# Computer Programs for Common Map Projections 

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# Computer Programs for Common Map Projections 

By G. D. Newton

A contribution of the regional aquifer systems analysis program

| Program Number: | None assigned |
| :--- | ---: |
| Equipment: | PRIME 750 |
| Operating System: | PRIMOS Rev 19.0 |
| Language: | FORTRAN 77 |

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# Computer Programs for Common Map Projections 

By G. D. Newton


#### Abstract

FORTRAN computer programs were originated to enable automated coordinate transformations between geodetic and rectangular coordinates within the American Polyconic, Lambert Conformal Conic, and Universal Transverse Mercator map projections. The programs facilitate processing large quantities of point data for ground-water modeling and were developed for use in the Snake River Plain Regional Aquifer Systems Analysis study.


## INTRODUCTION

This report is one in a series resulting from the U.S. Geological Survey's Snake River Plain RASA (Regional Aquifer Systems Analysis) study that began in 1979. The Snake River Plain study is one of a series of RASA studies made to evaluate the Nation's major aquifer systems. Each RASA study includes the development and use of digital computer ground-water flow models to aid in analysis.

Point data for the Snake River Plain RasA study are located by latitude and longitude coordinates and by the U.S. Bureau of Land Management's system of public lands subdivision. Data input to the computer models is based on a rectangular grid system. Nodal point values of parameter values are determined for each model grid cell from field data or as output from a computer model.

The U.S. Geological Survey, in cooperation with Idaho Department of Water Resources, determined irrigated areas on the Snake River Plain from Landsat imagery. The Landsat data consist of six scenes, each including 7.5 million pixels, or data points. A pixel represents about $4,452 \mathrm{~m}^{2}$. Each pixel is located by latitude and longitude and is assigned a numerical value that indicates whether the land is irrigated or nonirrigated. Total irrigated area for each model grid cell thus can be determined. Because of the large number of data points, manual methods of determining irrigated acreage for each model grid cell from the pixel data were impractical for the RASA study. Therefore, an automated method was developed using the programs documented in this report. Other large data sets, such as ground-water levels, specific capacities, irrigation diversions, and ground-water pumping, also can be processed more easily by automated methods.

The purpose of this study was to develop a set of computer programs to convert geodetic coordinates
from rectangular map coordinates for commonly used maps. FORTRAN programs were developed for common map projections: American Polyconic, Lambert Conformal Conic, and Universal Transverse Mercator. Both forward and inverse computations were included.

## generating an ellipse of a spheroid

A map projection is a planar representation of the curved surface of the Earth. Imposed upon the Earth's surface is a geodetic coordinate system of latitude, $\phi$, and longitude, $\lambda$.

A map projection is defined as a systematic method of drawing lines on a planar surface. Exact representation of the surface is impossible, but errors can be minimized, depending on the intended use of the map. Scale and shape are the fundamental considerations.

The surface of the Earth is approximated by a spheroid formed by rotating an ellipse about its minor axis. Dimensions for the generating ellipse of a spheroid are shown in figure 1 , where a is length of the major axis $O A$, and $b$ is length of the minor axis OB. Eccentricity, e, is a measure of the flattening of the ellipse and is defined by:

$$
\begin{equation*}
\mathrm{e}^{2}=1-\left(\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right) \tag{1}
\end{equation*}
$$

Estimates for the dimensions of the ellipse differ. Values used in this report are for the Clarke spheroid of 1866 (Birdseye, 1929):

$$
\begin{aligned}
& a=6,378,206.4 \mathrm{~m} \\
& b_{2}=6,356,583.8 \mathrm{~m}, \text { and } \\
& e^{2}=0.006768658
\end{aligned}
$$

The distance of a parallel from the equator measured along a meridian can be determined by considering the infinitely small arc, $\mathrm{PP}^{\prime}$, of the ellipse as an are of a circle with radius $R$. The length, ds, of the are is:

$$
\begin{equation*}
\mathrm{ds}=\mathrm{R}(\mathrm{~d} \phi) . \tag{2}
\end{equation*}
$$

In figure 1, P and $\mathrm{P}^{1}$ are two points on a meridian at the ends of an infinitely small are ds. The normal (PK) to the meridian is the radius of curvature ( R ) of the are ds; $\mathrm{P}^{\prime} \mathrm{K}^{\prime}$ is the radius of curvature ( N ) normal to the meridian.

The lengths $R$ and $N$ can be computed for any latitude by:
$R=a\left(1-e^{2}\right) /\left[1-e^{2} \sin ^{2}(\phi)\right]^{3 / 2}$,
$N=a /\left[1-e^{2} \sin ^{2}(\phi)\right]^{1 / 2}$
The distance from the equator $S$ is the integral of ds from zero to the latitude, $\phi$. The integral is approximated by the following series:

$$
\begin{align*}
S=A_{1} & +\frac{A_{2}}{2} \sin (2 \phi)+\frac{A_{3}}{2} \sin (4 \phi)  \tag{5}\\
& +\frac{A_{4}}{2} \sin (6 \phi)+\frac{A_{5}}{2} \sin (8 \phi)+\ldots
\end{align*}
$$

The distance between any two parallels, $\phi_{1}$ and
the meridian is: $\phi_{2}$ on the meridian is:

$$
\begin{align*}
& S=A_{1} \phi-A_{2} \cos (2 \phi) \sin (\Delta \phi)  \tag{6}\\
& +A_{3} \cos (4 \phi) \sin (2 \Delta \phi)-\ldots
\end{align*}
$$

where

$$
\begin{aligned}
\phi & =\left(\phi_{1}+\phi_{2}\right) / 2 \text { (radians) }, \\
\Delta \phi & =\left(\phi_{2}-\phi_{1}\right), \\
A_{1} & =6,367,399.6891 \mathrm{~m}, \\
\mathrm{~A}_{2} & =32,433,8882 \mathrm{~m}, \\
\mathrm{~A}_{3} & =34.4187 \mathrm{~m}, \text { and } \\
\mathrm{A}_{4} & =0.0454 .
\end{aligned}
$$

Development of these series was described by Birdseye (1929).


Figure 1. Elements of the generating ellipse of a spheroid.

## THE AMERICAN POLYCONIC PROJECTION

The American Polyconic projection is based on the development of a large number of cones tangent to the spheroid at parallels of latitude to be represented on the map. The projection was devised by Ferdinand Hassler (Thomas, 1952).

In this projection, a central meridian (AK) (fig. 2A) is drawn as a straight line, and the intersections of the parallels are spaced true to scale along the central meridian. Each parallel ( $\mathrm{K}_{1}, \mathrm{~K}_{2}, \ldots$ ) is developed separately on a cone whose base is tangent to the Earth's surface at the parallel, with the vertex of the developed cone on the extension of the central meridian (fig. 2B). Each parallel is represented by the arc of a circle ( PN ) with radius $\mathrm{KK}^{\prime}$ divided true to scale.

A.-- General development of Polyconic projection.


Figure 2. Elements of the American Polyconic projection.

The central meridian is a straight line and all other meridians are curves. The intersections of meridians and parallels are not at right angles except at the central meridian.

Errors in the meridian distances, areas, shapes, and angles of graticule (intersection of longitude and latitude) increase with the longitudinal limits of the polyconic projection and restrict its usage to largescale maps. However, the polyconic projection is simple to construct with little distortion of areas, shapes, distances, and azimuths for small areas at large scales. Mathematical development of the projection is described by Birdseye (1929).

## THE LAMBERT CONFORMAL CONIC PROJECTION

A conformal projection is one in which all angles are preserved and the scale factor is constant in any direction.

The Lambert Conformal Conic projection was devised by Johann Heinrich Lambert in about 1772. At large and medium scales, the Lambert projection provides a minimum of angular and scalar distortion. It is a conical type in which all meridians are straight lines that meet in a common point, $K$, beyond the limits of the map; and the parallels are concentric circles whose common center, $K$, is at the point of intersection of the meridians (fig. 3). The projection employs a cone intersecting the Earth at two parallels, $C D$ and $E F$, known as the standard parallels, which are chosen to minimize errors over the area of interest. On the standard parallels, ares of latitude are represented in their true lengths, or to exact scale. Between the standard parallels, the scale is too small; beyond them, the scale is too large.

At medium scale, the Lambert projection is best suited for maps with a dominating east-west dimension. The U.S. Geological Survey's 1:500,000and $1: 1,000,000$-scale map series are examples of the Lambert projection. Development of the equations was described by Adams (1918).


Figure 3. Secant cone for the Lambert Conformal Conic projection.

## THE UNIVERSAL TRANSVERSE MERCATOR PROJECTION

The UTM (Universal Transverse Mercator) projection is described as the projection of a spheroid upon a cylinder (fig. 4). The cylinder is tangent to the spheroid at the zero meridian. The distances along the tangent meridian are true distances; all other distances are distorted.

To reduce distortion, the UTM projection is developed by moving the cylinder into a secant position. The cylinder intersects the spheroid along lines $A B$ and EF. Lines AB and EF are common to both the cylinder and the sphere and are the same length. Between these lines, the distance between two points is shorter in the projection than on the spheroid. Outside these lines, the distances are longer in the projection than on the spheroid; when measuring distances on the map, a scale factor must be applied to obtain true distance.

The central meridian in the UTM map projection is a great circle along which there is no scale distortion; that is, the scale factor equals 1.0000 . A scale distortion or grid scale constant is applied along the central meridian of each zone to improve scaleretention characteristics of the projection. A scale factor of 0.9996 for the central meridian limits the scale error to $1 / 2,500$ within the zone. The effect is that scale distortions are spread more favorably.

The UTM projection of the Earth is divided into 60 zones, each extending $3^{0}$ east and $3^{\circ}$ west of a central meridian. The zones are numbered consecutively from 1 to 60 , beginning with the zone between longitudes $180^{\circ}$ and $174^{\circ}$ west (table 1). The central meridian is a great circle and is the only line of latitude or longitude represented by a straight line in the map projection.

A plane rectangular metric grid is superimposed on each zone assigning a $500,000-\mathrm{m}$ false easting ( X coordinate value) to the central meridian, a zero northing ( $Y$ coordinate value) to the equator for the northern hemisphere, and a false northing of $10,000,000 \mathrm{~m}$ to the equator for the southern hemisphere. This coordinate system eliminates negative coordinate values. A grid overlap between zones is customarily about $\mathbf{4 0 , 0 0 0} \mathrm{m}$ on either side of the zone boundary.


Figure 4. The Universal Transverse Mercator projection.

Table 1.--Universal Transverse Mercator zone numbers with central and bounding meridians

\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline Zone No. \& Central meridian \& Bounding meridians \& Zone No. \& Central meridian \& Bounding meridians \& Zone No. \& Central meridian \& Bounding meridians <br>
\hline 1 \& $177^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 180^{\circ} \\
& 174^{\circ} \mathrm{W}
\end{aligned}
$$ \& 21 \& $57^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 60^{\circ} \mathrm{W} \\
& 54^{\circ} \mathrm{W}
\end{aligned}
$$ \& 41 \& $63^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 60^{\circ} \mathrm{E} \\
& 66^{\circ} \mathrm{E}
\end{aligned}
$$ <br>
\hline 2 \& $171{ }^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 174^{\circ} \mathrm{W} \\
& 168^{\circ} \mathrm{W}
\end{aligned}
$$ \& 22 \& $51^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 54^{\circ} \mathrm{W} \\
& 48^{\circ} \mathrm{W}
\end{aligned}
$$ \& 42 \& $69^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 66^{\circ} \mathrm{E} \\
& 72^{\circ} \mathrm{E}
\end{aligned}
$$ <br>
\hline 3 \& $165^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 168^{\circ} \mathrm{W} \\
& 162^{\circ} \mathrm{W}
\end{aligned}
$$ \& 23 \& $45^{\circ} \mathrm{W}$ \& 48
$42^{\circ} \mathrm{W}$
4 \& 43 \& $75^{\circ} \mathrm{E}$ \& $72^{\circ} \mathrm{E}$
$78{ }^{\circ} \mathrm{E}$ <br>
\hline 4 \& $159{ }^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 162^{\circ} \mathrm{W} \\
& 156^{\circ} \mathrm{W}
\end{aligned}
$$ \& 24 \& $39^{\circ} \mathrm{W}$ \& $42^{\circ} \mathrm{W}$
36 \& 44 \& $81^{\circ} \mathrm{E}$ \& $78{ }^{\circ} \mathrm{E}$
$84{ }^{\circ} \mathrm{E}$ <br>
\hline 5 \& $153^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 156^{\circ} \mathrm{W} \\
& 150^{\circ} \mathrm{W}
\end{aligned}
$$ \& 25 \& $33^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 36^{\circ} \mathrm{W} \\
& 30^{\circ} \mathrm{W}
\end{aligned}
$$ \& 45 \& $87^{\circ} \mathrm{E}$ \& $84^{\circ} \mathrm{E}$
$90^{\circ} \mathrm{E}$ <br>
\hline 6 \& $147^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 150^{\circ} \mathrm{W} \\
& 144^{\circ} \mathrm{W}
\end{aligned}
$$ \& 26 \& $27^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 30^{\circ} \mathrm{W} \\
& 24^{\circ} \mathrm{W}
\end{aligned}
$$ \& 46 \& $93^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 90^{\circ} \mathrm{E} \\
& 96^{\circ} \mathrm{E}
\end{aligned}
$$ <br>
\hline 7 \& $141^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 144^{\circ} \mathrm{W} \\
& 138^{\circ} \mathrm{W}
\end{aligned}
$$ \& 27 \& $21^{\circ} \mathrm{W}$ \& $24^{\circ} \mathrm{W}$
$18^{\circ} \mathrm{W}$ \& 47 \& $99^{\circ} \mathrm{E}$ \& $$
\begin{array}{r}
96^{\circ} \mathrm{E} \\
102^{\circ} \mathrm{E}
\end{array}
$$ <br>
\hline 8 \& $135^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 138^{\circ} \mathrm{W} \\
& 132^{\circ} \mathrm{W}
\end{aligned}
$$ \& 28 \& $15^{\circ} \mathrm{W}$ \& $18^{\circ} \mathrm{W}$
$12^{\circ} \mathrm{W}$ \& 48 \& $105^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 102^{\circ} \mathrm{E} \\
& 108^{\circ} \mathrm{E}
\end{aligned}
$$ <br>
\hline 9 \& $129^{\circ} \mathrm{W}$ \& $132{ }^{\circ} \mathrm{W}$
$126{ }^{\circ} \mathrm{W}$ \& 29 \& $09^{\circ} \mathrm{W}$ \& $12^{\circ} \mathrm{W}$
06 \& 49 \& $111^{\circ} \mathrm{E}$ \& $108^{\circ} \mathrm{E}$
$114^{\circ} \mathrm{E}$ <br>
\hline 10 \& $123^{\circ} \mathrm{W}$ \& $126^{\circ} \mathrm{W}$
$120^{\circ} \mathrm{W}$ \& 30 \& $03^{\circ} \mathrm{W}$ \& 06
$00^{\circ} \mathrm{W}$ \& 50 \& $117^{\circ} \mathrm{E}$ \& $114^{\circ} \mathrm{E}$
$120^{\circ} \mathrm{E}$ <br>
\hline 11 \& $117^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 120^{\circ} \mathrm{W} \\
& 114^{\circ} \mathrm{W}
\end{aligned}
$$ \& 31 \& $03^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 00^{\circ} \\
& 06^{\circ} \mathrm{E}
\end{aligned}
$$ \& 51 \& $123^{\circ} \mathrm{E}$ \& $120^{\circ} \mathrm{E}$
$126{ }^{\circ} \mathrm{E}$ <br>
\hline 12 \& $111^{\circ} \mathrm{W}$ \& $114^{\circ} \mathrm{W}$
108 \& 32 \& $09^{\circ} \mathrm{E}$ \& 06

12 ${ }^{\circ} \mathrm{E}$ \& 52 \& $129^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 126^{\circ} \mathrm{E} \\
& 132^{\circ} \mathrm{E}
\end{aligned}
$$ <br>

\hline 13 \& $105^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 108^{\circ} \mathrm{W} \\
& 102^{\circ} \mathrm{W}
\end{aligned}
$$ \& 33 \& $15^{\circ} \mathrm{E}$ \& $12^{\circ} \mathrm{E}$

$18^{\circ} \mathrm{E}$ \& 53 \& $135^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 132^{\circ} \mathrm{E} \\
& 138^{\circ} \mathrm{E}
\end{aligned}
$$ <br>

\hline 14 \& $99^{\circ} \mathrm{W}$ \& $102{ }^{\circ} \mathrm{W}$
96 \& 34 \& $21^{\circ} \mathrm{E}$ \& $18^{\circ} \mathrm{E}$

$24^{\circ} \mathrm{E}$ \& 54 \& $141^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 138^{\circ} \mathrm{E} \\
& 144^{\circ} \mathrm{E}
\end{aligned}
$$ <br>

\hline 15 \& $93^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 96^{\circ} \mathrm{W} \\
& 90^{\circ} \mathrm{W}
\end{aligned}
$$ \& 35 \& $27^{\circ} \mathrm{E}$ \& $24^{\circ} \mathrm{E}$

30 \& 55 \& $147^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 144^{\circ} \mathrm{E} \\
& 150^{\circ} \mathrm{E}
\end{aligned}
$$ <br>

\hline 16 \& $87^{\circ} \mathrm{W}$ \& \[
$$
\begin{aligned}
& 90^{\circ} \mathrm{W} \\
& 84^{\circ} \mathrm{W}
\end{aligned}
$$

\] \& 36 \& $33^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 30^{\circ} \mathrm{E} \\
& 36^{\circ} \mathrm{E}
\end{aligned}
$$

\] \& 56 \& $153^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 150^{\circ} \mathrm{E} \\
& 156^{\circ} \mathrm{E}
\end{aligned}
$$
\] <br>

\hline 17 \& $81^{\circ} \mathrm{W}$ \& $$
\begin{aligned}
& 84^{\circ} \mathrm{W} \\
& 78^{\circ} \mathrm{W}
\end{aligned}
$$ \& 37 \& $39^{\circ} \mathrm{E}$ \& 36

$42^{\circ} \mathrm{E}$

4 \& 57 \& $159{ }^{\circ} \mathrm{E}$ \& $$
\begin{aligned}
& 156^{\circ} \mathrm{E} \\
& 162^{\circ} \mathrm{E}
\end{aligned}
$$ <br>

\hline 18 \& $75^{\circ} \mathrm{W}$ \& \[
$$
\begin{aligned}
& 78^{\circ} \mathrm{W} \\
& 72^{\circ} \mathrm{W}
\end{aligned}
$$

\] \& 38 \& $45^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 42^{\circ} \mathrm{E} \\
& 48^{\circ} \mathrm{E}
\end{aligned}
$$

\] \& 58 \& $165^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 162^{\circ} \mathrm{E} \\
& 168^{\circ} \mathrm{E}
\end{aligned}
$$
\] <br>

\hline 19 \& $69^{\circ} \mathrm{W}$ \& \[
$$
\begin{aligned}
& 72^{\circ} \mathrm{W} \\
& 66^{\circ} \mathrm{W}
\end{aligned}
$$

\] \& 39 \& $51^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 48^{\circ} \mathrm{E} \\
& 54^{\circ} \mathrm{E}
\end{aligned}
$$

\] \& 59 \& $171{ }^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 168^{\circ} \mathrm{E} \\
& 174^{\circ} \mathrm{E}
\end{aligned}
$$
\] <br>

\hline 20 \& $63^{\circ} \mathrm{W}$ \& \[
$$
\begin{aligned}
& 66^{\circ} \mathrm{W} \\
& 60^{\circ} \mathrm{W}
\end{aligned}
$$

\] \& 40 \& $57^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 54^{\circ} \mathrm{E} \\
& 60^{\circ} \mathrm{E}
\end{aligned}
$$

\] \& 60 \& $177^{\circ} \mathrm{E}$ \& \[

$$
\begin{aligned}
& 174^{\circ} \mathrm{E} \\
& 180^{\circ}
\end{aligned}
$$
\] <br>

\hline
\end{tabular}

The UTM system was developed for the military to satisfy the following requirements (U.S. Department of the Army, 1951) for a worldwide plane coordinate system:
(1) Directional errors must be minimized.
(2) There must be "continuity" over sizeable areas with a minimum number of zones
(3) Scale errors caused by the projection must not exceed a specified tolerance.
(4) A plane rectangular system of coordinates must have unique referencing for all zones.
(5) Transformation formulas from one zone to another must be uniform throughout the system.
(6) Meridianal convergence must not exceed $5^{\circ}$.

The UTM system is used between latitudes of $84^{\circ}$ N . and $80^{\circ} \mathrm{S}$. The polar regions are covered by the Universal Polar Stereographic System, which complements the UTM system but is independent of it. An overlap occurs along the boundary of the two systems.

## COMPUTER PROGRAM DOCUMENTATION

Programs in this report are written in FORTRAN 77 for PRIME $750^{1}$ computer systems. Some minor modifications may be necessary for other systems. The programs use double precision throughout. On other computers and for some applications on PRIME systems, double precision variables may not be necessary to achieve the desired accuracy.

Three sets of example runs are provided, one for each of the three map projections presented in this report. Each set will compute rectangular coordinates from latitude and longitude, as well as the inverse.

FORTRAN computer listings and an explanation of the variables for the American Polyconic, Lambert Conformal Conic, and UTM map projections are given in the attachments. An explanation of the variable names and the subroutines also is given in the attachments.

Subroutines for the American Polyconic Map Projection

Three subroutines for the American Polyconic map projection are called by the user:

CALL DATA (BASLAT,BASLON)
CALL XYTRAN (LAT, LON, X, Y) (forward

## transformation)

CALL INVERSE (LAT, LON, X,Y) (inverse
transformation)
BASLAT is the latitude of the origin (O) and BASLON is the longitude of the origin (fig. 5). LAT and LON are the latitude and longitude, in decimal degrees, of the point $(P)$ to be located; $X$ and $Y$ are the rectangular coordinates, in meters. The origin should be located on the central meridian, which is theoretically the only straight line on the map.

[^0]Subroutine DATA must be called first. It initializes basic parameters.

Subroutine XYTRAN determines the rectangular coordinates, in meters, for a point, given the latitude and longitude of the point (BASLAT, BASLON).

Subroutine INVERSE determines the geodetic coordinates from the rectangular coordinates, in meters. All geodetic coordinates are in decimal degrees.

The calculation of the inverse is an iterative process. The number of iterations required depends on the desired accuracy. Double precision variables may be necessary on some computers.

Subroutines for the Lambert Conformal Conic Map Projection

Three subroutines for the Lambert Conformal Conic map projection are called by the user:

CALL DATA (BASLAT, BASLON)
CALL XYTRAN (LAT, LON, X,Y) (forward transformation)
CALL INVERSE (LAT, LON, X, Y) (inverse transformation)

The subroutine DATA must be called first. It initializes the basic parameters. The standard parallels are set to $45^{\circ} \mathrm{N}$. and $33^{\circ} \mathrm{N}$. These values may be changed in subroutine DATA.

BASLAT is the latitude of the origin ( 0 ) and BASLON is the longitude of the origin (fig. 5). LAT and LON are the latitude and longitude, in decimal degrees, of the point ( P ) to be located; $X$ and $Y$ are the rectangular coordinates, in meters, from the origin.

Subroutine XYTRAN determines the rectangular coordinates, in meters, from point (BASLAT, BASLON) for the latitude and longitude of a point.


Figure 5. The rectangular coordinate system for the American Polyconic and the Lambert Conformal Conic map projections.

Subroutine INVERSE is an iterative routine that determines the geographical coordinates from the rectangular coordinates, in meters. All geographical coordinates are in decimal degrees.

Any meridian can be used as the $Y$ axis for the rectangular coordinate system because all meridians are represented as straight lines.

Subroutines for the Universal Transverse Mercator Map Projection

Three subroutines for the UTM map projection are called by the user:

CALL DATA (BASLAT, LAM)
CALL UTMF (LAT, LON, X, Y) (forward transformation)
CALL AUTMI (LAT, LON, X, Y) (inverse transformation)

The subroutine DATA must be called first. It initializes the basic parameters that are relative to the zone and the limits of the area being considered (fig. 6). BASLAT is the latitude user origin. LAM is the latitude of the user origin and is used to determine the UTM zone number and LAM0 (central meridian). The program computes LAMO for the standard UTM zones, but nonstandard zones may be used by replacing lines 189-192 in subroutine UTM with: C NONSTANDARD ZONES

LAMO = latitude of central meridian in radians
Subroutines UTMF or AUTMI are called for each data point to be transformed. Subroutine UTMF computes rectangular coordinates from geodetic coordinates and subroutine AUTMI computes the inverse. LAT and LON are the latitude and longitude,


Figure 6. Universal Transverse Mercator coordinate system.
in radians, of the point, and $X$ and $Y$ are the rectangular coordinates, in meters, from the origin $X=$ $500,000 \mathrm{~m}$ and $\mathrm{Y}=0 \mathrm{~m}$.

## EXAMPLES

Three examples given in the attachments show how to use the map projection routines. The examples show both the forward transformation and the inverse transformation. Each example will use the routines to compute the Cartesian coordinates ( $\mathrm{X}, \mathrm{Y}$ ) from a given geodetic coordinate (latitude, longitude). These Cartesian coordinates will then be used to compute a new geodetic coordinate and its Cartesian coordinates. Errors in the programs can be found by comparing the difference between old and new coordinate values.

The programs were compiled and executed on a PRIME 750 computer, as shown in example runs. Input and output are directed to the user's terminal.

## SUMMARY

Computer programs were developed to calculate geodetic and Cartesian coordinates for the American Polyconic, Lambert Conformal Conic, and Universal Transverse Mercator map projection systems.

Given the latitude and longitude of a point on the Earth, the programs calculate Cartesian coordinates. Given the Cartesian coordinates, the programs will calculate latitude and longitude.

## REFERENCES CITED

Adams, O. S., 1918, General theory of the Lambert Conformal Conic projection: U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication no. 52, 244 p.
Birdseye, C. H., 1929, Formulas and tables for the construction of polyconic projections: U.S. Geological Survey Bulletin 809,126 p.
Thomas, P. D., 1952, Conformal projections in geodesy and cartography: U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication no. 251, 150 p.
U. S. Department of the Army, 1951, The universal grid systems-Universal Transverse Mercator and Universal Polar Stereographic: Washington, U.S. Government Printing Office, 320 p.

## CONVERSION FACTORS

Conversion factors for terms used in this report are listed below:

| Divide <br> square meter <br> meter $(\mathrm{m})$ | $\left(\mathrm{m}^{2}\right)$ | $4, \frac{\text { By }}{047}$ |
| :--- | :---: | :---: |$\quad \frac{\text { To obtain }}{\text { acre }}$

Attachments $\mathbf{A}$ - C

Attachment A.--American Polyconic map projection source program listing.



```
C-------------------------------------------------------------------------
C SUBROUTINE TO COMPUTE LATITUDE OF PARALLEL A DISTANCE X FROM THE
C EQUATOR
                                    FUNCTION SINV (Y,PHI)
                                    IMPLICIT DOUBLE PRECISION (A-Z)
C
C
                                    COMMON /DATAl/ A,C,E,A0,Al,A2,A3,A4,EE,CF,CM,CL,ITER
C SAVE GUESS AT PHI
                OLD=Y
C BEGIN ITERATIONS
                DO 10 I=1,10
                S=Y0(PHI)
                DY=Y-S
                F=A*(1.D0-EE)
                D=(1.DO-EE*DSIN(PHI)*DSIN (PHI))
                Cl=-3.D0*EE/(F*F)
                Pl=D**(1.5)/F
                P2=C1*D*D*DCOS (PHI)
                PHI=(PHI+DY*P1+DY*DY*P2)
                IF(OLD.EQ.PHI) GOTO 11
                OLD=S
    10 CONTINUE
C ITERATION CONVERGED
11 SINV=PHI
        RETURN
    END
```

| SUBROUTINE: MAIN |  |
| :---: | :---: |
| PURPOSE: | To initialize constants and prompt user for input |
| Name | Description |
| LAT, LON | Latitude and longitude of origin in decimal degrees |
| DATA | Subroutine reference to initialize constants |
| L, M | Arrays containing latitude and longitude in deg-min-sec |
| XYTRAN | Subroutine reference to compute the X and Y coordinates from the latitude and longitude |
| INVERSE | Subroutine reference to compute the latitude and longitude from the $X$ and $Y$ coordinates |
|  |  |
| SUBROUTINE: DATA (BASLAT, BASLON) |  |
| PURPOSE: | To initialize constants |
| Name | Description |
| A, B | Lengths of the major and minor axis in meters |
| EE | Square of the eccentricity E |
| ITER | Number of iterations for computing inverse |
| C | Factor to convert degrees to radians |
| A0,A1,A2, Factors for computing the distance from the equator A3,A4 along a meridian |  |
| CM | Central meridian in radians |
| CL | Latitude of the origin in radians |
| BASLON | Longitude of the origin in degrees |
| BASLAT L | Latitude of the origin in degrees |
| *********** | *********************************************************** |


| SUBROUTINE | : XYTRAN (LAT,LON, $\mathrm{X}, \mathrm{Y}$ ) |
| :---: | :---: |
| PURPOSE: | To compute the $X$ and $Y$ coordinates from the latitude and longitude |
| Name | Description |
| LAT, LON | Latitude and longitude in degrees (input) |
| DLAT, DLON | Latitude and longitude in radians |
| PHI |  |
| RHO | Map radius of parallel at LAT,LON |
| X,Y | Rectangular coordinates in meters (output) |
| ******** | *********************************************************** |
| SUBROUTINE | : INVERSE (LAT,LON,X,Y) |
| PURPOSE: | To compute the latitude and longitude from the $X$ and $Y$ coordinates |
| Name | Description |
| LAT, LON | Latitude and longitude in decimal degrees on return |
| X,Y | Rectangular coordinates in meters |
| ********** |  |
| FUNCTION: | Y0 (LAT) |
| PURPOSE: | To compute the distance along a meridian between parallels at LAT and CL |
| Name | Description |
| DLAT | Difference in latitude between LAT and CL in radians |
| LAT | Factor in radians used in equation |
| Y0 | Distance in meters between the two parallels |

```
[F77 REV. 19.2.3]
0000 ERRORS [<MAIN> F77-REV 19.2.3]
0000 ERRORS [<DATA> F77-REV 19.2.3]
0000 ERRORS [<XYTRAN> F77-REV 19.2.3]
0000 ERRORS [<INVERSE> F77-REV 19.2.3]
0000 ERRORS [<Y0\rangle F77-REV 19.2.3]
0000 ERRORS [<SINV> F77-REV 19.2.3]
[SEG REV 19.2.2]
$ LO DPOLY2
$ LI
LOAD COMPLETE
$ SAVE
$ EXECUTE
    INPUT LATITUDE OF ORIGIN (DEG,MIN,SEC)
44,0,0
    INPUT LONGITUDE OF ORIGIN (DEG,MIN,SEC)
116,0,0
    INPUT LATITUDE OF POINT (DEG,MIN,SEC)
45,0,0
    ENTER LONGITUDE OF POINT (DEG,MIN,SEC)
115,0,0
    LATITUDE = 45.000000
    LONGITUDE = 115.000000
    X-COORDINATE IS 0.78847168E+05
    Y-COORDINATE IS 0.11160761E+06
    LATITUDE = 45.000000
    LONGITUDE = 115.000000
    X-COORDINATE IS 0.78847168E+05
    Y-COORDINATE IS 0.11160761E+06
    ANOTHER POINT? (YES OR NO)
NO
**** STOP
```

```
C*********************************
C BY GARTH D. NEWTON
C VERSION 04/27/84
C UPDATED 06/01/84 PRINT POLAR COORDINATES
C
        PROGRAM MAIN
C
    IMPLICIT DOUBLE PRECISION(A-Z)
        COMMON /DATA1/ E,EE,A,B,C,CM,CL,CF,LAT1,LAT2,THETAO,RO,XL,XK,PI
        DIMENSION L(3),M(3)
        CALL DATA
        CALL PARM
        PRINT *, 'INPUT LATITUDE'
        READ *, L(1),L(2),L(3)
        LAT=L (1) +L (2)/60.+L(3)/3600.
        PRINT *, 'INPUT LONGITUDE'
        READ *, M(1),M(2),M(3)
    LON=M(1)+M(2)/60.+M(3)/3600.
C
C
    PRINT *,'LATITUDE IS ',LAT
    PRINT *,'LONGITUDE IS ',LON
    PRINT 3,X,Y
    FORMAT(1H ,'X AND Y COORDINATES ARE',2E15.8)
3
C
C
    PRINT l,LAT
1 FORMAT(1H ,'LATITUDE IS ',F10.7)
    PRINT 2,LON
2 FORMAT(1H ,'LONGITUDE IS ',FlO.7)
    CALL XYTRAN (LAT,LON,X,Y)
    PRINT 3,X,Y
    STOP
    END
```



```
C***********************************************************************
C INITIALIZE PARAMETER VALUES
C
C SUBROUTINE PARM
    IMPLICIT DOUBLE PRECISION(A-Z)
C
C-----------------------------------------------------------------------
    COMMON /DATA1/ E,EE,A,B,C,CM,CL,CF,LAT1,LAT2,THETAO,RO,XL,XK,PI
    PRINT *,'INITIALIZING PARAMETERS'
    2l=TANZ (LAT1)
    Z2=TANZ(LAT2)
    N1=RHO(LAT1)
    N2=RHO(LAT2)
    XL=LOG(N1/N2*DCOS(LAT1)/DCOS(LAT2))/(Z2-Z1)
    XK=N1*DCOS (LAT1)/(XL*EXP(-XL*Z1))
    THETAO=DASIN (XL)
    R0=RHO (THETA0) /DTAN (THETAO)
C
    PRINT *,'ECCENTRICITY (E) IS ',E
    PRINT *,'MAJOR AXIS LENGTH IS ',A
    PRINT *,'MINOR AXIS LENGTH IS ',B
    PRINT *,'XL=',XL
    PRINT *,'XK=',XK
    PRINT *,'THETA0=',THETA0/C
    PRINT *,'RO=',RO
    RETURN
    END
C**************************************************************************
C COMPUTE THE X AND Y COORDINATES OF LAT,LON
    SUBROUTINE XYTRAN (LAT,LON,X,Y)
    IMPLICIT DOUBLE PRECISION(A-Z)
C
C--------------------------------------------------------------------------
    COMMON /DATAl/ E,EE,A,B,C,CM,CL,CF,LAT1,LAT2,THETAO,RO,XL,XK,PI
C
    LAT=LAT*C
    GAMMA=LON*C*XL
    R=XK*EXP(-XL*TANZ (LAT))
C
C WRITE POLAR COORDINATES
C
    WRITE(1,*) 'RHO IS ',R
    X=R*DSIN (GAMMA)
    Y=R0 - R*DCOS (GAMMA)
    LAT=LAT/C
    RETURN
    END
```

```
C**************************************************************************
C COMPUTE THE LATITUDE AND LONGITUDE FROM THE X AND Y COORDINATES
C
        SUBROUTINE INVERSE (LAT,LON,X,Y)
        IMPLICIT DOUBLE PRECISION(A-Z)
C
C
        COMMON /DATA1/ E,EE,A,B,C,CM,CL,CF,LAT1,LAT2,THETA0,RO,XL,XK,PI
        GAMMA=DATAN (X/(R0-Y))
        LON=GAMMA/XL
        R=X/DSIN (GAMMA)
C COMPUTE THE ISOMETRIC LATITUDE
        Z=-LOG (R/XK)/XL
        Z=EXP(Z)
C USE THETAO AS THE FIRST GUESS AT THE REAL LATITUDE
        LAT=THETAO
C ITERATE UNTIL THERE IS NO CHANGE IN THE VALUE OF THE LATITUDE (LNEW=LAT)
        PRINT *,'BEGINNING ITERATION
        DO 100 I=1,40
        P=((1.-E*DSIN(LAT))/(1.+E*DSIN(LAT)))**(E/2.)
        LNEW=2.* (DATAN (Z/P)-PI/4.)
C***** UPDATE 6/1/84 BY GDN
C***** FIX FLOATING POINT COMPARE
        DIFF=ABS (LNEW-LAT)
        IF(DIFF.EQ.O.) GOTO 200
        LAT=LNEW
100 CONTINUE
            PRINT *, 'SOLUTION DID NOT CONVERGE IN 40 ITERATIONS'
            STOP 200
200 PRINT *, 'SOLUTION CONVERGED'
        LAT=LAT/C
        LON=LON/C
            RETURN
                END
```

Description of variables for the Lambert Conformal Conic map projection.

| SUBROUTINE: | : MAIN |
| :---: | :---: |
| PURPOSE: T | To initialize data and prompt user for input |
| Name | Description |
| DATA | Subroutine reference to initialize constants |
| PARM | Subroutine reference to initialize constants dependent upon upper and lower latitude of the projection |
| L, M | Latitude and longitude in deg-min-sec |
| LAT,LON | Latitude and longitude in decimal degrees |
| INVERSE | Subroutine reference to compute latitude and longitude from $X$ and $Y$ coordinates |
| XYTRAN | Subroutine reference to compute $X$ and $Y$ coordinates from latitude and longitude |
| *********** | ***************************************************** |
| SUBROUTINE: | : DATA |
| PURPOSE: | To initialize constants |
| Name | Description |
| A, B | Lengths of major and minor axis in meters |
| EE | Square of the eccentricity E |
| E | Eccentricity |
| C | Factor to convert degrees to radians |
| LAT1, LAT2 L | Latitude and longitude of the upper and lower parallels for the Lambert Conformal Conic projection. They are usually 45 and 33 degrees |


| FUNCTION: PURPOSE: | TANZ(LAT) To compute the value of the isometric latitude for LAT |
| :---: | :---: |
| Name | Description |
| Z | Isometric latitude |
| TANZ | Function |
| LAT | Latitude in radians |
| ******************************************************************* |  |
| FUNCTION: | RHO (LAT) |
| PURPOSE: | To compute the map radius of the parallel at LAT |
| Name | Description |
| RHO | Map radius of the parallel at LAT |
| LAT | Latitude in radians |
| EE | Square of the eccentricity E |
|  |  |
| SUBROUTINE: PARM() |  |
| PURPOSE: | To initialize constants which depend upon the value of the upper and lower latitudes |
| Name | Description |
| TANZ | Function to compute the value of the isometric latitude |
| LAT1,LAT2 | Upper and lower latitudes in radians |
| N1,N2 | Map radius of the upper and lower latitudes |
| Z1, 22 | Isometric latitude for the upper and lower latitudes |
| XL, XK | Arbitrary parameters used in the transformation equations |
| THETA0 | Parallel of the corresponding one-standard-parallel projection in radians |
| R0 | Map radius of the parallel thetao |


| SUBROUTINE | : XYTRAN (LAT,LON, X,Y) |
| :---: | :---: |
| PURPOSE: | To compute the rectangular coordinates for the latitude and longitude |
| Name | Description |
| LAT | Latitude in decimal degrees |
| LON | Longitude from the central meridian in degrees |
| R | Map radius of the parallel at LAT |
| X,Y | Rectangular coordinates in meters |
| R0 | Map radius of the parallel thetao. Used as the grid origin |
| ****************************************************************** |  |
| SUBROUTINE: INVERSE (LAT, LON, $\mathrm{X}, \mathrm{Y}$ ) |  |
| PURPOSE: | To compute the latitude and longitude given the rectangular coordinates |
| Name | Description |
| GAMMA | Angle of intersection of the projected meridian and the $X$ axis |
| LAT, LON | Latitude and longitude in radians |
| R | Radius of intersection of the parallel LAT |
| Z | Isometric latitude of LAT |
| THETA0 | Parallel at the origin |
| LNEW | Latitude computed in iteration. Approaches the correct latitude as the iteration proceeds |

Example execution of the Lambert Conformal Conic map projection.

```
[F77 REV. 19.2.3]
0000 ERRORS [<MAIN> F77-REV 19.2.3]
0000 ERRORS [<DATA> F77-REV 19.2.3]
0000 ERRORS [<TANZ> F77-REV 19.2.3]
0000 ERRORS [<RHO\rangle F77-REV 19.2.3]
0000 ERRORS [<PARM> F77-REV 19.2.3]
0000 ERRORS [<XYTRAN> F77-REV 19.2.3]
0000 ERRORS [<INVERSE> F77-REV 19.2.3]
[SEG REV 19.2.2]
$ LO DLAMB
$ LI
LOAD COMPLETE
$ SAVE
$ EXECUTE
    INITIALIZING DATA
    INITIALIZING PARAMETERS
    ECCENTRICITY (E) IS 8.2271854223048E-0002
    MAJOR AXIS LENGTH IS 6378206.400000
    MINOR AXIS LENGTH IS 6356583.800000
    XL= 0.6304964577737
    XK= 12452706.25654
    THETA0= 39.08675978964
    R0= 7862673.103535
    INPUT LATITUDE
44,0;0
    INPUT LONGITUDE
116,0,0
    RHO IS 7276366.881771
    LATITUDE IS 44.00000000000
    LONGITUDE IS 116.0000000000
    X AND Y COORDINATES ARE 0.69635129E+07 0.57519853E+07
    BEGINNING ITERATION
    SOLUTION CONVERGED
    LATITUDE IS 44.0000000
    LONGITUDE IS $116.00000
    RHO IS 7276366.881771
    X AND Y COORDINATES ARE 0.69635129E+07 0.57519853E+07
**** STOP
```

PROGRAM MAIN

|  |  |
| :---: | :---: |
| C |  |
| C | PROGRAM WRITTEN BY GARTH NEWTON |
| C | FILE NAME IS DUTM.F77 |
| C | VERSION 04/27/84 |
| C |  |
| C | THIS PROGRAM IS AN EXAMPLE OF HOW TO USE THE UTM ROUTINES |
| C | TO COMPUTE LATITUDE AND LONGITUDE FROM RECTANGULAR COOR- |
| C | DINATES AND THE INVERSE. IT IS ALSO A GOOD WAY OF |
| C | TESTING THE ACCURACY OF THE ROUTINES. |
| C |  |
| C | THIS SUBROUTINE CALLS SUBROUTINES TO INITIALIZE DATA |
| C AND PROMPTS USER FOR INPUT |  |
| C |  |
|  | IMPLICIT DOUBLE PRECISION (A-Z) |
| C |  |
| C-------------------------- |  |
|  |  |
| 7 | PRINT *,' ENTER LATITUDE OF ORIGIN (DEG,MIN,SEC) READ *, LAT |
|  | IF (LAT (1).EQ.0.) GOTO 100 |
| 8 | FORMAT (3F3.0) |
|  | PRINT *,'ENTER LONGITUDE OF ORIGIN (DEG,MIN,SEC) ${ }^{\prime}$ |
|  | READ *,LON |
| 9 | FORMAT (F4.0,2F3.0) |
|  | CALL DEGREE (LAT, ALAT) |
|  | CALL DEGREE (LON,ALON) |
| C |  |
| C | INITIALIZE DATA DEPENDENT ON THE CHOICE OF ORIGIN. (EG. ZONE NUMBER) CALL UTM (ALAT,ALON) |
| C----- |  |
| C | INPUT LATITUDE AND LONGITUDE IN DEGREES-MINUTES-SECONDS |
| C | AND COMPUTE GRID COORDINATES. THEN COMPUTE NEW LATITUDE |
| C | AND LONGITUDE FROM GRID COORDINATES. THEN COMPUTE NEW |
| C | GRID COORDINATES AND COMPARE ALL RESULTS |
| C----- |  |
| 100 | PRINT *,'ENTER LATITUDE (DEG, MIN, SEC) FREE-FORM ${ }^{\text {¢ }}$ |
|  | PRINT *, 'TO STOP LATITUDE $=0,0,0{ }^{\circ}$ |
|  | READ *,LAT |
|  | IF (LAT (1). EQ.0.) GOTO 200 |
|  | PRINT *,'ENTER LONGITUDE (DEG,MIN,SEC)' |
|  | READ *,LON |
| C | CONVERT FROM DEG,MIN,SEC TO DEGREES |
|  | CALL DEGREE (LAT, ALAT) |
|  | CALL DEGREE (LON, ALON) |
| C--- | COMPUTE GRID COORDINATES |
|  | CALL UTMF (ALAT, ALON, X1, Yl) |
|  | COMPUTE NEW LATITUDE AND LONGITUDE |
|  | CALL AUTMI (BLAT, BLON, X1,Y1) |
|  | COMPUTE NEW GRID COORDINATES |
|  | CALL UTMF (BLAT, BLON, X2,Y2) |
|  |  |
|  | PRINT RESULTS |

```
C-------------------------------------------------------------------------------
    PRINT *,'************************
2 FORMAT(1H, 2F10.7,' OLD LATITUDE AND LONGITJDE')
    PRINT 2,ALAT,ALON
3 FORMAT(1H ,2F10.7,' NEW LATITUDE AND LONGITUDE')
        PRINT 3,BLAT,BLON
C A 'FALSE EASTING OF 500,000 METERS IS ADDED TO THE X COORDINATES BY
C CONVENTION. THIS PREVENTS ANY NEGATIVE VALUES WITHIN A ZONE
        Xl=Xl*.9996+500000.
        Yl=Yl*.9996
4 FORMAT(1H ,2F10.7,' OLD COORDINATES IN METERS')
        PRINT 4,Xl,Y1
5 FORMAT(1H ,2F10.7,' NEW COORDINATES IN METERS')
        PRINT 5,X2,Y2
        PRINT *,'GRID COORDINATES ARE IN METERS FROM THE ORIGIN*
        PRINT *,'************************'
        GOTO 100
200 STOP
    END
C**************************************************************************
C CONVERT DEG-MIN-SEC TO DEGREES
C SUBROUTINE DEGREE (A,B)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
    DIMENSION A(3)
    B=A(1)+A(2)/60.+A(3)/3600.
    RETURN
    END
C************************************************************************
C CONVERT DEGREES TO DEG-MIN-SEC
    SUBROUTINE DECIMAL(A,B)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
    DIMENSION A(3)
    A(1)=AINT (B+.5/3600.)
    A(2) =AINT ((B-A(1))*60.+.5/60.)
    A(3)=AINT((B-A (1)-A (2)/60.)* 3600.+.5)
    RETURN
    END
```

```
C**************************************************
C RECTANGULAR COORDINATES X AND Y
C
    SUBROUTINE AUTMI (LAT,LON,X,Y)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
C------------------------------------------------------------------------------
    COMMON /DATA1/ D2R,A,B,E2,EPS,A0,A1,A2,LAMO,ERR,MO,PHIO,YO
C
    X=(X-500000.)/M0
    Y=Y/M0
C
C----- ITERATE TO COMPUTE MERIDIANAL DISTANCE
C MAKE INITIAL GUESS
    PHI=SINV(Y,PHIO)
    OLD=Y
C BEGIN ITERATIONS
    DO 10 I=1,10
    PHI=SINV(Y,PHI)
C IF OLD VALUE EQUALS NEW VALUE STOP
    IF(OLD.EQ.YO) GOTO 11
    OLD=Y0
    CONTINUE
    S=DSIN (PHI)
    C=DCOS (PHI)
    T=S/C
    T2=T*T
    N2=EPS*C*C
    N=A/SORT(1.DO-E2*S*S)
    AA0 =X/N
    AAl=AA0** 3* (1.D0+2.D0*T2+N2)/6.D0
    AA2=AA0**5*(5.D0+28.D0*T2+24.D0*T2*T2+6.D0*N2+8.D0*T2*N2)
    #/120.D0
C
C COMPUTE LONGITUDE (RADIANS)
CC
    LON=LAMO- (1.DO/C) * (AA0-AAl +AA2)
C
    T3=T*T2
    T4=T*T3
    T5=T*T4
    T6=T*T5
    N4=N2*N2
    N6=N4*N2
    N8=N6*N2
    AAl=T* (1.D0+N2)*AA0*AA0
    AA2=T*(1.D0+N2)*AA0**4* (5.D0+3.D0*T2+N2-4.D0*N4-9.D0*N2*T2)
    AA 3=T*(1.D0+N2)*AA0**6*(61.D0+90.D0*T2+46.D0*N2+45.DO*T4
        # -252.D0*T2*N2
        #-3.D0*N4+100*N6-66*T2*N4-90.D0*T4*N2+88.D0*N8
        #+225.D0*T4*N4+84.D0*T2*N6-192.D0*T2*N8)
            AA4=T*(1.D0+N2)*AA0**8*(1385.D0+3633.D0*T2+4095.D0*T4+1575.D0*T6)
C
C COMPUTE LATITUDE (RADIANS)
C
    LAT=PHI-AAl/2.D0+AA2/24.D0-AA3/720.D0+AA4/40320.D0
C
```

```
C CONVERT TO DEGREES
    LAT=LAT /D2R
    LON=LON /D2R
C
    RETURN
    END
C**************************************************************************
C OF THE AREA BEING MAPPED
C
    SUBROUTINE UTM(BASLAT,LAM)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
    COMMON /DATAl/ D2R,A,B,E2,EPS,A0,A1,A2,LAMO,ERR,MO,PHIO,YO
    A=6378206.4
    B=6356584.8
    M0=0.9996
    D2R=.0174533
    E2=(A**A-B*B)/(A*A)
    EPS=(A*A-B*B)/(B*B)
    E4=E2*E2
    E6=E4*E2
    E8=E6*E2
    AE=A* (1.0-E2)
    A0=AE*(1.+(3./4.)*E2+(45./64.)*E4+(175./256.)*E6
    l +(l1025./16384.)*E8)
        Al=-AE/2.*((3./4.)*E2+(15./16)*E4+(525./512.)*E6
        l +(2205./2048.)*E8)
        A2=AE/4.*((15./64.)*E4+(105./256.)*E6+(2205./4096.)*E8)
        ERR=-AE/6.*((35./512.)*E6+(315./2048.)*E8)
        PRINT *,'A-CONSTANTS , A0,A1*2,A2*2,ERR*2
        PHIO=BASLAT*D2R
        IZONE=AINT((180.-LAM)/6.+1.)
        LAMO=(183.-(6.*FLOAT (IZONE)))*D2R
        PRINT *,'ZONE NUMBER IS ',IZONE
        PRINT *,'CENTRAL MERIDIAN IS ',LAMO/D2R
        RETURN
        END
C*********************************************************************
C CALCULATE THE DISTANCE FROM THE EQUATOR ALONG THE MERIDIAN
        FUNCTION SS(PHI)
        IMPLICIT DOUBLE PRECISION (A-Z)
C----------------------------------------------------------------------------
        COMMON /DATA1/ D2R,A,B,E2,EPS,A0,Al,A2,LAMO,ERR,MO,PHIO,YO
        LAT=PHI*D2R
        SS=(A0*LAT + Al*DSIN(2.DO*LAT) + A2*DSIN(4.DO*LAT) +
    * ERR*DSIN(6.DO*LAT))
        RETURN
        END
```

```
C COMPUTE THE RECTANGULAR COORDINATES X AND Y FROM THE
C LATITUDE AND LONGITUDE
C
    SUBROUTINE UTMF (LAT,LON,X,Y)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
    COMMON /DATAL/ D2R,A,B,E2,EPS,A0,Al,A2,LAMO,ERR,M0,PHIO,Y0
    D=LAM0-LON*D2R
    D2=D*D
    D3=D2*D
    D4=D3*D
    D5=D4*D
    D6=D5*D
    D7=D6*D
    D8=D7*D
C
    S=DSIN (LAT*D2R)
    C=DCOS (LAT*D2R)
    T=S/C
    T2=T*T
    T4=T2*T2
    T6=T4*'T2
    C2=C*C
    C3=C2*C
    C4=C3*C
    C5=C4*C
    C6=C5*C
    C7=C6*C
C
    N2=EPS*C*C
    N4=N2*N2
    N=A/SQRT(1.-E2*S*S)
    AAO =N*C
    AAl=N*C3* (1. -T2+N2)
    AA2=N*C5* (5.-18.*T2+T4+14.*N2
    1-58.*T2*N2)
    AA3=N*S*C7*(61.-479.*T2+179.*T4-T6)
    B0=SS (LAT)
    Bl=N*S*C
    B2=N*S*C3* (5.-T2+9.*N2+4.*N4)
    B3=N*S*C5*(61.-58.*T2+T4+270.*N2
    1-330.*T2*N2)
    B4=N*S*C7*(1385.-3111.*T2+543.*T4-T6)
C
    X=M0* (AA0*D+AA1*D3/6. +AA2*D5/120.+AA3*D7/5040.) +500000.
    Y=M0* (B0+B1*D2/2. +B2*D4/24.+B3*D6/720.+B4*D8/40320)
    RETURN
    END
```

```
C FUNCTION TO COMPUTE THE INVERSE OF FUNCTION SS. IT FINDS THE
C LATITUDE OF A POINT A DISTANCE Y FROM THE EQUATOR. PHI IS AN
C INITIAL GUESS AT THE LATITUDE OR A POINT NEAR Y
C
    FUNCTION SINV(Y,PHI)
    IMPLICIT DOUBLE PRECISION (A-Z)
C
    COMMON /DATAI/ D2R,A,B,E2,EPS,A0,A1,A2,LAMO,ERR,MO,PHIO,Y0
    Y0=SS (PHI/D2R)
    DY=Y-YO
    C=A* (1.-E2)
    D=(1.-E2*DSIN(PHI)*DSIN (PHI))
    Cl=-3.*E2/(C*C)
    Pl=D**(1.5)/C
    P2=Cl*D*D*DCOS (PHI)
    SINV=(PHI+DY*P1+DY*DY*P2)
    RETURN
    END
```

| SUBROUTINE | : INPUT () |
| :---: | :---: |
| PURPOSE: | To initialize transformation constants and prompt user for data |
| Name | Description |
| ALAT, ALON | Latitude and longitude in decimal degrees |
| AUTM I | Subroutine reference to compute latitude and longitude (inverse) |
| BLAT, BLON | Latitude and longitude in decimal degrees |
| DEGREE | Subroutine reference to convert deg-min-sec to decimal degrees |
| LAT | Latitude in deg-min-sec |
| LON | Longitude in deg-min-sec |
| UTM | Subroutine reference to initialize constants for coordinate conversion |
| UTMF | Subroutine reference to compute $X$ and $Y$ coordinates from latitude longitude (forward) |
| $\begin{aligned} & \mathrm{X} 1, X 2, \\ & \mathrm{Y} 1, \mathrm{Y} 2 \end{aligned}$ | Rectangular coordinates in meters ( $Y$ is 0 meters at the equator and x is 500,000 meters at the central meridian) |

SUBROUTINE: DEGREES (A,B)
PURPOSE: To convert deg-min-sec to decimal degrees
Name Description

A(3) Array containing latitude or longitude in deg-min-sec
B Result of subroutine. Value of latitude or longitude in decimal degrees


| SUBROUTINE | : UTM (BASLAT, LAM) |
| :---: | :---: |
| PURPOSE: | To compute data values used in the transformation equations |
| Name | Description |
| A, B | Lengths of the major and minor axis, respectively |
| A0, Al, A2 | Constants used in the equations to compute the distances of a point from the equator along a meridian |
| BASLAT | Latitude in decimal degrees of a point within the range of latitude of the points being considered |
| D2R | Factor to convert degrees to radians |
| DATAI | Common block |
| E2 | Square of the eccentricity E |
| IZONE | UTM zone number |
| LAM | Longitude of the origin |
| LAM0 | Longitude of the central meridian of the UTM zone |
| M0 | Scale factor used to adjust distortion owing to changes in scale in a zone |
| PHIO | BASLAT in radians |
| ********** | ***************************************************** |
| FUNCTION: PURPOSE: | SS (PHI) <br> To calculate the distance from the equator along the central meridian |
| Name | Description |
| LAT | Latitude in radians |
| PHI | Latitude in degrees |
| SS | Function result. Distance from the equator |


| SUBROUTINE | : UTMF(LAT,LON, X,Y) |
| :---: | :---: |
| PURPOSE: | To compute the $X$ and $Y$ coordinates in meters from the latitude and longitude |
| Name | Description |
| A, B | Length of the major and minor axis in meters |
| D2R | Factor to convert from degrees to radians |
| E2 | Square of the eccentricity E |
| LAMO | Longitude of the central meridian in radians |
| LAT, LON | Latitude and longitude in degrees |
| M0 | Scale factor |
| PHIO | Latitude of the origin in radians |
| SS | Function to compute distance from equator |
| X,Y | Rectangular coordinates in meters |
| ********** | ***************************************************** |
| FUNCTION: PURPOSE: | $\operatorname{SINV}(Y, P H I)$ <br> To compute the latitude of a point given the distance from the equator in meters |
| Name | Description |
| SINV | Latitude in radians of the point located $Y$ meters from the origin |
| PHI | Latitude of a point near $Y$. Starting value is usually PHIO |
| Y0 | $Y$ coordinate of a point near $Y$ |
| DY | Distance from $Y$ to the point $Y 0$. As point $Y 0$ moves closer to $Y$, the accuracy of the inverse increases. Eventually, $Y$ and $Y 0$ are equal |

Example execution of the
Universal Transverse Mercator map projection.

```
[F77 REV. 19.2.3]
0000 ERRORS [<MAIN> F77-REV 19.2.3]
0000 ERRORS [<DEGREE> F77-REV 19.2.3]
0000 ERRORS [<DECIMAL> F77-REV 19.2.3]
0000 ERRORS [<AUTMI> F77-REV 19.2.3]
0000 ERRORS [<UTM> F77-REV 19.2.3]
0000 ERRORS [<SS\rangle F77-REV 19.2.3]
0000 ERRORS [<UTMF> F77-REV 19.2.3]
0000 ERRORS [<SINV> F77-REV 19.2.3]
[SEG REV 19.2.2]
    LO DUTM
    LI
LOAD COMPLETE
    SAVE
    EXECUTE
    ENTER LATITUDE OF ORIGIN (DEG,MIN,SEC)
44,0,0
    ENTER LONGITUDE OF ORIGIN (DEG,MIN,SEC)
116,0,0
    A-CONSTANTS 6367400.188685 -32432.38821305
        34.41552738330 -4.5439906321459E-0002
    ZONE NUMBER IS 11.00000000000
    CENTRAL MERIDIAN IS 117.0000000000
    ENTER LATITUDE (DEG,MIN,SEC) FREE-FORM
    TO STOP LATITUDE = 0,0,0
45,0,0
    ENTER LONGITUDE (DEG,MIN,SEC)
115,0,0
    45.0000000 115.00000 OLD LATITUDE AND LONGITUDE
    45.0000000 115.00000 NEW LATITUDE AND LONGITUDE
        657635.30 4984682.5 OLD COORDINATES IN METERS
        657635.29 4984682.3 NEW COORDINATES IN METERS
    GRID COORDINATES ARE IN METERS FROM THE ORIGIN
    ***********************
    ENTER LATITUDE (DEG,MIN,SEC) FREE-FORM
    TO STOP LATITUDE = 0,0,0
0,0,0
**** STOP
```

.

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