# Bedload Equation for Ripples and Dunes

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SEDIMENT TRANSPORT IN ALLUVIAL CHANNELS

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*An examination of conditions under which bed-load transport rates may be determined from the dimensions and speed of shifting bed forms*

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## LIST OF SYMBOLS

| | |
|---|---|
| $C$ | Chezy coefficient |
| $C_1$ | A constant |
| $D$ | Mean depth |
| $g$ | Accleration due to gravity |
| $h$ | Average ripple or dune amplitude |
| $K_1, K_2, K_3, K_4, K_5$ | Constants |
| $q_b$ | Bedload, by volume or weight |
| $q_T$ | Bed material load |
| $S$ | Slope of the energy gradient |
| $t$ | Time |
| $V$ | Mean velocity |
| $V_0$ | Noneroding mean velocity |
| $V_s$ | Average velocity of ripples or dunes in the direction of flow |
| $x$ | Distance parallel to the direction of flow |
| $y$ | Elevation of the bed above an arbitrary horizontal datum |
| $\gamma$ | Unit weight of the fluid |
| $\delta$ | A transformation, $\delta = x - V_s t$ |
| $\tau_0$ | The shear stress at the bed |
| $\lambda$ | Porosity of the sand bed |
| $\partial$ | Partial derivative |

# BEDLOAD EQUATION FOR RIPPLES AND DUNES

By D. B. Simons, E. V. Richardson and C. F. Nordin, Jr.


ABSTRACT

A simple bedload transport equation based on the mean forward velocity and mean height of ripples and dunes is applied to 101 observations from flume studies by using a recirculating flume and four different sands. Where average dune velocities and heights are determined accurately with sonic equipment, and where suspension of bed material is negligible, the computed and observed transport rates of bed material agree reasonably well. General guidelines for adapting the procedure to field applications are given.


## INTRODUCTION

The bedload in most natural channels cannot be measured economically by presently available (1964) sampling techniques. However, where the size and shape of bed configurations—the sand waves or ripples and dunes which form on the channel bed—and the speed of shifting of the bed forms can be determined with reasonable accuracy, a simple relation based on the mean height and velocity of the sand waves can be used to determine bedload transport. It is the purpose of this paper to investigate the applicability and some of the limitations of such a relation.

The bedload equation presented herein was previously given by Richardson and others (1961), and presents no new ideas; in fact, the concept of determining bedload from the dimension and speed of the dunes and ridges in a channel dates from the 19th century (Hubbell, 1964). However, because of the recent technical advancements in electronic equipment (Richardson and others, 1961; Karaki and others, 1961) which is both portable and reliable and which permits rapid and accurate determination of bed configuration over large areas of a channel, it seems desirable to evaluate the applicability of the equation to the practical problem of determining bedload discharge.

## BEDLOAD TRANSPORT

When the velocity of flow in an alluvial channel is low, the channel bed configuration is ripples or dunes, or both, and a part of the transported material moves as bedload, in almost continuous contact with the bed. The bedload moves up the face of a ripple or a dune to the crest; the coarser particles then avalanche down the slope to be deposited on the downstream face of the dune or in the trough, and the finer particles may be deposited on the faces of downstream dunes or swept temporarily into suspension. The ripples and dunes move downstream owing to erosion from their upstream face and deposition on their downstream face.

The differential equation of bedload transport for the ripple and dune bed configuration (see fig. 1) may be written

$$\frac{\partial y}{\partial t}+\frac{1}{(1-\lambda)}\frac{\partial q_b}{\partial x}=0. \qquad (1)$$

Using the transformation

$$\delta=x-V_s t, \qquad (2)$$

$$\frac{\partial y}{\partial t}=\frac{\partial y}{\partial \delta}\frac{\partial \delta}{\partial t}=-V_s dy/d\delta \qquad (3)$$

and

$$\frac{\partial q_b}{\partial x}=\frac{\partial q_b}{\partial \delta}\frac{\partial \delta}{\partial x}=dq_b/d\delta. \qquad (4)$$

Substituting these expressions in equation 1,

$$-V_s\frac{dy}{d\delta}+\frac{1}{(1-\lambda)}\frac{dq_b}{d\delta}=0. \qquad (5)$$

Simplifying,

$$dq_b=(1-\lambda)V_s dy, \qquad (6)$$

from which

$$q_b=(1-\lambda)V_s y+C_1. \qquad (7)$$

Assuming that the dunes and ripples have triangular shapes, the final equation for bedload transport becomes

$$q_b=(1-\lambda)V_s h/2+C_1. \qquad (8)$$

Values of $q_b$ may be converted from volume to weight basis by multiplying equation 8 by the unit weight of the sediment.

In the above,

$q_b$=volume rate of bedload transport per unit width per unit of time,

$\lambda$=porosity of the sand bed,

$V_s$=average velocity of the ripples or dunes in the direction of flow,

$h$=average amplitude of the ripples or dunes,

$y$=elevation of the sand bed above an arbitrary horizontal datum,

$x$=distance parallel to the direction of flow,

$t$=time.

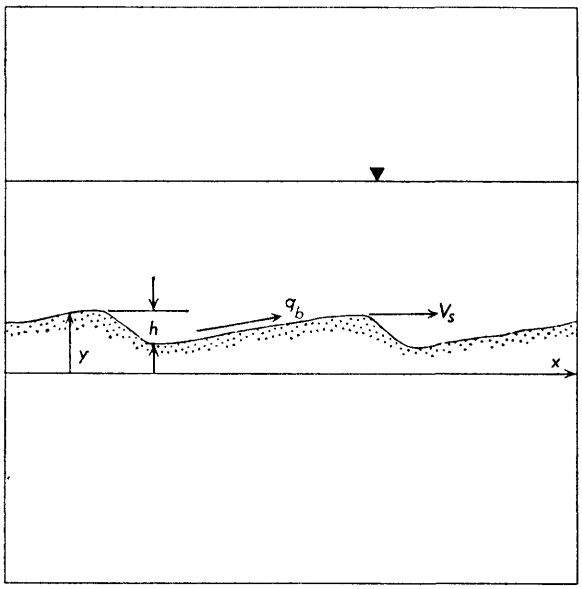


FIGURE 1.—Definition sketch of dunes.

The constant of integration ($C_1$) may be interpreted as that part of the bedload which does not enter into the propagation of dunes and ripples. Obviously, at the threshold of movement, $C_1=0$ and, so long as the bed is entirely covered with ripples and dunes, $C_1$ will remain equal to 0 (assuming that bedload is defined as the material that moves in almost continuous contact with the bed). But as the velocity of flow increases, the bed configuration changes from dunes through a transition region of multiple bed forms to a plane bed. For the plane bed condition, $h=0$ and $C_1=q_b$. For the transition region, $C_1$ is indeterminate.

Equation 8 was applied to 101 observations of equilibrium flows in an 8 foot-wide recirculating flume, where the average heights ($h$) and velocities ($V_s$) of the ripples and dunes were recorded (Simons and Richardson, 1961). Velocities and heights of dunes were determined by probing, by visual observation in the transparent wall of the flume, and by the use of the sonic depth sounder described by Karaki and others (1961). A description of the flume and of the general procedures used in collecting data has been presented (Simons and Richardson, 1962; Simons and others, 1961).

The bed-material median diameters, the number of observations for each bed material, and the range of unit discharge of water and of bed-material considered are summarized in table 1.

TABLE 1.—*Variables considered*

| Median diameter, in millimeters | Observations | Range of discharge per foot of width of— | |
|---|---|---|---|
| | | Water, in cubic feet per second | Bed material, in pounds per second |
| 0.19 | 17 | 0.428–2.75 | 0.0000276–0.226 |
| .28 | 32 | .520–2.75 | .0000069– .211 |
| .46 | 31 | .865–1.95 | .000086 – .350 |
| .93 | 21 | .862–2.84 | .00220 – .474 |

Figure 2 shows the computed bedload transport rate from equation 8 ($q_b$) plotted against the observed bed-material transport rate ($q_T$) for each of these observations. For the coarser material (the 0.93-mm median diameter sand), the agreement between the computed bedload and the observed bed-material load is excellent throughout the range of transport considered. The three points which deviate most from the equal line are for observations where the bed configurations were in a transition between dunes and a plane bed. Similarly, for the 0.46-mm sand, the agreement between computed bedload and observed bed-material load is reasonably good, except at the higher transport rates where an appreciable percentage of the bed-material load was in suspension, or where the bed was in a transition stage between a dune and a plane configuration.

For the finer sizes, the computed bedload is approximately equal to the observed bed-material load for the ripple-bed configurations. For the dune bed, the observed bed-material load is always greater than the computed bedload—a fact that indicates that a large part of the total transport was in suspension.

In figure 3, the bedload transport computed from equation 8 is plotted against observed bedload. This observed bedload was determined as the difference between the observed total bed-material load and the observed suspended load, for the 0.19- and the 0.28-mm sand. Suspended load was determined from the concentration of integrated samples collected to within about 0.1 foot from the bed. For the ripple-bed formation, the relation is not improved, probably because of inherent difficulties in obtaining representative suspended-sediment samples at the extremely low transport rates. For the dune-bed configuration, however, the relation of figure 3 is improved appreciably over that of figure 2. Again, the points which deviate most from the equal line are for bed conditions in transition between a dune and plane bed.

From figures 2 and 3, it is concluded that equation 8 is suitable for computing bedload transport rates for

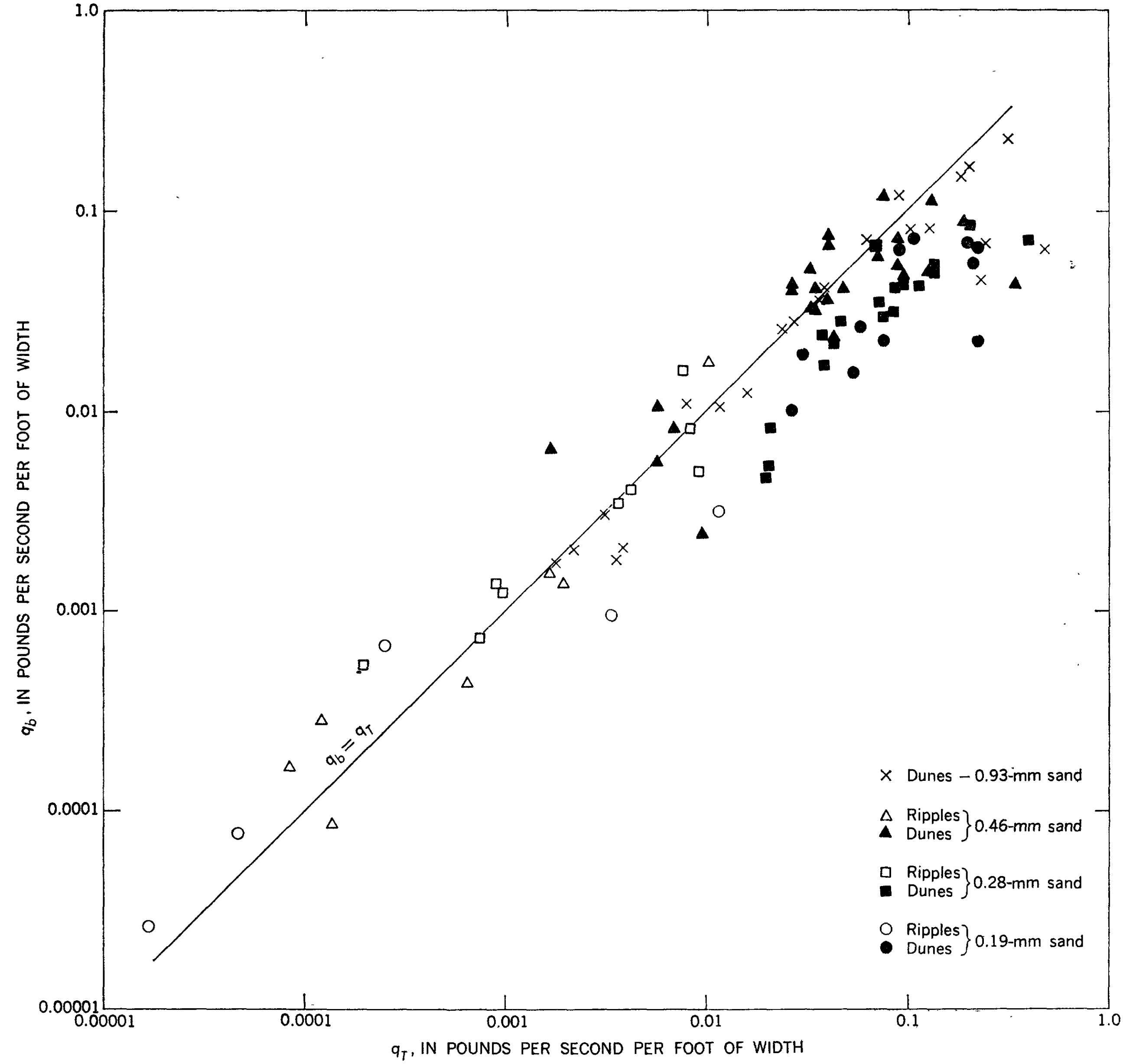


FIGURE 2.—Comparison of computed bedload ($q_b$) from equation 8 with observed bed-material load ($q_T$).

the entire range of conditions considered, excluding those observations when the bed configurations were in a transition from dune to plane bed. For the coarser material, where the suspended load was negligible, the equation may be used to estimate the total bed-material discharge, at least for shallow depths, that is, below approximately 3 feet. At greater flow depths, the suspended load perhaps would not be negligible.

Attempts have been made, notably by Russian investigators (Kondratév, 1962) to formulate relationships for estimating dune heights and dune velocities from the characteristics of the flow. Several empirical relations have been advanced. Znamenskaya (1962) proposed that $V_s$ could be evaluated from the equation

$$V_s = K_1 \frac{D}{h}(V - V_0) \tag{9}$$

and Barekyan (1962) presented the following relations:

$$V_s = K_2 \frac{V_3}{gD} \tag{10}$$

$$h = K_3 D \frac{g}{C^2}\left(\frac{V - V_0}{V_0}\right). \tag{11}$$

In the above,

$D$=mean depth of flow,
$V$=mean velocity,
$V_0$=noneroding mean velocity,
$g$=acceleration due to gravity,
$K_1$, $K_2$, and $K_3$ are constants,
$C$=Chezy coefficient, $V/\sqrt{gDS}$,
$S$=slope of the energy gradient.

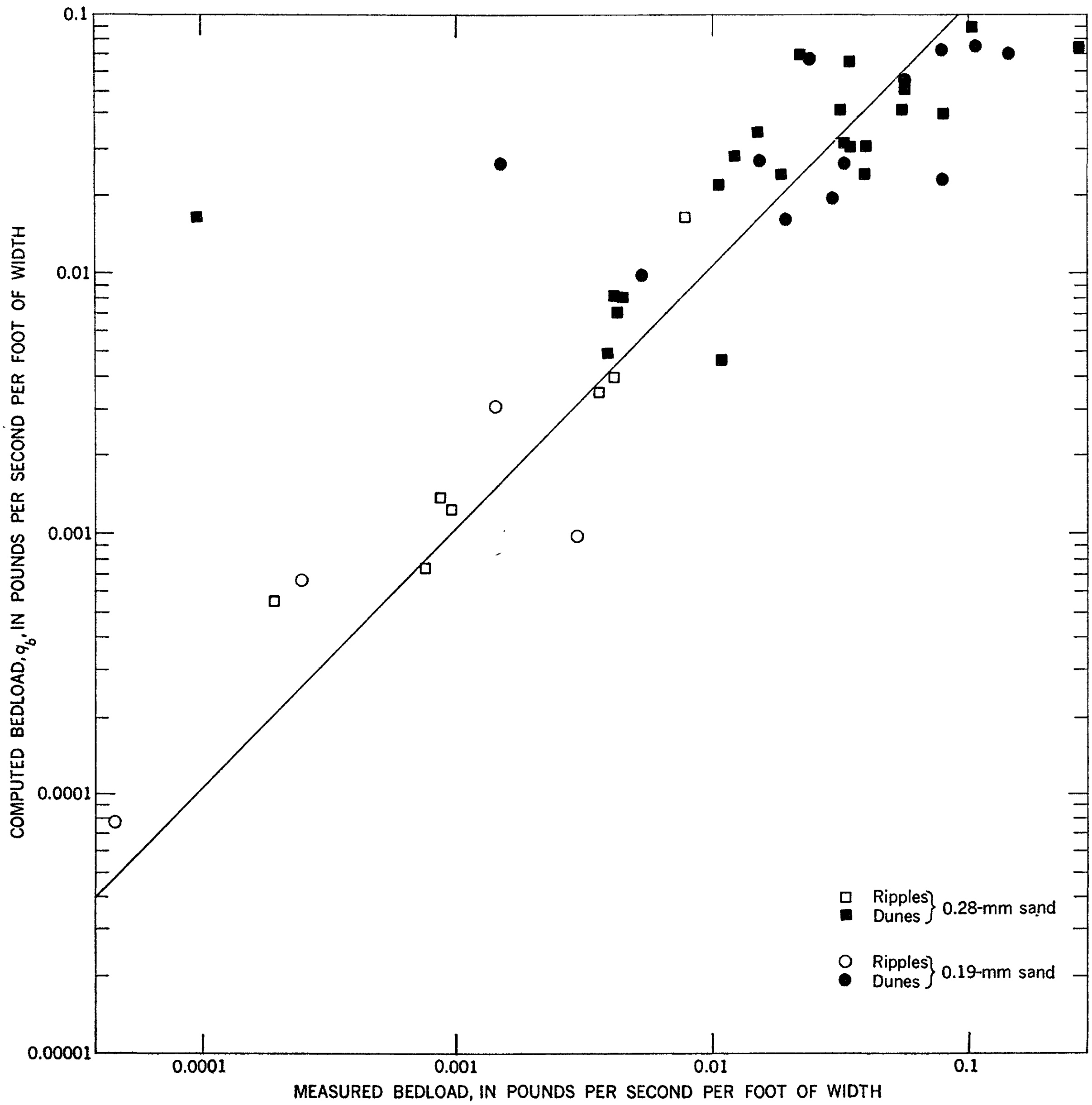


FIGURE 3.—Comparison of computed and observed bedload for the 0.19- and 0.28-mm sand.

When equations 8 and 9 are combined, the rate of bedload transport is found to be independent of dune height, and the transport relation of equation 8 simplifies to

$$q_b = K_4 D(V - V_0) \qquad (12)$$

when $K_4$ is a constant. Equation 12 is quite similar to the expression for stream capacity used by G. K. Gilbert (1914).

The velocity of dune movement ($V_s$) is plotted against $\frac{D}{h}(V-V_0)$ in figure 4. The solid line in the figure is equation 9 with the constant $K_1$, as given by Znemenskaya. Figure 4 shows that equation 9 is unsuited to express dune velocities for the data considered, irrespective of the value of $K$. However, it should be noted that equation 9 was given for the speed of shifting of "meso-forms" (large-scale bars and ridges in natural channels), so the equation probably is not applicable to the ripples and dunes formed in laboratory flumes.

In figure 5, where $V_s$ is plotted against $\frac{V^3}{gD}$, a trend line drawn through the plotted points could be represented by equation 10, but the constant $K_2$ is approximately 0.5, compared with 0.92 given by Barekyan, and the scatter of points about the line does not lend confidence to the relation.

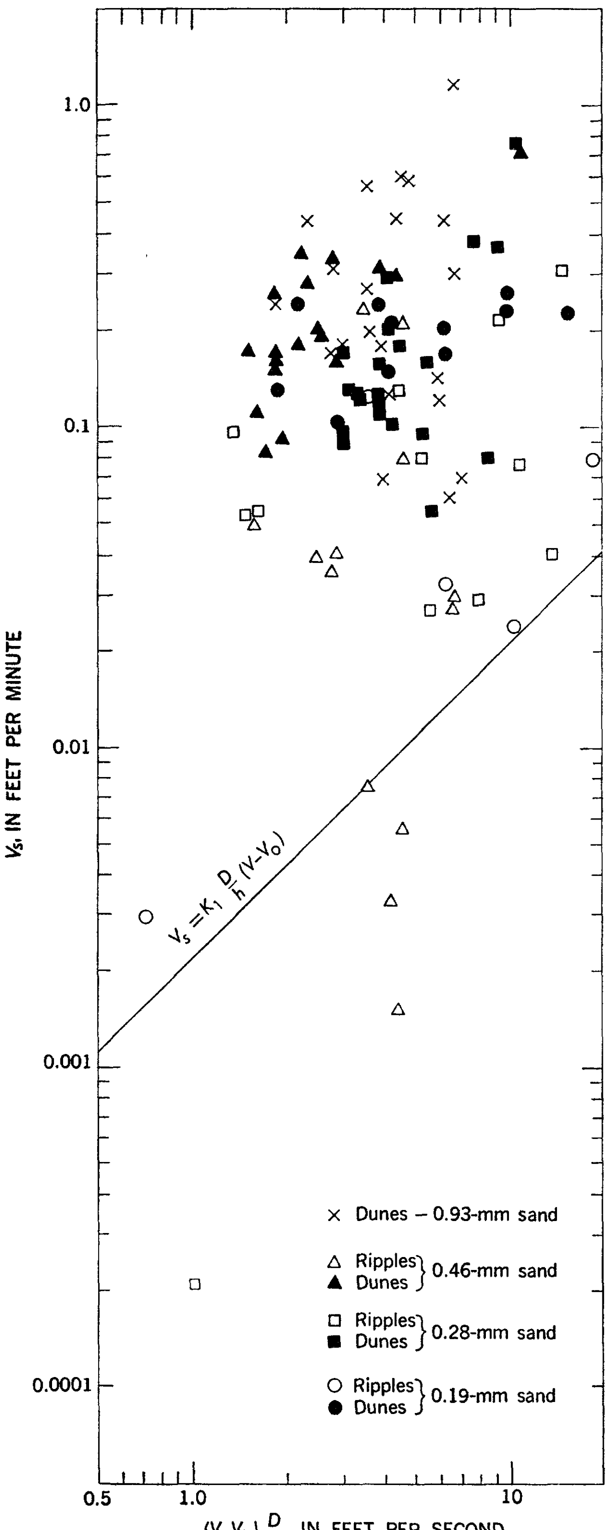


FIGURE 4.—Relation of $V_s$ to $(V-V_0)\frac{D}{h}$.

The relation of equation 11 for dune height ($h$) shown in figure 6 is somewhat better defined than the relation for dune velocity in figure 5, and the constant $K_3=20.6$ has the same value as the constant given by Barekyan.

Combining equations 10 and 11 with equation 8 yields

$$q_b=K_5\tau_0V\left(\frac{V-V_0}{V_0}\right) \qquad (13)$$

in which

$\tau_0$=shear stress at the bed ($\gamma DS$)
$\gamma$=unit weight of the fluid,
$K_5$=a constant.

The product of shear stress and velocity, sometimes called "stream power," has been shown to be an important factor in sediment transport (Bagnold, 1960; Cook, 1936), but the correlation of transport to stream power is better for flow over a plane bed with antidunes and standing waves than for flow over a dune bed (Colby, 1964).

Values of $q_T$ are plotted against $\tau_0V\left(\frac{V-V_0}{V_0}\right)$ in figure 7. The constant $K_5$, defined from the trend line through the plotted points, is approximately 0.3. Equation 13 does not fit the data for the ripple-bed configuration, and the scatter of points is again too great to provide confidence in equations 10, 11, or 13. It must be concluded that equation 8 is best suited for determining bedload when the average values of $V_s$ and $h$ can be determined quite accurately, as with the sonic depth sounder. The relations for determining heights and velocities of bed forms from flow parameters (eqs. 9–11) do not appear to yield reliable results, especially for the ripples.

It is important to note the limitations of equation 8 and to recognize the factors which influence its accuracy. Critical assumptions in the application of equation 8 were: (1) that the ripples and dunes are triangular in shape, (2) that the ripples and dunes are two-dimensional—that is, the average $V_s$ and $h$ are assumed applicable across the entire width of the channel, and (3) that the porosity ($\gamma$) is constant.

Simons and others (1961) have indicated that the bed with a dune configuration is softer than the bed with ripples; this difference suggests that the porosity is not constant, even for a given bed material. However, the porosity probably varies within rather narrow limits for natural river sand, and the introduction of a constant value should not introduce errors of appreciable magnitude.

According to Simons and Richardson (1962), dunes of the coarser material are more angular than are dunes of finer material. The influence of the difference in the shapes of the dunes is reflected in figure 2, where the relation of computed to measured bedload is some-

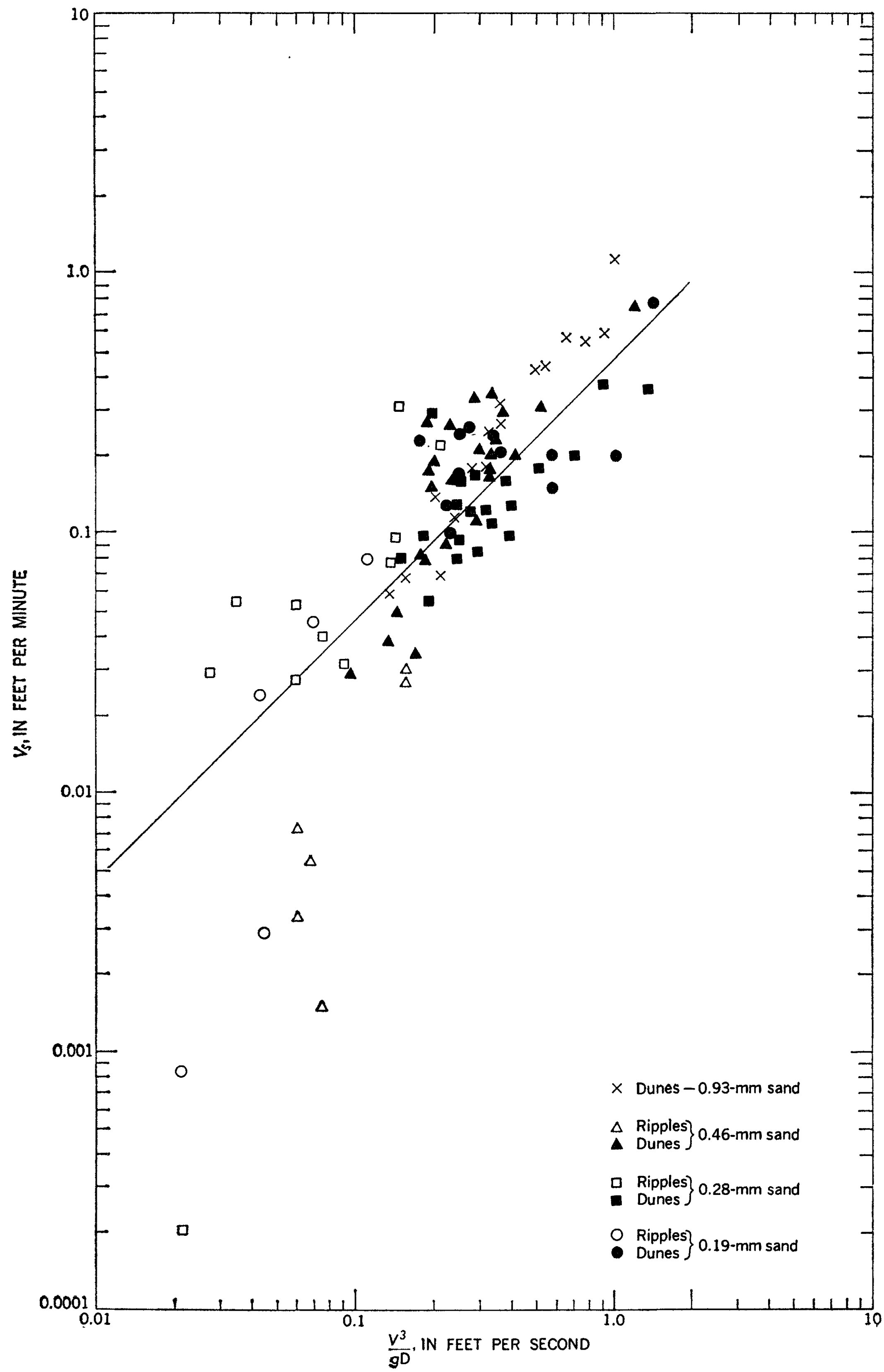


FIGURE 5.—Relation of $V_s$ to $\frac{V^3}{gD}$.

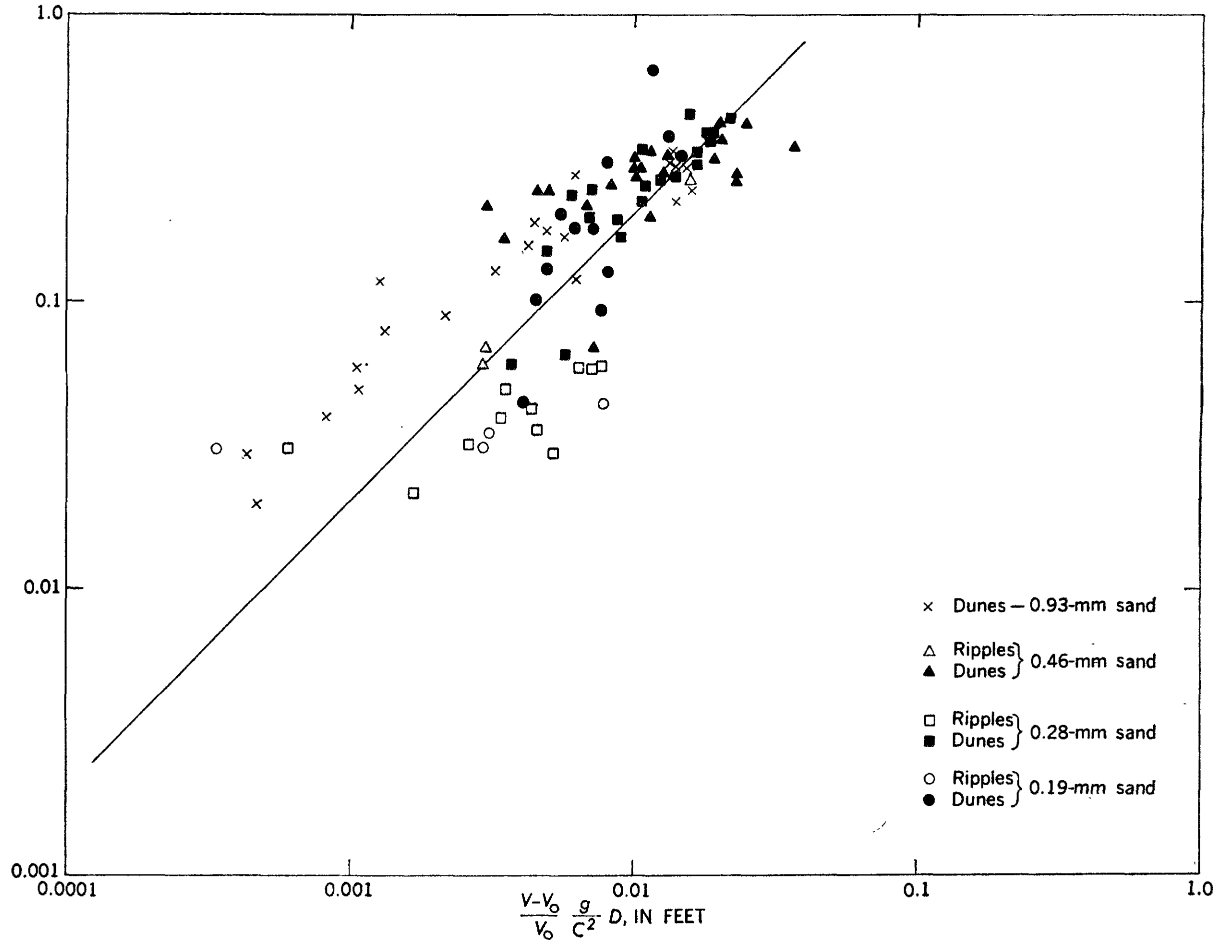


FIGURE 6.—Relation of $h$ to $\frac{V-V_0}{V_0} \frac{g}{C^2} D$.

what better for the 0.93 mm sand than for the finer sizes of bed material.

Figures 2 and 3 show that the relation of computed to measured bedload is better for the dune formation than for ripples, probably because the dunes are more nearly two-dimensional. The equation does not give satisfactory results for bed configurations in a transition stage between dunes and a plane bed. The unsatisfactory results are due to changes in the shape of the dune, to three-dimensional flow and bed configurations, and to the fact that not all of the bedload is active in propagating dune movement. Where these conditions exist, estimates of bedload must be obtained by other methods, such as by sampling (Hubbell, 1964) or by computation. Unfortunately, neither current (1964) sampling techniques nor existing bedload equations yield satisfactory results.

## CONCLUSIONS

Equation 8 should be useful in field applications when used in conjunction with a sonic depth sounder, which provides rapid and accurate determinations of bed configurations. Two proposed methods for estimating velocities of the bed forms from hydraulic characteristics were found to be inadequate. The analysis of flume data in this study gives some indications of the limitations of equation 8, and provides general guidelines for field applications. It is concluded that:

1. The equation is more suited for a dune-bed configuration than for ripples.
2. The equation will give more reliable results for coarse bed material than for fine material, because the dunes of coarse sand are more nearly triangular in shape than are the dunes of fine material.
3. The equation is applicable where average velocities and heights of the bed forms may be accurately determined and where the ripples or dunes cover the entire width of the channel. These conditions are probably met in straight, uniform channels of small width- to- depth ratios, where the flow is generally two-dimensional.

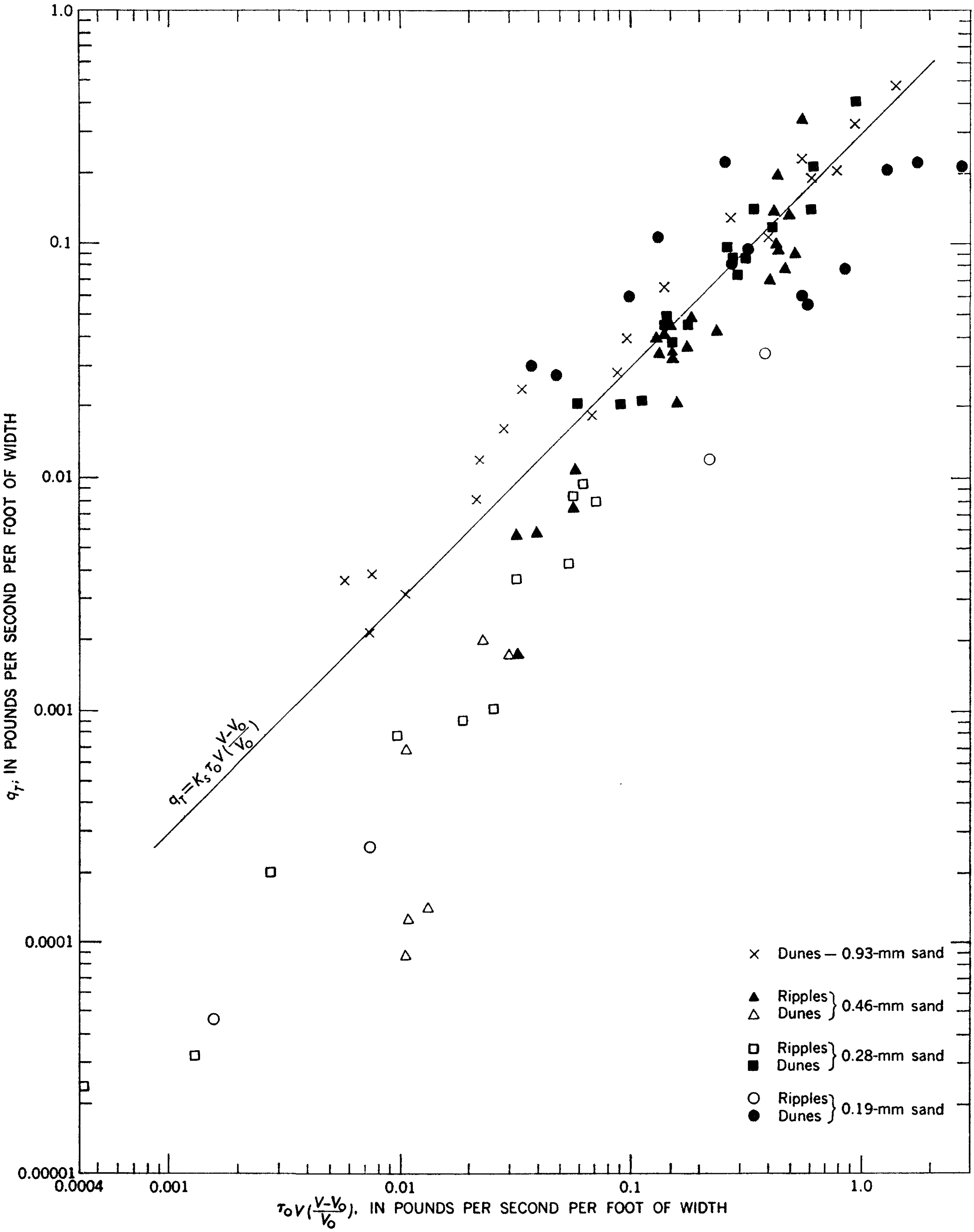


FIGURE 7.—Relation of $q_T$ to $\tau_0 V\left(\frac{V-V_0}{V_0}\right)$.

4. Within the above limitations, the equation provides reasonably accurate values of bedload transport rates. For shallow flow over coarse material in dune formation, or for flow over fine sand in ripple formation, where suspension is negligible, equation 8 may be used to estimate total bed material transport.

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