotor performance degradation.

## D. Propeller Performance Code

The propeller performance code which ia integrated into the performance degradation program is based upon a linearized inflow propeller strip analysis, developed by Cooper (34) in 1957. Strip analysis involves the calculation of aerodynamic forces at aelected apanwise blade locationa for a given operating condition and propeller geometry. From these differential forces, given in terms of thruat and torque coefficienta by

$$
\frac{d C_{T}}{d x}=k{ }_{D}^{M_{x}} x_{h}^{2}\left(C_{h} \cos \phi-C_{d} \sin \phi\right)
$$

where $k=900 a^{2} b$ $N^{2} D^{2}$
and

$$
\begin{equation*}
\frac{d C_{Q}}{d x}=\frac{x}{2} \frac{d C_{T}}{d x} \frac{C_{d} \cos \phi+C_{\ell} \sin \phi}{C_{\ell} \cos \phi-C_{d} \sin \phi} \tag{20}
\end{equation*}
$$

thruat and torque coefficienta may be obtained by integrating these terms along the blade. Propeller efficiency and power absorbed may then be calculated by

$$
\begin{equation*}
\eta=J \frac{C_{T}}{C_{p}}=\left(\frac{V}{n D}\right) \frac{C_{T}}{2 \pi C_{Q}} \tag{21}
\end{equation*}
$$

and

$$
B H P=\frac{C_{p} P \eta^{3} D^{8}}{550}
$$

Cooper's propeller performance analysis is unique due to the fact that it assumes, or actually approximates, a linear induced flow diatribution on the propeller blade. Typically

```
an iterative process is required to obtain the induced angle
Of attack and lift coefficient at a specific radial location
aince they are interrelated. However, Goldstein (35) has
solved for the radial distribution of circulation for a
lightly loaded propeiler having a finite number of bladea and
has established the relationship between }\mp@subsup{|}{P}{C}\mp@subsup{C}{A}{}\mathrm{ and * * ( Cooper
has then approximated the of col - oficurves by straight lines
and has plotted the slopes of these lines versus advance ratio
for varioua spanwise locations. From theae plots, and using
available aerodynamic data, it is then posaible to determine
values for an induced angle of attack. When experimental and
analytical results obtained using this method were compared,
agreement to within three percent was seen for over ninety
percent of the cases investigated.
```


## CHAPTER 4

## PERFORMANCE DEGRADATION CODE ASSEMBLY

In order to integrate each of the codes involved. into a single performance degradation code, heretofore known as ICEPERF, aeveral phasea of modificationa were required. These modifications were relatively minor in that they did not change the basic functions, operation, or logic of any of the codea but rather served to make the tranaition from one section to the next more simple and automatic. This automation then eliminated the neceasity of any user intervention in the run stream.

The user-friendly nature of the code was a primary objective of this effort, and although aeveral files are necessary to run the code very few changes are required from one run to another, for the same airfoil. The various components of the code were selected on the basis of availability and accuracy of reaulta. Each component was run and tested separately and extensively to demonstrate its capability to consiatently function properly, and all displayed good performance in these runs. Also of importance in the component selection and modification was a desire to maintain a high degree of modularity in the code, such that a different flowfield computation, new correlation, different propeller analysis, etc. could be ingerted with a minimum of changes to the code. This also permita the code, now applied to propellers, to be relatively easily extended to a
helicopter configuration. A flowchart denoting the current major elements of the program is provided in Appendix 1.

The firat phase in the code integration process involved combining the flowfield and droplet trajectory codea previously described. Because the droplet trajectory code was designed to operate using much of the output from the Theodorsen transformation code, this atep was relatively straightforward. The modified code has been set up to handle a maximum of four input propeller radial locations, and current runs of the code have been made with four input stations, at the $30,50,70$, and 90 percent radial locations. An investigation of the benefits of using more input stations, offaet by the additional cost due to the extra run time required, has not yet been performed.

Next, a set of subroutines was created to predict drag increment due to ice accretion, given the appropriate impingement efficiency values output from the trajectory code. The $\Delta C_{d}$ correlations of Gray, Bragg, and Flemming are included in the subroutines, with Flemming'a correlation elso being capable of determining lift and moment coefficient increments. The user may select any of these correlations in each run of the program, and the choice of correlation should be dependent on the type of ice formation expected for a given input condition.

The correlations of Gray and Bragg use values of total and maximum local collection efficiencies in computing drag coefficient increment. One value for each of these parametera
i! generated for each radial location input, so that a maximum of four sets of collection efficiencies are available as output from the trajectory calculation section. Howewer, it is best when using the propeller performance code to include considerably more input radial locations in the calculations (eleven stations are used in the analyais of a propeller utilizing double-cambered Clark-Y airfoil. aectiona which follows in Chapter 5). In order to obtain values for drag coefficient increment at each of these locations, the collection efficiencies at these stations must be known. Rather than running the Theodorsen transformation and droplet trajectory codes eleven times to obtain collection efficiency values at each of these stations however, a curve fit routine is employed which takes the values computed at each of the four input radial locations and fits a curve through them. Flemming'a correlations compute valuea of collection efficiencies empirically, so that it is not necessary to run the Theodorsen and droplet trajectory sections of the code if Flemming's correlations are to be employed.

In this code, a cubic spline fit routine is used to fit separate curves for total maximum local collection efficiencies between the $30 \%$ and $90 \%$ radial locations, or between the two most extreme sections input by the user. This method has been found to yield good results due to the fact that the collection efficiencies are both smooth, continuoum functions or propeller radial location. Due to the nature of the spline fit routine, it is applicable only between the

```
Innermost and outermost radial locations input. Outboard of
the outermost input station, and inboard of the innermost
station, some other technique is required to compute values
for the collection efficiencies as a function of radial
location. In this analysis a linear fit, computing the slopes
at the endpoints of the spline fit and extending atraight
lines inboard and outboard of these endpoints, is used to
approximate the curves where the spline fit ia not. applicable.
With this method, values for both total and maximum local
collection efficienciea are available for any radial location.
    Cooper's propeller performance code has been utilized to
calculate values of propeller thrust, torque, and efficiency
for a given condition. The previously mentioned drag
increment subroutines have been incorporated into the
propeller performance code to modify the drag coefficient and
values used by the code in computing iced propeller thrust and
torque. In Equations 18 and 20. the C Ch and C C terms are
modified by the factors (1+\DeltaC ) and (1+\DeltaC ( ) respectively, such
that
    C}\mp@subsup{C}{k}{}=\mp@subsup{C}{2}{\prime}(clean)*(1+\Delta\mp@subsup{C}{2}{\prime}
and
\[
\begin{equation*}
C_{d}=C_{d}(\operatorname{clean}) *\left(1+\Delta C_{d}\right) \tag{24}
\end{equation*}
\]
so that iced propeller thrust and torque coefficients as well as propeller efficiency may then be computed. The code first calculates clean, or non-iced, values for these quantities. and then calculates using the user-aelected aerodynamic
```

coefficient correlation the lced values for these same parameters. Various input parameters have been altered or omitted from the original code to meke it more conducive to the purposes of the present program, but the essaence and mechanics of the code remain unchanged.
The entire code then operatea by first computing clean propeller performance values for one advance ratio. Next. the Theodoraen tranaformation 1 p performed, or previously calculated reaulta are read for the firat input radial location. The droplet trajectory section then computess impingement efficiency values for the radial location using the flowfield from the Theodorsen transformation and a section angle of attack computed in the propelier performance section. Impingement efficienciea are then calculated for the remaining input radial stations following the same procedure. The code spline fits the impingement efficiency curves and computes using these values and the selected aerodynamic coefficient increment correlation the iced propeller performance data. This entire procedure is then repeated for each additional advance ratio input.

## CHAPTER 5

RESULTS

In the Neel and Bright teat, a C-46 twin-engine aircraft was used to obtain propeller efficiency losses as a result of ice formation. Ice was allowed to form on the propeller of the right engine while the left engine propeller was kept clean. Following thia procedure, data was recorded for twelve natural icing encounters and typical efficiency loases of four to ten percent were noted. The propellers on the $C-46$ utilized double-cambered Clark-Y airfoll bections, for which airfoil coordinates used in ICEPERF were obtained from Borat (36).

The Neel and Bright experimental data provided a data base with which to evaluate the ICEPERF iced propeller performance code. For ten of the twelve encountera documented in the teat program report (Table 3 ), calculations were performed for the propeller in both clean and iced configurations. Results and comparison of analytical and experimental performance values are here presented for each of the three correlations employed, specifically those of Bragg, Gray, and Flemming. It should be noted that in all performance calculations presented in this report, consideration was given to the radial extent of icing such that no ice was allowed to accrete analytically outboard of the experimentally obaerved radial icing extent. These results therefore differ from previous calculations shown by
this author (37) in which no restriction was placed analytically on the radial extent of icing.

The results presented in this paper may also be compared directly with calculations performed by Korkan, et. al (27). Their work involved generation of propeller performance predictions for the Neel and Bright data as well, using the Theordorsen transformation flowfield code, Bragg droplet trajectory code, and Cooper propeller performance codes separately. In this initial study by Korkan, et. al., presaure altitude was input and the atandard atmoapheric pressure, temperature, and density were then used in the calculations rather than the experimentally obtained values for these parameters. The use of actual flight conditions, as employed in all calculations presented in this report, was found to alter the computed thrust and power coefficients by roughly one percent. Potential differences in blade angle specification from study to study may also have a significant impact on the predictions. Finally, the initial constant in the original Bragg correlation was decreased in both studies in an attempt to produce reasonable performance values. This then introduces additional variation between the corresponding 1ced calculations.

Clean performance predictions were found to agree well with the experimental data for all of the encounters investigated. (Tablea 4-6 summarize the analytical and experimental data for Encounters 2, 7, and 12 respectively.) In ali casea, calculated thruat and power coefficients were
found to agree with the corresponding experimental values within an average of 12.5 percent. $C_{T}$ and $C_{p}$ for a given encounter and advance ratio tended to be underpredicted by roughly the same percentage, so that reaulting propeller efficiencies compared with experimental measurementa to within six percent. Since propeller efficiency is esaentially a ratio of thrust coefficient to power coefficient however, errors of similar magnitude in $C_{T}$ and $C_{p}$ should then cancel so that the resulting efficiency values would be much leas sensitive to error in performance calculations and would not be as good an indicator of analytical prediction accuracy. Therefore atatementa herein relate primarily to thruat and power coefficient calculations rather than propeller efficiency.

The results shown in this report all involved use of the actual blade angles as measured and reported in the experimental test program. It was found however that by increasing the reported blade angle setting by 0.7 degrees, much better agreement between theory and experiment could be obtained for all encounters. Figures 8-10 are illustrations of the effect of blade angle variation on clean thrugt and power coefficients for Encounter 2. This anomaly could be an indication of a deficiency in the performance calculations, but because a constant increase in blade setting produces much better correlation with experiment for several encounters over a range of flight conditions, it more likely aignifies the existence of a bias error in the experimental measurement of
blade angle. These calculations serve also to indicate the sensitivity of propeller performance predictiona to propeller geometry.

The code was found to often overpredict the clean propeller efficiency at the higher advance ratios. Typically. performance predictions agreed with experiment to within a few percent at the lowest advance ratios and gradually woraened to the point where they were often off by roughly twenty percent at the highest advance ratios investigated. It is postulated that this may be due to an inability of the code to adequately account for compressibility effects, which become more significant as advance ratio increases (via increased forward velocity for a fixed rpm). This effect ahould be atudied further in subsequent efforts.

When the reatriction on radial icing extent was imposed in the present study's calculations, better agreement with the experimental data was obtained than was previousiy presented. Currently there is no mechanism in the code for determining the actual radial icing extent. Rather, the user must specify the desired or assumed radial location inboard of which iced calculations are to be performed. In this study, approximate radial icing extent for each encounter was reported by Neel and Bright and. these values were then input for the lced performance calculations. The following discusaion detaila reaults obtained by each correlation for the several encounters investigated.

## A. BRAGG

Two forms of Bragg's basic rime ice correlation are available in ICEPERF, as diacuased in a previous aection. The first of these is a modified form of Bragg's original equation, in which the leading constant was changed from 0.01 to 0.0008 in an effort to produce more realistic drag coefficient increments. Thia change was made after initial runs of the code indicated that the original equation was producing quite unreaaonably large performance changea due to ice formation, as a result of inordinately large predicted $\Delta C_{d}$ values. The 0.0008 constant was arrived at by bestfitting the correlation's performance predictions to Encounter 2 data and was then found to perform acceptably for the other encounters as well. Future use of the correlation may auggeat additional modifications of this constant.

This modified form of Bragg's original correlation was then applied to all of the available Neel and Bright data regardless of assumed rime or glaze ice type, as was Bragg's new form of the correlation. Reasonably good agreement between experiment and theory was obtained using both correlations for all encounters. In all cases the original but modified Bragg correlation produced more accurate resulta than did the newer correlation, but the differences were generally amall. The fact that the newer unmodified correlation gave results almost as accurate as those of the older form which was specifically modified to better fit the propeller data indicatea that the changea made by Bragg in
forming the new correlation improved the quality of the correlation significantly. Both correlations handled the less severe icing encounters (icing extents below 60x) well (i.e., Encounter 7, Figures 14-16), much better than did the Gray or Flemming correlations. In the most severe case however (Encounter 12, Figures 17-19) both correlations seriously overpredict the degree of degradation, eapecially in terma of $C_{p}$, and produce $C_{p}-J$ curves which inflect upward at the lower advance ratios rather than downward as would be expected. It should also be noted that Bragg's equations are for $\Delta C_{d}$ only. Lift coefficient will also generally decrease as a reault of ice formation at the leading edge as the flow there is spoiled to some extent, but this is not accounted for in the Bragg equations. A factor was therefore introduced to provide a simple representation of this phenomenon by assigning the iced section lift coefficient a value equal to 95\% of its clean value. Although this aeems to be a reasonable typical value for $\Delta C_{A}$, it would be desirable to incorporate a more discriminating form of determining $\Delta C_{h}$ in future calculations.

## B. GRAY

As was the case with the Bragg correlations, the Gray correlation also provides its most accurate performance predictions when radial icing extent is small. It ia difficult to make any inference regarding the quality of the correlation under such conditions due to the fact that the
propeller is loaded most heavily at the tip and its performance will be most sensitive to geometry changes (i.e., ice accretions) in the tip region. Farther inboard, even quite severe ice growths will have much less effect on the overall performance and consequently any correlation, regardleas of how great its predicted $\Delta C_{d}$ values may be, will appear to work well in low radial icing extent casea. Thus the true measure of the quality of these correlations comes in the more severe icing encounters auch as Encountera 2 and 12. In theae two encountera, as can be aeen in Figurea 11-13 and 17-19, the Gray correlation fails to accurately predict the iced performance, yielding values which overpredict the degree of degradation. Indeed, upon examining the actual $\Delta C_{d}$ valuea predicted by Gray for Encounter 12 it was found that this correlation was predicting drag coefficient increases of typically 300\% at lower advance ratios to as much as 6000\% at the outer stations at high advance ratios. The Gray correlation, like that of Bragg, also predicts only drag coefficient increment, so the same five percent reduction in $C_{i}$ for the iced configuration was again employed when Gray'a correlation was used. The Gray correlation itaelf was unmodified for use in this study, and it is likely that introducing some constant into Gray's equation to affect a constant percentage decrease in ita calculated $\Delta C_{d}$ values, as was done with the original Bragg equation, would greatly


#### Abstract

improve its ability to calculate iced drag coefficient increment and hence iced propeller performance.


## C. FLEMMING

The Flemming correlationa were applied twice to each encounter, once with icing restricted by the user-input value and then allowing the Flemming correlations to predict the radial icing extent. The second runs were performed to check the no-icing capability of the correlations and to ascertain whether this capability would properly reflect exiatence of ice under the influence of $3-D$, rotating forces (kinetic heating and shedding). Inatead of properly predicting radial icing extent however, this feature dictated no icing in the first $30 \%$ of propeller radius for any encounter, and then overpredicted the radial icing extent outboard of the $30 \%$ station for eight of the ten encounters (Table 7).

In the case of the user-input icing extent, the no-icing problem manifests itself in an underprediction of the propeller performance degradation for all encounters. As the figures illustrate, many of the Flemming-predicted iced thrust and power coefficient values are identical or virtually identical to the clean values, indicating analytically little If any degradation. While these results may appear reasonable at first glance in cases where the actual degradation is minimal, they actually aerve only to mask the underlying problem and reason for the negligible degradation prediction. It should also be noted that in Figures 11 through 19, in cases where the clean theoretical curve is not easily viaible,
it has been hidden by the Flemming correlation curves which are of ten virtually coincident with the clean data curve.

When no reatriction is imposed upon radial icing extent, Flemming's correlations still tend to underpredict the degree of lced performance degradation in geveral cases. This occura even though Flemming's correlations predict more extensive icing radially than was aeen experimentally, which would indicate that the percentage decreasea in thrust and power coefficienta predicted by Flemming are actually amaller than the experimental values. For the encounters in which the nonrestricted Flemming correlations do predict $C_{T}$ and $C_{p}$ well, the predicted radial icing extent is alwaya greater than the measured extent, sometimes by as much as 60\%.

It would therefore appear that the Flemming correlations have two significant deficiencies. The first of these is the no-icing calculation which has failed in this study to properly indicate either ice existence or icing extent. In feirness to the correlations it should be pointed out though that various forces not stressed in the $2-D$ correlation development (i.e.. centrifugal forces leading to ice shedding) may have a significant effect on the radial icing extent, and in only one out of ten encounters did the flemming correlations actually underpredict the extent of ice. The second apparent shortcoming of the correlations is the underprediction of section drag and poasibly lift coefficient increments. This same trend toward underprediction of aection $C_{d}$ has also been seen in application of the flemming

```
correlations to available NACA and NASA 2-D icing data in a
separate study (38).
```

Application of the ICEPERF code to the Neel and Bright experimental data base has provided much insight into the code's performance prediction capability for both clean and iced configurations. The code has been ahown to run properly and to produce realistic thrust and power coefficient values, especially when radial icing extent is known and input. As a result of this study several areas of future development and potential improvement, discussed in the following paragraphs, have been identified.

Currently reatrictions and simplifications exist within the code which, if removed or modified. should yield better performance evaluations. Development of a $\Delta C_{A}$ correlation to be used in conjunction with the Bragg or Gray correlations would provide a more precise and complete representation of the effects of ice formation on section aerodynamics. Secondly, incorporation of some computational mechaniam of predicting the transition from rime to glaze ice, or vice versa, on the blades would enable the code to use the proper correlation at each radial computational atation, rather than assuming all rime or all glaze ice in any given encounter. Because of the generally moderate radial icing extent and only slight degree of ensuing degradation, this change will probably have only a very minor effect on the predicted propeller performance. This is in contrast to the helicopter rotor icing situation in which radial lcing extent and the
related performance degradation are typically more severe for a given icing condition.

Another potential area of improvement lies in the accuracy of the experimental data used for comparison with analyais. One such area with reapect to the Neel and Bright data is the actual radial icing extent for each encounter. As specified in the report, the true extent is somewhat unclear and may possibly vary by as much as 20\%. Values of radial icing extent presented in this report represent a best estimate of the extent as interpreted from the information in Neel and Bright's report.

The preaent study has also provided insight into the type and accuracy of experimental data which would be desirable in future teat programs. Additional propeller icing data of any type would be useful in further validating and analyzing the code at the present time. Future teats though should include, in addition to flight and atmospheric parameters and icing time, specification of icing extent as accurately as possible. Also desirable would be some photographic or other indication of the ice type so that the proper correlation could be applied at each radial location. Finally, sensitivity of propeller performance to blade angle setting also has indicated the need for quite accurate blade angle measurements. Certainly it is desirable to have a high degree of accuracy in the thrust and power measurements as well.

With respect to the empirical lift and drag correlations used in this code, results obtained in this atudy concurred with previous work done involving these correlations in indicating deficiencies which exiet in each correlation. Overall, the Bragg correlation(s) produced the most reasonable iced propelier performance predictions but overprediction of drag coefficient increment by Bragg's correlation (a) remaina a problem, as illustrated in the more, severe icing encounters. Overprediction also remaina a problem with Gray'a correlation, whereas Flemming's correlations typically underpredict the aerodynamic coefficient increments. It is evident then that much more $2-D$ validation of all these correlations is neceasary before iced propeller performance can be predicted consistently using this method and before results of this analyaia can be fully evaluated. It should be realized that any ultimate analytical treatment of ice accretion on rotating systems will potentially involve much less empiricism than is currently required and that the use of these correlations represents an interim approach to the problem, which will continue to be used until more sophisticated analytical procedures auch as Navier-Stokes or interactive boundary layer analyses of iced airfoil flowfields become available.

1. Corson, B. W. and Maynard, J. D., "The Effect of Simulated Ice on Propeller Performance," NACA TN 1084, 1946.
2. Preaton, C. M. and Blackman, C. D., "Effects of Ice Formation on Airplane Performance in Level Cruiaing Flight," NACA TN 1598. May 1948.
3. Neel, Jr., C. B. and Bright, G. L., "The Effect of Ice Formations on Propeller Performance," NACA TN 2212, October 1950.
4. Gray, V. H. and von Glahn, U. H.. "Effect of Ice and Frost Formations on Drag of NACA 65s-212 Airfoll for Various Modes of Thermal Ice Protection," NACA TN 2962, June 1953.
5. Gray, V. H. and von Glahn, U. H., "Aerodynamic Effecta Caused by Icing of an Unswept NACA 65A004 Airfoil," NACA TN 4155, Feb. 1958.
6. Brun, R. J., Gallagher, H. M., and Vogt, D. E.,
"Impingement of Water Droplets on NACA 65ı-208 and 651212 Airfoils at 4 Degrees Angle of Attack," NACA TN 2952. May 1953.
7. Brun, R. J., Gallagher, H. M., and Vogt, D. E..
"Impingement of Water Droplets on NACA 65A004 Airfoil and Effect of Change in Airfoil Thickness from 12 to 4 Percent at 4 Degrees Angle of Attack," NACA TN 3047, November 1953.
8. Brun, R. J., Gallagher, H. M., and Vogt, D. E., "Impingement of Water Droplets on NACA 65A004 Airfoil at 8 Degrees Angle of Attack," NACA TN 3155, July 1954.
9. Brun, R. J. and Vogt, D. E.. "Impingement of Water Droplets on NACA 65A004 Airfoil at O Degrees Arigle of Attack" NACA TN 3586, November 1955.
10. Gelder, T. F., Smyera, Jr., W. H., and von Glahn, U. H., "Experimental Droplet Impingement on Several TwoDimensional Airfoils with Thickness Ratios of 6 to 16 Percent," NACA TN 3859, December 1956.
11. Bergrun, N. R., "An Empirical Method Permitting Rapid Determination of the Area, Rate, and Distribution of Water-Drop Impingement on an Airfoil of Arbitrary Section at Subsonic Speeds," NACA TN 2476, September 1951.
12. Lewis, W. and Bergrun, N. R., "A Probability Analyais of the Meteorological Factors Conducive to Aircraft Icing in the United States," NACA TN 2738, July 1952.
13. Griy, V. H., "Correlations Among Ice Measurementa, Impingement Rates, Icing Conditions, and Drag Coefficients for Unswept NACA 65A004 Airfoil," NACA TN 4151, February 1958.
14. Gray, V. H., "Prediction of Aerodynamic Penalties Caused by Ice Formations on Various Airfoils," NASA TN-D 2166, February 1964.
15. Shaw, R. J., Sotos, R. G., and Solano, F. R., "An Experimental Study of Airfoil Icing Characteristics, AIAA 82-0282, January 1982.
16. Bragg, M. B. and Gregorek, G. M., "Aerodynamic Characteristics of Airfoils with Ice Accretions," AIAA 82-0282, January 1982.
17. Bragg, M. B., Zaguli, R. J., and Gregorek, G. M.. "Wind Tunnel Evaluation of Airfoil Performance Uaing Simulated Ice Shapes," NASA CR 167960, November 1982.
18. Flemming, R. J. and Lednicer, D. A., "High Speed Ice Accretion on Rotorcraft Airfoils," AHS A-83-39-04-0000, May 1983.
19. Korkan, K. D., Cross, Jr., E. J., and Cornell, C. C., "Experimental Study of Performance Degradation of a Model Helicopter Main Rotor with Simulated Ice Shapea." AIAA 84-0184, January 1984.
20. Korkan, K. D., Cross, Jr., E. J., and Cornell, C. C., "Experimental Aerodynamic Characteristica of a NACA 0012 Airfoil with Simulated Ice." to be publiahed in AIAA J. of Aircreft.
21. Korkan, K. D., Cross, Jr., E. J., and Miller. T. L.. "Performance Degradation of a Model Helicopter Main Rotor in Hover and Forward Flight with Generic Ice Shape," AIAA 84-0609. March 1984.
22. Korkan, K. D., Cross, Jr., E. J., and Miller, T. L., "Performance Degradation of a Model Helicopter Rotor with a Generic Ice Shape," AIAA J. of Aircraft, Vol. 21, No. 10, October 1984, pp. 823-830.
23. Bragg, M. B., Gregorek, G. M.', and Shaw, R. J., "An Analytical Approach to Airfoil Icing." AIAA 81-0403, Jenuary 1981.
24. Cansdale, J. T. and Gent, R. W., "Ice Accretion on Aerofoils in Two-Dimensional Compressible Flow - A Theoretical Model." RAE TR 82128, January 1983.
25. Flemming, R. J., and Lednicer, D. A., "High Speed Ice Accretion on Rotorcraft Airfoils," NASA CR 3910, August 1985.
26. Korkan, K. D.. Dadone, L., and Shaw, R. J., "Performance Degradation of Propeller Syatems Due to Rime Ice Accretion," AIAA J. Of Aircraft, Vol. 21, No. 1, January 1984, pp. 44-49.
27. Korkan, K. D'., Dadone, L., and Shaw, R. J., "Performance Degradation of Helicopter Rotor Systems in Forward Flight Due to Rime Ice Accretion," AIAA 83-0028, January 1983.
28. Miller, T. L., Korkan, K. D., and Shaw, R. J., "Statiatical Study of a Glaze Ice Drag Coefficient Correlation." SAE 830753, April 1983.
29. Shaw, R. J., Private Communication, NASA Lewis Research Center, Cleveland, Ohio, Auguat 1984.
30. Potapczuk, M. G. and Gerhart, P. M., "Progress in Development of a Navier-Stokes Solver for Evaluation of Iced Airfoil Performance, AIAA 85-0410, January 1985.
31. Bragg, M. B., Private Communication, The Ohio State University, Columbus, Ohio. December 1985.
32. Cooper, J. P., "The 'Linearized Inflow' Propeller Strip Analysia," WADC TR 56-615, March 1957.
33. Goldatein, S., "On the Vortex Theory of Screw Propellers," Proceedings of the Royel Aeronautical Society (London), Ser. A, Vol. 123, No. 792, April 6, 1929.
34. Borat, H. V., Private Communication, Henry V. Borat and Associatea, Wayne, Pennsylvania, June 1984.
35. Miller, T. L.. "Analytical Determination of Propeller Performance Degradation Due to Ice Accretion," AIAA 850339, January 1985.
36. Sanchez-Cantalejo. P. G.. Private Communication, NASA Lewis Research Center, Cleveland, Ohio, September 1985.


FIGURE 1. The two basic ice types


FIGURE 2. Velocity components of propeller section

FIGURE 3. Determination of airfoil impingement efficiency
FIGURE


FIGURE 4. Variation of local impingement efficiency with airfoil surface arc length; $r / R=0.9, J=1.18$


FIGURE 5: Radial variation of accumulation parameter, maximum local collection efficiency, and total collection efficiency for Encounter 2, J = 1.18


FIGURE 6. Radial variation of local angle of attack and Mach no. for Encounter 2, $J=1.18$


FIGURE 7. Comparison of experimental results with Gray's predicted $\Delta C_{d}$ values


FIGURE 8. Effect of propeller blade setting on predicted thrust coefficient, Encounter 2


FIGURE 9. Effect of propeller blade setting on predicted power coefficient, Encounter 2


FIGURE 10. Effect of propeller blade setting on predicted propeller efficiency, Encounter 2


FIGURE 11. Variation of thrust coefficient with advance ratio, clean and iced (iced to $r / R=0.7$ ), Encounter 2


FIGURE 12. Variation of power coefficient with advance ratio, clean and iced (iced to $r / R=0.7$ ), Encounter 2


FIGURE 13. Variation of propeller efficiency with advance ratio, clean and iced (iced to $r / R=0.7$ ), Encounter 2


FIGURE 14. Variation of thrust coefficient with advance ratio, clean and iced (iced to $r / R=0.4$ ), Encounter 7


FIGURE 15. Variation of power coefficient with advance ratio, clean and iced (iced to $r / R=0.4$ ), Encounter 7


FIGURE 16. Variation of propeller efficiency with advance ratio, clean and iced (iced to $r / R=0.4$ ), Encounter 7


FIGURE 17. Variation of thrust coefficient with advance ratio, clean and iced (iced to $r / R=0.9$ ), Encounter 12


FIGURE 18. Variation of power coefficient with advance ratio, clean and iced (iced to $r / R=0.9$ ), Encounter 12


FIGURE 19. Variation of propeller efficiency with advance ratio, clean and iced (iced to $r / R=0.9$ ), Encounter 12


```
ーNNMmのONON
```

Air
Time, min.





Encounter 2.
ICED iaing restricted
laing unrestricted icing restricted

${ }^{4}$



$$
\begin{aligned}
& v^{t} u^{a}=y^{t} u^{a}=v^{t} v^{a}=v^{t} u^{a}=v^{t} u^{a}=v^{t} v^{a}=v^{t} u^{a}=
\end{aligned}
$$

Encounter 7. ficing unrestricted
Fleming
Fleming restricted




$$
\begin{array}{lll}
0 & 0 & \circ \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}
$$

Table
CLEAN
NeelsBright
Encounter 12.
icing unrestricted icing restricted


ICED


eel\&Bright

.111
.140
.793
.102
.128
.829

CLEAN
$\begin{array}{lllll}0 & 0 & n & 0 & 0 \\ -1 & 0 & \infty & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{array}$

$\begin{array}{ccccccc}n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & - & 0 & 0 \\ 0 & 0 & - & -1 & -1 & -1 & -1\end{array}$

Table 7. Experimental and predicted (Flemming correlationa) radial extent for each encounter.


PROGRAM FLOWCHART




## APPENDIX 2

USER'S GUIDE

ICEPERF has been deaigned auch that user intervention in the run stream has been eliminated and the amount of user input is minimized. Still, with the wide scope of work and calculations which the code must perform, a substantial degree of input is required of the user. This information is provided through several files which are read at verioua points in the program and which are separated such that each contains input data relative to a particular phase of the program and to a particular radial location. As previously mentioned, the code handles four input radial locations, and a complete aet of input data is required at each radial location.

The actual parameters in each file are described in the Commenta section of the code listing which will follow. They are redescribed here to provide the user with a better idea of how to specify and use each variable.

Files 1 through 4 contain airfoil coordinates and other information related to the airfoil geometry at each radial locetion. These files are used in computing the flowfield about the airfoil. File 1 contains information for the most inboard radial station input, with files 2 through 4 containing input data for progressively more outboerd stations.

Card 1 of files 1 through 4 containg the title for the Theodorsen transformation input. This title is printed at the
beginning of the airfoil section geometry output for the particular radial location input. It may typically read similarly to, "CLARK-Y COORDINATES AT 30\% STATION".

Card 2 contains five variables: LPT, ITAU, IZ1N, TAU, and $X 21 N$, input in the format (3I5,2F10.0).

LPT represents the number of harmonics which the user deaires in the exponential tranaformation of the airfoil to the circle plane. A value of 50 to 60 is typically used for LPT to attain a satiafactory degree of accuracy.

ITAU is a flag which indicates whether or not the uaer wishes to input the trailing edge angle of the airfoil section. A value of 0 indicates that the trailing edge angle is not known or will not be input, and any value greater than 0 indicates that the user will input the section trailing edge angle.

IZ1N is a flag which indicates whether or not the user will input the nose singularity coordinates. An IZ1N value of O means that the noae aingularity wili not be input, and IZ1N should be set to any value greater than 0 to input the nose singularity coordinatea.

TAU is the trailing edge angle in degrees, to be input if ITAU is greater than O.

XZIN and YZ1N are the $x$ - and $y$-coordinates of the noae singularity, to be input if IZiN is greater than 0 .

Card 3 contains the variables $N U$ and $N L$, which are the number of upper and lower surface coordinates to be input, reapectively. This card has the format 215.

Cards 4 through the last contain the airfoll section coordinates, nondimensionalized with respect to chord. Eight values are listed on each line in format 8F10.0. Upper surface $x$-coordinatea are all listed firat, followed by upper surface $y$-coordinates, lower surface $x$-coordinatea, and finally lower aurface $y$-coordinates.

This then completes the input required to map the airfoil sections to the circle plane.

File 8 contains input parametera which are used to compute droplet impingement values for the airfoil. The same File 8 input data is used for each input radial location. $\therefore$ : Card 1 of file 8 containa the title for that particular radial location, and would typically read similarly to " $30 \%$ STATION TRAJECTORY INPUT." This title is printed at the beginning of the droplet trajectory aection at each radial location.

Card 2 contains the variables IPRINT, MSPL, XFIRST, XLAST, and EPS.

IPRINT ia a print option, with a value of 0 used for the shortened version of the trajectory summary output, and a value of 1 producing the complete printed output. The shortened version will print only the general input to the trajectory section, the tangent trajectories, and the impingement efficiencies. The trajectory tabulation and summary and surface impingement valuea are not printed when IPRINT equale 0.

MSPL is a variable which is used to select the method desired to calculate the impingement efficiency curve. Setting MSPL equal to o requesta the program to select the best of three methods possible for calculating the curve. An MSPL value of 1 causes the program to use a simple cubic apline routine to calculate the curve. An MSPL value of: 2 instructs the program to use a cubic spline but to piece in a linear fit where the cubic apine fails. An MSPL value of 3 causes a quadratic spline to be used, in which the coefficients are selected to optimize the fit of the curve. Setting MSPL equal to 0 has always produced satisfactory results in all cases run for this investigation.

XFIRST is the $x$-coordinate after which each step is checked for droplet impingement on the airfoil. A value of 0.05 is typically used for XFIRST.

XLAST is the x-coordinate at which droplet trajectory calculations are terminated. This value typically rangea from 0.5 to 1.0 , and if the user lacks a good feel for the impingement characteristics to be expected for a given case, a value of 1.0 should be used for XLAST to ensure that no trajectoriea are terminated prematurely.

EPS is the maximum error allowed in the step integration process. A value of $1 . \times 10^{-5}$ is generally used for EPS.

Card 3 contains the variables $X O, U D O, Y O, V D O$, and $F R$, in format 5F10.5.

XO is the initial droplet $x$-coordinate. This is the $x$ value of the point at which the trajectory inveatigation is
begun, and it is typically given a value of -5.0 , representing a starting point five chord lengths in front of the airfoil. UDO is the initial droplet s-velocity with respect to freestream, and is commonly set equal to 1.

Yo is the initial droplet y-coordinate. This is the starting point for the first trajectory calculation only, and its value will depend on the section angle of attack. For a positive angle of attack, it is suggested that to maximize the chance of initial droplet impingement a slightly negative value of $Y O$ be used, perhaps approximately -0.05.

VDO is the initial droplet y-velocity with respect to freestream, and is generally equal to 0 in the undisturbed flowfield five or more chordlengths in front of the airfoil. FR represents Froude number. This parameter may be set equal to 0 to ignore the force of gravity on the droplet trajectories, and this ia commonly done.

Card 4 contains four variables, DYO, CON, ERRY, and HGT, in the format 4F10.6.

DYO is the increment in $Y O$ used by the program to vary the beginning droplet $y$-coordinate. Values of DYO are commonly on the order of .001 to .01 . The code has been set up to assign a value for DYO if the user inputs a value of $O$ for this parameter. The internally assigned value will change with section angle of attack. If the user inputs a DY value however, this value will be used regardless of the section angle of attack. When internally assigned, DYO ia given a value of 0.003 for section angles of attack less than or equal
to 4.6 degrees and 18 set equal to 0.005 for section anglea of attack greater than 4.6 degrees.

CON is a multiplication constant for the impingement efficiency curve output. It is generally set equal to 1. ERRY is the maximum error acceptable in total collection efficiency. A value of 2 (in percent) has commonly been uaed. HGT is a reference height in the $y$-direction used to compute total collection efficiency. If this parameter 1 a set equal to $O$ the code will calculate and use for HGT the projected height of the airfoil section.

This then completes the droplet trajectory section input.

Files 12 through 15 contain binary flowfield informetion which is output by the flowfield computation section of the code when airfoil coordinates (files 1 through 4) are input. Since this flowfield information is independent of angle of attack and velocity, it is necessary to run the code with airfoil coordinates input only once for a particular geometry. In subsequent runs using the same geometry, the flowfield information calculated and stored in the initial run is read and used by the code, thereby eliminating the need for recalculation of propeller flowfield information in each run. The user must only set up the computer job control language (JCL) appropriately to exercise this option. For example, in the initial run of a particular geometry, units 12 through 15 must be set up to have the binary flowfield data written to them. In subsequent runs these files must be established as
existing data files from which data is to be read. When previously computed airfoil flowfield information is used, it is not necessary to input airfoil coordinetea, and the flowfield calculation section of the code will be bypasaed.

File 16 contains input data for the propeller
performance calculations. This file contains data for the entire propeller, so no duplicate files are required for different radial locations. Data for a maximum of 15 radial locations may be input.

Card 1 of thia file containa the filght and atmospheric variables VO, TO, DD, $W$, TAUM, and ALTUDE in format 6F10.4.

VO ia the freestream velocity in miles per hour.

TO is the freestream temperature in degrees Fahrenheit.

DD represents the volume mean droplet diameter of the icing cloud in microns.
$W$ is cloud liquid water content in grams per cubic meter.

TAUM is the icing exposure time in minutes.
Finally, ALTUDE is the pressure altitude in feet at which the icing encounter occurred.

Card 2 contains the variables NUMBLD, NUMSTA, INCOMP and NUMADV in format $4 I 5$.

NUMBLD repreaents the number of propeller blades.
NUMSTA is the number of radial locations to be input by the user (maximum is 15.)

INCOMP is a compressibility check flag which provides the user with the option of obtaining either a comprepsible or

```
incompressible solution. If set equal to 0, a compressible
solution is obtained and if set equal to l, an incompresaible
solution is computed. Recall however that the flowfield
around the various propeller sections was computed using a
method which assumes imcompressible flow.
    NUMADV is the number of advance ratios to be input. Set
equal to O if edvance ratio ia to be calculated uaing input
RPM and freestream velocity. NUMADV may have a maximum value
of 10.
Card 3 contains the variables RADHUB, BLSET, RPM, CPDESI, DESIHP, and DIVIS in format 6F10.5.
RADHUB is the propeller hub radius in feet.
BLDSET representa the propeller blade aetting in degrees. If either CPDESI or DESIHP are asaigned values other than \(O\), the program will derive a BLDSET value based upon these input parametera.
RPM is the number of propeller revolutions per minute. If specified, RPM will be held constant. Otherwise RPM will be calculated based on the input freestream velocity and advance ratio. Either RPM or advance ratio must be apecified in the input.
CPDESI ia the design power coefficient. If set equal to O, power coefficient will be calculated based upon the input value of BLDSET.
DESIHP is the design horsepower, and if it is input as O, a value for DESIHP will be calculated based on the input BLDSET value.
```

DIVIS is a convergence aid variable for the design mode In which a BLDSET value must be computed. It ie generally assigned a value between . 001 and 1 . When BLDSET 18 specified, the DIVIS variable ia not employed.

Card 4 containa the variablea CORTIP, RADIUS, STUBLT, CDINT, and CORHUB in format 5F10.5.

CORTIP is the propeller tip chord in feet.

RADIUS is the propeller radius in feet.

STUBLT is the propeller stub length in feet.

CDINT ia a blade shank correction factor which takea into account interference from the blade shank. If a value for CDINT is unknown, a value of 0.567 should be used.

CORHUB represents the propeller hub chord in feet.

Card 5 is a title card which denotes the variables on the card to follow exactly as shown in the code input comments:
$\mathrm{R} / \mathrm{RAD}=\quad \mathrm{BLDANG}=\quad \mathrm{CHORD}=\quad \mathrm{T} / \mathrm{C}=\quad \mathrm{ALPHAO}=\quad \mathrm{CLD}=$ VDIS $=\quad \mathrm{RC}=$

Cards 6 through NUMSTA contain values for these variables, in format $8 F 10.6$.

XPR is the nondimensional radial location to be input. BLDANG is the propeller blade angle in degrees of this section. Generally this is just the twist distribution built into the propeller.

CHORD is the propeller section chord in feet.

TC is the section thichness to chord ratio.

ALO is the angle of attack for zero lift, in degrees, referenced to the longest chordline.

CLD is the section design lift coefficient.
VDIS represents the velocity dietribution seen by the section, taking into account nacelle effects. This parameter equals the local velocity divided by that of freestream.

Finally RC is the section leading edge radius of curvature, nondimensional with respect to section chord.

The final card(a), in format $F 10.6$, with one value per card, are ADVRAT values.

ADVRAT is the propeller advance ratio, and as many as 10 values of ADVRAT may be input by the user, with performance calculations performed by the code for each advance ratio. If NUMADV is set equal to 0 , such that advance ratio is to be calculated by the code given RPM and freestream velocity, no ADVRAT values should be input.

Thia completes the propeller performance section input.
File 17 contains input for options available to the user, including choice of aerodynamic coefficient correlation to be used as well as print options.

Card 1 is the title card for the run. This title is printed at the beginning of each section of calculations within the program and so should be general in nature, such as, "ENCOUNTER 2 FROM NEEL AND BRIGHT REPORT."

Card 2 contains the variables NCD. RADEXT, IREAD, IPRCOR, and NSECT in format I5,F10.5.3I5.

NCD is the aerodynamic coefficient correlation selector. A value of 1 for $N C D$ calls for the use of Bragg' a rime ice $\Delta C_{d}$ correlation. A value of 2 calla for the uae of Gray'a glaze ice $\Delta C_{d}$ correlation. A value of 3 calls for the use of Flemming'a $\Delta C_{i}$ and $\Delta C_{d}$ correlations for rime and glaze ice.

RADEXT is the fractional radial extent of icing.
IREAD is a flag which determines whether or not the code will calculate the flowfield at the input radial locations. The option is to use previously calculated and stored flowfield data. For each radial location input, the code will write the flowfield information to a file if it has not been computed in a previous run. Set IREAD equal to o to calculate the flowfield, or set it equal to 1 to use the previously computed data. The user must also be sure that the corresponding JCL for each of these files is set up correctly, i.e., if IREAD equals 0 the JCL muat be write-oriented, and if IREAD equals 1 the JCL for these files must be read-oriented. IPRCOR is a flag which indicates whether or not the user wishes to have the airfoil coordinates at each of the four input radial locations printed. An IPRCOR value of 0 instructs the code to not print the coordinates, and an IPRCOR value of 1 causea the coordinates to be printed. NSECT is a variable which indicates the number of radial locations to be input, with a maximum of four possible. Cards 3 through 7 contains values for the variable RSTA in format F10.5.


#### Abstract

RSTA is the nondimensional radial location at which flowfield calculations are to be performed. A maximum of four radial locations may be input. These radial locations must for the current version of the code be the same as any four of the radial locations input in the propeller performance section.

It may be shown that the curves of total and maximum local collection efficiencies are relatively amooth functions of radial location, so that four points should provide a reasonable representation of the variance of theae impingement efficienclea with radial location. However, it is obvious that the fewer points one uses to fit these curves, the less accurate they will tend to be. It is recommended therefore that whenever posaible, no less than the maximum of four stations should be input to maximize the available accuracy in the impingement efficiency calculationa, which then affect the propeller performance computations for the iced configuration. This then completes the required input to the code.


## APPENDIX 3

## SAMPLE CASE

The following output was produced for Encounter 2 at an advance ratio of 1.18 . The iced calculations were performed uaing Bragg'a older, modified correlation. Thia aame format is output regardless of the user's choice of correlation. Use of the Bragg or Gray correlations will generate output similar in appearance to that shown here. When the Flemming correlations are used, no flowfield or droplet trajectory information is printed out.

The code requires approximately 570K of computer memory. Typical run times are roughly five minutes (CPU time) on an IBM 370 and 33 seconds on a CRAY-1S for one advance ratio using either the Bragg or Gray correlations. Use of the Flemming correlations significantly reduces run time because the flowfield and droplet trajectory calculation sections of the code are bypassed, resulting in typical run times for the Flemming runs of 8.4 seconds on the IBM 370 and 0.57 seconds on the CRAY-1S.

| $m$ |  |  | 0090090 |
| :---: | :---: | :---: | :---: |
|  | －000000 | Mnmintoo | 0090090 |
| $\bigcirc$ | U000600 | counaooo | InO00000 |
| 11 | －000000 | onnNomom | 0009090 |
| $\ldots$ | ONOOQOO | －6manさo | －NOOOOO |
| － | －omminco | ©0ヶnHro | －OHMina |
| 0 | ©OOOOOO | 0000000 | $\dot{000000}$ |
| $\cdots$ |  |  |  |
| ய | 0000000 | 1000000 | －000000 |
| $\vdash$ | U000000 | Mさかん， | 1000000 |
| $\checkmark$ | Noodooo | maOntMr | No60090 |
| Z | －00untur | NrmmNMo | －00intinun |
| H0 | －manへへN | がmamさn | OHのNへNN |
| 0 | －00Nポか | OOMmHmo | －00Nい6か |
| $\boldsymbol{\alpha}$ | －－．－ |  | －••••• |
| 0 | 0000000 | － | OOOOOOO |
| UON |  |  |  |



## 



OOOOQOO OOOOQOO GOOOOOO Foodo 0 oonmmNa
－0்○்க்

OOODOOO N000000 NoOOOOO ©minNunin OMNNNNN
－000000

200000






 －


| Nommout | OOOOOOO |
| :---: | :---: |
| NNいM去 | OOCOOOO |
| 心Nmさルmin | UOOOOOO |
| －6， $0 \times \infty$ | Nountume |
| NM， 0 ¢ ${ }^{\text {No }}$ | ONNNNNO |
| OHrn－o | OON＋ 0 の |



Nommomm owntocco $0 \infty \mathrm{HN} \boldsymbol{\omega} \boldsymbol{\infty}$
 00 OHANHO

wNNM OMm士 $5 N N O M O$ Nホホホ MNNMNNON － 0 ONNNM © 0 Hmmみo 0000000

ooooooo


000000
n000080 승으웅웅 omnNonN

－000000
©OOOOOOO U000000 ondininino OINMNMNM －© © © © ©
 60000 0 OOOOOOO omminmon

1111111
unownonn ingmamon NMいさMいま $\infty 60$ ation
 OOOCOOD 0000000
－ 00000 NH osoacno
 indownown oonow oNo 0monowin $000 \infty 00$

60mmmmu NN $N$ N $\cap \infty \infty$ MNoNーかN rovin or th ONONOMM $\therefore \dot{\circ} \dot{\circ} \dot{\circ}$－ 0000000


$0.10 D-05$
0.
0.

[^0]$$
\text { FILE } 16 \text { INPUT: }
$$

> FILE 17 INPUT:
> NEEL \& BRIGHT DATA, BRAGG CORRELATION
NEEL \& BRIGHT DATA, BRAGG CORRELATION
TEMPERATURE $=1.00$ DEG. F DROPLET DIAMETER $=18.00$ MICRONS
ATMOSPHERIC AND FLIGHT CONDITIONS:
FREESTREAM VELOCITY= 0.00 MPH
LIQUID WATER CONTENT $=0.41 \mathrm{G} / \mathrm{M} 3$
RADIAL ICING EXTENT $=0.70$
PROPELLER PERFORMANCE AND DESIGN SECTION

PROPELLER SECTION CHARACTERISTICS:
ADVANCE R.iIIO $=0.900$
PROPELLER CHARACTERISTICS :


| $R / R A D=$ | BLDANG= | CHORD= | $T / C=$ | ALPHAO $=$ | $C L D=$ | VDIS $=$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.2000 | 52.900 | 0.5130 | 0.6175 | $-3.998$ | 0.7900 | 0.85500 |
| 0.2500 | 47.200 | 0.6750 | 0.3900 | $-3.790$ | 0.7420 | 0.89750 |
| 0.3000 | 41.700 | 0.7750 | 0.2790 | $-3.513$ | 0.6930 | 0.92500 |
| 0.4000 | 32.800 | 0.9020 | 0.1715 | $-3.167$ | 0.6080 | 0.95000 |
| 0.5000 | 27.000 | 0.9290 | 0.1250 | $-2.720$ | 0.5250 | 0.96500 |
| 0.6000 | 23.200 | 0.8780 | 0.0975 | $-2.318$ | 0.4480 | 0.97750 |
| 0.7000 | 20.300 | 0.7720 | 0.0798 | -2.035 | 0.3820 | 0.98900 |
| 0.8000 | 17.800 | 0.6320 | 0.0725 | $-1.810$ | 0.3420 | 0.99850 |
| 0.9000 | 16.600 | 0.5280 | 0.0710 | $-1.721$ | 0.3300 | 0.99930 |
| 0.9500 | 15.900 | 0.3980 | 0.0695 | $-1.714$ | 0.3260 | 0.99950 |
| 0.9750 | 15.700 | 0.2460 | 0.0676 | $-1.699$ | 0.3210 | 0.99990 |



| 11 |  |
| :---: | :---: |
| $\underline{z}$ |  |
| 文 | กnamomonnNT |
| $\boldsymbol{\alpha}$ |  |

$C L / C D=$
かMーかONMかさす。
NGNommomno
Mom 0 N $\infty$ MNont
m七心納



Oominon
oongratmoummoo
 O $0^{\infty}$
 ONLNNONHONOHNo
 0
｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜｜\｜


## BY T．L．MILLER

LAST UPDATE 2／18／86





 00000000000000000000

जMmoOOOOOOOOOOOOOOQO
 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ Mo 엉ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 00000000000000000000





 000000000000000000000 00000000000000000000




 긍ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㄱㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ․ 00000000000111000010


$$
0
$$

$$
\begin{aligned}
& 0 \\
& 000 \\
& \text { NOO } \\
& 00 \\
& 10+
\end{aligned}
$$

$\qquad$ $0 \rightarrow 0$

| 46 |  |
| ---: | ---: |
| 1.0000 |  |
|  | -0 |（NE）$=$

CH，TAU，KAPA：YZ
CH，TAU，KAPA：
CH，TAU，KAPA：
号に
号に ．．mó ．．mó $C:$
31
ERO
79 $C:$
31
ERO
79



 $E P S O$（ANGLE
$C L=2$. $E P S O$（ANGLE
$C L=2$. $E P S O$（ANGLE
$C L=2$.PHA$)$

○OOOOOOOOOOOOOOOOOOO1111FOURIER COEFFICIENTS C（J）＝（A（J）＋I

EPSO

NELL \& BRIGHT DATA, BRAGG CORRELATION

## $R / R=0.30$

## ADVANCE RATIO =0.900

XLAST $=1.000000$
VO $=0.00000$.
HGT $=0.000000$
$\stackrel{1}{\leftarrow}$









 メ




$$
A K=
$$

 응ㅇㅇㅇ응응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
앙
DROPLET TRAJECTORY INPUT

$$
M S P L=0
$$

## .94

 .00000 ALPHA=$$
\mathrm{UDO}=
$$

$\mathrm{CON}=$
 0

 $\dot{\circ}$









| z |
| :--- |
| $\mathbf{2}$ | PROPELLER ICING ANALYSIS PROGRAM

$\qquad$





 00000000000000000000000000000000000000000


 10000000000000000000000000000000111111111110






 ©OOOOOOOOOOOOOOQOOOOOOOOQOOOOO


ロロロロロロロロロロロロロロロロロロロロロロー
 ज00000000000000000000000（1）

 tutttutMmmmiMmMMMMMMMMM





 MovinonantutnNoonmonomenno




00000000000000000000000
TANGENT TRAJECTORIES：
SF（UPPER）$=0.836704 D-01$
 ص000000000000000000000000000000 001111111

 ＊OGN

 －NNNNNONMnNo
 0000000000000000000000000000

응머멍멈ㅁㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
O1！







응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ윰FTVNT 00000000000000000000000010000100
 にすNo
 moumovavoanmunimoooonthrin minvo



 응응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ

 かめ－゙ッonのomno


 000000000000000000000000000000 11111111111111111111111











 1





























 00000000000000ㅇㅇㅇㅇㅇㅇㅇ




 mmmmNNNNNMFHHFONOTMNHN 00000000000000000000
-0000000000000~THTNHTNO 00000000000000 Ninioiniop -





 0000000000000000000000






 $0000000000000000000000^{\circ}$

응잉ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ

 omootammyonncoinm-omnominin
 MNNNNNNNNNNNNNNNNNNNNNN mmmmmmmmmmmmmmmmmmmmmmm -00000000000000000 111111111,11111111ipi11
ADVANCE RATIO＝0．900
CLARK－Y COORDINATES，R／R＝0． 5
 011767必经응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ जnio응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 능응응응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 응응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 응응응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ioopooopooopioiooopo
NMOOOOQ000000000000 nmoodono00000000000 mo 00000000000000000000 N0ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ioiooioiooioioioioioooc


Numoo000000000000000 NNM응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ NUOOOOOOOOOOOOOOOOOO
约응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ m응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 000000000000000000000001100 ＊
 जnvoio00000000000000 げ，ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ NH응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ M응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ M응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
 $-0$
00000008

## CH，TAU，KAPA：

 T，XZ $\infty$N
N
N
X
$\cdots$




00000000円m

 N\＆NHMNMINOGN
 $m$ in inntivnan ナMMNNHmm＠
0000000000
OOODOOOQOQ の日のロローロロの日 NumNundwNon













ロロのロロロロのーか
山保 0 onoono
سшшшшшшіш1， 1







OOOOOOOOOOOOOOOOOOOO












 000 O O O O O O O O O O O O O O inthininininininininininintinnintint


 リNの，ONom MNoNODN rommonn いのmNMホの ©000000 1111111
 Oロロの日ロの

 Hoostrn NNMmmmN

 の日のかのロの ナ以上NN世 NHOONNT $\infty N \sim M \rightarrow 0 \rightarrow$ かNNMUN゙ー in in in untino －000000

## H－NNHAT

 $\begin{array}{llll}0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1\end{array}$ MはNNNmか MNNはNが心 ○MNさNがH No ovかOo けNがNM以 0000000 0000000
－ロロの日ロの MNo $0000 \sim 1$ －Monoono NaNホMかN寸レNかの心が NNNN －000000

## -1 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0

90000000090
00000000 ロロロロロロロロロロロ $000000000 \infty 1$ 90000000000 60000000untm InOMOMOLOMNO
 NNNNNNNNNNm ©OOOOOOOOOO

 00000000000 00000000000 0000000000000000000000
 intninturnintuintin



7－12HMHO
白白！́！



 vintionrunrnu 0000000 のnの 111111101404 옹ㅇㅇ․ 0000000 ш

 Mホ ONmNHFNHト
 TrANNホGQona
－000000


ज0
 0011111111
 *






 $\begin{array}{lllllll}0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0\end{array}$






















 O $>-\infty$ 人
















 NNの






 NーGONけ


 1
0000000000000000000000000000000000000000000







 OOOOOOOOOOOOOOOOOOOOQOOOOOOOOOOOOOOOOOOOOO
Nonㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ N1ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ Nㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅁ응 승ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ응 Nㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 00000000000000000000 000000000000000000000 レー.1 •1 1


$R / R=0.70$
ADVANCE RATIO $=0.900$
TRAJECTORY CALCULATION GENERAL INPUT：
TEXAS A\＆M UNIVERSITY \＆SVERDRUP TECHNOLOGY，INC．
 にMMNM ONOONUN
 000000000000
 amoninmosarom
 THNNNNNNNNNN $000000000000^{\circ}$


$\stackrel{\Perp}{レ}$ Novinanかuminm
 inininininininninnin ○OOODOOOOOOO

MNNNHHTHaNov $000000000000^{\circ}$
 NGNNOMGMOOOM
 o心NmNoOONUOL
 $000000000000^{\circ}$
N MNNWHWH－wn＋w



 nenconanominamionNownN

 そくれそれく
anaraqNoMNNHFHNHFOOO шшшшшші



 0 － 00000000000 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
 ㅇㅇOOOOOOOOOOOOOOOQO 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ ＞ onnmuincinonimmoinolnaino
 1111111111111111111111

ดดดดดดดดดดดดดดดดดดดด ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ $\times 0$ ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
 00000000000000000000
 סóóoóoóoocooocóoo


OOOOCOOOOOOOOO 90006000000006 ロロロロロロロロロロロロロロ O00060000 000000000 min on in
 NMMt ininwo 0 on in in NNNNNNNNNNNFHー
 OOOOOOOOOQOQOO
 OOOOGOOOOOOOOQ

 0000000000000000 OOODOOOOOOOOOO 90000000000000000 ninunutumuntuntuntintin 00000000000000
 HrymrnNNN

OOOQOQOOOOQOOGOOROOOOOO





 NNNNNNNNNNNNNNHMmHFHmHF




 ONM o O 士 N NO ONMMOO


 00000000000000000000000 $\begin{array}{llllllllllllllllll}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{array}$
tangent trajectories：


620 0
00
$1 T=0.147522 \mathrm{D}$ 0



OQ



 नNm,


0000000000000
¢










































 O о о





















NEEL \＆BRIGHT DATA，BRAGG CORRELATION
.88851081
00247764
$\begin{array}{r}0 \\ \hline 8\end{array}$

## BY T．L．MILLER



 modigoㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ MODOㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ － 00 ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ oᄋOㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ



 NNㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ


 ¢ᄋOㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ － ， 11

ع0000000

$$
\begin{aligned}
& 214905 \\
& =1.48101880 \\
& =
\end{aligned}
$$

 ${ }^{*} \perp d 7^{*} I=r^{*}((f) 8$

$$
0 \text { ウウ乌0890.02 }
$$

$-0$ $C H, T A U, K A P A: ~ Y Z I N:$ 0.00056100 909500
FOURIER COEFFICIENTS C（J）＝（A（J）＋
TV000000000000000000
 ㄱㄱㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ 궁응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ
 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ iopoioiooooopioipoóoo
0m000000000000000000 ㅇNNㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇ

 응ㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅇㅡ 000융ㅇㅇㅇㅇㅇ000，0000


$$
\dot{O} \dot{0} 000000000000
$$








 ＜00000000000000000000火曰O O O O O O O O O O O O O O O O O O O O O




| $\frac{2}{2}$ |  |
| :---: | :---: |
|  |  |
|  |  |





OOOOOOOOOQODOQOOODOOOOOOODOOO




 NNNNNNNNNNNNNNNNHmmmmmmanmanmm












 Q00000000000000000000000000000000000 00111111111111111

 *





0000000000000000000000000000000000
01111111111111






 6000000





 0000000000000000000000000000000000 111111111111111111111111111111111111 00000000000000000000000000000000000
人Hoowowo
 ommmonunormnoninnonmwonmNonunoninnonin
 NNNNNNNNNNNMNNNNNNNNNNNNNNNNNNNNHE 0000000000000000000000000000000000



















 8080000000 ナームmーo
















 0000000000000000000011







 000000000000000000001
以MनNN以





 GOOOOOOOOOGOOOOOOOOOO





 0000000000000000000
 000000000000000000000






 ©OOOOOOOOOOOOOOOOOOOO 111111111111111111111
0.883 USING BRAGG CORRELATION 0.863 USING BRAGG CORRELATION NOII甘lヨy४OS 9อ४४g ONIS $526^{\circ} \mathrm{O}$ NOIL甘7ヨyy00 9ovyg onisn sゅI•I 1.545 USING BRAGG CORRELATION 2．202 USING BRAGG CORRELATION
 5．066 USING BRAGG CORRELATION
 10．925 USING BRAGG CORRELATION
 AT $R / R=0.200$ DELTA－CD $=$ AT $R / R=0.250$ DELTA－CD＝ AT R／R＝0．300 DELTA－CD＝ delta－cd＝ delta－cd＝ delta－cD＝ DELTA－CD＝ DELTA－CD＝ DELTA－CD＝ DELTA－CD＝ DELTA－CD＝ AT $R / R=0.400$
AT $R / R=0.500$
AT $R / R=0.600$
AT $R / R=0.700$
AT $R / R=0.800$
AT $R / R=0.900$
AT $R / R=0.950$
AT $R / R=0.975$

PROPELLER SECTION CHARACTERISTICS :



PROFELIER OPERATION CHARACTERISTICS :





[^1]Postage and Fees Paid National Aeronautics and Space Administration NASA-451


[^0]:    FILE 8 INPUT:

    | FILE 8 | INPUT: |  |
    | :--- | :--- | :--- |
    |  |  |  |
    | DROPLET |  |  |
    | 1 | 0 | -0.05 |
    | -5.0 | 1. | -0.10 |
    | 0.0 | 1. | 2. |

[^1]:    *For sale by the National Technical Information Service, Springfield, Virginia 22161

