UAH Report No. 570

GRANT/MARSHALL
IN-70
39524 - C.R

36P

New Experiments to Constrain the Coefficients of the Robertson Transformations for Inertial Systems

Contract NAG8-047

Final Report for the Period September 30, 1985, to September 30, 1986

(November, 1986)

Prepared By

Douglas G. Torr
Principal Investigator

Larry L. Smaley
Co-Principal Investigator

School of Science
The University of Alabama in Huntsville
Huntsville, Alabama 35899

Prepared For
George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812

(NASA-CR-179908) NEW EXPERIMENTS TO CONSTRAIN THE COEFFICIENTS OF THE ROBERTSON TRANSFORMATIONS FOR INERTIAL SYSTEMS Final Report, 30 Sep. 1985 - 30 Sep. 1986 (Alabama Univ., Huntsville.) 36 p CSCL 20C G3/70

N87-12320

Unclas
44925

Final Report on NASA/MSFC Grant NAG8-047, entitled:

"New Experiments to Constrain the Coefficients of the Robertson Transformations for Inertial Systems"

SUMMARY

The work proposed under NASA/MSFC Grant Number NAG8-047 was successfully completed. Under this contract, a feasibility study was conducted to evaluate a measurement of the variability in the one-way speed of light by a direct-time-of-flight approach. The proposed experiment design was successfully completed and the initial tests indicated that the approach is viable. The experiment is now being carried out under new funding.

1. Introduction

The purpose of Grant NAG8-047 was to provide for the design and commencement of a laboratory version of an experiment to measure variations in the one-way velocity of light, a topic in fundamental physics that is receiving increasing attention in the literature.* The experiment had previously been performed at Utah State University (USU), in Logan, Utah.

* G. Spavieri, Phys. Rev. A 34, 1708 (1986) and "Accuracy of Time Transfer in Satellite Systems", Report by the National Academy of Science (National Academy Press, Washington, D.C., 1986).

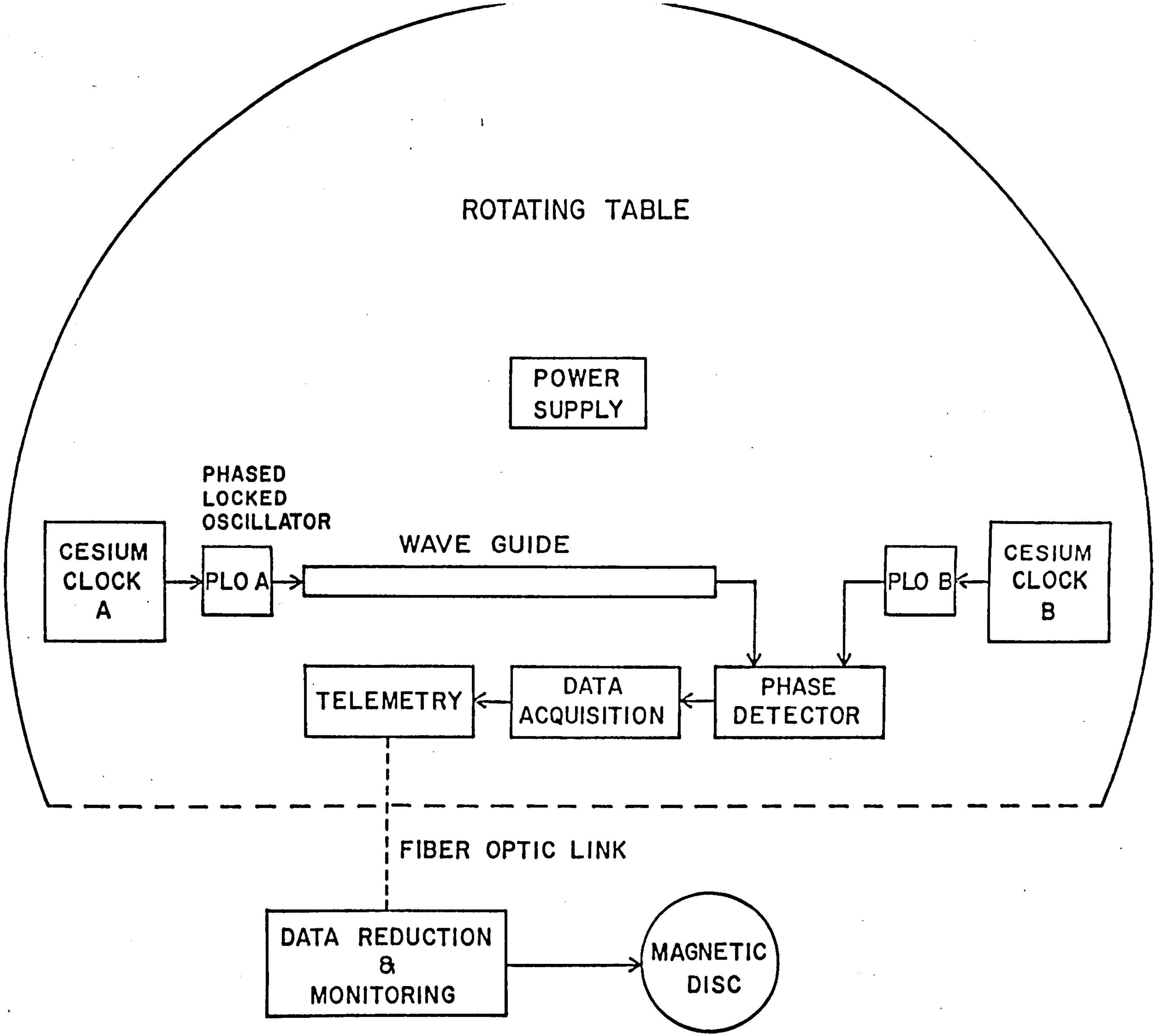

ROTATING TABLE
POWER SUPPLY
PHASED LOCKED OSCILLATOR
CESIUM CLOCK A
PLO A
WAVE GUIDE
PLO B
CESIUM CLOCK B
TELEMETRY
DATA ACQUISITION
PHASE DETECTOR
FIBER OPTIC LINK
DATA REDUCTION & MONITORING
MAGNETIC DISC

Figure 1

The USU version of the experiment lacked the rigid controls that can be achieved in a laboratory environment. A new version of the experiment was designed with the collaboration of Mr. Dave Gagnon (Naval Weapons Center, China Lake, California). Work on the actual construction of the experimental setup was started. The effort made under this project continues at present under Grant NAG8-067.

2. Experimental Setup

Numerous discussions were conducted among members of the group until the experimental setup of Figure 1 was finally adopted. The essence of the experiment consists in comparing the output of two cesium time standards after the signal of one of them goes through a ceramic waveguide with high refractive index. Phase locked oscillators were included in the system to operate at higher energies than those provided by the standards. This feature was expected to improve the signal to noise ratio. All this was mounted on a rotating table together with data acquisition and telemetry systems. Data were transmitted to external data reduction and monitoring systems through a fiber optic link.

3. Tasks Performed

3.1. Cesium Standards

Cesium standards ("clocks") with high performance tubes were rented from Hewlet-Packard under a Lease-Purchase agreement. The frequency stability of the clocks was characterized at 100 KHz and 5 MHz.

3.2. Rotating Table

The desired characteristics of the rotating system were determined. An air turbine was chosen to perform the rotation of the table. Electric motors were discarded as they produce undesirable electromagnetic noise. The construction of the rotating mechanics was subcontracted with Reisz Engineering, Huntsville, Alabama.

3.3. Waveguide System

A microwave guide was designed by Mr. Gagnon. The dielectric substance chosen was barium titanate because of its extremely high dielectric constant. A company in Lambertville, New Jersy, was contacted for the construction of the wave guide. RFD, Ltd., of Tampa, Florida, provided us with high reliability phase locked frequency multipliers (oscillators). 5 MHz power attenuators were constructed to match the power requirements of the PLO's with the power output from the clocks.

3.4. Data Acquisition

A data acquisition system was designed. The search started for a system that would have the specifications of the design. Appropriate microprocessor controller and 12 bit digital to analog converter were ordered.

3.5. Telemetry System

Bidirectional optical compilers were constructed. An RS-232/optical coupler circuit diagram for the interface control system was designed.

3.6. Data Storage

A microcomputer with appropriate interfacing and data storage requirements was selected.

3.7. Signal Processing

Method and Software packages for data analyses were selected.

4. Data Taking

Clock data were taken (not the actual data for the present experiment) in order to improve on the analysis of the above mentioned experiment at

Utah State University. Basically, the noise of the clocks was restudied and used as input for some refinements in the analysis of that previous experiment.

5. Theoretical Work

Further theoretical work was done which resulted in a paper which is presently under consideration for publication by the journal Foundations of Physics (See attached copy).

6. Budget Information

A greater part of the budget than had been proposed was used for paying the rental of the clocks until a new grant from this NASA center was obtained to provide for the bulk of the hardware of the experiment.

TENTATIVE VERIFICATION OF ROBERTSON'S PRIVILEGED "REST FRAME" HYPOTHESIS

D. G. Torr and Tsao Chang[1]

Department of Physics
The University of Alabama in Huntsville
Huntsville, Alabama 35899

[1] On leave from Shanghai University of Science and Technology, Shanghai, China.

Abstract

Experimental results are reported in a companion paper[13] which, although marginal, suggests a possible detection of anisotropies in the one-way velocity of light which are consistent with the motion of the solar system inferred from measurements of Smoot et al., of the 2.7K cosmic background radiation. We use Robertson's approach to theoretically interpret these observations using Maxwell's equations for anisotropic space. We find that the results are consistent with the null results of other experiments, if the metric is velocity dependent. The physical cause of the anisotropy appears to be related to the Fresnel convection of light in a rotating system. We argue briefly that the results may provide experimental evidence for ten-dimensional space.

1. Introduction

Robertson[1] has devised a theoretical framework which has been generally used in analyzing experimental tests of Special Relativity (SR). His approach is based on the assumption that there exists *a priori* a reference frame $S(x^{\mu})$ which intrinsically possesses the properties of any "rest frame" of special relativity. In this frame light propagates isotropically at velocity c in vacuo. Space is assumed to be homogeneous so that the coordinate transformation equations between $S(x^{\mu})$ and any arbitrary inertial frame $S'(x'^{i})$

moving with relative velocity $\underline{v}$ with respect to S are given by

$$x^{\mu} = a_{1}{}^{\mu} x'^{1} + a^{\mu} \tag{1}$$

Robertson[1] then reduces the 16 unknown coefficients $a_{1}{}^{\mu}$ of (1) to 3 in the following way: (1) by the introduction of additional symmetry arguments; (2) by suitable choices of origins and coordinate axes; (3) by the adoption of Einstein's clock-synchronization in S and S'. Robertson[1] then shows that if the Michelson-Morley (MM), Kennedy-Thorndike (KT), experiments yield exactly null results, and Ives-Stilwell (IS) the second order relativistic doppler effect, then the Lorentz-Einstein transformations are derived.

Vargas[2,3] expressed the transformation equations in the most general form allowed by observational constraints taking into account errors of measurement, namely

$$\begin{aligned} x' &= a(x - vt) \qquad & y' &= ey \\ t' &= hx + jt \qquad & z' &= ez \end{aligned} \tag{2}$$

where the coefficients a, e, h and j form a four parameter family of Robertson-Vargas (RV) transformations. Precise knowledge of a, e, h and j determines the mechanics, and electrodynamics associated with the transformations, since each family of coefficients uniquely defines an associated physical theory, each of which must closely resemble SR. Lorentz[4] derived two sets of experimentally equivalent transformations referred to by Rindler[5] as LTA and LTB. LTB are the Lorentz-Einstein transformations. The values of the coefficients for LTA[6,7] and LTB are given in Table 1.

TABLE 1

Coefficient	LTA	LTB (SR)
a	γ*	γ
e	1	1
h	0	$\pm\gamma v/c^2$
j	γ^{-1}	γ

*Where $\gamma = (1 - v^2/c^2)^{-1/2}$

The presence of terms in LTA[4] which violate the symmetries associated with LTB became the primary factor which drove the physics of the early twentieth century towards LTB. At that time the group properties of LTB had become apparent, as had the apparent absence of group structure in LTA. Thus along with LTB arose a certain formalism which, it was thought[8], would be seriously disturbed by the anisotropies of LTA, however small they might be.

While the RV transformations do not possess group properties in four dimensions, they do in seven dimensional space, which is the carrier space for the transformations. Vargas[9] finds that both LTA and LTB are isomorphic in the RV group, and that rather than constraining the physics, the RV group structure expands the possible physical effects to be observed, and might be important for other areas in physics. For example, of particular significance is the fact that according to Vargas[9], space-time is at least seven-dimensional. Vargas finds that a passive rotation of any arbitrary coordinate system is not the same as an active rotation of a physical system, and this adds three more dimensions raising the total dimensionality of space to ten. Thus any experimental test which confirms the existence of Robertson's hypothesized rest frame, also implies a space-time of dimensionality equal to at least ten. Because it is impractical to consider testing all possible values of the Robertson-Vargas family of coefficients a, e, h and j, we restrict ourselves, at this stage, to testing the LTA transformations.

The LTA transformations which are derived from the coefficients given above in vector form are

$$\begin{aligned} \underline{r} &= \underline{R} + (\gamma - 1)\ (\underline{R} \cdot \underline{v})\ \underline{v}/v^2 - \gamma\underline{v}T \\ t &= T/\gamma \end{aligned} \tag{3}$$

Here ($\underline{R}$,T) are space and time coordinates in Robertson's rest frame S; ($\underline{r}$,t) pertain to any other arbitrary inertial frame $\underline{F}$; $\underline{v}$ is the velocity of F with respect to S. In order to avoid confusion with superscript notation, we do not use primes here and in Section 2.

Chang[10] has derived Maxwell's equations for anisotropic space from Equations (3). Below we show that potentially these equations predict a detectable variation in the one-way velocity of light as a function of the orientation of the propagation path, for propagation under non-vacuum conditions.

2. Theoretical Analysis of the Propagation of EM Waves

In terms of Equations (3), the expression for four-line element ds^2 in F is

$$ds^2 = (cdt - \underline{v} \cdot d\underline{r}/c)^2 - d\underline{r}^2 \quad (4)$$

Equation (4) can be rewritten in tensor form:

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \quad (5)$$

here $x^{\mu} = (x, y, z, ct)$ and $g_{\mu\nu}$ is the covariant metric tensor. Then the contravariant metric tensor is given as:

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & v_x/c \\ 0 & 1 & 0 & v_y/c \\ 0 & 0 & 1 & v_z/c \\ v_x/c & v_y/c & v_z/c & -1+v^2/c^2 \end{bmatrix} \quad (6)$$

For an electromagnetic (EM) wave in free space, the EM wave equation will be

$$g^{\mu\nu} \frac{\partial^2 A}{\partial x^\mu \partial x^\nu} = 0 \tag{7}$$

where A is a component of electric field $\underline{E}$ or magnetic field $\underline{B}$. Substituting (6) into (7), we get

$$\nabla^2 A + 2\left(\frac{\mathrm{v}}{c^2} \cdot \nabla\right) \frac{\partial A}{\partial t} - \left(1 - \frac{\mathrm{v}^2}{c^2}\right) \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = 0 \tag{8}$$

This is an explicit expression of the EM wave equation in anisotropic free space.

Chang[10] has proposed two theories for deriving the wave equation. Although the theory published in 1983 is not rigorous, it gives a simple derivation of the EM wave equation in an infinite medium without sources. Chang will present a rigorous treatment of the derivation in a separate paper.

In terms of (4), a notation x° can be introduced:

$$x^\circ = ct - (1/c)\underline{v} \cdot \underline{r} \tag{9}$$

which denotes the zeroth space coordinate in four dimensional space. Then (4) can be rewritten in a form which has four dimensional symmetry:

$$ds^2 = (dx^\circ)^2 - [(dx^1)^2 + (dx^2)^2 + (dx^3)]^2 \qquad (10)$$

Here x° is not equivalent to the time coordinate in the F frame (i.e., except for the privileged frame). The significance of this formulation is that physical laws have the same form in any inertial frame if x° is used as the zeroth space component. However, the same physical laws will have different forms in different frames if x° is given by the quantity $[ct - (\underline{v} \cdot \underline{r})/c]$. The four dimensional symmetry is broken when we use $(\underline{r},t)$ as variables. We recognize that if no physical consequence arises from this approach, this will be equivalent to a resynchronization.

We need to derive the wave equation for propagation in a homogeneous medium of refractive index η. Following chang's[10] 1983 illustrative approach, equations (9) and (10) yield the following relations for the differential operators:

$$\partial/\partial x^\circ = (1/c)\partial/\partial t \qquad (11a)$$

$$\nabla_3 = \nabla + (v/c^2)\partial/\partial t \qquad (11b)$$

The notations $(\partial/\partial x^\circ, \nabla_3)$ in (11) have the same definition as in special relativity, but they must be changed into the operators shown on the right hand sides of (11). Following the above rule, one may write the wave equation in the form:

$$\nabla_3{}^2 A - \eta^2 \frac{\partial^2 A}{(\partial x^\circ)^2} = 0 \tag{12}$$

for an EM wave propagating in a uniform medium without a source, where η is the refractive index. Now we replace the notations $(\nabla_3, \partial/\partial x^\circ)$ by using (11), and (12) becomes:

$$\left(\nabla + \frac{\underline{v}}{c^2} \frac{\partial}{\partial t}\right)^2 A - \frac{\eta^2}{c^2} \frac{\partial^2 A}{\partial t^2} = 0 \tag{13}$$

Equation (13) can be rewritten as:

$$\nabla^2 A + \frac{2}{c^2} (\underline{v} \cdot \nabla) \frac{\partial A}{\partial t} - \left(\eta^2 - \frac{v^2}{c^2}\right) \frac{\partial^2 A}{c^2 \partial t^2} = 0 \tag{14}$$

Equation (14) is a fundamental equation for an EM wave propagating in an infinite homogeneous medium in anisotropic space. We assume that the solution for a plane wave in orm:

$$A = A_0 e^{i(\omega t - k \cdot r)} \tag{15}$$

Substituting (15) into (14), we get:

$$\left(\eta^2 - \frac{v^2}{c^2}\right) \frac{\omega^2}{c^2} + \frac{2}{c^2} (\underline{v} \cdot \underline{k})\omega - k^2 = 0 \tag{16}$$

We define the group velocity as: $\underline{u}_g = \left(\frac{\partial\omega}{\partial k_x}, \frac{\partial\omega}{\partial k_y}, \frac{\partial\omega}{\partial k_z}\right)$

Equation (16) gives the solution:

$$u_g = \frac{c}{\eta + v \cos \theta / c} \tag{17}$$

Here θ is the angle between $\underline{v}$ and the direction of the EM wave. If $\eta = 1$, u_g is the expression for the one-way velocity of light in vacuum. According to (17), it is easy to see that the roundtrip time interval for a plane EM wave in an infinite stationary medium (or in a vacuum) is direction independent:

$$t_{rt} = \frac{L}{u_g(\theta)} + \frac{L}{u_g(\theta + \pi)} = \frac{2L\eta}{c} \qquad (18)$$

where L is the propagation distance. Therefore, the wave equations for propagation in a homogeneous infinite space (which corresponds to the situation for most optical experiments such as those of Brillet and Hall[11], and Trimmer et al.[12], to cite two examples) yield null results. Equation (18) also gives a method to measure the index of refraction η. The expression for the group velocity of light for propagation in a moving medium in the S frame is derived in Appendix I.

3. Experimental Arrangement and Measurements

Details of the experimental arrangement are given in a companion paper by Kolen and Torr[13]. Only a summary description is given here.

Measurements of the one-way time-of-flight of EM waves were made by propagating a 5 MHz wave generated by a stable Cesium atomic frequency standard with high performance beam tubes. The 5 MHz signal was injected into a 1 km coaxial cable filled with dry nitrogen maintained at a pressure of 2 psi above ambient. The center conductor was supported by a polyurethane spiral helical structure. The refractive index was 1.04. The roundtrip time-of-flight was also measured simultaneously by reflecting back part of the signal by an impedance mismatch at the end of the cable. The phase of the source and reflected signals was compared using a linear phase comparator which yielded an equivalent detection threshold for the time-of-flight of $\simeq 2 \times 10^{-12}$ seconds.

To make the one-way measurement, a second identical frequency standard was located at the west end of the cable, which was oriented in the east-west direction. The arrival time of a point of constant phase was compared at the west end with the phase of an identical signal generated by the west-end clock. Ideally, the difference in arrival times between the phase points at the phase comparator represents an arbitrary "turn-on" phase difference between the two unsynchronized standards at t = 0 plus an additional time difference directly dependent on the one-way time-of-flight of the phase point propagated through the cable. A detailed technical description of the experiment has been given elsewhere[13,14].

Laboratory calibration at zero separation yeilded a mean value of $\Delta\nu/\nu = 2.5 \times 10^{-14}$ frequency stability over a diurnal cycle. The clock housings were isolated from external electrical and magnetic influences and the temperature was actively controlled. Active modulation of the thermal environment was introduced via random cycling of the heating and cooling periods. Clock and cable temperatures were monitored every 10 minutes. The thermally induced variations in frequency were computed using a predetermined functional relationship. Analysis of the thermally induced variations confirmed that over a period of approximately one month, the randomization procedure reduced the average value to below detection threshold ($\Delta\nu/\nu$ = 3.5 x 10^{-15} for one month's integration). Data were taken for 150 days over 1 1/2 years. The detection threshold was estimated to be $\Delta\nu/\nu \simeq 1 \times 10^{-15}$. Data were averaged over 149 sidereal cycles.

4. Tentative Analysis of the One-Way Time-of-Flight Results

Results for the one-way time-of-flight are shown in Figure 1. The experiment yielded a departure from a constant value for the time-of-flight parameterized by (see Figure 1) $\delta t = 80 \times 10^{-12} \sin(\omega t)$ seconds for L = 1 km, where here ω is the angular velocity of the earth.

From the perspective of an observer in the Robertson frame, the oscillator frequencies of the clocks are retarded according to (3), and following the same procedure used in special relativity to derive the doppler shifted received frequency, ν_r, we find it is given by (Appendix II)

$$\nu_r = \nu_o \sqrt{\frac{(1 - V_s^2/c^2)}{(1 - V_r^2/c^2)}} \; \frac{(1 - \underline{V}_r \cdot \underline{K}_r/\Omega)}{(1 - \underline{V}_s \cdot \underline{K}_s/\Omega)} \tag{19}$$

Where $\underline{K}_r$, $\underline{K}_s$, and Ω are the wave vectors and angular frequency respectively observed from the S frame. The square root factor in (19) comes from the effect of time dilation due to the motion of the source and receiver. And we have

$$\underline{V}_s = \underline{v} + \underline{u}_s \qquad \underline{V}_r = \underline{v} + \underline{u}_r \tag{20}$$

where (20) effectively defines the relative velocities $\underline{u}_s$ and $\underline{u}_r$, between source, receiver and the laboratory measured in the S frame respectively. We also note that (20) is the appropriate velocity addition law in the Robertson rest frame.

In terms of (19), (20) and (III.4) in Appendix III, the frequency difference recorded by the receiver, i.e. calculated for the receiver frame is given to second order in v/c by

$$\frac{\Delta\nu}{\nu_o} = \frac{\nu_r - \nu_o}{\nu_o} = -\frac{(\underline{u}_r - \underline{u}_s)\cdot\underline{e}}{c^*} - \frac{(\underline{u}_r - \underline{u}_s)\cdot\underline{e}(\underline{V}_s\cdot\underline{e})}{c^{*2}}$$
$$+ \frac{(\eta^2 - 1)}{c^2}(\underline{V}_r\cdot\underline{v}_r - \underline{V}_s\cdot\underline{v}_s)$$
$$+ \frac{(\underline{u}_r - \underline{u}_s)\cdot v}{c^2} + \frac{u_r^2 - u_s^2}{2c^2} \tag{21}$$

where $c^* = c/\eta$.

Note that we introduce here $\underline{v}_r$ and $\underline{v}_s$, the velocities respectively of the medium near the receiver and source with respect to the S frame, $\underline{e}$ is the unit vector in the direction of propagation.

Here the fourth and fifth terms on the right hand side of (21) come from the expansion of the square root portion of (19). To the first order, the above four equations are valid for a wave propagating in a co-axial cable. From (3) and (17) we derive the following relations to the first order in v/c:

$$\begin{aligned}
&\underline{e} = \underline{e}' + \eta\underline{v}/c - \eta\underline{e}'(\underline{e}' \cdot \underline{v})/c, \\
&\underline{u}_r - \underline{u}_s = \underline{u}'_r - \underline{u}'_s, \\
&u_r^2 - u_s^2 = u'^2_r - u'^2_s = 0, \\
&(\underline{u}'_r - \underline{u}'_s) \cdot \underline{e}' = 0
\end{aligned} \tag{22}$$

and from (21) and (22),

$$\frac{\Delta\nu}{\nu_o} = \frac{1 - \eta^2}{c^2}(\underline{u}'_r - \underline{u}'_s) \cdot \underline{v} + \frac{\eta^2 - 1}{c^2}(\underline{V}_r \cdot \underline{v}_r - \underline{V}_s \cdot \underline{v}_s) \tag{23}$$

Suppose the medium is at rest in the laboratory system, i.e. $\underline{v}_s = \underline{v}_r = \underline{v}$. Then (23) reduces to null result:

$$\frac{\Delta\nu}{\nu_0} = 0 \tag{24}$$

The reason for this is given below. In deriving (17) for the group velocity, we used (14) where the metric is determined solely by the uniform Robertson velocity of the laboratory. When the medium actually moves with source and receiver, the motion within the laboratory frame affects the metric locally. In this case the metric depends on $\underline{V}_s$ and $\underline{V}_r$, i.e., on $\underline{u}_r$ and $\underline{u}_s$. Since Chang's[10] metric yields a null result, we suggest that the group velocity of light propagating in the moving medium depends only partially on the earth's rotation. The physical implication would be that light is only partially Fresnel convected in the case of rotational motion. In this case $\underline{v}_s$ and $\underline{v}_r$ can be generalized as

$$\begin{aligned} \underline{v}_s &= \underline{v} + \alpha\underline{u}_s \\ \underline{v}_r &= \underline{v} + \alpha\underline{u}_r \end{aligned} \tag{25}$$

In order to represent the partial Fresnel convection, the coefficient α is introduced which has a range $0 \le \alpha \le 1$. It is obvious that if $\alpha = 0$, $\underline{v}_s = \underline{v}_r = \underline{v}$, which is the null result case. On the other hand, if $\alpha = 1$, i.e., if there is no difference between the convection and rotational motion, $\underline{v}_s = \underline{V}_s$, $\underline{v}_r = \underline{V}_r$, and (23) becomes

$$\frac{\Delta\nu}{\nu_o} = \frac{1-\eta^2}{c^2}(\underline{u}'_r - \underline{u}'_s)\cdot\underline{v} + \frac{\eta^2-1}{c^2}(\underline{V}_r^2 - \underline{V}_s^2)$$

$$= \frac{\eta^2-1}{c^2}(\underline{u}'_r - \underline{u}'_s)\cdot\underline{v} \tag{26}$$

Since $(\underline{u}'_r - \underline{u}'_s)\cdot\underline{v} = -\omega L v \cos\omega' t'$, we obtain

$$\frac{\Delta\nu}{\nu_o} = -\frac{\omega L v}{c^2}(\eta^2 - 1)\cos\omega' t' \tag{27}$$

where L is the distance between the receiver and source. Then

$$\delta t' = \int_0^{t'} \frac{\Delta\nu}{\nu_o}\,dt' = -\frac{L}{c}\frac{v}{c}(\eta^2 - 1)\sin\omega' t' \tag{28}$$

For $\delta t' = 80 \pm 65$ ps, we find $\underline{v}$ in the equatorial plane to be less than 163 kms^{-1} with the vector pointing to ≃11:30 hours R.A. If $0 \le \alpha \le 1$, we have partial Fresnel convection of light and v > 163 km. The value of α must be determined by experiment. In terms of (25) the following result is derived for the fractional frequency variation predicted:

$$\frac{\Delta \nu}{\nu} = - \frac{\omega L v}{c^2} (\eta^2 - 1) \alpha \cos \omega' t' \tag{29}$$

which is a generalized form of (27). We also have

$$\delta t' = - Lv (\eta^2 - 1) \alpha \sin \omega' t' / c^2 \tag{30}$$

If we assume v = 390 km/s measured by Smoot, et al.[15], Equation (30) yields $\delta t' \simeq 80 \sin \omega' t'$ ps for $\alpha = 0.46$. The physical implication is that the rotational motion of the earth results in only partial Fresnel convection of the EM wave. As mentioned above, this sensitivity of the metric to angular motion means that space-time exhibits velocity dependent properties in rotating homogeneous media. This result suggests the interesting conclusion: that within the context of the Robertson preferred frame hypothesis, a Riemannian metric is consistent with the null predictions of special relativity, whereas if space-time is velocity dependent, the associated velocity dependent metric allows for the non-null case. It should be noted that this space-time is a more general depiction of the usual Riemannian case. Equation (30) which is an experimental result, if confirmed, will define the form of the metric. The reason the dependence on rotational motion only shows up in the one-way results is because, in this case, we rely on the rotational motion to reverse the orientation of the experiment. These results are in good agreement in phase and amplitude with the results of Smoot, et al.[15] In the case of the rocket-borne hydrogen maser experiment of Vessot and Levine[16], and others where $\eta = 1$, null results are derived.

5. Discussion

The theoretical significance of these results lies in the interpretation of what is the quantity that clocks actually measure. Since no clock synchronization was used in the experiments reported here, the results serve to determine the appropriate temporal coordinate to be identified with the measurement of time i.e., the results, if confirmed, show that $x^0 = ct - (1/c)\, \underline{v} \cdot \underline{r}$ and not ct. The actual character of space-time, however, remains unchanged, even though the appearance of the metric becomes frame dependent. The invariance of the line element is preserved, although we must expect small new terms to appear in the equations of mechanics, electrodynamics, gravitation, particle physics, quantum mechanics, and other areas of Physics which, nevertheless, closely resemble relativity in numerical values. Vargas and Vargas and Torr[9] discusses the group properties of the LTA group in the context of the larger RV group and the impact on other areas of physics in detail.

Acknowledgements

This work was supported by NASA Grant NAGW-119 and NAG8-047. We thank M. R. Torr for laboratory support, Dr. V. Gordon Lind for departmental support at Utah State University, Y. Clifton and J. G. Vargas for useful discussions.

APPENDIX I

The Velocity of Light in a Moving Medium

According to Equation (14), the four-line element in a medium-rest frame F ($\underline{r}$,t) is

$$ds^2 = g_{\mu\nu} d^2 x^\mu dx^\nu = (d\underline{r})^2 - 1/\eta^2 (cdt - \underline{v} \cdot d\underline{r}/c)^2 \qquad (I.1)$$

Using the transformation Equation (3)

$$\underline{r} = \underline{R} + (\gamma - 1)(\underline{R} \cdot \underline{v})\underline{v}/v^2 - \gamma \underline{v} T \qquad (Eq.3)$$

$$t = T/\gamma$$

We obtain

$$ds^2 = (d\underline{R})^2 + \gamma^2 (1 - 1/\eta^2)(cdT - \underline{v} \cdot d\underline{R}/c)^2 - c^2 dT^2 \qquad (I.2)$$

Where $\underline{R}$,t are the space and time coordinate in S frame. For the propagation of light, let $ds^2 = 0$, Equation (I.2) yields

$$U_g = c\left[1 + \left(\frac{\underline{v} \cdot \underline{e}}{c}\right)^2 \gamma^2(1 - 1/\eta^2)\right]^{-1}$$

$$\left[\sqrt{1 - \gamma^2(1 - 1/\eta^2)\left[1 - \left(\frac{\underline{v} \cdot \underline{e}}{c}\right)^2\right]} \right. \tag{I.3}$$

$$\left. + \gamma^2\left(1 - \frac{1}{\eta^2}\right)\left(\frac{\underline{v}}{c} \cdot \underline{e}\right)\right]$$

where the upper case U_g refers to the velocity of light in S. Here $\underline{e}$ is the unit vector of the velocity of light.

To the first order of $(\underline{v}/c)$, (I.3) reduces to

$$U_g = (c/\eta) + (1 - 1/\eta^2)(\underline{v} \cdot \underline{e}) \tag{I.4}$$

which is the same as Fresnel's formula for the convection of light. The vector form of (I.4) is

$$\underline{U}_g = \frac{c}{\eta}\,\underline{e} + \left(1 - \frac{1}{\eta^2}\right)(\underline{v} \cdot \underline{e})\underline{e} \tag{I.5}$$

APPENDIX II

The Transformation of the Frequency and Wave Vector

Consider that the phase angle is an invariant

$$\Phi = \Omega T - \underline{K} \cdot \underline{R} = \omega t - \underline{k} \cdot \underline{r} \tag{II.1}$$

Where Ω and $\underline{K}$ are the angular frequency and wave vector in Robertson rest frame S, and ω and $\underline{k}$ are in frame F. Using the transformation Equation (3), we obtain

$$\omega = \gamma(\Omega - \underline{v} \cdot \underline{K}) = \gamma\Omega(1 - \underline{v} \cdot \underline{K}/\Omega) \tag{II.2}$$

$$\underline{k} = \underline{K} - \frac{1 - \gamma^{-1}}{v^2} (\underline{v} \cdot \underline{K})\underline{v} \tag{II.3}$$

Assume a source with frequency ν_o moves with velocity $\underline{V}_s$ with respect to S, Equation (II.2) becomes

$$2\pi\nu_o = \gamma_s\Omega(1 - \underline{V}_s \cdot \underline{K}_s/\Omega) \tag{II.4}$$

Assume the receiver moves with velocity $\underline{V}_r$ with respect to S, the Equation (II.2) gives

$$2\pi\nu_r = \gamma_r\Omega(1 - \underline{V}_r \cdot \underline{K}_r/\Omega) \tag{II.5}$$

Equation (II.5) divided by (II.4) yeilds

$$\nu_r = \nu_o\sqrt{\frac{1 - V_s{}^2/c^2}{1 - V_r{}^2/c^2}}\left[\frac{1 - \underline{V}_r \cdot \underline{K}_r/\Omega}{1 - \underline{V}_s \cdot \underline{K}_s/\Omega}\right] \tag{II.6}$$

This is the Doppler formula [Equation (19)].

APPENDIX III

The Relationship Between $\underline{K}$ and $\underline{e}$

We start with (16)

$$\left(\eta^2 - \frac{v^2}{c^2}\right)\frac{\omega^2}{c^2} + \frac{2}{c^2}(\underline{v} \cdot \underline{k})\omega - k^2 = 0$$

Replace ω, $\underline{k}$ by using (II.2) and (II.3). We obtain

$$\left(\eta^2 - \frac{v^2}{c^2}\right)\gamma^2\left(\frac{\Omega}{c} - \frac{\underline{v}}{c} \cdot \underline{K}\right)^2 + 2\left(\frac{\underline{v}}{c} \cdot \underline{K}\right)\left(\frac{\Omega}{c}\right)$$

$$- \left(\frac{\underline{v}}{c} \cdot \underline{K}\right)^2 - (\underline{K})^2 = 0 \qquad \text{(III.1)}$$

In terms of the definition of group velocity,

$$\underline{U}_g = \left(\frac{\partial\Omega}{\partial\underline{K}_x}, \frac{\partial\Omega}{\partial\underline{K}_y}, \frac{\partial\Omega}{\partial\underline{K}_z}\right)$$

where the upper case $\underline{U}_g$ represents the velocity of light in the S frame. We have

$$\underline{U}_g = \frac{c^2\left(\dfrac{\underline{K}}{\Omega}\right) - \underline{v}}{\left(\eta^2 - \dfrac{v^2}{c^2}\right)\gamma^2\left(\dfrac{1}{c} - \dfrac{\underline{v}}{c} \cdot \dfrac{\underline{K}}{\Omega}\right) + \dfrac{\underline{v}}{c} \cdot \dfrac{\underline{K}}{\Omega}} + \underline{v} \qquad \text{(III.2)}$$

Solving for K/Ω, to first order,

$$\frac{\underline{K}}{\Omega} = \frac{\eta^2}{c^2}\underline{U}_g\left[1 - (\eta^2 - 1)\left(\frac{\underline{v}}{c^2} \cdot \underline{U}_g\right)\right] - (\eta^2 - 1)\frac{\underline{v}}{c^2} \qquad \text{(III.3)}$$

Substituting (I.5) into (III.3), and evaluating K/Ω at the location of the receiver and the source, we get

$$\frac{\underline{K}_{r,s}}{\Omega} = \frac{\eta}{c}\underline{e} - (\eta^2 - 1)\frac{\underline{v}_{r,s}}{c^2} = \frac{\underline{e}}{c^*} - (\eta^2 - 1)\frac{\underline{v}_{r,s}}{c^2} \qquad \text{(III.4)}$$

where $v_{r,s}$ are the velocities of the medium with respect to the S frame and $c^* = c/\eta$. Substituting (III.4) into (19), we obtain

$$\nu_r = \nu_o \sqrt{\frac{1 - V_s^2/c^2}{1 - V_r^2/c^2}}\; \frac{1 - (\underline{V}_r \cdot \underline{e})/c^* + (\eta^2 - 1)(\underline{V}_r \cdot \underline{v}_r)/c^2}{1 - (\underline{V}_s \cdot \underline{e})/c^* + (\eta^2 - 1)(\underline{V}_s \cdot \underline{v}_s)/c^2}.$$

REFERENCES

[1]H. P. Robertson, Rev. Mod. Phys., 21, 378, 1949.

[2]J. G. Vargas, Revised Robertson's Test Theory of Special Relativity, Found. of Phys., 14, 625, 1984.

[3]J. G. Vargas, Lett. Nuovo Cimento, 28, 289, 1980.

[4]H. A. Lorentz, The Theory of Electrons, Dover Publications, Inc., New York, 1909.

[5]W. Rindler, Am. J. Phys., 38, 111, 1970.

[6]H. A. Lorentz, Amst. Proc., 6, 809, 1904.

[7]J. G. Vargas, Nonrelativistic Para-Lorentzian Mechanics, Found. of Phys., 11, 235, 1981.

[8]W. Pauli, Theory of Relativity, Dover Publications, Inc., New York, 1981.

[9]J. G. Vargas, Structure and Effects of the Theories that Result from Relaxing the First Principle of Relativity, Ph.D. Thesis, Utah State University, 1986; J. Vargas, Revised Robertson's Test Theory on Special Relativity: Supergroups and Superspace, in press, Foundations of Physics, 1986; J. Vargas and D. G. Torr, Revised Robertson's Test Theory of Special Relativity: Space-Time Structure and Dynamics, in press, Foundation of Physics, 1986.

[10]T. Chang, Maxwell's Equations in Anisotropic Space, Phys., Lett., 70A, 1, 1979; Found. of Phys., 13, 1-13, 1983.

[11]A. Brillet and J. L. Hall, Phys. Rev. Lett., 42, 549, 1979.

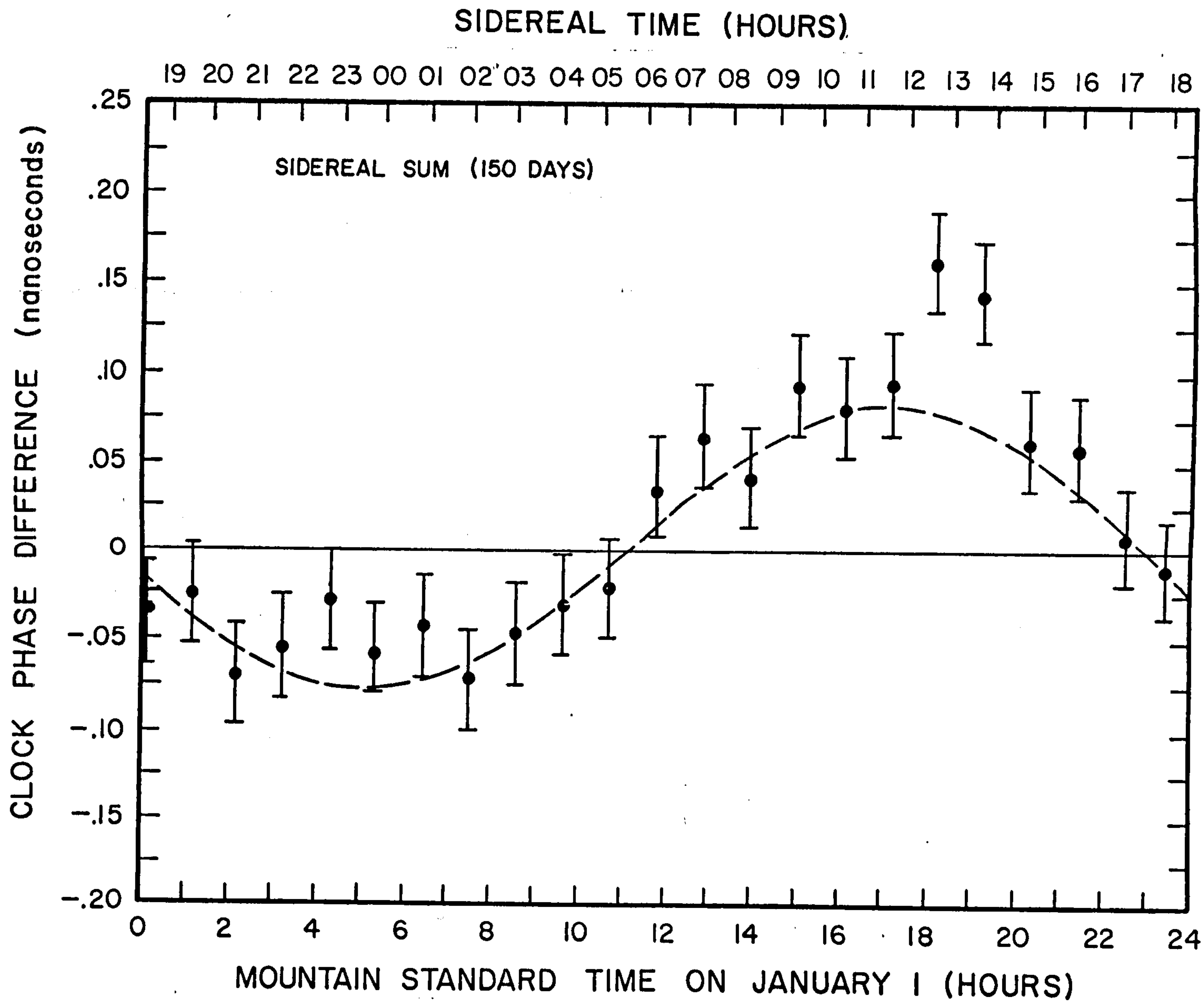
[12]W. S. N. Trimmer, R. F. Baierlein, J. E. Faller, and H. A. Hill, Phys. Rev. D, 8, 3321, 1973.

[13]P. T. Kolen and D. G. Torr, Test of the Dimensionality of Space by Direct One-Way Time-of-Flight Measurement, companion paper this issue, Phys. Rev., 1986.

[14]D. G. Torr and P. Kolen, Found. of Phys., 12, 265, 1982.

[15]G. F. Smoot, et al., Phys. Rev. Let., 39, 898, 1977.

[16] R. F. C. Vessot and M. W. Levine, General Relativity and Gravitation, 10, 181, 1979.



OBSERVED VARIATION IN THE ONE-WAY TIME-OF-FLIGHT OF A 5 MHz WAVE OVER A DISTANCE OF 1km IN A COAXIAL WAVEGUIDE OF REFRACTIVE INDEX 1.04