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ANALYSIS OF SHELL-TYPE STRUCTURES SUBJECTED TO TIME-DEPENDENT MECHANICAL AND THERMAL LOADING

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INTRODUCTION

The objective of the present research is to develop a general mathematical model and solution methodologies for analyzing structural response of thin, metallic shell-type structures under large transient, cyclic or static thermomechanical loads. Among the system responses, which are associated with these load conditions, are thermal buckling, creep buckling and racheting. Thus, geometric as well as material-type nonlinearities (of high order) can be anticipated and must be considered in the development of the mathematical model. Furthermore, this must also be accommodated in the solution procedures.

SUMMARY OF PROGRESS

The progress to date has been elaborated upon in an interim scientific report submitted to the sponsor during the summer of 1986, and in a series of semiannual progress reports. The most recent of these is dated April, 1988.

A complete true ab-initio rate theory of kinematics and kinetics for continuum and curved thin structures, without any restriction on the magnitude of the strains or the deformation, was formulated. The time dependence and large strain behavior are incorporated through the introduction of the time rates of the metric and curvature in two coordinate systems; a fixed (spatial) one and a convected (material) coordinate system. The relations between the time derivative and the covariant derivatives (gradients) have been developed for curved space and motion, so that the velocity components supply the

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connection between the equations of motion and the time rate of change of the metric and curvature tensors.

The metric tensor (time rate of change) in the convected material coordinate system is linearly decomposed into elastic and plastic parts. In this formulation, a yield function is assumed, which is dependent on the rate of change of stress, metric, temperature, and a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermoelastic part of the deformation.

A time and temperature dependent viscoplastic model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by phenomenological thermodynamic considerations.

The nonisothermal elastic-viscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators (in the area of viscoplasticity) employ plastic strains as state variables. Our study shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed by all previous investigators lead to the condition that all plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice, which, when phenomenologically considered, causes hardening. Both limitations are not present in our formulation, because of the inclusion of the "thermodynamic state" equations.

The obtained complete rate field equations consist of the principles of the rate of the virtual power and the rate of conservation of energy, of the constitutive relations, and of boundary and initial conditions. These formulations provide a sound basis for the formulation of the adopted finite element solution procedures.

The derived shell theory, in the least restricted form, before any simplifying assumptions are imposed, has the following desirable features:

(a) The two-dimensional, impulse-integral form of the equations of motion and the Second Law of Thermodynamics (Clausius-Duhem inequality) for a shell follow naturally and <u>exactly</u> from their three-dimensional counterparts.

(b) Unique and concrete definitions of shell variables such as stress resultants and couples, rate of deformation, spin and entropy resultants can be obtained in terms of weighted integrals of the three-dimensional quantities through the thickness.

(c) There are <u>no</u> series expansions in the thickness direction.

(d) There is <u>no need</u> for making use of the Kirchhoff Hypotheses in the kinematics.

(e) <u>All</u> approximations can be postponed until the descretization process of the integral forms of the First Law of Thermodynamics

(f) A by-product of the descent from three-dimensional theory is that the two-dimensional temperature field (that emerges) is not a through-the-thickness average, but a true point by point distribution. This is contrary to what one finds in the literature concerning thermal stresses in the shell.

To develop geometrically nonlinear, doubly curved finite shell elements the basic equations of nonlinear shell theories have to be transferred into the finite element model. As these equations in general are written in tensor notation, their implementation into the finite element matrix formulation requires considerable effort.

The nonlinear element matrices are directly derived from the incrementally formulated nonlinear shell equations, by using a tensor-oriented procedure. The classical thin shell theory based on the Kirchoff-Love hypotheses (Formulation D in Appendix A) was employed for this purpose. For this formulation, we are using the "natural" degrees of freedom per mid-surface shell node: three incremental velocities and the rates of rotations about the material coordinates in a mixed form.

A description of the developed element and related finite element code are given in Appendix B. This exposition provides information concerning the formulation, the finite element and how it is employed in the solution of shell-like configurations. Moreover a complete description including program flow chart, listing, input instructions to the user and explanation of output are also included in Appendix B.

The quasi-linear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain V. In the present instance, the choice has been made in favor of a simple first order expansion in time for the construction of incremental solutions from the results of finite element spatial integration of the governing equations.

The procedure employed permits the rates of the field formulation to be interpreted as increments in the numerical solution. This is particularly convenient for the construction of incremental boundary condition histories.

Even under the condition of static external loads and slowly growing creep effects, the presence of snap-through buckling makes the inertial effects significant. In dynamic analyses, the applied body forces include inertial forces. Assuming that the mass of the body considered is preserved, the mass matrix can be evaluated prior to the time integration using the initial configuration.

Finite element solution of any boundary-value problem involves the solution of the equilibrium equations (global) together with the constitutive equations (local). Both sets of equations are solved simultaneously in a step by step manner. The incremental form of the global and local equations can be achieved by taking the integration over the incremental time step $t=t_{i+1}-t_i$. The rectangular rule has been applied to execute the resulting time integration.

Clearly, the numerical solution involves iteration. A simplified version of the Riks-Wempner constant-arc-length method has been utilized. This iteration procedure which is a generalization of the displacement control method also allows to trace the nonlinear response beyond bifurcation points. In contrast to the conventional Newton-Raphson techniques, the iteration of the method takes place in the velocity and load rate space. The load step of the first solution in each increment is limited by controlling the length ds of the tangent. Either the length is kept constant in each step or it is adapted to the characteristics of the solution. In each step the triangular-size stiffness matrix has to be checked for negative diagonal terms, indicating that a critical point is reached.

One of the most challenging aspects of finite strain formulations is to locate analytical solution with which to compare the proposed formulation. Typically, as a first problem, a large strain uniaxial test case was analyzed. The case considered examines the rate-dependent plastic response of a bar to a deformation history that includes segments of loading, unloading, and reloading, each occurring at varying strain and temperature rates. Moreover, it was shown that the proposed formulation generates no strain energy under a pure rigid body rotation. These are surely important demonstrations but they only represent a partial test because the principal stretch directions remain constant. Finally, a problem which was discussed by Nagtegaal and de Jong, and others too, as a problem which demonstrates limitations of the constitutive models in many strain formulation, is the Couette flow problem. This problem is solved as a third example. The results of these test problems show that:

- The formulation can accommodate very large strains and rotations.
- The formulation does not display the oscillatory behavior in the stresses of the Couette flow problem.
- The model incorporates the simplification associated with rate-insensitive elastic response without losing the ability to model rate temperature dependent yield strength and plasticity.

The problem of buckling of shallow arches under transient thermomechanical load was investigated next. The analysis was performed with the aid of 24 paralinear isoparametric elements. The paralinear isoparametric element is such that the thickness is small compared to other dimensions. The characteristics of the element are defined by the geometry and interpolation functions, which are linear in the thickness direction and parabolic in the longitudinal direction. Consequently, the element allows for shear strain energy since normals to a mid-surface before deformation remain straight, but not necessarily normal to the midsurface after deformation.

The developed solution scheme is capable of predicting response which includes pre- and post-buckling with thermal creep and plastic effects. The solution procedure was demonstrated through several examples which include both creep and snap-through behavior.

The last set of problems which are under investigation consists of creep or thermal buckling, with plastic effects, of shells of revolution.

In addition, following a more traditional approach, a method was developed for bounding the response (solution) of bars and beams of (linear) visoelastic material behavior, based on nonlinear kinematic relations.

In connection with the progress to date, two papers were published by the AIAA Journal in 1986 and 1987. Moreover, a paper entitled, "Non-Isothermal Elastoviscoplastic Analysis of Planar Curved Beams" was presented at the 3rd Symposium on Nonlinear Constitutive Relations for High-Temperature Applications, held at the University of Akron, on June 11-13, 1986. A descriptive abstract of this paper was published in the meeting proceedings and the full paper appeared in a special publication (NASA CP 10010). Copies of the above have been sent to the sponsor.

In addition, the two papers presented at the 28th AIAA/ASME/ASCE/AHS SDM Conference and published in the proceedings of this conference, have been accepted for publication; both by the AIAA Journal. These two papers deal with applications to snap-through and creep buckling of bars and arches. Most of this work was also presented at the NASA-Lewis Conference on Structural Integrity and Durability of Reusable Space Propulsion Systems on May 1987 in Cleveland, Copies of these papers will be forwarded to the Sponsor, as soon as they appear in print.

In connection with the more traditional approach a paper accepted for presentation and for publication in the Proceedings of the special Symposium on Constitutive Equations at the ASME Winter Annual Meeting, Chicago, IL., November 28 - December 2, 1988. The title of the paper is "Creep Analysis of Beams and Arches Based on a Hereditary Visco- Elastic- Plastic Constitutive Law". Copies of this paper have been forwarded to the sponsor in December, 1988.

Moreover, one paper was published in the the AIAA/ASME/.../AHS 30th SDM Conference. A copy of this paper is attached, herewith.

Finally, a finite element has been developed and it is currently being tested for a less restricted shell formulation (Formulation C; see Appendix A). A description of this newly developed element and the related finite element code will be submitted to the sponsor as soon as the testing is completed and reliable results have been obtained.

FUTURE TASKS

The main thrust of the additional tasks is to develop a finite element and select a code, which will be made available to all users and which will be based on the most general (but practical) nonlinear shell formulation possible and nonlinear constitutive relations to predict the response of shell-like structures, when subjected to time-dependent thermomechanical loads with large excursions.

This should be completed in the next six months and the complete package will be delivered to the sponsor.

APPENDIX A

The various shell theory approximations (formulations) are based on the use of certain simplifying assumptions regarding the geometry and kinematics of the shell configuration.

These are:

Assumption I: The material points which are on the normal to the reference surface before deformation will be on the same normal after deformation.
Assumption II: The shell is sufficiently thin so that we can assume linear dependence of the position of any material point (in the deformed state) to the normal (to the reference surface) coordinate (in the deformed state). The linear dependence can easily be changed to parabolic, cubic, or any desired degree of approximation.
Assumption III: The rate of change of the velocity gradients with respect to in-plan coordinates on the two boundary shell surfaces is negligibly small.
Assumption IV: The rate of change of the distance of a material from the reference surface is negligibly small.

On the basis of the above four simplifying assumptions, several formulations result, for the analysis of thin shells.

Formulation A: This formulation makes use of Assumption I, only.
Formulation B: This formulation employs Assumptions I and II.
Formulation C: This formulation employs Assumption I, II and III.
Formulation D: This is the classical thin shell theory based on the Kirchhoff-Love hypotheses of Assumptions I, II, III, IV, as applied to large deformation theory.

These formulations are arranged in such a manner that we move from the least restrictive (A) to the most restrictive (D).

In addition to this a fifth formulation (E) can easily be devised and this formulation in terms of order of restriction is similar to Formulation A. Formulation E makes use of Assumption II only.

APPENDIX B FINITE ELEMENT AND RELATED CODE FOR FORMULATION D

In this Appendix, a description of the developed finite element and the related finite element code is presented. First, the essentials of the element are described and then the complete solution procedure is presented with sufficient detail. This includes a flow chart, a line by line listing of the computer program, input data information, and explanation of the output.

B.1. THE SHELL ELEMENT

A brief description highlighting the essential features of the shell element development and the related code used in this work is given here.

In order to derive discrete algorithm based on the finite element displacement method we approximate the velocity field by index-oriented notation, which allows the separate representation of the shape functions (the specific expression depends on the decided upon degress - of - freedom, Lagrangian, Hermitian, etc.) for the tangential velocities v_α and for the normal velocity v_3 .

$$v_\alpha = v_\alpha^M V_M = V^M v_{\alpha M} \tag{1}$$

$$v_3 = v_3^M V_M = V^M v_{3M} \tag{2}$$

Upper indices imply the columns, lower indices the rows of a matrix expression, and the summation is carried out spanning the number of degrees-of-freedom. V^M and V_M represent, therefore, the vector of nodal velocities by the row and by the column respectively. We get the shape functions for the partial derivatives of the velocity shape functions v_α^M , v_3^M :

$$v_{\alpha,\beta} = (v_\alpha^M)_{,\beta} V_M = v_{\alpha,\beta}^M V_M \tag{3}$$

$$v_{3,\beta} = (v_3^M)_{,\beta} V_M = v_{3,\beta}^M V_M \tag{4}$$

The main idea of this formulation is the development of shape functions for further mechanical and thermal variables by the application of well-known tensor procedure on the basic shape functions (3) and (4). Taking, for example, the covariant derivative

$$v_{\alpha;\beta} = v_{\alpha,\beta} - v_\lambda \Gamma_{\alpha\beta}^\lambda \tag{5}$$

and inserting (3) and (1), we can define the shape function of the covariant derivative:

$$v_{\alpha;\beta} = \underbrace{(v_{\alpha,\beta}^M - v_\lambda^M \Gamma_{\alpha\beta}^\lambda)}_{v_{\alpha;\beta}^M} V_M = v_{\alpha;\beta}^M V_M \tag{6}$$

In the same way we receive the shape function of the internal variables, for example the rate of deformation and the spin tensors:

$$d_{(\alpha\beta)} = \frac{1}{2}(v_{\alpha;\beta} + v_{\beta;\alpha} - 2b_{\alpha\beta} v_3) =$$

$$\underbrace{\frac{1}{2}(v_{\alpha;\beta}^M + v_{\beta;\alpha}^M - 2b_{\alpha\beta} v_3^M)}_{d_{(\alpha\beta)}^M} V_M = d_{(\alpha\beta)}^M V_M \tag{7}$$

$$\omega_{(\alpha\beta)} = -\left(v_{3;\alpha\beta} + v_{\lambda;\alpha}\, b^{\lambda}_{\beta} + v_{\lambda;\beta}\, b^{\lambda}_{\alpha} + v_{\lambda}\, b^{\lambda}_{\alpha;\beta} - v_{3}\, b_{\lambda\alpha}\, b^{\lambda}_{\beta}\right) =$$

$$-\underbrace{\left(v^{M}_{3;\alpha\beta} + v^{M}_{\lambda;\alpha}\, b^{\lambda}_{\beta} + v^{M}_{\lambda;\beta}\, b^{\lambda}_{\alpha} + v^{M}_{\lambda}\, b^{\lambda}_{\alpha;\beta} - v^{M}_{3}\, b_{\lambda\alpha}\, b^{\lambda}_{\beta}\right)}_{\omega^{M}_{(\alpha\beta)}} V_{M} = \tag{8}$$

$$= \omega^{M}_{(\alpha\beta)}\, V_{M}$$

The same procedure is now applied to the shift tensor

$$\mu^{-1}{}^{\Lambda}_{\Gamma} = \left(\delta^{\gamma}_{\beta} + \xi_{0}\, b^{\gamma}_{\beta}\right) \delta^{\beta}_{\Gamma}\, \delta^{\Lambda}_{\gamma} + \delta^{3}_{\Gamma}\, \delta^{M}_{3} \tag{9}$$

which is responsible for the "exact" distributions over the thickness.

Finally the shape functions of the internal forces and temperature variables can be derived from the shape functions of the rate of deformation and spin tensor via the constitutive relations; for example:

$$N^{(\alpha\beta)} = \underbrace{D\, H^{\alpha\beta\rho\lambda}\, v^{M}_{(\rho\lambda)}}_{N^{(\alpha\beta)M}} V_{M} = N^{(\alpha\beta)M}\, V_{M} \tag{10}$$

All these expression are now substituted into the rate of the first law of thermodynamics to obtain the element "stiffness" equations. The developed element matrices are implemented into the global relation of the complete shell structure by standard assemblage process considering incidence and boundary conditions.

The present curvilinear formulation of the element enables the precise description of geometry, external loads and temperatures and the fulfillment of the convergence criteria, while the rigid body motion condition can only be satisfied in an approximate manner. The tensor oriented formulation renders the optional use of various shape functions for the tangential velocities v and the normal velocity v_3.

The shell element which have been used up to today is based on the bicubic Hermite polynomial with 4x12 generalized velocities and 4 temperatures. Numerical integration spanning the element domain was applied (16 points of integration), whereby area and boundary integrals were replaced by double integration with respect to the curvilinear θ^{α} - coordinates.

$$dA = \sqrt{a}\; d\theta^{1}\, d\theta^{2} \tag{11}$$

$$dS = \sqrt{a_{\alpha\beta}\, d\theta^{\alpha}\, d\theta^{\beta}} \tag{12}$$