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Abstract

Stress rupture is not a critical failure mode for most composite structures, but there are a few applications where it can be critical. One application where stress rupture can be a critical design issue is in Composite Overwrapped Pressure Vessels (COPV's), where the composite material is highly and uniformly loaded for long periods of time and where very high reliability is required. COPV's are normally required to be proof loaded before being put into service to insure strength, but it is feared that the proof load may cause damage that reduces the stress rupture reliability. Recently, a fiber breakage model was proposed specifically to estimate a reduced reliability due to proof loading. The fiber breakage model attempts to model physics believed to occur at the microscopic scale, but validation of the model has not occurred. In this paper, the fiber breakage model is re-derived while highlighting assumptions that were made during the derivation. Some of the assumptions are examined to assess their effect on the final predicted reliability. By improving some assumptions at the micro-mechanics level, the predicted reliability was shown to improve with proof loading instead of being drastically reduced, as predicted by the original model. The fiber breakage model may be useful for understanding the underlying mechanism of stress rupture failures. However, until validation of the model at both the micro and macro scales can be accomplished, it should not be used for programmatic reliability estimates. Even in the improved form, the fiber breakage model is undependable because it still relies on some assumptions at the micro-mechanics level that have not been validated and to which the predicted reliability is quite sensitive. Therefore, the effect of proof loading on reliability is still unknown, and testing should be performed to understand the true effect of proof loading.

Nomenclature

α – the Weibull shape parameter for composite (strand or vessel) strength distribution

β – the Weibull shape parameter for a stress rupture distribution

γ – shear strain in composite matrix

δ – overload region in fiber breakage model

ς – shape factor for the strength distribution of individual fibers

μ – matix shear modulus

σ – stress (composite stress unless otherwise noted)

σ_f – nominal stress in fiber

σ_{vessel}^{o} – composite stress in the vessel when the vessel is pressurized to its burst scale parameter pressure

$\sigma_{\delta_e}^{o}$ – scale strength level for fibers with length δ_e

θ – power law exponent in a visco-elastic creep model

ρ – classic model parameter describing stress rupture scale parameter changes with stress ratio

ζ – the Weibull shape parameter for individual fiber strength

τ – shear stress in composite matrix

D – ratio of composite stress to fiber stress

j – cluster size

j_c – critical cluster size leading to vessel or strand failure

j_p – critical cluster size needed for continued growth after proof

$J(t)$ – visco-elastic compliance of matrix material

J_e – elastic compliance of matrix material

K – stress concentration around a cluster of broken fibers

ℓ – characteristic length used to describe individual fiber strength

n – number of fibers immediately adjacent to a cluster of broken fibers

N – number of characteristic lengths of fiber in a structure (strand or vessel)

P – Probability of Failure

P_j – Probability of a cluster growing from size j-1 to j

$P_{1 \to j}$ – Probability of a cluster initiating and growing to size j

R – Reliability (Probability of Survival)

s – stress ratio defined as nominal stress divided by the strength scale parameter (stress ratio can be defined at the fiber or composite level)

t – time

t_c – time constant for matrix creep

t_{ref} – scale parameter for stress rupture failure time at s=1

t_{safe} – time required for a danger area to initiate along a fiber neighboring a broken fiber cluster

$\xrightarrow[probability]{small}$ – indicates a small probability approximation (1-e^{-x}≈x when x<0.1)

$\xrightarrow[t>>0]{t>>t_c}$ – simplification of equations at long time periods

Superscript

cp – conditional on proof survival (conditional reliability given survival of proof loading)

ip – including the chance of failure during proof loading

o – characteristic value (Weibull scale parameter value which corresponds to a 63.2% chance of survival)

p – related to a proof load

Subscript

e – related to elastic response

f – related to an individual fiber

j – related to a cluster of fibers of size j

o – nominal value, e.g. stress in fibers away from a fiber break.

p – related to a proof load

vessel – related to vessel reliability or probability of failure

1→j – combined probability of cluster growing from cluster size 1 to j

Acronyms

SR - Stress Ratio

WSTF -White Sands Test Facitilty

LLNL -Lawrence Livermore National Laboratory

NESC –NASA Engineering and Safety Center

Introduction

Stress rupture of composite material is a critical failure mode in a few composite applications such as Composite Overwrapped Pressure Vessels (COPV's) where the composite material is highly and uniformly loaded for long periods of time. Although time delayed failures under constant load are quite rare, applications that demand high reliabilities must address the stress rupture mechanism. Traditionally, the probability of stress rupture failure is addressed by empirical models where the probability of failure measured at very high stress levels is extrapolated to lower stress levels where the chance of failure is too low to be adequately measured.

There are a number of empirical models that are generally quite similar which can be used to estimate reliability as a function of time and stress level. Equation 1 is an example of an empirical model that will be called the "classic" model[1]. This model is in the mathematical form and notation proposed by Phoenix and predicts the life of a COPV as a Weibull distribution where the scale parameter is a powerlaw function of the stress ratio.

$$P_{vessel}(t) = 1 - e^{-\left(\frac{t}{s^{-\rho} t_{ref}}\right)^{\beta}} \qquad (1)$$

$$\text{where stress ratio, } s = \frac{\text{nominal stress}}{\text{strength scale parameter}} = \frac{\sigma}{\sigma^{o}}$$

This classic model has three curve fitting parameters (ρ, β and t_{ref}) which are fit to experimental data at high stress ratios. The model then allows predictions at much lower stress levels where the probability of failure is too low to be adequately measured. COPV are often designed around stress ratios of 0.5. An example of estimates from the classic model are shown in Figure 1. The results in Figure 1 are plotted as the number of 9's of reliability. Reliability is the probability of not having a failure, and since one wants to design for high reliability (low chance of failures), the reliability value gets very close to 1, e.g. 0.999 or 0.99999. Here three 9's of reliability implies that 1 out of 1000 articles are predicted to fail or five 9's is 1 out of 100,000. By using
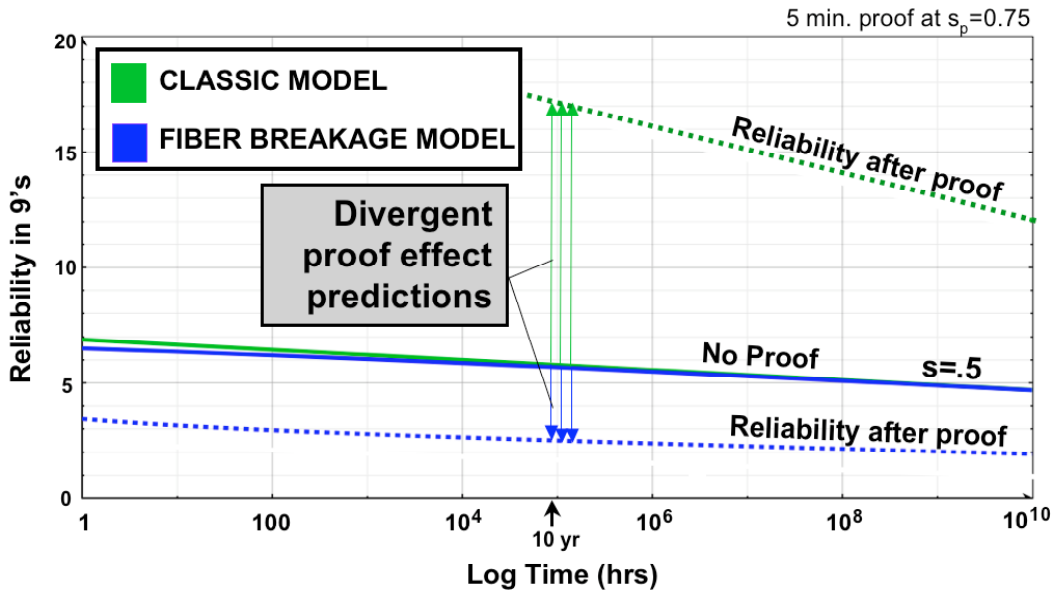

Figure 1. Weibull plot of reliability including proof loading effects.

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Table 1. T1000 composite parameter input values for the classic and fiber breakage models

	Classic Model	Fiber Breakage Model
Model Parameters	t_{ref}=0.001 hr β=0.22 ρ=114	t_c=0.01 time constant for matrix power law creep θ=0.11 exponent for matrix power law creep j_c=5 critical cluster size ζ=5 shape parameter for individual fiber strength α=25 shape parameter for vessel strength (which is j_cζ so not independent in this case)

the scale of 9's of reliability, predictions in the high reliability region can be more easily distinguished. Precisely, the scale is

$$Weibull\ Scale = -\left(Log\left[-Ln(R)\right]\right) = -\left(Log\left[-Ln(1-P)\right]\right) \qquad (2)$$

where R is the reliability and P is the probability of failure.

This scale is a Weibull scale because a Weibull distribution will appear linear. However, it is a specific Weibull scale where above 1, it can be directly interpreted as the number of 9's of reliability as indicated on the figure and at zero it is the Weibull scale parameter. From the figure, one can see that at a stress ratio of 0.5 the predicted reliability after 10 yrs ($10^{4.9}$ hrs) is approximately six 9's or 1 in a million chance of failure. These predictions were made with the curve fitting parameters given in Table 1, which have previously been used for T1000/Epoxy composites [2].

In 2004, the NASA Engineering and Safety Center (NESC) sponsored an engineering review of how the risk of stress rupture failures has been addressed [1]. During that study, a critical issue was identified that COPV's in service are exposed to proof loading prior to use. It is feared that a proof load could cause damage in the COPV that would actually reduce its reliability. This effect cannot be modeled with traditional models of carbon vessels. One can account for proof loading with the classic model, but because of the low beta found when fitting carbon data (β=0.22 used here), the model predicts an ever decreasing rate of failure with time. A reliability after proof can be calculated with the classic model, but because stress at a higher level behaves as a lower stress for a much longer period of time, the decreasing failure rate of carbon composites will cause the predicted reliabilities after proof to be much higher. In the example shown in Figure 1, a 5 min. proof load at 0.75 stress ratio causes the predicted reliability to increase from six 9's to greater than fifteen 9's. This is a nine order of magnitude increase in reliability. Other traditional models, like the classic model, will never be able to predict a decreased reliability due to a proof load. Because the effect of proof loading in traditional models is to drastically increase the reliability estimate, and this increase cannot be verified, the effect of proof loading is generally omitted when making reliability estimates. So in the current example, the estimated reliability would be six 9's for a COPV in service for 10 yrs.

To address the fear that a proof load could damage a COPV and result in lower predictions of reliability, a fiber breakage model was proposed [2]. The fiber breakage model attempts to model physics thought to occur at the microscopic scale, but validation of the model has not occurred. In the example presented in Figure 1, the predicted reliabilities from the fiber breakage model, dropped from six 9's to two 9's due to the proof load. (Fiber breakage model equations and

parameters used in the example problem will be presented later in the paper.) This dramatic decrease in the reliability estimate (four orders of magnitude), of course caused concern among project managers responsible for managing risk in programs that use COPV's. In this paper, the fiber breakage model is re-derived while highlighting assumptions that were made during the derivation. Some of the assumptions are examined to assess their effect on the final predicted reliability.

General description of the fiber breakage model

The fiber breakage model [2] assumes that the stress rupture response is due to individual fiber breaks at the microscopic level as shown in Figure 2. When an individual fiber breaks, the load that had been carried by that fiber must be transferred through the matrix to the neighboring fibers thus causing a stress increase in the neighboring fibers over some distance close to the fiber break. If all the fibers had essentially the same strength, the failure of one fiber
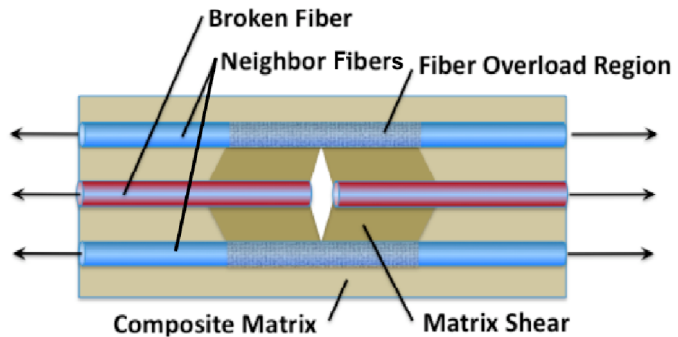

Figure 2. Fiber break geometry.

would cause an overload in the neighboring fibers. This would instantaneously cause a cascade of fiber failures leading to the failure of the composite. The strength of carbon fibers has been shown to exhibit a high degree of scatter [3].   Therefore, when the weakest spot in a group of fibers fails, there is a significant probability that the surrounding fibers will all have significantly higher strengths and be able to carry the elevated stress level caused by that first failure. When the composite stress is raised high enough, there will be many individual failed fibers and some of these failed fibers will have created an overload in neighboring fibers that is large enough to fail a neighboring fiber, thus causing an even higher overload in the remaining fibers.   This results in groups of failed fibers in the composite (also called clusters).   Finally, the stress concentrations created by the larger groups of failed fibers becomes large enough that it is likely that the remaining fibers will not be able to withstand the elevated stress level, resulting in an unstoppable cascade of fiber failures and failure of the composite. The strength of the composite is therefore described by the stress level required to create a broken fiber cluster of adequate size to initiate an unstoppable cascade of failures. This micro-mechanical description of progressive composite failure has been proposed by many researchers and is the prevailing micro-mechanical theory regarding composite strength [4].

The fiber breakage stress rupture model then introduces a time dependent response due to the visco-elastic response of the matrix. Because the matrix transfers the load from a broken fiber to its neighbors through shear and because polymeric matrix materials will creep over time, the displacement will increase with time under constant stress. For a broken fiber in a composite, this means that the load that was carried by the broken fiber gets transferred to the neighboring fibers, and over time, the length of the neighboring fibers that are overloaded will increase from $2\delta_e$ to 2δ as shown in Figure 3.   The amount of stress increase in the neighboring fiber can be characterized by a stress concentration (K) that is a function of the number of neighboring fibers that share the load and the number of fibers in a broken fiber cluster. With the increasing length, there is a chance that a newly overloaded portion of the neighboring fiber will have a weak spot and fail. This newly overloaded region will be referred to as the danger area, the area where additional damage could occur with time. So, without any increase in global stress, the increasing

danger area creates the potential for clusters of broken fibers to grow and eventually creates a critical cluster size that propagates to failure.

The fiber breakage model predictions of reliability after proof are quite different from the classic model predictions. With the fiber breakage model, a detrimental effect of proof loading can be

Figure 3. Change in fiber stress near fiber break with time.

predicted because the proof loading will create many more broken fibers and larger cluster sizes, all of which cause elevated stresses in neighboring fibers. The danger area associated with each break then grows with time. Therefore, the proof load causes an increased total danger area along neighboring fibers because of the additional number of breaks. However, the elevated stress region creating a danger area around a fiber break after proof is more complicated because all of the fibers endured an elevated load during proof. For the neighboring fiber to still be unbroken, the fiber strength must be higher than the overload that occurred during proof as illustrated in Figure 4. For small cluster sizes, the stress concentration caused by the broken cluster may not be larger than the stress endured by the neighboring fiber during proof (i.e. $K\sigma_f < \sigma_f^p$), and therefore these clusters of broken fibers are predicted to never grow with time. In contrast, larger fiber clusters, where $K\sigma_f < \sigma_f^p$, may grow with time and have the potential to eventually cause a stress rupture failure.
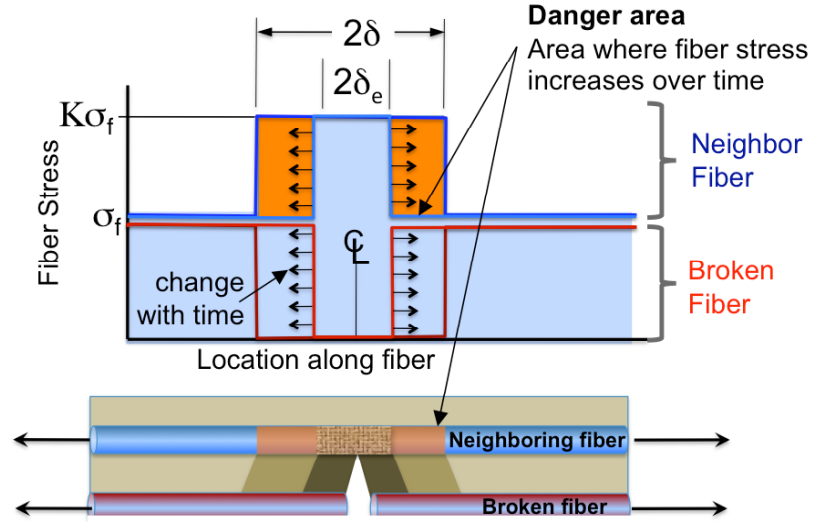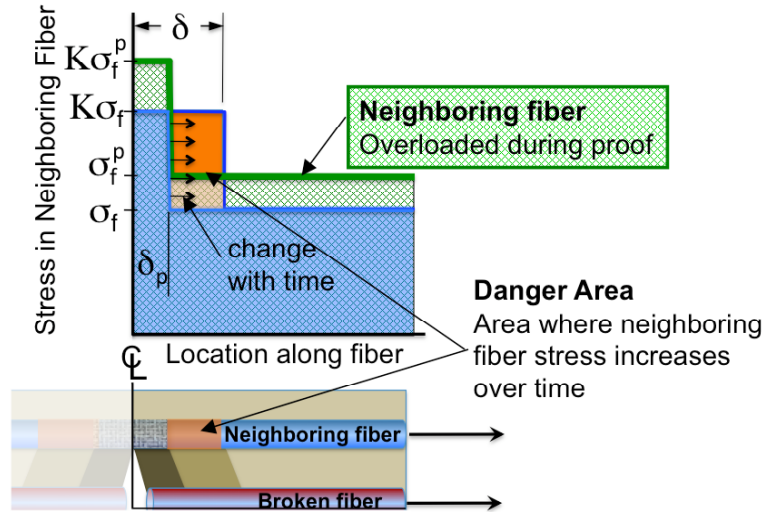

Figure 4. Change in neighboring fiber danger area after proof loading.

Mathematical derivation of the fiber breakage model.

In this section, the equations used to calculate the fiber breakage model reliabilities will be re-derived in order to highlight the assumptions that are made during the derivation. The next section will look at the effect of some of these assumptions.

Visco-elastic response of matrix

The fiber breakage model assumes the matrix obeys a power law creep compliance given by

$$J(t) = J_e \left[1 + \left(\frac{t}{t_c} \right)^{\theta} \right] \tag{3}$$

where $J(t)$ is the matrix compliance as a function of time
                 J_e is the elastic compliance of the matrix
                 t_c is a time constant for the matrix creep
                 θ is a power law exponent

In a simple elastic shear lag model [5], a characteristic distance δ_e over which the broken fiber stress is shared with neighboring fibers is related to the effective shear modulus of the matrix μ by:

$$\delta_e \propto \frac{1}{\sqrt{\mu}} = \sqrt{J_e} \tag{4}$$

The fiber breakage model assumes that the time dependent length of the overload region can be described by simply substituting the time dependent compliance into the elastic equation for the characteristic distance.

$$\frac{\delta(t)}{\delta_e} = \frac{\sqrt{J(t)}}{\sqrt{J_e}} = \frac{\sqrt{J_e}\sqrt{1 + \left(\frac{t}{t_c} \right)^{\theta}}}{\sqrt{J_e}} \implies \delta(t) = \delta_e \sqrt{1 + \left(\frac{t}{t_c} \right)^{\theta}} \xrightarrow[t>>0]{t>>t_c} \delta_e \left(\frac{t}{t_c} \right)^{\frac{\theta}{2}} \tag{5}$$

where δ_e is the overload distance due to the elastic response of the matrix
                 $\delta(t)$ is the overload distance as a function of time
                 $\xrightarrow[t>>0]{t>>t_c}$ simplification of equations at long times periods

One critical assumption inherent in these equations is that:
- the visco-elastic response of the matrix can be represented by an elastic model with a time dependent compliance.

The implications of critical assumptions will be examined later in the paper.

Probability of broken fiber cluster growth

By using the time dependent deformation in the overload region, the stress rupture response can be related to the following parameters:

1. the increased overload distance as a function of time, $\delta(t)$
2. the likelihood of having an additional break in the overload distance
3. the size of the broken fiber cluster, j
4. the total number of broken fiber clusters that could grow
5. the probability of cluster growth under constant load

Assuming the strength of a fiber obeys a Weibull distribution, the probability of a failure in a fiber of length, ℓ, exposed to a fiber stress level σ_f is

$$P_{fib} = 1 - e^{-\left(\frac{\sigma_f}{\sigma_\ell^o}\right)^\zeta} \xrightarrow[probability]{small} P_{fib} = \left(\frac{\sigma_f}{\sigma_\ell^o}\right)^\zeta \tag{6}$$

where $\qquad$ σ_ℓ^o is the strength scale parameter for fibers of length ℓ

$\qquad\qquad$ ζ $\quad$ is the fiber strength shape parameter

$\xrightarrow[probability]{small}$ indicates a small probability assumption which is used extensively in this derivation and is valid whenever the exponent in the probability equation is small ($1-e^{-x} \approx x$ when x<0.1). This approximation greatly simplifies the probability equations.

Note that the probability of the fiber surviving (i.e. the reliability) is just the converse of the probability of failure.

$$R_{fib} = 1 - P_{fib} = e^{-\left(\frac{\sigma_f}{\sigma_\ell^o}\right)^\zeta} \tag{7}$$

Because the strength of a fiber would obey a weakest link behavior, the strength distribution of a fiber would change with the fiber length. The probability of fiber failure with length δ is

$$P_{fib} = 1 - e^{-\frac{\delta}{\ell}\left(\frac{\sigma_f}{\sigma_\ell^o}\right)^\zeta} \tag{8}$$

Eq. 8 is equivalent to Eq. 6 when the strength scale parameter is defined in terms of length δ fibers as follows

$$\sigma_\delta^o = \left(\frac{\ell}{\delta}\right)^{\frac{1}{\zeta}} \sigma_\ell^o \tag{9}$$

To begin the fiber breakage model derivation, it is convenient to calculate the probability of an initial failure in a fiber of length δ_e. (δ_e is the characteristic overload distance due to elastic deformation along a neighboring fiber which will enter the derivation later).

$$P_{\delta_e} = 1 - e^{-\left(\frac{\sigma_f}{\sigma_{\delta_e}^o}\right)^\zeta} \xrightarrow[probability]{small} P_{\delta_e} = \left(\frac{\sigma_f}{\sigma_{\delta_e}^o}\right)^\zeta \tag{10}$$

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Once an initial break occurs, the neighboring fibers will see elevated stresses. The probability of a second break occurring in a neighboring fiber due to the initial break is:

$$P_2 = 1 - e^{-n_1 \frac{2\delta(t)}{\delta_e}\left(\frac{K_1\sigma_f}{\sigma_{\delta_e}^o}\right)^{\zeta}} \xrightarrow[\substack{small \\ probability}]{} P_2 = n_1 \frac{2\delta(t)}{\delta_e}\left(\frac{K_1\sigma_f}{\sigma_{\delta_e}^o}\right)^{\zeta} \tag{11}$$

where K_1 is the stress concentration in a neighboring fiber caused by a cluster size $j=1$
$\quad\quad\quad n_1$ is the number of neighboring fibers for a cluster size $j=1$
$\quad\quad\quad 2\delta(t)$ is the length along a neighboring fiber that is overloaded

The combined probability of having an initial break followed by the second break is then

$$P_{1\to 2} = P_{\delta_e} P_2 \xrightarrow[\substack{small \\ probability}]{} P_{1\to 2} = 2n_1 \frac{\delta(t)}{\delta_e}\left(\frac{K_1\sigma_f}{\sigma_{\delta_e}^o}\right)^{\zeta}\left(\frac{\sigma_f}{\sigma_{\delta_e}^o}\right)^{\zeta} \tag{12}$$

which can be generalized to the probability of a cluster of size j forming in time t as

$$P_{1\to j} = P_{\delta_e} P_2 P_3 ... P_j \xrightarrow[\substack{small \\ probability}]{} P_{1\to j} = (n_1 n_2 . n_{j-1})\left(\frac{2\delta(t)}{\delta_e}\right)^{j-1}\left(K_1 K_2 ... K_{j-1}\right)^{\zeta}\left(\frac{\sigma_f}{\sigma_{\delta_e}^o}\right)^{j\zeta} \tag{13}$$

A conservative assumption is made in Eq. 13 that the entire time is available at each new cluster size for the overload zone to grow.
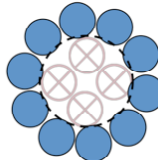
K and n are related and somewhat compensating parameters. If one assumes fewer neighboring fibers, then the elevation in stress in the neighboring fibers increases. To simplify equations, a planar array of fibers was assumed. Thus, two intact fibers on either side of a line of broken fibers carry the load of the broken fiber cluster. The two neighbors are assumed to carry most of load of the broken cluster, but not all the load as would be expected from a fracture mechanics type decaying stress state around a sharp notch. Therefore, n and K for any cluster of size, j, are assumed to be:

$$n_j = 2 \quad and \quad K_j = \sqrt{1 + \frac{\pi j}{4}} \quad\quad (2D\ assumption) \tag{14}$$

Although a planar array of fibers is generally assumed in the fiber breakage model, a 3D array of fibers was also been postulated in the original derivation [6]. By assuming a circular cluster of broken fibers, the following equations were derived,

$$n_j = \pi\left(\sqrt{\frac{4j}{\pi}} + 1\right) \quad and \quad K_j = \sqrt{1 + \sqrt{\frac{4j}{\pi^3}}} \quad (3D\ assumption) \tag{15}$$

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Table 2. Cluster size influence on n, K and δ_{safe}/δ_e.

Cluster size j	2D Cluster					3D Cluster				
	n_j	K_j	δ_{safe}/δ_e (Eq 39)			n_j	K_j	δ_{safe}/δ_e (Eq 39)		
		(Eq 14)	$\sigma_P/\sigma=$	1.25	1.5		(Eq 15)	$\sigma_P/\sigma=$	1.25	1.5
1	2	1.34		3.90	∞	6.7	1.17		∞	∞
2	2	1.60		1.71	5.84	8.2	1.23		∞	∞
3	2	1.83		1.43	2.51	9.3	1.27		11.58	∞
4	2	2.04		1.32	1.93	10.2	1.31		5.11	∞
5	2	2.22		1.26	1.69	11.1	1.34		3.69	∞
6	2	2.39		1.22	1.56	11.8	1.37		3.07	∞
7	2	2.55		1.19	1.48	12.5	1.40		2.71	∞
10	2	2.98		1.14	1.34	14.4	1.46		2.18	∞
20	2	4.09		1.26	1.69	19.0	1.61		1.69	5.37

The values of the number of surrounding fibers (n) and stress concentration values (K) for the 2D and 3D assumptions are compared in Table 2. With the 3D assumption, the number of neighboring fibers is higher which decreases the stress concentration for a given cluster size.

Critical assumptions in this section include:
- Stress state in the overload region assumed constant at a level Kσ.
- A 2D arrangement of fibers adequately represents the damage state.
- Each cluster grows for the entire time.

The implications of critical assumptions will be examined later in the paper.

Probability of strand or vessel failure due to clusters of broken fibers

The final step is realizing that there is a large number (N) of δ_e length fibers in a strand or vessel that could originate the cluster. This derivation will assume a scale up to a vessel, but the same equations would apply to strands by substituting the scale parameter of a strand (σ^o_{strand}) for the scale parameter of the vessel (σ^o_{vessel}). The reliability of the vessel failing is the probability that none of the fibers initiate a cluster that grows to failure. The probability that none of the δ_e lengths initiate a critical cluster is:

$$R_{vessel} = \left(R_{1 \to j}\right)^N = \left(1 - P_{1 \to j}\right)^N \xrightarrow[\text{probability}]{small} R_{vessel} = \left(1 - \left(1 - e^{-P_{1 \to j}}\right)\right)^N$$

$$R_{vessel} = e^{-N P_{1 \to j}}$$

(16)

The probability of a vessel failure is just the converse of reliability.

$$P_{vessel} = 1 - e^{-N P_{1 \to j}}$$

(17)

Note that in Eq. 16, $P_{1 \to j}$ is assumed small therefore $P_{1 \to j}$ can be expressed as

$P_{1 \to j} = 1 - e^{-P_{1 \to j}}$ and substituting Eq. 13 into Eq. 17

$$P_{vessel} = 1 - e^{-\left[N\,2^{j-1}(n_1 n_2 ... n_{j-1})(K_1 \cdot K_2 \cdot K_3 \cdot ... K_{j-1})^\varsigma \left(\frac{\delta(t)}{\delta_e}\right)^{(j-1)} \left(\frac{\sigma_f}{\sigma^o_{\delta_e}}\right)^{j\varsigma} \right]}$$

(18)

The assumption in this step is that the probability of any one δ_e length initiating a critical cluster ($P_{1 \to j}$) is small. However, because N will be large, it is possible for P_{vessel} to be large.

Assuming that:

1. the strength of the vessel obeys a Weibull distribution $P_{vessel} = 1 - e^{-\left[(s)^\alpha\right]}$
   where $s \equiv \sigma \Big/ \sigma^o_{vessel}$
2. the strength is measured when t is sufficiently small so $\delta(t) = \delta_e$
3. the composite and fiber nominal stresses are proportional $\sigma = D\,\sigma_f$
4. the vessel fails when a critical cluster of size j_c is formed
5. the critical cluster size is not a function of σ

Eq. 18 can be shown to reduce to a Weibull distribution by making the following substitutions:

$$\sigma^o_{vessel} = \frac{D\sigma^o_{\delta_e}}{N^{\frac{1}{j_c\varsigma}}\,2^{\frac{j_c-1}{j_c\varsigma}}\,(n_1 n_2 ... n_{j_c-1})^{\frac{1}{j_c\varsigma}}\,(K_1 \cdot K_2 \cdot K_3 \cdot ... K_{j_c-1})^{\frac{1}{j_c}}}$$

(19)

$$\alpha = j_c\varsigma$$

The time dependent probability of failure and reliability become

$$P_{vessel}(t) = 1 - e^{-\left[\left(\frac{\delta(t)}{\delta_e}\right)^{(j_c-1)}(s)^{j_c\varsigma}\right]}$$

(20)

$$R_{vessel}(t) = e^{-\left[\left(\frac{\delta(t)}{\delta_e}\right)^{(j_c-1)}(s)^{j_c\varsigma}\right]}$$

(21)

Substituting the visco-elastic model of Eq. 5 in Eq. 21 produces

$$R_{vessel}(t) = e^{-\left[\sqrt{1+\left(\frac{t}{t_c}\right)^\theta}^{(j_c-1)}(s)^{j_c\varsigma}\right]} \xrightarrow[t>>0]{t>>t_c} R_{vessel}(t) = e^{-\left[\left(\frac{t}{t_c}\right)^{\frac{\theta(j_c-1)}{2}}(s)^{j_c\varsigma}\right]}$$

(22)

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Eq. 22 can be used to make reliability predictions with the fiber breakage model when no proof loading is applied, and has modeling parameters t_c, θ, j_c, ζ, and the inputs are s and t. σ and σ_{vessel}^o are also needed to calculate s. Common parameters used for the T1000 fiber based composite are presented in Table 1.

At large t, the fiber breakage model can be shown to be equivalent to the classic model given in Eq. 1 by making the following substitution into Eq. 22:

$$\beta = \frac{\theta}{2}(j_c - 1) \qquad\qquad t_{ref} = t_c$$

$$\rho = \frac{2 j_c \varsigma}{(j_c - 1)\theta} \qquad\qquad s = \frac{\sigma}{\sigma_{vessel}^o} \tag{23}$$

Therefore, the fiber breakage model predictions can be comparable to the classic model predictions at long times, when a proof load is not applied. However, the fiber breakage model is based on micro-mechanics that requires numerous assumptions and more modeling parameter inputs than required in the classic model. The two models are only comparable in their predictions if specific values of the fiber breakage parameters are chosen, and these values may or may not be representative of the physical properties that they are meant to represent.

Figure 1 shows the fiber breakage model and the classic model reliability predictions for a T1000 composite before proof, and it is clear that the models agree at longer times. The models use the input parameters listed in Table 1, that have been suggested for this composite [2]. However, the models agree because the fiber breakage model parameters were chosen to fit the classic model predictions instead of being measured directly from the physical phenomenon that the parameters are intended to represent.

Critical assumptions in this section are that:
- Input parameters back calculated from the classic model adequately represent the actual micro-mechanical response.
- The critical cluster size, j_c, remains constant over a wide range of load levels.

The implications of critical assumptions will be examined later in the paper.

Strand or vessel reliability after proof loading
The predictions of the classic model and the fiber breakage model diverge once proof loading is applied. From Eq. 13, the probability of a critical cluster of size j forming during proof loading from a δ_e section of fiber is

$$P_{1 \to j_c}^p \xrightarrow[probability]{small} P_{1 \to j_c}^p = P_{\delta_e}^p P_2^p P_3^p ... P_{j_c}^p$$

$$= (n_1 n_2 .. n_{j_c-1}) \left(\frac{2\delta(t)}{\delta_e}\right)^{j_c-1} \left(K_1 K_2 .. K_{j_c-1}\right)^\xi \left(\frac{\sigma_f}{\sigma_{\delta_e}}\right)^{j_c \xi} \tag{24}$$

After the proof load, only clusters creating an overload stress larger than the proof loading $(\sigma_f K_{j-1} \geq \sigma_f^p)$ can grow to size j. As seen in Figure 4, smaller clusters with smaller K's will never develop a danger area. Assuming the 2D load sharing of Eq. 14 leads to

$$\frac{\sigma_f^p}{\sigma_f} < K_{j-1} = \sqrt{1 + \frac{\pi(j-1)}{4}} \tag{25}$$

Solving for the minimum cluster size that would be required to form during proof that could eventually lead to a stress rupture failure under normal load produces the following equation

$$j_p = \frac{4}{\pi}\left(\left(\frac{\sigma_f^p}{\sigma_f}\right)^2 - 1\right) + 1 \tag{26}$$

Assuming the nominal stress in the composite is proportional to the nominal fiber stress ($\sigma = D\sigma_f$), j_p can also be expressed in terms of the composite stress ratio, s, as

$$j_p = \frac{4}{\pi}\left(\left(\frac{s_p}{s}\right)^2 - 1\right) + 1 \tag{27}$$

where j_p is not the cluster size created under proof, but is instead the cluster size that would be needed for further growth under nominal stress. Thus, j_p is a function of both s and s_p. Furthermore, since the vessel did not burst, $j_p < j_c$.

The probability of a flaw in a δ_e long section of fiber growing to j_p during proof, and then on to j_c during use pressure, is

$$P_{1 \to j_p}^p P_{(j_p+1) \to j_c} = P_{\delta_e}^p P_2^p ... P_{j_p}^p * P_{j_p+1} P_{j_p+2} ... P_{j_c} \tag{28}$$

Note that $\delta(t)$ in P_{j_p} must be adjusted for the growth that already occurred in the proof test cycle

$$P_{j_p+1} = 2n_{j_p} \frac{\delta(t) - \delta(t_p)}{\delta_e}\left(\frac{K_{j_p}\sigma_f^p}{\sigma_{\delta_e}^o}\right)^\varsigma = 2n_{j_p} \frac{\delta(t_p)}{\delta_e}\left(\frac{\sigma_f^p}{\sigma_{\delta_e}^o}\right)^\varsigma K_{j_p}^\varsigma \left[\left(\frac{\delta(t)}{\delta(t_p)} - 1\right)\left(\frac{\sigma_f}{\sigma_f^p}\right)^\varsigma\right] \tag{29}$$

When calculating reliability after a proof loading, two numbers can be calculated. One can calculate the reliability based on all failures, R^{ip} (including the chance of failures during proof loading). Alternatively, a conditional reliability, R^{cp}, can be calculated (conditional on survival of proof loading) where only the failures that occur due to post proof loading are counted. The conditional reliability that ignores the risk of proof loading failures is the more relevant number for long-term reliability estimates because proof tests are conducted so that proof failures do not cause significant damage to surrounding equipment or personnel. Although the R^{cp} is the more

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relevant number, calculating R^{ip} and the associated total probability of failure P^{ip} are helpful in calculating, $R^{cp.}$.

P^{ip} is the probability of a cluster of size j_p being created during proof and then either growing to failure under proof loading or later under nominal stress and is given by

$$P^{ip}_{1 \to j_c} \xrightarrow{\substack{small \\ probability}} P^{p}_{1 \to jc} + P^{p}_{1 \to j_p} P_{(j_p +1) \to j_c} = P^{p}_{1 \to j_p} (P^{p}_{(j_p +1) \to j_c} + P_{(j_p +1) \to j_c}) \qquad (30)$$

$$P^{ip}_{1 \to j_c} \xrightarrow{\substack{small \\ probability}} 2^{j_c -1} (n_1 n_2 ... n_{j_c -1})(K_1 \cdot K_2 \cdot K_3 \cdot ... K_{j_c -1})^{\varsigma} \left(\frac{\delta(t_p)}{\delta_e}\right)^{j_p -1} \left(\frac{\sigma_f^p}{\sigma_{\delta_e}^o}\right)^{\varsigma j_p} *$$
$$\left(\left[\left(\frac{\delta(t_p)}{\delta_e}\right)\left(\frac{\sigma_f^p}{\sigma_{\delta_e}}\right) + \left(\frac{\delta(t_p)}{\delta_e}\right)\left(\frac{\sigma_p}{\sigma_{\delta_e}}\right)\left[\left(\frac{\delta(t)}{\delta(t_p)} -1\right)\left(\frac{\sigma_f}{\sigma_f^p}\right)^{\varsigma}\right]\right]\right)^{(j_c -j_p)} \qquad (31)$$

$$P^{ip}_{1 \to j_c} \xrightarrow{\substack{small \\ probability}} \left(2^{j_c -1} (n_1 n_2 ... n_{j_c -1})(K_1 \cdot K_2 \cdot K_3 \cdot ... K_{j_c -1})^{\varsigma} \left(\frac{\delta(t_p)}{\delta_e}\right)^{j_c -1} \left(\frac{\sigma_f^p}{\sigma_{\delta_e}^o}\right)^{\varsigma j_c}\right) *$$
$$\left[1+\left(\frac{\delta(t)}{\delta(t_p)} -1\right)\left(\frac{\sigma_f}{\sigma_f^p}\right)^{\varsigma}\right]^{(j_c -j_p)} \qquad (32)$$

$$P^{ip}_{1 \to j_c} \xrightarrow{\substack{small \\ probability}} P^{p}_{1 \to j_c}\left[1+\left(\frac{\sigma_f}{\sigma_f^p}\right)^{\varsigma}\left(\frac{\delta(t)}{\delta(t_p)} -1\right)\right]^{j_c -j_p} \qquad (33)$$

The conditional proof (cp) reliability (probability of failing during service after surviving proof loading) is:

$$R^{cp}_{1 \to j_c} = \frac{R^{ip}_{1 \to j_c}}{R^{p}_{1 \to jc}} = \frac{1 - P^{ip}_{1 \to j_c}}{1 - P^{p}_{1 \to jc}} \xrightarrow{\substack{small \\ probability}} \frac{1-(1-e^{-P^{ip}_{1 \to j_c}})}{1-(1-e^{-P^{p}_{1 \to jc}})} = \frac{e^{-P^{ip}_{1 \to j_c}}}{e^{-P^{p}_{1 \to jc}}} = e^{-(P^{ip}_{1 \to j_c} -P^{p}_{1 \to j_c})} \qquad (34)$$

The conditional proof (cp) probability is then just the converse of the reliability.

$$P^{cp}_{1 \to j_c} = 1 - R^{cp}_{1 \to j_c} = 1 - e^{-(P^{ip}_{1 \to j_c} -P^{p}_{1 \to j_c})} \xrightarrow{\substack{small \\ probability}} P^{ip}_{1 \to j_c} - P^{p}_{1 \to j_c} = P^{p}_{1 \to j_c}\left(\left[1+\left(\frac{\sigma_f}{\sigma_f^p}\right)^{\varsigma}\left(\frac{\delta(t)}{\delta(t_p)} -1\right)\right]^{j_c -j_p} -1\right) \qquad (35)$$

Scaling to the vessel level as was done in Eq. 16-22, the vessel reliability including proof becomes

$$R_{vessel}^{ip} = e^{-\left[\left(\frac{\delta(t_p)}{\delta_e}\right)^{(j_c-1)}(s_p)^{j_c\varsigma}\right]\left(\left[1+\left(\frac{s}{s_p}\right)^{\varsigma}\left(\frac{\delta(t)}{\delta(t_p)}-1\right)\right]^{j_c-j_p}\right)}$$

(36)

Conditional reliability given proof survival

$$R_{vessel}^{cp} = e^{-\left[\left(\frac{\delta(t_p)}{\delta_e}\right)^{(j_c-1)}(s_p)^{j_c\varsigma}\right]\left(\left[1+\left(\frac{s}{s_p}\right)^{\varsigma}\left(\frac{\delta(t)}{\delta(t_p)}-1\right)\right]^{j_c-j_p}-1\right)}$$

(37)

Substituting the visco-elastic model of Eq. 5 and writing the equation in terms of stress ratio

$$R_{vessel}^{cp} = e^{-\left[\left(\sqrt{1+\left(\frac{t_p}{t_c}\right)^{\theta}}\right)^{(j_c-1)}(s_p)^{j_c\varsigma}\right]\left(\left[1+\left(\frac{s}{s_p}\right)^{\varsigma}\left(\frac{\sqrt{1+\left(\frac{t}{t_c}\right)^{\theta}}}{\sqrt{1+\left(\frac{t_p}{t_c}\right)^{\theta}}}-1\right)\right]^{j_c-j_p}-1\right)}$$

(38)

which is equivalent to equation 46 in the ITAI report [6] given for the conditional vessel reliability. Note that the original equation was expressed with α the scale parameter on composite strength which was assumed equal to $j_c\varsigma$.

Eq. 38 like Eq. 22 has modeling parameters t_c, θ, j_c, ς, and the inputs are s, t, but has the additional input parameters, s_p and t_p, to describe be proof loading. Note that j_p is not an independent parameter because it is a combination of the input parameters as given by Eq. 27. Note also that if α is used in the equation, it is an additional modeling parameter if $\alpha \neq \varsigma\, j_c$ since ς and j_c are used elsewhere in the equation. The fiber breakage model therefore has four or five independent parameters as opposed to the classic model which has three.

Using the fiber breakage model and parameters in Table 1, the reduction in predicted reliability due to proof loading can be seen to be dramatic as shown in Figure 1 and, therefore, reason for concern. For the example of a 5 min. proof at stress ratio 0.75, the predicted reliability is only two 9's or one in 100 being predicted to fail in 10 yrs. This is a reduction in reliability of four orders of magnitude due to the proof loading. However, there are numerous assumptions that were made during this derivation that have not been validated. The effect that these assumptions have on the reliability predictions will be addressed in the next section.

Effect of Simplifying Assumptions

As pointed out in the last section, numerous assumptions were made in the derivation of the fiber breakage model equations to make the equations tractable. In this section the effect of these

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simplifying assumptions will be discussed. The more critical assumptions highlighted in the previous sections are listed here in the order in which they will be discussed:

- Stress state in the overload region assumed constant.
- Input parameters back calculated from classic model adequately represent the actual micro-mechanical response.
- A 2D arrangement of fibers adequately represents the damage state.
- The visco-elastic response of the matrix can be represented by an elastic model with a time dependent compliance.
- Critical cluster size, j_c, assumed independent of nominal stress.
- Each cluster grows for entire time.

Assumption: Stress state in the overload region assumed constant.

A very important part of the fiber breakage model relates to the way load is shared by a neighboring fiber in the area of a broken fiber. The mathematical derivation assumes a characteristic distance δ over which the load is transferred from the broken fiber to the neighboring fiber. The δ is assumed to grow over time due to the visco-elastic deformation of the matrix material. In the mathematical derivation, δ is related to the creep modulus through Eq. 5 which comes from an elastic shear lag model.

However, to make the statistical model tractable, the load in the neighboring fibers is assumed to be constant over the δ distance. This is in contrast to the results of a shear lag model where the stress in the first unbroken fiber is predicted to decay rapidly as one moves along the fiber but away from the location of a nearby fiber break. Assuming that the stress jumps from the nominal level to the constant elevated level at a δ boundary is actually quite unrealistic because a change in fiber load must occur due to matrix shear stress which is compliant compared to the fiber. Therefore the change in fiber load should be more gradual. This simplification has been recognized and debated in the literature but was used in the fiber breakage model to simplify the equations because it was believed to cause a negligible effect [4, 7, 8]. This section will look at the merit of this assumption and compare it to other possible simplifying assumptions.

The fiber breakage model is primarily used to make predictions of reduced reliability after proof loading. The reduced reliability is due to "danger areas" along the neighboring fiber where stress increases over time to a higher level than was experienced during the short exposure to proof loading. These danger areas are therefore the areas where there is some chance of additional fiber damage that can eventually lead to a stress rupture failure. Although the probability of cluster growth from a given cluster after proof may be small, the overall reliability after proof may be reduced because the proof load may have created a larger number of clusters. The assumption of a constant load in the δ region leads to a simple characterization of the "danger area" because all of the fibers in the region are elevated by a constant amount, and the growth of the danger area is then completely characterized by how δ grows with time.

A number of other possible assumptions could be made as shown in Figure 5. One could assume that plastic deformation occurs in the matrix. If one assumes that because of plasticity, shear stress in the matrix remains constant near the break, then the change in fiber stress should be linear. This leads to a triangular elevated stress distribution in the danger area with the region expanding over time as shown in Figure 5. After proof loading the elevated stress region δ must grow significantly before a danger area is created. Using the geometry shown in Figure 6, the following expression for when the danger area is created can be derived.
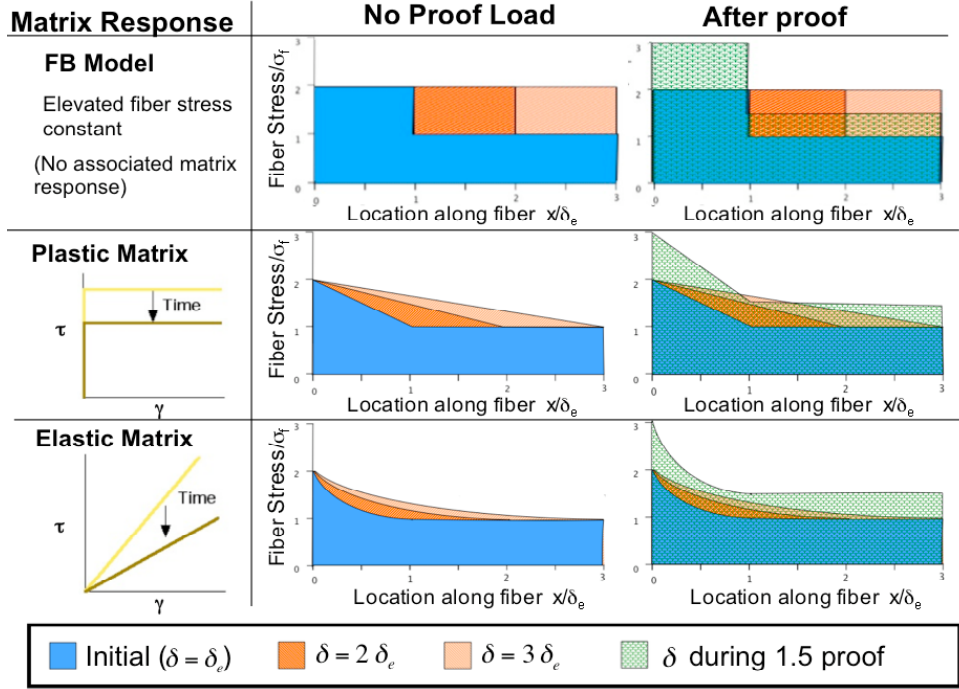
Figure 5. Change in fiber stress near fiber break with time with and without proof loading (K=2 and s_p/s=1.5 assumed).

$$K - \frac{(K-1)}{\delta/\delta_e}(1) = \frac{\sigma_P}{\sigma} \tag{39}$$

Solving for δ/δ_e and assuming the δ growth law of Eq. 5 produces

$$\frac{\delta(t_{safe})}{\delta_e} = \sqrt{1 + \left(\frac{t_{safe}}{t_c}\right)^\theta} = \frac{K-1}{K - \frac{\sigma_P}{\sigma}} \tag{40}$$

where t_{safe} is the time required for a danger area to form.

Solving Eq. 40 for t_{safe} produces

$$t_{safe} = t_c \left(\left[\frac{K-1}{K - \frac{\sigma_P}{\sigma}}\right]^2 - 1\right)^{\frac{1}{\theta}} \tag{41}$$

Until this time, the model would predict no chance of continued damage growth and 100% conditional reliability after surviving proof loading. This safe time, t_{safe}, is clearly related to the level of proof and the cluster size (through K). After t_{safe}, the danger area would grow much more slowly than predicted by assuming the elevated stress was constant.

The t_{safe} is smaller for higher stress concentrations. The worst case stress concentration occurs at the last break before a cluster becomes critical (j_c-1). As shown in Table 2 for a 2D cluster, the critical stress concentration would be K=2.04 assuming j_c=5, and the effective overload zone would need to grow to 1.32 or 1.93, for proof loads of $s_p/s =$ 1.25 and 1.5, respectively. Therefore, the effective overload region would have to grow by nearly 1/3 before there is any chance of failure. The effect of the assumed visco-elastic model and input parameters on t_{safe} will be discussed in the next section.
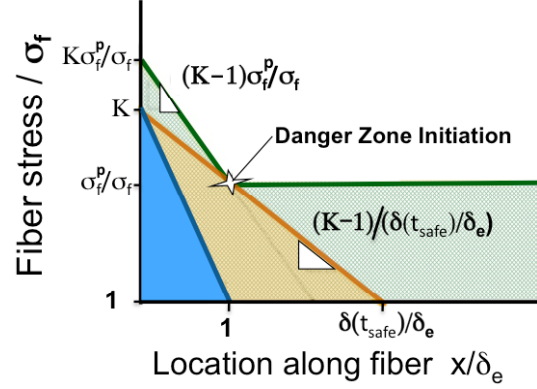

Figure 6. Geometry governing initiation of danger zone assuming plastically deforming matrix.

Assumption: Input parameters back calculated from classic model adequately represent the actual micro-mechanical response.

The current model assumes a t_c=.01 hrs and θ=.11 in order to match the classic Weibull model predictions before proof.   The predicted growth of the overload region using these parameters is shown in Figure 7. Overload lengths of 1.32 and 1.93 would lead to t_{safe} times of .0006 hrs (2 sec) and 90 hrs, given s_p/s = 1.25 and 1.5 proof loads, respectively. However, the predictions that the deformations would grow by 1/3 due to creep in 1 sec does not seem credible for the cured epoxy matrix. More realistic values might be t_c=98,000 hr, θ=0.247 [9], which is the measured creep compliance of a unidirectional composite (T300/5208) with fibers 10° to the loading direction. This type of test primarily loads the matrix in shear. With these parameters, the t_{safe} becomes 30,000 (3.4 yrs) and 6,000,000 hrs (680 yrs) for proof levels of s_p/s = 1.25 and 1.5, respectively. With more reasonable visco-elastic constants, the model predicts no chance of failure for significant periods and reverses the prediction that higher loads are detrimental. These predictions are much closer to the predictions of the classic model that predicts extremely high reliability following proof and better results with higher proof.   But with these updated parameters, the fiber breakage model would not replicate the classic Weibull model predictions without a proof load.

The long safe periods also rely on the relatively small j_c=5. As shown in Table 2, the value of K can significantly increase with cluster size. With larger j_c values, there could be larger clusters that would not have the long safe periods that were predicted here.

The parameters in reference 1 are tied to the original matrix creep parameters to match the classic Weibull model without
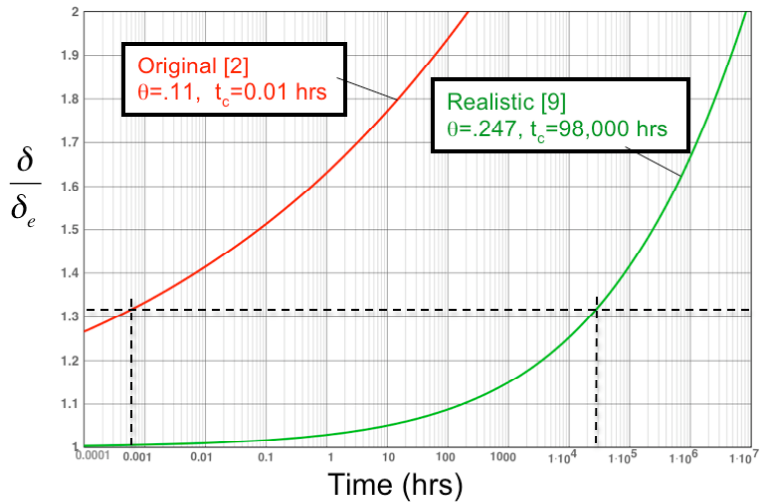

Figure 7. Effect of matrix creep parameters on predicted response.

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accounting for proof loading. Fitting the parameters to the classic model allowed a direct comparison between the two models. However, fitting the parameters to the global response invalidates the claim of a physics based model if the resulting values are not realistic in representing the response of the phenomenon they are supposed to represent. In this case, it also indicates that some of the many assumptions made during the derivation may not have been valid. Even basic questions, such as "Does the fiber breakage model represent the right physical phenomenon?" may be questioned. Although the mechanism that is being modeled by the fiber breakage model does appear to be a reasonable suggestion for the stress rupture phenomenon, there is no proof that it is, in fact, the controlling phenomenon.

Assuming a constant plastic stress in the matrix is just one possible assumption that could have been made. Figure 5 shows other possible assumptions. An elastic model would create a more gradual decay in stress at the end of the elevated stress region. This would require even more growth in δ before the proof and after proof curves would cross and result in longer values of t_{safe}.

A debond along the fiber could also be assumed. This would tend to elongate the region of elevated stress in the neighboring fiber. Then, one would need to postulate why the elevated stress in the neighboring fiber would change with time. There are two possible ways in which the debonds could grow with time:

1) *Visco-elastic debond growth*. This does not seem likely because the debond stopped growing under proof loading. Crack growth (i.e. the debond) is driven by strain energy release rate, which is normally proportional to the load squared. The reduced loading under use conditions would therefore have a dramatically reduced driving force on the crack. To explore this possibility a visco-elastic crack growth model would be required.

2) *Debonding under proof followed by visco-elastic matrix deformation*. Another possibility is that the fiber debonds under proof loading, increasing the length of the overload region, but then under nominal loading, the matrix deforms visco-elastically. This would result in values of t_{safe} as produced earlier because the change in stress at the end of the debond would closely resemble the change in stress without a debond. An inconsistency still exists in that visco-elastic constants are being assumed while the shape of the overload region is assumed to be plastic. Since the plastic deformation was assumed to be conservative, and epoxy resins are fairly brittle, large regions of plastic deformation are unlikely.

By making small improvements to the micro-mechanics assumed by the fiber breakage model, the predicted reliability after proof changes dramatically and the response is more in line with the extremely high reliability predicted by the classic model after proof loading.

Assumption: A 2D arrangement of fibers adequately represents the damage state.
A 2D arrangement of broken fibers was assumed. A 3D approximation has also been proposed that appears more realistic. The 2D approximation tends to underestimate the number of neighboring fibers and then overestimate the stress concentration. In the original formulation of the fiber breakage model, these errors would tend to cancel each other. In the shear lag fiber breakage model, the safe period is only a function of the stress concentration so the overestimation of the stress concentration would tend to significantly underestimate the safe period. As seen in Table 2, the required growth in the overload regions required for the formation of a damage zone is much larger in a 3D model than in the 2D model for a given size of cluster. The larger zone growth would translate into much longer safe periods.

Assumption: The visco-elastic response of the matrix can be represented by an elastic model with a time dependent compliance.

The stress field around a broken cluster is a complex stress state [10]. Substituting a visco-elastic compliance for the elastic compliance assumes that all loading remains constant in the structure. With plastic, visco-elastic and elastic deformations occurring in the matrix surrounding the fiber, the more heavily loaded parts of the matrix will most certainly shed load into less highly loaded regions as the matrix creeps. The substitution of the visco-elastic compliance is therefore invalid. Although stress redistribution might tend to slow the time dependent effects, the actual response at a given location along a fiber is difficult to measure or predict.

Assumption: Critical cluster size, j_c, assumed independent of nominal stress.

The fiber breakage model assumes that the critical cluster size is constant regardless of global stress level. This appears to be a poor assumption because as the nominal stress level is reduced the critical cluster size would likely increase. Because the fiber breakage model is quite sensitive to the cluster size parameters, this would tend to significantly increase the reliability at lower stress levels above what is currently predicted by the fiber breakage model. This would be true with or without proof.

Assumption: Each cluster grows for entire time.

The fiber breakage model assumes that at each increase in cluster size, the entire time of loading is available for increased overload region growth. One exception is that proof time is separated from time after proof. The time dependent growth of the overload region would actually be divided between the various steps. Allowing all the time at each step certainly yields a conservative approximation, but it may not be as conservative as it first appears because growth rate decreases after loading. If the majority of time is consumed with growth at one crucial cluster size, the approximation would be rather accurate.

Conclusion

The fiber breakage model examined was an attempt at a physics based model for stress rupture of composite material that would account for proof loading history. The fiber breakage model equation was re-derived while highlighting assumptions required for the derivation. Some of these assumptions were then examined. The assumptions, in general, were found to be conservative but a few adjustments to the original assumptions completely changed the predicted response after proof loading.

The original fiber breakage model postulated a significant decrease in reliability due to damage resulting from elevated proof loading. The improved micro-mechanics model does not support this postulation but still relies on large numbers of assumptions of the micro-mechanics behavior that cannot be validated. Therefore, a damaging effect of proof loading cannot be ruled out by analysis.

The sensitivity of the current fiber breakage model to small changes in assumed behavior at the micro-mechanics level make it an inappropriate model for predicting stress rupture. However, it may be a useful tool for understanding the mechanics causing the stress rupture phenomenon and may eventually be improved and substantiated so that reliability predictions can be made. Until validation of a fiber breakage model is accomplished at both a micro- and macro-level, it should not be used for reliability estimates. However, the effects of proof loading are still unknown, and the present work does not alleviate the general concern. This issue must be addressed by test and further analytical work.

References

1. NESC, *Shelf Life Phenomenon and Stress Rupture Life of Carbon/Epoxy Composite Overwrapped Pressure Vessels*. Report RP-06-83**,** 2006, NASA Engineering and Safety Center.

2. Phoenix, S.L. and L.N.P. Murthy, "Pro's and Cons of Proof Testing Carbon Composite Overwrapped Pressure Vessels: A Comparison of Two Mathematical Models," AIAA Paper No. 2007-2325, presented at *48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*. 2007. Honolulu, HI.

3. Grimes-Ledesma, L., "A Comparative Study of the Stress-Rupture Lifetimes of High-Strength Carbon Fibers," dissertation from *Materials Science and Engineering Dept*. 2006, U of California: Los Angeles.

4. Phoenix, S.L. and I.J. Beyerlien, "Statistical Strength Theory for Fibrous Composite Materials," Chapter 19 of *Comprehensive Composite Materials*. *Vol. 1*, Elsevier Science Ltd. 2000. pp. 559-639.

5. Nairn, J.A., "Fracture Mechanics of Unidirectional Composites Using the Shear Lag Model, I: Theory". *Journal of Composite Materials*, 1988. **22**(6): pp. 561-588.

6. NASA Engineering and Safety Center, "Development of the Fiber Breakage Model and Implications of Proof Testing on the Reliability of COPVs," in *Shelf Life Phenomenon and Stress Rupture Life of Carbon/Epoxy Composite Overwrapped Pressure Vessels*. Report RP-06-83, 2006.

7. Lagoudas, D.C., C.Y. Hui, and S.L. Phoenix, "Time Evolution of Overstress Profiles Near Broken Fibers in a Composite With A Viscoelastic Matrix." *Int J Solids Structures,* 1989. **25**(1): pp. 45-66.

8. Phoenix, S.L., P. Schwarz, and H.H. Robinson, "Statistics for the Strength and Lifetime in Creep-Rupture of Model Carbon/Epoxy Composites," *Composites Science and Technology*, 1988. **32**: pp. 81-120.

9. Tuttle, M.E. and H.F. Brinson, "Accelerated Viscoelastic Characterization of T300/5208 Graphite-Epoxy Laminates." NASA CR 3871, March 1985.

10. Zhou, C.H., I.J. Beyerlein, and L.S. Schadler, "Time-Dependent Micromechanical Behavior in Graphite/Epoxy Composites Under Constant Load at Elevated Temperatures." *Journal of Material Science*, 2003. **38**: pp. 877-884.

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14. ABSTRACT

Stress rupture is not a critical failure mode for most composite structures, but there are a few applications where it can be critical. One application where stress rupture can be a critical design issue is in Composite Overwrapped Pressure Vessels (COPV's), where the composite material is highly and uniformly loaded for long periods of time and where very high reliability is required. COPV's are normally required to be proof loaded before being put into service to insure strength, but it is feared that the proof load may cause damage that reduces the stress rupture reliability. Recently, a fiber breakage model was proposed specifically to estimate a reduced reliability due to proof loading. The fiber breakage model attempts to model physics believed to occur at the microscopic scale, but validation of the model has not occurred. In this paper, the fiber breakage model is re-derived while highlighting assumptions that were made during the derivation. Some of the assumptions are examined to assess their effect on the final predicted reliability.

15. SUBJECT TERMS
Composite materials; Stress rupture; Reliability

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