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# Demonstration Optimization Analyses of Pumping from Selected Arapahoe Aquifer Municipal Wells in the West-Central Denver Basin, Colorado, 2010–2109

Scientific Investigations Report 2012–5140

**U.S. Department of the Interior**
**U.S. Geological Survey**

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Suggested citation:
Banta, E.R., and Paschke, S.S., 2012, Demonstration optimization analyses of pumping from selected Arapahoe aquifer municipal wells in the west-central Denver Basin, Colorado, 2010–2109: U.S. Geological Survey Scientific Investigations Report 2012–5140, 37 p.

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# Figures

# Tables

# Conversion Factors

## Inch/Pound to SI

| Multiply | By | To obtain |
|---|---|---|
| | Length | |
| foot (ft) | 0.3048 | meter (m) |
| mile (mi) | 1.609 | kilometer (km) |
| | Area | |
| square mile ($mi^2$) | 2.590 | square kilometer ($km^2$) |
| | Volume | |
| cubic foot ($ft^3$) | 0.02832 | cubic meter ($m^3$) |
| | Flow rate | |
| cubic foot per second ($ft^3/s$) | 0.02832 | cubic meter per second ($m^3/s$) |
| cubic foot per day ($ft^3/d$) | 0.02832 | cubic meter per day ($m^3/d$) |
| acre-foot per year (acre-ft/yr) | 1,233 | cubic meter per year (m3/yr) |

Vertical coordinate information is referenced to the North American Vertical Datum of 1988 (NAVD 88).

Altitude, as used in this report, refers to distance above the vertical datum.

# Demonstration Optimization Analyses of Pumping from Selected Arapahoe Aquifer Municipal Wells in the West-Central Denver Basin, Colorado, 2010–2109

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## Abstract

Declining water levels caused by withdrawals of water from wells in the west-central part of the Denver Basin bedrock-aquifer system have raised concerns with respect to the ability of the aquifer system to sustain production. The Arapahoe aquifer in particular is heavily used in this area. Two optimization analyses were conducted to demonstrate approaches that could be used to evaluate possible future pumping scenarios intended to prolong the productivity of the aquifer and to delay excessive loss of saturated thickness. These analyses were designed as demonstrations only, and were not intended as a comprehensive optimization study.

Optimization analyses were based on a groundwater-flow model of the Denver Basin developed as part of a recently published U.S. Geological Survey groundwater-availability study. For each analysis an optimization problem was set up to maximize total withdrawal rate, subject to withdrawal-rate and hydraulic-head constraints, for 119 selected municipal water-supply wells located in 96 model cells. The optimization analyses were based on 50- and 100-year simulations of groundwater withdrawals.

The optimized total withdrawal rate for all selected wells for a 50-year simulation time was about 58.8 cubic feet per second. For an analysis in which the simulation time and head-constraint time were extended to 100 years, the optimized total withdrawal rate for all selected wells was about 53.0 cubic feet per second, demonstrating that a reduction in withdrawal rate of about 10 percent may extend the time before the hydraulic-head constraints are violated by 50 years, provided that pumping rates are optimally distributed.

Analysis of simulation results showed that initially, the pumping produces water primarily by release of water from storage in the Arapahoe aquifer. However, because confining layers between the Denver and Arapahoe aquifers are thin, in less than 5 years, most of the water removed by managed-flows pumping likely would be supplied by depleting overlying hydrogeologic units, substantially increasing the rate of decline of hydraulic heads in parts of the overlying Denver aquifer.

## Introduction

The Denver Basin bedrock aquifers are a primary source of water for much of the population of the southern part of the Denver metropolitan area and adjacent rural areas. Because of the slow rate of natural recharge, groundwater in the Denver Basin commonly is regarded for practical purposes as nonrenewable. Municipal-well pumping in the west-central part of the basin has contributed to water-level declines and, in some areas, declining well yields. Concerns related to the rates of water-level decline and the longevity of the usefulness of the aquifers (Topper and Raynolds, 2007) have prompted investigations (Hydrosphere Resource Consultants, Inc., and others, 1999; Black and Veatch and others, 2003) with the goal of prolonging productivity of the aquifers for beneficial use. A recent report on groundwater availability (Paschke, 2011) presents a three-dimensional groundwater flow model of the Denver Basin and describes the decline of water levels in wells in these areas as the rate of withdrawal of water from wells in the area has increased.

The U.S. Geological Survey (USGS), in cooperation with the Colorado Water Conservation Board (CWCB), developed a series of groundwater-flow simulations to demonstrate how optimization techniques could be used to design pumping schemes that may prolong the productivity of the Arapahoe aquifer. These analyses were designed as demonstrations only and were not intended as a comprehensive optimization study. The optimization analyses were based on the groundwater-flow model of the Denver Basin bedrock aquifers and overlying alluvial aquifer described by Banta and others (2011). The analyses demonstrate the applicability of optimization technology to the problem of maximizing production of water while maintaining saturated thickness of the Arapahoe aquifer of at least 90 percent of the aquifer thickness for 50 or 100 years.

## Purpose and Scope

This report describes results of a groundwater-flow modeling investigation designed to demonstrate the application of optimization techniques to understand the effects of various possible future pumping schemes and to delay excessive loss of saturated thickness in the Arapahoe aquifer. The optimization goals were developed in cooperation with CWCB and the Colorado Division of Water Resources (CDWR). The investigation focused on pumping from the Arapahoe aquifer in the metropolitan area south of Denver in the west-central part of the Denver Basin. The other bedrock aquifers of the Denver Basin and the overlying alluvial aquifer were included in the modeling simulations because of the hydrologic connections among the aquifers. These analyses were designed as demonstrations only and were not intended as a comprehensive optimization study.

## Study Area

The project study area is the area underlain by the Denver Basin bedrock aquifer system, which occupies about 6,700 $mi^2$ in eastern Colorado (fig. 1). The metropolitan area south of Denver (hereinafter, the south Denver metropolitan area) is the main area of interest for this study because it contains a large number (119) of municipal wells completed in the upper and lower Arapahoe aquifers (fig. 2). For the purposes of this study, the south Denver metropolitan area is considered to include northern Douglas County and southwestern Arapahoe County. The groundwater model encompasses the entire study area; however, the optimization analyses only involve wells operated by municipal water providers in the south Denver metropolitan area. Two wells included in the optimization analyses are east of Denver and somewhat isolated from the other selected wells, but they belong to a water provider that operates wells within the area of greatest interest.

# Denver Basin Bedrock Aquifer System

As residential development has expanded beyond Denver, particularly to the south of Denver, newly developed areas have become increasingly dependent on water from Denver Basin bedrock aquifers. Estimated production from all wells completed in the Denver Basin bedrock aquifers increased from about 15 $ft^3/s$ in 1958 to about 41 $ft^3/s$ in 1978 (Robson, 1987) and about 118 $ft^3/s$ in 2003 (Paschke and others, 2011a). In 2003, municipal water providers in the study area withdrew an estimated 48 $ft^3/s$ (35,000 acre-ft/yr) from wells completed in the upper and lower Arapahoe aquifers, the aquifers of interest in this investigation (Paschke and others, 2011a). A detailed description of the hydrogeologic framework is provided by Paschke and others (2011b) and is summarized in the following paragraph.

The bedrock aquifers of the Denver Basin are in water-yielding sandstones of Cretaceous and Tertiary age in a 6,700-$mi^2$ area in eastern Colorado located between Greeley on the north, Colorado Springs on the south, Limon on the east, and the Rocky Mountain Front Range on the west (fig. 1). Rules relating to withdrawal of groundwater of the Denver Basin (Colorado Division of Water Resources, 1985) are based on a hydrogeologic framework in which four primary bedrock aquifers (the Dawson, Denver, Arapahoe, and Laramie–Fox Hills aquifers) are defined; the Dawson and Arapahoe aquifers locally are further differentiated into upper and lower aquifers (fig. 2, table 1). The aquifers generally are separated by confining units. The lower Dawson, Denver, and Arapahoe aquifers represent the synorogenic deposition of sediments in the Denver Basin during Laramide uplift of the Rocky Mountain Front Range (Raynolds, 2002). The prominent Wildcat Mountain alluvial fan mapped in the lower Arapahoe aquifer is also present in the Denver and lower Dawson portions of the sequence, and the Denver confining units are thin (mean thicknesses of 50 ft) compared to the thickness of the Denver aquifer (mean thickness of 740 ft). A confining unit separates the upper Arapahoe aquifer from the lower Arapahoe aquifer in approximately the northern one-third of the basin (fig. 3). In the southern two-thirds, the confining unit is absent and, for the purposes of this study, the Arapahoe aquifer is considered undifferentiated. Alluvial sediments of the South Platte River and numerous tributaries of the South Platte and Arkansas Rivers, some of which are ephemeral or intermittent, overlie substantial areas of the bedrock basin and, where saturated, form an unconfined alluvial-aquifer system. This investigation uses the same hydrogeologic framework as the groundwater-flow model described by Paschke and others (2011b).

# Numerical Model

For the optimization analyses, the MODFLOW-2000 (Harbaugh and others, 2000) model of the Denver Basin aquifer system (Banta and others, 2011) was converted to a MODFLOW-2005 (Harbaugh, 2005) format. The finite-difference model grid (fig. 4) encompasses the Denver Basin bedrock-aquifer system and immediately adjacent areas of the alluvial aquifer of the South Platte River (fig. 2). The grid has 84 columns and 124 rows of square cells; each cell represents a 1-mile by 1-mile area. In the vertical dimension, the aquifers and confining units are represented by 12 model layers, as indicated in table 1. Details of model conceptualization and parameterization are fully described in Banta and others (2011). This section only describes aspects of the model that are particularly relevant to the optimization analyses or that differ from the model described in that report.

The calibration period for the Denver Basin groundwater model extended from 1880 through 2003 (Banta and others, 2011). In that model, one MODFLOW-2000 (Harbaugh and others, 2000) steady-state stress period was used to simulate

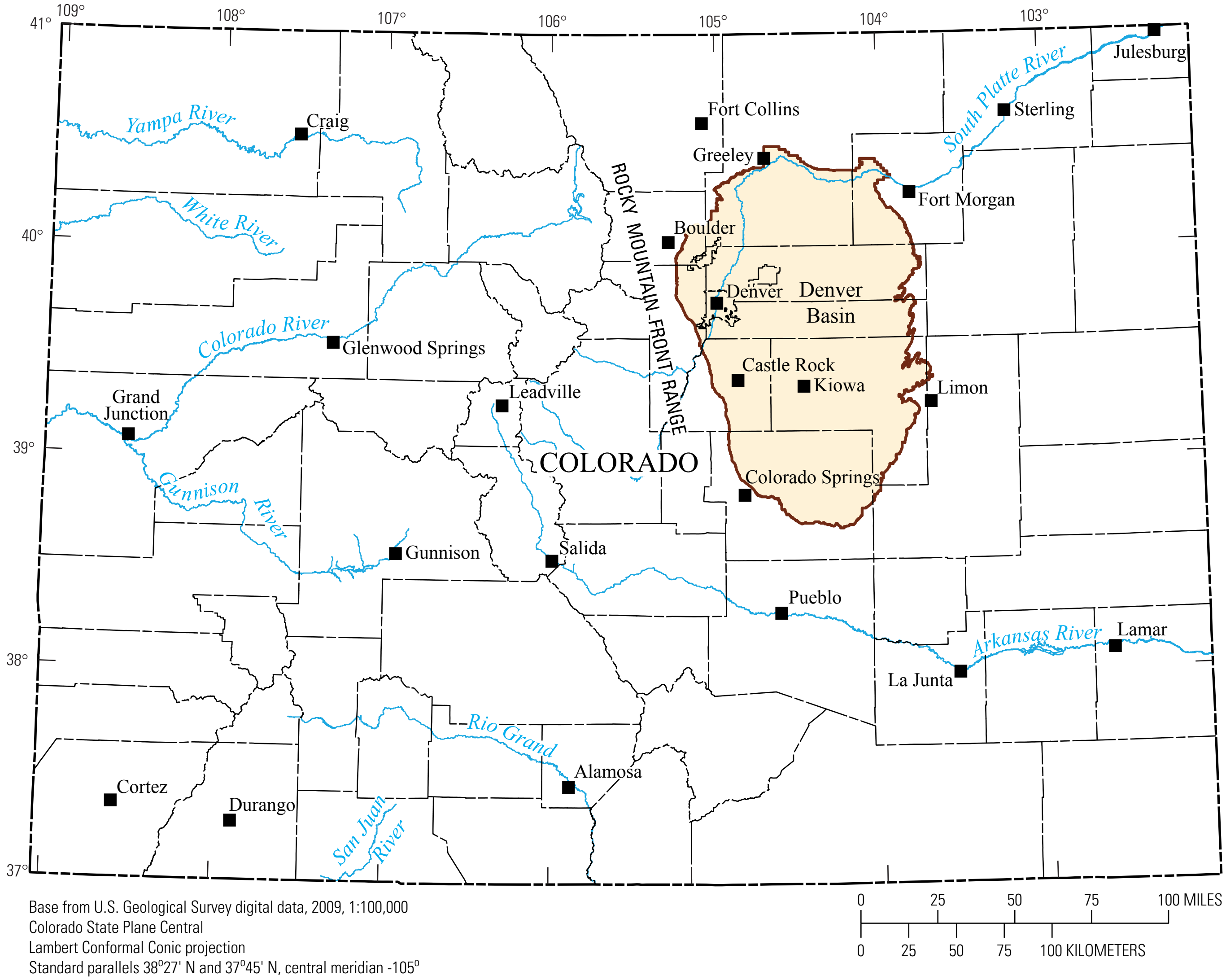



**Figure 1.** Location of Denver Basin in Colorado.

groundwater conditions before 1880, and 15 transient-state stress periods simulated conditions from January 1, 1880, through December 31, 2003. The lengths of the stress periods ranged from 2 to 27 years. The final 4-year stress period of that model represented January 1, 2000, through December 31, 2003. Transient stress periods are subdivided into multiple time steps for accuracy. In each stress period, simulated stresses, for example pumping rates, are held constant. However, individual model cells can be deactivated at any time step if, during solution of the groundwater-flow equation, the approximated head is below the bottom of a model cell. If a cell where a pumping well is simulated is deactivated, the discharge associated with that well is no longer simulated.

The simulation periods for the optimization analyses described in this report begin January 1, 2010. To bridge the gap between the end of the calibration period and the beginning of the optimization-analysis period, a 6-year period (2004–2009) was simulated in which all simulated hydraulic stresses on the system were identical to those used for 2000–2003, the final stress period of the calibration period. These stresses included simulated recharge from precipitation and irrigation, evapotranspiration, withdrawals from wells, exchange with streams and reservoirs, discharge to springs, and alluvial-aquifer outflow (Banta and others, 2011). Hydraulic heads calculated for the end of 2009 were used as starting heads for the optimization model runs. Simulated 2009 potentiometric (hydraulic head) surfaces for the upper (model layer 8) and lower (model layer 10) Arapahoe aquifers are shown in figure 5. For the optimization analyses, the model was run with constant stresses for either 50 or 100 years.

Maps showing the height of the layer-8 simulated potentiometric surface relative to the top of the Arapahoe aquifer are provided at various simulation times to facilitate interpretation of the optimization analyses. Figure 6 shows the height of the layer 8 simulated 2009 potentiometric surface above the top of the Arapahoe aquifer in the area of interest.

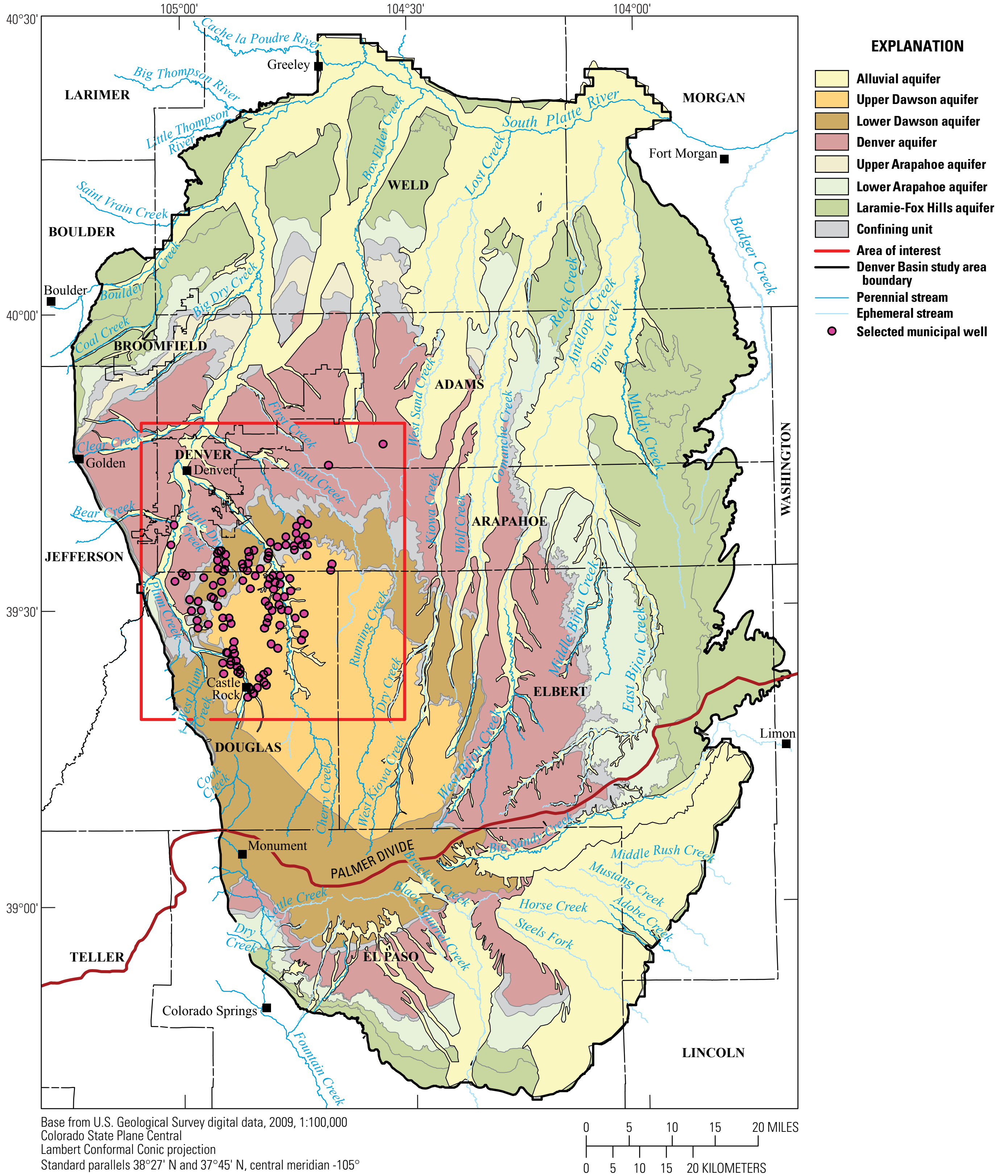



**Figure 2.** Denver Basin aquifer outcrops and selected municipal wells.

**Table 1.** Hydrogeologic units of the Denver Basin.

| Hydrogeologic unit | Age | Stratigraphic unit | Model layer | Lithologic description | Thickness (feet) | Hydrogeologic description |
|---|---|---|---|---|---|---|
| Alluvial aquifer where saturated | Quaternary | Alluvial, flood-plain, terrace, colluvial, and eolian sand, gravel, and clay deposits | 1 | Unconsolidated sand and gravel with clay lenses | 0 to 175 | Productive unconfined alluvial aquifer where saturated |
| upper Dawson aquifer | Eocene | Dawson Formation | 2 | Arkosic fluvial sandstone and conglomerate with interbedded claystone | 200 to 840 | Productive unconfined to confined aquifer |
| Dawson confining unit | Paleocene | Dawson Formation | 3 | Claystone | 1 to 350 | Claystone confining unit in northern part of Dawson extent |
| lower Dawson aquifer | Paleocene | Dawson Formation | 4 | Mixed arkosic and andesitic fluvial sandstone with interbedded claystone, lignite, and volcanics | 100 to 570 where undifferentiated | Productive confined aquifer |
| upper Denver confining unit | Paleocene | Denver Formation | 5 | Predominantly claystone with interbedded sandstone, lignite, and volcanics | 1 to 150 | Fine-grained confining unit in upper Denver Formation |
| Denver aquifer | Late Cretaceous | Denver Formation | 6 | Mixed arkosic and andesitic fluvial sandstone | 280 to 1,100 | Confined to unconfined aquifer. Lower hydraulic conductivity and less productive than Arapahoe aquifers |
| lower Denver confining unit | Late Cretaceous | Denver Formation | 7 | Predominantly claystone with andesitic fluvial sandstone | 5 to 150 | Fine-grained confining unit at base of Denver Formation |
| upper Arapahoe aquifer[1] | Late Cretaceous | Arapahoe Formation | 8 | Arkosic fluvial sandstone with interbedded claystone | 50 to 300 | Productive confined aquifer above Arapahoe confining unit in northern one-third of basin |
| Arapahoe confining unit[1] | Late Cretaceous | Arapahoe Formation | 9 | Predominantly claystone | 50 to 180 | Claystone confining unit in northern one-third of basin |
| lower Arapahoe aquifer[1] | Late Cretaceous | Arapahoe Formation | 10 | Alluvial fan conglomerate and sandstone with interbedded claystone | 200 to 600 where undifferentiated | Productive confined aquifer. Greatest thickness and hydraulic conductivity in west-central part of basin |
| Laramie confining unit | Late Cretaceous | Laramie Formation | 11 | Gray to black shale, coal, siltstone, and sandstone | 100 to 500 | Confining unit in upper part of Laramie Formation |
| Laramie-Fox Hills aquifer | Late Cretaceous | Laramie Formation | 12 | Poorly consolidated delta-front fluvial sandstone | 100 to 500 | Productive confined to unconfined aquifer composed of sandstones of lower Laramie Formation and Fox Hills Sandstone |
| | | Fox Hills Sandstone | | Yellow-brown marine delta-front and beach sandstone | | |
| Pierre Shale confining unit | Late Cretaceous | Pierre Shale | not simulated | Dark to light gray marine shale | 5,200 | Low-permeability base of Denver Basin aquifer system |

[1]Where Arapahoe confining unit is absent, upper Arapahoe and lower Arapahoe aquifers are considered undifferentiated.

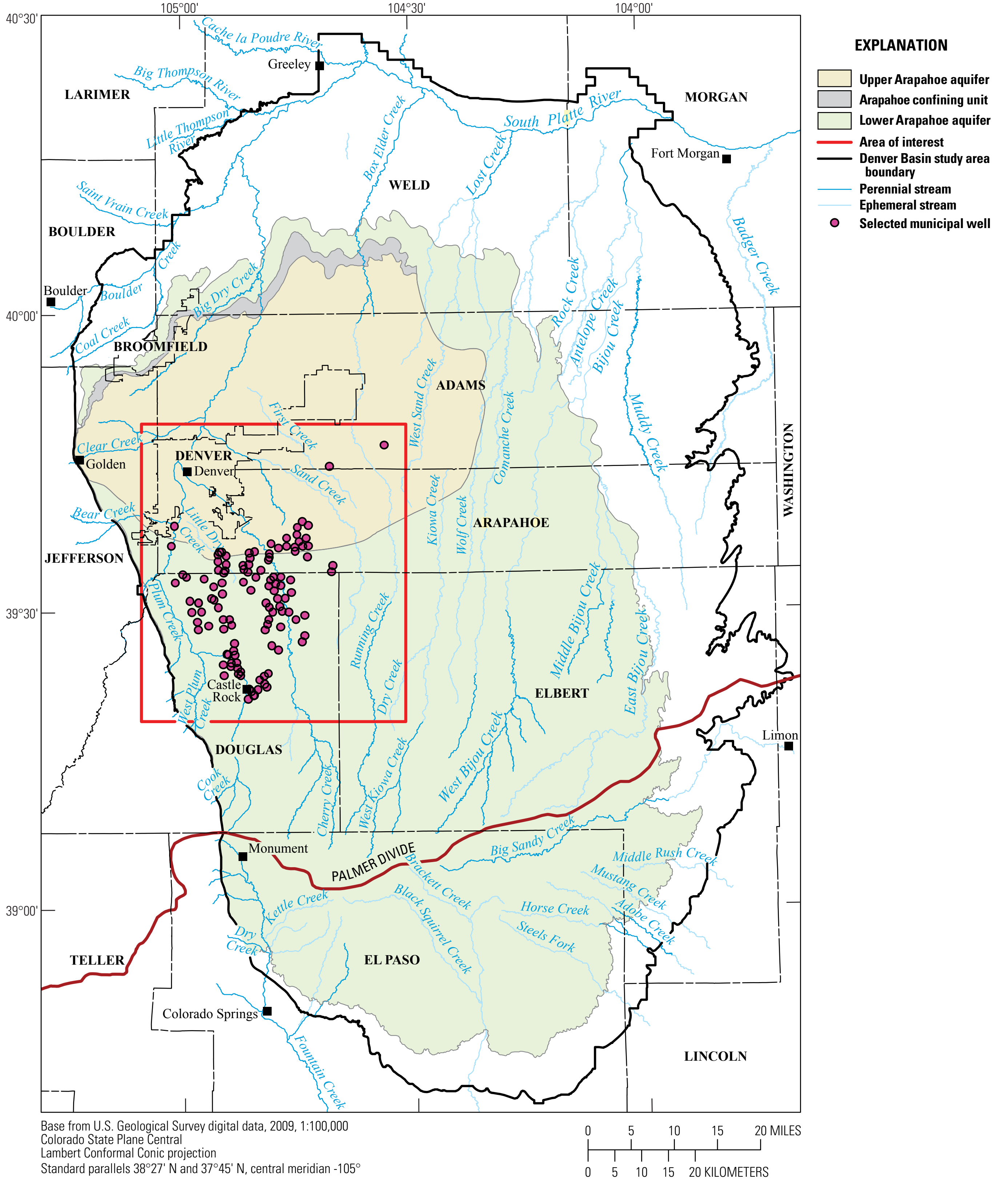



**Figure 3.** Extent of upper and lower Arapahoe aquifers.

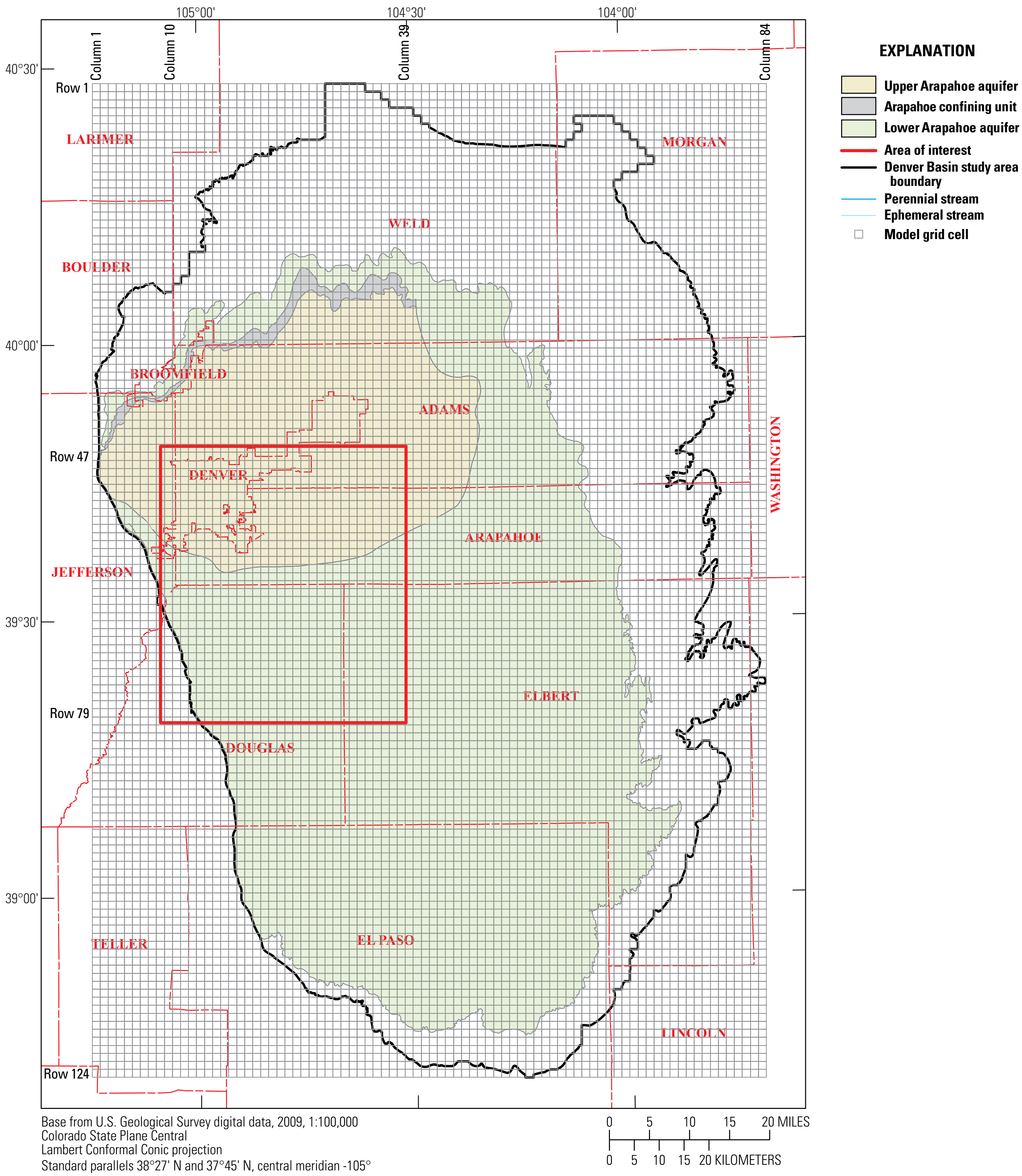



**Figure 4.** Denver Basin model grid.

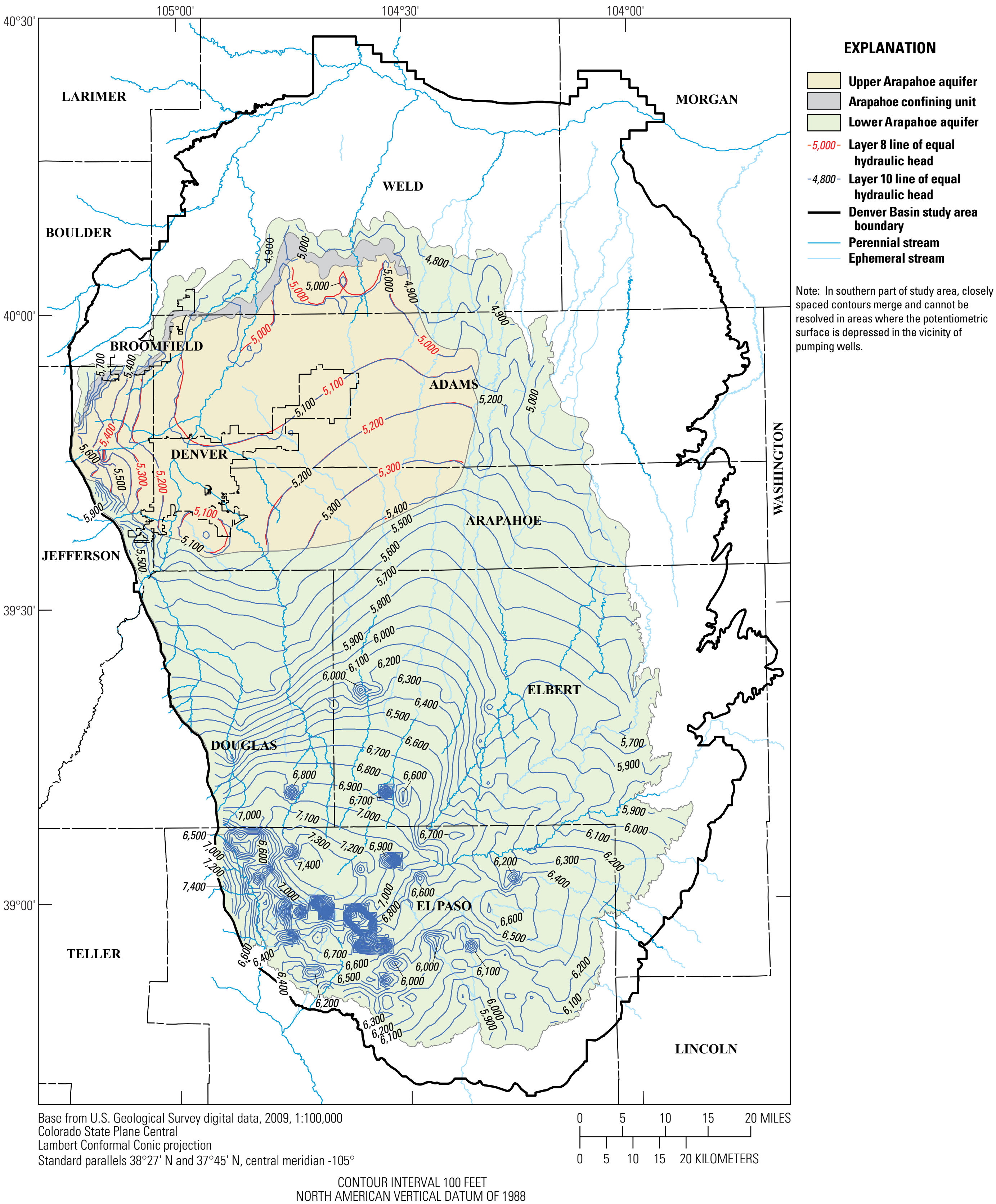



**Figure 5.** Simulated 2009 potentiometric (hydraulic-head) surfaces for upper and lower Arapahoe aquifers.

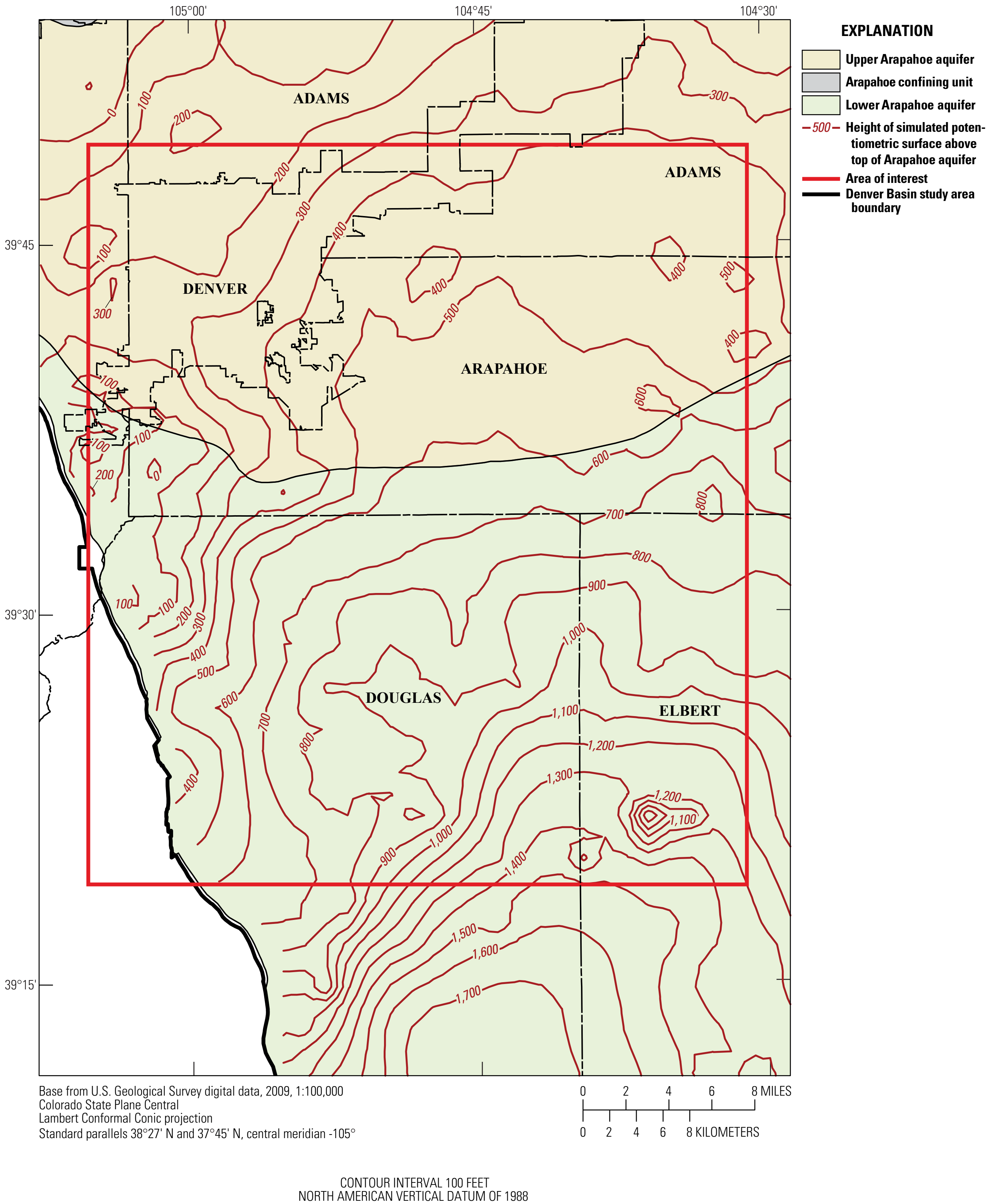



**Figure 6.** Height of layer 8 simulated 2009 potentiometric (hydraulic-head) surface relative to top of Arapahoe aquifer.

Maps showing simulated saturated thickness of the Arapahoe aquifer provide additional information related to the effects of sustained pumping on the aquifer at various simulation times. Figure 7 shows the simulated 2009 combined saturated thickness of the upper and lower Arapahoe aquifers. Combined saturated thickness was calculated as the sum of the saturated thickness of the upper Arapahoe aquifer and the saturated thickness of the lower Arapahoe aquifer; where the Arapahoe confining unit is present, the thickness of the confining unit is not included in the combined saturated thickness. Because 2009 simulated heads are above the top of the upper Arapahoe aquifer nearly everywhere in the area of interest, figure 7 also illustrates (nearly everywhere) the combined thickness of the upper and lower Arapahoe aquifers.

# Demonstration of Optimization Analyses of Pumping from Municipal Wells

Optimization analyses were conducted to demonstrate the potential benefits of using available optimization-modeling capabilities to address questions related to prolongation of the productivity of the Arapahoe aquifer and to limitation of the loss of saturated thickness of the Arapahoe aquifer. These analyses were designed as demonstrations only and were not intended as a comprehensive optimization study. Potential benefits that could be expected to result from an appropriately designed and executed comprehensive study include: (1) extended usefulness of the aquifers as a municipal water source; (2) a cost/benefit evaluation of alternative water-supply strategies, including infrastructure costs; (3) evaluation of trade-offs between the use of potential surface-water sources and the continued use of groundwater; or (4) an analysis of the benefits of aquifer storage and recovery or artificial recharge. The primary benefit of this demonstration project is to illustrate the general usefulness of some of the capabilities of optimization analysis as they could be applied to distribution of possible future pumping from municipal wells in the Denver Basin. A discussion of possible additional analyses and technical issues that may be applicable or of concern in a more comprehensive study also is provided.

## Software and Analytical Framework

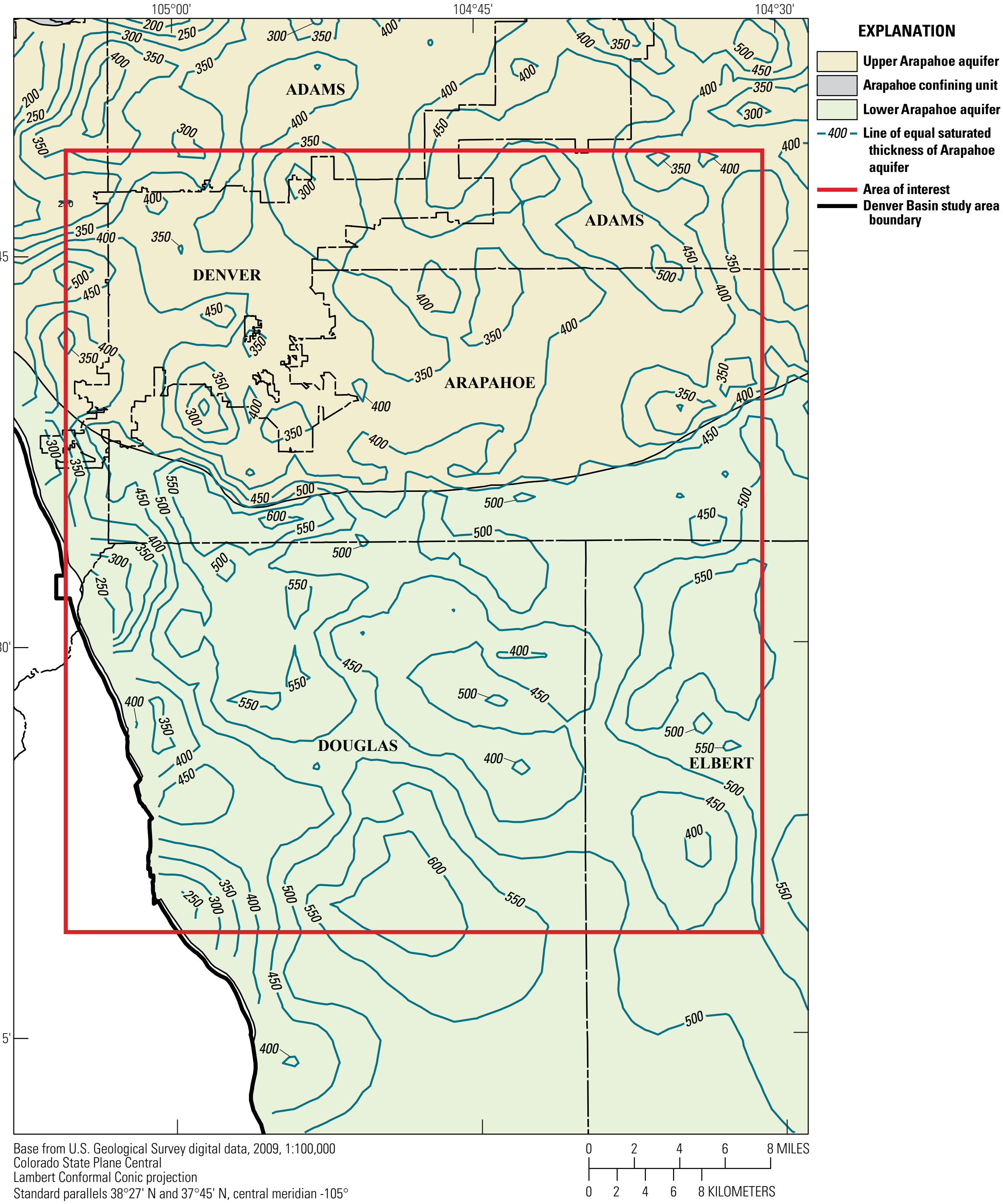
Optimization analyses described in this report utilized the MODFLOW-based Groundwater Management Process (GWM) (Ahlfeld and others, 2005, 2009). Ahlfeld and Mulligan (2000) describe key concepts and examples of optimization methods as applied to groundwater modeling. In optimization modeling, a problem is formulated by defining a set of decision variables, a set of constraints, and an objective function. Decision variables are used to control model input. Constraints are defined to assign acceptable limits on decision variables and model results. Values extracted or derived from model output commonly are referred to as state variables. The objective function is a mathematical expression that quantifies the effects of management decisions on outcomes of the managed system. A goal of optimization is to maximize or minimize the objective function. An optimization algorithm manipulates decision variables and executes model runs so as to maximize or minimize the objective function without violating any constraints.

Decision variables supported by GWM include flow-rate decision variables, which are used to control a withdrawal (discharge) or injection (recharge) flow rate at a managed-well (or well-field) location. As noted by Ahlfeld and others (2005), the decision variables of a management problem are the quantifiable controls that are to be determined by the mathematical model of the management decision-making process. Each decision variable in the analyses described in this report was defined to control the simulated withdrawal rate from one or more wells in a model cell.

GWM supports several options for defining constraints. Two types of constraints were specified in the optimization models developed for this study: upper- and lower-bound constraints on withdrawal rates controlled by the flow-rate decision variables, and lower limits on simulated heads at selected model cells. The lower limit for managed withdrawal rates was zero. In each analysis the objective was to maximize the sum of withdrawal rates controlled by all decision variables involved in the analysis, subject to the specified constraints. With this approach the maximized total withdrawal might be obtained when one or more flow rates at individual cells is set to zero. This situation can happen when calculated head at a constraint location declines to the head-constraint value even though the decision variable controlling pumping at that location is zero.

## Selection of Wells

Optimization of pumping distribution can be expected to have the most substantial benefits in areas where well-interference effects are most pronounced. In contrast, in areas where spacing between wells is so large that well-interference effects are small or negligible, optimization of pumping distribution is unlikely to be worthwhile. The west-central part of the Denver Basin was selected for analysis because the large concentration of municipal wells in the area produces conditions in which well-interference effects are most likely to have an effect on productivity of adjacent wells. In all, 119 wells (fig. 8) were used to define locations at which simulated withdrawal rates were to be controlled with the flow-rate decision variables. All municipal wells completed in the Arapahoe aquifer and currently being used in the south Denver metropolitan area were considered for inclusion in the analyses; wells were eliminated from consideration only if available data were insufficient to define the well location or aquifer of completion. Two wells in southwestern Adams County also were included because they are operated by one of the major water providers of western




Base from U.S. Geological Survey digital data, 2009, 1:100,000
Colorado State Plane Central
Lambert Conformal Conic projection
Standard parallels 38°27' N and 37°45' N, central meridian -105°

CONTOUR INTERVAL 50 FEET
NORTH AMERICAN VERTICAL DATUM OF 1988

**Figure 7.** Combined 2009 simulated saturated thickness of upper and lower Arapahoe aquifers.

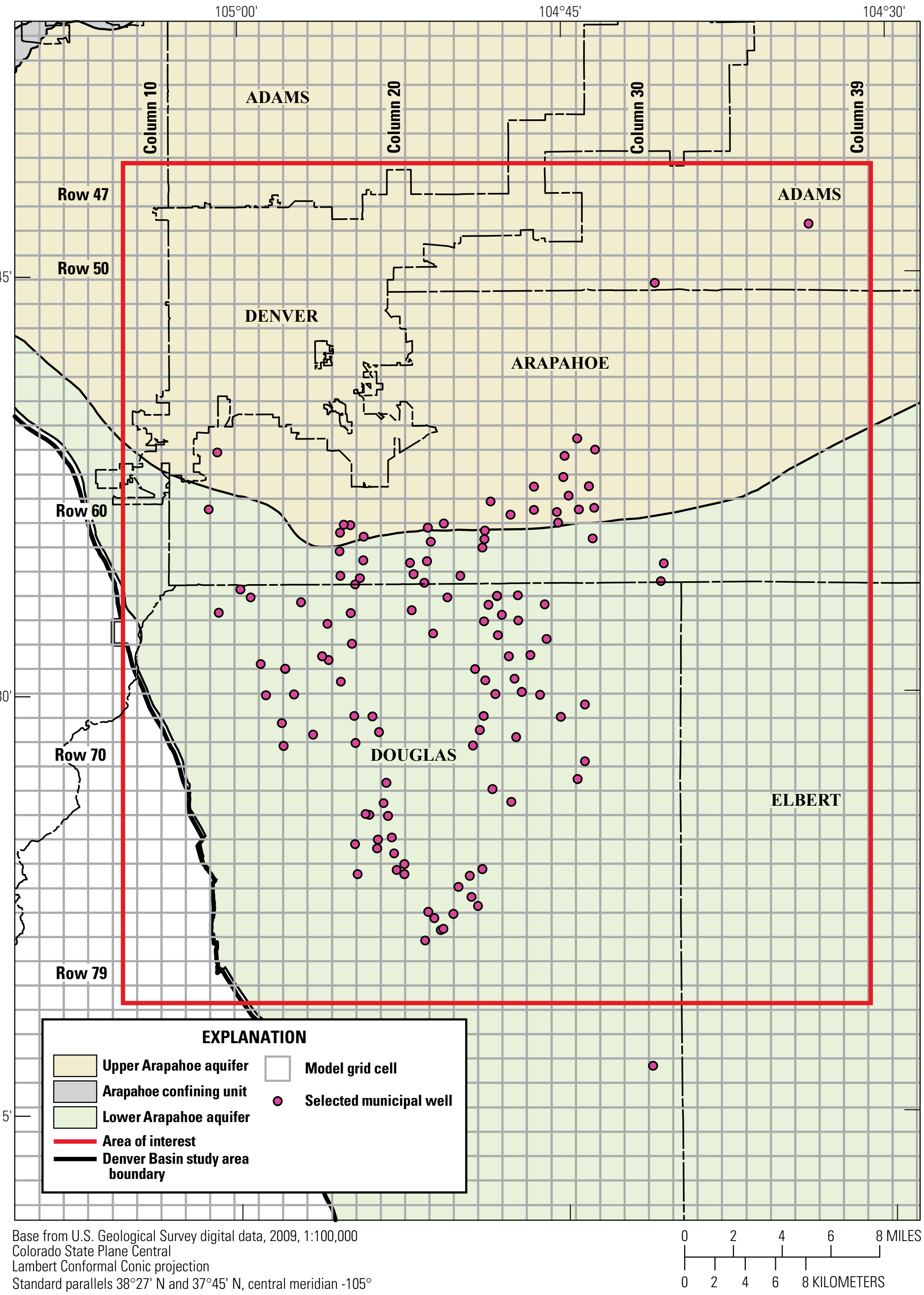



**Figure 8.** Selected municipal wells.

Arapahoe County. Of the 119 wells, 78 were simulated in the 2004–2009 model and were converted to be represented as managed wells for the optimization analyses. The remaining 41 managed wells were newly constructed or otherwise not included in the 2004–2009 simulation.

## Application of Optimization Analysis to Arapahoe Aquifer Pumping

The concept behind the project documented in this report was to use the groundwater model of Banta and others (2011) in conjunction with GWM (Ahlfeld and others, 2005, 2009) to demonstrate methods to investigate pumping options that may prolong the productivity of the Arapahoe aquifer. As water-level declines cause increasing desaturation of an aquifer in the vicinity of pumping wells, well yields, and thus productivity of the aquifer, necessarily decline. Any measures taken to delay excessive drawdowns will tend to prolong productivity. Optimization analysis can be helpful in the development of such measures.

Two optimization analyses were designed to address issues related to the excessive loss of saturated thickness in the Arapahoe aquifer caused by pumping for municipal water supplies. In addition a baseline, non-optimized simulation was performed. The common goal in both optimization analyses was to maximize withdrawals from the Arapahoe aquifer while constraining drawdowns of water levels in the Arapahoe aquifer such that dewatering of the aquifer was limited to less than a specified percentage of the overall aquifer thickness at specified locations. In consultation with CWCB and CDWR, the constraint was defined as 10 percent, so that at least 90 percent of the overall aquifer thickness would remain saturated. This percentage allows unconfined yield to be considered in the analyses, and it produces optimization results in which pumping rates are substantially constrained to avoid violating the head constraints. The two analyses differed in the simulation time at which the head constraints applied.

In these analyses the term “managed-flow cell” refers to a model cell where simulated withdrawal rate is controlled by a flow-rate decision variable and determined as part of the solution to the optimization formulation. In many instances, a managed-flow cell contains a single municipal well. In other instances, a single managed-flow cell contains multiple municipal wells because multiple wells are located within the area represented by the model cell. Ninety-six managed-flow cells were required to include all 119 wells and, thus, 96 flow-rate decision variables were defined. Depending on the completion interval of a well, the corresponding managed-flow cell was either in model layer 8 or layer 10 (table 1).

For each managed well included in the analysis, a maximum withdrawal rate was assigned. The bases for the maximum withdrawal rates for the various wells depended on data availability, according to the following order of preference:

1. The maximum pumping rate supplied by the municipal water provider that operates the well;
2. The “pump_rate” entry obtained from the WellView Web online data base maintained by the CDWR (Colorado Division of Water Resources, 2009);
3. The “yield” entry obtained from a well-permit data base provided by the CDWR (Brian Ahrens, written commun., 2004); or
4. Calculation from reported production volumes or meter readings.

Any maximum withdrawal rate based on the data sources above that exceeded the decreed annual rate provided by CDWR was reassigned as the decreed rate. Decreed rates conform to the one-percent annual withdrawal limitation (James Heath, Colorado Division of Water Resources, oral commun., 2011) established by the Colorado statewide nontributary groundwater rules (Colorado Division of Water Resources, 1986).

A decision variable was defined for each model cell corresponding to one or more managed municipal wells, and a maximum withdrawal rate was defined for each decision variable (table 2) from the maximum withdrawal rate(s) for any managed-flow well(s) in that cell. Each flow-variable name is composed of “FV” (where “FV” stands for “flow variable”), followed by a string of characters identifying the location of the corresponding managed-flow cell, in the format LL_RR_CC, where LL is the layer number in the model grid, RR is the row number, and CC is the column number. If a cell contained multiple municipal wells, the decision-variable maximum withdrawal rate was determined as the sum of the maximum withdrawal rates of all managed-flow wells within the cell. Wells included in the simulations but not included in any decision variable are referred to as “unmanaged” wells. Withdrawal rates for unmanaged wells were specified at the rates that applied for the 2004–2009 period.

In the Denver Basin, most wells are completed in a single aquifer; however, some wells are completed in multiple aquifers. In the model simulations, pumping from wells completed in a single aquifer is simulated using the Well Package of MODFLOW-2005 (Harbaugh, 2005). Pumping from wells completed in multiple aquifers is simulated using the Multi-Node Well (MNW) Package (Halford and Hanson, 2002). For the 2004–2009 simulation, the simulated pumping rate for all wells completed in the upper, lower, or undifferentiated Arapahoe aquifer and simulated by the Well Package was 43.4 $ft^3/s$; this rate is for wells of all water uses (not just municipal) in the entire Arapahoe aquifer (not just the area of interest).

Some wells represented by the MNW Package are completed in the upper and lower Arapahoe aquifers; some are completed in the Denver aquifer in addition to the upper and lower Arapahoe aquifers, and some are completed in aquifers other than the Arapahoe. Even though MNW withdrawal rates are held constant for 2004 and later, interaquifer flow simulated by the MNW Package changes slightly during the course of a simulation in response to changing hydraulic heads near the MNW-simulated wells. However, the variability in

**Table 2.** Flow-rate decision variables and results of optimization analyses.

[$ft^3/s$, cubic feet per second; yrs, years]

| Decision-variable name | Number of wells | Layer | Row | Column | Maximum withdrawal rate ($ft^3/s$) | Analysis 1 (50 yrs) optimized withdrawal rate ($ft^3/s$) | Analysis 2 (100 yrs) optimized withdrawal rate ($ft^3/s$) |
|---|---|---|---|---|---|---|---|
| FV08_58_13 | 1 | 8 | 58 | 13 | 0.111 | 0.111 | 0.111 |
| FV08_61_18 | 1 | 8 | 61 | 18 | 0.044 | 0.000 | 0.000 |
| FV08_66_17 | 1 | 8 | 66 | 17 | 0.423 | 0.423 | 0.423 |
| FV08_72_19 | 1 | 8 | 72 | 19 | 0.279 | 0.279 | 0.279 |
| FV08_75_20 | 1 | 8 | 75 | 20 | 0.579 | 0.579 | 0.579 |
| FV08_76_21 | 1 | 8 | 76 | 21 | 0.134 | 0.134 | 0.134 |
| FV08_77_22 | 2 | 8 | 77 | 22 | 0.690 | 0.690 | 0.690 |
| FV10_48_37 | 1 | 10 | 48 | 37 | 0.099 | 0.099 | 0.099 |
| FV10_51_31 | 1 | 10 | 51 | 31 | 0.076 | 0.076 | 0.076 |
| FV10_57_28 | 2 | 10 | 57 | 28 | 0.500 | 0.500 | 0.500 |
| FV10_58_27 | 1 | 10 | 58 | 27 | 0.102 | 0.102 | 0.102 |
| FV10_59_26 | 1 | 10 | 59 | 26 | 0.322 | 0.322 | 0.322 |
| FV10_59_27 | 2 | 10 | 59 | 27 | 0.617 | 0.617 | 0.617 |
| FV10_59_28 | 1 | 10 | 59 | 28 | 0.668 | 0.668 | 0.668 |
| FV10_60_12 | 1 | 10 | 60 | 12 | 0.067 | 0.067 | 0.067 |
| FV10_60_24 | 1 | 10 | 60 | 24 | 0.516 | 0.516 | 0.516 |
| FV10_60_25 | 1 | 10 | 60 | 25 | 0.414 | 0.414 | 0.414 |
| FV10_60_26 | 1 | 10 | 60 | 26 | 0.607 | 0.607 | 0.607 |
| FV10_60_27 | 2 | 10 | 60 | 27 | 0.985 | 0.985 | 0.985 |
| FV10_60_28 | 2 | 10 | 60 | 28 | 1.131 | 1.131 | 1.131 |
| FV10_61_18 | 2 | 10 | 61 | 18 | 0.651 | 0.630 | 0.122 |
| FV10_61_19 | 1 | 10 | 61 | 19 | 0.331 | 0.331 | 0.331 |
| FV10_61_21 | 1 | 10 | 61 | 21 | 0.889 | 0.721 | 0.619 |
| FV10_61_22 | 2 | 10 | 61 | 22 | 1.492 | 1.090 | 0.894 |
| FV10_61_24 | 2 | 10 | 61 | 24 | 0.825 | 0.825 | 0.825 |
| FV10_61_28 | 1 | 10 | 61 | 28 | 0.791 | 0.791 | 0.791 |
| FV10_62_18 | 1 | 10 | 62 | 18 | 0.442 | 0.000 | 0.000 |
| FV10_62_19 | 1 | 10 | 62 | 19 | 0.442 | 0.347 | 0.257 |
| FV10_62_21 | 2 | 10 | 62 | 21 | 0.552 | 0.397 | 0.229 |
| FV10_62_24 | 1 | 10 | 62 | 24 | 0.261 | 0.261 | 0.261 |
| FV10_62_31 | 1 | 10 | 62 | 31 | 0.557 | 0.557 | 0.557 |
| FV10_63_14 | 1 | 10 | 63 | 14 | 0.066 | 0.066 | 0.066 |
| FV10_63_18 | 2 | 10 | 63 | 18 | 1.255 | 0.463 | 0.348 |
| FV10_63_19 | 1 | 10 | 63 | 19 | 0.442 | 0.442 | 0.442 |
| FV10_63_21 | 2 | 10 | 63 | 21 | 1.104 | 1.104 | 0.971 |
| FV10_63_23 | 1 | 10 | 63 | 23 | 0.469 | 0.469 | 0.469 |
| FV10_63_25 | 1 | 10 | 63 | 25 | 0.276 | 0.276 | 0.276 |
| FV10_63_31 | 1 | 10 | 63 | 31 | 0.557 | 0.557 | 0.557 |
| FV10_64_13 | 1 | 10 | 64 | 13 | 0.145 | 0.145 | 0.145 |
| FV10_64_14 | 1 | 10 | 64 | 14 | 0.663 | 0.275 | 0.180 |
| FV10_64_16 | 1 | 10 | 64 | 16 | 0.663 | 0.663 | 0.583 |
| FV10_64_18 | 1 | 10 | 64 | 18 | 0.663 | 0.663 | 0.663 |
| FV10_64_21 | 1 | 10 | 64 | 21 | 0.345 | 0.345 | 0.345 |
| FV10_64_22 | 1 | 10 | 64 | 22 | 0.345 | 0.345 | 0.345 |
| FV10_64_24 | 4 | 10 | 64 | 24 | 1.519 | 1.519 | 0.927 |
| FV10_64_26 | 1 | 10 | 64 | 26 | 0.276 | 0.276 | 0.276 |
| FV10_65_17 | 1 | 10 | 65 | 17 | 0.663 | 0.663 | 0.663 |
| FV10_65_18 | 1 | 10 | 65 | 18 | 0.663 | 0.663 | 0.663 |
| FV10_65_22 | 1 | 10 | 65 | 22 | 0.414 | 0.414 | 0.414 |

**Table 2.** Flow-rate decision variables and results of optimization analyses.—Continued

[ft³/s, cubic feet per second; yrs, years]

| Decision-variable name | Number of wells | Layer | Row | Column | Maximum withdrawal rate (ft³/s) | Analysis 1 (50 yrs) optimized withdrawal rate (ft³/s) | Analysis 2 (100 yrs) optimized withdrawal rate (ft³/s) |
|---|---|---|---|---|---|---|---|
| FV10_65_24 | 2 | 10 | 65 | 24 | 0.525 | 0.525 | 0.525 |
| FV10_65_25 | 1 | 10 | 65 | 25 | 0.207 | 0.207 | 0.207 |
| FV10_65_26 | 1 | 10 | 65 | 26 | 1.560 | 1.560 | 1.560 |
| FV10_66_15 | 1 | 10 | 66 | 15 | 0.668 | 0.668 | 0.501 |
| FV10_66_16 | 1 | 10 | 66 | 16 | 0.377 | 0.377 | 0.377 |
| FV10_66_17 | 1 | 10 | 66 | 17 | 0.663 | 0.663 | 0.429 |
| FV10_66_25 | 1 | 10 | 66 | 25 | 0.780 | 0.780 | 0.780 |
| FV10_66_26 | 1 | 10 | 66 | 26 | 0.128 | 0.128 | 0.128 |
| FV10_67_16 | 1 | 10 | 67 | 16 | 0.291 | 0.291 | 0.291 |
| FV10_67_18 | 1 | 10 | 67 | 18 | 0.663 | 0.663 | 0.663 |
| FV10_67_23 | 1 | 10 | 67 | 23 | 0.197 | 0.197 | 0.197 |
| FV10_67_24 | 1 | 10 | 67 | 24 | 0.051 | 0.051 | 0.051 |
| FV10_67_25 | 2 | 10 | 67 | 25 | 0.058 | 0.058 | 0.058 |
| FV10_68_15 | 1 | 10 | 68 | 15 | 0.600 | 0.600 | 0.455 |
| FV10_68_16 | 1 | 10 | 68 | 16 | 0.851 | 0.828 | 0.609 |
| FV10_68_18 | 1 | 10 | 68 | 18 | 1.154 | 1.154 | 1.039 |
| FV10_68_19 | 1 | 10 | 68 | 19 | 0.446 | 0.446 | 0.446 |
| FV10_68_24 | 2 | 10 | 68 | 24 | 3.509 | 2.041 | 1.382 |
| FV10_68_26 | 1 | 10 | 68 | 26 | 0.891 | 0.891 | 0.891 |
| FV10_68_27 | 1 | 10 | 68 | 27 | 0.118 | 0.118 | 0.118 |
| FV10_68_28 | 1 | 10 | 68 | 28 | 0.367 | 0.367 | 0.367 |
| FV10_69_15 | 1 | 10 | 69 | 15 | 0.847 | 0.847 | 0.847 |
| FV10_69_17 | 1 | 10 | 69 | 17 | 0.769 | 0.769 | 0.769 |
| FV10_69_19 | 1 | 10 | 69 | 19 | 2.225 | 1.972 | 1.276 |
| FV10_69_24 | 1 | 10 | 69 | 24 | 1.950 | 1.662 | 1.148 |
| FV10_69_25 | 1 | 10 | 69 | 25 | 1.894 | 1.585 | 1.063 |
| FV10_70_16 | 1 | 10 | 70 | 16 | 0.831 | 0.831 | 0.831 |
| FV10_70_18 | 1 | 10 | 70 | 18 | 1.038 | 1.038 | 1.038 |
| FV10_70_23 | 1 | 10 | 70 | 23 | 1.000 | 1.000 | 1.000 |
| FV10_70_28 | 1 | 10 | 70 | 28 | 0.362 | 0.362 | 0.362 |
| FV10_71_20 | 1 | 10 | 71 | 20 | 0.552 | 0.552 | 0.552 |
| FV10_71_24 | 1 | 10 | 71 | 24 | 0.456 | 0.456 | 0.456 |
| FV10_71_28 | 1 | 10 | 71 | 28 | 0.668 | 0.668 | 0.668 |
| FV10_72_19 | 1 | 10 | 72 | 19 | 0.552 | 0.552 | 0.552 |
| FV10_72_20 | 1 | 10 | 72 | 20 | 0.552 | 0.552 | 0.552 |
| FV10_72_25 | 1 | 10 | 72 | 25 | 0.380 | 0.380 | 0.380 |
| FV10_73_20 | 2 | 10 | 73 | 20 | 1.215 | 1.215 | 1.215 |
| FV10_74_18 | 1 | 10 | 74 | 18 | 0.388 | 0.388 | 0.388 |
| FV10_74_19 | 2 | 10 | 74 | 19 | 1.008 | 1.008 | 1.008 |
| FV10_74_20 | 1 | 10 | 74 | 20 | 1.025 | 1.025 | 1.025 |
| FV10_75_19 | 1 | 10 | 75 | 19 | 0.695 | 0.695 | 0.695 |
| FV10_75_20 | 2 | 10 | 75 | 20 | 2.400 | 1.191 | 0.857 |
| FV10_75_23 | 2 | 10 | 75 | 23 | 1.318 | 1.318 | 1.318 |
| FV10_75_24 | 1 | 10 | 75 | 24 | 0.276 | 0.276 | 0.276 |
| FV10_76_23 | 2 | 10 | 76 | 23 | 0.906 | 0.906 | 0.906 |
| FV10_77_22 | 2 | 10 | 77 | 22 | 0.652 | 0.652 | 0.489 |
| FV10_78_21 | 1 | 10 | 78 | 21 | 0.690 | 0.690 | 0.690 |
| | | | | Totals | 64.874 | 58.820 | 52.971 |

discharge from the Arapahoe aquifer caused by changes in interaquifer flow was small. The rate of withdrawals from the upper and lower Arapahoe aquifers simulated by the MNW Package, rounded to the nearest 0.1 $ft^3/s$, remained at 4.9 $ft^3/s$; again, this rate is for wells of all water uses in the entire Arapahoe aquifer. The total well withdrawal rate for wells of all water uses in the entire Arapahoe aquifer, including single-aquifer and multi-aquifer wells, was, therefore, about 48.3 $ft^3/s$. The optimization analyses did not involve management of multi-aquifer wells. For this reason, discussion of the optimization focuses primarily on the single-aquifer wells.

For the non-optimized simulation and for the optimization analyses, a subset of wells simulated by the Well Package was removed and simulated instead with rates controlled by GWM. During these simulations, the total pumping rate for all Arapahoe aquifer wells simulated by the Well Package was 37.7 $ft^3/s$, rather than the 43.4 $ft^3/s$ of the 2004–2009 simulation. The difference in withdrawals simulated by the Well Package, 5.7 $ft^3/s$, is the rate simulated during 2004–2009 for 78 wells that were removed from the Well Package and included in decision variables for the non-optimized simulation and optimization analyses. The other 41 of the 119 wells included in decision variables are wells that either were newly constructed or otherwise were absent from the set of wells identified and simulated as Arapahoe aquifer municipal wells in the Denver Basin model (Banta and others, 2011).

A single set of altitudes was defined and incorporated into hydraulic-head constraints for the optimization analyses. Two head-constraint altitudes were defined for each managed-flow cell location: one in layer 8 and one in layer 10. In much of the area affected by pumping from wells in the west-central Denver Basin, the confining bed between the upper and lower Arapahoe aquifers is absent, and, for the purposes of this investigation, the Arapahoe aquifer is considered undifferentiated.

Head-constraint altitudes were chosen for each flow-rate decision-variable location such that, depending on the layer and the presence or absence of the Arapahoe confining bed, if the head for the cell was at the constraint head, the saturated thickness of either the upper, lower, or undifferentiated Arapahoe aquifer would be 90 percent of the aquifer thickness. Where the Arapahoe confining bed is present, a constraint was defined for the layer-8 cell at an altitude that would represent 90 percent of the thickness of the layer-8 cell, and a constraint was defined for the layer-10 cell at an altitude that would represent 90 percent of the thickness of the layer-10 cell. In areas where the Arapahoe confining bed is absent, the bottom of layer 8 was set equal to the midpoint altitude of the Arapahoe aquifer, and the bottom of layer 9 was assigned to be 0.1 ft below the bottom of layer 8 (Banta and others, 2011). The head constraint for both the layer 8 and the layer 10 cells were defined at an altitude that would represent 90 percent of the overall thickness of layers 8 through 10.

Locations and head values for all head constraints are listed in table 3. The constraint names were generated as “HC” followed by a string of characters in the format LL_RR_CC, where LL is the layer number, RR is the row number, and CC is the column number of the model cell where the constraint applied. For the two optimization analyses, the total simulation time was either 50 or 100 years. All pumping rates remained constant throughout the 50- or 100-year simulation period. In each analysis the constraints applied at the end of the simulation time. The height of the 2009 simulated head above each constraint altitude (table 3), which ranges from 65.1 to 966.9 ft, indicates the amount of drawdown available at each constraint location for the 50- or 100-year simulation period.

Because of nonlinear computational behavior during solution of the Denver Basin model (Banta and others, 2011), the sequential linear programming (SLP) approach of GWM (Ahlfeld and others, 2005, 2009) was selected to solve the optimization problems. MODFLOW and SLP solver settings and convergence criteria were defined to calculate model results with sufficient precision to enable the SLP algorithm to generate reliable results. Details of the SLP method and solver settings are provided in Ahlfeld and others (2005).

Table 2 lists the flow-rate decision variables used in the optimization analyses. The purpose of each analysis was to determine the optimum spatial distribution of pumping among all wells for either 50 or 100 years, subject to the maximum withdrawal-rate constraints and head constraints. The formulation of each optimization analysis can be expressed in terms of the following objective function and constraints:

Maximize objective function: $\sum_{i=1}^{NV} Q_i$

subject to maximum withdrawal constraints:

$Q_i \le Q_i^u, i = 1, NV$

and head constraints: $H_j \ge H_j^l, j = 1, NC$ at the end of the simulation,

where

$NV$ is the number of flow-rate decision variables;

$Q_i$ is the withdrawal rate for decision variable *i*;

$Q_i^u$ is the maximum withdrawal rate for decision variable *i;*

$H_j$ is the simulated head corresponding to head constraint *j* at the end of the simulation;

$H_j^l$ is a lower head constraint listed in table 3; and

$NC$ is the number of head constraints.

## Non-Optimized Simulation

To provide a basis for evaluating the optimization analyses, a 100-year simulation using non-optimized pumping rates was performed, representing Jan. 1, 2010, through Dec. 31, 2109. For this simulation, pumping rates for all decision variables were assigned as the maximum withdrawal rates (table 2). Other stresses were assigned at the 2004–2009 simulated values.

**Table 3.** Head constraints and simulated heads.

[deactiv., cell was deactivated; --, constraint was not violated]

| Constraint name | Layer | Row | Column | Cell top elevation | 2009 simulated hydraulic head | Lower head constraint (feet) | Height of 2009 simulated head above constraint (feet) | Non-optimized (2109) simulated hydraulic head | Head-constraint violation year (non-optimized simulation) | Analysis 1 (2059) simulated hydraulic head | Analysis 2 (2109) simulated hydraulic head |
|---|---|---|---|---|---|---|---|---|---|---|---|
| HC08_48_37 | 8 | 48 | 37 | 4,809.5 | 5,235.4 | 4,790.6 | 444.9 | 5,166.6 | -- | 5,182.9 | 5,167.0 |
| HC10_48_37 | 10 | 48 | 37 | 4,516.9 | 5,215.1 | 4,494.2 | 720.9 | 5,141.7 | -- | 5,158.2 | 5,142.1 |
| HC08_51_31 | 8 | 51 | 31 | 4,772.7 | 5,218.8 | 4,753.4 | 465.5 | 5,100.3 | -- | 5,123.1 | 5,100.9 |
| HC10_51_31 | 10 | 51 | 31 | 4,434.9 | 5,213.8 | 4,417.8 | 796.0 | 5,081.9 | -- | 5,104.9 | 5,082.5 |
| HC08_57_28 | 8 | 57 | 28 | 4,693.7 | 5,260.5 | 4,674.7 | 585.8 | 4,955.8 | -- | 4,993.9 | 4,956.5 |
| HC10_57_28 | 10 | 57 | 28 | 4,413.9 | 5,257.7 | 4,394.5 | 863.2 | 4,908.6 | -- | 4,946.9 | 4,909.3 |
| HC08_58_13 | 8 | 58 | 13 | 4,987.0 | 5,106.1 | 4,974.8 | 131.3 | 4,992.5 | -- | 5,008.1 | 4,997.5 |
| HC10_58_13 | 10 | 58 | 13 | 4,785.8 | 5,107.1 | 4,757.0 | 350.1 | 4,993.7 | -- | 5,009.5 | 4,998.7 |
| HC08_58_27 | 8 | 58 | 27 | 4,682.4 | 5,245.9 | 4,664.7 | 581.3 | 4,875.5 | -- | 4,915.7 | 4,876.0 |
| HC10_58_27 | 10 | 58 | 27 | 4,420.4 | 5,244.2 | 4,397.3 | 846.8 | 4,848.3 | -- | 4,888.7 | 4,848.8 |
| HC08_59_26 | 8 | 59 | 26 | 4,662.2 | 5,233.2 | 4,643.3 | 589.9 | 4,797.1 | -- | 4,837.6 | 4,796.8 |
| HC10_59_26 | 10 | 59 | 26 | 4,385.3 | 5,224.3 | 4,362.5 | 861.8 | 4,743.8 | -- | 4,784.6 | 4,743.5 |
| HC08_59_27 | 8 | 59 | 27 | 4,687.1 | 5,246.4 | 4,669.2 | 577.3 | 4,799.4 | -- | 4,844.0 | 4,800.0 |
| HC10_59_27 | 10 | 59 | 27 | 4,439.2 | 5,244.6 | 4,415.3 | 829.4 | 4,737.3 | -- | 4,781.8 | 4,738.0 |
| HC08_59_28 | 8 | 59 | 28 | 4,671.6 | 5,269.5 | 4,656.6 | 612.9 | 4,852.7 | -- | 4,898.3 | 4,853.7 |
| HC10_59_28 | 10 | 59 | 28 | 4,437.9 | 5,268.3 | 4,415.6 | 852.7 | 4,788.6 | -- | 4,834.0 | 4,789.7 |
| HC08_60_12 | 8 | 60 | 12 | 5,108.5 | 5,128.4 | 5,062.5 | 65.9 | 5,094.6 | -- | 5,104.8 | 5,095.2 |
| HC10_60_12 | 10 | 60 | 12 | 4,878.4 | 5,127.6 | 5,062.5 | 65.1 | 5,093.7 | -- | 5,103.9 | 5,094.3 |
| HC08_60_24 | 8 | 60 | 24 | 4,627.8 | 5,199.2 | 4,606.8 | 592.4 | 4,731.1 | -- | 4,759.6 | 4,724.7 |
| HC10_60_24 | 10 | 60 | 24 | 4,360.0 | 5,195.0 | 4,335.4 | 859.6 | 4,689.3 | -- | 4,718.6 | 4,683.2 |
| HC08_60_25 | 8 | 60 | 25 | 4,605.0 | 5,219.3 | 4,584.7 | 634.6 | 4,732.6 | -- | 4,771.8 | 4,730.8 |
| HC10_60_25 | 10 | 60 | 25 | 4,342.2 | 5,218.3 | 4,318.3 | 900.0 | 4,689.6 | -- | 4,729.4 | 4,688.1 |
| HC08_60_26 | 8 | 60 | 26 | 4,619.2 | 5,235.6 | 4,600.3 | 635.2 | 4,735.9 | -- | 4,780.9 | 4,736.1 |
| HC10_60_26 | 10 | 60 | 26 | 4,338.2 | 5,234.6 | 4,318.3 | 916.3 | 4,669.1 | -- | 4,714.9 | 4,669.6 |
| HC08_60_27 | 8 | 60 | 27 | 4,635.6 | 5,254.9 | 4,621.4 | 633.6 | 4,745.1 | -- | 4,794.6 | 4,746.4 |
| HC10_60_27 | 10 | 60 | 27 | 4,420.2 | 5,254.1 | 4,394.5 | 859.6 | 4,665.6 | -- | 4,715.0 | 4,666.9 |
| HC08_60_28 | 8 | 60 | 28 | 4,653.8 | 5,276.4 | 4,638.9 | 637.5 | 4,796.2 | -- | 4,846.4 | 4,797.8 |
| HC10_60_28 | 10 | 60 | 28 | 4,446.2 | 5,276.0 | 4,421.0 | 854.9 | 4,729.1 | -- | 4,779.1 | 4,730.8 |
| HC08_61_18 | 8 | 61 | 18 | 4,790.2 | 5,103.3 | 4,766.5 | 336.8 | 4,696.8 | 2039 | 4,766.9 | 4,788.2 |
| HC10_61_18 | 10 | 61 | 18 | 4,460.1 | 5,091.3 | 4,439.3 | 652.1 | 4,595.6 | -- | 4,674.3 | 4,761.9 |
| HC08_61_19 | 8 | 61 | 19 | 4,788.6 | 5,113.7 | 4,759.6 | 354.1 | 4,697.2 | 2044 | 4,760.5 | 4,764.6 |
| HC10_61_19 | 10 | 61 | 19 | 4,407.1 | 5,105.6 | 4,384.6 | 721.0 | 4,636.4 | -- | 4,704.9 | 4,713.4 |
| HC08_61_21 | 8 | 61 | 21 | 4,684.8 | 5,157.4 | 4,635.4 | 522.0 | 4,515.2 | 2033 | 4,636.3 | 4,636.1 |
| HC10_61_21 | 10 | 61 | 21 | 4,437.7 | 5,157.3 | 4,635.4 | 522.0 | 4,514.6 | 2031 | 4,635.4 | 4,635.3 |
| HC08_61_22 | 8 | 61 | 22 | 4,612.8 | 5,160.5 | 4,571.4 | 589.1 | deactiv. | 2025 | 4,572.4 | 4,572.5 |
| HC10_61_22 | 10 | 61 | 22 | 4,405.7 | 5,160.4 | 4,571.4 | 589.0 | deactiv. | 2025 | 4,571.4 | 4,571.6 |
| HC08_61_24 | 8 | 61 | 24 | 4,572.8 | 5,193.1 | 4,526.9 | 666.2 | 4,612.6 | -- | 4,649.0 | 4,610.2 |
| HC10_61_24 | 10 | 61 | 24 | 4,343.2 | 5,192.9 | 4,526.9 | 666.0 | 4,611.9 | -- | 4,648.2 | 4,609.4 |
| HC08_61_28 | 8 | 61 | 28 | 4,636.4 | 5,298.6 | 4,588.3 | 710.2 | 4,785.3 | -- | 4,841.3 | 4,788.0 |
| HC10_61_28 | 10 | 61 | 28 | 4,396.2 | 5,298.5 | 4,588.3 | 710.2 | 4,784.8 | -- | 4,840.8 | 4,787.5 |
| HC08_62_18 | 8 | 62 | 18 | 4,844.4 | 5,141.7 | 4,781.6 | 360.1 | 4,624.8 | 2027 | 4,782.3 | 4,781.8 |
| HC10_62_18 | 10 | 62 | 18 | 4,530.2 | 5,141.4 | 4,781.6 | 359.8 | 4,623.6 | 2027 | 4,781.6 | 4,781.3 |
| HC08_62_19 | 8 | 62 | 19 | 4,794.8 | 5,151.8 | 4,733.9 | 417.8 | 4,632.0 | 2039 | 4,734.9 | 4,734.3 |
| HC10_62_19 | 10 | 62 | 19 | 4,490.4 | 5,151.6 | 4,733.9 | 417.6 | 4,631.1 | 2039 | 4,733.9 | 4,733.5 |
| HC08_62_21 | 8 | 62 | 21 | 4,687.3 | 5,168.5 | 4,633.7 | 534.8 | 4,529.9 | 2039 | 4,634.4 | 4,633.8 |
| HC10_62_21 | 10 | 62 | 21 | 4,419.5 | 5,168.3 | 4,633.7 | 534.6 | 4,529.3 | 2039 | 4,633.7 | 4,633.3 |

**Table 3.** Head constraints and simulated heads.—Continued

[deactiv., cell was deactivated; --, constraint was not violated]

| Constraint name | Layer | Row | Column | Cell top eleva-tion | 2009 simulated hydraulic head | Lower head con-straint (feet) | Height of 2009 simulated head above constraint (feet) | Non-optimized (2109) simulated hydraulic head | Head-constraint violation year (non-optimized simulation) | Analysis 1 (2059) simulated hydraulic head | Analysis 2 (2109) simulated hydraulic head |
|---|---|---|---|---|---|---|---|---|---|---|---|
| HC08_62_24 | 8 | 62 | 24 | 4,559.6 | 5,228.3 | 4,512.4 | 715.9 | 4,605.2 | -- | 4,659.3 | 4,617.7 |
| HC10_62_24 | 10 | 62 | 24 | 4,323.4 | 5,228.2 | 4,512.4 | 715.9 | 4,605.0 | -- | 4,659.0 | 4,617.4 |
| HC08_62_31 | 8 | 62 | 31 | 4,726.6 | 5,380.7 | 4,678.7 | 702.0 | 5,009.0 | -- | 5,062.6 | 5,011.2 |
| HC10_62_31 | 10 | 62 | 31 | 4,486.9 | 5,380.7 | 4,678.7 | 702.0 | 5,008.7 | -- | 5,062.3 | 5,010.9 |
| HC08_63_14 | 8 | 63 | 14 | 5,171.3 | 5,356.1 | 5,119.8 | 236.3 | 5,112.9 | 2099 | 5,152.3 | 5,140.7 |
| HC10_63_14 | 10 | 63 | 14 | 4,913.4 | 5,356.0 | 5,119.8 | 236.2 | 5,112.3 | 2096 | 5,151.7 | 5,140.2 |
| HC08_63_18 | 8 | 63 | 18 | 4,759.6 | 5,187.4 | 4,703.3 | 484.1 | deactiv. | 2022 | 4,704.3 | 4,703.9 |
| HC10_63_18 | 10 | 63 | 18 | 4,478.0 | 5,187.1 | 4,703.3 | 483.8 | 4,415.4 | 2021 | 4,703.3 | 4,703.0 |
| HC08_63_19 | 8 | 63 | 19 | 4,706.0 | 5,199.9 | 4,656.8 | 543.0 | 4,598.7 | 2070 | 4,696.5 | 4,678.1 |
| HC10_63_19 | 10 | 63 | 19 | 4,460.0 | 5,199.6 | 4,656.8 | 542.8 | 4,597.8 | 2070 | 4,695.6 | 4,677.3 |
| HC08_63_21 | 8 | 63 | 21 | 4,594.5 | 5,200.9 | 4,548.2 | 652.7 | 4,481.3 | 2066 | 4,567.6 | 4,548.9 |
| HC10_63_21 | 10 | 63 | 21 | 4,363.0 | 5,200.7 | 4,548.2 | 652.4 | 4,480.4 | 2066 | 4,566.4 | 4,547.9 |
| HC08_63_23 | 8 | 63 | 23 | 4,581.3 | 5,215.5 | 4,533.0 | 682.5 | 4,548.3 | -- | 4,594.3 | 4,569.3 |
| HC10_63_23 | 10 | 63 | 23 | 4,339.4 | 5,215.2 | 4,533.0 | 682.3 | 4,547.6 | -- | 4,593.6 | 4,568.6 |
| HC08_63_25 | 8 | 63 | 25 | 4,627.5 | 5,276.2 | 4,574.1 | 702.0 | 4,627.7 | -- | 4,702.9 | 4,645.2 |
| HC10_63_25 | 10 | 63 | 25 | 4,360.4 | 5,276.1 | 4,574.1 | 702.0 | 4,627.3 | -- | 4,702.6 | 4,644.9 |
| HC08_63_31 | 8 | 63 | 31 | 4,712.0 | 5,417.8 | 4,664.9 | 752.9 | 5,036.5 | -- | 5,095.0 | 5,039.2 |
| HC10_63_31 | 10 | 63 | 31 | 4,476.8 | 5,417.8 | 4,664.9 | 752.9 | 5,036.2 | -- | 5,094.6 | 5,038.9 |
| HC08_64_13 | 8 | 64 | 13 | 5,225.8 | 5,384.0 | 5,190.5 | 193.5 | 5,193.9 | -- | 5,222.2 | 5,215.6 |
| HC10_64_13 | 10 | 64 | 13 | 5,049.3 | 5,383.8 | 5,190.5 | 193.3 | 5,193.4 | -- | 5,221.7 | 5,215.1 |
| HC08_64_14 | 8 | 64 | 14 | 5,187.1 | 5,394.7 | 5,137.9 | 256.8 | 4,998.3 | 2025 | 5,138.8 | 5,138.5 |
| HC10_64_14 | 10 | 64 | 14 | 4,941.0 | 5,394.4 | 5,137.9 | 256.5 | 4,997.2 | 2024 | 5,137.9 | 5,137.9 |
| HC08_64_16 | 8 | 64 | 16 | 4,897.5 | 5,350.5 | 4,849.4 | 501.1 | 4,815.7 | 2063 | 4,855.1 | 4,850.3 |
| HC10_64_16 | 10 | 64 | 16 | 4,656.8 | 5,350.3 | 4,849.4 | 500.9 | 4,814.7 | 2059 | 4,854.0 | 4,849.3 |
| HC08_64_18 | 8 | 64 | 18 | 4,723.5 | 5,288.0 | 4,674.1 | 613.9 | 4,614.7 | 2063 | 4,701.6 | 4,678.9 |
| HC10_64_18 | 10 | 64 | 18 | 4,476.1 | 5,287.8 | 4,674.1 | 613.8 | 4,613.7 | 2063 | 4,700.6 | 4,678.0 |
| HC08_64_21 | 8 | 64 | 21 | 4,591.1 | 5,251.6 | 4,542.9 | 708.6 | 4,583.7 | -- | 4,646.4 | 4,601.1 |
| HC10_64_21 | 10 | 64 | 21 | 4,350.2 | 5,251.4 | 4,542.9 | 708.4 | 4,583.1 | -- | 4,645.9 | 4,600.6 |
| HC08_64_22 | 8 | 64 | 22 | 4,580.7 | 5,247.0 | 4,533.1 | 713.9 | 4,564.8 | -- | 4,621.0 | 4,577.6 |
| HC10_64_22 | 10 | 64 | 22 | 4,342.2 | 5,246.9 | 4,533.1 | 713.8 | 4,564.3 | -- | 4,620.6 | 4,577.1 |
| HC08_64_24 | 8 | 64 | 24 | 4,581.6 | 5,253.1 | 4,532.4 | 720.8 | 4,428.8 | 2066 | 4,545.4 | 4,533.0 |
| HC10_64_24 | 10 | 64 | 24 | 4,335.4 | 5,252.9 | 4,532.4 | 720.5 | 4,427.9 | 2066 | 4,543.8 | 4,532.1 |
| HC08_64_26 | 8 | 64 | 26 | 4,621.2 | 5,319.0 | 4,571.8 | 747.1 | 4,649.3 | -- | 4,735.7 | 4,662.0 |
| HC10_64_26 | 10 | 64 | 26 | 4,374.2 | 5,318.9 | 4,571.8 | 747.1 | 4,649.0 | -- | 4,735.3 | 4,661.7 |
| HC08_65_17 | 8 | 65 | 17 | 4,784.2 | 5,386.7 | 4,733.0 | 653.7 | 4,704.8 | 2079 | 4,753.1 | 4,734.7 |
| HC10_65_17 | 10 | 65 | 17 | 4,528.3 | 5,386.5 | 4,733.0 | 653.5 | 4,703.9 | 2079 | 4,752.1 | 4,733.8 |
| HC08_65_18 | 8 | 65 | 18 | 4,728.1 | 5,366.2 | 4,675.7 | 690.5 | 4,665.6 | 2103 | 4,724.8 | 4,697.3 |
| HC10_65_18 | 10 | 65 | 18 | 4,466.0 | 5,366.0 | 4,675.7 | 690.3 | 4,664.6 | 2103 | 4,723.7 | 4,696.4 |
| HC08_65_22 | 8 | 65 | 22 | 4,629.1 | 5,295.1 | 4,578.3 | 716.8 | 4,614.6 | -- | 4,686.0 | 4,621.9 |
| HC10_65_22 | 10 | 65 | 22 | 4,374.9 | 5,295.0 | 4,578.3 | 716.7 | 4,613.9 | -- | 4,685.4 | 4,621.2 |
| HC08_65_24 | 8 | 65 | 24 | 4,560.1 | 5,274.2 | 4,516.1 | 758.1 | 4,517.4 | -- | 4,592.7 | 4,540.1 |
| HC10_65_24 | 10 | 65 | 24 | 4,340.2 | 5,274.0 | 4,516.1 | 757.8 | 4,516.7 | -- | 4,592.1 | 4,539.4 |
| HC08_65_25 | 8 | 65 | 25 | 4,536.8 | 5,295.7 | 4,496.1 | 799.6 | 4,536.9 | -- | 4,612.4 | 4,551.0 |
| HC10_65_25 | 10 | 65 | 25 | 4,332.9 | 5,295.6 | 4,496.1 | 799.5 | 4,536.6 | -- | 4,612.1 | 4,550.7 |
| HC08_65_26 | 8 | 65 | 26 | 4,571.6 | 5,325.6 | 4,528.4 | 797.2 | 4,529.5 | -- | 4,601.2 | 4,540.4 |
| HC10_65_26 | 10 | 65 | 26 | 4,355.4 | 5,325.5 | 4,528.4 | 797.1 | 4,528.3 | 2109 | 4,599.9 | 4,539.1 |

**Table 3.** Head constraints and simulated heads.—Continued

[deactiv., cell was deactivated; --, constraint was not violated]

| Constraint name | Layer | Row | Column | Cell top elevation | 2009 simulated hydraulic head | Lower head constraint (feet) | Height of 2009 simulated head above constraint (feet) | Non-optimized (2109) simulated hydraulic head | Head-constraint violation year (non-optimized simulation) | Analysis 1 (2059) simulated hydraulic head | Analysis 2 (2109) simulated hydraulic head |
|---|---|---|---|---|---|---|---|---|---|---|---|
| HC08_66_15 | 8 | 66 | 15 | 5,033.9 | 5,449.4 | 4,984.3 | 465.0 | 4,939.4 | 2066 | 4,991.0 | 4,985.1 |
| HC10_66_15 | 10 | 66 | 15 | 4,785.9 | 5,449.0 | 4,984.3 | 464.7 | 4,938.6 | 2066 | 4,990.0 | 4,984.3 |
| HC08_66_16 | 8 | 66 | 16 | 4,903.0 | 5,462.0 | 4,852.0 | 609.9 | 4,843.2 | 2103 | 4,886.9 | 4,874.1 |
| HC10_66_16 | 10 | 66 | 16 | 4,648.3 | 5,461.9 | 4,852.0 | 609.9 | 4,842.6 | 2099 | 4,886.2 | 4,873.5 |
| HC08_66_17 | 8 | 66 | 17 | 4,799.4 | 5,444.0 | 4,748.6 | 695.3 | 4,691.7 | 2059 | 4,749.2 | 4,748.8 |
| HC10_66_17 | 10 | 66 | 17 | 4,545.5 | 5,443.9 | 4,748.6 | 695.2 | 4,691.4 | 2059 | 4,748.8 | 4,748.6 |
| HC08_66_25 | 8 | 66 | 25 | 4,555.1 | 5,340.0 | 4,513.4 | 826.6 | 4,538.1 | -- | 4,610.8 | 4,541.0 |
| HC10_66_25 | 10 | 66 | 25 | 4,346.5 | 5,340.0 | 4,513.4 | 826.6 | 4,537.3 | -- | 4,610.0 | 4,540.2 |
| HC08_66_26 | 8 | 66 | 26 | 4,565.3 | 5,335.5 | 4,525.4 | 810.1 | 4,624.8 | -- | 4,713.4 | 4,630.5 |
| HC10_66_26 | 10 | 66 | 26 | 4,365.5 | 5,338.3 | 4,525.4 | 813.0 | 4,624.6 | -- | 4,713.1 | 4,630.3 |
| HC08_67_16 | 8 | 67 | 16 | 4,968.5 | 5,501.5 | 4,917.8 | 583.7 | 4,900.3 | 2090 | 4,942.7 | 4,927.9 |
| HC10_67_16 | 10 | 67 | 16 | 4,714.8 | 5,499.9 | 4,917.8 | 582.1 | 4,899.8 | 2090 | 4,942.1 | 4,927.4 |
| HC08_67_18 | 8 | 67 | 18 | 4,799.5 | 5,480.4 | 4,747.3 | 733.1 | 4,744.9 | 2109 | 4,798.2 | 4,773.9 |
| HC10_67_18 | 10 | 67 | 18 | 4,538.6 | 5,480.3 | 4,747.3 | 732.9 | 4,744.1 | 2109 | 4,797.4 | 4,773.1 |
| HC08_67_23 | 8 | 67 | 23 | 4,581.8 | 5,407.4 | 4,538.6 | 868.8 | 4,731.5 | -- | 4,788.4 | 4,693.5 |
| HC10_67_23 | 10 | 67 | 23 | 4,365.9 | 5,407.3 | 4,538.6 | 868.7 | 4,731.3 | -- | 4,788.1 | 4,693.2 |
| HC08_67_24 | 8 | 67 | 24 | 4,570.8 | 5,405.2 | 4,525.9 | 879.2 | 4,671.3 | -- | 4,708.5 | 4,631.1 |
| HC10_67_24 | 10 | 67 | 24 | 4,346.4 | 5,405.1 | 4,525.9 | 879.2 | 4,671.2 | -- | 4,708.2 | 4,630.9 |
| HC08_67_25 | 8 | 67 | 25 | 4,587.5 | 5,399.9 | 4,544.6 | 855.3 | 4,648.1 | -- | 4,714.0 | 4,637.7 |
| HC10_67_25 | 10 | 67 | 25 | 4,373.1 | 5,399.9 | 4,544.6 | 855.3 | 4,648.0 | -- | 4,713.8 | 4,637.5 |
| HC08_68_15 | 8 | 68 | 15 | 5,103.3 | 5,546.1 | 5,054.2 | 491.9 | 5,019.9 | 2070 | 5,061.2 | 5,054.8 |
| HC10_68_15 | 10 | 68 | 15 | 4,857.7 | 5,546.0 | 5,054.2 | 491.7 | 5,019.3 | 2066 | 5,060.5 | 5,054.2 |
| HC08_68_16 | 8 | 68 | 16 | 5,000.8 | 5,552.8 | 4,949.7 | 603.1 | 4,902.6 | 2059 | 4,950.6 | 4,950.3 |
| HC10_68_16 | 10 | 68 | 16 | 4,745.3 | 5,552.7 | 4,949.7 | 603.0 | 4,901.9 | 2056 | 4,949.7 | 4,949.7 |
| HC08_68_18 | 8 | 68 | 18 | 4,834.5 | 5,534.9 | 4,781.9 | 753.0 | 4,741.4 | 2079 | 4,809.3 | 4,782.9 |
| HC10_68_18 | 10 | 68 | 18 | 4,571.5 | 5,534.9 | 4,781.9 | 752.9 | 4,740.4 | 2079 | 4,808.1 | 4,781.9 |
| HC08_68_19 | 8 | 68 | 19 | 4,797.1 | 5,504.5 | 4,741.9 | 762.5 | 4,746.5 | -- | 4,816.2 | 4,791.0 |
| HC10_68_19 | 10 | 68 | 19 | 4,521.0 | 5,504.3 | 4,741.9 | 762.4 | 4,745.7 | -- | 4,815.6 | 4,790.3 |
| HC08_68_24 | 8 | 68 | 24 | 4,587.1 | 5,446.4 | 4,543.1 | 903.3 | deactiv. | 2020 | 4,544.9 | 4,544.2 |
| HC10_68_24 | 10 | 68 | 24 | 4,366.8 | 5,446.3 | 4,543.1 | 903.2 | deactiv. | 2019 | 4,543.0 | 4,542.9 |
| HC08_68_26 | 8 | 68 | 26 | 4,630.5 | 5,423.5 | 4,584.9 | 838.5 | 4,637.4 | -- | 4,720.4 | 4,637.4 |
| HC10_68_26 | 10 | 68 | 26 | 4,402.5 | 5,423.4 | 4,584.9 | 838.4 | 4,636.5 | -- | 4,719.3 | 4,636.4 |
| HC08_68_27 | 8 | 68 | 27 | 4,614.3 | 5,419.3 | 4,567.6 | 851.7 | 4,791.2 | -- | 4,890.3 | 4,796.8 |
| HC10_68_27 | 10 | 68 | 27 | 4,380.4 | 5,419.1 | 4,567.6 | 851.6 | 4,791.0 | -- | 4,890.1 | 4,796.5 |
| HC08_68_28 | 8 | 68 | 28 | 4,595.9 | 5,479.0 | 4,556.5 | 922.5 | 4,919.7 | -- | 5,012.2 | 4,926.1 |
| HC10_68_28 | 10 | 68 | 28 | 4,399.1 | 5,479.0 | 4,556.5 | 922.5 | 4,919.3 | -- | 5,011.8 | 4,925.7 |
| HC08_69_15 | 8 | 69 | 15 | 5,105.6 | 5,581.2 | 5,060.3 | 520.9 | 5,052.5 | 2103 | 5,091.6 | 5,067.0 |
| HC10_69_15 | 10 | 69 | 15 | 4,879.0 | 5,581.1 | 5,060.3 | 520.8 | 5,051.8 | 2099 | 5,090.8 | 5,066.3 |
| HC08_69_17 | 8 | 69 | 17 | 4,937.2 | 5,573.8 | 4,883.6 | 690.1 | 4,903.0 | -- | 4,940.6 | 4,921.7 |
| HC10_69_17 | 10 | 69 | 17 | 4,669.2 | 5,573.6 | 4,883.6 | 690.0 | 4,902.2 | -- | 4,939.8 | 4,920.9 |
| HC08_69_19 | 8 | 69 | 19 | 4,834.8 | 5,536.8 | 4,778.8 | 758.0 | 4,633.2 | 2050 | 4,780.8 | 4,780.0 |
| HC10_69_19 | 10 | 69 | 19 | 4,554.4 | 5,536.7 | 4,778.8 | 757.9 | 4,632.0 | 2047 | 4,778.8 | 4,778.7 |
| HC08_69_24 | 8 | 69 | 24 | 4,621.2 | 5,498.1 | 4,576.1 | 922.0 | deactiv. | 2039 | 4,577.8 | 4,577.2 |
| HC10_69_24 | 10 | 69 | 24 | 4,395.8 | 5,498.1 | 4,576.1 | 922.0 | deactiv. | 2039 | 4,576.1 | 4,576.0 |
| HC08_69_25 | 8 | 69 | 25 | 4,649.6 | 5,480.4 | 4,603.0 | 877.4 | deactiv. | 2042 | 4,604.9 | 4,604.2 |
| HC10_69_25 | 10 | 69 | 25 | 4,416.9 | 5,480.4 | 4,603.0 | 877.3 | 4,250.2 | 2039 | 4,603.0 | 4,602.9 |

**Table 3.** Head constraints and simulated heads.—Continued

[deactiv., cell was deactivated; --, constraint was not violated]

| Constraint name | Layer | Row | Column | Cell top elevation | 2009 simulated hydraulic head | Lower head constraint (feet) | Height of 2009 simulated head above constraint (feet) | Non-optimized (2109) simulated hydraulic head | Head-constraint violation year (non-optimized simulation) | Analysis 1 (2059) simulated hydraulic head | Analysis 2 (2109) simulated hydraulic head |
|---|---|---|---|---|---|---|---|---|---|---|---|
| HC08_70_16 | 8 | 70 | 16 | 5,089.6 | 5,612.1 | 5,034.6 | 577.5 | 5,055.5 | -- | 5,091.9 | 5,064.4 |
| HC10_70_16 | 10 | 70 | 16 | 4,814.6 | 5,612.0 | 5,034.6 | 577.4 | 5,054.7 | -- | 5,091.2 | 5,063.6 |
| HC08_70_18 | 8 | 70 | 18 | 4,883.8 | 5,576.3 | 4,827.9 | 748.4 | 4,870.8 | -- | 4,930.7 | 4,883.0 |
| HC10_70_18 | 10 | 70 | 18 | 4,604.4 | 5,576.1 | 4,827.9 | 748.2 | 4,869.7 | -- | 4,929.6 | 4,881.9 |
| HC08_70_23 | 8 | 70 | 23 | 4,626.3 | 5,550.9 | 4,584.0 | 966.9 | 4,763.8 | -- | 4,800.4 | 4,715.3 |
| HC10_70_23 | 10 | 70 | 23 | 4,414.8 | 5,550.8 | 4,584.0 | 966.8 | 4,762.9 | -- | 4,799.4 | 4,714.4 |
| HC08_70_28 | 8 | 70 | 28 | 4,645.9 | 5,536.7 | 4,630.0 | 906.7 | 4,967.5 | -- | 5,055.8 | 4,973.3 |
| HC10_70_28 | 10 | 70 | 28 | 4,425.1 | 5,536.6 | 4,630.0 | 906.6 | 4,966.6 | -- | 5,054.9 | 4,972.4 |
| HC08_71_20 | 8 | 71 | 20 | 4,675.2 | 5,584.3 | 4,624.2 | 960.1 | 4,847.9 | -- | 4,930.1 | 4,859.3 |
| HC10_71_20 | 10 | 71 | 20 | 4,420.4 | 5,584.2 | 4,624.2 | 960.0 | 4,847.2 | -- | 4,929.4 | 4,858.7 |
| HC08_71_24 | 8 | 71 | 24 | 4,652.9 | 5,560.4 | 4,609.7 | 950.7 | 4,859.4 | -- | 4,913.2 | 4,827.6 |
| HC10_71_24 | 10 | 71 | 24 | 4,437.0 | 5,560.4 | 4,609.7 | 950.6 | 4,858.8 | -- | 4,912.5 | 4,827.0 |
| HC08_71_28 | 8 | 71 | 28 | 4,666.0 | 5,578.9 | 4,619.9 | 959.0 | 4,932.9 | -- | 5,013.6 | 4,937.6 |
| HC10_71_28 | 10 | 71 | 28 | 4,435.4 | 5,578.8 | 4,619.9 | 958.9 | 4,931.3 | -- | 5,012.0 | 4,936.0 |
| HC08_72_19 | 8 | 72 | 19 | 4,785.6 | 5,609.4 | 4,736.2 | 873.2 | 4,881.3 | -- | 4,949.5 | 4,884.6 |
| HC10_72_19 | 10 | 72 | 19 | 4,538.5 | 5,609.4 | 4,736.2 | 873.1 | 4,880.9 | -- | 4,949.1 | 4,884.2 |
| HC08_72_20 | 8 | 72 | 20 | 4,719.3 | 5,597.2 | 4,669.9 | 927.4 | 4,815.2 | -- | 4,882.4 | 4,818.3 |
| HC10_72_20 | 10 | 72 | 20 | 4,471.9 | 5,597.1 | 4,669.9 | 927.3 | 4,814.5 | -- | 4,881.7 | 4,817.6 |
| HC08_72_25 | 8 | 72 | 25 | 4,654.9 | 5,576.7 | 4,613.0 | 963.6 | 4,971.4 | -- | 5,046.1 | 4,963.5 |
| HC10_72_25 | 10 | 72 | 25 | 4,445.7 | 5,576.5 | 4,613.0 | 963.5 | 4,970.7 | -- | 5,045.4 | 4,962.8 |
| HC08_73_20 | 8 | 73 | 20 | 4,803.9 | 5,608.0 | 4,753.7 | 854.3 | 4,774.9 | -- | 4,815.0 | 4,772.6 |
| HC10_73_20 | 10 | 73 | 20 | 4,552.9 | 5,607.9 | 4,753.7 | 854.2 | 4,773.5 | -- | 4,813.6 | 4,771.2 |
| HC08_74_18 | 8 | 74 | 18 | 4,900.9 | 5,671.6 | 4,860.1 | 811.4 | 5,101.8 | -- | 5,137.9 | 5,080.4 |
| HC10_74_18 | 10 | 74 | 18 | 4,697.0 | 5,671.5 | 4,860.1 | 811.4 | 5,101.5 | -- | 5,137.6 | 5,080.1 |
| HC08_74_19 | 8 | 74 | 19 | 4,832.3 | 5,637.8 | 4,790.3 | 847.5 | 4,876.7 | -- | 4,897.7 | 4,846.7 |
| HC10_74_19 | 10 | 74 | 19 | 4,622.2 | 5,637.7 | 4,790.3 | 847.4 | 4,875.9 | -- | 4,896.8 | 4,845.9 |
| HC08_74_20 | 8 | 74 | 20 | 4,783.2 | 5,641.6 | 4,739.7 | 901.9 | 4,766.0 | -- | 4,782.7 | 4,756.4 |
| HC10_74_20 | 10 | 74 | 20 | 4,565.6 | 5,641.6 | 4,739.7 | 901.9 | 4,764.9 | -- | 4,781.6 | 4,755.2 |
| HC08_75_19 | 8 | 75 | 19 | 4,911.6 | 5,684.9 | 4,867.9 | 817.1 | 5,044.8 | -- | 4,991.1 | 4,949.8 |
| HC10_75_19 | 10 | 75 | 19 | 4,692.7 | 5,684.8 | 4,867.9 | 817.0 | 5,044.1 | -- | 4,990.3 | 4,949.0 |
| HC08_75_20 | 8 | 75 | 20 | 4,814.7 | 5,667.2 | 4,772.9 | 894.3 | deactiv. | 2017 | 4,773.9 | 4,773.4 |
| HC10_75_20 | 10 | 75 | 20 | 4,605.7 | 5,667.1 | 4,772.9 | 894.2 | deactiv. | 2017 | 4,772.9 | 4,772.9 |
| HC08_75_23 | 8 | 75 | 23 | 4,771.9 | 5,667.2 | 4,719.6 | 947.6 | 4,741.6 | -- | 4,769.3 | 4,732.4 |
| HC10_75_23 | 10 | 75 | 23 | 4,510.5 | 5,667.1 | 4,719.6 | 947.5 | 4,738.1 | -- | 4,765.7 | 4,728.9 |
| HC08_75_24 | 8 | 75 | 24 | 4,780.4 | 5,687.0 | 4,724.6 | 962.4 | 4,999.3 | -- | 5,050.7 | 4,996.4 |
| HC10_75_24 | 10 | 75 | 24 | 4,501.1 | 5,687.0 | 4,724.6 | 962.4 | 4,998.1 | -- | 5,049.5 | 4,995.1 |
| HC08_76_21 | 8 | 76 | 21 | 4,866.0 | 5,671.3 | 4,814.9 | 856.4 | 5,078.3 | -- | 5,036.2 | 4,993.9 |
| HC10_76_21 | 10 | 76 | 21 | 4,610.6 | 5,671.0 | 4,814.9 | 856.1 | 5,078.1 | -- | 5,035.9 | 4,993.6 |
| HC08_76_23 | 8 | 76 | 23 | 4,856.2 | 5,652.9 | 4,796.0 | 856.9 | 4,828.3 | -- | 4,852.4 | 4,823.5 |
| HC10_76_23 | 10 | 76 | 23 | 4,555.0 | 5,652.4 | 4,796.0 | 856.4 | 4,824.5 | -- | 4,848.5 | 4,819.7 |
| HC08_77_22 | 8 | 77 | 22 | 4,885.3 | 5,658.5 | 4,828.5 | 830.0 | 4,796.0 | 2079 | 4,838.9 | 4,829.0 |
| HC10_77_22 | 10 | 77 | 22 | 4,601.1 | 5,657.7 | 4,828.5 | 829.2 | 4,795.2 | 2079 | 4,837.8 | 4,828.5 |
| HC08_78_21 | 8 | 78 | 21 | 4,984.4 | 5,724.9 | 4,930.4 | 794.5 | 5,098.0 | -- | 5,121.1 | 5,073.6 |
| HC10_78_21 | 10 | 78 | 21 | 4,714.3 | 5,724.1 | 4,930.4 | 793.6 | 5,095.2 | -- | 5,118.2 | 5,070.8 |

For the non-optimized simulation, 63 of the 182 head constraints were not met, and 10 of the head-constraint cells were deactivated (converted to no-flow cells) because the estimated head in the cell dropped below the cell bottom during solution of the groundwater-flow equation. These 10 deactivated head-constraint cells (table 3) are associated with 6 managed-flow cells (table 2). The head constraints that were violated during the non-optimized simulation are identified in table 3 by the presence of a year in the column labeled "Head-constraint violation year (non-optimized simulation)." Head constraints were violated in years ranging from 2017 to 2109.

If no managed-flow cells had been deactivated, the total pumping rate for managed-flow cells would have been 64.9 $ft^3/s$ throughout the non-optimized simulation (table 4). However, four of the managed-flow cells were deactivated; as a result, the total pumping rate for managed-flow cells at the end of the simulation was 54.9 $ft^3/s$. The total rate of withdrawals from the Arapahoe aquifer was 107.5 $ft^3/s$ (table 4) at the beginning of the simulation, when no cells had been deactivated.

## Analysis 1: Optimization of Pumping for Managed Withdrawals Over 50-Year Period

Analysis 1 was designed to establish an optimal set of constant withdrawal rates for 96 decision variables representing 119 municipal wells. In this 50-year analysis, the optimal solution was obtained when 81 of the 96 decision variables were assigned (by GWM) at their maximum withdrawal rates, and 2 of the decision variables were assigned (by GWM) a withdrawal rate of zero. The other 13 decision variables had nonzero values less than their maximum rates. No constraints were violated. Table 2 lists the optimized withdrawal rates for individual decision variables for this analysis. The total optimized withdrawal rate for managed wells for the 50-year analysis was about 58.8 $ft^3/s$ (table 4). This rate is about 9 percent smaller than the sum of the maximum withdrawal rates for managed wells, which is about 64.9 $ft^3/s$. Of the 58.8 $ft^3/s$ total managed withdrawals, 39.4 $ft^3/s$ was associated with wells that were included in the 2004–2009 simulation and were converted to be represented as managed wells. The remaining 19.4 $ft^3/s$ was associated with wells that were newly constructed or otherwise not included in the 2004–2009 simulation. The rate of pumping from the Arapahoe aquifer as a whole for this analysis was 101.4 $ft^3/s$ (table 4).

The 2059 potentiometric surfaces for the upper and lower Arapahoe aquifers, as simulated using the withdrawal rates from the 50-year optimization analysis, are shown in figure 9. The difference in altitude between the simulated 2059 layer-8 potentiometric surface and the top of the Arapahoe aquifer is shown in figure 10. Areas where the 2059 potentiometric surface is below the top of the Arapahoe aquifer (unconfined conditions) are shaded in figure 10. The combined 2059 saturated thickness of the upper and lower Arapahoe aquifers is shown in figure 11. Figures 12–14 (orange curves) show simulated hydrographs for layer-8 cells at three managed-flow cell locations selected as being representative of a range of simulated conditions. The hydrographs illustrate that the optimization results in heads being limited by the head constraints at cell (8, 63, 18) (fig. 12). At cells (8, 73, 20) (fig. 13) and (8, 64, 24) (fig. 14), the head constraints are not limiting and do not affect the optimization outcome. However, the simulated head at cell (8, 64, 24) at the end of the 50-year analysis is only 13 ft above the head constraint (table 3).

**Table 4.** Pumping rates for managed and unmanaged wells, by model package used to simulate the wells.

[$ft^3/s$, cubic feet per second; GWM, Groundwater Management Process; MNW, Multi-Node Well Package]

| Managed or unmanaged | Package or process | Pumping rate for indicated simulation ($ft^3/s$) | | | |
|---|---|---|---|---|---|
| | | 2004–2009 | Non-optimized 2010–2109 | Analysis 1 2010–2059 | Analysis 2 2010–2109 |
| Managed | GWM[1] | 0.0 | [2]64.9 | 58.8 | 53.0 |
| Unmanaged | Well[3] | 43.4 | 37.7 | 37.7 | 37.7 |
| Unmanaged | MNW[4] | 4.9 | 4.9 | 4.9 | 4.9 |
| Total | | 48.3 | [5]107.5 | 101.4 | 95.6 |

[1]Ahlfeld and others (2009).

[2]Initial pumping rate. Because four managed-flow cells were deactivated during the simulation, the total pumping rate for managed wells was 54.9 $ft^3/s$ at the end of the simulation.

[3]Harbaugh (2005) for wells completed in a single aquifer.

[4]Halford and Hanson (2002) for wells completed in multiple aquifers.

[5]Initial total pumping rate. Because four managed-flow cells were deactivated during the simulation, the total pumping rate was 97.5 $ft^3/s$ at the end of the simulation.

## Analysis 2: Optimization of Pumping for Managed Withdrawals Over 100-Year Period

Analysis 2 was designed to provide insight into the question "How much would the total withdrawal rate need to be reduced to extend the time before the hydraulic-head constraints are violated from 50 to 100 years, given the same constraints as in Analysis 1?" Analysis 2 used the same set of 96 decision variables as in Analysis 1, and the maximum withdrawal rate for each decision variable was the same as in Analysis 1. The only differences between Analysis 1 and Analysis 2 were the simulation duration and the simulation time at which the head constraints applied. In Analysis 2 the head constraints applied at the end of the 100-year (2010 through 2109) simulation rather than at the end of the 50-year simulation.

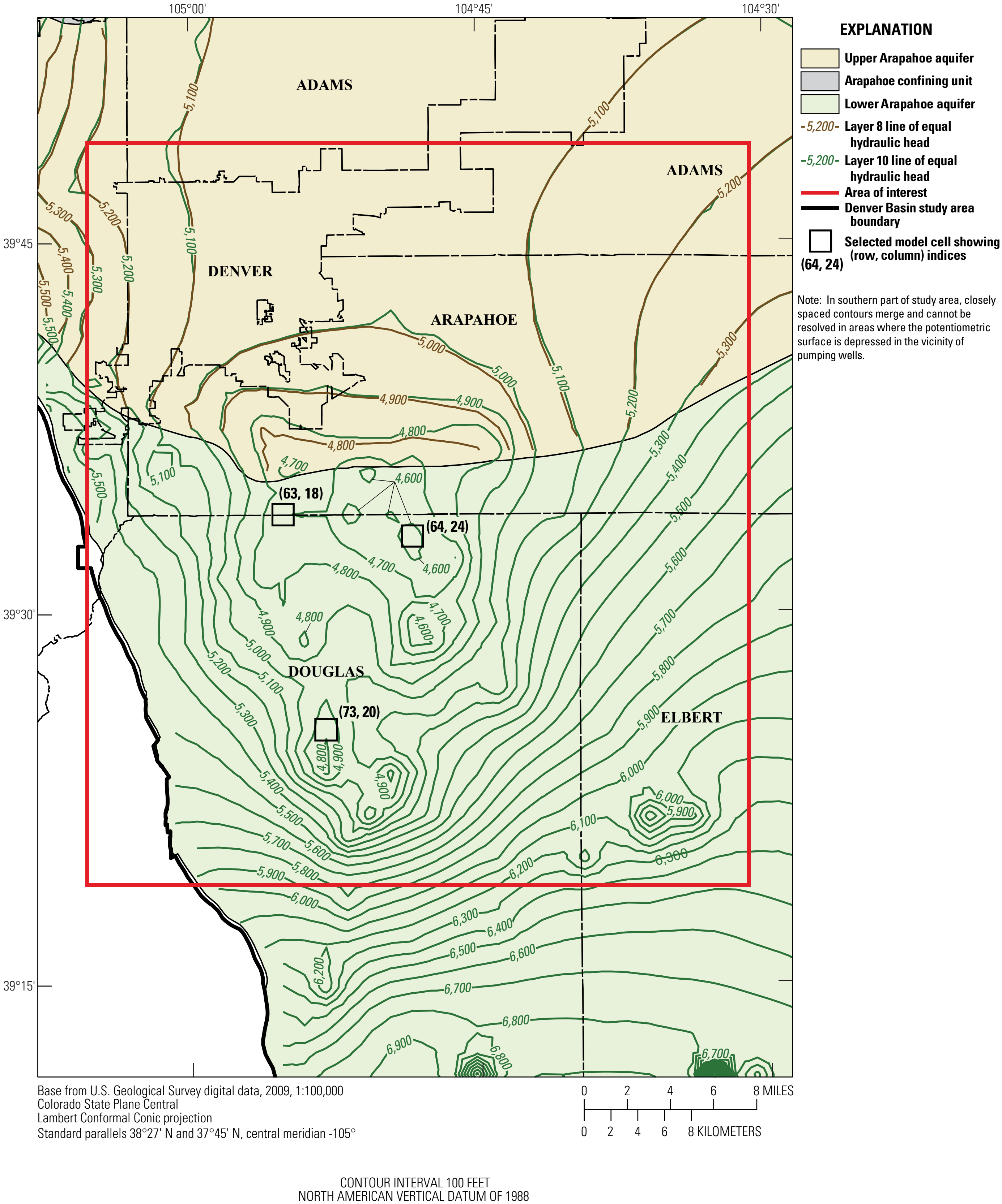



**Figure 9.** Simulated 2059 potentiometric (hydraulic head) surfaces for upper and lower Arapahoe aquifers.

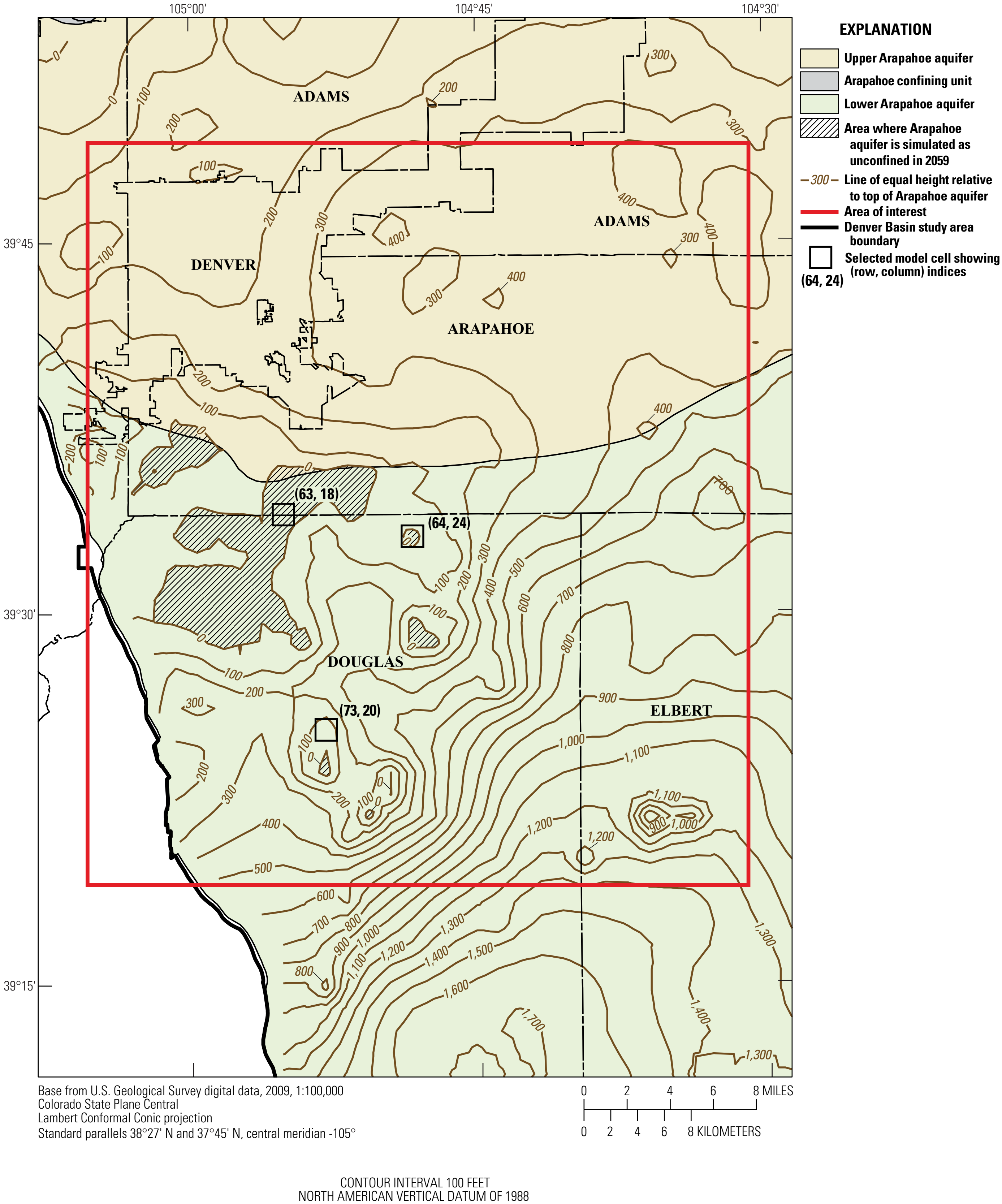
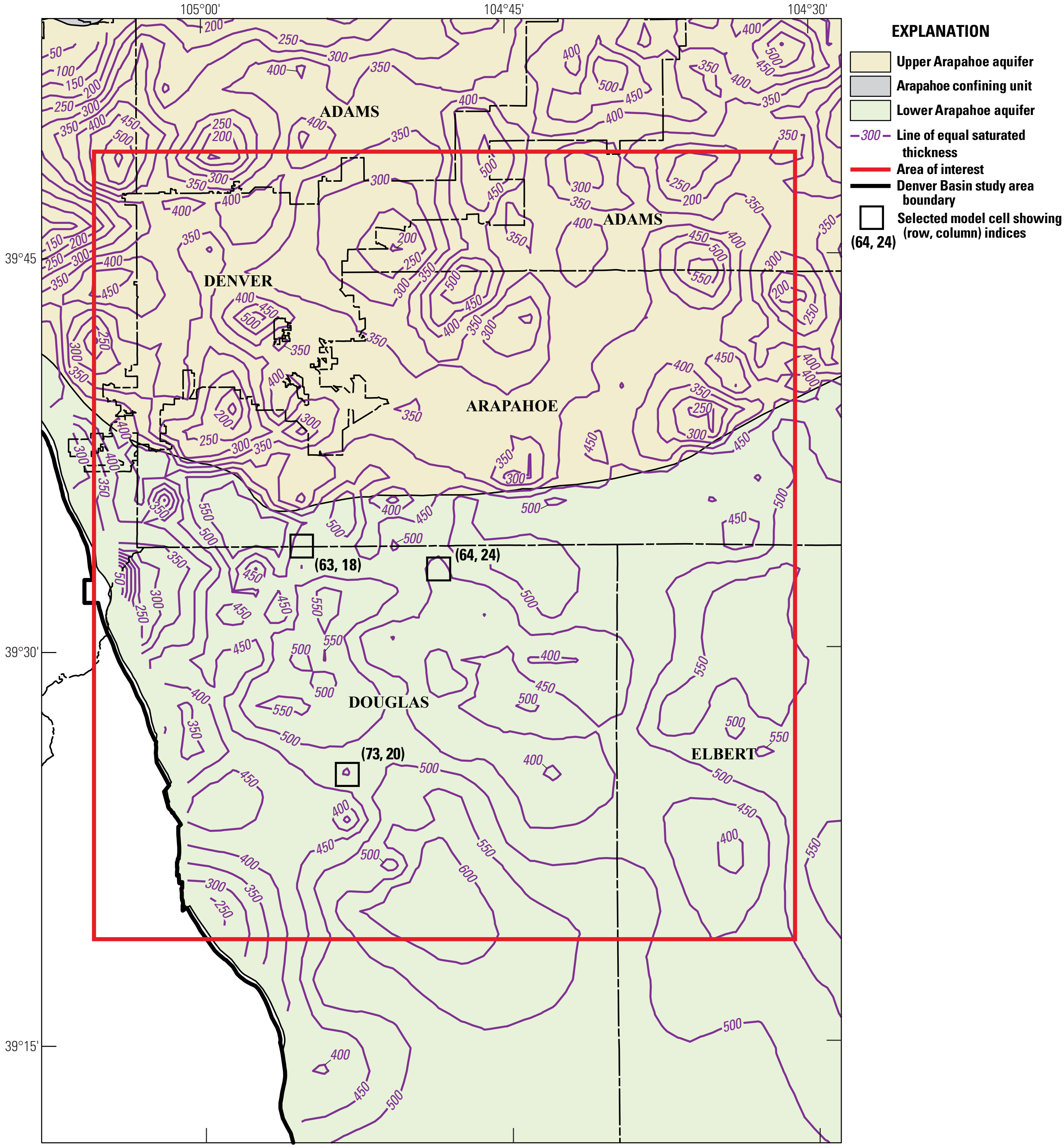



**Figure 10.** Height of layer 8 simulated 2059 potentiometric (hydraulic head) surface relative to top of Arapahoe aquifer.




Base from U.S. Geological Survey digital data, 2009, 1:100,000
Colorado State Plane Central
Lambert Conformal Conic projection
Standard parallels 38°27' N and 37°45' N, central meridian -105°

CONTOUR INTERVAL 50 FEET
NORTH AMERICAN VERTICAL DATUM OF 1988

**Figure 11.** Combined 2059 saturated thickness of upper and lower Arapahoe aquifers.

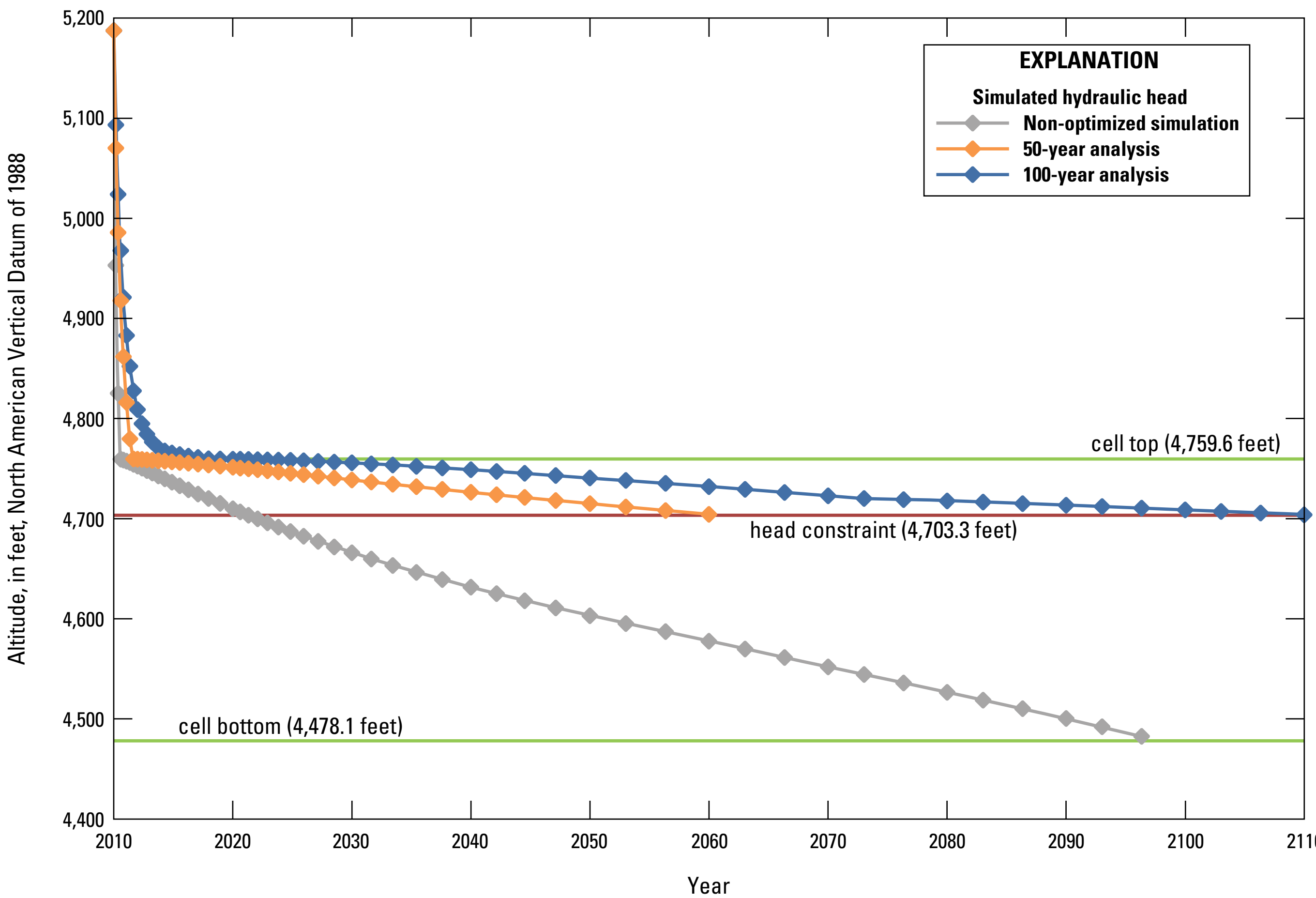



**Figure 12.** Simulated hydrographs and head constraint for cell (8, 63, 18; layer, row, column) for optimization analyses and for model results using non-optimized pumping rates.

In this analysis the optimal solution was obtained when 73 of the 96 decision variables were assigned (by GWM) at their maximum withdrawal rates and 2 of the decision variables were assigned (by GWM) a withdrawal rate of zero. The other 21 decision variables had nonzero values less than their maximum rates. No constraints were violated. The optimized withdrawal rates for managed wells are listed in table 2. The total optimized withdrawal rate for managed wells for the 100-year analysis was about 53.0 $ft^3/s$. This withdrawal rate represents a reduction in the optimized withdrawal rate from managed wells of about 10 percent, relative to the Analysis 1 result. The total optimized withdrawal rate from managed wells is about 18 percent smaller than the sum of the maximum withdrawal rates for managed wells. Of the 53.0 $ft^3/s$ total managed withdrawals, about 35.3 $ft^3/s$ was associated with wells that were included in the 2004–2009 simulation and were converted to be represented as managed wells. The remaining 17.7 $ft^3/s$ was associated with wells that were newly constructed or otherwise not included in the 2004–2009 simulation. The rate of pumping from the Arapahoe aquifer as a whole for this analysis was 95.6 $ft^3/s$ (table 4).

The 2109 potentiometric surfaces for the upper and lower Arapahoe aquifers, as simulated using the withdrawal rates from the 100-year optimization analysis, are shown in figure 15. The difference in altitude between the simulated 2109 potentiometric surface and the top of the Arapahoe aquifer is shown in figure 16. The area where the 2109 potentiometric surface is below the top of the Arapahoe aquifer (unconfined conditions, shaded in fig. 16) is somewhat larger than for Analysis 1 (fig. 10). Even so, this area is mainly limited to the vicinity of the municipal wells. The combined 2109 saturated thickness of the upper and lower Arapahoe aquifers is shown in figure 17. In general, the maps showing the results of the 100-year analysis (figs. 15–17) are substantially similar to those showing the results of the 50-year analysis (figs. 9–11). The similarity results from the similarity in design of the two analyses, which used identical decision-variable and head constraints. Figures 12–14 show simulated hydrographs for layer-8 cells at selected managed-flow cell locations. The hydrographs illustrate that the optimization solution results in heads being limited by the head constraints at cells (8, 63, 18) (fig. 12) and (8, 64, 24) (fig. 14) but not at cell (8, 73, 20) (fig. 13).

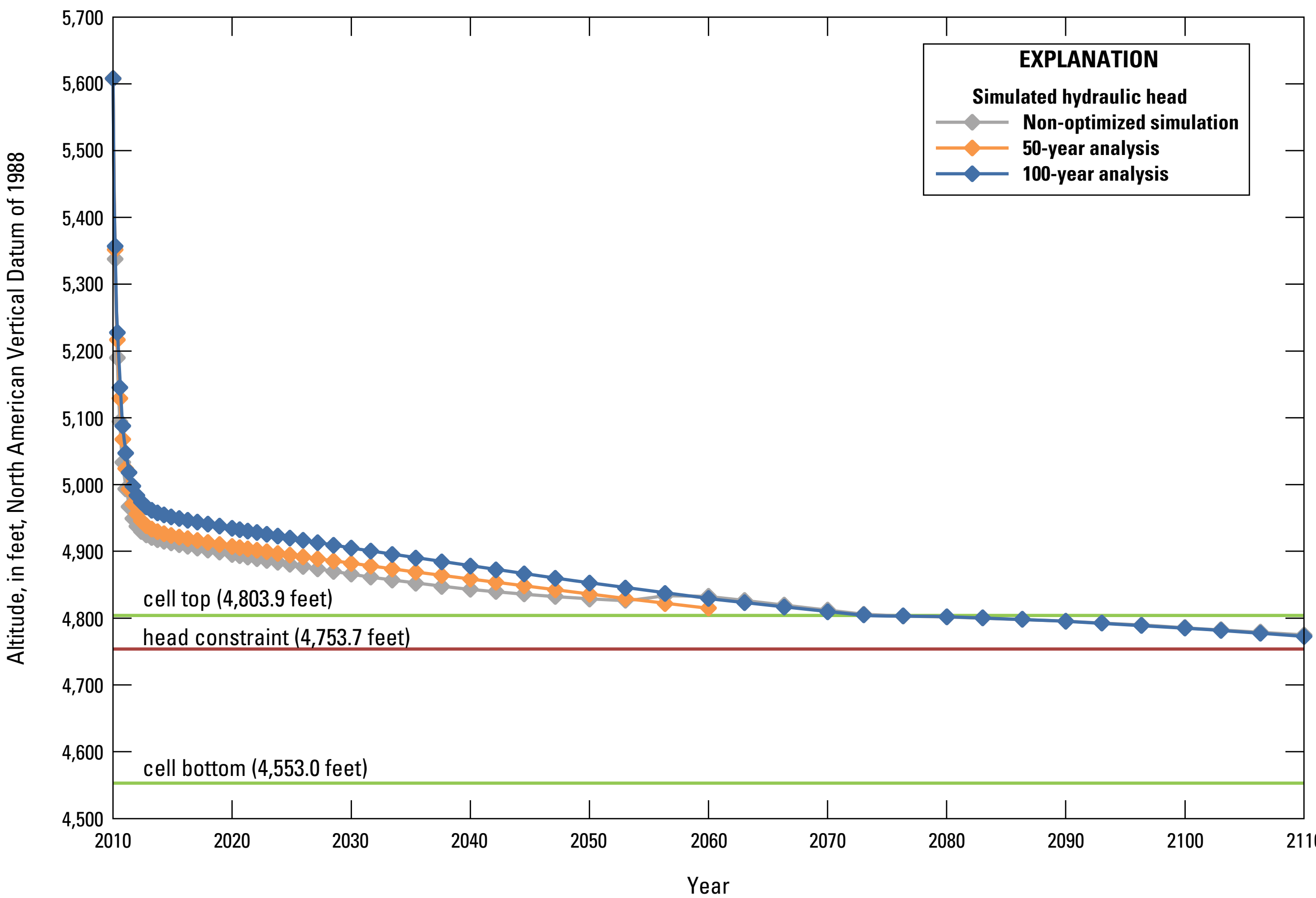



**Figure 13.** Simulated hydrographs and head constraint for cell (8, 73, 20; layer, row, column) for optimization analyses and for model results using non-optimized pumping rates.

## Comparison with Non-Optimized Simulation

Hydrographs for the non-optimized simulation are plotted in gray along with hydrographs generated by the optimization analyses in figures 12–14. The differences among the hydrographs vary depending on location of the cell of interest.

The differences between the hydrograph for the non-optimized simulation and the hydrographs for the optimized simulations are dramatic at cell (8, 63, 18) (fig. 12). For the non-optimized simulation, the head in this cell violates the head constraint in 2022, and the cell is deactivated at the first time step in 2100. Optimizing the pumping prevents simulated heads at this cell (and other cells at managed-flow and head-constraint locations) from violating head constraints and, therefore, prevents cell deactivation.

At first glance the benefit of optimization is not obvious at cell (8, 73, 20) (fig. 13). The hydrograph for the non-optimized simulation is just below the hydrograph generated by the 50-year optimization analysis until 2056, at which point the hydrograph for the non-optimized simulation rises a few feet before declining again and closely following the hydrograph generated by the 100-year optimization analysis. This apparent anomaly is an artifact, however, resulting from the deactivation of managed-flow cells and the resulting elimination of pumping associated with those cells.

At cell (8, 64, 24) the hydrograph generated by the non-optimized simulation again is slightly below the hydrograph for the 50-year optimization analysis. The benefit of optimization becomes apparent when the non-optimized hydrograph is compared with the hydrograph for the 100-year optimization analysis. In the non-optimized simulation, the head in cell (8, 64, 24) violates the head constraint (4,532.4 ft) in 2066 and declines to 4,428.8 ft at the end of the simulation, more than 100 ft below the head constraint. In the 100-year optimization analysis, in contrast, the head constraint is not violated.

## Effects of Managed-Flows Pumping

As described in the section titled “Application of Optimization Analysis to Arapahoe Aquifer Pumping,” the total 2004–2009 simulated withdrawal rate for Arapahoe aquifer wells was 48.3 ft$^3$/s, and for both optimization analyses, pumping from unmanaged wells was 42.6 ft$^3$/s (table 4). For

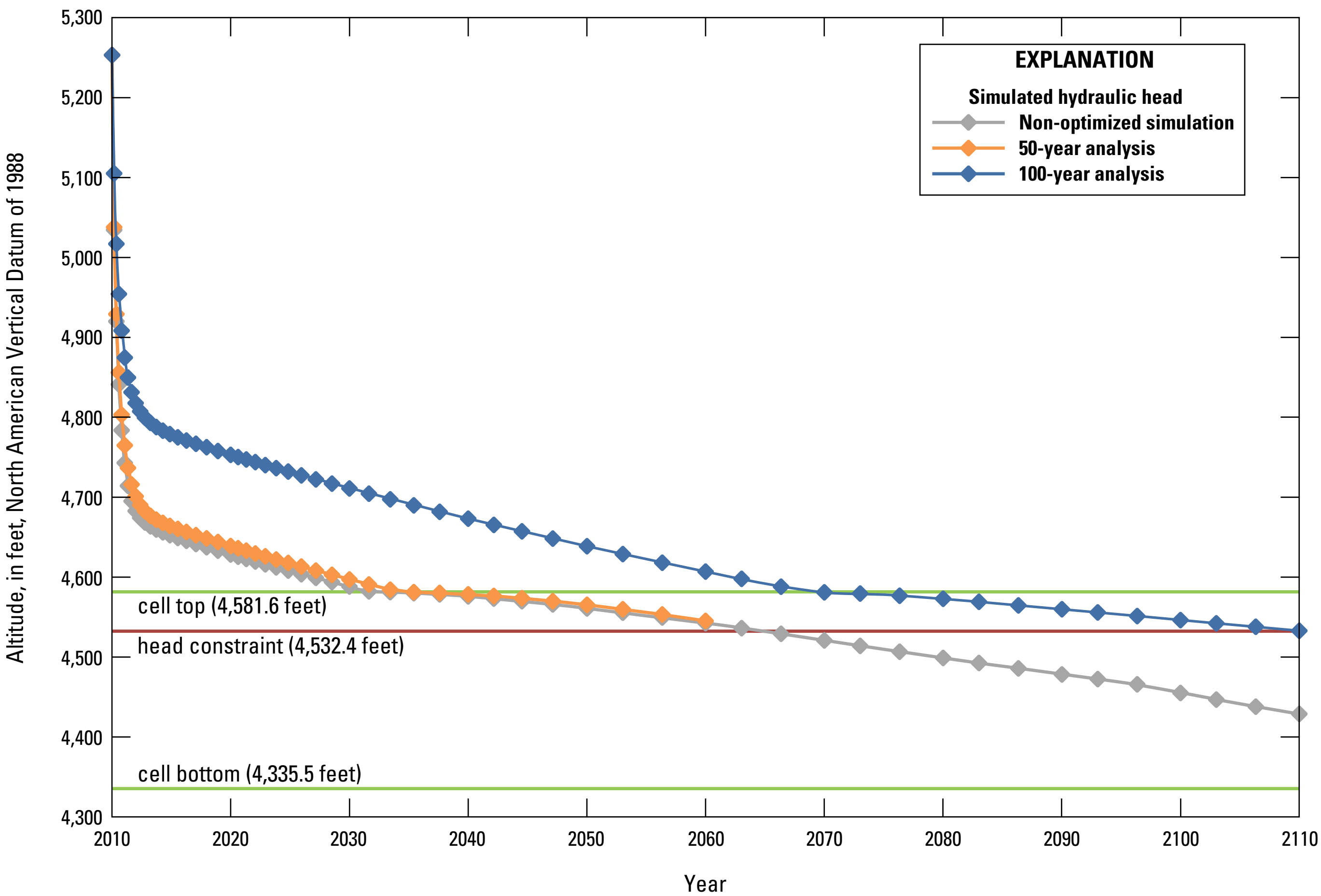



**Figure 14.** Simulated hydrographs and head constraint for cell (8, 64, 24; layer, row, column) for optimization analyses and for model results using non-optimized pumping rates.

Analysis 1, optimized pumping from managed wells was 58.8 $ft^3/s$, and the total of pumping from unmanaged and managed wells was 101.4 $ft^3/s$ (table 4). For Analysis 2, optimized pumping from managed wells was 53.0 $ft^3/s$ (table 2), and the total was 95.6 $ft^3/s$ (table 4). In comparison to the 2004–2009 rate of 48.3 $ft^3/s$, these total well-pumping rates for the optimization analyses represent a substantial increase in pumping. This section describes the simulated effects of the increased pumping.

Water-budget analyses using the program Zonebudget (Harbaugh, 1990) illustrate the effects of the managed-flows pumping rate simulated in Analysis 1. For the Zonebudget analyses, the model domain was divided into three zones: model layers (1–7) representing units that overlie the Arapahoe aquifer, model layers (8–10) representing the Arapahoe aquifer and the Arapahoe confining unit, and model layers (11–12) representing units that underlie the Arapahoe aquifer (table 1). Figure 18 illustrates selected components of the volumetric budget for all time steps of the 2004–2009 simulation and the 2010–2059 Analysis 1 simulation, using optimized flow values for the managed wells. Components not shown account for less than 1 percent of the volumetric budget. The horizontal axis represents time, in years, relative to January 1, 2010, the beginning of the Analysis 1 simulation time. In figure 18 flow rates for the 2004–2009 simulation plot to the left of zero, and flow rates for the 2010–2059 simulation plot to the right of zero.

The “managed withdrawals” line of figure 18 represents the total flow rate for all managed flows, as optimized in the 50-year analysis. The “unmanaged withdrawals” curve represents well pumping simulated using the Well Package. The other curves shown in figure 18 illustrate the response of selected other water-budget components to the optimized managed-flows pumping. The “storage in” curve represents water released from confined and unconfined storage in the Arapahoe aquifer and Arapahoe confining unit. The “from overlying units” curve of figure 18 represents flow into the Arapahoe aquifer across its upper boundary; the values are a combination of all sources of water in those units, including removal of water from storage, induced recharge, and capture of discharge. Similarly, the “from underlying units” curve represents flow into the Arapahoe aquifer across its lower boundary. The “to overlying units” curve represents flow out of the Arapahoe aquifer across its upper boundary.

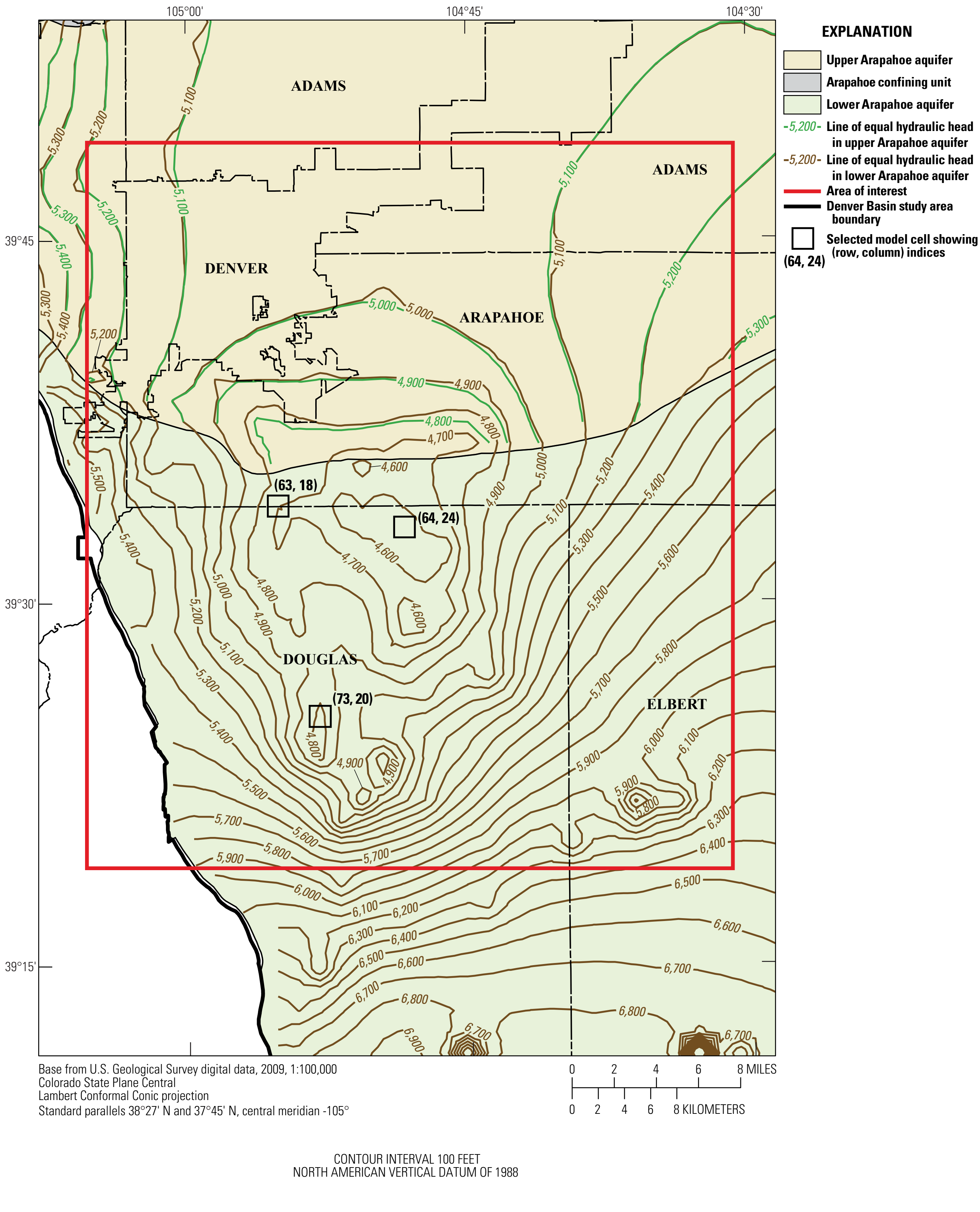



**Figure 15.** Simulated 2109 potentiometric (hydraulic head) surfaces for upper and lower Arapahoe aquifers.

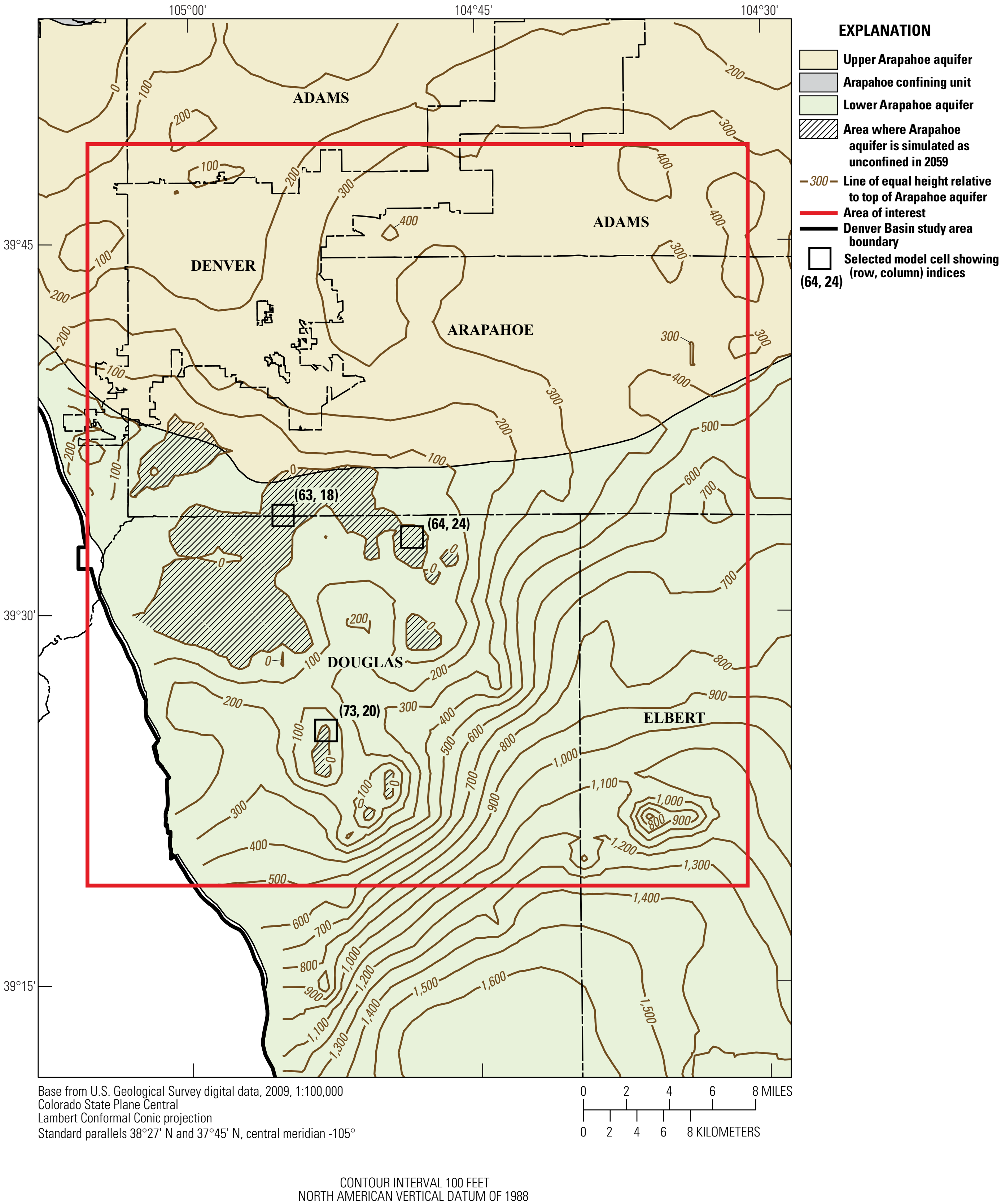



**Figure 16.** Height of layer 8 simulated 2109 potentiometric (hydraulic head) surface relative to top of Arapahoe aquifer.

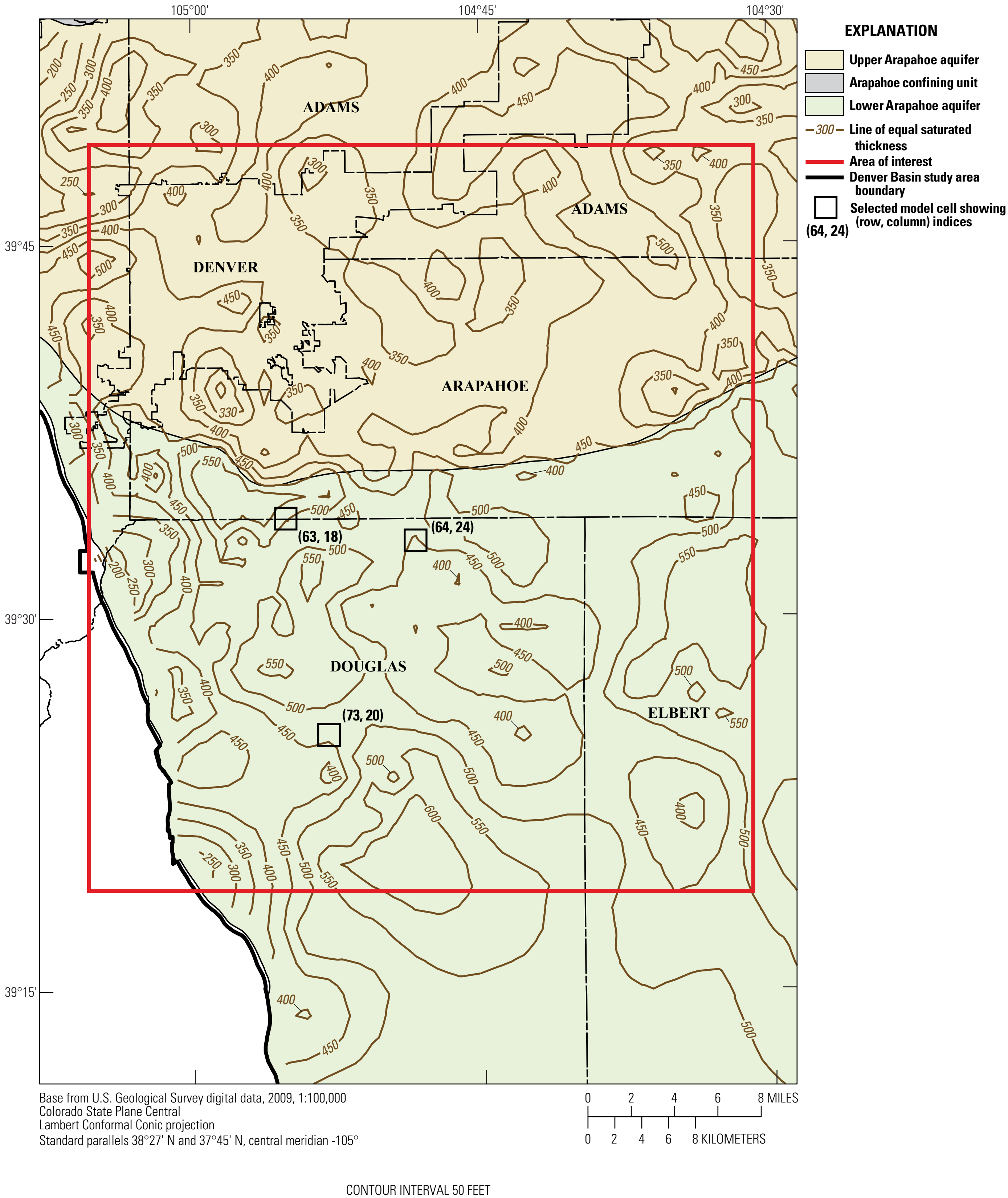



**Figure 17.** Combined 2109 saturated thickness of upper and lower Arapahoe aquifers.

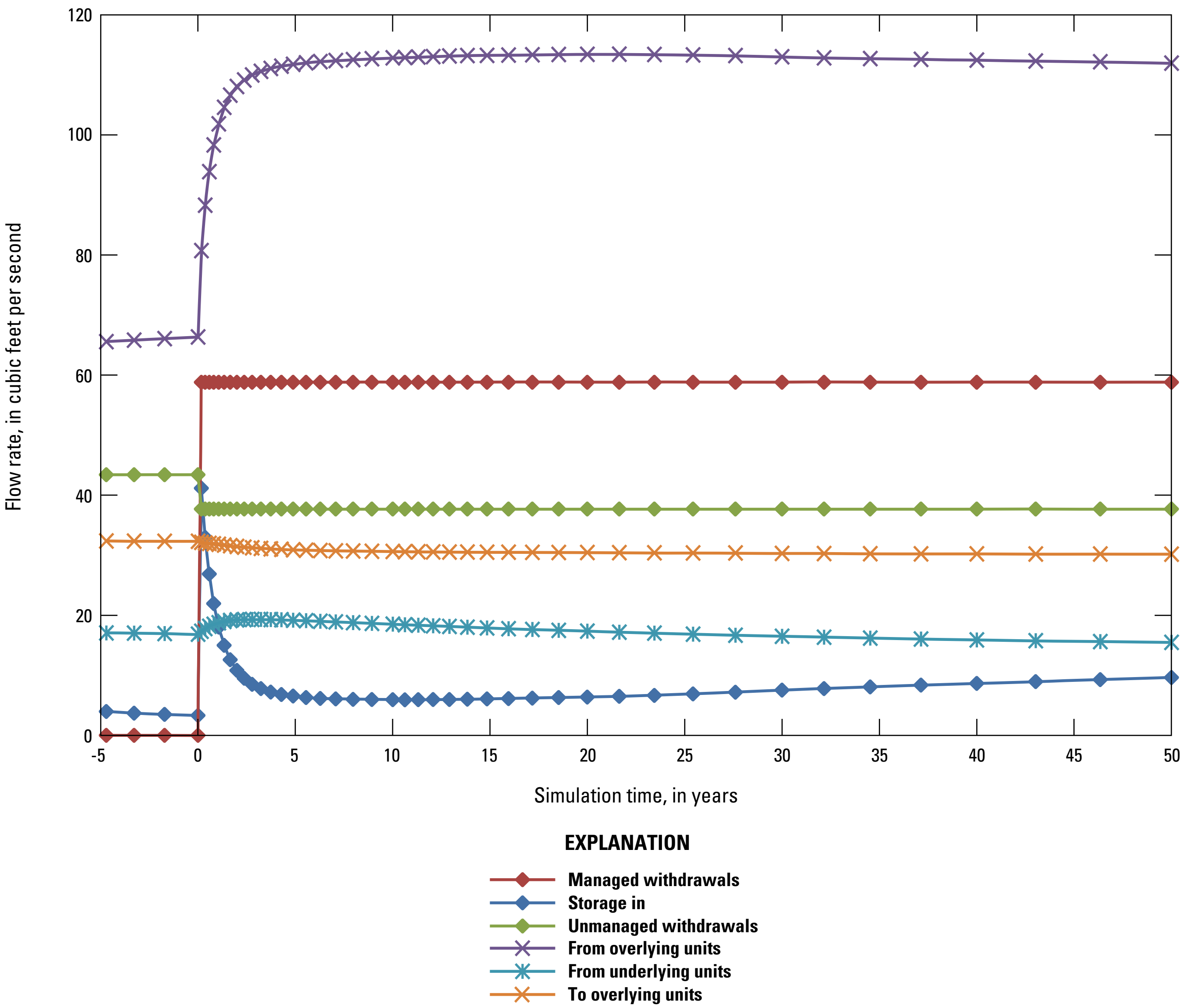



**Figure 18.** Simulated flow rates for selected Arapahoe aquifer (model layers 8–10) water-budget components, showing (2004–2009) antecedent rates (indicated by simulation times less than zero) and (2010–2059) responses to managed flows of Analysis 1.

The "managed withdrawals" line of figure 18 has a value of zero during 2004–2009 and increases to 58.8 $ft^3/s$ in 2010. The "unmanaged withdrawals" curve, as described above, decreases from 43.4 $ft^3/s$ during 2004–2009 to 37.7 $ft^3/s$ in 2010. The difference, 5.7 $ft^3/s$, supplies a small part of the managed-flows rate. In the first time step after time zero, the "storage in" curve increases from 3.3 $ft^3/s$ to 41.2 $ft^3/s$; this change in flow from storage is associated with a rapid initial decline in hydraulic heads in the Arapahoe aquifer in response to the onset of managed-flow withdrawals. The rate of release from storage in layers 8–10 decreases rapidly after the first time step, however, and by 4.9 years into the Analysis 1 simulation period, the rate of release from storage is 6.5 $ft^3/s$. This decline in the rate of release from storage is indicative of a change in the source of water for managed flows from storage to other sources of water, primarily flow from adjacent units. Flow from overlying units increases from 66.3 $ft^3/s$ at the beginning of the Analysis 1 simulation to 111.8 $ft^3/s$ at 4.9 years. Other budget components are affected by the managed-flows pumping, but much less substantially.

The large increase of flow from units overlying the Arapahoe aquifer resulting from managed-flows pumping is indicative of substantial changes in hydrologic conditions in overlying units resulting from the increase in withdrawals from the Arapahoe aquifer. The most significant change is the effect on hydraulic heads in the Denver aquifer (model layer 6), which is attributed, in part, to the thinness of the confining bed separating the Denver and Arapahoe aquifers, especially on the west side of the basin (Paschke and others, 2011a). Results for the calibration period of the groundwater-flow model (1880–2003) and predictive simulations indicate substantial simulated vertical connection between these layers and downward flow induced by the increase in pumping in the Arapahoe aquifers (Paschke and others, 2011b). For comparison purposes, a model run was set up to simulate aquifer response to stresses that were used in the 2004–2009 simulation but extended through 2109 with the stress-period and time-step lengths matching those of the Analysis 1 and Analysis 2 runs; this simulation had no managed-flows pumping. Figure 19 is a hydrograph for the cell in model layer 6 (Denver aquifer), row 63, column 18. Cell (6, 63, 18) is in an area of substantial drawdown in the Arapahoe aquifer resulting from the managed-flows pumping (figs. 8–11 and 15–17). The top curve in figure 19, for the simulation without managed-flows pumping (using 2004–2009 stresses), shows the calculated head in the Denver aquifer at this location declining during the 106-year simulation from an initial value of 5,368 ft to 4,998 ft in 2109, which is 215 ft above the cell bottom at 4,783 ft. The other two curves are for the Analysis 1 and Analysis 2 simulations. In the Analysis 2 simulation, this cell in model layer 6 is converted to inactive as the calculated head approaches the cell bottom.

Figure 18 also indicates that the response of volumetric-budget components to managed-flows pumping changes over time. Another Zonebudget (Harbaugh, 1990) analysis was performed using the results of the simulation without managed-flows pumping for comparison with the Analysis 1 results. Differences between the two simulations in the flow rates for selected budget components for all time steps are shown in figure 20. Components not shown account for less than 1 percent of the managed flows. The differences show the extent to which each selected budget component contributes to the managed-flows pumping. Three pie charts (fig. 20) illustrate how these differences change over time. Figure 20*A* is for 64 days, figure 20*B* for 4.9 years, and figure 20*C* for 50 years after managed-flows pumping begins. As the sequence of charts in figure 20 shows, most of the managed-flows pumping initially comes from release of water from storage in the Arapahoe aquifer. However, the primary source of water rapidly shifts, so that by 4.9 years after the start of managed-flows pumping and through the rest of the simulation, most of the water comes from increased flow from overlying hydrogeologic units, primarily layer 6, which represents the Denver aquifer.

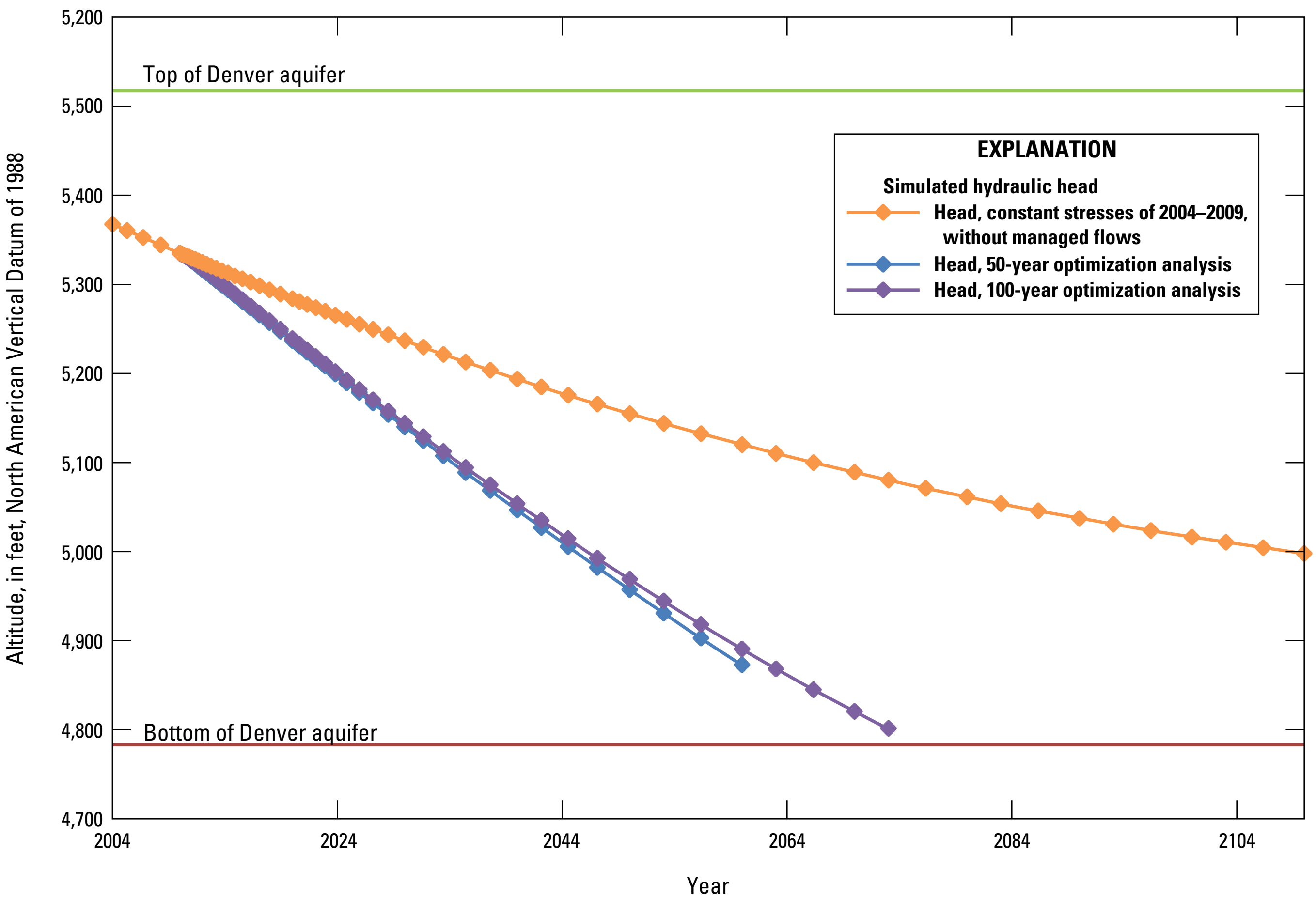



**Figure 19.** Simulated hydrographs for cell in layer 6, row 63, column 18.

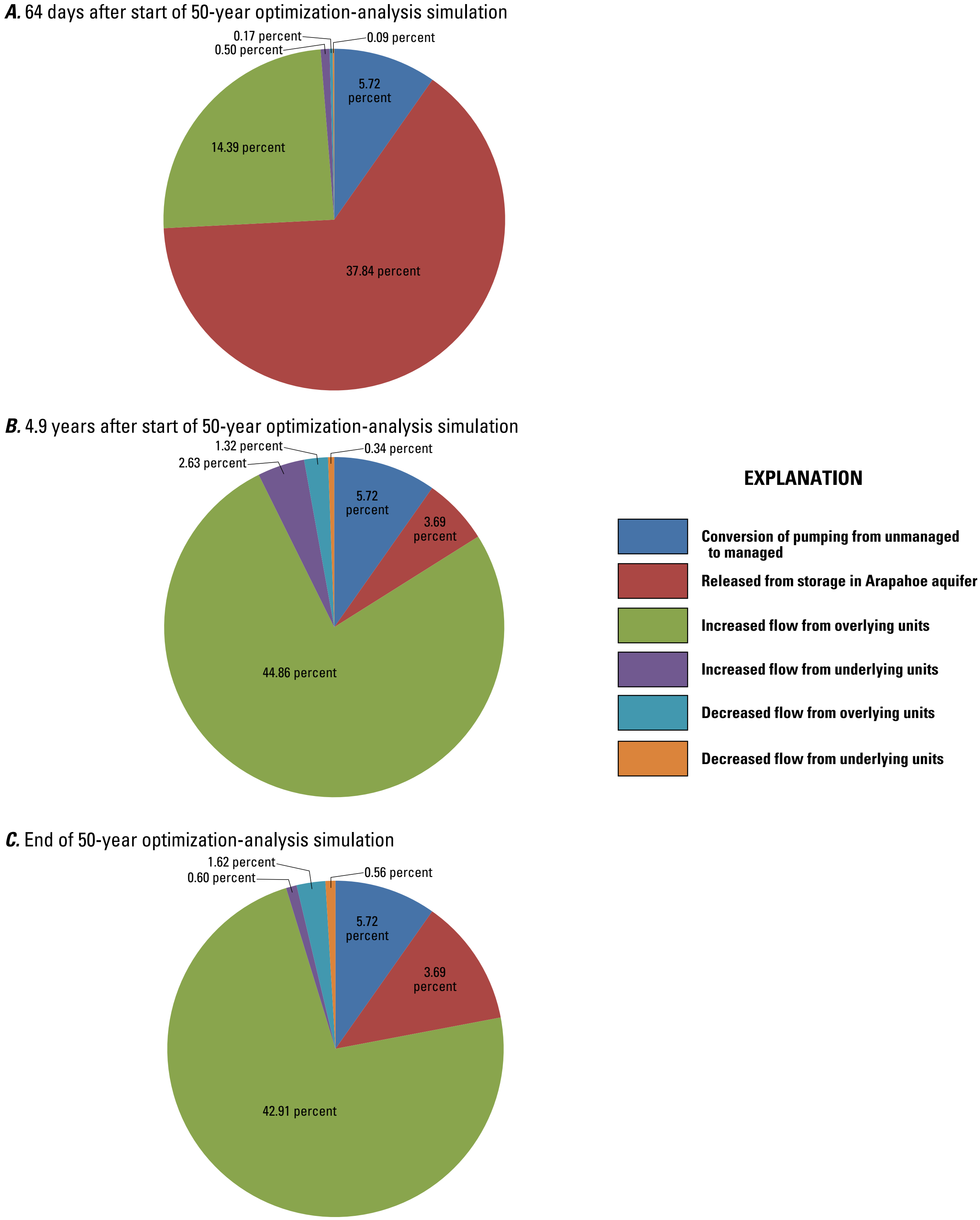



**Figure 20.** Sources of water for managed flows, relative to simulation without managed flows, at *A,* 64 days; *B,* 4.9 years; and *C,* end of 50-year optimization-analysis simulation. Data are flow rates, in cubic feet per second, based on Zonebudget (Harbaugh, 1990) analyses of Arapahoe aquifer (model layers 8–10). Sources not shown supply less than 1 percent of combined managed flows.

## Limitations

As with any modeling analysis, appropriate interpretation and use of analysis results necessarily are limited by practical considerations. It is important for readers to recognize these limitations and resist the temptation to over-interpret simulation and optimization results. This section describes some of those considerations.

The groundwater-flow simulations on which the optimization analyses rely are based on assumptions regarding future stresses on the aquifer system. The accuracy of the simulations also are limited by the degree to which the groundwater-flow model approximates the physical aquifer system. Banta and others (2011) present a pertinent discussion of model uncertainty and limitations. Simulation results and optimization-analysis results are not intended to be interpreted as absolute predictions of future conditions. These results are intended to be most informative in the context of comparison of starting and final aquifer conditions and of comparison between results of the two optimization analyses.

The head constraints as implemented in the optimization analyses are surrogates for field conditions of concern in the operation of municipal water wells. In particular, the optimization analyses use model-calculated hydraulic head in 1-mile by 1-mile model grid cells instead of the water level in a pumping well, which would be more appropriate from an operational standpoint. The head to which the simulated head is compared, either the altitude of 90-percent saturated thickness of the Arapahoe aquifer or the altitude of 90-percent saturated thickness of the upper or lower Arapahoe aquifer, was selected in recognition that the head in a pumping well would be substantially lower than the average head in a one-square-mile area of aquifer. A more rigorous approach would be to use an analytical tool, possibly the Multi-Node Well (MNW2) Package (Konikow and others, 2009), to simulate heads in pumping wells and to use pump-intake altitudes as head constraints, but such a high-resolution approach was beyond the scope of the current project. Given the approach that was used, it is recognized that the maps and hydrographs based on simulated head and presented in this report represent only a coarse resolution of possible future conditions that could be expected to result from the long-term average pumping rates that are described. For example, apparent uniformity of the large shaded area in figure 16 where head is below the top of the Arapahoe aquifer but where saturated thickness is at least 90 percent of the aquifer thickness (indicated by non-violation of the head constraints) is an artifact generated by the coarseness of the grid cells relative to the scale of drawdown effects in the vicinity of a pumping well. Although it may be possible to obtain such a saturated-thickness distribution by distributing the simulated pumping much more evenly, for example by installing a large number of wells in each square-mile cell and pumping each at a small fraction of the total pumping rate indicated for the cell, for practical purposes this distribution needs to be interpreted as a spatial and temporal generalization. Actual field conditions would be much less uniform.

The discretization of the model domain into rows and columns is insufficiently refined to allow withdrawals from municipal wells to be resolved to a degree that would allow for reasonably accurate well-interference effects. In many instances, the area represented by a single model cell contains multiple municipal wells with interwell spacing that is far smaller than the 1-mile cell dimensions. Finer discretization could reduce this problem, but practical limitations related to model discretization (for example, computer memory requirement and execution time) likely would prevent elimination of the problem.

Model stress periods were of multiyear duration rather than some duration appropriate for day-by-day, month-by-month, or season-by-season well operations. This temporal discretization of stresses prevents the model from predicting seasonal fluctuations of aquifer hydraulic heads and substantially limits the accuracy with which simulation results can be expected to represent actual field conditions.

The performance of wells completed in the bedrock aquifers of the Denver Basin under unconfined conditions has been a subject of debate among water providers, hydrologists, and engineers in Colorado. Of particular concern is the potential effect of numerous shale and mudstone intervals having relatively small permeability interlayered with more productive, higher permeability layers in the various aquifers, including the Arapahoe aquifer. Preliminary analyses (Mark Palumbo, HRS Water Consultants, Inc., written commun., 2002) indicate that effects of such layering on water levels (increased drawdown) in, and in the immediate vicinity of, pumping wells may be substantial. The analyses described in this report do not consider layering in the aquifers and its effects on hydraulic heads in the vicinity of pumping wells. As aquifer conditions convert from confined to unconfined in the vicinity of pumping wells, water providers may find it necessary to install and produce from multiple wells to achieve production rates comparable to production rates attained under confined conditions.

# Possible Additional Analyses

The analyses documented in this report demonstrate examples of possible formulations of optimization problems designed to address excessive loss of saturated thickness due to municipal pumping in the Arapahoe aquifer. These examples are only a small subset of the possible optimization approaches that could be used to analyze the costs, effects, and benefits of municipal pumping. This section presents a number of possibilities for additional optimization analyses.

## Address Issue of Effects of Pumping in Arapahoe Aquifer on Denver Aquifer

Results of the optimization analyses described in the section titled “Effects of managed-flows pumping” indicate that pumping from Arapahoe wells can produce substantial drawdowns in the Denver aquifer. The analyses described in this report did not include any mechanism for constraining the effects of Arapahoe withdrawals on the Denver aquifer. In a future analysis, it may be desirable to include a mechanism by which constraints on the effects on the Denver aquifer would be incorporated into the optimization scheme. One possible mechanism, currently supported by GWM (Ahfeld and others, 2009), would be to define drawdown constraints for selected cells representing locations in the Denver aquifer. Alternatively, a mechanism involving the use of Zonebudget (Harbaugh, 1990) and separate optimization software could be developed to implement a constraint on changes to flow from the Denver aquifer to the Arapahoe aquifer, compared to a base simulation.

### Refine Spatial and (or) Temporal Discretization

Accuracy of modeled effects of pumping could be improved by refining the spatial and (or) temporal discretization of the Denver Basin model. The Local Grid Refinement Package (Mehl and Hill, 2005, 2010) could be used to refine the model grid in the area of greatest interest to improve resolution of well-interference effects.

More accurate simulation of pumping effects could be achieved by defining stress periods and corresponding well withdrawals on a seasonal or monthly basis. This would require a substantial effort to estimate seasonal or monthly pumping rates. It also would require similar revisions to at least part of the calibration period (1880–2003) of the Banta and others (2011) model and recalibration of that model. The benefit of refined spatial and temporal discretization would be that calculated hydraulic heads would enable the model to more closely approximate the conditions that limit the operation of wells.

### Simulate and Place Constraints on Water Levels in Production Wells

The MNW2 Package (Konikow and others, 2009) can be used with MODFLOW-2005 to simulate water levels in pumping wells. Note that, despite the “multi-node” descriptor in the package name, the MNW2 Package also can be used to simulate a well connected to a single model cell with the benefit that water level in the well can be calculated. The version of GWM (Ahlfeld and others, 2005, 2009) used for the analyses described in this report does not support constraints on water levels in wells calculated by MNW2. To include this type of constraint in an optimization analysis would require the use of alternative optimization software or addition of support for MNW2-based head constraints in GWM. Appropriate use of the MNW2 Package also would require analyses and (or) calibration to establish settings and realistic coefficients that determine the head loss between a well and the model cell in which the well is located. As for the suggested refinement of spatial and temporal discretization, simulation of water levels in wells and basing constraints on those simulated water levels would enable the model and the optimization algorithm to more closely approximate the conditions that limit the operation of wells.

### Include Infrastructure Costs in Optimization Analysis

Implementation of optimized pumping distributions, such as those described in this report, could involve construction of additional infrastructure, in which case municipal-water providers would incur substantial costs. Optimization software, including GWM, generally is designed to support analyses involving objective functions of various kinds. An objective function could be written in terms of financial costs, for example, rather than withdrawal rates. An analysis that considers construction, operation, and maintenance costs as well as hydrologic benefits and effects would be useful in a cost/benefit analysis related to potential infrastructure construction.

## Summary and Conclusions

Optimization analyses were performed to demonstrate approaches that might be used to address issues related to operation of municipal wells in an area of interest of the Arapahoe aquifer in the west-central part of the Denver Basin. The analyses used a previous conceptual model and groundwater-flow model of the Denver Basin. These analyses were designed as demonstrations only and were not intended as a comprehensive optimization study. For each analysis an optimization problem was set up to maximize a total withdrawal rate, subject to withdrawal-rate and hydraulic-head constraints. The head constraints were at the 90-percent saturated thickness altitude of the Arapahoe aquifer where it is undifferentiated. Where the Arapahoe aquifer is divided into upper and lower parts by a confining unit, head constraints were defined at the 90-percent saturated thickness altitudes of the upper and lower Arapahoe aquifers.

In Analysis 1 an optimal withdrawal distribution for 96 flow-rate decision variables representing 119 wells was determined using a 50-year simulation period. The distribution was determined by setting up an optimization problem to maximize the sum of withdrawals for all decision variables, subject to a set of minimum head constraints at the end of the 50-year period. The optimized total withdrawal rate for all managed wells was 58.8 $ft^3/s$.

Analysis 2 was similar to Analysis 1, except that the simulation time was 100 years instead of 50 years, and the head constraints applied at 100 years after the beginning of the analysis period, rather than at 50 years after the beginning of the analysis period. For this analysis, the optimized total withdrawal rate for all managed wells was 53.0 $ft^3/s$. These results indicate that with optimal pumping distributions, the time until violation of the 90-percent saturated-thickness constraints might be extended from 50 years to 100 years by a 10-percent decrease in total pumping rates of the managed wells.

Analyses of the simulated groundwater volumetric budget show time-dependent effects on various budget components in response to managed-flows pumping. Initially, the pumping produces water primarily by release of water from storage in the Arapahoe aquifer. In less than 5 years, however, most of the water removed by managed-flows pumping likely would be supplied from overlying hydrogeologic units.

Comparison of Denver aquifer simulated heads generated from model runs with optimized pumping with heads generated from a run using pumping rates unchanged from the 2004–2009 simulation showed that the optimized pumping rates could result in an increase in the rate of decline of hydraulic heads in parts of the Denver aquifer because of the simulated hydraulic connection between the Denver and Arapahoe aquifers. Additional analyses would be needed to determine the extent to which Arapahoe aquifer pumping rates would need to be modified to limit drawdown effects in the Denver aquifer to acceptable amounts.

## Acknowledgments

The authors extend thanks to Andy Moore (CWCB) and Brian Ahrens, Mary Halstead, and James Heath (CDWR) for suggestions related to the development of optimization scenarios and for assistance in database development. Thanks also are extended to the municipal water providers for providing data related to wells and historical water withdrawal rates, including: Arapahoe County Water and Wastewater Authority, Castle Pines Metropolitan District, Centennial Water and Sanitation District, City of Englewood, City of Littleton, Parker Water and Sanitation District, and Pinery Water and Wastewater District.

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Publishing support provided by
Denver and Lafayette Publishing Service Centers

Banta and Paschke—**Demonstration Optimization Analyses of Pumping from Selected Arapahoe Aquifer Municipal Wells**—SIR 2012–5140