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The major accomplishments during this period are summarized in the enclosed abstract which was submitted for the Aerospace Sciences meeting. Other approaches to turbulence modeling are being pursued.

A One Equation Turbulence Model
for Transonic Airfoil Flows

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Abstract

A one equation turbulence model based on the turbulent kinetic energy equation is presented. The model is motivated by the success of the Johnson-King model and incorporates a number of features uncovered by Simpson's experiments on separated flows. Based on the results obtained , the model duplicates the sucesses of algebraic models in attached flow regions and out preforms the two equation models in detached flow regions.

Introduction

This work is the first phase of an on going research effort designed to develop turbulence models for separated turbulent flows over transonic airfoils. It

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is generally accepted that algebraic turbulence models are not suited to describe non-equilibrium turbulent flows. This was clearly brought out by the extensive work of Simpson and his associates (see reference 1 for a summary). The next logical step is to use a higher order closure model. Unfortunately, the work of Patel, Rodi and Scheurer [2] showed that most of the $k - \epsilon$ models and related two equation models are unable to predict some simple flows. Moreover, the work of Coakley [3] showed that predictions of two equation models are not much better than algebraic models in the presence of separation.

The recent model of Johnson and King [4], which is based on a simplified version of the tubulent kinetic energy equation, preformed well in regions where non-equilibrium turbulent effects are important. However, it did not always preform well in regions where equilibrium turbulent flow exists. This negative result does not diminish the significance of their contribution. Rather, it suggests that more physics exists in the turbulent kinetic energy equation that is yet to be explored and exploited.

Because of the above, an effort was undertaken to develop a one equation turbulence model based on the turbulent kinetic energy equation for both attached and detached flows. Simpson made two relevant conclusions. First, scaling within and outside separated flow regions is not the same. Second, the eddy viscosity assumption is incorrect in the backflow region of a separating boundary layer. Thus, the model developed retains the accepted form of the turbulent kinetic energy equation but adjusts the tubulent shear stress and the scaling depending on the local flow conditions.

Most of the previous work using one equation models employed the same length scale for turbulent stresses and turbulent dissipation. This led to numerical difficulties in the near wall region with the result that most researchers lost interest in such a model. The difficulties can be avoided by using two different length scales in the near wall region, one for the turbulent stresses and one for turbulent dissipation. Specification of the length scales requires close attention to scaling near the wall.

The Navier-Stokes code employed is based on that used by Swanson and Turkel [5]. The turbulent kinetic energy equation is coupled to the conservation equations and a four-stage Runge-Kutta time stepping scheme is employed. The Reynolds averaged Navier-Stokes equations that use the Johnson-King model or higher order closure models cannot be started from

a uniform flow condition. The usual procedure is to use an initial solution obtained from a flow solver that incorporates an algebraic turbulence model. This innefficient starting proceedure is not necessary in the present work.

Approach

The equations governing turbulent flow past airfoils are the Reynolds averaged conservation equations of mass, momentum and energy and the turbulent kinetic energy equation. When Favre's mass averaging and indicial notation are used the conservation equations are ;

$$\bar{\rho}_{,t} + (\bar{\rho}\tilde{u}_j)_{,j} = 0. \tag{1}$$

$$(\bar{\rho}\tilde{u}_i)_{,t} + \left[\bar{\rho}\tilde{u}_j\tilde{u}_i + \delta_{ij}\bar{P} - \left(\bar{\tau}_{ij} - \overline{\rho u_i'' u_j''}\right)\right]_{,j} = 0. \tag{2}$$

$$\left(\bar{\rho}\tilde{E}\right)_{,t} + \left[\tilde{u}_j\left(\bar{\rho}\tilde{E} + \bar{P}\right) + \bar{q}_j + \overline{\rho u_j'' h''} - \tilde{u}_i\left(\bar{\tau}_{ij} - \overline{\rho u_i'' u_j''}\right)\right]_{,j} = 0 \tag{3}$$

where

$$\bar{\tau}_{ij} = \mu\left[\tilde{u}_{i,j} + \tilde{u}_{j,i} - \frac{2}{3}\delta_{ij}\tilde{u}_{m,m}\right] \tag{4}$$

$$\bar{q}_j = -\lambda\bar{T}_{,j} \tag{5}$$

$$\bar{P} = \bar{\rho}(\gamma - 1)\left[\tilde{E} - \frac{1}{2}\tilde{u}_i\tilde{u}_i\right] \tag{6}$$

where $\bar{\rho}$ is the density, $\tilde{u}_i$ is the mean velocity in the direction of x_i, $\tilde{E}$ and h are the total energy and enthalpy per unit mass, $\bar{P}$ is the mean pressure. μ , λ and $\bar{T}$ are respectively, the molecular viscosity and conductivity and temperature. In attached flow regions, the turbulent stress and heat flux are determined from the eddy viscosity (Bousinessq) approximation.

$$\overline{\rho u_i'' u_j''} = -\mu_t\left[\left(\frac{\partial\tilde{u}_j}{\partial x_i} + \frac{\partial\tilde{u}_i}{\partial x_j}\right) - \frac{2}{3}\delta_{ij}\frac{\partial\tilde{u}_m}{\partial x_m}\right] + \frac{2}{3}\delta_{ij}\bar{\rho}\tilde{k} \tag{7}$$

$$\overline{\rho u_i'' h''} = \frac{\mu_t}{Pr_t}\left(\frac{\partial\tilde{h}}{\partial x_i}\right) \tag{8}$$

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Pr_t is the turbulent Prandtl number. μ_t is the eddy viscosity and $\tilde{k}$ is the mean turbulent kinetic energy per unit mass.

$$\bar{\rho}\tilde{k} = \frac{1}{2}\overline{\rho u_i'' u_i''} \tag{9}$$

For detached flow the above relations are replaced by correlations obtained from Simpson's experimental data.

The modeled turbulent kinetic energy equation is;

$$\left(\bar{\rho}\tilde{k}\right)_{,t} + \left[\bar{\rho}\tilde{u}_j\tilde{k} - \nu\left(\bar{\rho}\tilde{k}\right)_{,j} - c_k\frac{\tilde{k}}{\bar{\rho}\epsilon}\overline{\rho u_i'' u_j''}\left(\bar{\rho}\tilde{k}\right)_{,i}\right]_{,j} = -\overline{\rho u_i'' u_j''}\tilde{u}_{i,j} - \bar{\rho}\epsilon \tag{10}$$

where ϵ is the turbulent kinetic energy dissipation rate.

$$\epsilon = \frac{\tilde{k}^{\frac{3}{2}}}{\ell_\epsilon} \tag{11}$$

c_k is a constant (0.1). ℓ_ϵ is the dissipation length scale and ν is the molecular kinematic viscosity. In a one equation turbulence model the eddy kinematic viscosity is given by

$$\nu_t = C_\mu\sqrt{\tilde{k}}\ell_\mu \tag{12}$$

where C_μ is a constant and ℓ_μ is the turbulent length scale.

Simpson emphasized the importance of normal turbulent stresses in separating flows regions. Because of this, the governing equations retain all normal stresses.

The Length Scales

To complete the formulation of the model, the length scales must be defined. The expressions are summarized here and a complete derivation will be given in the paper. All variables are non-dimensionalized with the chord being the characteristic length scale and $(P_\infty/\rho_\infty)^{\frac{1}{2}}$ the characteristic velocity. Letting

$$L_\epsilon = C_l y\left[1 - exp\left(\frac{-R_\epsilon\sqrt{k}y}{A_\epsilon}\right)\right]$$

$$L_\mu = C_l y\left[1 - exp\left(\frac{-R_\epsilon\sqrt{k}y}{A_\mu}\right)\right]$$

$$F(y) = y\mid\omega\mid\left[1 - exp\left(\frac{-R_\epsilon\sqrt{k}y}{A_P}\right)\right]$$

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then

$$\ell_\epsilon \;=\; min\left(L_\epsilon, y_{max}\right)$$
$$\ell_\mu \;=\; min\left(L_\mu, y_{max}\right)$$

where y_{max} is the value of y at which $F(y)$ is maximum. ω is the vorticity and R_ϵ is the Reynolds number. The constants in the definition are

$$C_l \;=\; \kappa C_\mu^{-\frac{3}{4}}$$
$$C_\mu \;=\; 0.09$$
$$\kappa \;=\; 0.41$$
$$A_\epsilon \;=\; 2C_l$$
$$A_\mu \;=\; 76.$$
$$A_F \;=\; 47.47$$

Results and Discussion

The results presented here are for a flate plate at zero angle of attack and for an NACA 0012 airfoil at a variety of Mach and Reynolds numbers and angles of attack. Fig. 1 shows flat plate results for a free stream Mach number, M_∞, of 0.5, $R_e = 10^6$ and an angle of attack, α, of zero. As is seen in the figure, good agreement is indicated for the skin friction, Fig. 1a, velocity distribution in the near wall region, Fig. 1b, and the one seventh power law, Fig. 1c.

The next set of calculations are for an NACA 0012 airfoil. All calculations employed a 160 X 79 C-grid. The normal spacing of the first point off the wall was $2X10^{-5}$. The first case considered is that for $M_\infty = 0.5$, $R_e = 2.89X10^6$ and $\alpha = 0..$ Fig. 2(a) compares the pressure distribution with the measurements of Ref. 6. Fig. 2(b) compares the skin friction coefficient with a boundary layer calculation given in Ref. 7. As is seen in the figure, good agreement is indicated. The difference in the skin friction coefficients at the leading and trailing edges are a result of much finer resolution in the boundary layer calculation. Another comparison with the

experiment of Ref. 6 is given for $M_\infty = 0.756$, $\alpha = 0.$ and $R_e = 4.01X10^6$. Again excellent agreement with experiment is obtained.

All of the above cases are for $\alpha = 0.$. Fig. 4 compares predictions of this theory with the experiment of Ref. 8 for $\alpha = 1.49^\circ$ and $R_e = 9.X10^6$. As is seen in the figure, both upper and lower surface presure distributions are well predicted by the present one equation model.

Simpson noted that in the backflow region of a separating boundary layer, the eddy viscosity assumption is incorrect. Because of this, Simpson's measurements are used to develop correlations for the turbulent stresses in the backflow region. Unfortunately, there is scatter in the data . Bacause of this, the results presented in Fig. 5 ($M_\infty = 0.799$, $R_e = 9.X10^6$, $\alpha = 2.26^\circ$) should be considered preliminary.

Details of the separated models together with comparisons for the RAE 2822 and the Cast 10 airfoils will be presented in the paper. Based on the results obtained to date, it is concluded that one equation models, properly formulated, are capable of out preforming two equation models at reduced computational cost.

References

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Computer Program Assessment, May 1979.

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[8] Harris, C. D., "Two-Dimensional Aerodynamic Characteristics of the NACA 0012 Airfoil in the Langley 8-Foot Transonic Pressure Tunnel," NASA TM 81927, 1981.

Acknowledgments
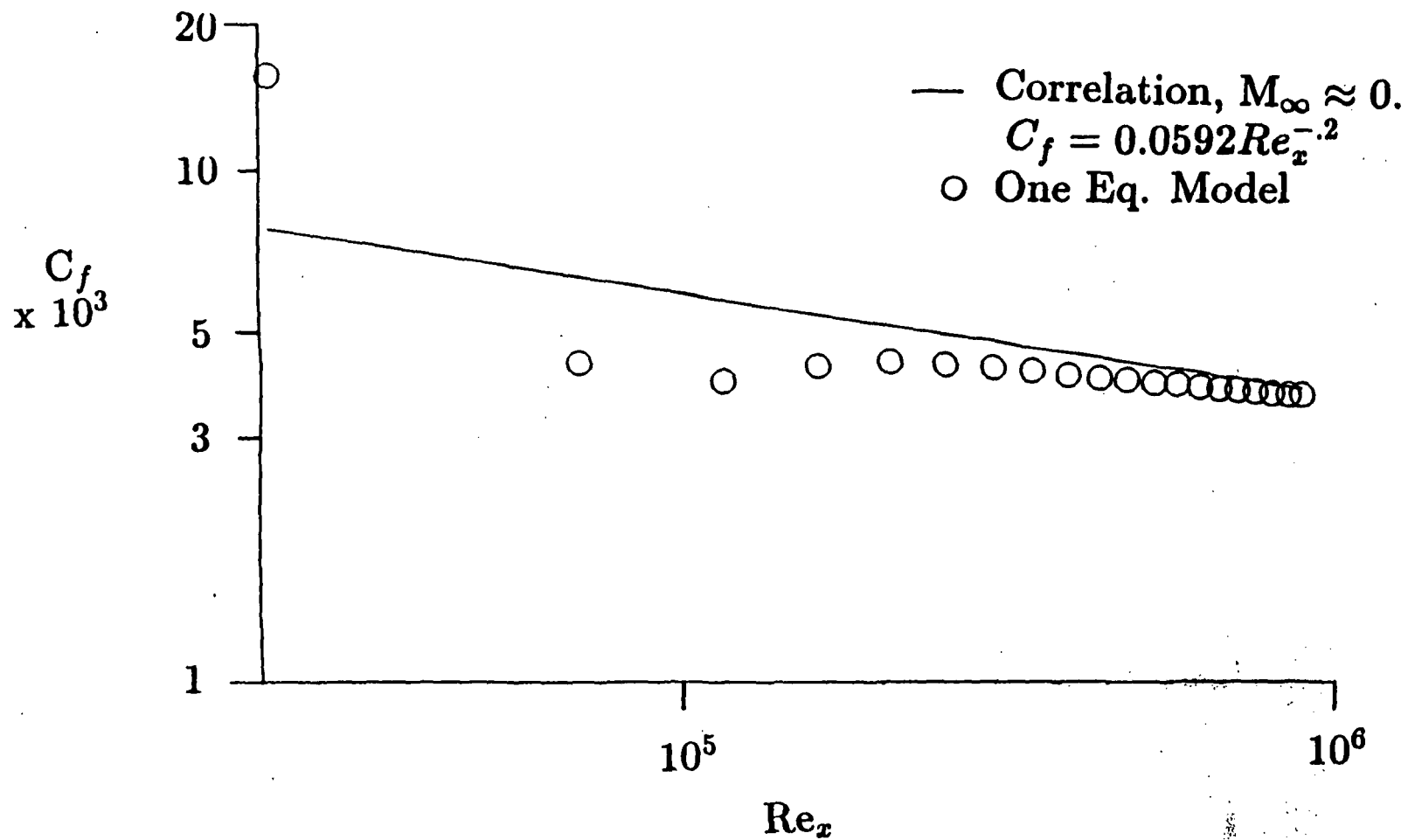
Figure 1a. Turbulent flat plate flow (M = 0.5, Re = 10^6):
comparison of skin friction distributions.
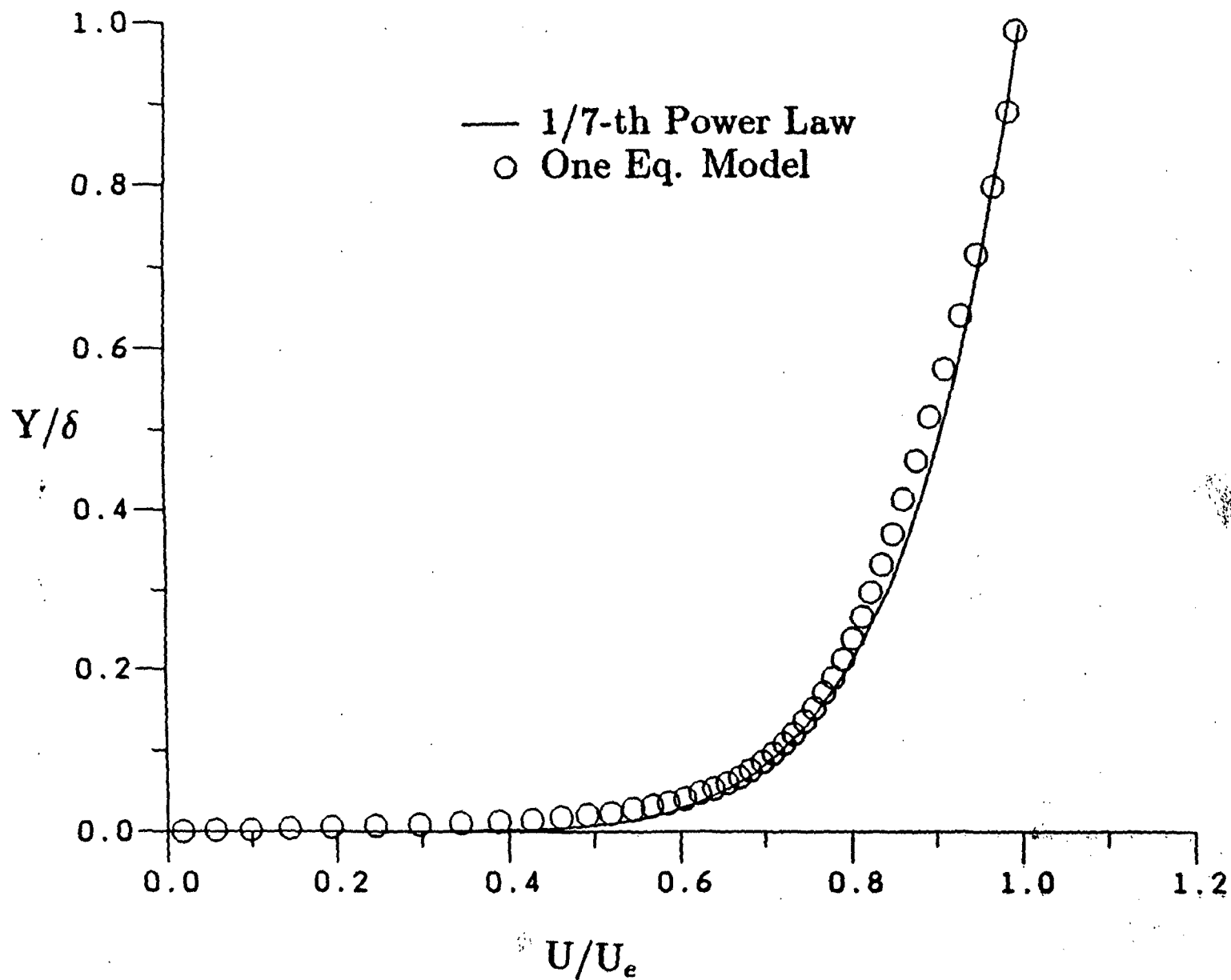
Figure 1b. Turbulent flat plate flow (M = 0.5, Re = 10^6): comparison of velocity profiles.
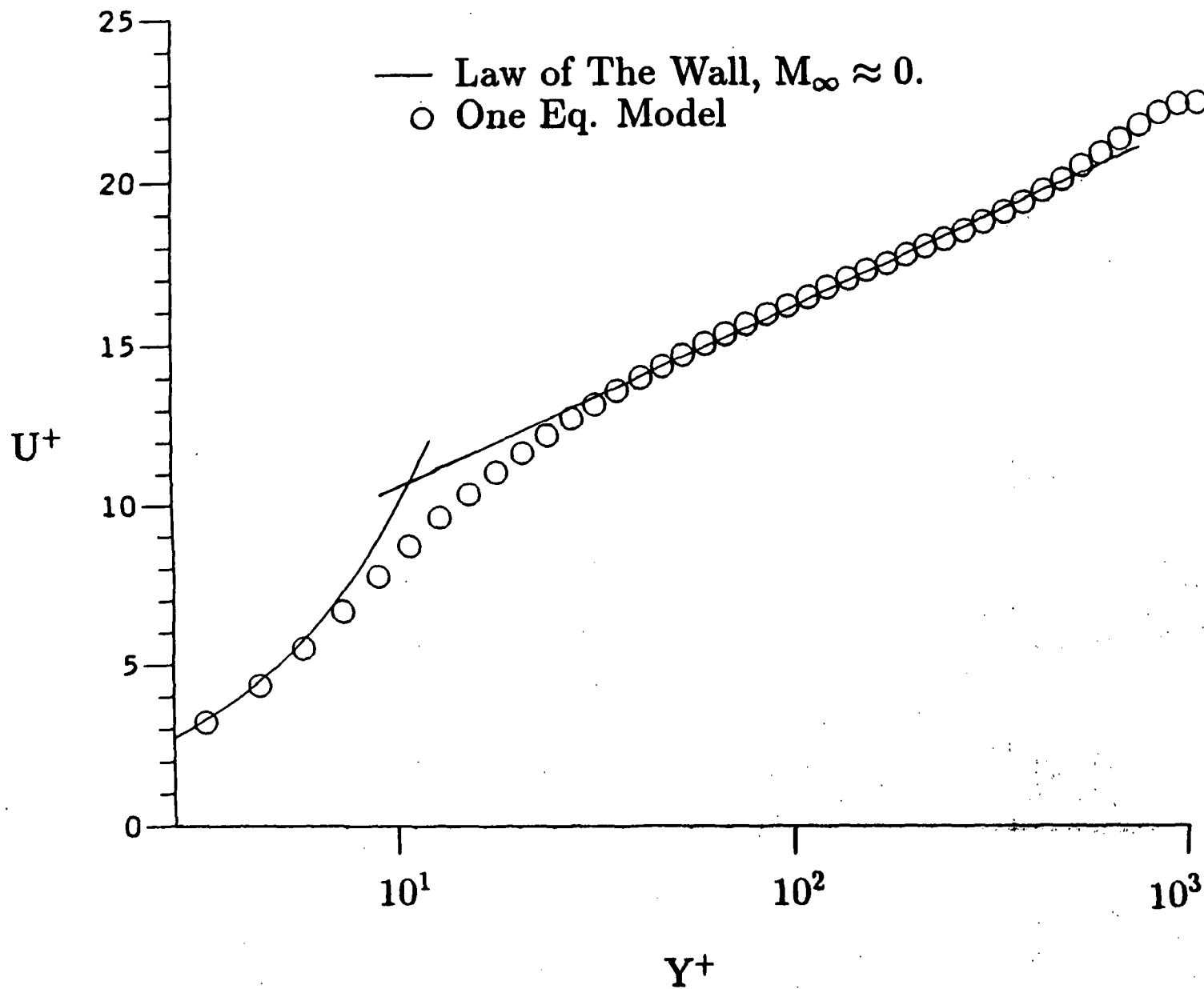
Figure 1c. Turbulent flat plate flow (M = 0.5, Re = 10^6):
comparison of near wall velocity profiles.
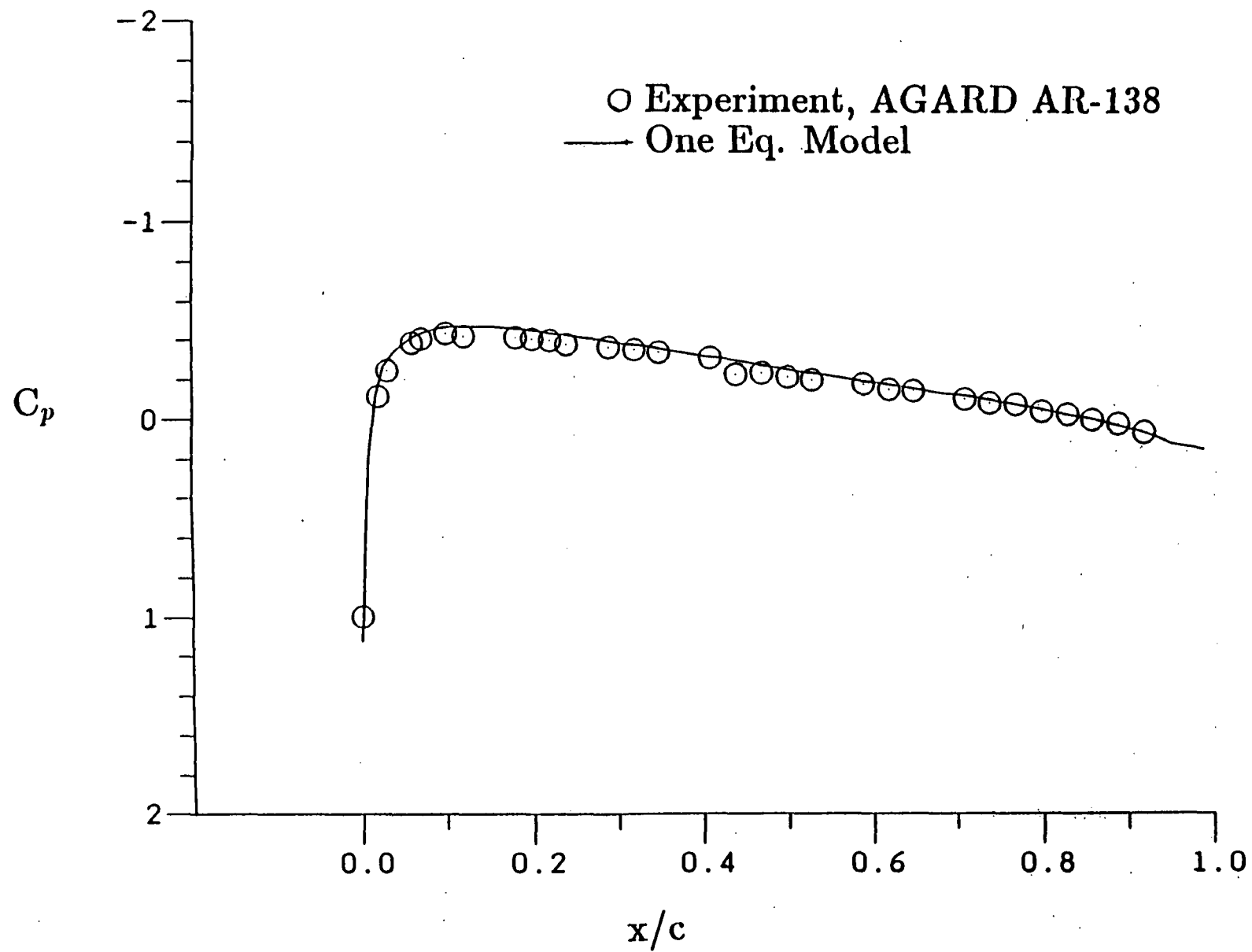
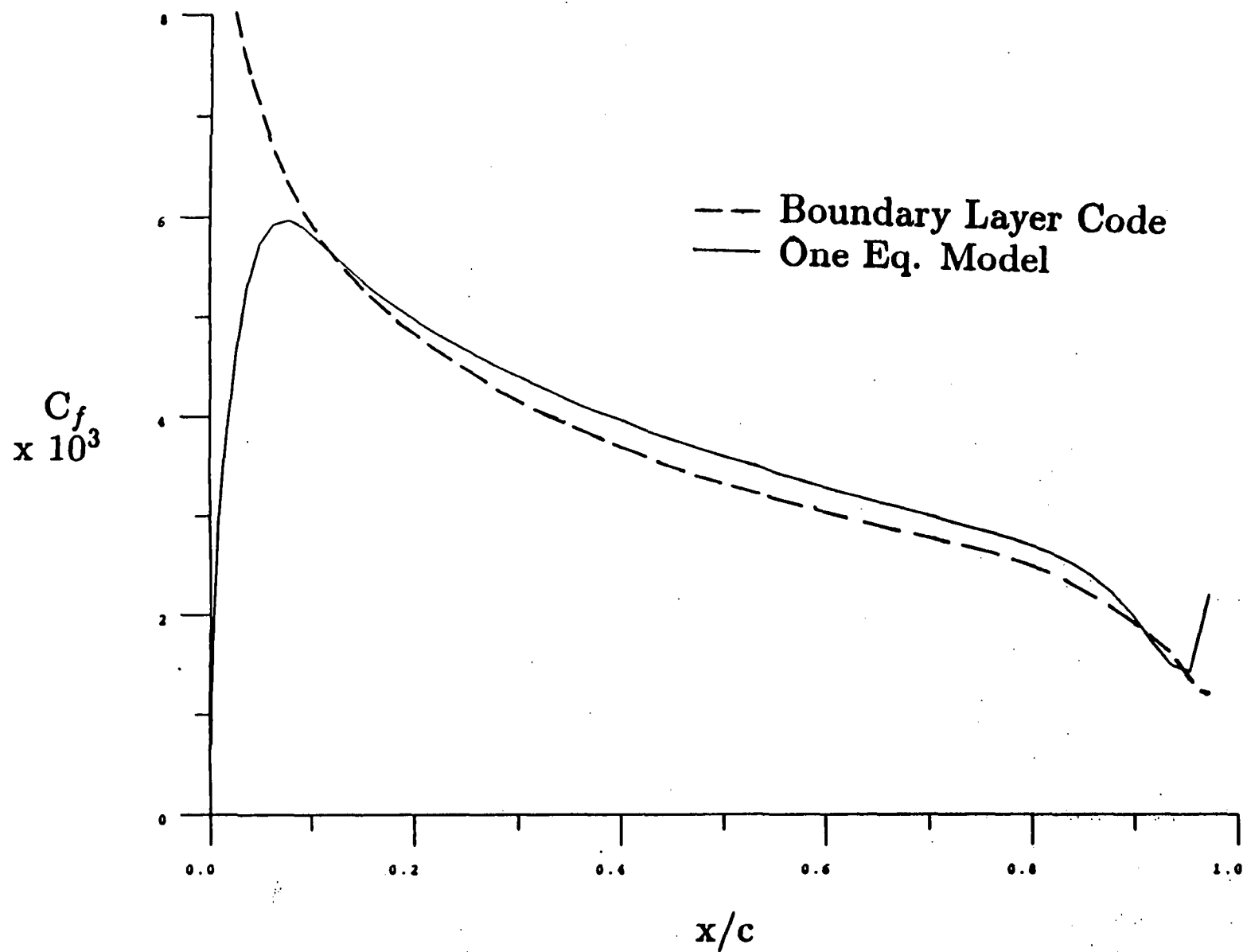Figure 2a. Turbulent flow over a NACA 0012 airfoil (M = 0.5, Re = 2.89x10^6): pressure distributions.

Figure 2b. Turbulent flow over an NACA 0012 airfoil (M = 0.5, Re = 2.89x10^6): skin friction distributions.
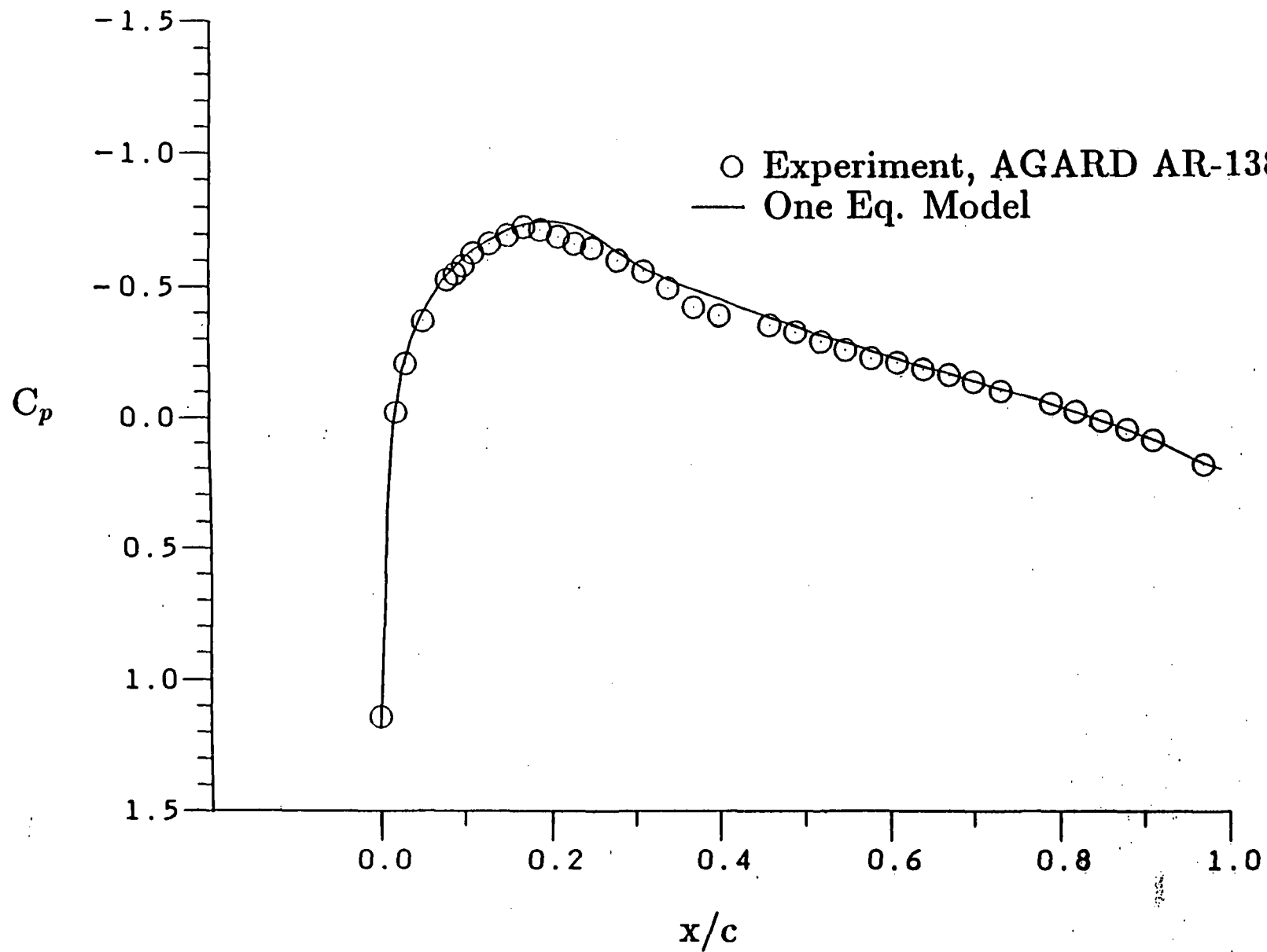
Figure 3.   Turbulent flow over an NACA 0012 airfoil (M = 0.756,
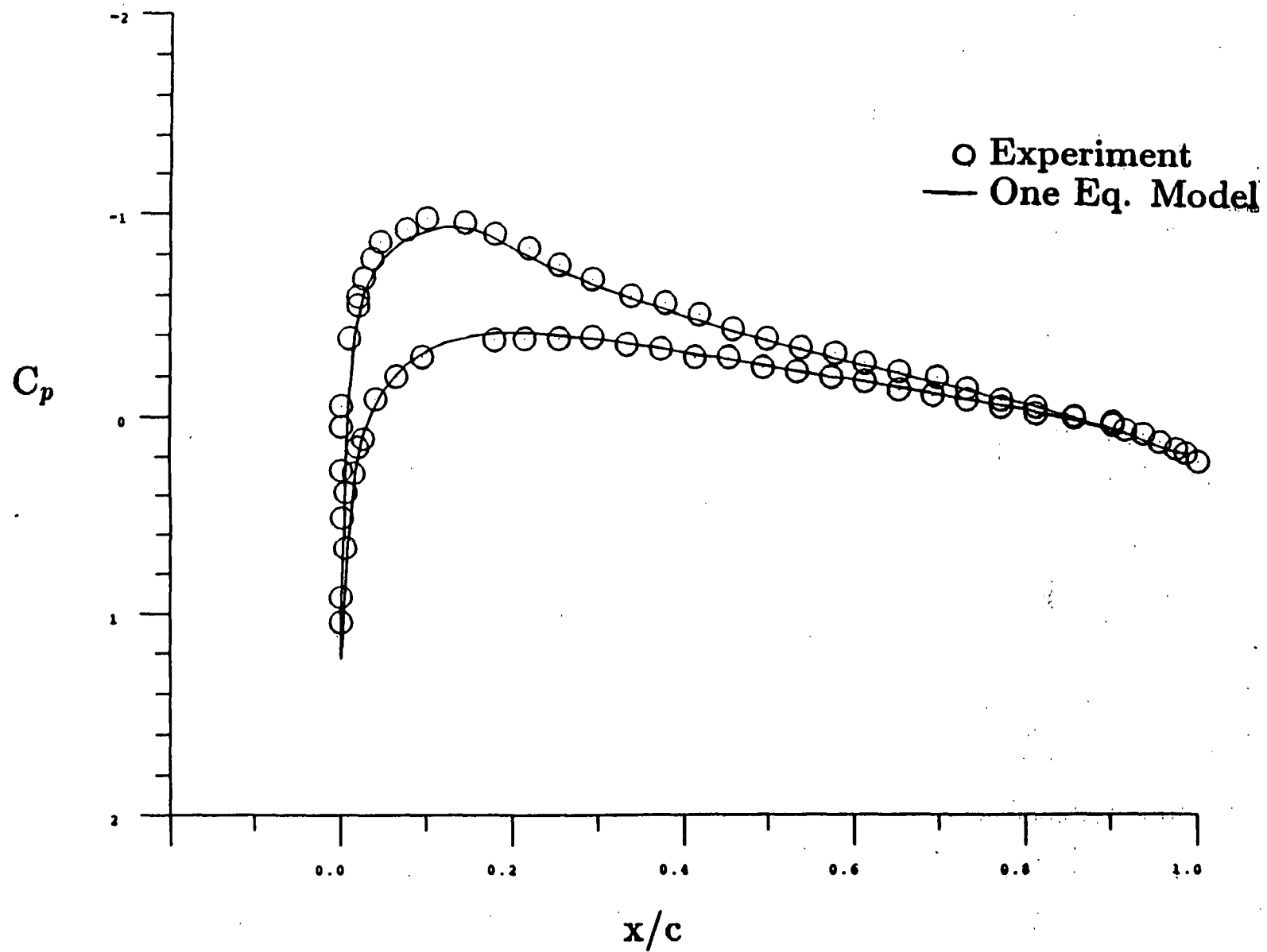α =   0 $^\bullet$, Re = 4.01x10^6): pressure distributions.

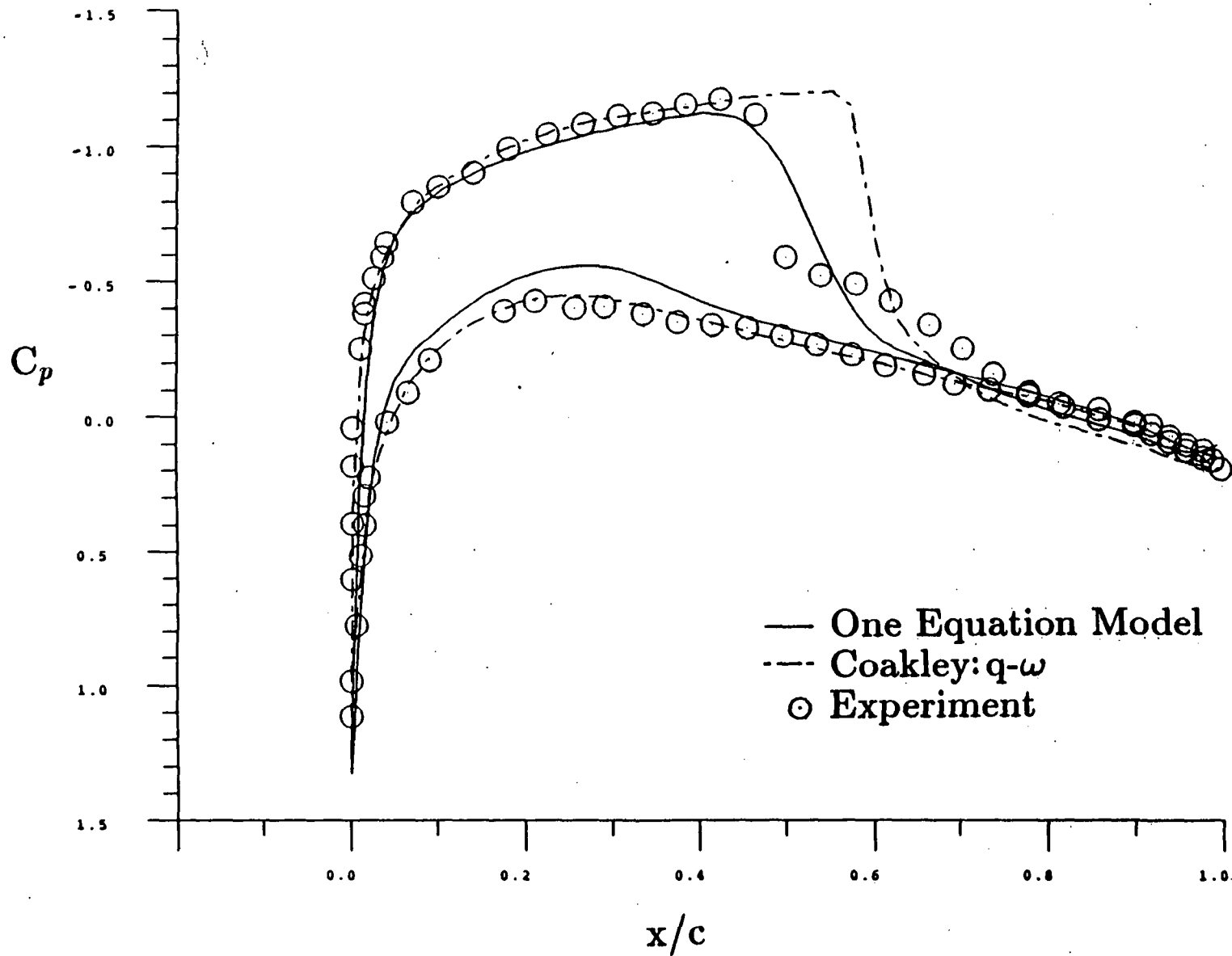Figure 4.   Turbulent flow over an NACA 0012 airfoil (M = 0.7, $\alpha = 1.49°$, Re = 9.x10^6): pressure distributions.

Figure 5a. Turbulent flow over an NACA 0012 airfoil (M = 0.799,
α = 2.26° , Re = 9.x10^6): pressure distributions

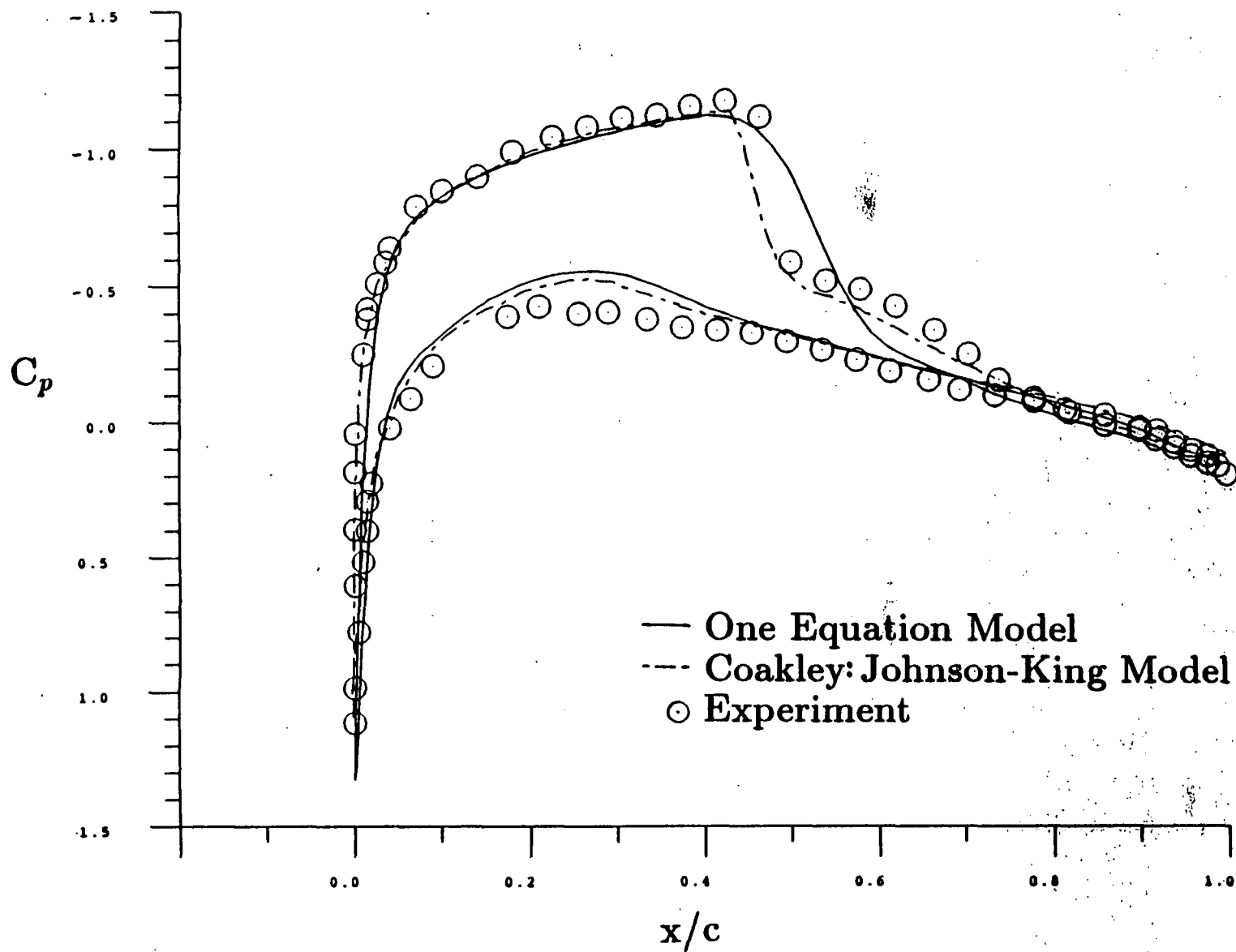Figure 5b.   Turbulent flow over an NACA 0012 airfoil (M = 0.799,

α = 2.26° , Re = 9.x10[6]): pressure distributions