Classification, Delineation and Measurement of Nonparallel Folds

GEOLOGICAL SURVEY PROFESSIONAL PAPER 314-E

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By JOHN B. MERTIE, Jr.

CONTRIBUTIONS TO GENERAL GEOLOGY

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CLASSIFICATION, DELINEATION, AND MEASUREMENT OF NONPARALLEL FOLDS

By John B. Mertie, Jr.

ABSTRACT

Simple folds are divided primarily into two classes, cylindrical and noncylindrical. The true nature of the stratigraphic surfaces of all folds is unknown, but it is the thesis of this paper that they may be described approximately as cylinders and quadrics. The traces of the stratigraphic surfaces are likewise unknown, but they are represented empirically in selected profiles by algebraic curves. Cylindrical and noncylindrical folds are divided into four genera that depend upon the character of their stratigraphic traces. These are designated as parallel, similar, cognate, and composite folds.

All cylindrical folds have axial lines. They are subdivided into species according to whether the fold has an axial plane or a curved axial surface. Cylindrical parallel folds are unique in that they necessarily have axial planes. Cylindrical similar folds are most simply defined as those whose traces, in a plane normal to the axial line, are curves that are reproducible from one another by nondistortional enlargement or reduction. Similar traces are illustrated by elliptic arcs of the same eccentricity, at the same or different scales. Cylindrical cognate folds are defined as those whose traces, in the selected profile, are nonparallel nonsimilar curves that are analytically related but differ in the values of their assigned parameters. Cognate traces are represented by ellipses with different eccentricities. Cylindrical composite folds are of three kinds. One has related but mixed stratigraphic traces; a second consists of hybrid traces, illustrated by quarter ellipses, either similarly placed or rotated 90° to one another, that are joined where their tangents are parallel; and a third has analytically unrelated traces.

Noncylindrical folds are defined to include quaquaversal folds, elongate or canoe-shaped domes, and the plunging ends of cylindrical folds. All such structures are characterized by an absence of axial lines, though they have analogous indices that are described as apical lines. Canoe-shaped folds have principal sections that are comparable to the axial planes and surfaces of cylindrical folds and to the cross sections normal thereto. The apical lines are either plane or space curves; the principal sections may be either plane or curved surfaces. Plunging folds have some of the characteristics of canoe-shaped folds. The stratigraphic surfaces of noncylindrical folds are illustrated by spheroids, ellipsoids, and modifications thereof. The traces of the stratigraphic surfaces in the principal sections are analogous to those in the selected profiles of cylindrical folds.

Parallel folds, both cylindrical and noncylindrical, are represented in cross section by higher plane curves that may be precisely generated by means of evolutes and involutes. All nonparallel folds, however, must be shown approximately in cross section by empirical curves. Ellipses were chosen for this purpose, first, because they are simple curves of the second degree; second, because their eccentricities are variable; third, because the arcs of an ellipse at the ends of its major and minor axes are parallel, and this permits the delineation of dips of 90° at the base of a fold; and finally, because this parallelism facilitates the construction of hybridized traces. The empirical curves thus generated by the use of ellipses are extensively modified by variations in the eccentricity, by change of scale, and by several types of linear translation, to simulate the various kinds of nonparallel folds that are required to fit the geological data.

Many ellipses of different eccentricity and scale are needed in the empirical representation of nonparallel folds. These are generated as glissettes, by application of the trammel of Archimedes, the construction and use of which are described. As a further aid, 38 prolate and oblate ellipses of different eccentricity are illustrated.

Stratigraphic thickness, in the selected profiles, is defined as the area between two elliptic traces, divided by the length of a medial elliptic arc. The area between the elliptic arcs is obtained either by graphical integration or by formula. The length of an elliptic arc requires an evaluation of Legendre's E function, as given in a table of elliptic functions. Graphic linear integration may also be used. As an aid in this work, the lengths of 99 semiellipses, with major semiaxes of 10, and minor semiaxes ranging from 9.9 to 0.1, have been computed and tabulated. This table, with a trivial amount of interpolation, gives the semiperimeters of all ellipses, regardless of the lengths of their semiaxes. An empirical formula for the length of an ellipse is also presented.

Methods for the mathematical analysis of the traces of cylindrical and noncylindrical folds are not treated at length in this paper, but the value of such work is stressed. Some elementary methods are outlined, and attention is called to the applicability of intrinsic equations for the simplification of such work.

INTRODUCTION

The surfaces of folds are not simple geometrical figures, but they do correspond approximately, within certain limits, to recognizable geometric patterns, such that they may be studied, described, and measured in simplified geometrical terms, with the understood proviso that all results are approximations. Most field geologists appreciate these limitations. The terms "parallel" and "similar" folds and folding are those most commonly used in geologic descriptions; but these terms are so loosely applied that it behooves a writer on such topics to define accurately, or as accurately as the facts permit, the meaning of his nomenclature. Folds whose stratigraphic surfaces approach parallelism are the most amenable to exact delineation and measurement. Nonparallel folds, of which similar folds will be shown to be merely one genus, are much more difficult to analyze and delineate.

Parallel folds have been so systemized and conventionalized by the method of evolute and involutes that any group of geologists, applying this method to the same structural data, will reconstruct identical geologic sections. Likewise, different geologists will obtain the same values for the stratigraphic thickness, depth or distance to a stratum, and other stratigraphic dimensions. It would be desirable if such concordant results could be obtained for nonparallel folds; but this is impossible because, unlike the involutes that represent the traces of parallel stratigraphic surfaces, the geometrical character of nonparallel traces is unknown. Such traces, however, may be conventionalized and represented approximately in accordance with the structural data in possession of the geologist. For reasons later to be stated, this empirical representation is attempted by the use of elliptic arcs that are so selected as to match approximately the dips in a profile and the geologist's interpretation of such dips. Nonparallel folds that are reconstructed in this manner should be easier to draw and should be more comparable with one another; and as the traces are drawn as definite curves, it should be easier to compute stratigraphic thickness and related measurements. More precise analytical methods could also be used advantageously in the classification of folds and in studies pertaining to the mechanics of folding, but such work has not yet been attempted. A few elementary methods, however, are suggested on pages 118–121 of this paper.

The following definitions and inferences are meant to apply to simple arches, cylindrical or noncylindrical, that may be upright, tilted, recumbent, or reversed, with or without axial lines or axial planes. A simple arch is defined as a fold whose stratigraphic surfaces attain either a maximum or a minimum curvature along a line or at a point but have no points or zones at which discontinuities, singularities, or inflections occur. The stratigraphic traces of a simple arch, in a section selected to show maximum or minimum curvature, are continuous curves without discontinuities, singularities, or points of the first or higher orders of inflection. Stated in this manner, these definitions include tilted and recumbent folds without reference to any derived curves and without recourse to any transformation of coordinates. A generalization of the definition of a simple arch, to include surfaces with zones of inflection, would permit the inclusion of monoclinal folds. By combining numbers of such simple arches, more complex folds may be simulated.

PLANE CURVES

The thesis of this paper is that the stratigraphic traces of most nonparallel folds, in sections selected to show the maximum or minimum curvature, may be represented approximately by one or more families of curves that are analytically related. The selection of a suitable family of curves for a particular class or genus of folds is largely a matter of convenience, as the true curvature of folds has not been investigated. Attention naturally centers on the conic sections, as these are simple curves that are easy to construct.

The use of circular arcs is both unrealistic and impracticable. The parabola is unsuitable for two reasons. Its eccentricity is constant, so that it is impossible to show two or more conic traces with different eccentricities; and second, its limbs approach parallelism only at infinity, so that it is impossible to show dips of 90° on both flanks of an upright symmetrical structure. The hyperbola meets the first objection but is vulnerable to the second, as its limbs nowhere approach parallelism. The ellipse, however, meets the necessary specifications, as its eccentricity is variable and its arcs are parallel at the ends of the major and minor axes. These properties lend variability to the delineation of stratigraphic traces, permit the desired charting of dips of 90° on both flanks of certain folds, and also facilitate the hybridization of the traces. The ellipse has therefore been selected to illustrate theoretically the types of folds that may exist and to reconstruct empirically the actual folds that occur in nature.

The terms "parallel," "similar," "cognate," and "composite," applied in this paper to the traces and surfaces of folds, must be defined. With the exception of concentric circles, no two conics of the same species may be drawn parallel to one another. For example, it is impossible to construct two parallel ellipses, though a series of curves of another family may be drawn parallel to an ellipse and to one another, according to Leibnitz's definition of parallelism. This construction is shown in figure 16*A*, where 3 curves are drawn parallel to a central ellipse with an eccentricity of $k=0.800$. These 3 parallel curves have an algebraic equation of the eighth degree with 4 constants, whereas the central ellipse is a curve of the second degree with only 2 essential constants. It is generally true that parallel curves, excepting concentric circles, have rather involved equations, so that parallel traces and parallel surfaces are best treated geometrically.

For practical purposes, however, certain ellipses may be constructed that simulate parallelism, as shown in figure 19*C*. Strange though it may seem, parallel curves may intersect one another, or may actually be self-intersecting. Both these conditions are illustrated in figure 16*B*, where one curve is drawn parallel to and outside an ellipse with an eccentricity of $k=0.923$; and two other intersecting and self-intersecting curves are drawn parallel to and inside the same ellipse. Necessarily, such intersecting curves are excluded in the consideration of parallel stratigraphic traces; and only involutes are used, which conform with the intuitive concept of parallelism.

Similar curves may be defined either analytically or geometrically. In general, similar curves are produced if the original curve is subjected to a transformation of similitude, as if the plane containing the curve were an elastic membrane that was stretched uniformly in all directions. Referring to the general equation of the conic—

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0 \tag{1}$$

two conics are similar if—

$$\frac{B^2-4AC}{4(A+C)^2}=\frac{B'^2-4A'C'}{4(A'+C')^2} \tag{2}$$

and they are both similar and similarly placed if—

$$\frac{A}{A'}=\frac{B}{B'}=\frac{C}{C'} \tag{3}$$

Moreover, the expressions B^2-4AC and $A+C$ are invariant under a rotation of axes, or under a change in origin. Therefore, similarity depends only upon the constants of the terms of the second degree. In general, families of ellipses and families of hyperbolas are not similar, but as B^2-4AC vanishes for every parabola, equation 2 likewise vanishes, and it follows that all parabolas are similar. The central equations for the circle and the equilateral hyperbola, respectively $x^2\pm y^2=a^2$, have one constant that obviously affects both variables equally and thus meet the condition of similarity. Ellipses and hyperbolas, with central equations, respectively of $\frac{x^2}{a^2}\pm\frac{y^2}{b^2}=1$, may be similar only if their major and minor semiaxes are identical, or if their ratio a : b is constant.

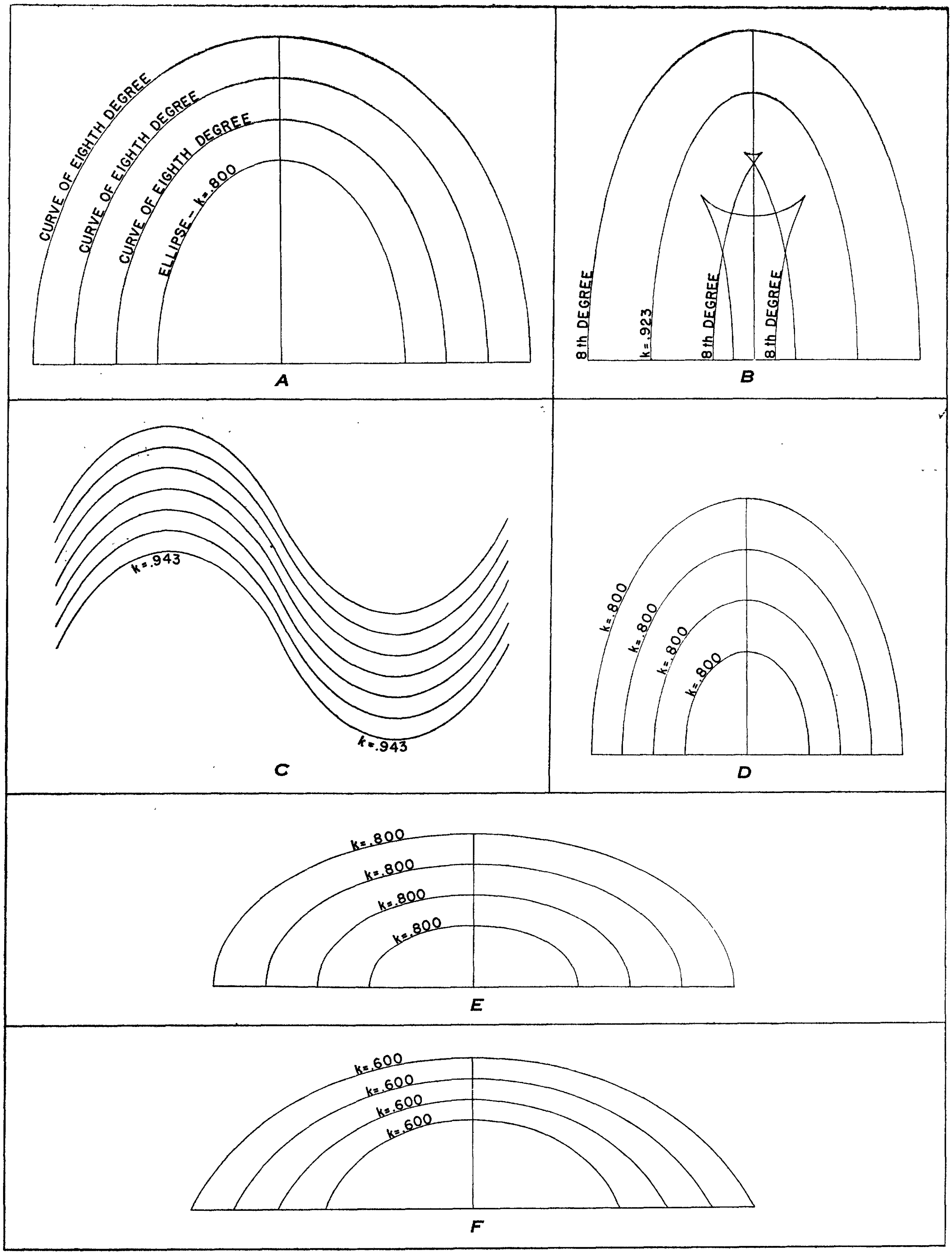
The geometric definition of similarity is simpler. Two curves are similar and similarly placed if radii vectors drawn to the first from some selected point are in constant ratio to parallel radii vectors drawn to the second from some other point. If two such points exist, an unlimited number of others can be found. More simply stated, similar curves may be produced from one another by uniform enlargement or reduction; and congruent or identical curves are a special type of similar curve, where the enlargement or reduction is unity. Therefore the transformation of one curve to a similar curve alters the sizes of the figures but preserves their shapes.

Circles, parabolas, and equilateral hyperbolas are similar conics because they can be transformed by uniform enlargement or reduction without change of shape. Two ellipses or two hyperbolas, however, are similar only if they are congruent or if they have the same eccentricity. Similar conics are similarly placed if they have the same or parallel axes. In general, all parallel plane sections of a quadric surface are similar conics.

A term is needed to describe two or more dissimilar curves that are nonparallel, noncongruent, and nonsimilar, but are analytically related by having the same general equation. A set of ellipses with different eccentricities or a family of parabolic catenaries will illustrate the type. For curves of this kind the writer proposes and in this paper utilizes the designation cognate curves, with special application to ellipses. The adjective cognate, thus applied, has no recognized mathematical acceptance but is used merely for descriptive convenience.

Composite curves are also utilized. These are produced by joining arcs of ellipses with the same or different eccentricities at points where their tangents are parallel. The principal application is in the construction of the so-called hybrid folds and their traces. Thus, two quarter ellipses, with the same or different eccentricities, may be joined at the ends of their major or minor axes or at other points where their tangents are parallel. The same operation may also be performed after one quarter ellipse has been revolved 90° with regard to another. Two such quarter ellipses, after they are joined, constitute a composite curve that is no longer of the second degree but instead is a higher plane curve of unknown character. These hybrid curves cannot be represented exactly by any analytical expression but are known as piecewise functions that may be characterized within 1 range by 1 equation and within another range by a different equation. Two joined quarter ellipses are thus represented piecewise by two elliptic equations having different and possibly reversed constant parameters. They might be described, however, as closely as desired, by some empirical equation. Hybrid curves are illustrated in this paper by arcs of ellipses, but they may also be constructed from other curves. Cassinian ovals, for example, could be similarly utilized.

The parallel, similar, cognate, and composite curves discussed in this paper are similarly placed, though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ELLIPSE - k=.800
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some are afterwards rotated; and initially they are constructed with collinear major and minor axes, in vertical or horizontal positions. Initially they are also equally spaced, representing the traces of strata of the same thickness, though in actual usage they will be unevenly spaced to represent beds of different thickness. Linear translation from their original positions, parallel to either or both axes, results in the production of curves that are partly or wholly noncollinear. Such translations are used to delineate strata that thicken, thin, or disappear entirely, either at the apices or along the flanks of folds, and also to illustrate folds with inclined axial planes or curved axial surfaces. The pinching out, or complete disappearance, of strata, either at the apex or along the flanks of a fold, implies the tangency of adjacent stratigraphic traces in some part or parts of the fold. The types of folds in which this structural feature is possible are later described.

All the folds illustrated in the following pages are not claimed to exist in nature. Some freakish profiles are shown merely to demonstrate the versatility of the method of empirical charting by the use of elliptic arcs.

CLASSIFICATION OF FOLDS

Simple folds may be divided into two principal classes, cylindrical and noncylindrical. The existence of flexures that are essentially cylindrical in character is so commonplace in regions of geosynclinal orogeny as to require no special proof. Noncylindrical folds that are essentially cupolas or domes with approximately a circular plan are likewise generally recognized under the designation of quaquaversal folds. These two classes, however, do not exhaust the field but instead may be regarded as the end members of a much larger group of structures, to which no suitable name can be applied. Quaquaversal folds, for example, grade into domes that have a pronounced elongation, designated in this paper as canoe-shaped folds or elongate domes. Cylindrical folds, on the other hand, become noncylindrical as they plunge, provided the plunge has not been caused by tilting subsequent to the folding. Such plunging folds are intermediate in character between cylindrical and canoe-shaped folds and have some of the properties of both.

The axial line is the most fundamental of the various stratigraphic dimensions, because most of the others depend directly or indirectly upon it; and any classification of folds will depend upon the presence or absence of axial lines. An unequivocal definition is therefore needed. On every stratigraphic surface of a cylindrical fold, there exists a right line along which the curvature of the surface attains either a maximum or a minimum value, depending, respectively, on whether the cross section of the fold is a prolate or an oblate figure. Many stratigraphic surfaces, however, exist in a cylindrical fold, and therefore many axial lines are present; but these are parallel and may be regarded vectorially as the single axial line of the fold. Thus the axial line has the dimensional properties of a lineation, with a direction and possibly a plunge but without spatial position. It should be emphasized that the axial line is defined in terms of a cylindrical fold, and the inference follows that axial lines are absent in noncylindrical folds. Curved lines that lie along the crests of noncylindrical stratigraphic surfaces are analogous to axial lines and are here designated as apical lines. These may be plane or space curves and may or may not be parallel, though ordinarily they are nonparallel.

The axial plane is a property of cylindrical folds that must be defined in terms of the axial line. If a set of axial lines is parallel and coplanar, they lie in and define an axial plane. If they are parallel and noncoplanar, they lie in and define a cylindrically curved surface, which is here designated as the "axial surface." Figures 18*A*, *B*, *C*, *F*; 20*B*, *C*; 21*D*; and 24 show the traces of axial surfaces of this kind.

Noncylindrical folds include quaquaversal folds, canoe-shaped or elongate folds, and the plunging ends of cylindrical folds. A quaquaversal fold is approximately equidimensional in horizontal outline, with an unlimited number of apical lines; hence no unique apical line exists. Similarly an unlimited number of planes or surfaces normal to the stratigraphic surfaces may be constructed to pass through the apex of the dome. Therefore the terms "axial plane" and "axial surface" are also inapplicable.

An elongate dome, however, has one set of apical lines, either plane or space curves, which follow the crests of the successive stratigraphic surfaces in the same direction as the elongation of the fold. These apical lines are the loci of the minimum or elongate curvature of the strata; and the totality of such lines

FIGURE 16.—Central semiellipses, congruent hybrid elliptic arcs, and prolate and oblate similar semiellipses exemplifying certain types of structural features.
A, central semiellipse, bounded by three parallel curves of the eighth degree. Represents the profile of one type of parallel fold.
B, central semiellipse, with 1 outer and 2 intersecting inner parallel curves of the eighth degree. Illustrates parallel curves with discontinuities that are unsuitable for the reconstruction of folds.
C, congruent hybrid elliptic arcs illustrating the profile of a congruent similar fold. Diagram comparable to Van Hise's original drawing of a similar fold.
D, prolate similar semiellipses Represents the profile of a similar fold, with dips of 90° everywhere along its base. Strata thin proportionately from apex to base of fold.
E, oblate similar semiellipses. Represents the profile of a similar fold, with dips of 90° everywhere along its base. Strata thicken proportionately from apex to base of fold.
F, oblate similar elliptic arcs, with noncoaxial horizontal axes. Represents the profile of a similar fold, in which dips of 90° are absent.

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defines a surface that is called the major or longitudinal section of the fold. A surface that passes through the vertices of all the stratigraphic surfaces and intersects orthogonally all the elongate apical lines, constitutes the minor or transverse section of the fold. Collectively these two upright surfaces are called the principal sections of a canoe-shaped or elongated dome, and obviously either or both may be plane or curved surfaces. A third section, not generally a principal section, is indicated by the horizontal plan of the fold. In conformity with these definitions, a canoe-shaped fold is one whose elongate apical lines, lying in the major section, are continuously curved and grade nowhere into linear segments. If the principal section of a canoe-shaped fold is a curved surface, a plane surface that passes through the apex of some selected stratigraphic surface and is normal to an elongate apical line at that point, will show the general character of the fold. Principal plane sections drawn parallel and normal to coplanar apical lines are illustrated in figure 21*E*.

The plunging end of a cylindrical fold is generally noncylindrical, but the cylindrical and noncylindrical sectors are continuous and are threfore difficult to treat separately. The length of a cylindrical fold, however, is ordinarily much greater than its plunging end. The composite structure may best be regarded and described as a cylindrical fold over some stated longitudinal interval, and a noncylindrical fold over some other specified range. The plunging end of a cylindrical fold has the general characteristics of an elongate dome, but owing to structural modifications in the ends of such folds, the apical lines are commonly neither parallel nor coplanar. Yet it is possible for nonparallel but coplanar apical lines to lie within and define a plane that is an extension of the axial plane of the cylindrical fold. With such a structural relationship, the term "axial plane" is warranted. Generally, however, the terms "axial plane" and "axial surface" are not strictly applicable to the plunging ends of cylindrical folds.

Profiles drawn across a plunging fold, where it has lost its cylindrical character, do not have the properties of analogous profiles along the principal sections of a canoe-shaped fold. Both are normal to the stratigraphic surfaces, and generally both are curved surfaces; but the transverse section of a canoe-shaped fold passes through the apices of all the stratigraphic surfaces, whereas the analogous section across the plunging part of a cylindrical fold passes through some arbitrarily selected point on an arbitrarily selected apical line, and is normal to the other apical lines at random points. A plane profile, however, may be drawn normal to some apical line and tangent to the curved profile, or better still, as a secant plane intersecting the curved surface. A profile of this sort will closely approximate the true cross section of the plunging fold, and will be analogous to the cross section of the cylindrical part of the fold.

Cylindrical folds have heretofore been divided into "parallel" and "similar" folds, but these two terms are commonly undefined and are used differently than in this paper. The term "concentric" fold is used by some authors as a synonym for "parallel" folds, but only circles and spheres are concentric; and it is easy to show from a sequence of three or more dips in a profile that stratigraphic traces are generally noncircular. A fourfold division of cylindrical folds is here recognized, based upon the geometry of the stratigraphic surfaces and their traces. These four genera are designated as parallel, similar, cognate, and composite folds, all of which have axial lines. A cylindrical parallel fold necessarily has parallel stratigraphic surfaces and an axial plane. Cylindrical similar, cognate, and composite folds have, respectively, similar, cognate, and composite stratigraphic surfaces but may have either axial planes or axial surfaces. Cylindrical similar and cognate folds are subdivided into species according to whether they have axial planes or axial surfaces. Cylindrical composite folds, however, are subdivided first into subgenera on the basis of their stratigraphic traces and thereafter into species as stated above. Noncylindrical folds are likewise divided into four genera, but further subdivisions into specific types appears to be too difficult and involved to be useful.

A classification of folds that accords with the definitions already stated is presented herewith:

- I. Cylindrical folds
  - A. Parallel folds
  - B. Similar folds
    - 1. With axial plane
    - 2. With curved axial surface
  - C. Cognate folds
    - 1. With axial plane
    - 2. With curved axial surface
  - D. Composite folds
    - 1. Mixed but related stratigraphic traces
      - a. With axial plane
      - b. With curved axial surface
    - 2. Hybrid stratigraphic traces
      - a. Unrotated
        - aa. With axial plane
        - bb. With curved axial surface
      - b. Rotated 90°
        - aa. With axial plane
        - bb. With curved axial surface
    - 3. Unrelated stratigraphic traces
      - a. With axial plane
      - b. With curved axial surface

II. Noncylindrical folds
 A. Parallel domed surfaces
 B. Similar domed surfaces
 C. Cognate domed surfaces
 D. Composite domed surfaces

CYLINDRICAL FOLDS

Cylindrical folds have cylindrical stratigraphic surfaces over some stated or implied longitudinal limit. No assumption is made regarding the cross section of a cylindrical fold, and therefore no assumption is made regarding the character of its stratigraphic traces in a section normal to the axial line. These curved traces may be any of the myriads of continuous curves, here classified as parallel, similar, cognate, and composite.

The term "symmetry," as applied to a cylindrical fold, should mean that an axial plane exists which is approximately a plane of symmetry, such that the same stratigraphic curvature, sequence, and thickness of beds appear on both sides of it. Otherwise stated, the half of the fold on one side of the axial plane must be identical with the half on the other side. Symmetry vanishes if a curved axial surface is present. Symmetrical similar, cognate, and composite cylindrical folds should therefore be less common than symmetrical parallel folds.

The major and minor axes of the ellipses used in this paper to illustrate and simulate cylindrical folds are drawn initially as vertical and horizontal lines. The traces of cylindrical folds constructed in this manner represent upright anticlinal arches that have vertical axial planes. But these figures may be rotated so as to illustrate inclined, recumbent, or synclinal folds, as required by the structural data. All dips will be modified numerically by rotation, but generically and intrinsically they will remain invariant. Thus the comments regarding dips of 90° apply only to upright folds; and such dips can appear on both flanks of an upright fold only where complete semiellipses are utilized.

Five variables may be used in the reconstruction of nonparallel folds by the application of elliptic arcs. These are, first, a choice in the eccentricity of the elliptic arcs; second, change of scale, defined geometrically as uniform or nondistortional enlargement or reduction; third, linear translation of the stratigraphic traces parallel either to the vertical or horizontal axes of the ellipses; fourth, a choice between proportional and differential linear translation of each trace; and fifth, rotation of the elliptic axes and stratigraphic traces.

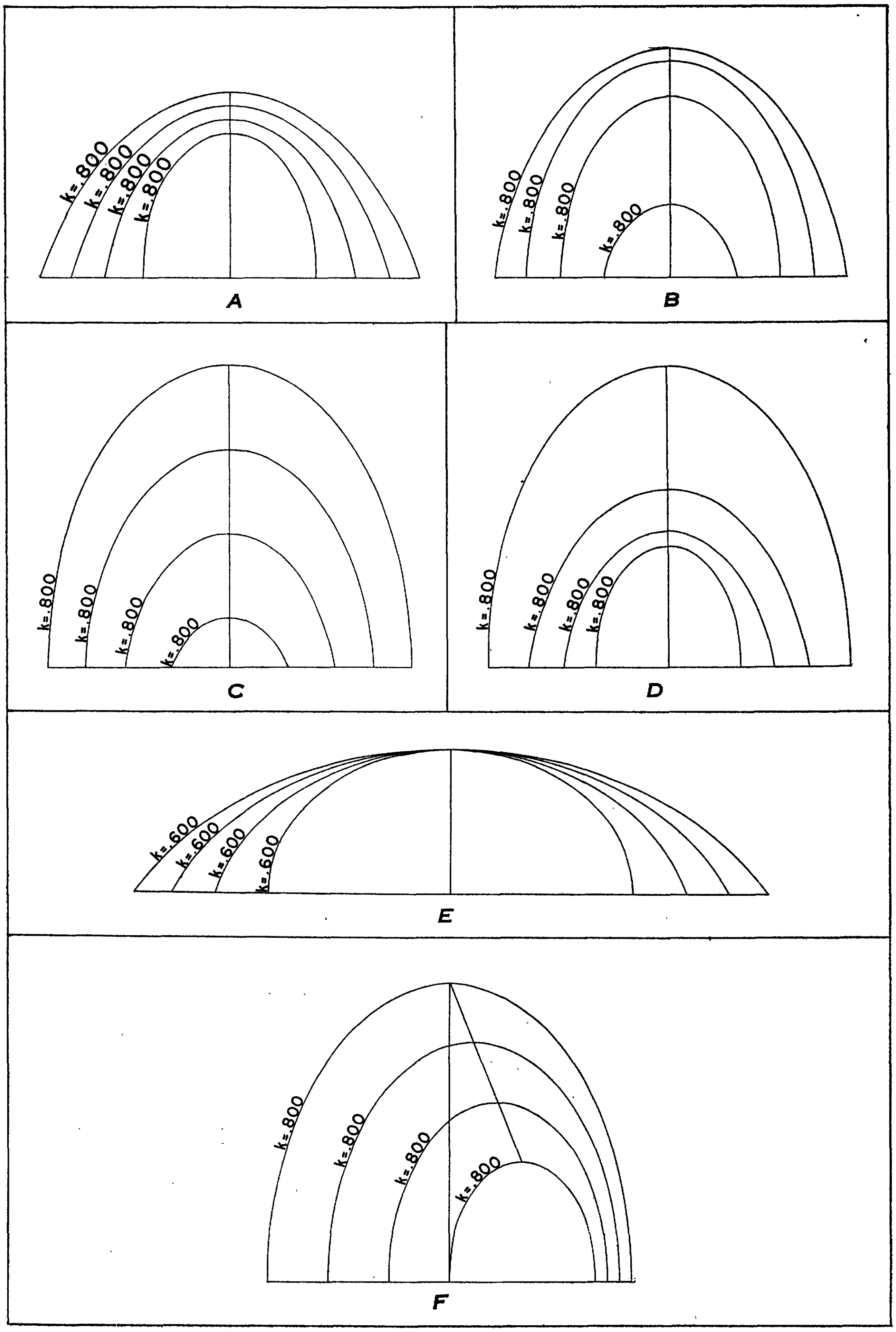
The first two of these variables require no further explanation. Linear translation results when one or both semiaxes depart from coaxiality without rotation. Vertical and horizontal translations produce markedly different effects. Proportional linear translation means a displacement parallel to either or both axes, such that the stratigraphic intervals on one or both axes are increased or decreased proportionately. Differential linear translation results in corresponding displacements that are nonproportional to the initial stratigraphic intervals. These are called differential displacements. A rotatory displacement of 90° of the elliptic axes is also permissible, but any other rotation would cause the stratigraphic traces to intersect.

The major types of linear translation are shown below.

I. Translation parallel to the vertical axes, thus displacing the horizontal axes.
 A. Translation proportional to the thickness of beds.
 B. Translation nonproportional, or differential.
II. Translation parallel to the horizontal axes, thus displacing the vertical axes.
 A. Translation proportional to the thickness of beds.
 B. Translation nonproportional, or differential.
III. Translation parallel to and displacing both axes.
 A. Translation proportional to the thickness of beds.
 B. Translation nonproportional, or differential.
 C. Translation both proportional and differential.
  1. Proportional vertical and horizontal displacements.
  2. Proportional vertical and differential horizontal displacements.
  3. Differential vertical and proportional horizontal displacements.
  4. Differential vertical and horizontal displacements.

PARALLEL FOLDS

Parallel folds are defined as folds whose stratigraphic surfaces are essentially parallel. The traces of such surfaces in sections normal to the axial line are involutes, which constitute one class of parallel lines. The strata of all folds must be stretched in relation to their original lengths; but this is an overall effect that may be accompanied locally by stratigraphic thickening as well as thinning. Parallel folds are conceived to be those in which the strata have not been thickened but instead have been uniformly stretched and thinned. It is doubtful if all the strata of any fold are uniformly stretched, so that truly parallel folds may not exist in nature. Yet in actual folding, particularly in open folding, many competent beds show so little differential thickening and thinning that for practical purposes their thickness may be regarded as constant. The terms "parallel folds" and "folding" therefore appear to be warranted, and descriptions and measurements based upon this interpretation are permissible and useful.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Cylindrical parallel folds may also be defined as those whose stratigraphic surfaces, over some limited longitudinal range, may be generated by two or more parallel coplanar right lines that move parallel to themselves, and in such a manner that the plane in which they lie remains normal to the surfaces generated. This condition is best visualized in a plane section normal to the axial line, wherein the traces of the stratigraphic surfaces are parallel curves, the traces of the generating lines are collinear points, and a right line connecting these points is normal to each of the stratigraphic traces. No assumption is made regarding the cross section of a parallel fold, except that the stratigraphic traces must be parallel or approximately so. These traces may thus be any continuous curves, though it is improbable that they are ever concentric circular arcs. The delineation and measurement of parallel folds have been treated by the writer (1940, 1944, 1947, 1948) in four earlier papers. It suffices here to emphasize that the traces of parallel folds, in sections normal to the axial line, are not represented empirically as elliptic or any other kind of preselected arcs. Instead they are accurately delineated as involutes derived from an evolute that is generated as the envelope of a set of lines drawn normal to the dips charted on a geologic profile.

SIMILAR FOLDS

Cylindrical nonparallel folds have axial lines over some stated or implied longitudinal limit, but the traces of the stratigraphic surfaces, in a plane normal to the axial line, are commonly either similar or cognate curves, though these traces may also be one of several types of composite curves.

The term "similar fold," meaning in reality a similar cylindrical fold, was introduced by Van Hise (1896) and has been used by many geologists to describe all sorts of nonparallel folds. Van Hise's drawing of the traces of a similar fold, in a plane normal to the axial line, consists of congruent or identical curves of unspecified character that are similarly placed but are separated from one another by a linear translation parallel to the trace of the axial plane. An equivalent type of folding is shown in figure 16C, where the congruent curves are formed by joining arcs of an ellipse having an eccentricity of 0.943. These congruent curves are composite and hybrid and are therefore higher plane curves. Dips of 90° may be shown for such folds along the flanks of the outer stratigraphic surface but not elsewhere in the fold. Only moderate dips appear in figure 16C, because complete semiellipses are not used. Instead only arcs of the semiellipse were chosen and translated.

Congruent similar folds, however, constitute only one type of similar folding. If the traces of the stratigraphic surfaces, in the specified section, are noncongruent, nonintersecting, similar curves that are similarly placed, the fold is likewise similar. In three dimensions, similar surfaces, cylindrical or noncylindrical, are defined as those having point-to-point correspondence, such that the distance between any two points of one surface is invariably the same multiple of the distance between two corresponding points on the other surface. It is doubtful whether many cylindrical folds have stratigraphic surfaces that meet exactly the requirements of similarity. It is permissible, however, to utilize the term for folds that meet approximately the required conditions, just as we use the term "parallel folds" for those whose traces are only approximately parallel.

Similar folds are represented in this discussion by elliptic arcs of the same eccentricity. Therefore an initial choice exists in the selection of some particular semiellipse, but no further choice is permitted. Three other variables, however, are commonly utilized in the construction of similar traces. These are, first, change of scale; second, linear translation of the stratigraphic traces parallel either to the vertical or horizontal elliptic axes, or to both; and third, a choice between proportional and differential linear translation for each trace. It is also possible to have a similar fold represented by hybridized stratigraphic traces, if one such trace is repeated at different scales.

FIGURE 17.—Prolate and oblate similar elliptic arcs and similar semiellipses exemplifying certain types of structural features.

A, prolate similar elliptic arcs, with noncoaxial horizontal axes and an innermost semiellipse. Represents the profile of a similar fold, with dips of 90° only at the inner base of the fold. Strata thicken proportionately from apex to base.

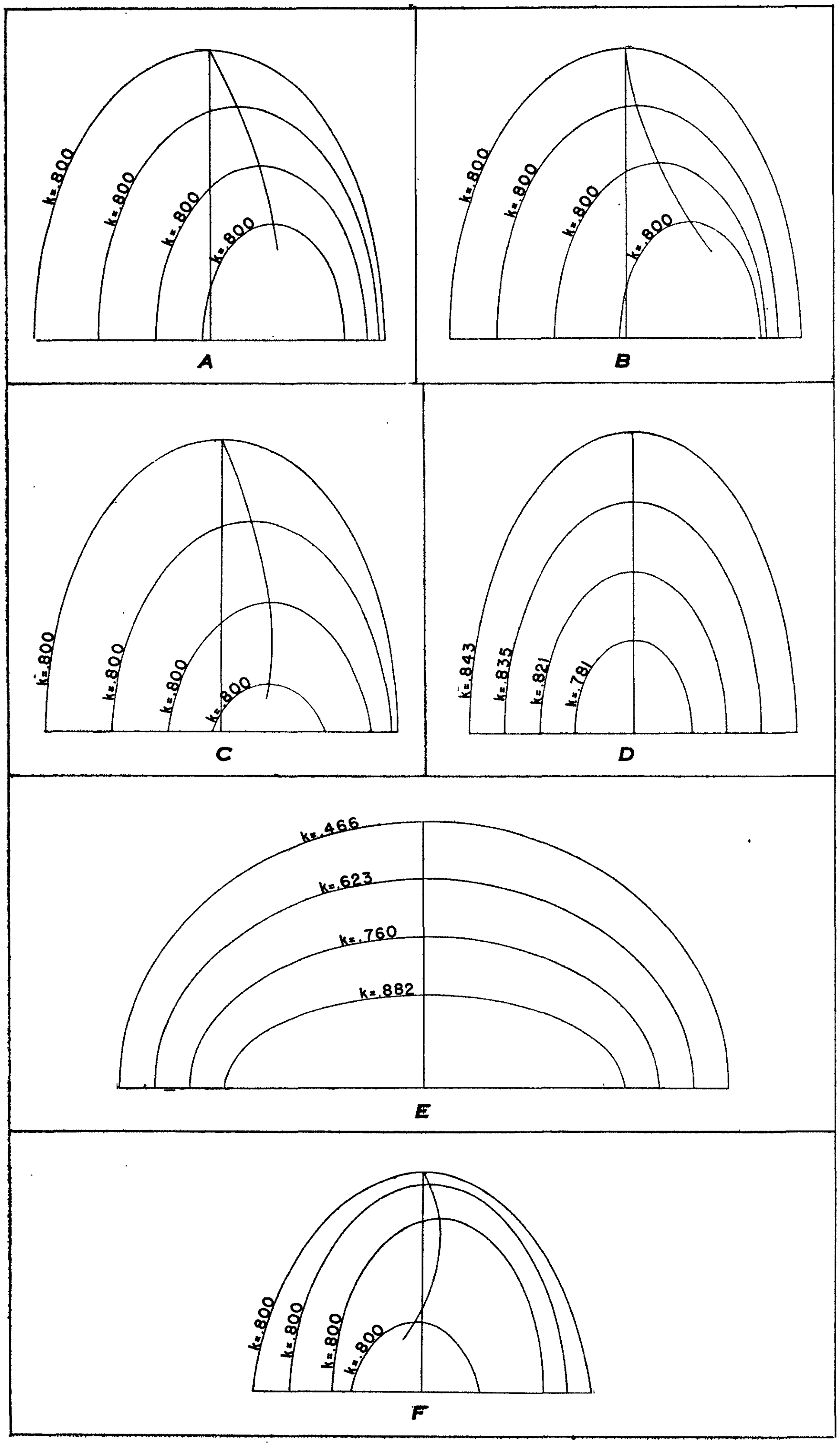
B, prolate similar elliptic arcs, with noncoaxial horizontal axes and a semiellipse next to the innermost curve. Represents the profile of a similar fold, in which the thickness of the strata changes differentially from the apex to the base.

C, prolate similar elliptic arcs, with noncoaxial horizontal axes and an outermost semiellipse. Represents a similar fold with dips of 90° only at the outer base of the fold. Strata thin proportionately from apex to base.

D, prolate similar elliptic arcs, with noncoaxial horizontal axes and outermost and innermost semiellipses. Represents the profile of a similar fold, in which the thickness of the strata changes differentially from the apex to the base.

E, oblate similar elliptic arcs, with noncoaxial horizontal axes, an innermost semiellipse, and all curves tangent to one another at one end of their minor semiaxes. Represents the profile of a similar fold, in which the strata pinch out entirely at the apex. This is the limiting form of a similar supratenuous fold.

F, prolate similar semiellipses, with noncoaxial vertical axes. Represents the profile of a similar fold, with dips of 90° everywhere along its base, and an inclined axial plane. Shows proportional thinning of the strata on the right side of the fold and proportional thickening on the left side. Configuration produced by proportional horizontal translation.


k=.800
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k=.843
k=.835
k=.821
k=.781
D
k=.466
k=.623
k=.760
k=.882
E
k=.800
k=.800
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F

The eccentricity of the elliptic traces of a similar fold is first selected. Thereafter change of scale is necessarily used, in order to illustrate the number and thickness of the strata. In most of the accompanying diagrams, the successive strata are shown as equal in thickness, in order to facilitate comparisons between different constructions, though in the reconstruction of actual folds, the strata will have different thicknesses. If the dips along a profile cannot be made to correspond with those indicated by the chosen elliptic arcs, a new set of arcs with a different eccentricity must be tried. If translation is not used, the construction requires coaxial semiellipses, though the complete semiellipses may not be used; but for either construction the resulting fold will be symmetrical to the vertical axis.

A construction using only changes in scale is first attempted. If the major axes of the semiellipses are vertical, such a fold will show strata that are uniformly thicker along the trace of the axial plane than at the base of the fold; but if they are horizontal, the reverse will be true. These conditions are illustrated, respectively, in figures 16*D* and *E*. Under this construction the complete disappearance of the strata, either at the apex or along the outer flanks of the fold, cannot be shown; but dips of 90° may be shown entirely across the base of the fold. Dips of 90°, however, are not necessarily present, as only parts of the semiellipses may be utilized; and where this is done, the dips that do appear at the base of the fold will depend partly upon the eccentricity of the ellipses and partly upon the lengths of the elliptic arcs that are used. This is exemplified in figure 16*F*, where only parts of semiellipses with an eccentricity of 0.600 are shown. This feature is likewise illustrated in the congruent similar curves of figure 16*C*.

Linear translation introduces the second variable in the representation of similar folds. Case I–A is first considered. The major axes of a set of similar noncongruent ellipses illustrated in figure 16*D* are used again in a vertical position in figure 17*C*; but each of the three inner curves has been translated vertically downward, in proportion to the original stratigraphic intervals, leaving the outer curve untranslated. Under this construction the thickness of each stratum is proportionately increased, but dips of 90° appear only along the outer flanks of the fold at its base. If, however, the three outer curves of figure 16*D* are translated proportionately downward, leaving the inner curve untranslated, the thickness of each stratum along the vertical axis will be proportionately decreased, as shown in figure 17*A*; or if desired, the strata may be made to disappear entirely at the apex of the fold, as shown in figure 17*E*. In figure 17*A* and *E*, dips of 90° may be shown only along the inner base of the fold. All constructions that produce proportional thickening or thinning along the vertical axis produce a nonproportional thinning of strata along the base of the fold, from the center outward to the flanks. Also, under the assumption of type I–A, no cross section of a similar fold with elliptic traces may show a dip of 90° along its base at any point intermediate between the center of the fold and its outer flanks.

Figures 17*A* and *C* portray opposite effects, and it therefore follows that a proper choice of stratigraphic intervals along the vertical axis can be made to match approximately those along the horizontal axis; this produces the traces of a pseudoparallel fold. The approximation to stratigraphic parallelism can be made closer by utilizing a slightly nonproportional spacing on the vertical axis, so that each stratigraphic interval on the vertical axis matches exactly its corresponding interval on the horizontal axis; but such a construction would introduce a slight nonuniformity in the stratigraphic spacing on both axes. The approximation to parallel folding may be made still closer by the use of cognate curves, as shown in figure 19*C*. Any of these three constructions would be difficult to discriminate from parallel folding, except by accurate measurement.

The third variable in the construction of similar folds is introduced by type I–B, where the linear trans-

FIGURE 18.—Prolate similar semiellipses and elliptic arcs and prolate cognate and oblate cognate semiellipses exemplifying certain types of structural features.

A, prolate similar semiellipses, with noncoaxial vertical axes. Represents the profile of a similar fold with dips of 90° everywhere along its base, and a curved axial surface Shows differential thickening and thinning of strata at base of fold. Configuration produced by differential horizontal translation.

B, prolate similar semiellipses with noncoaxial vertical axes. Represents the profile of a similar fold with dips of 90° everywhere along its base, and a curved axial surface in reverse of illustration *A*. Shows differential thickening and thinning of strata at base of fold. Configuration produced by differential horizontal translation.

C, prolate similar elliptic arcs with noncoaxial vertical and noncoaxial horizontal axes and an outermost semiellipse. Represents a curved axial surface and the profile of a similar fold with dips of 90° only at the outer base of the fold. Shows differential thickening and thinning of strata at base of fold. Configuration produced by proportional vertical and differential horizontal translation.

D, prolate cognate semiellipses. Represents the profile of a cognate fold, with dips of 90° everywhere along its base. Shows proportional thinning of strata from apex to base of fold.

E, oblate cognate semiellipses. Represents the profile of a cognate fold, with dips of 90° everywhere along its base. Shows proportional thinning of strata from apex to base of fold.

F, a semiellipse next to the innermost curve and prolate similar elliptic arcs with noncoaxial vertical and noncoaxial horizontal axes. Represents a strongly curved axial surface and the profile of a similar fold with limited basal dips of 90°. Shows differential thickening and thinning of strata at base of fold. Configuration produced by differential vertical and differential horizontal translation.

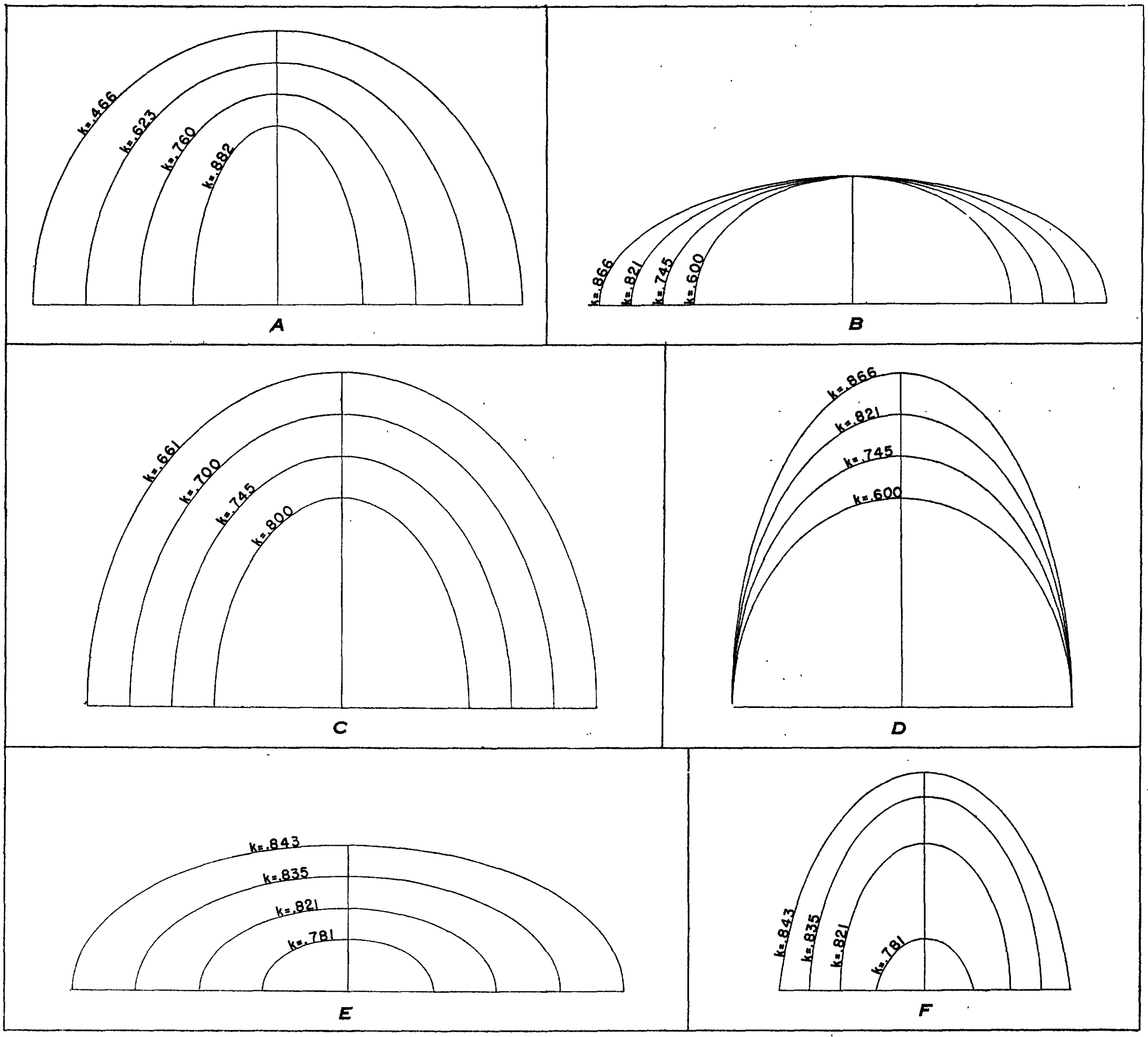

FIGURE 19.—Prolate cognate and oblate cognate semiellipses and prolate cognate elliptic arcs exemplifying certain types of structural features.

A, prolate cognate semiellipses. Represents the profile of a cognate fold, with dips of 90° everywhere along its base. Shows proportional thickening of strata from apex to base of fold.

B, oblate cognate semiellipses that are tangent to one another at one end of their minor semiaxes. Represents the profile of a cognate fold, in which the strata pinch out entirely at the apex. This is the limiting form of a cognate supratenuous fold.

C, prolate cognate semiellipses with equal increments of length for the major and minor semiaxis. Represents the profile of a cognate fold that simulates a parallel fold.

D, prolate cognate semiellipses that are tangent to one another at both ends of their minor semiaxes. Represents the profile of a cognate fold, in which the strata pinch out entirely at the base on both flanks.

E, oblate cognate semiellipses. Represents the profile of a cognate fold, with dips of 90° everywhere along its base. Shows proportional thickening of strata from apex to base of fold.

F, prolate cognate elliptic arcs with noncoaxial horizontal axes, and a semiellipse next to the innermost curve. Represents the profile of a cognate fold with limited basal dips of 90°. Shows differential changes in thickness of strata from apex to base of fold. Configuration produced by differential vertical translation.

lation is nonproportional. Vertical differential translation makes it possible to show dips of 90° along the base of a fold, either on the outer or inner stratigraphic surfaces, on both the outer and inner surfaces, on one or more of the intermediate surfaces, or on none of these surfaces. The only limitation is that dips of 90° may not be shown entirely across the base of a fold. In these various delineations the thickness of the strata may increase along the vertical axis downward, upward, or in some erratic manner that may be required to simulate a stratigraphic sequence. Figure 17*B* illustrates a set of stratigraphic traces in which only the surface next to the innermost one shows a dip of 90° at the base of the fold, and the thickness of the strata increase differentially downward. Figure 17*D* illustrates a fold in which only the outer and inner stratigraphic surfaces show dips of 90° at the base of the fold, and the stratigraphic intervals decrease nonproportionately downward.

Proportional horizontal translation, as in type II–A, results in additional effects; and these are further diversified in type II–B, where the horizontal translation is differential. Proportional horizontal translation is illustrated in figure 17*F*, which is derived from figure 16*D* merely by moving the four semiellipses laterally by amounts proportional to the original stratigraphic separations along the horizontal axis. This movement produces an inclined axial plane, whose trace appears in the figure; but this position of the axial plane, with regard to the stratigraphic traces, is different from what would have been produced if figure 16*D* had merely been tilted. Stratigraphic asymmetry has been introduced; and dips of 90° are also preserved everywhere across the base of the fold. By still further horizontal translation, all the strata can be made to thin uniformly to zero, or in other words to disappear completely on one outer flank of the fold.

Nonproportional, or differential, horizontal translation of similar ellipses is shown in figure 18*A* and *B*. These traces are produced from figure 16*D* by a differential lateral movement of the semiellipses, done in such a manner that the strata on the right flank are thinned respectively by negative and positive increments. Both figures show the traces of curved axial surfaces, but these traces are reversed in their general inclinations. The delineation of dips of 90° across the base of the fold remains unchanged. Figure 16*E*, where the minor axes of the semiellipses are vertically placed, could likewise be transformed to show either a tilted axial plane or a curved axial surface. Further variations could be produced by operating in the same way on figure 17*A–D*.

Combinations of vertical and horizontal translations constitute type III; but as these displacements may be proportional, differential, or both, several major combinations may exist, with numerous minor variations depending upon the vertical or horizontal placement of the axes of the semiellipses, the preexisting thickening or thinning of stratigraphic intervals, the directions and amounts of the dips, and other factors. The principal combinations are tabulated as types III–C–1, III–C–2, III–C–3, and III–C–4. Only two illustrations are given. Figure 17*C*, which shows proportional vertical translation, has been transformed by differential horizontal translation, to produce the fold shown in figure 18*C*. Similarly figure 17*B*, which shows one type of differential vertical translation, has been transformed by differential horizontal translation to produce the fold shown in figure 18*F*. Folds that are represented by figure 17*C*, where only parts of the semiellipses are used, can likewise be transformed by vertical, horizontal, or both vertical and horizontal translations, either proportionately, differentially, or both proportionately and differentially. Thus it appears that with linear translations the number of similar folds that may be constructed is very large; and no general statements are therefore warranted regarding all the effects that may be produced.

COGNATE FOLDS

Cylindrical cognate folds comprise nonparallel stratigraphic surfaces whose traces, in a plane normal to the axial line, are cognate curves. Three variables may be used in the delineation of cognate folds by the use of ellipses; first, a changing eccentricity in each of the stratigraphic surfaces; second, linear translation of the traces parallel either to the vertical or the horizontal axis, or to both; and third, the choice between proportional and differential linear translation. Change of scale and congruency are inadmissible, as either of these conditions would produce similar traces and folds. Variable eccentricity, however, is so effective in producing a variety of folds that it eliminates in large measure the need for translation parallel to the vertical axis. Probably more cognate than similar cylindrical folds occur in nature. A simple test, regardless of the nature of the curvature, is to determine whether all the traces of the stratigraphic surfaces, in a specified section, can be made to superpose on one another, with or without change of scale, followed by linear translation. If this is possible, the traces are similar; if not, they are probably cognate, though the possibility remains that they may be composite.

Thickening, thinning, or the complete disappearance of strata, either at the apex or along the flanks of a symmetrical fold, may readily be shown in cross section by the use of cognate curves, whereas the complete disappearance of strata cannot be shown along both flanks of a symmetrical fold by the use of similar curves. Moreover, the degree of thickening or thinning can be made to vary between much wider limits than for similar folds. Dips of 90° may be shown concurrently with these effects, if desired, entirely across the base of a cognate fold. Finally, the eccentricities may be chosen to produce stratigraphic traces which resemble parallel curves so closely that the difference is determinable only

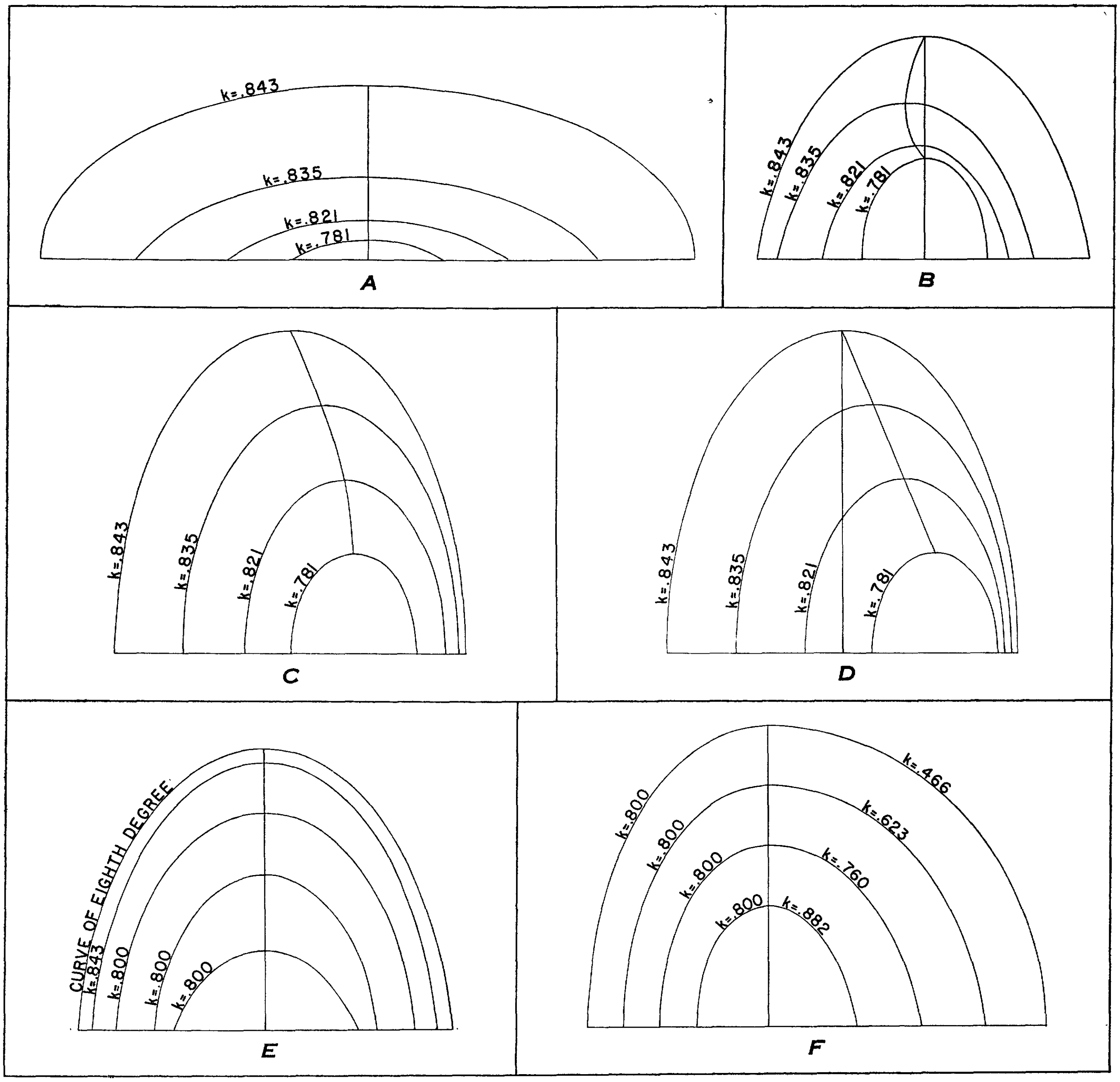

FIGURE 20.—Prolate cognate semiellipses and elliptic arcs and oblate cognate elliptic arcs exemplifying certain types of structural features.

A, oblate cognate elliptic arcs with noncoaxial horizontal axes, and an outermost semiellipse. Represents the profile of a cognate fold, with dips of 90° only at the outer base of the fold. Shows differential changes in the thickness of strata from apex to base of fold. Configuration produced by differential vertical translation.

B, prolate cognate elliptic arcs with noncoaxial vertical and noncoaxial horizontal axes and an innermost semiellipse. Represents the profile of a cognate fold with a strongly curved axial surface and dips of 90° only at the inner base of the fold. Shows differential thickening and thinning of strata throughout the fold. Configuration produced by differential vertical and differential horizontal translation.

C, prolate cognate semiellipses with noncoaxial vertical axes. Represents the profile of a cognate fold with a curved axial surface and dips of 90° everywhere along its base. Shows differential thickening and thinning of strata along base of fold. Configuration produced by differential horizontal translation.

D, prolate cognate semiellipses with noncoaxial vertical axes. Represents the profile of a cognate fold with dips of 90° everywhere along its base, and an inclined axial plane. Shows proportional thinning of strata on right side of fold and proportional thickening on left side. Configuration produced by proportional horizontal translation.

E, four prolate elliptic arcs bounded by a curve of the eighth degree. The three inner curves are similar; the third and fourth are cognate; and the fourth and fifth are parallel. Represents the profile of a composite fold of mixed but related stratigraphic surfaces.

F, one set of prolate similar quarter ellipses joined to a set of prolate cognate elliptic arcs. Represents the profile of a hybrid composite fold, with dips of 90° along the base of the left half of the fold.

by close measurement. Horizontal translation, or a combination of horizontal and vertical translations, however, are needed to represent folds with curved axial surfaces. Only a few of the unlimited types of cognate folds will be illustrated.

Thickening of strata at the apices of cognate folds is illustrated in figure 18*D* and *E*, and thinning at the apices, in figure 19*A* and *E*. The arcs of figure 18*D* and 19*E* are identical, but one set is rotated 90° with respect to the other. The same is true of figures 18*E* and 19*A*. Disappearance of strata at the apex and along the outer flanks of a fold are shown, respectively, in figures 19*B* and 19*D*. The simulation of parallel folding is accomplished in figure 19*C* by utilizing equal increments of the major and minor semiaxes of the ellipses. Thus the four curves shown in this figure are parallel to one another at the ends of the elliptic axes, but not elsewhere, so that the deviation from complete parallelism is scarcely noticeable. Figures 18*D*, *E*, and 19*A–D* show dips of 90° across the entire base of the folds. This type of folding may be considered unrealistic; but if not applicable, dips of moderate degree may be shown at the base of the fold by utilizing only parts of the semiellipses, as shown in figure 17*C*, or by using vertical linear translation, either proportional or differential.

It has been shown that strata may thin or disappear completely at the apices of both similar and cognate folds that they may be made to disappear along one outer flank of a similar fold by horizontal translation, and that they may be caused to disappear along both outer flanks of a cognate fold without recourse to translation. Where the thinning or disappearance occurs at the apex of a fold, the term "supratenuous fold" has been applied by Nevin (1942). Hills (1953), however, has pointed out that supratenuous folds may originate from several causes, so that they cannot be regarded as a distinct genetic type. Such flexures may be simulated as similar folds, by the use of linear vertical translation; or as cognate folds, by variable eccentricity, without or with such translation. Supratenuous folds are therefore not a distinct geometric type but instead are merely a subspecies of similar or of cognate folds.

Linear translation, vertical, horizontal, or both, and either proportional or differential, lead to an unlimited number of cognate structures. Added to these variables is also the use of partial instead of entire semiellipses. Figure 19*F*, for example, is constructed from figure 18*D*, by the use of differential vertical translation, accompanied by the utilization of 1 semiellipse and 3 partial semiellipses. A dip of 90° is thus depicted only for the trace whose eccentricity is 0.821. Figure 20*A*, on the other hand, is constructed from figure 19*E* by the same method, but the strata in this figure thicken upward instead of downward, and a dip of 90° at the flanks is shown only for the outer trace, whose eccentricity is 0.843.

Figures 20*B*, *C*, and *D* are produced from figure 18*D*. Figure 20*D*, by proportional horizontal translation, shows proportional thinning on one flank of a fold, and proportional thickening on the other. Figure 20*C*, produced by differential horizontal translation, shows differential thinning on one flank of a fold and differential thickening on the other. Figure 20*D* has an axial plane; figure 20*C* shows a curved axial surface: and both are drawn to show dips of 90° entirely across the base of the fold. Figure 20*B* is produced by a combination of differential vertical and differential horizontal translation, and the use of partial semielliptic arcs, with the development of a sharply curved axial surface, and the delineation of a dip of 90° only on the inner stratigraphic trace. Innumerable other constructions may be made.

COMPOSITE FOLDS

Three principal types of composite cylindrical folds are recognized, each of which may be modified by the application of vertical, horizontal, or combined vertical and horizontal translations; and such translation may either be proportional or differential. Horizontal translation results in tilted axial planes or curved axial surfaces. Moreover, the first and third types may be modified extensively by the methods utilized to produce the second type. The subject is thus a large one that cannot be treated exhaustively; but the permissible and practical types will occur to field geologists in the investigation of particular folds.

The first type comprises folds in which the traces of the stratigraphic surfaces, in a plane normal to the axial line, are various combinations of parallel, similar, and cognate curves. The possible combinations may be parallel and similar, parallel and cognate, similar and cognate, or parallel, similar, and cognate. An example of the latter type is shown in figure 20*E*. The two lowest traces in this diagram are congruent; the second and third, moving upward in the figure, are noncongruent but similar; the third and fourth are cognate; and the fourth and fifth are parallel, the fifth being a curve of the eighth degree parallel to the ellipse whose eccentricity is 0.843. None of the first four curves is a complete semiellipse, and therefore no dips of 90° are shown at the base of the fold.

Hybrid stratigraphic traces may produce many types of composite folds, all of which will be unsymmetrical. Hybridization is produced by joining two quarter ellipses at the ends of one or the other of their semiaxis or at other points where their tangents are parallel. This construction may be achieved by either of two general methods, of which one does not require rotation of the quarter ellipses, whereas the other requires the rotation of one set of quarter ellipses through an angle of 90° with regard to the other set. Numerous modifications of each of these two subspecies are possible.

Some of the cross sections of folds that may be produced without rotation are now mentioned. Consider the traces of any of the folds heretofore illustrated by semiellipses or parts thereof. Bifurcate these traces along the vertical or near-vertical axis of the ellipses, and by translation parallel to this axis move one set of quarter ellipses upward or downward with respect to the other, joining the stratigraphic traces as required. Hybrid curves will result that no longer have elliptical curvature but instead are higher plane curves. Hybrid curves generated in this way are such that the quarter-elliptical traces remain parallel, similar, cognate, or of whatever type they originally were, so that each half of the fold retains its original classification. The whole fold, however, belongs to none of these original types but is instead a hybridized variety of a composite fold.

Hybridized composite folds produced without rotation may be further generalized. Thus, a set of quarter ellipses of one kind may be joined to a similarly placed set of quarter ellipses of another kind, along their vertical or near-vertical common axis. Such a fold is illustrated in figure 20*F*, where a set of similar quarter ellipses are joined to a set of cognate quarter ellipses. The resulting cross section resembles somewhat those shown in figures 17*F* and 20*D*, but the axis is vertical instead of inclined. Figure 20*F* however, may be further modified, if desired, by vertical translation, either proportional or differential. After the composite fold has been constructed, it may be still further modified by additional vertical translation, by horizontal translation, or by both, in the manner shown in figures 17*F*; 18*A*, *B*, *C*, *F*; and 20*B*–*D*.

Rotation of the quarter ellipses adds another variable that greatly diversifies the types of hybridized composite folds, but in general the effect is to produce folds of marked asymmetry. The traces that are transformed may be either identical or different, and their junction may be made either at the ends of the major or minor axes or at other points where their tangents are parallel. Consider the similar fold illustrated in figure 16*D*. If two sets of quarter ellipses are suitably revolved, translated, and connected, they can be so arranged as to produce the asymmetrical hybrid fold illustrated in figure 21*A*. Figure 21*B* shows two sets of similar quarter ellipses of different eccentricity that have been revolved, translated, and joined in the same way. In this construction, however, the quarter ellipses have been translated proportionately, both vertically and horizontally, to produce a tilted axial plane.

The traces of cognate folds may also be hybridized by rotation, with or without translation, to produce composite folds. Thus in figure 21*C*, 4 traces of eccentricities 0.484, 0.533, 0.600, and 0.700 have been combined with another set of revolved traces having eccentricities of 0.843, 0.835, 0.821, and 0.781. Proportional horizontal translation has also been used to produce the inclined axial plane whose trace is shown in the figure. Differential horizontal translation of the same curves results in a fold with a curved axial surface, as illustrated in figure 21*D*. No illustration, except figure 16 *C*, is presented of cognate or similar traces that are joined at points other than the ends of the major and minor axes, though this procedure is altogether feasible.

A third type of composite folds has a different origin. Cognate folds have been required by definition to possess the same equation, with parametric variations. The different strata in a fold have been deformed by the same forces, and it is therefore probable that successive traces are analytically related. Yet it may be possible to have stratigraphic surfaces that are analytically unrelated. Thus the traces of the surfaces of a cylindrical fold, in a plane normal to the axial line, might conceivably be arcs of an ellipse, a parabolic catenary, a cassinian oval, and a sine curve. Such a possibility needs to be observed and mapped before folds of this type can be listed as proved species. If existent, it is obvious that related types can be produced by the different methods of translation.

SUMMARY

Four genera of cylindrical folds have been discussed, with species that depend primarily upon whether they have axial planes or curved axial surfaces. Parallel folds are determined by the test of constant, or sensibly constant, stratigraphic thickness of each bed; and until this test has been applied, they should be interpreted as similar or cognate. From 19*C* illustrates a pseudoparallel fold that is really cognate. Likewise, similar folds should be regarded as cognate until the test has been applied of reproducibility of the stratigraphic traces by nondistortional enlargement or reduction, including congruency. Composite folds have been classified into three subgenera, which indeed might be further expanded; and each of these may be divided into species, subspecies, and varieties. The term "composite folds," as defined above, is broad enough to include

all simple cylindrical folds in nature that may not be classified as parallel, similar, or cognate, or approximately so. Many geologists will object that adequate data are not ordinarily available for utilization of this classification. This is true; but the classification may stimulate the acquisition of the necessary data.

The traces of nonparallel cylindrical folds, in a plane normal to the axial line, are not claimed to have elliptical curvatures; but ellipses are the simplest figures that can be used to match approximately the stratigraphic curvatures and tangents thereto that exist in nature. One cannot ordinarily measure the curvature of stratigraphic surfaces or traces thereof; but strikes and dips can be translated into curvature and approximately into elliptic curvature. Stratigraphic thickness, or other stratigraphic measurements in folded beds are most commonly made indirectly, either numerically or graphically, but whatever method is used must be based upon some assumption regarding stratigraphic curvature. The theory of evolute and involutes provides a perfect assumption for parallel folds; but the reconstruction of nonparallel folds, unfortunately, cannot be based upon such a unique theorem. Elliptic arcs, however, will provide a satisfactory first approximation to the true curvature, and this permits a logical and at the same time conventional method for the reconstruction of folds and for making consistent stratigraphic measurements. Nonparallel folds have not yet been studied from an analytical or even a geometrical point of view, and such studies must precede a proper understanding of the genesis of folding. No genetic interpretations are therefore associated with the geometrical classification of folds presented in this paper. The mechanics of folding awaits an adequate analysis of folds.

NONCYLINDRICAL FOLDS

Noncylindrical folds have been defined to include quaquaversal folds, elongate domes or canoe-shaped folds, and the plunging ends of cylindrical folds. Canoe-shaped folds and plunging folds have been shown to have apical lines that are analogous to the axial lines of cylindrical folds, but these lines may be either plane or space curves. A canoe-shaped fold has major and minor sections, known as principal sections, that may be either plane or curved surfaces. The minor section of a canoe-shaped fold is analogous to the profile of a cylindrical fold, normal to the axial line. The terms "axial plane" and "axial surface" are not applied to canoe-shaped folds. The medial surface of a plunging fold, however, may possibly be designated as an axial plane or axial surface, if continuity exists with such surfaces in a cylindrical fold.

Quaquaversal and canoe-shaped folds comprise the flexures under present discussion. The maximum curvature of a canoe-shaped fold will generally appear in its minor section, and the complimentary minimum curvature in its major section. The structure of an elongate dome must therefore be determined in two principal sections normal to one another, whereas the structure of a cylindrical fold can be understood from numerous cross sections parallel to one another. Such sets of orthogonal profiles render necessary a three-dimensional evaluation of canoe-shaped folds.

A group of surfaces, analogous to cylinders, is needed to describe and illustrate empirically the stratigraphic surfaces of domed structures. Three of the 5 fundamental quadric surfaces, together with 3 surfaces of revolution, are suitable for this purpose. These, together with their principal Cartesian sections, are given in the following tabulation:

Surfaces	*Principal sections*
1. Ellipsoid of revolution (spheroid)	Circle; unlimited congruent ellipses.
2. Paraboloid of revolution	Circle; unlimited congruent parabolas.
3. Biparted hyperboloid of revolution	Circle; unlimited congruent hyperbolas.
4. Ellipsoid (triaxial)	Three ellipses.
5. Elliptic paraboloid	Ellipse; two parabolas.
6. Biparted hyperboloid	Ellipse; two hyperbolas.

One principal section for each quadric surface will have to lie in a horizontal or nearly horizontal plane; and it therefore follows that surfaces 1, 2, and 3, if their circular sections were made horizontal, could be used advantageously to represent quaquaversal folds. It is equally obvious that surfaces 4, 5, and 6, if an elliptic section were placed in a horizontal position, could be used to represent elongate or canoe-shaped folds. Reasons have been given, however, why elliptic sections are preferred in the delineation of the stratigraphic traces of cylindrical folds. If this limitation is extended to domes, surfaces 1 and 4 appear to be the most suitable, the first for quaquaversal folds and the fourth for canoe-shaped folds. In the delineation of domes it is commonly assumed that the applied force acted vertically upward; but if the applied force acted obliquely upward, or if its direction of application changed during the formation of the dome, or if the dome was tilted during or after its formation, the principal sections will be either inclined planes or curved surfaces.

Domed surfaces with a circular horizontal outline may be generated as surfaces of revolution, but canoe-shaped surfaces may be produced only by more involved modes of graphical or mechanical assembly. All sur-

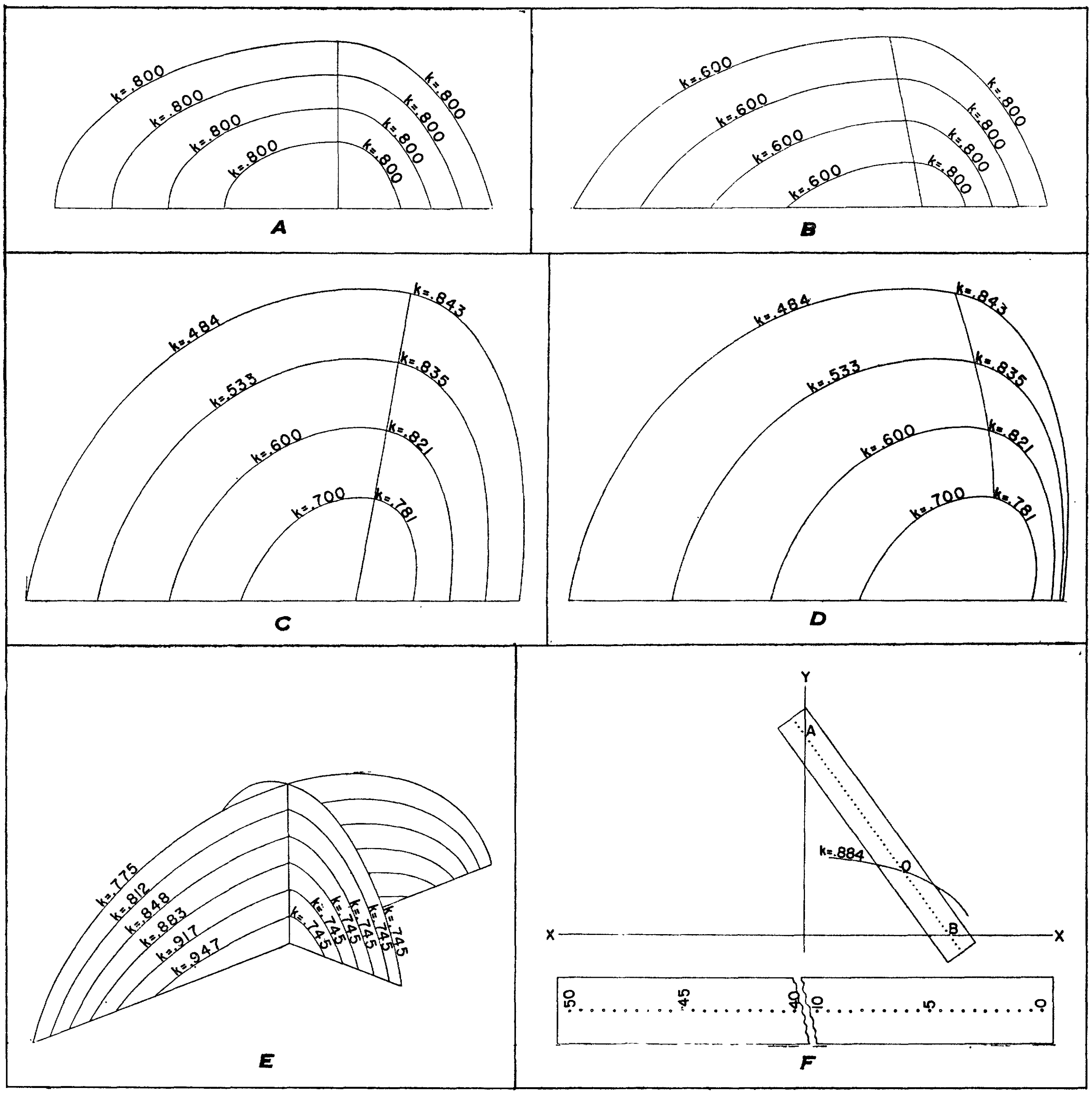

FIGURE 21.—Oblate similar quarter ellipses, oblate similar and prolate cognate elliptic arcs, sections of a composite elongate dome, and application of trammel of Archimedes in construction of ellipses.

A, one set of oblate similar quarter ellipses joined to a set of prolate similar elliptic arcs of the same eccentricity. Represents the profile of a hybrid composite fold with dips of 90° along the base of the left half of the fold.

B, one set of oblate similar elliptic arcs joined to a set of prolate similar elliptic arcs. Represents the profile of a hybrid composite fold with an inclined axial plane. Configuration produced by proportional horizontal translation.

C, one set of prolate cognate elliptic arcs joined to another set of prolate cognate elliptic arcs. Represents the profile of a hybrid composite fold with an inclined axial plane. Configuration produced by proportional horizontal translation.

D, one set of prolate cognate elliptic arcs joined to another set of prolate cognate elliptic arcs. Represents the profile of a hybrid composite fold with a curved axial surface. Configuration produced by differential horizontal translation.

E, two principal sections of a composite elongate dome, drawn in perspective. The major section consists of oblate cognate elliptic arcs; the minor section consists of prolate similar elliptic arcs.

F, application of the trammel of Archimedes in projecting as a glissette an oblate ellipse with an eccentricity of $k=0.884$. The semidiameters are $a=30$ and $b=14$.

faces of revolution, regardless of whether the generating curves are parallel, similar, cognate, or composite, show an unlimited number of plane sections that intersect in the axis of revolution. Thus a dome with an equidimensional horizontal outline may be represented approximately as a set of prolate or oblate surfaces, of which at least one may be constructed as a spheroid; the shapes of the others will depend upon the nature of the stratigraphic traces. If the dome is elongate, the best choice for the first stratigraphic surface is an ellipsoid, bounded by other surfaces that may or may not be ellipsoidal.

The exact configuration of domes is not definitely known. If they were thought to have the shapes of quadric and related surfaces, noncylindrical folds could be divided primarily into subclasses according to the preceding tabulation. But quadrics are simpler surfaces than cylinders, as their sections yield curves of the second degree, whereas the sections of cylinders may show higher plane curves. True cylindrical folds are therefore more likely to occur in nature than are quadric folds; and in any classification the concept of cylindrical folding will withstand the acquisition of new knowledge, whereas the idea of quadric folding probably will not. Hence, it seems best, until more information is available, to subdivide noncylindrical folds primarily upon the nature of their stratigraphic surfaces in principal sections and to utilize the concept of quadric folding only secondarily for descriptive purposes.

The existence of two principal sections in domes introduces a complication not present in cylindrical folds. Horizontal translation, or combinations of vertical and horizontal translations, of the curves that represent stratigraphic traces in cylindrical folds have been shown to produce inclined axial planes and curved axial surfaces. But canoe-shaped folds have two principal or orthogonal sections, and quaquaversal folds have an unlimited number. Hence, vertical and horizontal translations of similar, cognate, and composite curves can be applied independently along two sections normal to each other. The translations, moreover, need not be in the same sense, or of the same magnitude; and they may thus generate not merely curved but warped or twisted sections lying between the principal sections. This effect can apply both to canoe-shaped and to quaquaversal folds, producing marked asymmetry in such structures.

Asymmetry may also be produced in another way. Transverse and longitudinal sections of a canoe-shaped fold, or two orthogonal sections in a quaquaversal fold, are not necessarily related in their general geometric configurations. Thus one section may show similar stratigraphic traces, and the other may show cognate or composite traces. By using the three types of composite surfaces, it appears that at least 10 such general combinations are possible. Twisted as well as simply curved profiles may thus result in sections lying between the principal or orthogonal sections. Such flexures would really be composite domes, because mixed stratigraphic traces are present in orthogonal sections. A simple type of canoe-shaped fold with a vertical axis, showing similar curvature in one principal section and cognate curvature in the other, is illustrated in figure 21*E*.

Domes may thus include many kinds of unsymmetrical domed surfaces; for if n types of curvature or translation are possible in one principal section and n types in another, it follows that n^2 types of composite curvature may characterize such folds as a whole. No attempt is made in this paper to enumerate and describe these many species of domes. Instead, related types of curvature are utilized in both principal sections; and translation, if any, is conceived to be applied equally. It is desirable also to avoid complex figures, for which reason the principal sections of all domes are interpreted as plane profiles.

PARALLEL DOMED SURFACES

Parallel domed surfaces may be constructed by either of two general methods. Two orthogonal sections of a quaquaversal fold or the two principal sections of a canoe-shaped fold may be drawn as involutes, generated from evolutes derived from the dips along these sections. This is the most realistic method, if the geological data are available, as it involves no assumption regarding elliptic or any other curvature for the stratigraphic traces.

The second method requires that one stratigraphic trace be fitted to an elliptic arc, after which parallel traces are drawn. No two ellipses can be parallel, and it therefore is impossible to have a domed structure with parallel spheroidal or ellipsoidal surfaces. But just as it is possible to have an ellipse bordered by a set of curves that are parallel to the ellipse and to one another, so it is possible to have a spheroid or an ellipsoid bounded by surfaces that are parallel to the given surface and to one another. Surfaces parallel to the other conicoids could likewise be constructed; and in fact, stratigraphic traces may be drawn parallel to any continuous surface that itself meets the stratigraphic requirements.

Surfaces parallel to a semispheriod are easily generated as surfaces of revolution. Thus if figure 16*A* is rotated 180°, either about its vertical or its horizontal axis, parallel surfaces will be generated. With the first construction, a prolate parallel quaquaversal fold will be simulated; but from oblate parallel curves, oblate quaquaversal folds may also be produced. Surfaces of revolution may likewise be generated parallel to a paraboloid of revolution or to a hyperboloid of revolution; and more generally any algebraic or transcendental surface of revolution could be used as the original stratigraphic surface.

Surfaces parallel to a semiellipsoid cannot be produced as surfaces of revolution but are conceived to be generated as follows. If normals to the ellipsoid are constructed at many closely adjacent points and are extended both inward and outward from the ellipsoidal surface, a set of nonparallel lines will result, resembling penetrating quills. If fixed distances are laid off equally on all these normals, both inside and outside the ellipsoid, the resulting points will define the loci of a set of surfaces parallel to the ellipsoid. The inner surfaces may or may not qualify as stratigraphic surfaces, as shown in figure 16*B;* but all surfaces of this kind generated outside the ellipsoid, and some of those inside the ellipsoid, may be used as parallel stratigraphic surfaces to illustrate parallel canoe-shaped folds. By the same method, surfaces may be constructed parallel to an elliptic paraboloid, a biparted hyperboloid, or any other continuous algebraic or transcendental surface.

Parallel domed surfaces, either quaquaversal or canoe-shaped, necessarily have plane orthogonal or principal sections. Such profiles, however, may be inclined for either of two reasons. First, the direction of the force applied to produce the dome may have been inclined to the vertical; and second, the dome may have been tilted subsequent to its formation. Any variation in the direction of the applied force during its period of operation, or any differential in the direction of later tilting, would probably have destroyed the parallelism of the stratigraphic surfaces. Parallel domed surfaces therefore have at least a negative significance in the mechanics of folding.

SIMILAR DOMED SURFACES

Similar surfaces are defined as those which have a point-to-point correspondence such that the distance between any two points on one surface is invariably the same multiple of the distance between two corresponding points on the second surface. Also the areas of similar surfaces are to each other as the squares of the corresponding distances between them. Similar spheroids and ellipsoids conform with these general requirements.

Three variables were shown to apply in the delineation of cylindrical similar folds; and these apply equally well in the construction of quaquaversal and canoe-shaped similar folds. The effects are thickening, thinning, or the disappearance of strata at the apex of a fold, and a thickening or thinning of strata along the outer flanks; differential thickening or thinning of strata along the axial plane or at the base of the fold; the production of asymmetry, accompanied either by proportional or differential thickening or thinning of strata in one-half of the fold; and the generation of tilted axial planes or curved axial surfaces.

Similar domed surfaces may be constructed as spheroidal surfaces of revolution or as assemblies of ellipsoids. Any of the sets of symmetrical similar curves shown in figures 16*D–F* and 17*A–D* could be revolved 180° about their vertical axes to produce either prolate or oblate quaquaversal structures, depending upon the character of the original profiles. They may be rotated about their horizontal axes only if dips of 90° are shown everywhere across the base of the fold, for otherwise the resulting surfaces would show discontinuities. But such sets of surfaces would have one principal section showing circular stratigraphic traces, which are inadmissible. The same geometric difficulty applies also to parallel, cognate, and composite curves that might otherwise be revolved about their horizontal axes. Thus the similar curves shown in figures 16*D*, 16*E*, 17*F*, 18*A*, and 18*B*, and the parallel curves of figure 16*A*, should not be revolved about their horizontal axes to illustrate domal structures. Figures 17*F*, 18*A*, *B*, *C*, and *F* cannot be revolved about any vertical lines.

Similar quaquaversal folds may also comprise sets of similar paraboloids of revolution or similar hyperboloids of revolution. And in general, similar quaquaversal folds may be produced from sets of any similar algebraic or transcendental surfaces that can be generated as surfaces of revolution. All the similar surfaces of revolution heretofore mentioned are symmetrical in 1 or 2 orthogonal sections; but completely asymmetrical folds may be produced by various combinations of vertical and horizontal translations of these surfaces.

Elongate similar domes are not constructed as surfaces of revolution. Similar ellipsoids, for example, are ellipsoids whose principal sections are similar ellipses; that is, ellipses with the same or proportional semiaxes. If the semiaxes are identical, the ellipsoids are congruent. But an ellipsoid whose semiaxes are 15, 12, and 9 is similar to another ellipsoid with semiaxes of 10, 8, and 6. A dome consisting of an assembly of ellipsoidal surfaces conforming to this restriction illustrates an elongate similar fold; and such a fold may be constructed either as a prolate or an oblate fold. Commonly the longest and median axes of canoe-shaped folds lie approximately in the plane of the horizon; but this is not necessarily so, as the axes of greatest and least lengths may occupy this position.

Elongate similar folds may likewise be constructed from similar elliptic paraboloids, similar biparted hyperboloids, or in general from many sets of similar surfaces of degree higher than the second. The principal sections of all these similar canoe-shaped folds may be plane or curved, depending upon the deviation from coaxiality of the sets of surfaces that are utilized.

COGNATE DOMED SURFACES

Cognate curves have been shown to produce a greater variety of cylindrical folds than similar curves, owing principally to the permissible variation in the eccentricity of each stratigraphic trace. Cognate curves are also useful in permitting dips of 90° in many more folds than is possible with similar curves; and in addition it is possible to show the complete disappearance of strata along both outer flanks of a fold. All these effects can likewise be produced in cognate domed surfaces.

Cognate quaquaversal domes may be constructed as sets of spheroidal surfaces of revolution with cognate ellipses as the generating elements. Figures 18*D*, *E*; 19*A*, *C*, *E*, *F*; and 20*A*, for example, may be rotated about their vertical axes to simulate domed surfaces of revolution. But in order to avoid circular stratigraphic traces, figures 18*D*, *E*, and 19*A*, *C*, *E* should not be revolved about their horizontal axes. Figures 20*B*, *C*, *D* illustrate unsymmetrical curves that cannot be revolved about any vertical lines.

Cognate quaquaversal surfaces of revolution may also comprise cognate hyperboloids of revolution, but not paraboloids of revolution, as the generating elements of the latter are parabolas, which are similar curves. Cognate quaquaversal or canoe-shaped folds may also comprise sets of algebraic or transcendental surfaces, represented by equations that differ only in the values of their constant parameters. A family of catenoids will illustrate this possibility in the generation of a quaquaversal fold. Parenthetically, the catenoids are the only minimal surfaces of revolution, for which reason they may have some structural significance if they are found to exist in quaquaversal folds.

Elongate cognate domes may be constructed by assembling similarly placed cognate ellipsoids, which may or may not be coaxial, depending on whether or not linear translation is utilized. The simplest construction is the assembly of coaxial cognate ellipsoids. Four such ellipsoids, for example, might have semiaxes 16, 12, and 8; 13, 10, and 7; 10, 8, and 6; and 7, 6, and 5. The eccentricities of the elliptic arcs in all the principal sections of such surfaces would be different; and in fact, the same would be true of their horizontal sections. The halves of these ellipsoids, oriented with the major axes either in a vertical or in a horizontal position, will constitute two sets of cognate stratigraphic surfaces, illustrating respectively a prolate and an oblate canoe-shaped cognate fold.

Elongate cognate domes could likewise be represented by an assembly of cognate biparted hyperboloids; and these might also be constructed from numerous sets of related surfaces of higher degree. All cognate domes produced with or without rotation, in the manner outlined above, could be greatly modified by different combinations of vertical and horizontal translation, to produce many types of unsymmetrical folds. It is clear that the number of quaquaversal and canoe-shaped cognate folds that are possible is very large.

COMPOSITE DOMED SURFACES

Composite domes, like composite cylindrical folds, have at least three types of surfaces, which are classified according to the character of the stratigraphic traces. Some of the curves that simulate these traces are amenable to rotation, so that the resulting figures may be used to illustrate specialized types of domes. An example is figure 20*E*, which can be revolved about its vertical axis to produce a prolate composite quaquaversal dome, though it cannot be revolved about the horizontal line at its base. A difference in the original profile could have resulted in an oblate composite quaquaversal dome. Figures 20*F* and 21*A*–*D*, on the other hand, cannot be rotated about any horizontal or vertical line without producing discontinuities.

Composite curves may also be assembled without rotation to produce a large variety of unsymmetrical quaquaversal and elongate domes, with principal sections that are either plane or curved surfaces. These sections are not necessarily related in their geometrical configurations, so that composite domes will be much more numerous and complex than similar or cognate cylindrical folds. For all practical purposes, one is constrained to study mainly the principal sections, which are represented conventionally and approximately as plane profiles.

Quaquaversal and canoe-shaped composite domes are thus constructed by all the methods heretofore described in the production of composite cylindrical folds. The number of possible domal structures is too large to attempt their enumeration. Figure 21*E*, drawn in perspective, illustrates theoretically a simple elongate composite dome with two kinds of plane principal sections. The major section, parallel to the elongation of the fold, comprises cognate ellipses, having eccentricities ranging from 0.775 to 0.947; the minor or cross section consists of a set of similar ellipses at different scales, each with an eccentricity of 0.745. It should be noted that neither principal section in this figure comprises composite stratigraphic traces; but the fold as a whole is composite because these two sections illustrate different species of profiles.

CONSTRUCTION AND APPLICATION OF ELLIPSES

Little is really known of the curvature of stratigraphic surfaces, cylindrical or noncylindrical, first because complete profiles of folds are rare and second because the geometry of actual folds has not been studied. The traces of the stratigraphic surfaces of some folds, however, are available in natural outcrops, either in profiles normal to the axial line or in profiles that do not depart materially from this orientation, so that they could be transformed with slight error into the desired plane. The geometrical measurement and analytical classification of such traces should constitute a noteworthy step in an understanding of the mechanics of folding and would also serve as a guide or check for some of the experimental work now being done on the deformation of rocks. Most folds, however, have to be reconstructed without the aid of such complete data, for commonly only a vertical profile is available, on which are charted a set of dips, together with lithologic data.

Ellipses have been shown to be particularly suitable for the approximate delineation of stratigraphic traces in profiles selected to show the maximum or minimum apical curvature; and four general types of cylindrical and noncylindrical folds have been illustrated, mainly by the use of elliptic arcs. A large number of semiellipses are required in this work. Various methods are known for drawing ellipses, including the use of articulated linkages. The well-known method of drawing an ellipse with a string of constant length, anchored at the two foci, is impracticable, because it is difficult to prepare a string of a stated length and still more difficult to avoid stretching the string while drawing the ellipse. All ellipses illustrated in this paper were drawn by means of the trammel of Archimedes. This method, though discovered 2,200 years ago, remains still the most practical means for point-to-point elliptic charting. The ellipse is constructed as a glissette, by sliding a bar of length $(a+b)$ between 2 coordinate axes, with the point of projection at the junction of the 2 semiaxes. Rectangular coordinates are used for simplicity, but an ellipse will still be produced if the axes are oblique. The mechanism and method are shown in figure 21*F*. A strip of lucite 3 centimeters wide and 51 centimeters long is perforated by 101 holes of size 60, that are 0.5 centimeter apart. Holes of this size permit the insertion of a sharply pointed hard pencil. It is possible with this instrument to construct 2,450 ellipses with eccentricities ranging from 0.999 to 0.199; but if the holes were drilled 0.2 centimeter apart, 16,500 ellipses could be constructed. In figure 21*F* the major semiaxis, with a length of 30 units, is laid off on the strip of lucite as OA; the minor semiaxis, with a length of 14 units, as OB. The resulting ellipse has an eccentricity of $k=0.884$, as determined by the equation—

$$k=\frac{(a^2-b^2)^{1/2}}{a}.$$

The points on the ellipse are laid off as closely as seems necessary for accurate sketching of its locus.

A limited number of semiellipses have been prepared in graphical form for structural reference. These were drawn originally with major semiaxes of 20 centimeters and minor semiaxes ranging from 1 to 19 centimeters, and this provided 19 prolate semiellipses, as shown in figure 22. These were also drawn in oblate form, as shown in figure 23. But such figures, in order to be usable, must also be available at different scales; and the semiellipses of figures 22 and 23 were reproduced at reduced scales, resulting in 18 additional figures. These, however, could not be presented in this paper; and therefore those wishing to use these methods will have to provide a series of reductions of figures 22 and 23. The scale of the profile must be adjusted to fit a set of such semiellipses, unless the geologist wishes to prepare other semiellipses of predetermined eccentricity and scale to fit a profile of fixed size.

The utilization of these ellipses and elliptic arcs will depend upon the type of fold that is to be reconstructed. If the fold is known to have parallel or approximately

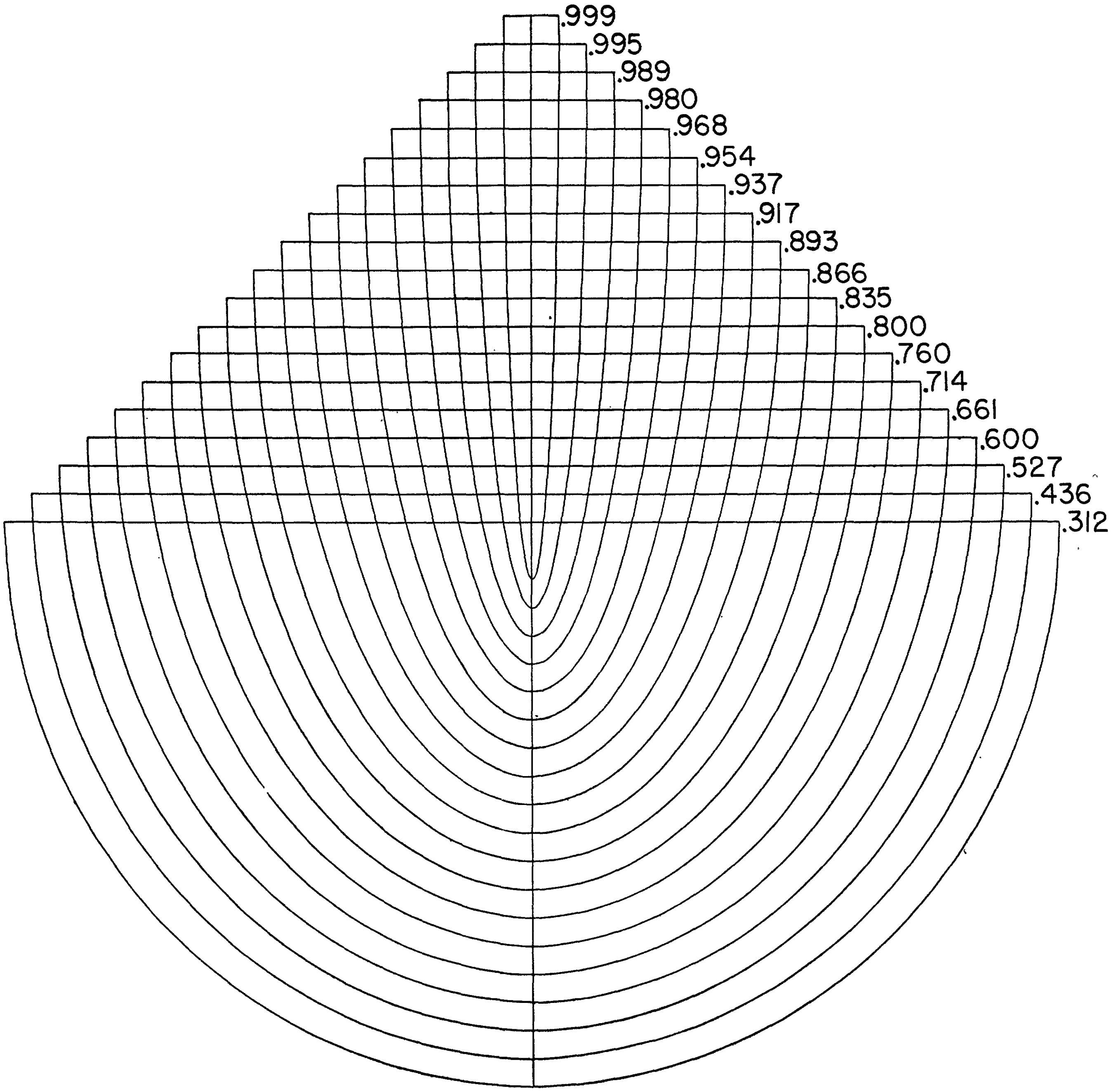

FIGURE 22.—Nineteen prolate semiellipses with eccentricities ranging from $k=0.312$ to $k=0.999$. The major semiaxes have the same length.

parallel surfaces, in whole or in part, these curves will not apply; instead the reconstruction will be accomplished by application of the method of evolute and involutes. But if the fold has nonparallel surfaces, one must first determine its departure from parallelism, and then select the genus of the nonparallel fold. The first decision requires an evaluation of the localization and degree of the stratigraphic thickening and thinning, as revealed by the available geologic data. The second decision will have to be made experimentally by fitting the structural data to the simplest curvilinear traces that will serve the required purposes.

Let a vertical profile be given on which are plotted a series of dips. If the plane of the profile is oblique to the axial plane or surface, another plane profile may be drawn that intersects the original and is normal to the axial plane or surface. The given dips may be transformed in position and magnitude from the first to the second plane with little distortion, by well-known methods that require no description. The dips are thus

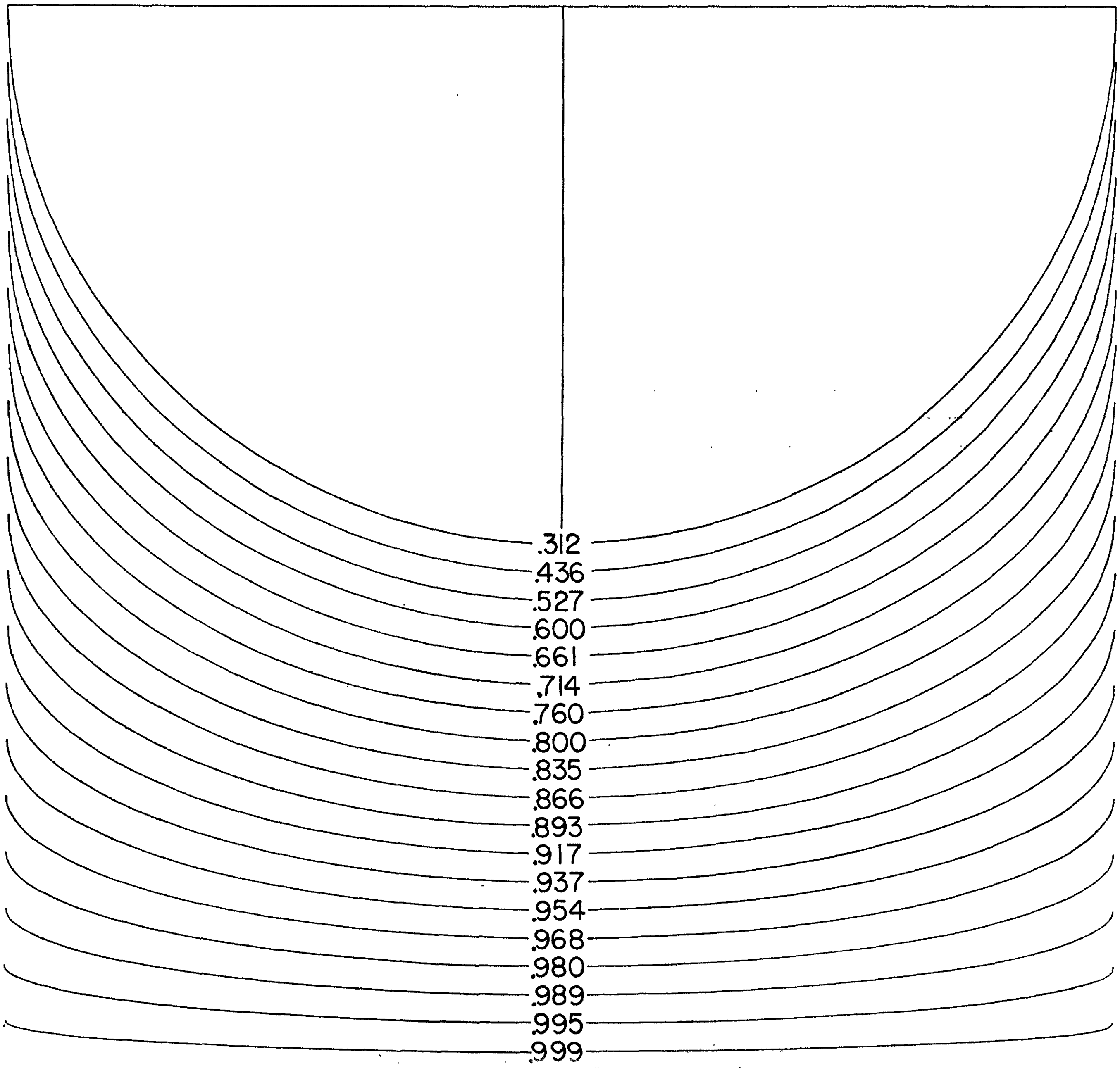

FIGURE 23.—Nineteen oblate semiellipses with eccentricities ranging from $k=0.312$ to $k=0.999$. The major semiaxes have the same length.

finally assembled on the desired profile, in their proper relation to original separation and to relief. These dips, however, may not have the optimum spacing to yield the best structural interpretation. The best structural spacing is generally a uniform one, but unequal spacing may be required to place the dips at contacts between beds, or for other reasons. Any desired spacing may be obtained by use of a method given by the writer in an earlier publication (1947). Thus a profile of suitable character and orientation is now ready to be used in the reconstruction of a fold.

The fold is known or assumed to be nonparallel. If the nature of the axial plane or surface is not already known, this information cannot be deduced from any assemblage of dips along a profile. But some idea of the dip of the axial plane or the configuration of an axial surface may be gained by the following method. Any assemblage of dips along a profile may be reconstructed, rightly or wrongly, as a parallel fold, which necessarily has an axial plane. One may therefore assume as a first approximation that the fold is parallel and then construct such a fold by the method of evolute and involutes. The approximate tilt of an axial plane is thus determined, to serve as a guide in reconstructing the nonparallel fold. This axial plane may be usable as deduced, but more commonly it will need to be modified materially in fitting elliptic arcs to the given dips of a nonparallel fold.

The construction of parallel folds by the method of evolute and involutes has been described by the writer in earlier publications, (1940, 1944, 1947, 1948), and needs no repetition. It should be stated, however, that discontinuities in the evolute are to be expected, not only where the dips along a profile pass through values of 0° and 90° but also at other places where the increment of dip becomes markedly irregular. Thus a set of dips that is increasing gradually from west to east may suddenly begin to increase more slowly or more rapidly, or to decrease. Any of these conditions may cause an interruption in the formation of an envelope that will result in a discontinuity in the evolute. Such discontinuities, however, will not interfere with the generation of the involutes.

The nonparallel fold is reconstructed by selecting semiellipses or smaller elliptic arcs that will fit the real and interpolated dips along a profile and will come closest to fitting the dip of the axial plane deduced from an analogous parallel fold. A single semiellipse from a series of reductions of figures 22 or 23, moved in a direction parallel to its vertical axis and repeated at equal or unequal intervals, as determined by the stratigraphy, will represent the traces of a congruent similar fold, of the type shown originally by Van Hise. But a series of semiellipses of the same eccentricity at different scales, or arcs thereof, will represent the traces of a noncongruent similar fold; and a selection of semiellipses or elliptic arcs of different eccentricity will lead to the construction of the cross section of a cognate fold. If all these possibilities fail, a composite fold must be constructed. The magnitude and localization of the stratigraphic thickening or thinning is further controlled by a choice of vertical or horizontal translation, or both, either proportional or differential. Semiellipses with the eccentricities shown in figures 22 or 23 may be inadequate for the desired curvature; if they are inadequate, other semiellipses or elliptic arcs may be drawn by the method above described.

A transverse geologic profile of a cognate fold, constructed in the manner described above, is illustrated in figure 24. Six beds are shown with 12 dips that occur either naturally, or have been interpolated, at equal intervals along the contacts of the strata. These beds are known to be thicker at the trough of the syncline than along its flanks; and the geologic mapping shows that the thicknesses on one flank are greater than on the other. More than one solution of these require-

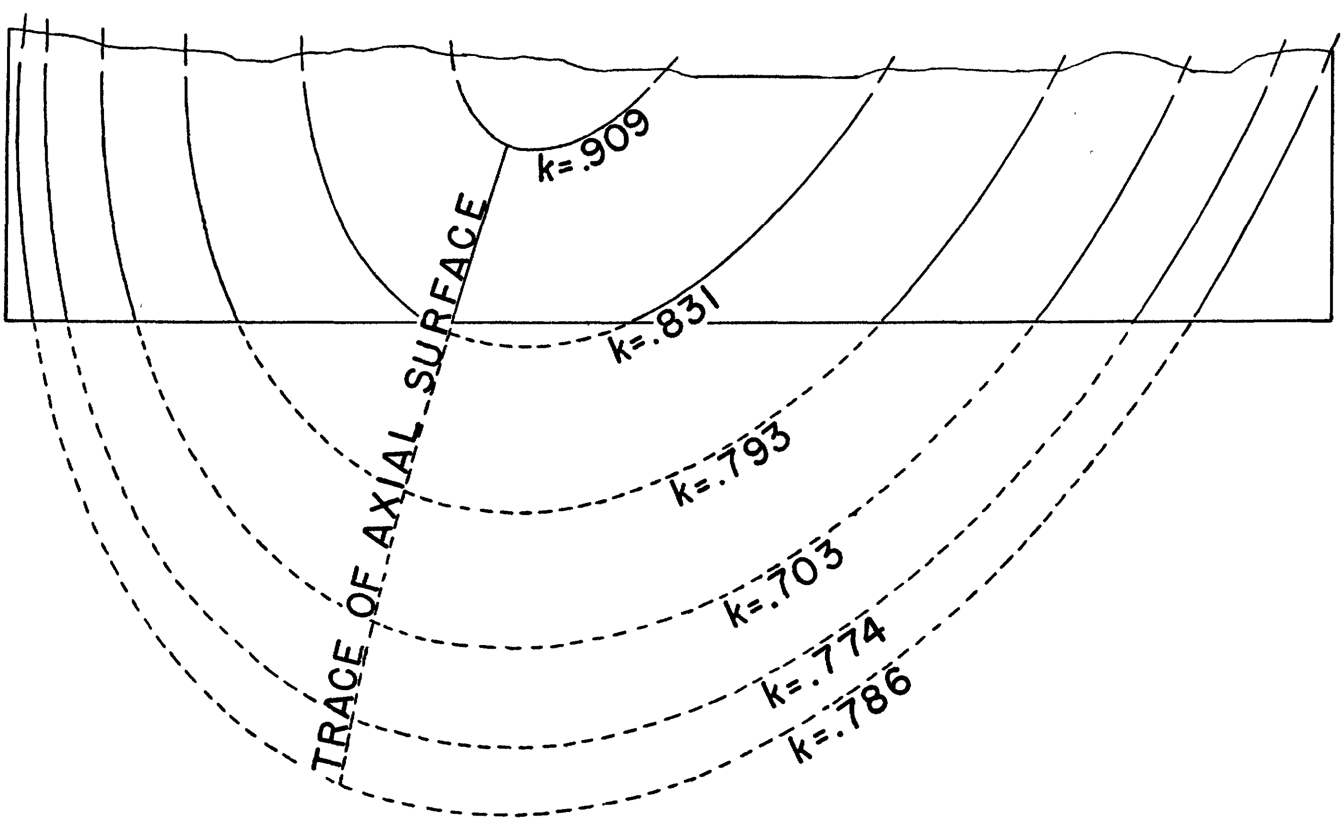

FIGURE 24.—Transverse geologic profile of a cognate syncline with a slightly curved axial surface. Constructed from dips and contacts plotted on a section normal to the axial line.

ments may be obtained, but the one illustrated is produced by 6 elliptic arcs, having eccentricities of 0.909, 0.831, 0.793, 0.703, 0.774, and 0.786. In the preparation of this profile, the tilt of a tentative axial plane was first determined. Six arcs from a series of semiellipses of the type shown in figure 22 were then selected and oriented collinearly with their major axes along the trace of a tentative axial plane and their minor axes parallel at equally spaced intervals. These arcs were next translated differentially along the line of their major axes to fit stratigraphic intervals assumed to exist at the trough of the fold. The traces were then translated differentially in a direction parallel to the minor axes, in an attempt to fit the given dips and contacts. Failing in this, elliptic arcs of different eccentricity were used, on a trial and error basis, until the dips were finally duplicated as tangents to the elliptic arcs. This construction produced a slightly curved axial surface, as shown by its trace in figure 24.

The reconstruction of nonparallel folds, by the methods described in this paper, is evidently accomplished by an experimental and empirical technique that cannot yield a unique structural solution. These methods thus differ fundamentally from the method of evolute and involutes that produces one and only one reconstruction of a parallel fold. What, then, are the stratigraphic advantages? A geologist may indeed prepare a structural section of a nonparallel fold that is not made in accord with any established plan; or he may try to adjust the structural data to fit conventional curves that correspond approximately to the stratigraphic traces. The utilization of elliptic arcs yields at least a solution that is amenable to measurement and checking; and it is the thesis of this paper that a conventional though empirical solution of this kind is preferable to stratigraphic conjecture. One of the measurements that is facilitated by the use of predetermined curves is that of stratigraphic thickness, now to be described.

STRATIGRAPHIC THICKNESS

Stratigraphic thickness, or other stratigraphic dimensions, can be measured from a structural profile either graphically or by numerical computation. The stratigraphic thickness of a nonparallel fold must first be defined, however, before it can be measured by either method. The utilization of predetermined curves in a profile, such as elliptic arcs, facilitates both the definition and the measurements. Assume that the section of a stratum within a fold is bounded by two semiellipses, or by arcs thereof. The stratigraphic thickness may then be defined as the area of the section delineated in the profile, divided by the length of another elliptic arc that lies midway between the two bounding arcs. Such a median arc can be constructed either graphically or according to an analytic formula.

A graphic measurement of the stratigraphic thickness requires two machines; a line integrator for the measurement of the median arc and a polar planimeter for measurement of the area between two bounding elliptic arcs. If such instruments are available and the geologist prefers the graphic method, the resulting measurement of thickness will doubtless be within the limits of accuracy of the constructed profile. But if an analytical measurement is preferred, certain methods need to be described.

The area of an ellipse is given by the simple expression πab; and as an ellipse is bisymmetrical, the areas of the semiellipse and of the quarter ellipse are obvious. Many of the illustrations of nonparallel folds, however, utilize elliptic arcs shorter than a quarter ellipse, which may either be coaxial or noncoaxial. If the two arcs are coaxial, the area between them is obtained by two integrations and a subtraction of the resulting integrals. The general case has to do with an area between two elliptic arcs, one of which has been translated both vertically and horizontally with regard to the other. In other words, if both arcs are treated as central conics, their origins are different. This condition is illustrated in figure 25.

Let the arc PQ be represented by the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, referred to an origin at O_1; and let the arc RT be represented as $\frac{X^2}{A^2}+\frac{Y^2}{B}=1$, be referred to an origin at O_2. Also let S_1 be the area between a segment of the arc PQ, the ordinates $x=c$ and $x=d$, and the x axis; and let S_2 be the area between a segment of the arc RT, the ordinates $X=m$ and $X=n$, and the X axis. If the ordinates of any point in the two systems of coordinates are y_r and Y_r, the required area is—

$$S=|S_1-S_2|\pm(c-d)\cdot|y_r-Y_r|$$

depending upon the relative positions of the two origins. Writing the equation of the arc PQ as—

$$y=\frac{b}{a}(a^2-x^2)^{1/2}$$

we have—$S_1=\int_d^c \frac{b}{a}(a^2-x^2)^{1/2}dx$

$$S_1=\frac{b}{a}\left[\frac{x(a^2-x^2)^{1/2}}{2}+\frac{a^2}{2}\text{arc sin}\frac{x}{a}\right]_d^c$$

$$S_1=\frac{b}{2a}\left[c(a^2-c^2)^{1/2}-d(a^2-d^2)^{1/2}+a^2\left(\text{arc sin}\frac{c}{a}-\text{arc sin}\frac{d}{a}\right)\right]$$

The area S_2 is similarly found, and from S_1 and S_2 the required area S is readily computed.

No exact formula exists for the perimeter of an ellipse. Soreau (1921) has presented an empirical

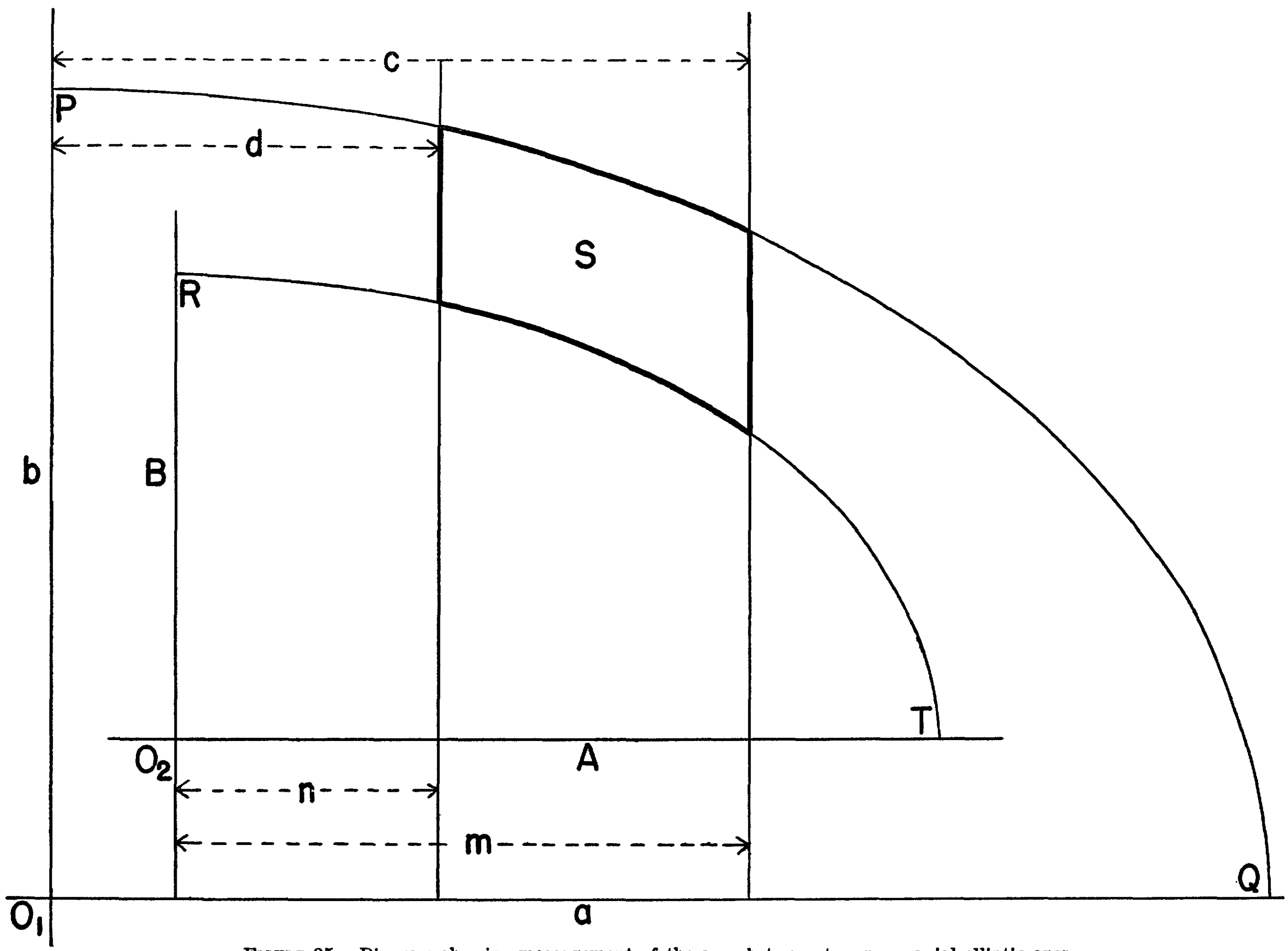

FIGURE 25.—Diagram showing measurement of the area between two noncoaxial elliptic arcs.

formula that gives a close approximation to the perimeter, as follows:

$$p=4a\cdot\frac{C\pi}{\sin C\pi}, \quad \text{if} \quad C=\frac{b}{a+b}$$

where p is the perimeter, and a and b are respectively the semimajor and semiminor axes. A still closer approximation is obtained if—

$$C=\frac{b}{a}\cdot\frac{a+0.03b}{0.97a+1.09b}$$

The perimeter may also be expressed as an infinite algebraic series in the eccentricity (k), but this converges so slowly for large values of the eccentricity as to be practically useless. The perimeter is most accurately determined by an evaluation of the elliptic integral of the second kind, called by Legendre the E function.

The derivation of the length of an elliptic arc follows. Elliptic arcs are measured clockwise from the minor semiaxis, as shown in figure 26. Let—

$s=QP$, an elliptic arc
$a=OR=OA$, the major semiaxis
$b=OQ=OB$, the minor semiaxis
$k=\frac{(a^2-b^2)^{1/2}}{a}$, the eccentricity
$\psi=$the eccentric angle
$\phi=$the minor eccentric angle
$x=a \sin \phi$
$y=b \cos \phi$
$\theta=\text{arc} \sin k$, the modular angle

The length of any plane curve that is parametrically defined as $x=x(u)$ and $y=y(u)$ is given by the integral—

$$s=\int\left[\left(\frac{dx}{du}\right)^2+\left(\frac{dy}{du}\right)^2\right]^{1/2}du$$

For an ellipse, let—

$$x=a \sin \phi, \text{ and } y=b \cos \phi$$

Then—

$$\frac{dx}{d\phi}=a \cos \phi, \text{ and } \frac{dy}{d\phi}=-b \sin \phi$$

$$s=\int [a^2 \cos^2 \phi+b^2 \sin^2 \phi]^{1/2}d\phi$$

which, with the limits of integration inserted, may be written as—

$$s=a\int_0^{\phi}(1-k^2\sin^2\phi)^{1/2}d\phi=aE(\phi,k)=aE(\phi,\theta)$$

The length of a quarter ellipse, within the limits $\phi=\frac{\pi}{2}$ and $\phi=0$ is $p/4=aE$.

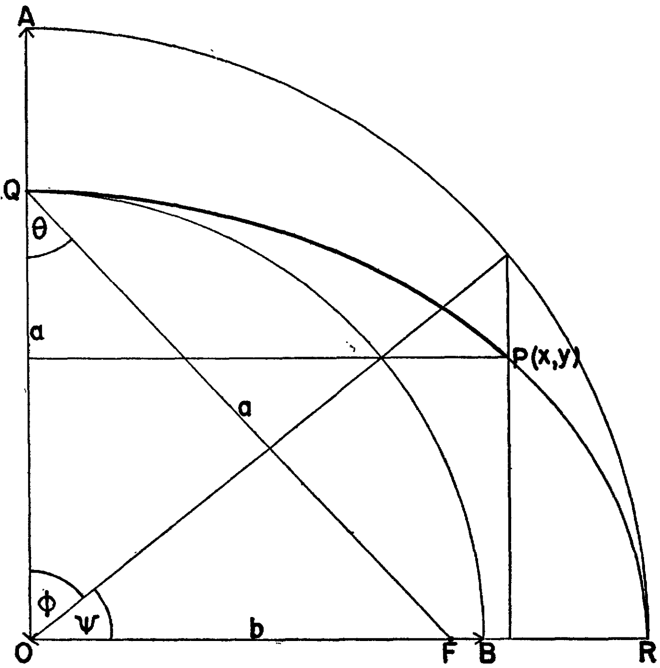

FIGURE 26.—Diagram showing measurement of the length of an elliptic arc. The minor eccentric angle, ϕ, and the modular angle, θ, are the arguments used in modern tables of elliptic functions.

The quadrature of the E function cannot be performed in terms of any of the elementary functions; but it may be computed numerically to any required degree of accuracy; this makes it as available for practical work as other tabulated functions. The values of E and of the other elliptic functions have been computed to 12 decimals by the Spenceleys (1947). As two variables, ϕ and θ, comprise the arguments, a double interpolation is required. The angular values of ϕ must also be converted to radians. By taking the values of E (ϕ, θ) from these tables, the lengths of elliptic arcs are not difficult to obtain, though some arithmetical computation is necessary. It is obvious from figure 26 that the length of any elliptic arc is the distance measured clockwise from the minor semiaxis to the farthest terminal point of the arc, minus the corresponding distance to its nearest terminal point.

The lengths of semiellipses and quarter ellipses, however, may be computed and tabulated. Thus the writer has computed the lengths of 99 semiellipses, with major semiaxes of 10 and minor semiaxes ranging from 9.9 to 0.1; and for reference the corresponding eccentricities are also given. In the reconstruction of a fold, it may be feasible to utilize semiellipses within this range of centimeters, inches, or other units of length. If possible, the semiperimeters may be read directly from the table. If, however, it is necessary to assume other lengths for the semiaxes, the corresponding semiperimeters may still be obtained from the table with a trivial amount of interpolation, and without the computation of eccentricity. Thus the semiperimeter of an ellipse with semiaxes of $a=17$ and $b=13$ is 1.7 times that of an ellipse with semiaxes of $a=10$ and $b=\frac{10\times 13}{17}$ or $b=7.647$. Interpolating in the table between $b=7.6$ and $b=7.7$, it is found that the semiperimeter of such an ellipse is 27.84. Therefore the semiperimeter of an ellipse with semiaxes of $a=17$ and $b=13$ is $27.84\times 1.7=47.33$. The table of semiperimeters follows.

Semiperimeters of ellipses, a=*10*

b	k	$p/2$	b	k	$p/2$	b	k	$p/2$
9.9	0.1411	31.26	6.6	0.7513	26.35	3.3	0.9440	22.24
9.8	.1990	31.10	6.5	.7599	26.21	3.2	.9474	22.14
9.7	.2431	30.95	6.4	.7684	26.07	3.1	.9507	22.03
9.6	.2800	30.79	6.3	.7766	25.93	3.0	.9539	21.93
9.5	.3122	30.64	6.2	.7846	25.80	2.9	.9570	21.83
9.4	.3412	30.48	6.1	.7924	25.66	2.8	.9600	21.73
9.3	.3676	30.33	6.0	.8000	25.53	2.7	.9629	21.63
9.2	.3919	30.17	5.9	.8074	25.39	2.6	.9656	21.54
9.1	.4146	30.02	5.8	.8146	25.06	2.5	.9682	21.45
9.0	.4359	29.87	5.7	.8216	25.13	2.4	.9708	21.36
8.9	.4560	29.71	5.6	.8285	24.99	2.3	.9732	21.27
8.8	.4750	29.56	5.5	.8352	24.86	2.2	.9755	21.18
8.7	.4931	29.41	5.4	.8417	24.73	2.1	.9777	21.09
8.6	.5103	29.26	5.3	.8480	24.60	2.0	.9798	21.01
8.5	.5268	29.11	5.2	.8542	24.48	1.9	.9818	20.93
8.4	.5426	28.96	5.1	.8602	24.35	1.8	.9837	20.85
8.3	.5578	28.81	5.0	.8660	24.22	1.7	.9854	20.78
8.2	.5724	28.66	4.9	.8717	24.10	1.6	.9871	20.70
8.1	.5864	28.51	4.8	.8773	23.97	1.5	.9887	20.63
8.0	.6000	28.36	4.7	.8827	23.85	1.4	.9902	20.56
7.9	.6131	28.22	4.6	.8879	23.72	1.3	.9915	20.50
7.8	.6258	28.07	4.5	.8930	23.60	1.2	.9928	20.44
7.7	.6380	27.92	4.4	.8980	23.48	1.1	.9939	20.38
7.6	.6499	27.77	4.3	.9028	23.36	1.0	.9950	20.32
7.5	.6614	27.63	4.2	.9075	23.25	.9	.9959	20.27
7.4	.6726	27.48	4.1	.9121	23.13	.8	.9967	20.22
7.3	.6834	27.34	4.0	.9165	23.01	.7	.9975	20.17
7.2	.6940	27.20	3.9	.9208	22.90	.6	.9982	20.13
7.1	.7042	27.05	3.8	.9250	22.79	.5	.9987	20.10
7.0	.7141	26.91	3.7	.9290	22.67	.4	.9992	20.07
6.9	.7238	26.77	3.6	.9330	22.56	.3	.9995	20.04
6.8	.7332	26.63	3.5	.9367	22.45	.2	.9998	20.02
6.7	.7424	26.49	3.4	.9404	22.35	.1	.9999	20.01

ANALYSIS OF FOLDS

The analytical classification and measurement of folds can be done only for complete or partial flexures, whose stratigraphic traces or parts thereof are visible and mappable in satisfactory profiles. Two general alternatives exist for approaching this problem. The mapped arcs representing stratigraphic traces may be fitted empirically to preselected curves, using some system of coordinates; or these arcs may be represented analytically in terms of invariants of such curves, without reference to any system of coordinates. As no folds are simple geometric structures, the analysis of stratigraphic traces by either method will invariably result in approximations, but these can be made as accurate as the work requires. Such analytical methods may be useful in certain practical applications, such as subsurface geologic or geophysical prospecting; but the principal application will probably be in the classification of folds and in theoretical studies relating to the mechanics of folding.

The conics, being relatively simple curves of the second degree, are first considered in the analytical representation of stratigraphic traces. Any 5 points, of which 3 or more are not collinear, may be fitted to an ellipse or to a hyperbola; 4 points to a parabola; and 3 points to a circle. Mertie (1948) has shown that 5 points on a stratigraphic trace may be fitted graphically to an ellipse by the application of Brianchon's theorem. Analytically the procedure is as follows. Assume a random origin of rectangular coordinates, and select 5 points on the mapped stratigraphic trace, preferably about equally spaced along the arc. The coordinates of these points are then measured in terms of the selected origin. The equation of a conic passing through these 5 points is expressed as follows:

$$\begin{vmatrix} x^2 & xy & y^2 & x & y & 1 \\ x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5y_5 & y_5^2 & x_5 & y_5 & 1 \end{vmatrix} = 0$$

where (x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), and (x_5, y_5) are the coordinates of the 5 points. This determinant is reducible order by order by either of the 2 methods of condensation given by Muir and Metzler (1933). It becomes finally the general equation of the conic—

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

in which the 6 constants are numerically determined. The discriminant of the general conic is—

$$\Delta=4ACF+BDE-AE^2-CD^2-FB^2$$

If $\Delta \neq 0$, the conic is an ellipse, hyperbola, or parabola, depending respectively whether B^2-4AC is less, greater, or equal to zero. The equation is unlikely to represent a parabola, as it is not possible in general to pass a parabola through 5 random points. The equation of the ellipse or hyperbola may now be simplified to one whose axes coincide with the axes of coordinates. The coordinates of the central conics are— $\alpha=\frac{2CD-BE}{B^2-4AC}$ and—

$$\beta=\frac{2AE-BD}{B^2-4AC}$$

and the angle between the old and new systems of coordinates is—$\theta=1/2 \text{ arc tan } \frac{B}{A-C}$.

The resulting simplified equation will have the form of—

$$A'x^2 \pm B'y^2-1=0$$

where—

$$A'=a^{-2} \text{ and } B'=b^{-2}.$$

If by any chance the general equation of the conic should represent a parabola, a conversion to the standard form $y^2=2mx$ may be accomplished by equally well-known methods of analytical geometry.

The simplified equation of the ellipse or hyperbola may need to be further modified to fit more closely the entire stratigraphic trace. To accomplish this objective, measure and record the coordinates of a considerable number of equally spaced points, including the five original points, in terms of the new origin of coordinates. An equal number of residual equations are then derived, having the general form of—

$$v_n=A'x_n^2 \pm B'y_n^2-1$$

which are linear in A' and B'. From these, two normal equations will be obtained, which may readily be solved for A' and B'. Computation of the residuals will determine the closeness of fit.

The fitting of stratigraphic traces to preselected curves is a problem that differs from the fitting of observed data to empirical curves, as described in many textbooks. Observed measurements are automatically charted in relation to an origin of coordinates. Stratigraphic traces as premapped arcs do not determine or suggest where the origin of coordinates should be placed. In the method of fitting traces to conics, as outlined above, random origin is assumed, but the resulting equation of the conic contains the information needed for a close placement of the simplest origin of coordinates. This method cannot be used advantageously for cubics or algebraic curves of higher degree. Thus, nine points are required to define a cubic, and the resulting equation would have to be written as a determinant of the tenth order. The labor involved in clearing such a determinant is prohibitive; the analysis that would be required for reducing the equation to its simplest form, by the transformation of coordinates, is difficult; and the final least square adjustment of nine coefficients would likewise be impracticable. Neither can the method be applied to transcendental curves.

Higher plane curves may be used, however, if a random origin of coordinates is used. Many higher plane curves are available, some of which simulate parabolic, hyperbolic, or elliptic curvature, whereas others are exponential, logarithmic, trigonometric, or hyperbolic in nature. In particular, the polynomial $y=a+bx+cx+dx^3+ \ . \ . \ . \ .$ may be adjusted by least squares to fit any nonperiodic curve, and further diversity may be added by substituting for y any function of y. Such curve fitting, referred to an arbitrary origin of coordinates, may be useful in practical work such as underground exploration, but is of little value for the classification of stratigraphic traces or in other theoretical applications, because curves referred to different systems of coordinates are not comparable.

The utilization of invariants that are independent of any system of coordinates is a much better method for the classification of stratigraphic traces. Algebraic relations between such invariants are called intrinsic equations. Figure 27 illustrates these invari-

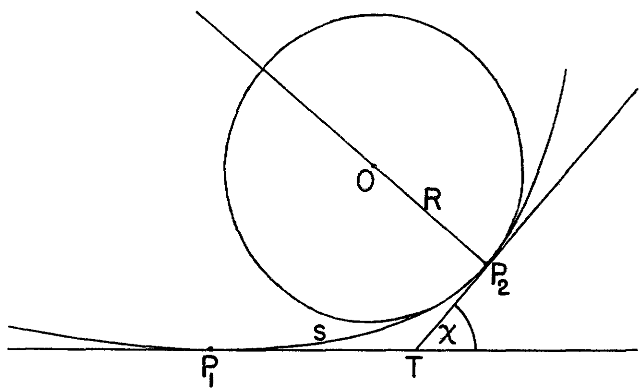

FIGURE 27.—Diagram illustrating angle of contingence (χ), length of arc (s), and radius of curvature (R), which are used in formulating intrinsic equations. The curvature (K), where $K=\frac{1}{R}$, may be used instead of the radius of curvature.

ants, which constitute the length of an arc, $s=P_1P_2$, the corresponding angle of contingence, χ, and the radius of curvature, $R=OP_2$. The curvature K is the reciprocal of the radius of curvature. Any two of these variables will determine the shape of a curve, without locating it in a plane. Equations in s and χ are called Whewell equations; those in s and R (or K) are called Cesàro equations; and those in χ and R (or K) are called Euler equations. It is easier to measure lengths of arcs and angles of contingence than to measure R or K, for which reason Whewell equations are the most practical for the present purpose.

The method consists in measuring the lengths of arcs and the corresponding angles of contingence of stratigraphic traces in the profile of some actual fold, covering if possible the entire fold for each trace. The photograph of a section of a fold, transformed if necessary into a plane normal to an axial or an apical line, will serve as the original data. These measurements are then fitted to the intrinsic equation of some assumed curve by the method of least squares. The residual equations are first formed, from which the normal equations and the values of the constant parameters are determined by well-known methods. Computation of the residuals will determine how closely the assumed curve can be made to fit the stratigraphic traces. If the parameter or parameters of the equation are not linear and no algebraic transformation will lead to a linear relation, other least square methods are available for determining the values of the parameters. Such methods are clearly presented by Scarborough (1950).

Cartesian or other coordinates are not used in the methods above described, but when it is desired to utilize Whewell equations, it generally is necessary to derive them from conventional equations. The procedure for obtaining a Whewell equation from an explicit equation in cartesian coordinates is as follows: Let the given equation be—

$$y=f(x)$$

Then—

$$\tan \chi=f'(x)$$

But the differential of the arc of any plane curve is—

$$ds^2=dx^2+dy^2$$

Therefore—

$$ds=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{½}dx$$

and—

$$s=\int[1+(y')^2]^{½}dx$$

After this integration is performed, we have—

$$s=F(x)$$

Eliminating x from equations—

$$\tan \chi=f'(x)$$

and—

$$s=F(x)$$

we obtain the desired Whewell equation—

$$s=W(\chi)$$

Other methods are available for the conversion of implicit cartesian, polar, and parametric equations to Whewell equations.

Intrinsic equations of the hyperbola and ellipse are not well suited to this method, unless one is versed in the use of elliptic functions. The Whewell equation of the parabola $y^2=4ax$, however, is—

$$s=a[\sec \chi \tan \chi+\log (\sec \chi+\tan \chi)]$$

Probably the best application of Whewell equations in matching stratigraphic traces is in the use of higher plane curves, particularly transcendental curves, with arcs that have a general resemblance to the conics. A few such equations that might prove of interest, followed by their Whewell equations, are given below.

Parabolic catenary: $y=a \cosh\left(\frac{x}{a}\right)$

$s=a \tan \chi$

Catenary of uniform strength: $y=a \log \sec\left(\frac{x}{a}\right)$

$s=a \log \tan\left(\frac{\pi}{4}+\frac{\chi}{2}\right)$

Semicubical parabola: $3ay^2=2x^3$

$9s=4a(\sec^3 \chi-1)$

Cycloid: $x=a \operatorname{vers}^{-1}\left(\frac{y}{a}\right)+(2ay-y^2)^{1/2}$

$s=4a \sin \chi$

Epicycloid: $x=(a+b) \cos t+b \cos\left[(a+b)\cdot\frac{t}{b}\right]$

$y=(a+b) \sin t+b \sin\left[(a+b)\cdot\frac{t}{b}\right]$

$s=\frac{4b(a+b)}{a} \sin\left[\frac{a}{a+2b}\cdot\chi\right]$

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