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# High-Performance Systems Biology and Associated Combinatorial Scientific **Computing Problems**

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## **Project Participants and Funding**

Collaborators:

- NREL SCG: Chris Chang, Peter Graf, and Kwiseon Kim<sup>\*</sup>
- NREL Photobiology: Mike Seibert<sup>\*</sup>
- Summer Student from CU Boulder: David Biagioni

Participating institutions:

- National Renewable Energy Laboratory
- Colorado School of Mines
- Stanford University

Funding through SciDAC (OASCR and OBER)

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## Metabolism and Metabolic Modeling

#### Metabolism

- Chemical reactions occurring in living cells
- Reactions catalyzed by enzymes
- Metabolic species (e.g., glucose, pyruvate) produced and consumed

#### **Reaction Modeling**

- Several models
- Michaelis-Menten kinetics

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## High-Performance Systems Biology

#### In a nutshell

- Model complete metabolism of Chlamydomonas reinhardtii
- Develop high-performance software to explore metabolism kinetics

### Example problems

- Parameter estimation (data fitting)
- Sensitivity minimization
- Parameter space characterization



## Metabolic Model

For metabolic reaction:

$$\frac{dy}{dt} = f(y, k, E)$$

- y vector of metabolite concentrations
- *k* vector of kinetic parameters
- *E* vector of enzyme concentrations

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## Metabolic Model

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## Metabolic Model

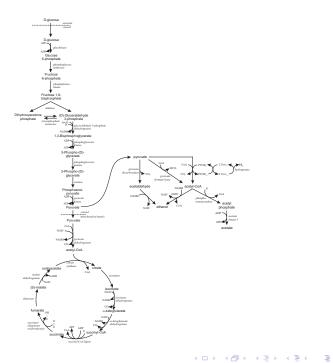
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Kinetic parameters:

- Time consuming to determine experimentally
- Essential to understanding metabolic kinetics





## **Parameter Estimation**

- Find set of k such that species concentrations match target values
- Expressed as optimization problem:

$$\min_k g(k),$$

where  $g(k) = ||r||_2^2$  and r is vector of differences between target and simulated values

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## Sensitivity Minimization

- Find set of k to minimize sensitivity
- Applies to organism engineering issues and general "nature of life" questions
- Objective function to minimize:

$$h(k) = \|J_y(k)\|_F^2$$
$$= \sum_{i=1}^n \sum_{j=1}^m \left(\frac{\partial y_i}{\partial k_j}\right)^2$$

#### Parameter estimation:

$$g(k) = \sum_{i=1}^{n} (y_i - \bar{y}_i)^2$$

 $y_i$ : simulated metabolite concentration  $\bar{y}_i$ : target concentration

Single entry of  $\nabla g$ :

$$\frac{\partial g}{\partial k_j} = 2\sum_{i=1}^n (y_i - \bar{y}_i) \frac{\partial y_i}{\partial k_j}$$

No problems computing  $\nabla g$  (adjoint sensitivity analysis)

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Computing second derivative term expensive

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**Project Overview** 

# Second Derivative Computation CSC Problem Slide #1

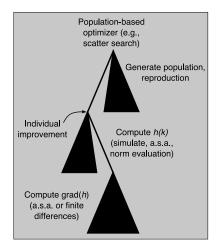
- Finite differences
- Automatic differentiation
- Collaboration with Paul Hovland
- Collaboration with Radu Serban (formerly at LLNL):
  - Combines adjoint sensitivity analysis and AD (via Tapenade)
  - Parallelized
  - Received code two weeks ago (i.e., nothing yet to show)

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#### Project Overview

## Sensitivity Analysis Cost Substantial

- ≈1000 dimension parameter space (ultimately)
- Each evaluation of h(k) inexpensive (similar to cost of  $\nabla g$ )
- Evaluation cost of  $\nabla h$  adds up



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- The hierarchical parallelism in this application generates load balance issues
- More of the scheduling problem variety
- Want to hear more on this

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## **Kinetics Complications**

- Many gaps to fill regarding metabolic kinetic rates
- Requires much experimentally-obtained data
- Relevant experimental data difficult (tedious?) and expensive to generate
- Switch gears and discuss another systems biology (metabolomics) problem

## **Stoichiometric Matrix**

Stoichiometry refers to numbers in chemical formulae

$$2A + B \longrightarrow C + 3D$$
$$C + 2E \longrightarrow F$$

• Stoichiometric matrix stores stoichiometry for all reactions

$$S = \begin{pmatrix} -2 & 0 \\ -1 & 0 \\ 1 & -1 \\ 3 & 0 \\ -2 & 0 \\ 0 & 1 \end{pmatrix}$$

#### **Project Overview**

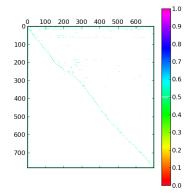
## Stoichiometric Matrix and Flux Analysis

- Previously said dy/dt = f(y, k, E)
- Can now also say

$$\frac{dy}{dt} = Sv,$$

where *v* is vector of *reaction velocities* 

- No kinetic parameters to get in the way!
- At steady state, dy/dt = 0 (almost)
- Now looking at Sv = 0



## **Problem Definition**

• Say we want to maximize concentration of species *y<sub>i</sub>*:

## $\max_v y_i$

- This problem trivial without constraints
- Possible constraints:
  - Thermodynamics (an arbitrarily large reaction rate not possible)
  - Sv = 0 at steady state (almost)
  - All v<sub>i</sub> ≥ 0 (reactions do not happen in reverse not exactly true)

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## Add Another Layer

- Suppose goal is to dispose of some reactions or pathways
- Which pathways are more critical?
- Conceptual connections to electrical grid modeling?

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### Molecular Dynamics Shameless plug for another SciDAC project

Application:

- Study function of CBH I enzyme in "digesting" crystalline cellulose
- Processivity?

Challenges:

- CHARMM does not scale well, has everything needed for application
- Other packages scale well, do not have everything needed
- If CHARMM: load balancing

