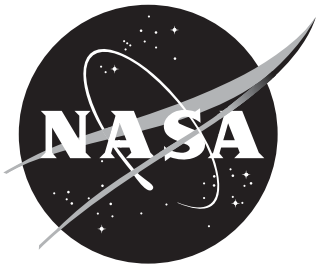
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Exploratory Study for Continuous-Time Parameter Estimation of Ankle Dynamics

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May 2014

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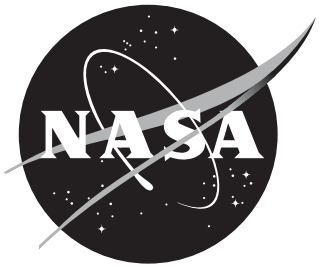
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Abstract

Recently, a parallel pathway model to describe ankle dynamics was proposed. This model provides a relationship between ankle angle and net ankle torque as the sum of a linear and nonlinear contribution. A technique to identify parameters of this model in discrete-time has been developed. However, these parameters are a nonlinear combination of the continuous-time physiology, making insight into the underlying physiology impossible. The stable and accurate estimation of continuous-time parameters is critical for accurate disease modeling, clinical diagnosis, robotic control strategies, development of optimal exercise protocols for longterm space exploration, sports medicine, etc.

This paper explores the development of a system identification technique to estimate the continuous-time parameters of ankle dynamics. The effectiveness of this approach is assessed via simulation of a continuous-time model of ankle dynamics with typical parameters found in clinical studies. The results show that although this technique improves estimates, it does not provide robust estimates of continuous-time parameters of ankle dynamics. Due to this we conclude that alternative modeling strategies and more advanced estimation techniques be considered for future work.

Nomenclature

Acronyms

IRF- Impulse Response Function
NARMAX- Nonlinear AutoRegressive, Moving Average eXogenous
ELS - Extended Least-Squares
PEI - Prediction Error Identification
SLS - Standardized Least-Squares
PRBS - Pseudo-Random Binary Sequence
SNR - Signal-to-Noise Ratio
NF - Noise-Free
CT - Continuous-Time
FIR - Finite Impulse Response
IIR - Infinite Impulse Response
LTI - Linear Time Invariant

Symbols

$\mathscr{F}^l$ - nonlinear mapping
u - exogenous input
z - output
e - innovation, or uncontrolled input
n - sample index point
l - maximum polynomial order
n_z - maximum output lag
n_u - maximum input lag
n_e - maximum error (innovation) lag
I- inertia
B - viscosity
K - elasticity
Δ - reflex time delay
ζ - damping parameter
ω - natural frequency
g - gain
c_0 - zeroth-order coefficient of polynomial nonlinearity
c_1 - first-order term coefficient of polynomial nonlinearity
c_2 - second-order term coefficient of polynomial nonlinearity
s - Laplace variable
$U(s)$ - ankle angle
$V(s)$ - ankle velocity
$V(s)e^{-\Delta s}$ - delayed ankle velocity
$X(s)$ - output of half-wave rectifier
$Y_L(s)$ - noise-free linear path output
$Y_{NL}(s)$ - noise-free nonlinear path output
$Y(s)$ - noise-free net torque

$E(s)$ - measurement noise
$\theta_0 - \theta_{11}$ - discrete-time coefficients of ankle dynamics model
τ - discrete-time delay
$\upsilon(n)$ - terms of discrete-time ankle model associated with nonlinear polynomial coefficient c_1
$\chi(n)$ - terms of discrete-time ankle model associated with nonlinear polynomial coefficient c_2
T - sampling rate
$\hat{\varepsilon}$ - prediction errors
$\mathbb{R}$ - set of real numbers
N - data length
$\mathbf{Z}$ - vector of measured outputs
$\hat{\mathbf{Z}}$ - vector of predicted outputs
$\hat{\theta}$ - extended least-squares estimate of the system parameters
p - number of system parameters
Ψ - partitioned regressor matrix
Ψ_{zu} - regressor matrix that is a function of z and u only
$\Psi_{zu\hat{\varepsilon}}$ - regressor matrix that represents all the cross products involving $\hat{\varepsilon}$
$\Psi_{\hat{\varepsilon}}$ - regressor matrix that is a polynomial function of the prediction errors
$\widetilde{\Psi}$ - centered and standardized variate of Ψ
$\widetilde{\mathbf{Z}}$ - centered and standardized variate of $\mathbf{Z}$
Σ_Ψ - diagonal matrix of standard deviations
$\Sigma_{\Psi k}$ - standard deviation of the kth column of Ψ
μ_Ψ - matrix who's kth column has all entries equal to the mean of column k of Ψ
$\widetilde{\theta}$ - centered and standardized system parameters
$\min_{\widetilde{\theta}} \frac{1}{2} \left\| \left(\widetilde{\mathbf{Z}} - \widetilde{\Psi}\widetilde{\theta} \right) \right\|_2^2$ - standardized least-squares objective function
Hz - Hertz
ms - millisecond
dB - decibel

1 Introduction

In a zero or low-g environment it is critical to keep astronauts healthy and functional. To maintain bone and muscle mass, astronauts need to exercise everyday [1]. Under terrestrial 1-g conditions gravity works against muscles and bones, which requires the neuromuscular system to maintain enough muscle and bone mass to support body weight. In an altered gravity-inertial environment the forces of gravity are significantly reduced. As a result, astronauts lose muscle mass and bone density since it is not required to support body weight [2].

Under current technological limitations (e.g. lack of artificial gravity through inertial forces), rigorous physical activity is the only successful way to compensate for the lack of gravity [2]. Nevertheless, even with rigorous exercise, astronauts have typically lost 0.4-1% of their bone density per month in space [3]. Upon return to Earth, with a thorough physical therapy regime astronauts progressively recover muscle tissue and much of the bone mass lost during a semi-long duration (i.e. 4-6 months) in low Earth orbit. However, throughout the mission span it is important that astronauts are strong enough to perform strenuous activities in space, such as spacewalks and emergency procedures during landing. Due to these possible vital mission activities, a regular exercise routine in orbit prepares astronaut for such situations and accelerates a reconditioning period to recover muscle and bone loss.

A parallel pathway model of ankle dynamics has been proposed that measures changes to intrinsic, reflex and muscle properties [4]. These characteristics change, for example, due to a lack of effective exercise. Therefore, astronauts health and exercise effectiveness can be monitored in orbit by evaluating deviations of these parameters from optimal pre-orbit measures. In this work, we propose to use this parallel pathway model as a foundation to assess lower limb health during orbit to maintain an optimal exercise program.

Traditional approaches to nonlinear system identification of human ankle dynamics have relied on quasi-linear methods, e.g. impulse response function (IRF) method [5]. These methods provide convenient, robust means of characterizing the dynamics of nonlinear systems without requiring a priori assumptions regarding the system structure. However, nonparametric techniques may require many parameters to describe even simple systems and can be difficult to relate to the parameters of the underlying physiological system.

The NARMAX (Nonlinear AutoRegressive, Moving Average eXogenous) model structure has been shown to be well suited to modeling the input-output behavior of ankle dynamics [6]. The unknown model parameters can be estimated in discrete-time using the extended least-squares (ELS) algorithm [7–9]. However, discrete-time parameters are a nonlinear combination of the continuous-time physiology, making insight into the underlying mechanisms difficult. The primary objective of biological system identification is to provide parameters that give insight and relevance to the underlying biology, which are represented in continuous-time.

The problem of continuous-time system identification from sampled input-

output data can be divided into two broad approaches: (i) indirect methods, where a discrete-time model is estimated from sampled data; then an equivalent continuous-time model is calculated and (ii) direct methods, where a continuous-time model is obtained directly without going through the intermediate step of first determining a discrete-time model; based on concepts of approximate numerical integration to recreate time-derivatives needed in continuous-time formulations [10].

The central problem for indirect methods is that of estimating stable and robust continuous-time parameters from discrete-time estimates. For both linear and nonlinear systems, the inverse mapping problem often results in large bias or sign flips. To address these issues several indirect techniques have been developed to modify the estimators, keeping the computational complexity low [11–13]. In addition, a method for estimating the natural logarithm of a square matrix to map discrete-time parameters to continuous-time has been proposed [14]. Most other procedures are variants of these two approaches.

Direct methods avoid the inverse mapping problem by identifying a continuous-time model directly from sampled data. This raises several technical issues. Unlike a difference equation model, a differential equation model contains time-derivative terms that may be required but not available from measurements. Many techniques have been devised to deal with the need to reconstruct these time-derivatives (see e.g. [10, 15–17]). However, these approaches are challenged by a lack of robust implementation for numerical computation of derivatives, selection of filter cutoffs to suppress high frequency noise, filter order, etc. These problems are more complex when the system under study is nonlinear, which is inherently the case for biological systems.

We propose to investigate the problem of continuous-time system identification by developing methodology using an indirect approach. One such approach relies on matrix preconditioning techniques, such as standardized least-squares, to improve the spectral properties of the regressor matrix [18, 19]. The transformed matrix will have a smaller spectral condition number, and eigenvalues clustered around one. Often when the clustered spectrum is away from zero it results in rapid and robust convergence, especially when the preconditioned matrix is close to normal. We deem this will provide more robust solutions and greater consistency for nonlinear biological systems by circumventing issues of implementing numerical derivatives and filter selection required by direct techniques, which are more challenging in a nonlinear framework. Here, we focus on one critical issue, namely, that of mapping the underlying continuous-time system to discrete-time for estimation, then inverse mapping back to continuous-time to provide physiological relevance and insight.

The organization of this paper is as follows. The NARMAX model structure is described in section 2. Section 3 discusses a continuous-time representation of a parallel pathway model describing ankle dynamics and its NARMAX representation. In Section 4 we summarize the parameter estimation method implemented in this study, namely, extended least-squares. Section 5 shows how to standardized the identification data to formulate a standardized least-squares approach, which we investigate for its utility as an indirect technique

to estimate continuous-time parameters of ankle dynamics. Section 6 illustrates the results of our proposed approach on a simulated ankle model. Section 7 provides a discussion of our findings and suggestions for future direction. Section 8 summarizes the conclusions of our study.

2 NARMAX Model Description

The input-output relationship of many nonlinear dynamic systems can be written as the nonlinear difference equation

$$\begin{aligned} z(n) &= \mathscr{F}^l[z(n-1),\cdots,z(n-n_z),u(n),\cdots,u(n-n_u), \\ &\quad e(n-1),\cdots,e(n-n_e)]+e(n) \end{aligned} \tag{1}$$

where $\mathscr{F}^l$ is a nonlinear mapping, u is the exogenous input, z is the output, and e is the innovation, or uncontrolled input. This structure describes both the stochastic and deterministic components of a system. This nonlinear mapping may include a variety of nonlinear terms, such as terms raised to an integer power (e.g. $u^2(n-3)$), products of present and past inputs (e.g. $u(n)u(n-1)$), past outputs (e.g. $z(n-1)z(n-2)$), or cross-terms (e.g. $u^2(n-1)z(n-2)$). This system description encompasses most forms of nonlinear difference equations that are linear-in-the-parameters.

3 Parallel Pathway Model of Ankle Dynamics

The Neuromuscular Control Laboratory (Department of Biomedical Engineering, McGill University, Montréal Canada), has developed a parallel pathway model (Fig. 1) to describe ankle dynamics [4]. This model provides a relationship between ankle angle (rad) and net ankle torque (Nm) as the sum of a linear and nonlinear contribution. The upper, linear pathway models intrinsic stiffness as a second-order system with parameters corresponding to inertia (I), viscosity (B) and elasticity (K). The lower, nonlinear pathway models reflex stiffness as a cascade of a derivative, a reflex time delay, a static nonlinearity (i.e. half-wave rectifier) and a low-pass system representing muscle activation. The parameters associated with the low-pass system are damping parameter (ζ), natural frequency (ω) and gain (g). This model is ideally suited to study the effects of exercise (in space) because it quantifies both the intrinsic and reflex stiffness components. The parameters of these two paths will deviate from normal a priori terrestrial measurements due to a suboptimal exercise regime. A method to monitor changes of these parameters could be used to develop individualized exercise protocols for astronauts.

Kukreja *et al.* [6] showed that a second-order static polynomial ($c_0+c_1x(n)+c_2x^2(n)$) provided a good approximation to the half-wave rectifier which gave

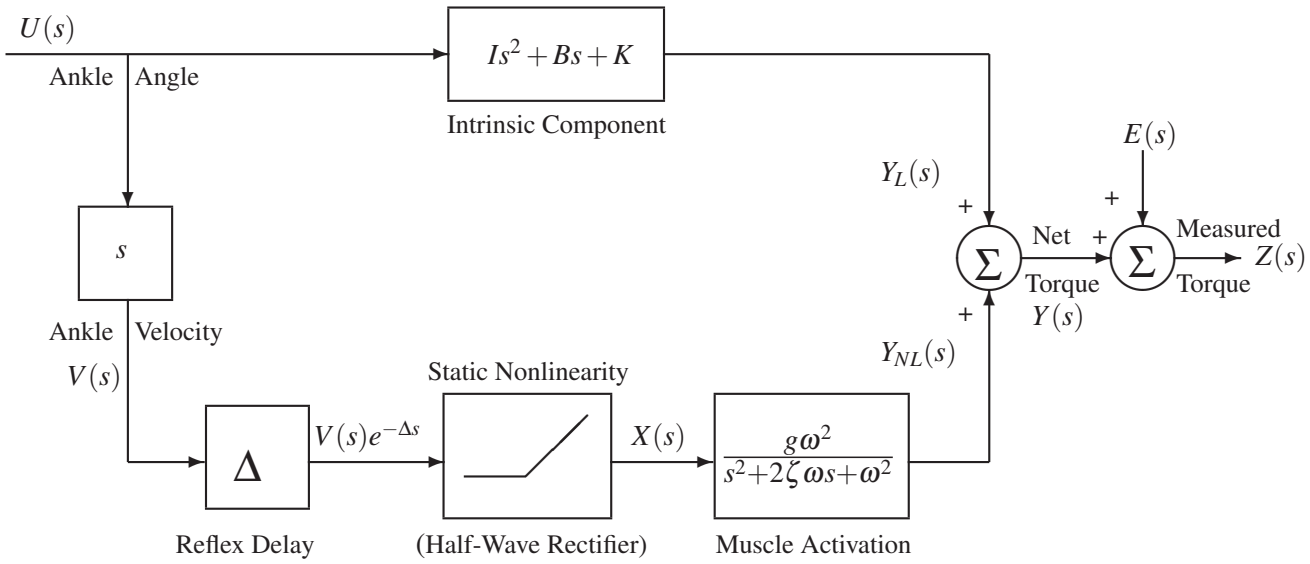


Figure 1. Continuous-time morphological model describing ankle dynamics. Upper (linear) path: intrinsic stiffness. Lower (nonlinear) path: reflex stiffness.

a NARMAX representation for this model of ankle dynamics as

$$\begin{aligned} z(n) &= \theta_0 + \theta_1 z(n-1) + \theta_2 z(n-2) + \theta_3 u(n) + \theta_4 u(n-1) + \theta_5 u(n-2) + \theta_6 u(n-3) \\ &+ \theta_7 u(n-4) + \theta_8 [u(n-\tau) + u(n-\tau-1) - u(n-\tau-2) - u(n-\tau-3)] \\ &+ \theta_9 [u^2(n-\tau) + 3u^2(n-\tau-1) + 3u^2(n-\tau-2) + u^2(n-\tau-3) \\ &- 2u(n-\tau)u(n-\tau-1) - 4u(n-\tau-1)u(n-\tau-2) - 2u(n-\tau-2)u(n-\tau-3)] \\ &+ \theta_{10} e(n-1) + \theta_{11} e(n-2) + e(n) \\ &= \theta_0 + \theta_1 y(n-1) + \theta_2 y(n-2) + \theta_3 u(n) + \theta_4 u(n-1) + \theta_5 u(n-2) + \theta_6 u(n-3) \\ &+ \theta_7 u(n-4) + \theta_8 \upsilon(n) + \theta_9 \chi(n) + \theta_{10} e(n-1) + \theta_{11} e(n-2) + e(n) \end{aligned} \tag{2}$$

where τ is the discrete-time delay,

$$\upsilon(n) = u(n-\tau) + u(n-\tau-1) - u(n-\tau-2) - u(n-\tau-3) \tag{3}$$

and

$$\begin{aligned} \chi(n) &= u^2(n-\tau) + 3u^2(n-\tau-1) + 3u^2(n-\tau-2) + u^2(n-\tau-3) \\ &- 2u(n-\tau)u(n-\tau-1) - 4u(n-\tau-1)u(n-\tau-2) \\ &- 2u(n-\tau-2)u(n-\tau-3). \end{aligned} \tag{4}$$

Table 1 gives the relationships of these discrete-time NARMAX parameters (Eqn. 2) to the underlying continuous-time coefficients (the "extra" coefficient T denotes the sampling rate). An estimate of unknown system parameters can be obtained using standard prediction error identification (PEI) techniques, such as extended least-squares.

NARMAX Coefficient	Relationship to Continuous-time Coefficient
θ_0	$\frac{4c_0 g\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T}$
θ_1	$-\frac{-8+2\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T}$
θ_2	$-\frac{-4\zeta\omega T+4+\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T}$
θ_3	$\frac{I}{T^2}+\frac{B}{T}+K$
θ_4	$(\frac{-2I}{T^2}-\frac{B}{T})-((-\frac{-8+2\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{I}{T^2}+\frac{B}{T}+K))$
θ_5	$(\frac{I}{T^2})-((-\frac{-8+2\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{-2I}{T^2}-\frac{B}{T}))-((-\frac{-4\zeta\omega T+4+\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{I}{T^2}+\frac{B}{T}+K))$
θ_6	$-(-\frac{-8+2\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{I}{T^2})-((-\frac{-4\zeta\omega T+4+\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{-2I}{T^2}-\frac{B}{T}))$
θ_7	$-((-\frac{-4\zeta\omega T+4+\omega^2 T^2}{4+\omega^2 T^2+4\zeta\omega T})(\frac{I}{T^2}))$
θ_8	$\frac{g\omega^2 T^2 c_1}{(4+\omega^2 T^2+4\zeta\omega T)T}$
θ_9	$\frac{g\omega^2 T^2 c_2}{(4+\omega^2 T^2+4\zeta\omega T)T^2}$

Table 1. Theoretical relationship of NARMAX model parameters to continuous-time system coefficients.

4 Parameter Estimation

Many parameter estimation techniques are based on least-squares theory. However, ordinary least-squares algorithms for linear systems cannot be applied here because they assume that the noise terms in the model are independent and the regressor matrix is deterministic [20]. Both of these conditions are violated in Eqn. 2, when one considers that only a noisy measure of the output is available.

Clearly, the regressors in Eqn. 2 contain correlated error terms and noisy data. To obtain unbiased parameters other estimation techniques based on least-squares must be used. One such method that is applicable to NARMAX model estimation is extended least-squares. This technique provides unbiased parameter estimates and is iterative [19, 21, 22].

4.1 Extended Least-Squares

Extended least-squares is one method, appropriate for NARMAX models that easily enables unbiased estimates to be computed. ELS addresses the bias problem by modeling the lagged errors to obtain unbiased parameter estimates. ELS for linear systems has been widely studied and is also referred to as Panuska's method, the extended matrix method, or approximate maximum likelihood [7–9].

In general, since the noise sequence is a realization of a stochastic process, it is not possible to solve for the noise source e, and it will not be equal to the

prediction errors [23]. The prediction errors, $\hat{\varepsilon} \in \mathbb{R}^{N\times 1}$, are defined as

$$\hat{\varepsilon} = \mathbf{Z} - \hat{\mathbf{Z}} \tag{5}$$

where $\mathbf{Z} \in \mathbb{R}^{N\times 1}$ is the measured output and $\hat{\mathbf{Z}} = \Psi\hat{\theta} \in \mathbb{R}^{N\times 1}$ is the predicted output. In ELS, the NARMAX formulation of Eqn. 1 is redefined into a prediction error model by replacing e with $\hat{\varepsilon}$; making it a deterministic least-squares problem.

The ELS formulation is defined as

$$\hat{\theta} = (\Psi^T\Psi)^{-1}\Psi^T\mathbf{Z}; \quad \text{where} \quad \Psi = [\Psi_{zu}\Psi_{zu\hat{\varepsilon}}\Psi_{\hat{\varepsilon}}]. \tag{6}$$

$\hat{\theta} \in \mathbb{R}^{p\times 1}$ is an ELS estimate of the system parameters, $\Psi \in \mathbb{R}^{N\times p}$ is a partitioned regressor matrix where Ψ_{zu} is a function of z and u only, $\Psi_{zu\hat{\varepsilon}}$ represents all the cross products involving $\hat{\varepsilon}$, and $\Psi_{\hat{\varepsilon}}$ is a polynomial function of the prediction errors only [23].

Often, ELS is considered a pseudolinear approach to parameter estimation [7,9,24]. Strictly speaking, the introduction of prediction errors into the model formulation no longer makes the model linear-in-the-parameters because the prediction errors depend on the model output, which is a function of all model parameters. The ELS technique solves a nonlinear optimization problem by ignoring the nonlinear character of the model and employing a least-squares approach. Essentially, ELS uses an approximate gradient of the model output with respect to the model parameters as a regression vector.

5 Standardized Least-Squares

In many nonlinear systems there are large numerical differences of the regressors due to the basis function(s) used to estimate the static map. The large differences lead to the regressor matrix being ill-conditioned and results in unstable matrix inversion and poor parameter estimates [25]. To alleviate ill-conditioning we propose using standardized least-squares (SLS). The SLS technique is summarized as follows.

Given a matrix of independent variables Ψ and of dependent variables $\mathbf{Z}$ compute the mean and standard deviation of each variable and replace Ψ and $\mathbf{Z}$ with the centered and standardized variate as

$$\widetilde{\Psi} = (\Psi - \mu_\Psi)\,\Sigma_\Psi^{-1} \quad \text{and} \quad \widetilde{\mathbf{Z}} = (\mathbf{Z} - \mu_Z)\,\Sigma_Z^{-1} \tag{7}$$

where Σ_Ψ is a diagonal matrix of standard deviations with $\Sigma_{\Psi k}$ denoting the standard deviation of the kth column of Ψ and μ_Ψ is a matrix whose kth column has all entries equal to the mean of column k of Ψ. Substituting Eqn. 7 into 6 yields a SLS formulation. The SLS objective function is compactly expressed as

$$\min_{\widetilde{\theta}} \frac{1}{2}\left\|\left(\widetilde{\mathbf{Z}} - \widetilde{\Psi}\widetilde{\theta}\right)\right\|_2^2. \tag{8}$$

This is an unbiased estimator and converges asymptotically to the true system parameters.

6 Simulations

The efficacy of the SLS ELS technique was assessed to provide stable estimates of continuous-time parameters describing ankle dynamics by simulating the parallel pathway model in continuous-time using Simulink (Fig. 1). The inputs were bandlimited 30 Hz pseudo-random binary sequences (PRBS) which were 0.02 rad (peak-to-peak) and had a 35 ms switching rate. A PRBS input was used to excite the system dynamics since it is standard practice in experimental settings. The continuous-time coefficients used in this study correspond to values found in experiments and are shown in Table 2. One thousand Monte-

CT Coefficient	Value
I	0.015 Nm/rad/s^2
B	0.8 Nm/rad/s
K	150 Nm/rad
ω	40.0
ζ	1.00
g	10.00 Nm/rad/s
Δ	0.045 s
T	0.005 s

Table 2. Continuous-time coefficient values. I: inertia, B: viscosity, K: elasticity, ω: natural frequency, ζ: damping parameter, g: reflex stiffness gain, Δ: reflex delay and T: sampling interval.

Carlo simulations were generated in which each input-output realization was unique, and had a unique Gaussian, white, zero-mean, noise sequence added to the output. Excluding the noise-free (NF) case the signal-to-noise ratio (SNR) of the noise sequence was decreased from 20 to 0 dB in increments of 5 dB. For identification, the data length was $N = 4,000$ points.

6.1 Continuous-Time Parameter Estimation

The system parameters were estimated as outlined in Eqns. 6 – 8. Specifically, for each input-output realization, we analyzed the ankle dynamics model as follows:

1. *Non-Standardized Extended Least-Squares*: Ankle dynamics were analyzed using the ELS estimator.

2. *Standardized Extended Least-Squares*: Ankle dynamics were analyzed using the ELS estimator and standardized data.

Fig. 2 shows a typical input-output realization used for this analysis. Employing our approach, continuous-time parameters were computed from discrete-time NARMAX estimates using the theoretical relationships given in Table 3.

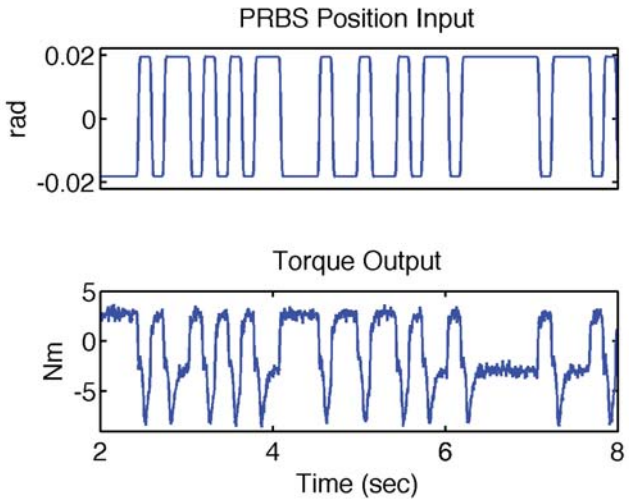


Figure 2. Typical PRBS ankle position input and torque output with 20 dB SNR used for Monte-Carlo analysis.

CT Coefficient	DT Relationship
$I =$	$\theta_7 \times \frac{T^2}{-\theta_2}$
$B =$	$\frac{\theta_6+\theta_1\frac{I}{T^2}}{-\theta_2} = -(\frac{2I}{T^2}+\frac{B}{T})+\frac{2I}{T^2} = -\frac{B}{T} \times -T$
$K =$	$\theta_3 - (\frac{I}{T^2}+\frac{B}{T})$
$\omega =$	$\frac{-4+4\theta_2+4\theta_1}{-1+\theta_2-\theta_1} = \sqrt{(\omega^2T^2) \times \frac{1}{T^2}}$
$\zeta =$	$\frac{-2-2\theta_2}{-1+\theta_2-\theta_1} = \frac{\zeta\omega T}{\omega T}$
$g =$	$\theta_8 \times (4+\omega^2T^2+4\zeta\omega T) \times T = \frac{gc_1\omega^2T^2}{c_1\omega^2T^2}$

Table 3. Discrete to continuous-time relationships for parameters I, B, K, ω, ζ and g of the parallel pathway ankle model.

The results of this study are presented in Fig. 3. The panels show plots of standard deviation about the mean for estimated continuous-time parameters of the linear and nonlinear paths (I, B, K, g, ω and ζ) using the non-standardized and standardized ELS identification techniques. Column one shows the linear path parameters. The continuous-time estimates using non-standardized ELS and relationships given in Table 3 provided a mean value that was close to the true parameter and had monotonically increasing variance from 20-10 dB SNR. However, with decreasing SNR (5-0 dB) the parameter mean was biased away from the true with non-monotonically increasing variance that included estimates with incorrect sign. Using standardized data with the ELS algorithm, mean and variance estimates for the linear path parameters improved for all SNR levels but experienced a similar non-monotonically increasing variance for 5-0 dB SNR and encompassed values with opposite sign. Column two represents the nonlinear path estimates. Applying non-standardized and standardized ELS to the nonlinear path illustrates that both approaches gave similar results. For the damping (ζ) and gain (g) parameters standardized data provided estimate that were worse than non-standardized data. The natural frequency (ω) estimates were moderately improved using standardized data. Results show that, for the linear path, the parameters were close to the true mean but variance estimate was non-monotonic and included estimates with incorrect sign for low SNR levels. However, for the nonlinear path, continuous-time parameters computed using both approaches were significantly biased from the true mean for all SNR levels but did not include parameter estimates with incorrect sign.

7 Discussion

In this section, we summarize the findings of this study and discuss possible future approaches. The discussion highlights major features of our approach and offers possible explanations for the unexpected errors. The suggestion for future direction provided two broad alternatives to our solution, namely, an alternative identification strategy and discretization approach.

7.1 Continuous-Time Parameter Estimation

Estimation of continuous-time parameters of ankle dynamics demonstrates that, with noise-free input and output the both the ELS and SLS ELS approaches provided estimates that agree with the true known values. However, with output additive noise the linear path parameter estimates had non-monotonically increasing variance and encompassed values with opposite sign (see column 1, Fig. 3). For the nonlinear path, the parameters were biased and the standard deviation did not encompass the true value (see column 2, Fig. 3). Although the nonlinear path parameters were biased, they did not experience sign flips as the linear path parameters did.

The reason for the parameter bias may be due to the opposing nature of

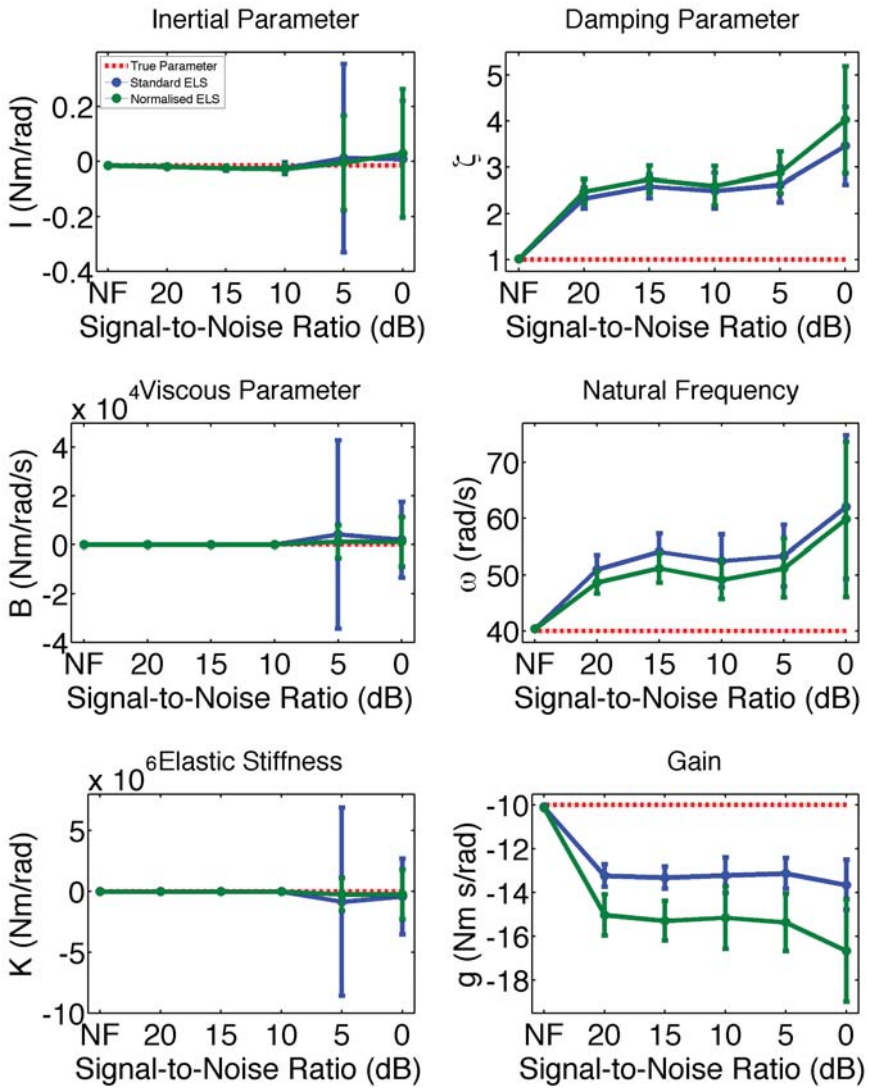


Figure 3. Non-standardized and standardized ELS approaches to continuous-time parameter estimation of ankle dynamics. Column 1: Linear path parameters, I, B, K. Column 2: Nonlinear path parameters, ζ, ω, g. Ordinate: STD about mean. Abscissa: Output SNR = NF, 20, 15, 10, 5, 0 dB, where NF denotes noise-free case. (Note that the abscissa is shown in decreasing SNR which corresponds to increasing noise amplitude.)

the two paths. The linear path is described by a finite impulse response (FIR) system, which has theoretical poles close to the Nyquist frequency; requiring a high frequency signal to excite the dynamics sufficiently. The nonlinear path is described by a combination of a nonlinear static function and linear dynamics. The nonlinear function is described as a half-wave rectifier. This is considered a hard nonlinearity because it generates multiple harmonics and its output cannot be written analytically. To limit the effect of higher order harmonics and avoid internal aliasing at the nonlinearity output the input signal was bandlimited to be of low frequency. The opposing requirements of the two paths are central to this estimation problem and difficult to satisfy simultaneously. It is not possible to make more definitive comments about this approach without further detailed investigation and analysis. Below we outline a few alternative approaches to solve this identification problem.

7.2 Closed-Loop Simulation

A possible solution to continuous-time parameter estimation for ankle dynamics is to identify the model in closed-loop, removing the effects of derivatives (FIR realization of the linear path). In feedback, intrinsic stiffness is represented as a compliance model (pure infinite impulse response (IIR) realization). Although it may be possible to pose the identification problem as a feedback model, the true system is represented from position input to torque output. The recorded output torque has observation noise associated with it. When identifying the model in feedback the noisy output is considered the input. Noise on the input violates assumptions and conditions for least-squares to yield unbiased parameter estimates. Implementing a feedback identification approach may require advanced methods such as total-least squares [26, 27].

7.3 Discrete to Continuous-Time Parameter Mapping

Another possibility for the source of error for continuous-time parameters computed using our NARMAX approach may be related to the nonlinear relationships between discrete-time NARMAX parameters and continuous-time parameters of the physical system (see Table 3). A small deviation from the true parameter value in discrete-time, due to noise or numerical error, may appear as a significant error in the estimated continuous-time coefficients. As a result it may be advantageous to study these model parameters only in discrete-time as a combination of physiological effects.

7.4 Continuous to Discrete-Time Transformations

To derive a NARMAX model formulation the bilinear transform and Newton's backwards formula were used to convert the continuous-time linear dynamics to discrete-time. These transforms were implemented since both require only a simple substitution to convert a continuous-time system to discrete-time. Two other techniques that give better approximation for linear time invariant

(LTI) systems are linear extrapolation and linear interpolation methods [28,29]. However, these techniques are seldom used due to added complexity for little gain.

Linear extrapolation gives an improper transfer function (i.e. more zeros than poles) and linear interpolation gives a transfer function that is strictly proper (i.e. equal zeros and poles) [28, 29]. These methods produce a discrete-time transfer function that gives a better output response than the bilinear transform or Newton's backwards formula. However, the pitfall of these methods is that it is difficult to derive the coefficients of the discrete-time linear system. This is the main reason that almost all engineering text books and literature only discussed the bilinear transform and Newton's backwards formula.

Using linear interpolation or linear extrapolation to derive a NARMAX representation of ankle dynamics it may be possible to derive a better approximation to the derivative than the one used i.e. Newton's backwards formula. This may give a better approximation to the derivative and provide simulation data that matches the continuous-time process better. However, one drawback is that it may give a discrete-time approximation to the derivative that is higher than first order thereby increasing the complexity of the NARMAX representation [28, 29]. Another limitation is that the continuous to discrete mapping of the continuous-time parameters will be more complicated since it involves exponential functions. This may result in more sensitivity for continuous-time parameter estimation since small deviations from the true value in discrete-time will result in large errors in continuous-time.

8 Conclusions

Clearly, much work remains to be done to resolve the possible source(s) of error for continuous-time parameter estimation for the model of ankle dynamics. Our results show that the proposed standardized extended least-squares method provided some improvement for estimating the continuous-time parameters of the linear path. However, the parameter variance was too high because it included parameters with incorrect sign. For the nonlinear path, this technique did not offer any improvement over the non-standardized extended least-squares approach. A possible explanation for these results is due to opposing constrains needed to properly excite the dynamics of both paths simultaneously. The linear path requires a high frequency signal while the nonlinear path needs low frequency. It is unclear how to satisfy both concurrently. It is not possible to make more definitive remarks about the root of this error and remains an open research question. We have offered several suggestions for alternative ways forward to estimate the continuous-time parameters of the parallel pathway model of ankle dynamics. Of these, we deem that identifying this model in closed loop and implementing more advanced estimation techniques such as total least-squares is likely the best way forward.

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14. ABSTRACT

Recently, a parallel pathway model to describe ankle dynamics was proposed. This model provides a relationship between ankle angle and net ankle torque as the sum of a linear and nonlinear contribution. A technique to identify parameters of this model in discrete-time has been developed. However, these parameters are a nonlinear combination of the continuous-time physiology, making insight into the underlying physiology impossible. The stable and accurate estimation of continuous-time parameters is critical for accurate disease modeling, clinical diagnosis, robotic control strategies, development of optimal exercise protocols for longterm space exploration, sports medicine, etc.
This paper explores the development of a system identification technique to estimate the continuous-time parameters of ankle dynamics. The effectiveness of this approach is assessed via simulation of a continuous-time model of ankle dynamics with typical parameters found in clinical studies. The results show that although this technique improves estimates, it does not provide robust estimates of continuous-time parameters of ankle dynamics. Due to this we conclude that alternative modeling strategies and more advanced estimation techniques be considered for future work.

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