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## Abstract

A well-clear volume is a key component of NASA's Separation Assurance concept for the integration of UAS in the NAS. This paper proposes a mathematical definition of the well-clear volume that uses, in addition to distance thresholds, a time threshold based on time to entry point (TEP). The mathematical model that results from this definition is more conservative than other candidate definitions of the well-clear volume that are based on range over closure rate and time to closest point of approach.

# 1 Introduction

The *Traffic Alerting and Collision Avoidance System* (TCAS) is a family of airborne devices that are designed to reduce the risk of mid-air collisions between aircraft with operating transponders [4]. TCAS II, the current generation of TCAS devices, provides resolution advisories (RAs) that direct pilots to maintain or increase vertical separation when aircraft distance and time parameters cross designed system thresholds. TCAS II RAs are based on a mathematically-derived time *tau* denoted  $\tau$ . The time  $\tau$  is an approximation to the time of closest point of approach (TCPA, denoted  $t_{\text{cpa}}$ ) between two aircraft, and an important element of the resolution advisory logic in TCAS II is to check whether  $\tau$  falls below a certain threshold value. A formal definition of a well-clear volume based on the alerting logic of TCAS was presented in [2].

The present paper further refines the definition of the well-clear volume, which is a key component of NASA's Separation Assurance concept for the UAS in the NAS project [1]. The refinement presented in this paper uses distance and time thresholds as in [2]. However, it addresses some drawbacks of using  $\tau$  or the related quantity *modified*  $\tau$  ( $\tau_{\text{mod}}$ ), which appeared in later versions of TCAS. For instance, it is well known that for certain scenarios,  $\tau$  diverges as the aircraft converge upon the closest point of approach [2]. Thus, in general  $\tau$  approximates TCPA only for sufficiently large values.

Instead of  $\tau$ ,  $\tau_{\text{mod}}$ , or TCPA, the definition of the well-clear volume presented in this paper relies on the time to violation of the horizontal threshold. This time is called *time to entry point* (TEP, denoted  $t_{\text{ep}}$ ). Figure 1 shows a comparison of  $\tau$ , modified  $\tau$ , TCPA, and TEP for a  $90^\circ$  encounter, where the aircraft separation at the closest point of approach is zero nautical miles. In this particular encounter scenario,  $\tau$  is perfectly coincident with  $t_{\text{cpa}}$ . It can also be noted that  $t_{\text{ep}} \leq \tau_{\text{mod}} \leq t_{\text{cpa}}$ . Indeed, this inequality holds for every scenario. Hence, a well-clear concept based on TEP is more conservative than a similar concept based on  $\tau$ ,  $\tau_{\text{mod}}$ , or  $t_{\text{cpa}}$ .

# 2 Preliminaries

Throughout this paper, letters in **bold-face** denote two-dimensional (2-D) vectors. Vector operations such as addition, subtraction, scalar multiplication, dot product, i.e.,  $\mathbf{s} \cdot \mathbf{v} \equiv s_x v_x + s_y v_y$ , and the norm of a vector, i.e.,  $\|\mathbf{s}\| \equiv \sqrt{\mathbf{s} \cdot \mathbf{s}}$ , are defined in a 2-D Euclidean geometry. By convention, vectors  $\mathbf{s}$  and  $\mathbf{v}$  denote relative positions and velocities, respectively. Furthermore, the function **root**, defined by Formula (1), computes the roots of the quadratic equation  $ax^2 + bx + c = 0$ . For completeness, it is defined such that it returns the value 0 when the roots are undefined. In this paper, the values returned by **root** are only used in a context where  $a \neq 0$  and  $b^2 - 4ac \geq 0$ .

$$\mathbf{root}(a, b, c, \epsilon) \equiv \begin{cases} \frac{-b + \epsilon \sqrt{b^2 - 4ac}}{2a} & \text{if } a \neq 0 \text{ and } b^2 - 4ac \geq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Assuming constant velocities, the horizontal positions of the ownship and in-

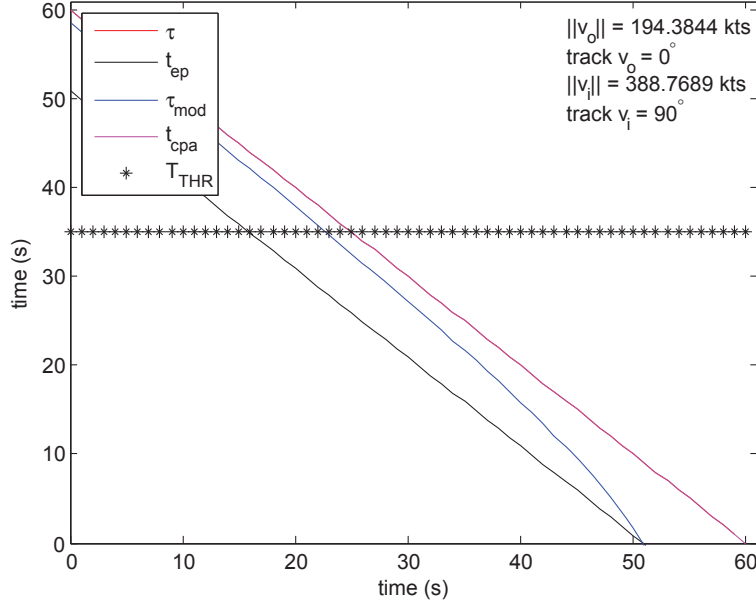


Figure 1. Time comparison for 90° encounter

truder aircraft at a time  $t \geq 0$ , are given by

$$\begin{aligned} \mathbf{s}_o(t) &\equiv \mathbf{s}_o + t\mathbf{v}_o, \\ \mathbf{s}_i(t) &\equiv \mathbf{s}_i + t\mathbf{v}_i, \end{aligned} \quad (2)$$

respectively. As it simplifies the mathematical development, some definitions in this paper use a relative coordinate system where the intruder is static at the center of the system. In this relative system, the ownship is located at  $\mathbf{s} = \mathbf{s}_o - \mathbf{s}_i$  and moves at relative velocity  $\mathbf{v} = \mathbf{v}_o - \mathbf{v}_i$ . Therefore, the relative horizontal position of the ownship with respect to the traffic aircraft at any time  $t$  can be defined as follows.

$$\mathbf{s}(t) \equiv \mathbf{s} + t\mathbf{v}. \quad (3)$$

Given the relative position  $\mathbf{s}$  and velocity  $\mathbf{v}$ , the time of horizontal closest point of approach, denoted  $t_{\text{cpa}}$ , is the time  $t$  that satisfies  $\|\mathbf{s}(t)\| = 0$ . By convention,  $t_{\text{cpa}}$  is defined as 0 when the velocities of the ownship and intruder aircraft are identical, i.e., when  $\|\mathbf{v}\|^2 = 0$ . Hence,

$$t_{\text{cpa}}(\mathbf{s}, \mathbf{v}) \equiv \begin{cases} -\frac{\mathbf{s} \cdot \mathbf{v}}{\|\mathbf{v}\|^2} & \text{if } \|\mathbf{v}\|^2 \neq 0, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

The dot product  $\mathbf{s} \cdot \mathbf{v}$  characterizes whether the aircraft are horizontally diverging, i.e.,  $\mathbf{s} \cdot \mathbf{v} > 0$ , or horizontally converging, i.e.,  $\mathbf{s} \cdot \mathbf{v} < 0$ . It can be seen from Formula (4) that when the aircraft are converging,  $t_{\text{cpa}}$  is always positive.

The time to co-altitude  $t_{\text{coa}}$  is the time when the relative altitude and vertical speed of the aircraft satisfies  $s_z + t_{\text{coa}}v_z = 0$ , where  $s_z \equiv s_{oz} - s_{iz}$  and  $v_z \equiv v_{oz} - v_{iz}$ .

Similar to the horizontal case, the product  $s_z v_z$  characterizes whether the aircraft are vertically diverging, i.e.,  $s_z v_z > 0$ , or vertically converging, i.e.,  $s_z v_z < 0$ . In this paper, time to co-altitude is only relevant when the aircraft are vertically converging. By convention, when the aircraft are not vertically converging, time to co-altitude is defined as -1. Formally, time to co-altitude is defined as follows.

$$t_{\text{coa}}(s_z, v_z) \equiv \begin{cases} -\frac{s_z}{v_z} & \text{if } s_z v_z < 0, \\ -1 & \text{otherwise.} \end{cases} \quad (5)$$

The following functions are used to compute the times when the linear relative trajectory of the ownship with respect to the intruder will intersect a circular area of radius  $D$  around the intruder.

$$\Delta(\mathbf{s}, \mathbf{v}, D) \equiv D^2 \|\mathbf{v}\|^2 - (\mathbf{s} \cdot \mathbf{v}^\perp)^2, \quad (6)$$

$$\Theta(\mathbf{s}, \mathbf{v}, D, \epsilon) \equiv \text{root}(\|\mathbf{v}\|^2, 2(\mathbf{s} \cdot \mathbf{v}), \|\mathbf{s}\|^2 - D^2, \epsilon). \quad (7)$$

When  $\Delta(\mathbf{s}, \mathbf{v}, D) \geq 0$ , the function  $\Theta$  computes the times when the aircraft will lose separation, if  $\epsilon = -1$ , or regain separation, if  $\epsilon = 1$ , with respect to  $D$ . In this paper, time to entry point is only relevant when the aircraft are horizontally converging, i.e., when  $\mathbf{s} \cdot \mathbf{v} < 0$ . By convention, when the aircraft are not horizontally converging, time to entry point is defined as -1. Therefore, given a distance  $D$ , TEP is formally defined as follows.

$$t_{\text{ep}}(\mathbf{s}, \mathbf{v}, D) \equiv \begin{cases} \Theta(\mathbf{s}, \mathbf{v}, D, -1) & \text{if } \mathbf{s} \cdot \mathbf{v} < 0, \\ -1 & \text{otherwise.} \end{cases} \quad (8)$$

### 3 Well-Clear Volume Based on Time to Entry Point

The definition of the well-clear volume presented in this paper uses a horizontal distance threshold (D\_THR), a vertical distance threshold (Z\_THR), and a time threshold (T\_THR). Formula (11) provides the formal definition of the well-clear volume, given absolute state information, in vector form, for the ownship and intruder aircraft. The volume consists of horizontal and vertical components, whose definitions are provided by Formula (9) and Formula (10), respectively.

$$\begin{aligned} \text{horizontal\_WCV\_tep}(\mathbf{s}, \mathbf{v}) &\equiv \|\mathbf{s}\| \leq \text{D\_THR} \text{ or} \\ &(\|\mathbf{s} + t_{\text{cpa}}(\mathbf{s}, \mathbf{v})\mathbf{v}\| \leq \text{D\_THR} \text{ and} \\ &0 \leq t_{\text{ep}}(\mathbf{s}, \mathbf{v}, \text{D\_THR}) \leq \text{T\_THR}). \end{aligned} \quad (9)$$

$$\text{vertical\_WCV}(s_z, v_z) \equiv |s_z| \leq \text{Z\_THR} \text{ or } (0 \leq t_{\text{coa}}(s_z, v_z) \leq \text{T\_THR}), \quad (10)$$

$$\begin{aligned} \text{WCV\_tep}(\mathbf{s}_o, s_{oz}, \mathbf{v}_o, v_{oz}, \mathbf{s}_i, s_{iz}, \mathbf{v}_i, v_{iz}) &\equiv \\ \text{let } \mathbf{s} = \mathbf{s}_o - \mathbf{s}_i, \mathbf{v} = \mathbf{v}_o - \mathbf{v}_i, s_z = s_{oz} - s_{iz}, v_z = v_{oz} - v_{iz} \text{ in} & \\ \text{horizontal\_WCV\_tep}(\mathbf{s}, \mathbf{v}) \text{ and vertical\_WCV}(s_z, v_z). & \end{aligned} \quad (11)$$



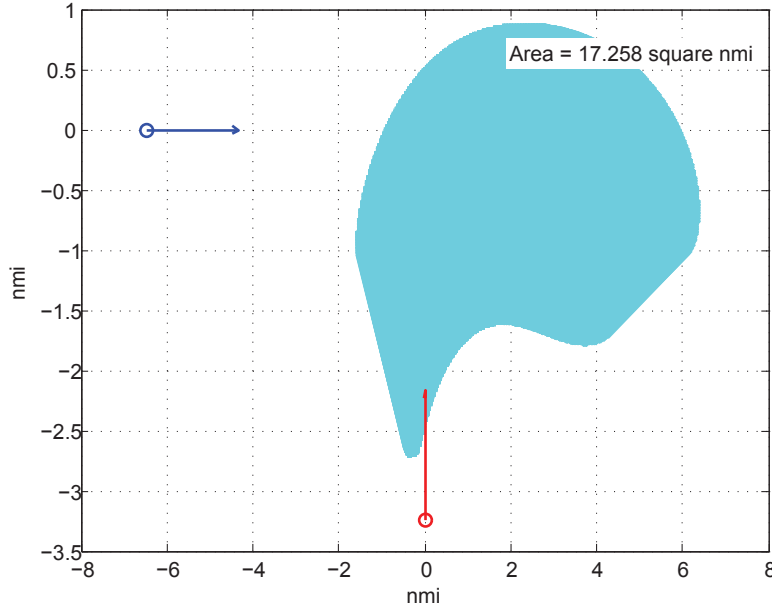


Figure 2. Two-dimensional projection of conflict volume for 90° encounter

It should be noted that the conflict volume defined by Formula (11) is symmetric from the point of view of the ownship and traffic aircraft. In fact, the following proposition can be formally proven.

**Proposition 1.** *For all two-dimensional vectors  $\mathbf{s}_o$ ,  $\mathbf{v}_o$ ,  $\mathbf{s}_i$ ,  $\mathbf{v}_i$  and real numbers  $s_{oz}$ ,  $v_{oz}$ ,  $s_{iz}$ ,  $v_{iz}$ ,*

$$WCV\_tep(\mathbf{s}_o, s_{oz}, \mathbf{v}_o, v_{oz}, \mathbf{s}_i, s_{iz}, \mathbf{v}_i, v_{iz}) = WCV\_tep(\mathbf{s}_i, s_{iz}, \mathbf{v}_i, v_{iz}, \mathbf{s}_o, s_{oz}, \mathbf{v}_o, v_{oz}).$$

Figure 2 shows the two-dimensional projection of the conflict volume for the encounter geometry associated with Figure 1. The conflict volume was determined with  $D\_THR = 2037.2$  m,  $Z\_THR = 213.36$  m, and  $Z\_THR = 35$  s.

## 4 Conclusion

This paper presents a well-clear definition based on time to entry point (TEP) which exploits the RA logic in TCAS II. Relevant intermediate definitions and concepts are developed, and a mathematical definition of this well-clear concept is provided. The TEP approach has been shown to always provide earlier alerting times than similar approaches based on  $\tau$ ,  $\tau_{mod}$ , or  $t_{cpa}$ . Thus, TEP is a more conservative approach in terms of alerting time and providing a larger well-clear boundary.

The mathematical development presented in this paper has been mechanically verified in the Prototype Verification System (PVS) [3]. This level of rigor is justified by the safety-critical nature of the well-clear volume in the concept of integration of Unmanned Aerial Vehicles in the National Aerospace System.

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