NASA-CR-204779

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FINAL REPORT

SOLAR FLARE PHYSICS

June 8, 1997

NAG 5-2001 (UMD 01-5-26120)
A Grant from the Goddard Space Flight Center,
National Aeronautics and Space Administration

January 1, 1997 to June 30, 1997

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Solar Flare Physics

We have continued our previous efforts in studies of fourier imaging methods applied to hard X-ray flares. We have performed physical and theoretical analysis of rotating collimator grids submitted to GSFC for future space or suborbital missions involving the imaging of solar flares in hard X-rays. In particular, we have simulated the performance of the High Energy Solar Spectroscopic Imager (HESSI), using pseudo-flare images provided by Dr. David Alexander at Lockheed-Martin Palo Alto Research Lab. We have computed count rates that HESSI would record for these simulated flares, and reconstructed images from the count rates. These results were presented at the "HXT Image Reconstruction Workshop, January 13-16, 1997, at Lockheed-Martin Palo Alto Research Lab.

The following pages show the parameters of the HESSI simulation (Section I), the mathematics of image synthesis (Section II), and the results of reconstructing the images provided by Dr. Alexander. In general, from this study we conclude that the HESSI telescope will provide better imaging of solar flares than previous hard X-ray imagers.

SECTION I
THE PARAMETERS OF THE HESSI SIMULATION

HESSI IMAGING PARAMETERS

- PITCHES: 2, 3.5, 6, 10.4, 18, 31.2, 54,93.5, 162 arc sec
- ROTATION RATE: 15 RPM => All Fourier components in 2 sec
- FIELD OF VIEW: 1/50 radian = 1.1 degree
- PHOTON TAGGING: ~ 2 microsec

SIMULATION OF MODEL HESSI COUNT RATES

- Select N_{coll} collimators from the 7 (or possibly 9) grid pairs.
- Select number of count bins (N_{bins}) appropriate to pitch, time interval and photon flux:
  - Mean count rate ~ 1/4 photon flux on subcollimator
  - Number of bins < 1/10 count rate x time interval
  - Number of pitch cycles=radial distance/pitch
  - Number of bins > 2 x number of pitch cycles
- Transparancy Matrix: 64 x 64 x N_{bins} x N_{coll}
- Randomize photons by one of the following equivalent methods:
  - Introduce exponentially distributed time delays
  - Shift the photons' trajectories
  - Poisson transform the model source
  - Poisson transform the count rate (preferred method)
- Multiply the model source by transparancy matrix for each collimator and time bin.
- Total the products for each time bin to obtain binned count rate.

RECONSTRUCTION OF IMAGES

Back Projection and Spatially-Oriented Methods

- Multiply each Transmission Matrix by count rate and sum, producing a "Dirty Map"
- Construct a Point-Spread Function (PSF) from the Transmission Matrix.
- Deconvolve the Dirty Map using one of the following:

  - Cornwell MEM
  - Hogbom CLEAN
  - Richardson-Lucy Maximum Likelyhood
  - Fourier Inversion

- Disadvantages:

  - Not straightforward to include Chi-squared statistics, which are based on the count rates in the time domain.
  - Requires high count rates to define a useful PSF.
  - PSF is not exactly spatially invariant.

Reconstruction Constrained by Count Rate Chi-Squared

- Sakao's Method:

  - Maximize Entropy for a model of the source;
  - Iteratively modify the source model, minimizing the Chi squared of the count
  - Stop when Chi squared equals 0.5.

- Advantages:

  - Convergence is relatively fast;
  - Can watch the convergence process;
  - The Chi-squared value can be used for goodness of fit.

- Disdvantages:

  - Convergence depends on many parameters;
  - Convergence is not unique: one chisquared, many entropies;
  - Some parts of the count-rate profiles are fit better than others.

1. HESI collimators

The High Energy Spectroscopic Imager (HESI) is a Rotation Modulation Collimator (RMC) telescope proposed to be launched by NASA to image hard X-ray and gamma ray flares during solar maximum around the turn of the century. One of its possible configurations is 9 parallel collimators with grids that subtend angles ranging from 2 to 162 arc seconds. The image below shows small segments of the grids.

Fig. 1: Image of 9 grids with 77 deg relative rotations

The above image is shown with poor resolution, and the finer grids are highly aliased. Click it to see the larger and more detailed version.

The grid pitches in the above figure are (reading from the bottom up):2.00, 3.46, 6.00, 10.39, 18.00, 31.18, 54.00, 93.53, 162.00 arc seconds. The field of view of each square is 128 x 128 arcseconds.

The pitches are derived from a powerlaw with base b=sqrt(3). That is, pitch(N)=2*b^N, where N=0,1,2,3,4,5,6,7,8. This means that every other grid in the sequence have pitches in a 1:3 ratio.

HESSI Imaging Parameters

- PITCHES: 2, 3.5, 6, 10.4, 18, 31.2, 54,93.5, 162 arc sec
- ROTATION RATE: 15 RPM => All Fourier components in 2 sec
- FIELD OF VIEW: 1/50 radian = 1.1 degree

- PHOTON TAGGING: ~ 2 microsec

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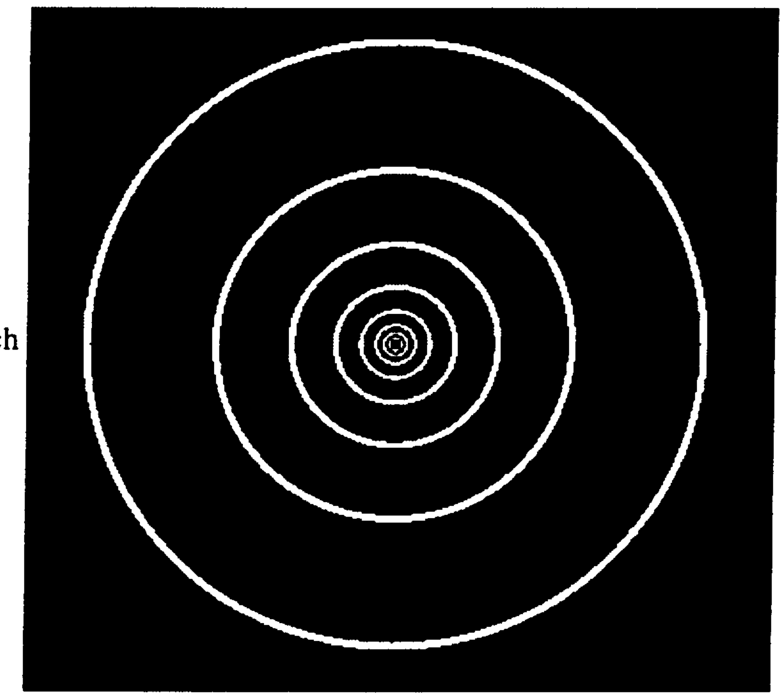
2. UV plane and comparison with HXT

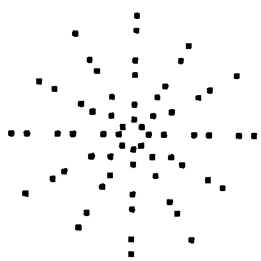
The spatial frequencies that an RMC is sensitive to are inversely proportional to the pitches of its collimator grids. For a given instantaneous orientation of a collimator, the sensitive frequency is represented by a point in the wave number plane (called the U,V plane). As the collimator rotates, the locus of sensitive spatial frequencies traces out a circle in the U,V plane. For HESI, there are 9 such circles, as shown below.

Fig. 2a: HESI: Circles at radii=1/pitch



The spatial frequencies that the Hard Xray Telescope (HXT) on Yohkoh is sensitive to are shown below.

Fig. 2b: HXT: U,V Points



Since the HXT telescope does not rotate, the spatial frequencies it is sensitive to are represented by discrete points in the U.V plane. There are 64 such points, compared with the potential thousands of points in HESI's U,V plane.

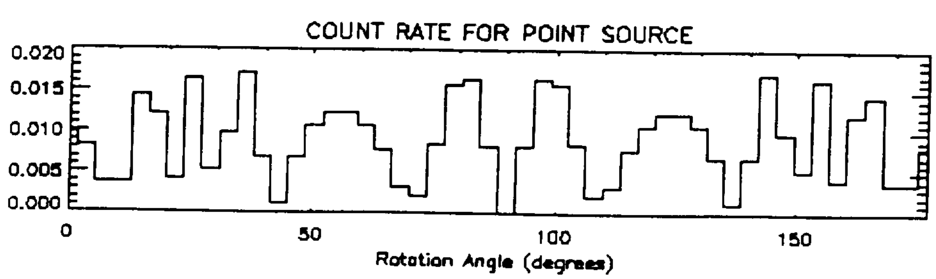
3. How modulation occurs (high photon rates)

For a point source with high flux, the detectors behind each collimator record a "chopped" signal which provides the modulation profile for that collimator. If the source is constant in time and space, the modulation is regular and almost sinusoidal. To compute this, we take the projected image of a collimator against the sky, and select a single source point of that projected image. As the image rotates, the brightness of that source point changes with time, going from zero to 100% of the peak brightness. The projected field seen through an RMC is shown for 100 rotation angles in the figure below.

Fig. 3: The projected field of a single collimator at 100 rotation angles between 0 and 180 degrees.

4. Modulation for a point source

To illustrate the modulation of an extended source by the HESI collimators, we use a point source model. The modulation is computed by multiplying the instantaneous point-source response by the model map and totaling the product for that time. The RMCs are assumed to rotate at 15 RPM, so the collimators rotate 180 degrees in 2.0 s. The total is obtained at 50 intervals during the 180 degrees of rotation.



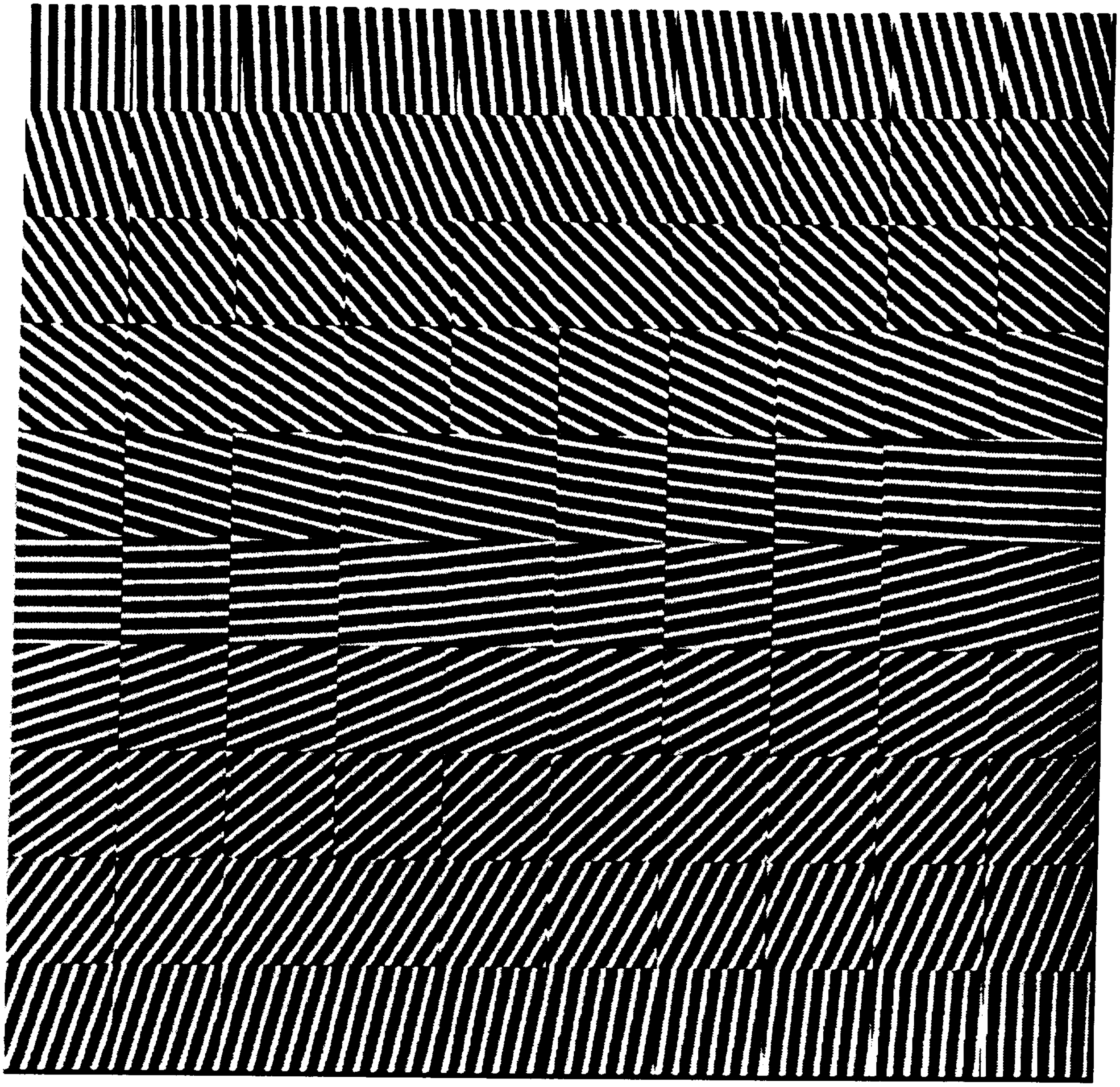


Fig. 4: Plot of modulation vs time for a single collimator.

In general, for extended sources, modulation profiles show the following characteristics:

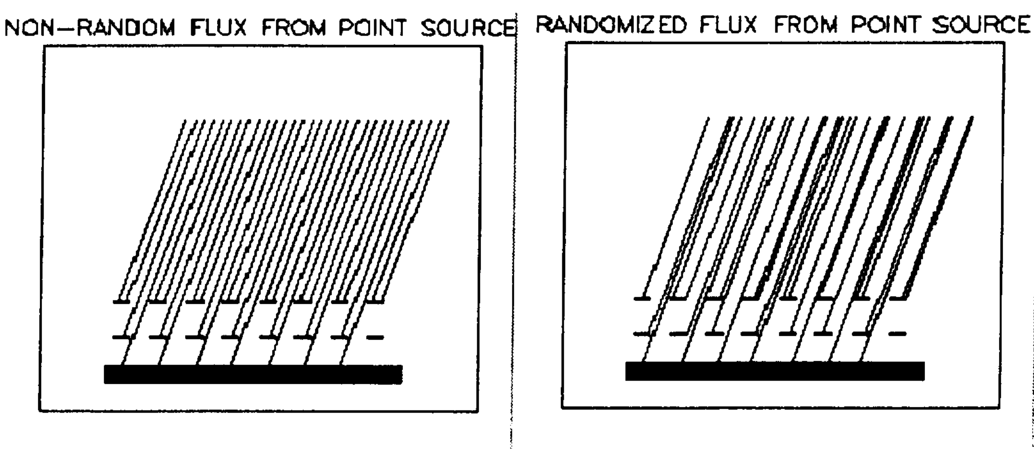
- The coarsest collimator shows the highest modulation efficiency;
- The mean value for each 0.5 s interval is close to the true flux;
- The modulation amplitude goes to zero for the finest grids because an extended source is over resolved.

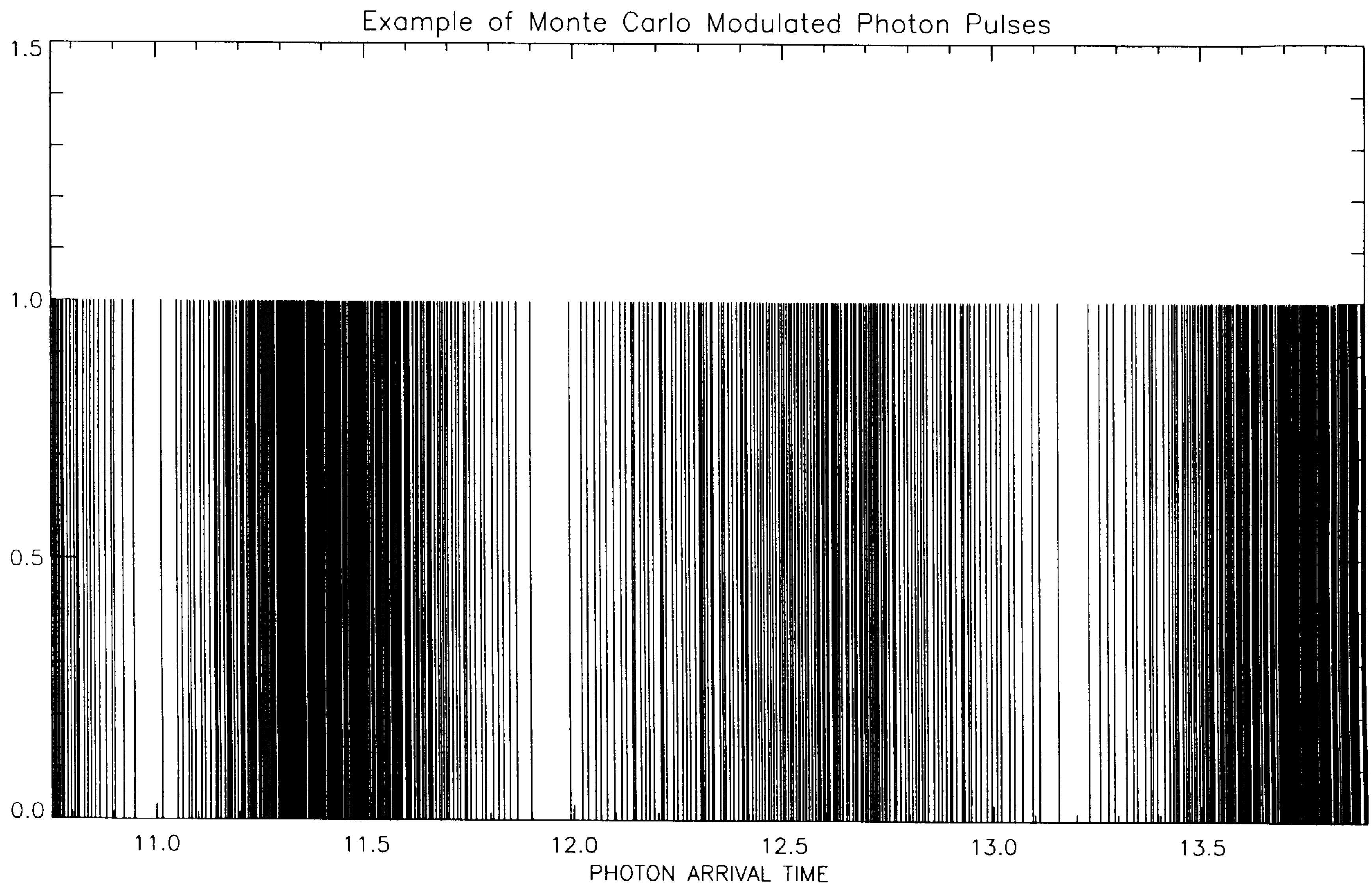
5. HESI time-tagged photons

Because the HESI instrument records the time of arrival and the energy of photons, sections 3 and 4 above are really a "swindle". To pass from a model image to time-tagged photon lists, one must go through several steps. We assume the image is for a 0.5 s period.

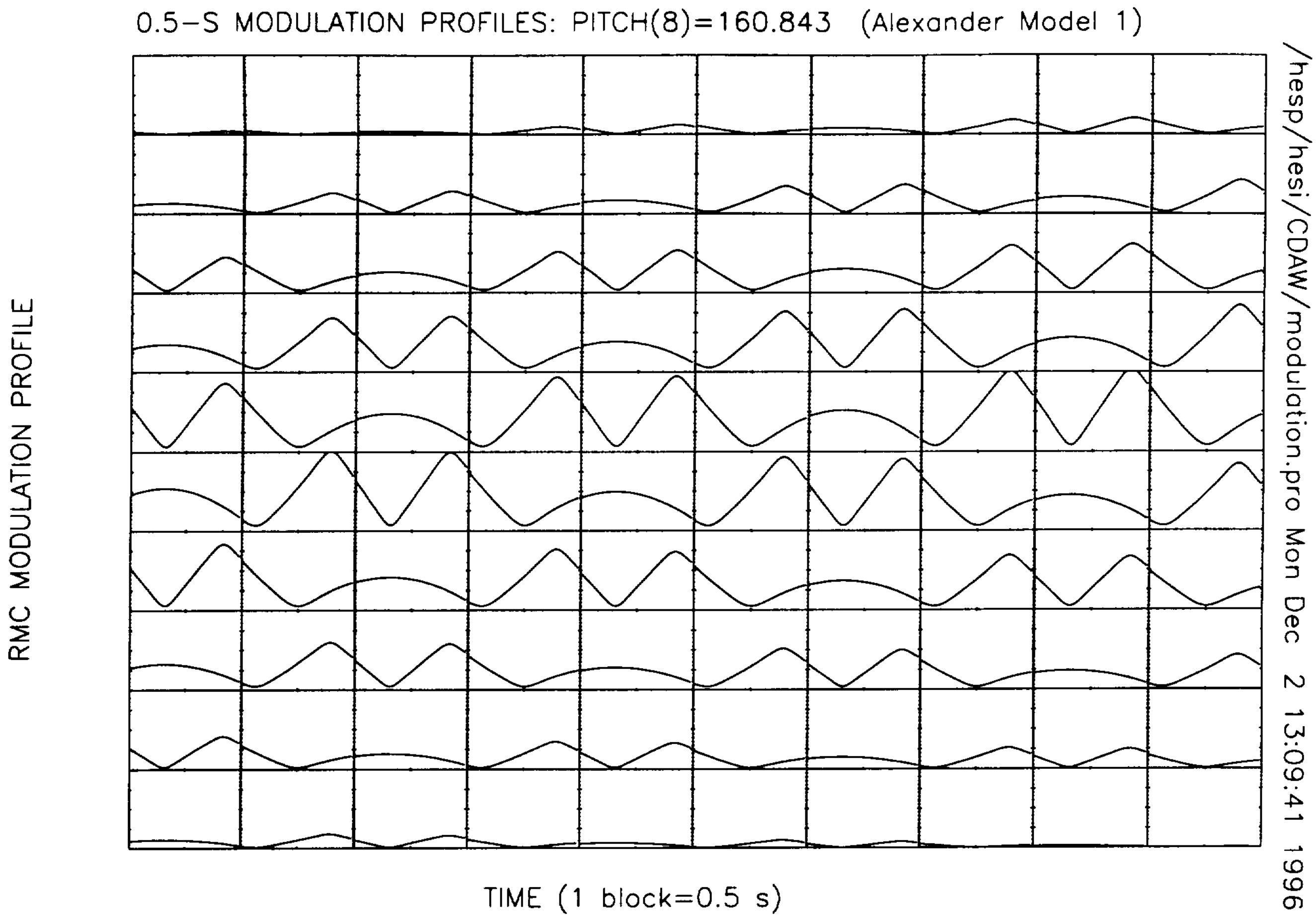
1. For each pixel of the image, take the photons and spread them out at random times in the 0.5 s interval. (This process introduces Poisson statistics into the data).
2. Spread the time-randomized photons, while maintaining their vectorial directions, shift their paths to random set of points of incidence into the collimators.
3. Trace the path of the photons, and list the times of arrival of those photons that successfully pass through both grids.

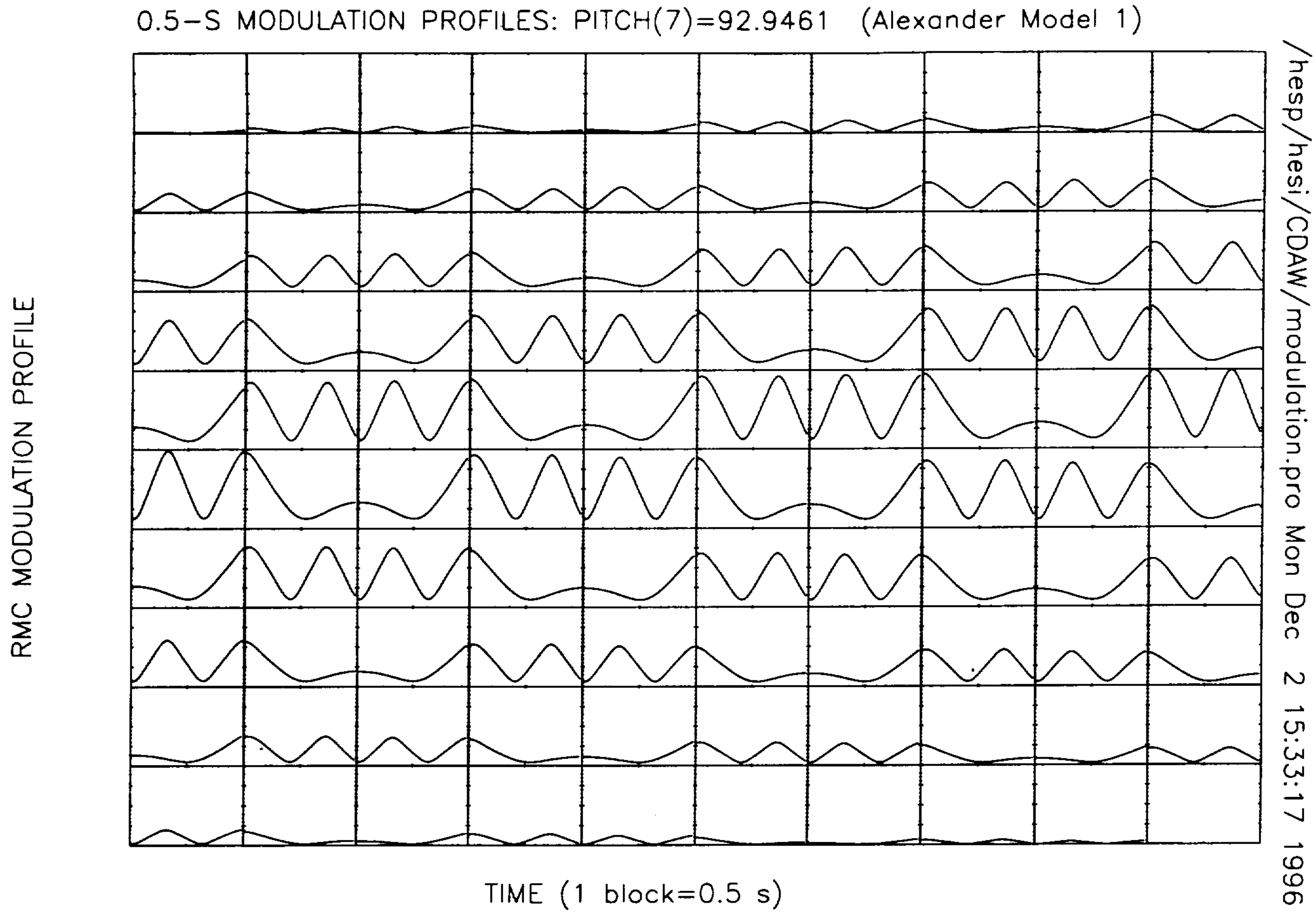
The figure below illustrates step 2.

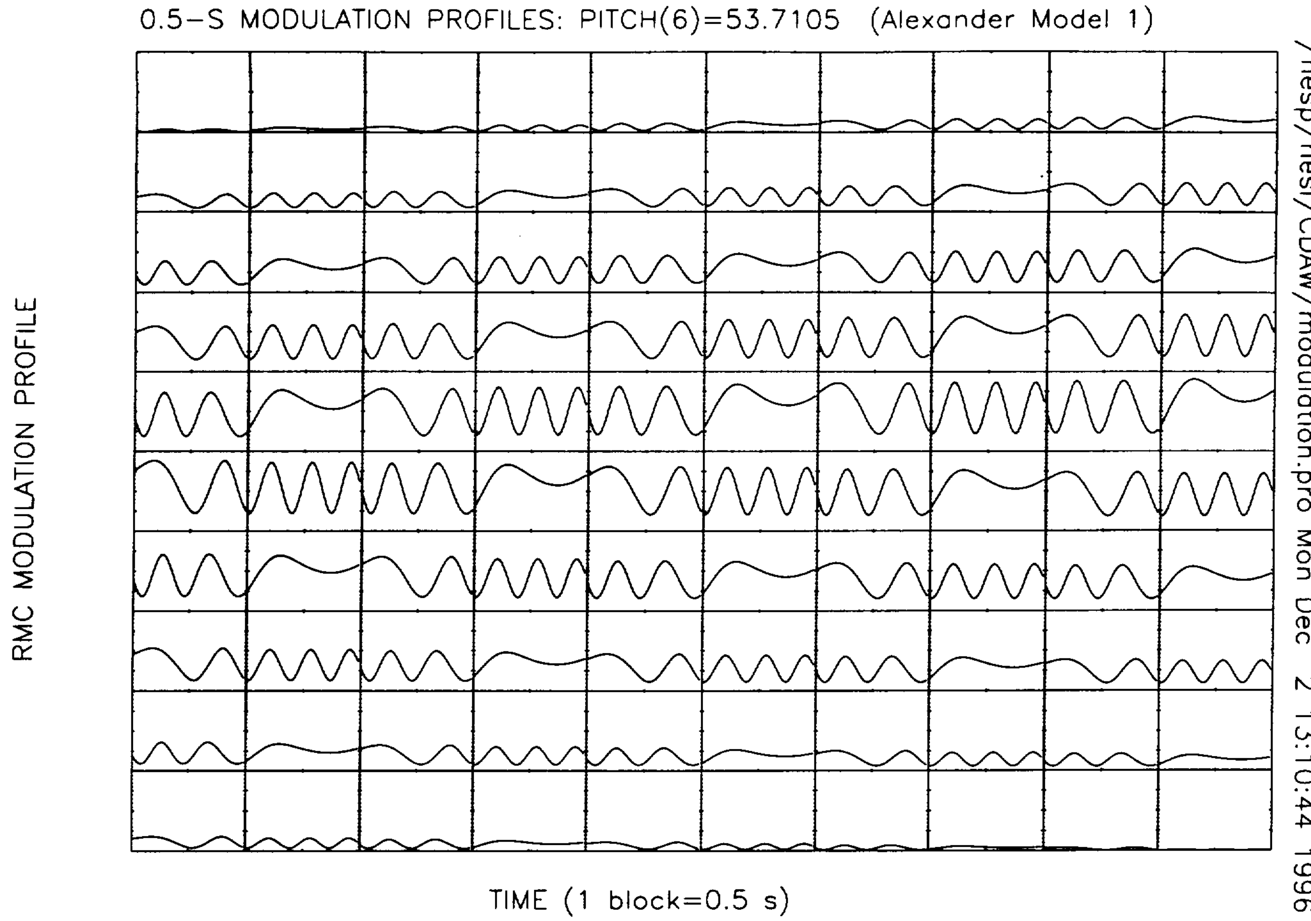


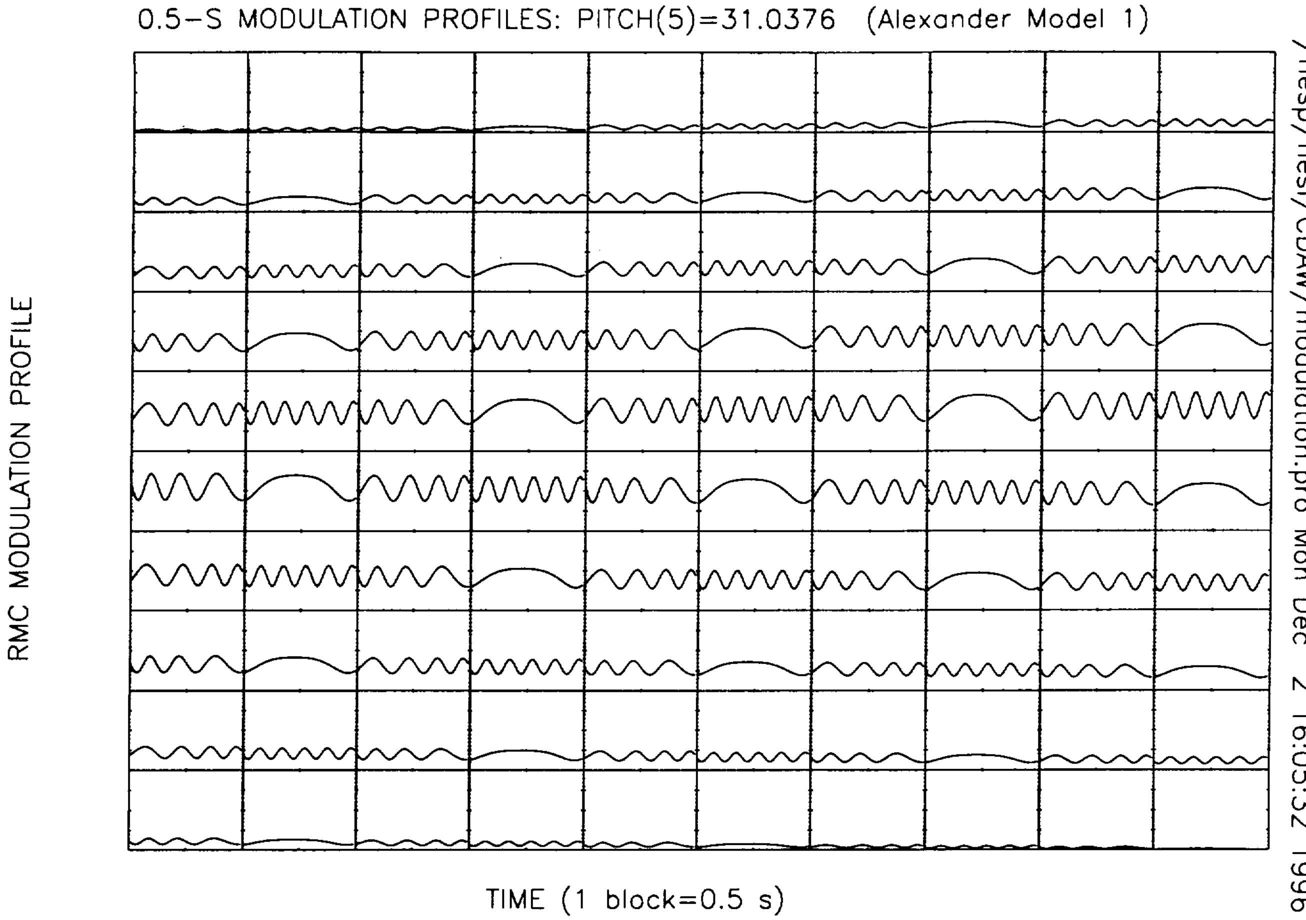


Example of Monte Carlo Modulated Photon Pulses
1.5
1.0
0.5
0.0
11.0
11.5
12.0
12.5
13.0
13.5
PHOTON ARRIVAL TIME

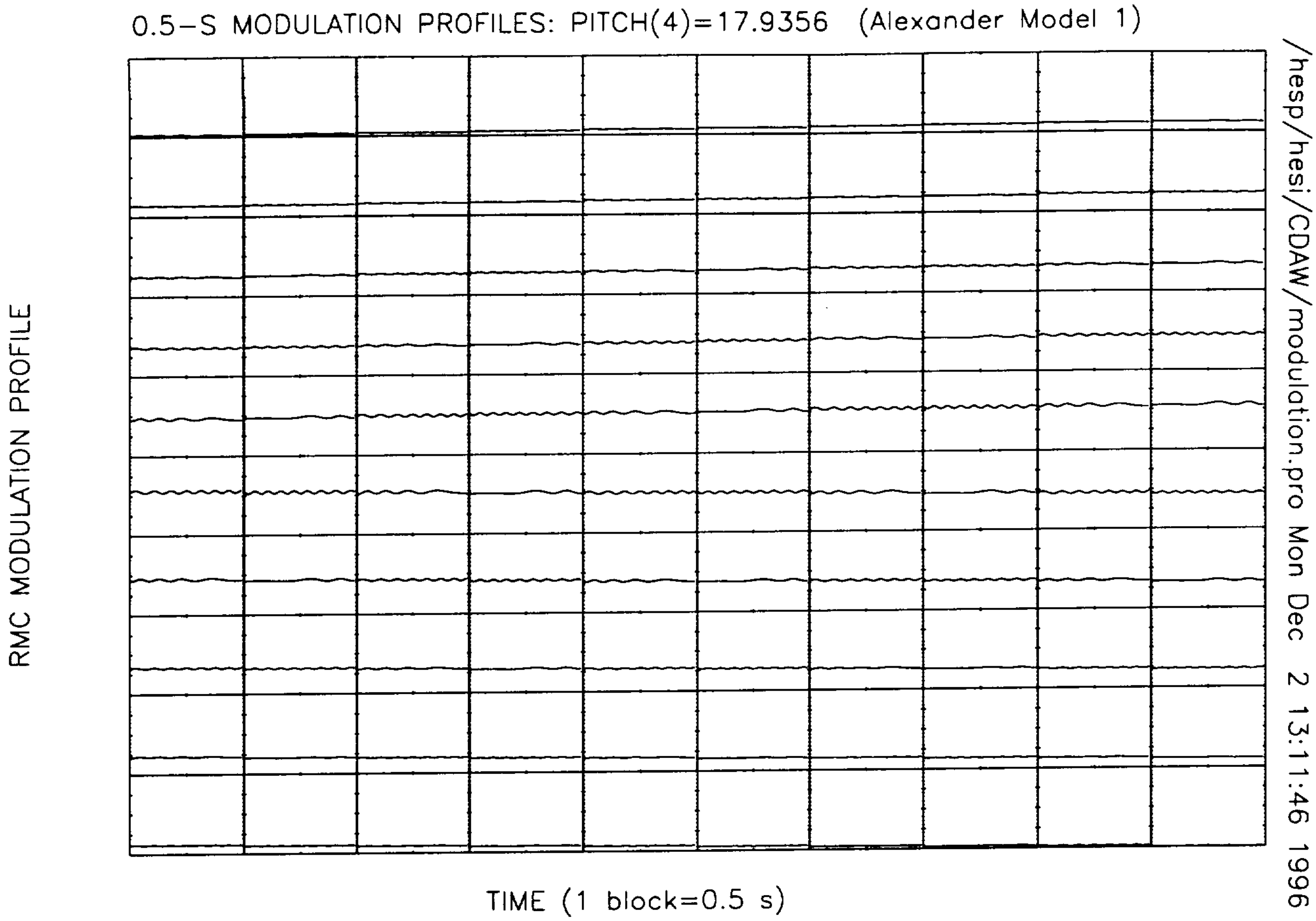

0.5-S MODULATION PROFILES: PITCH(8)=160.843 (Alexander Model 1)
RMC MODULATION PROFILE
TIME (1 block=0.5 s)
/hesp/hesi/CDAW/modulation.pro Mon Dec 2 13:09:41 1996


0.5-S MODULATION PROFILES: PITCH(7)=92.9461 (Alexander Model 1)
RMC MODULATION PROFILE
TIME (1 block=0.5 s)
/hesp/hesi/CDAW/modulation.pro Mon Dec 2 15:33:17 1996


0.5-S MODULATION PROFILES: PITCH(6)=53.7105 (Alexander Model 1)
RMC MODULATION PROFILE
TIME (1 block=0.5 s)
/hesp/hesi/CDAW/modulation.pro Mon Dec 2 13:10:44 1996



0.5-S MODULATION PROFILES: PITCH(5)=31.0376 (Alexander Model 1)
RMC MODULATION PROFILE
TIME (1 block=0.5 s)
/hesp/hesi/CDAW/modulation.pro Mon Dec 2 16:05:32 1996


0.5-S MODULATION PROFILES: PITCH(4)=17.9356 (Alexander Model 1)
RMC MODULATION PROFILE
TIME (1 block=0.5 s)
/hesp/hesi/CDAW/modulation.pro Mon Dec 2 13:11:46 1996

SECTION II
THE MATHEMATICS OF IMAGE SYSTHESIS

T. SAKAO (1994)

The principle of the image synthesis is to obtain a two-dimensional brightness distribution B_{ij} which maximizes the entropy S among those which gives b_K consistent with the observations b_K^0, *i.e.*,

$$\boxed{\chi^2_{\nu=64} \equiv \frac{1}{64} \sum_{K=1}^{64} \frac{(b_K - b_K^0)^2}{{\sigma_K^0}^2} \sim 1.0 \,,} \tag{5.19}$$

where σ_K^0 is a standard deviation of the photon count error which includes both Poisson and systematic errors. Then, this principle becomes equivalent with maximizing the following quantity $\tilde{S}$,

$$\boxed{\tilde{S} = S - \frac{\lambda}{2}\chi^2 \,.} \tag{5.20}$$

Here λ (or $\lambda/2$) is a parameter which compromises relative weight between observations (expressed as χ^2) and entropy S. To increase λ means to put more weight on the observations than the entropy term or smoothness of images, hence χ^2 value decreases while synthesized images (B_{ij}) contain increased structure. The final image is obtained when the χ^2 value satisfies eq.(5.19). By taking partial derivatives of eq.(5.20) with respect to B_{ij} to be zero, we obtain

$$\boxed{B_{ij} = \overline{B^0} \exp\left[-S_{\text{unit}} + \lambda \overline{B^0} \sum_{K=1}^{64} \left\{ \frac{1}{{\sigma_K^0}^2} \left(b_K - \sum_{m,n=1}^{N} P_{mn,K} B_{mn} \right) P_{ij,K} \right\}\right] \,.} \tag{5.21}$$

Here S_{unit} is the unit entropy per pixel, *i.e.* $S_{\text{unit}} = S/N^2$. Starting from a uniform initial brightness distribution $B_{ij}^{(0)}$, or so-called gray map, we obtain the maximum entropy image B_{ij} at a given λ by successively taking weighted average of iteration in eq.(5.21) as follows :

T. SAKAO (1994)

$$\boxed{B_{ij}^{(l+1)} = (1-\gamma)B_{ij}^{(l)} + \gamma\overline{B^0}\exp\left[-S_{\rm unit}^{(l)} + \lambda\overline{B^0}\sum_{K=1}^{64}\left\{\frac{1}{\sigma_K^{0\,2}}\left(b_K - \sum_{m,n=1}^{N} P_{mn,K}B_{mn}^{(l)}\right)P_{ij,K}\right\}\right]}$$

$$(l = 0,1,2,3,\ldots) \qquad (5.22)$$

Here $B_{ij}^{(l)}$ is the image after l–th iteration, and γ is a weighting factor called *iteration gain*. Note that a larger γ makes the iteration more stable but the convergence slower. Usually $\gamma \sim 0.02-0.1$ gives satisfactory convergence of images within reasonable number of iteration.

In many cases, the iteration eq.(5.22) gives satisfactory convergence. However, the exponential term in eq.(5.22) may sometimes cause convergence of the iteration difficult, since small changes in the index of the exponential would result in large changes in $B_{ij}^{(l+1)}$. To make the convergence more stable, if the total flux in the resultant $B_{ij}^{(l)}$ differs significantly from the observations, then $B_{ij}^{(l)}$ is practically re–normalized to have the observed total flux $B_{\rm tot}^0$:

$$\boxed{B_{ij}^{(l)} \Longrightarrow \left(\frac{B_{\rm tot}^0}{\sum B_{ij}^{(l)}}\right)B_{ij}^{(l)}\,.} \qquad (5.26)$$

As λ in eq.(5.22) cannot be determined from the observations, we assume certain values of λ in the iteration. In the actual iteration we start with $\lambda \sim 0.2$ and when the iteration converges, increment λ by a certain amount (typically ~ 0.1) and restarts the iteration (eq.(5.22)) with the new λ. Converged B_{ij} obtained by the last sequence of iterations (with the old λ) is used as the initial brightness distribution $B_{ij}^{(0)}$ in the new iteration sequence.

T. SAKAO (1994)

The observed X-ray count rate b_K^0 for the K-th modulation collimator, due to the brightness distribution B_{ij}, is given by using two-dimensional *response matrix* $P_{ij,K}$ as follows :

$$\boxed{b_K^0 = \sum_{i,j=1}^{N} P_{ij,K} B_{ij}^0 + n_K^0 \ ,} \tag{5.9}$$

where n_K^0 represents uncertainties in b_K^0 due to Poisson statistics and systematic errors inevitably included in observations.

Now let us return to eq.(5.9). The maximum entropy solution which corresponds to eq.(5.9) can be written as

$$\boxed{b_K = \sum_{i,j=1}^{N} P_{ij,K} B_{ij} \ ,} \tag{5.14}$$

where B_{ij} gives MEM image of hard X-ray sources. We adopt the entropy expression given originally by Frieden (1972) :

$$\boxed{S = - \sum_{i,j=1}^{N} \left(\frac{B_{ij}}{\overline{B^0}} \right) \cdot \ln \left(\frac{B_{ij}}{\overline{B^0}} \right) \ ,} \tag{5.15}$$

and $\overline{B^0}$ is the average incident hard X-ray photon counts per pixel :

$$\overline{B^0} = \frac{1}{N^2} B_{tot}^0 \ . \tag{5.16}$$

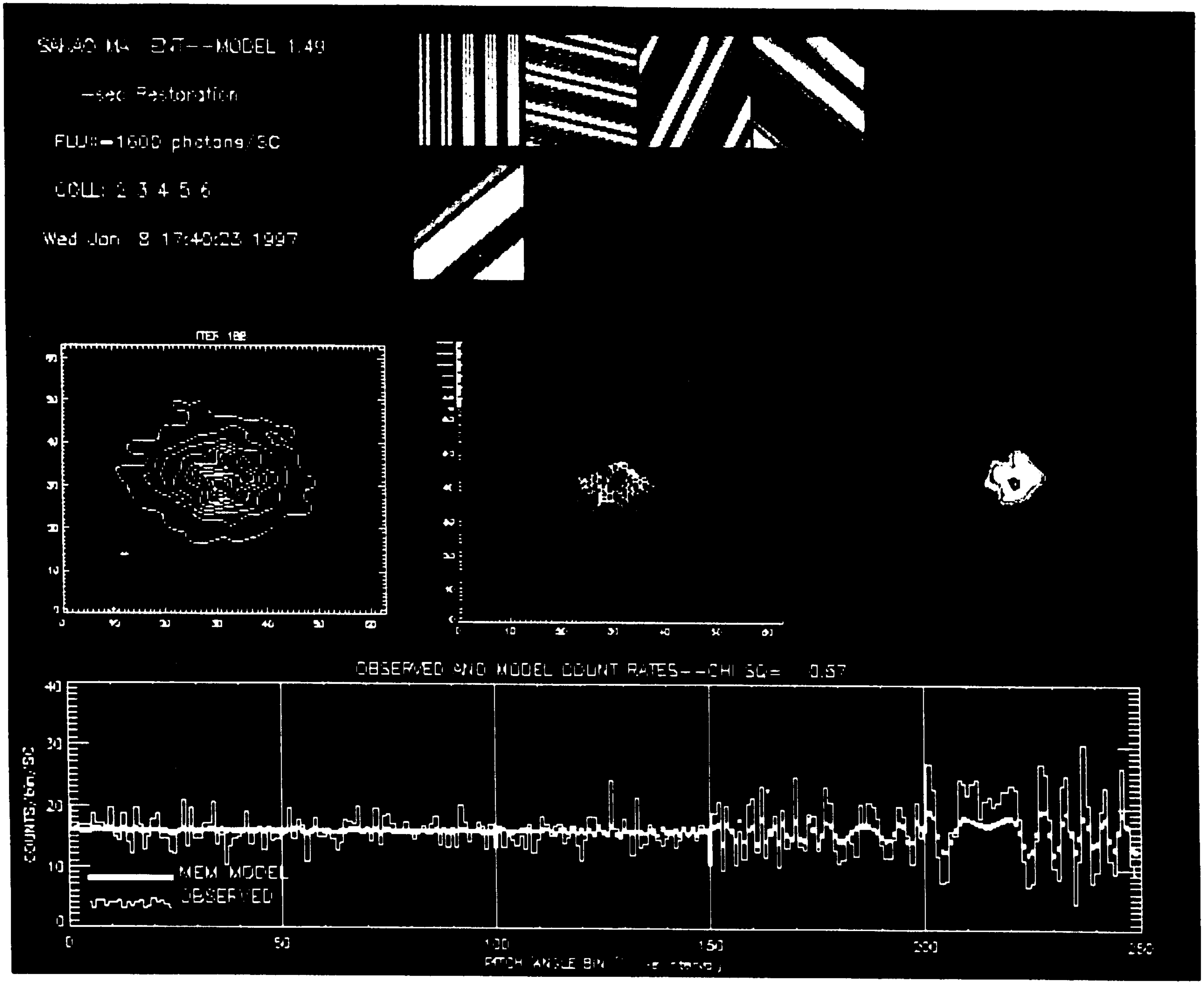
Larger entropy S implies less structures in the image (or, I can say, smoother images), *i.e.*, maximizing entropy under no observational constraints simply gives flat brightness distribution : B_{ij} = const. In eq.(5.15), $B_{ij}/\overline{B^0}$ is the normalized pixel counts and hence the above entropy expression is independent of the absolute value of B_{ij} itself.

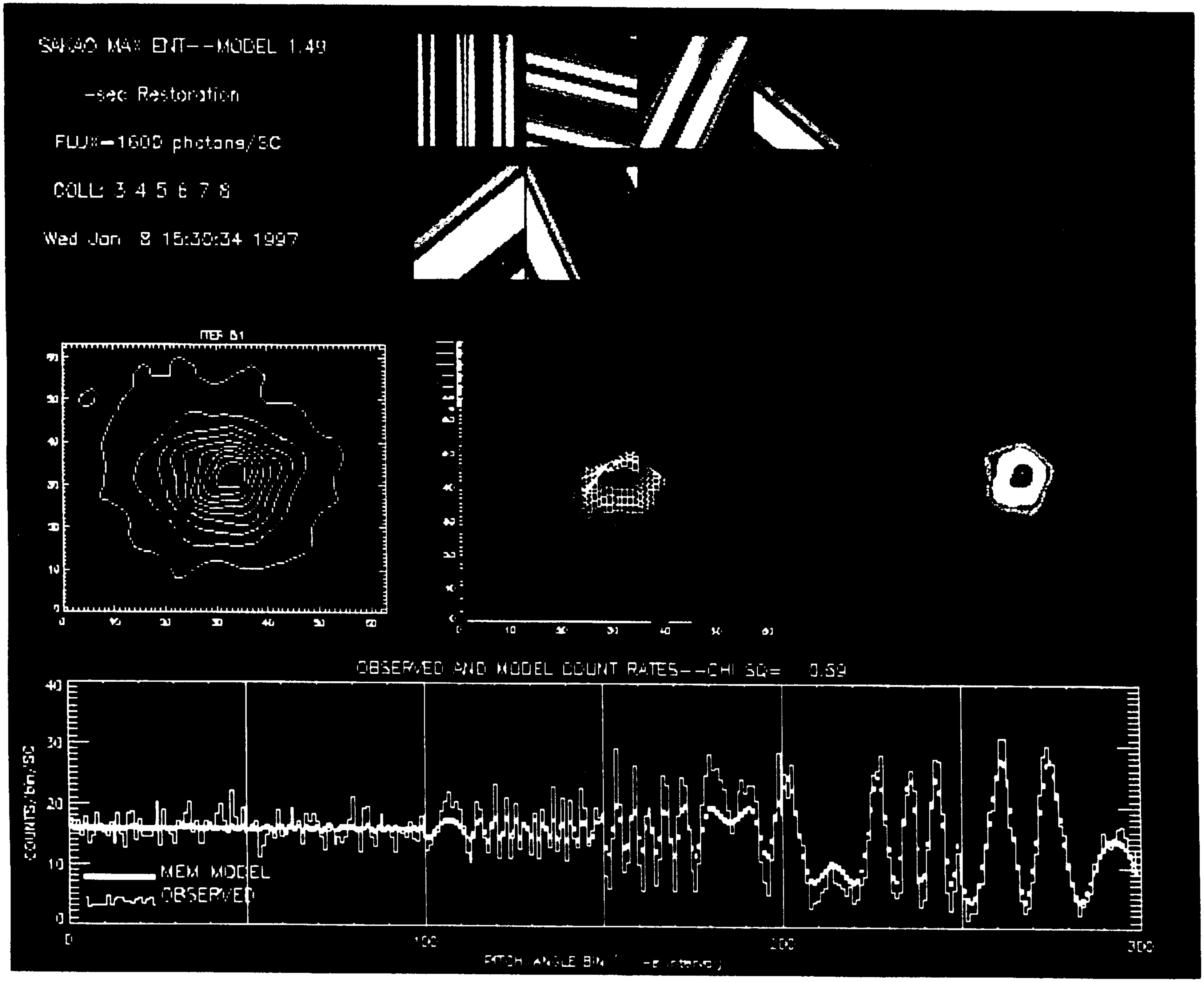
SECTION I
RECONSTRUCTING PSEUDO IMAGES

MODEL 1

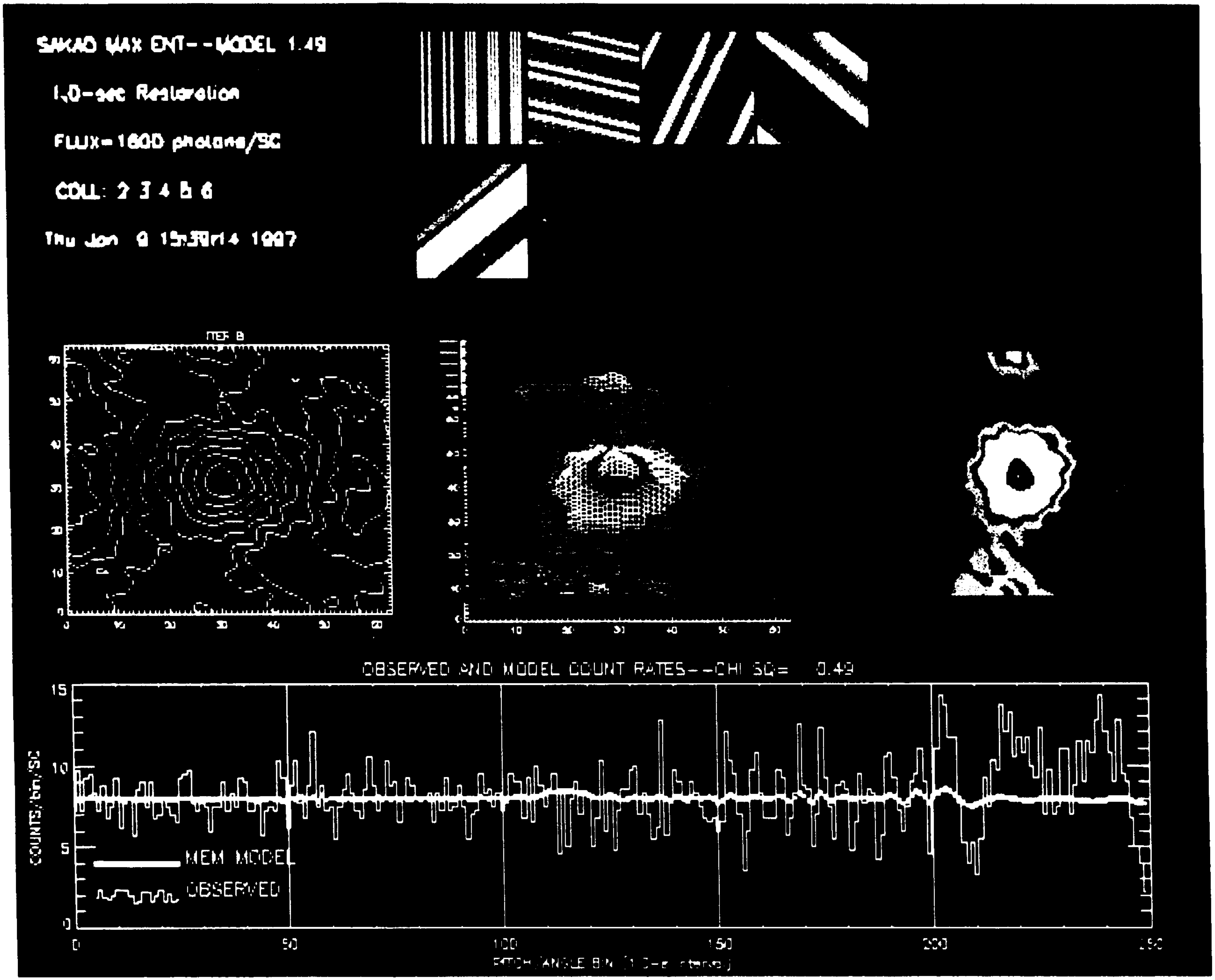
FRAME 49

HESSI 1- AND 2-S SIMULATIONS


-sec Restoration
FLUX=1600 photons/SC
COLL: 2 3 4 5 6
Wed Jan 8 17:40:23 1997
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.57
COUNTS/bin/SC
MEM MODEL
OBSERVED


SAKAO MAX ENT--MODEL 1.49
-sec Restoration
FLUX=1600 photons/SC
COLL: 3 4 5 6 7 8
Wed Jan 8 15:30:34 1997
ITER 61
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.59
COUNTS/bin/SC
MEM MODEL
OBSERVED
PITCH ANGLE BIN

※ 2.00 ct/s/SC Frame #1

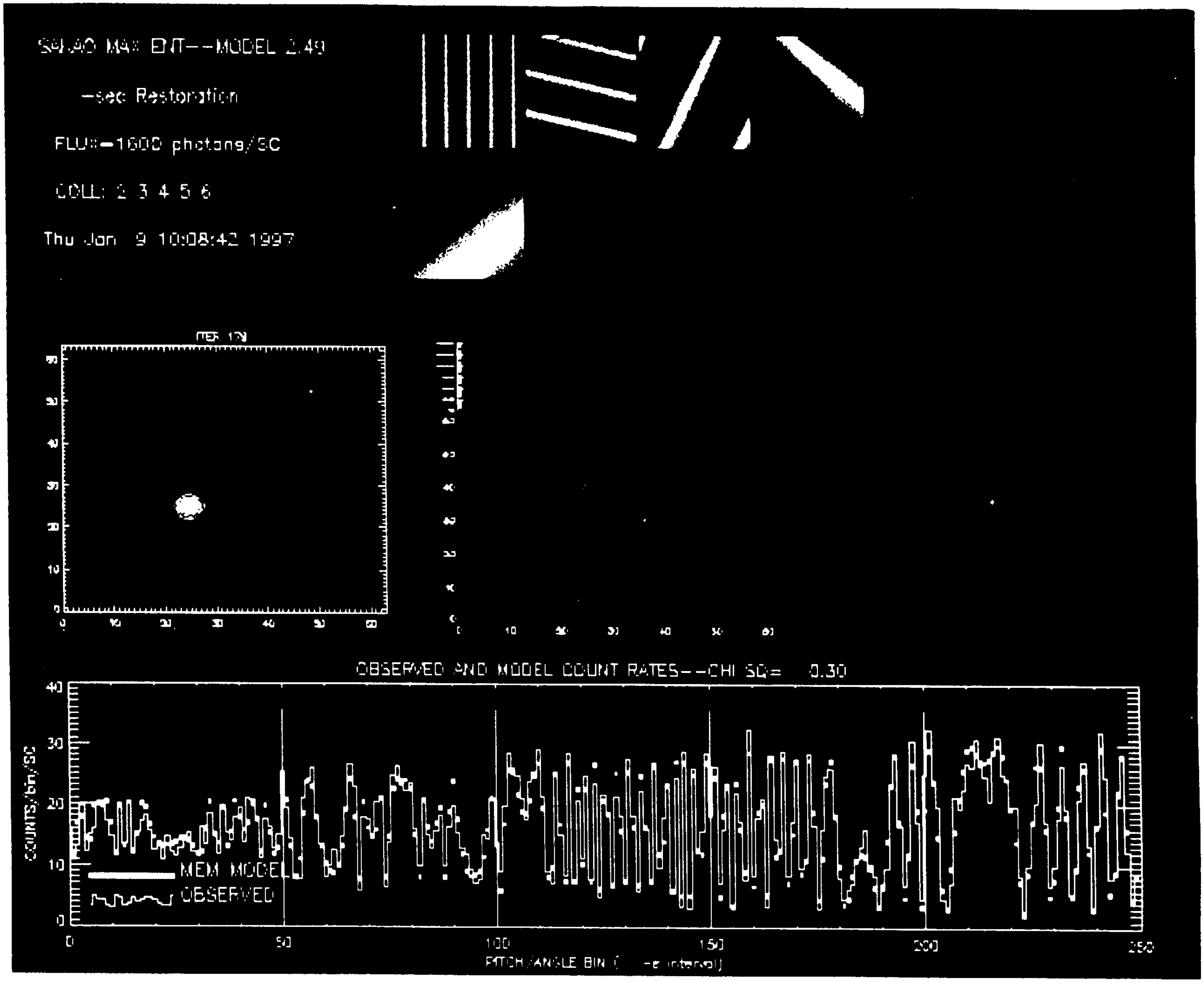


MODEL 2

FRAME 49

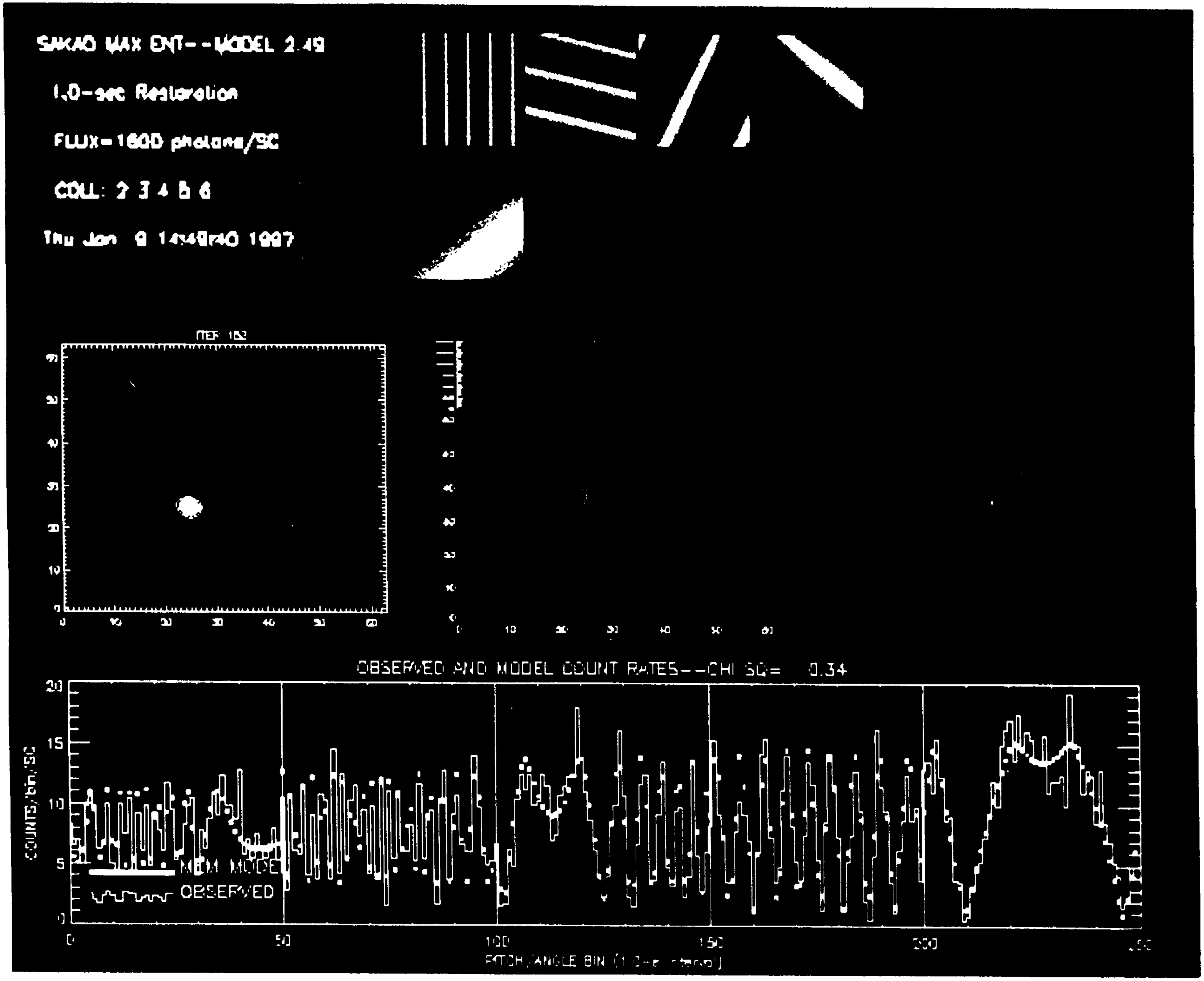
HESSI 1- AND 2-S SIMULATIONS

≈ 100 cts/s/SC



SMAO MAX ENT--MODEL 2.49
-sec Restoration
FLUX=1600 photons/SC
COLL: 2 3 4 5 6
Thu Jan 9 10:08:42 1997
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.30
COUNTS/bin/SC
MEM MODEL
OBSERVED
PITCH ANGLE BIN

473/SC


SAKAO MAX ENT--MODEL 2.49
1.0-sec Restoration
FLUX=1600 photons/SC
COLL: 2 3 4 5 6
Thu Jan 9 14:49:40 1997
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.34
COUNTS/bin/SC
MEM MODEL
OBSERVED

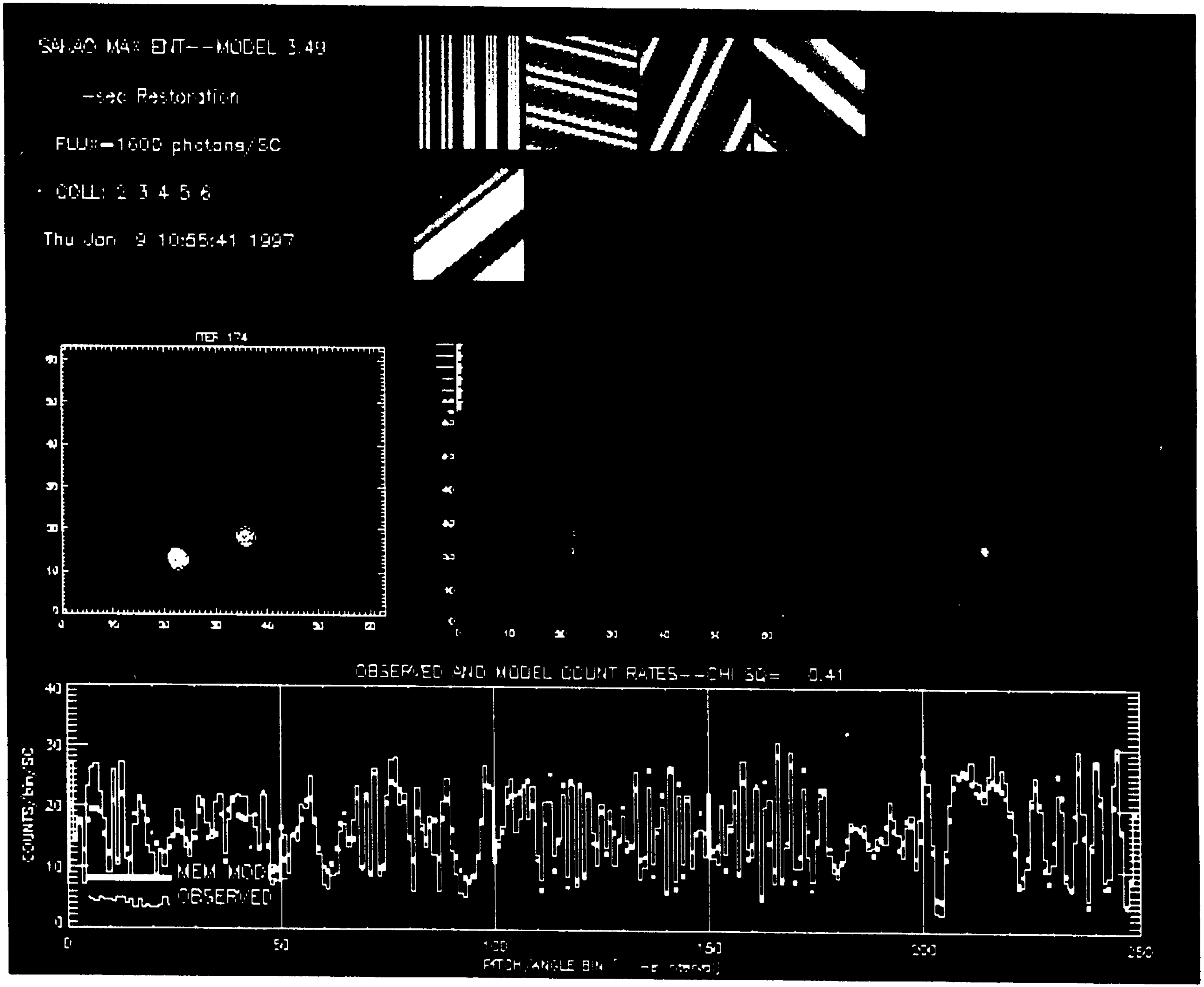
MODEL 3

FRAME 49

HESSI 1- AND 2-S SIMULATIONS

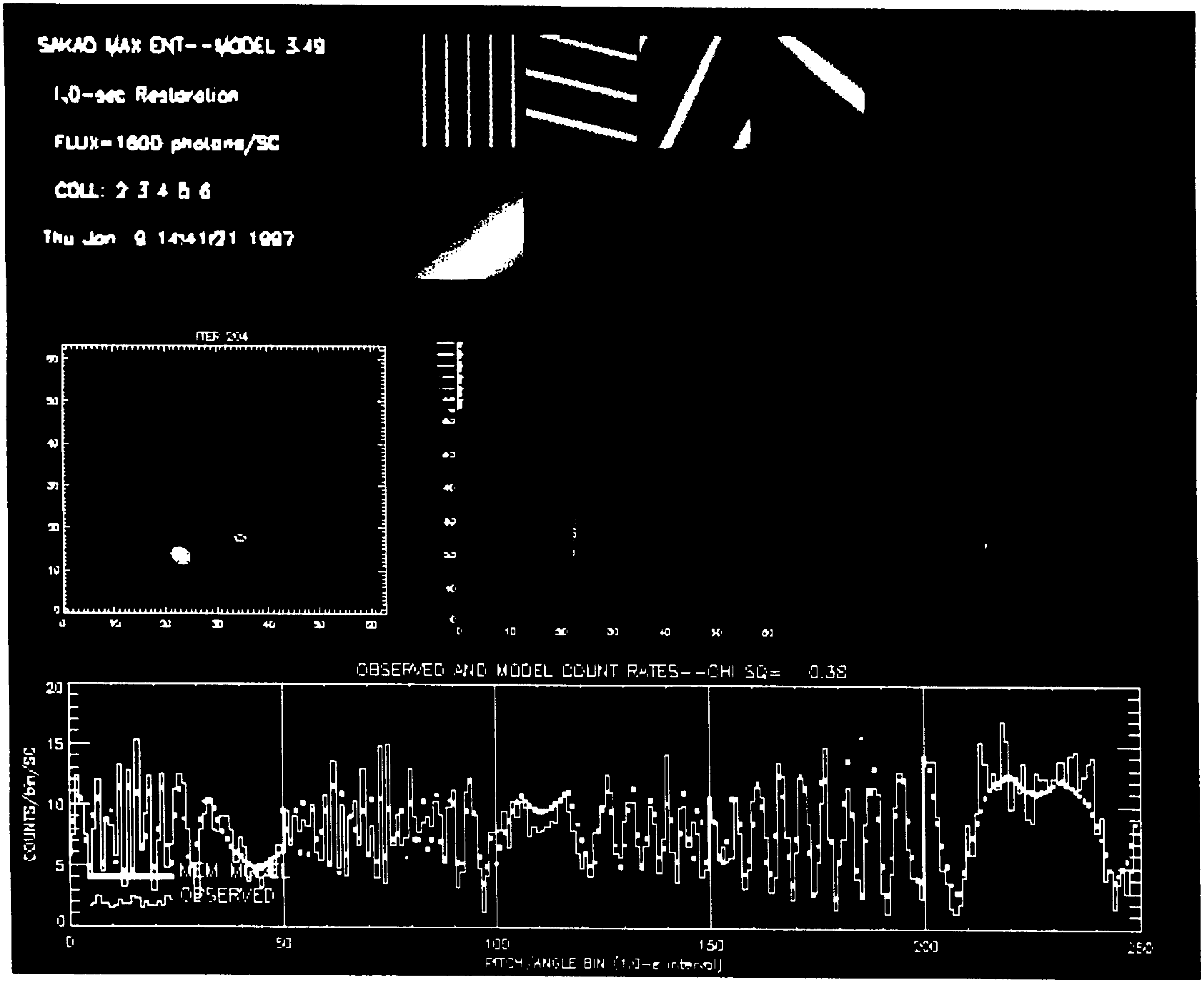
≈ 100 cts/s/SC

Frame 4.9



9200 cts/s/sc

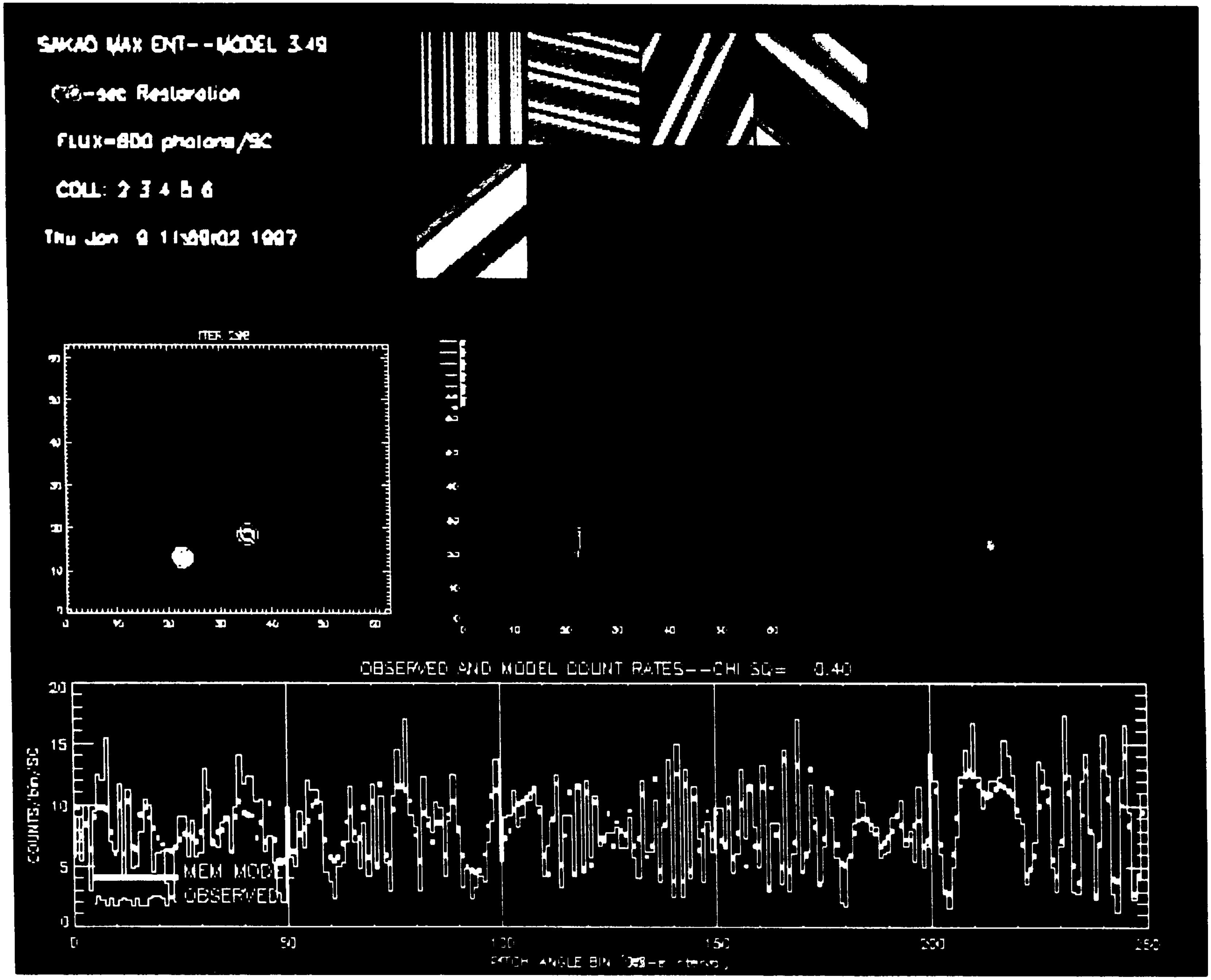
Frame 49



2-5

= 50 cts/s/SC

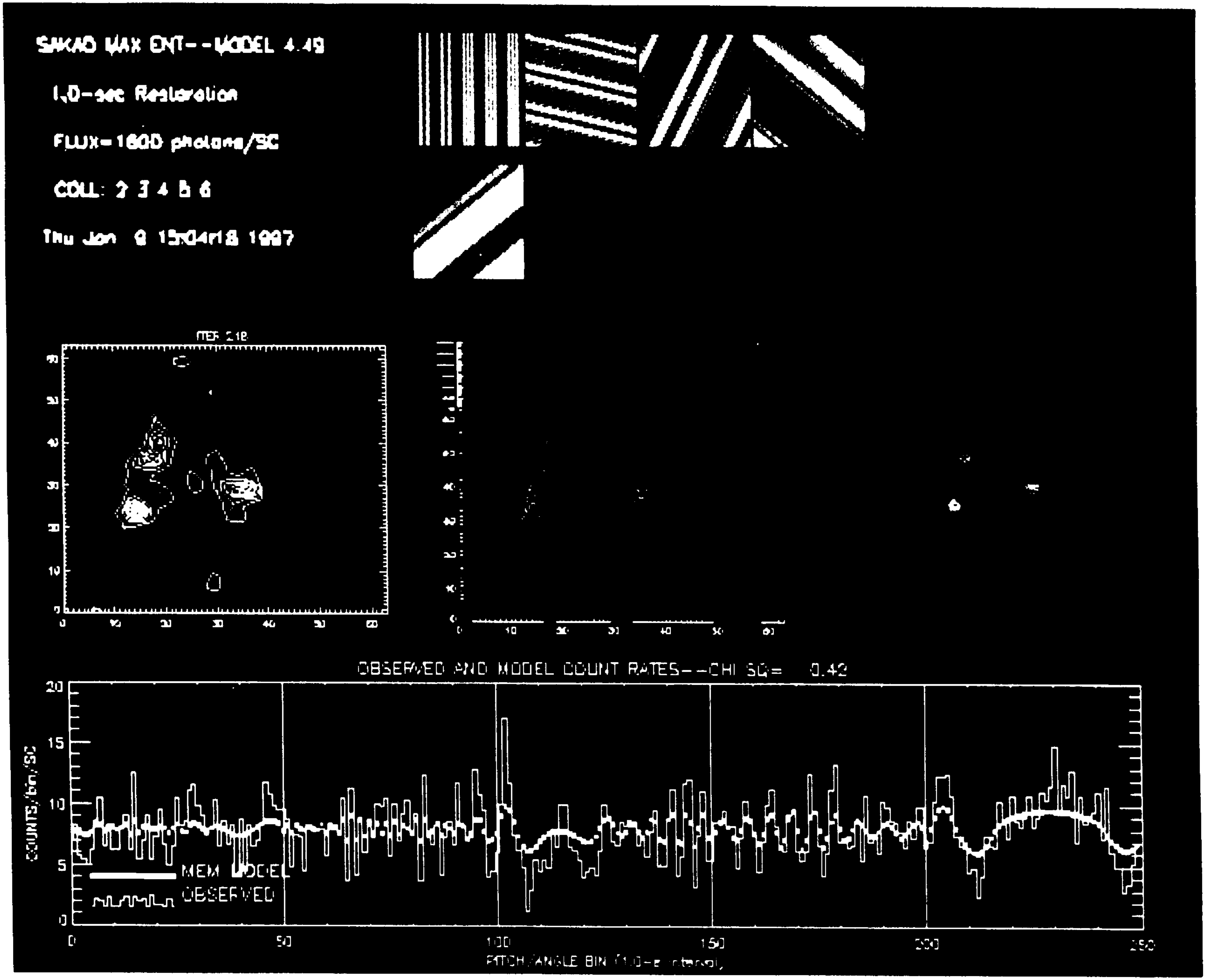
Frame 49



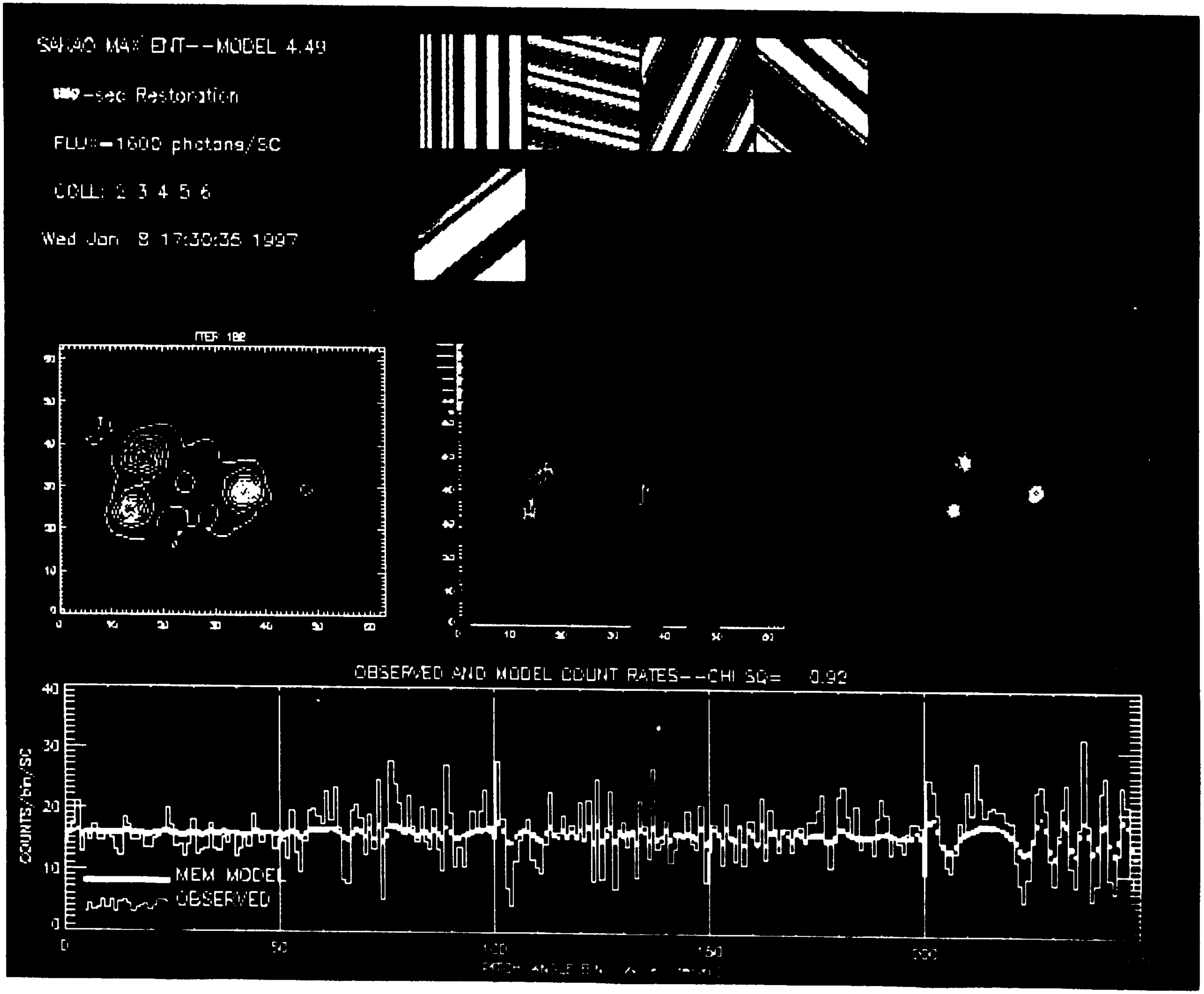
MODEL 4

FRAME 49

HESSI 1- AND 2-S SIMULATIONS


SAKAO MAX ENT--MODEL 4.49
1.0-sec Restoration
FLUX=1600 photons/SC
COLL: 2 3 4 5 6
Thu Jan 9 15:04:18 1997
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.42
MEM MODEL
OBSERVED
COUNTS/bin/SC

Model 4, Frame 49

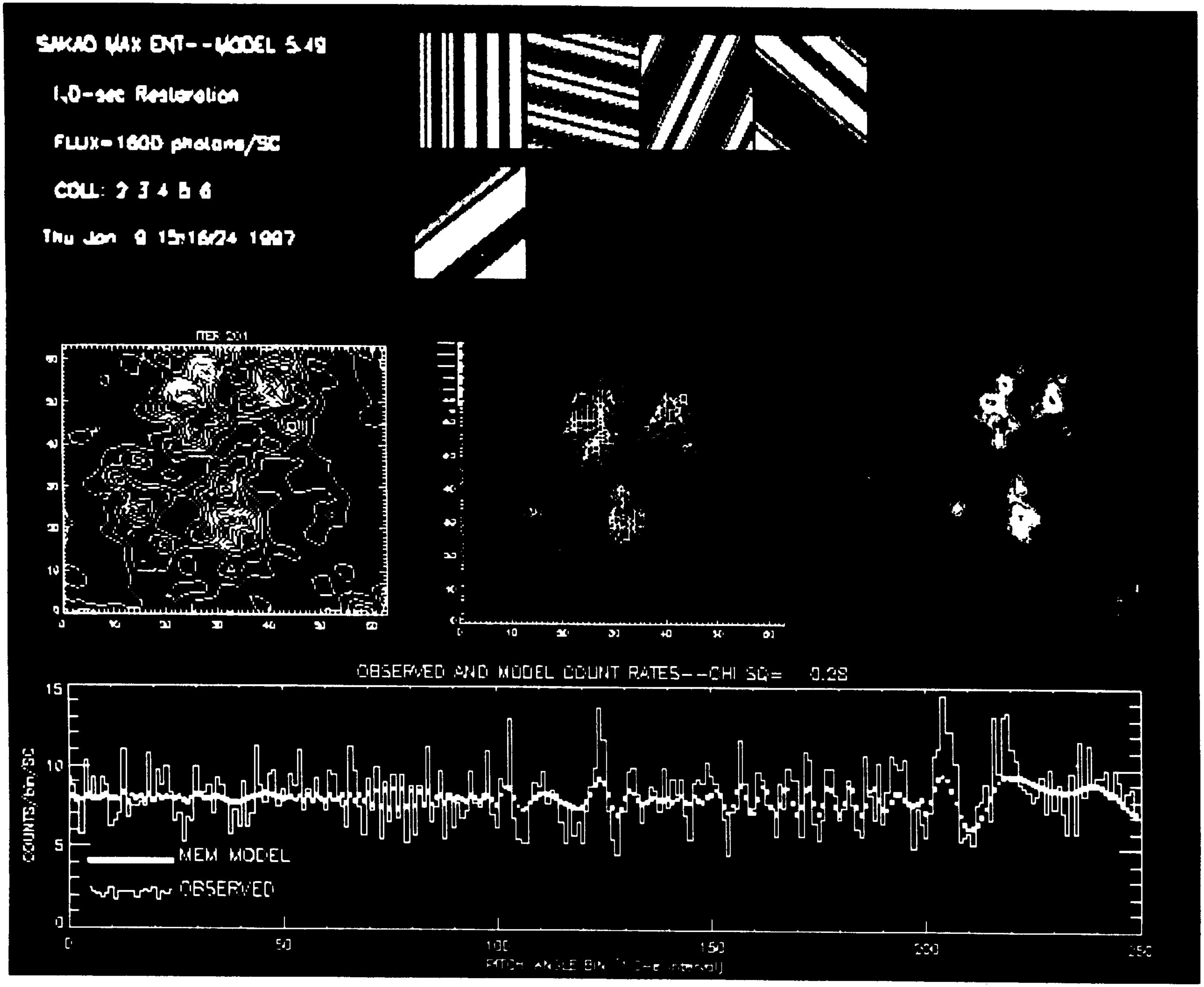


MODEL 5

FRAME 49

HESSI 1-S SIMULATIONS

Model 5



SAKAO MAX ENT--MODEL 5.49
1.0-sec Restoration
FLUX=1600 photons/SC
COLL: 2 3 4 5 6
Thu Jan 9 15:16:24 1997
ITER 201
OBSERVED AND MODEL COUNT RATES--CHI SQ= 0.28
COUNTS/bin/SC
MEM MODEL
OBSERVED

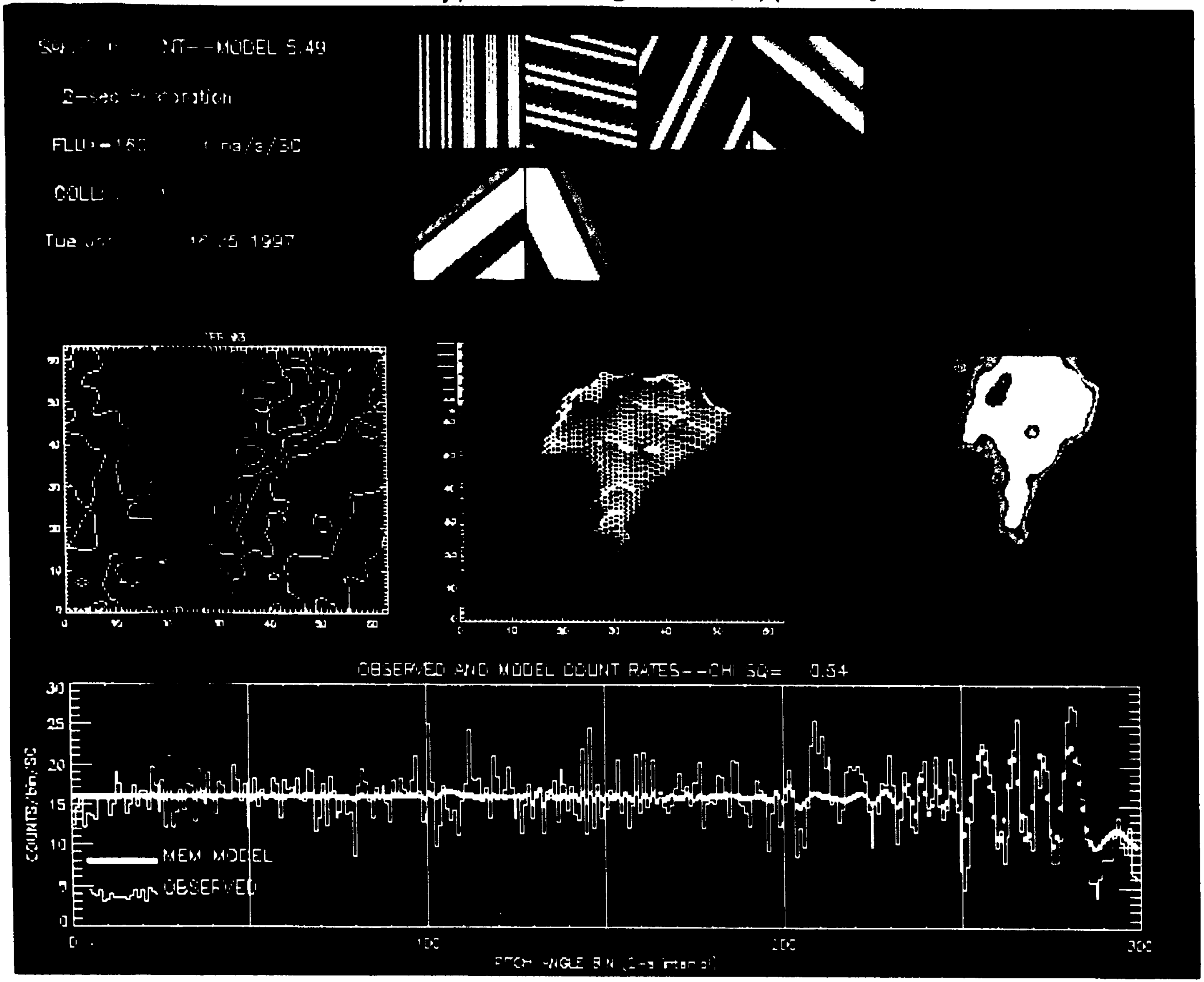
MODEL 5

FRAME 49

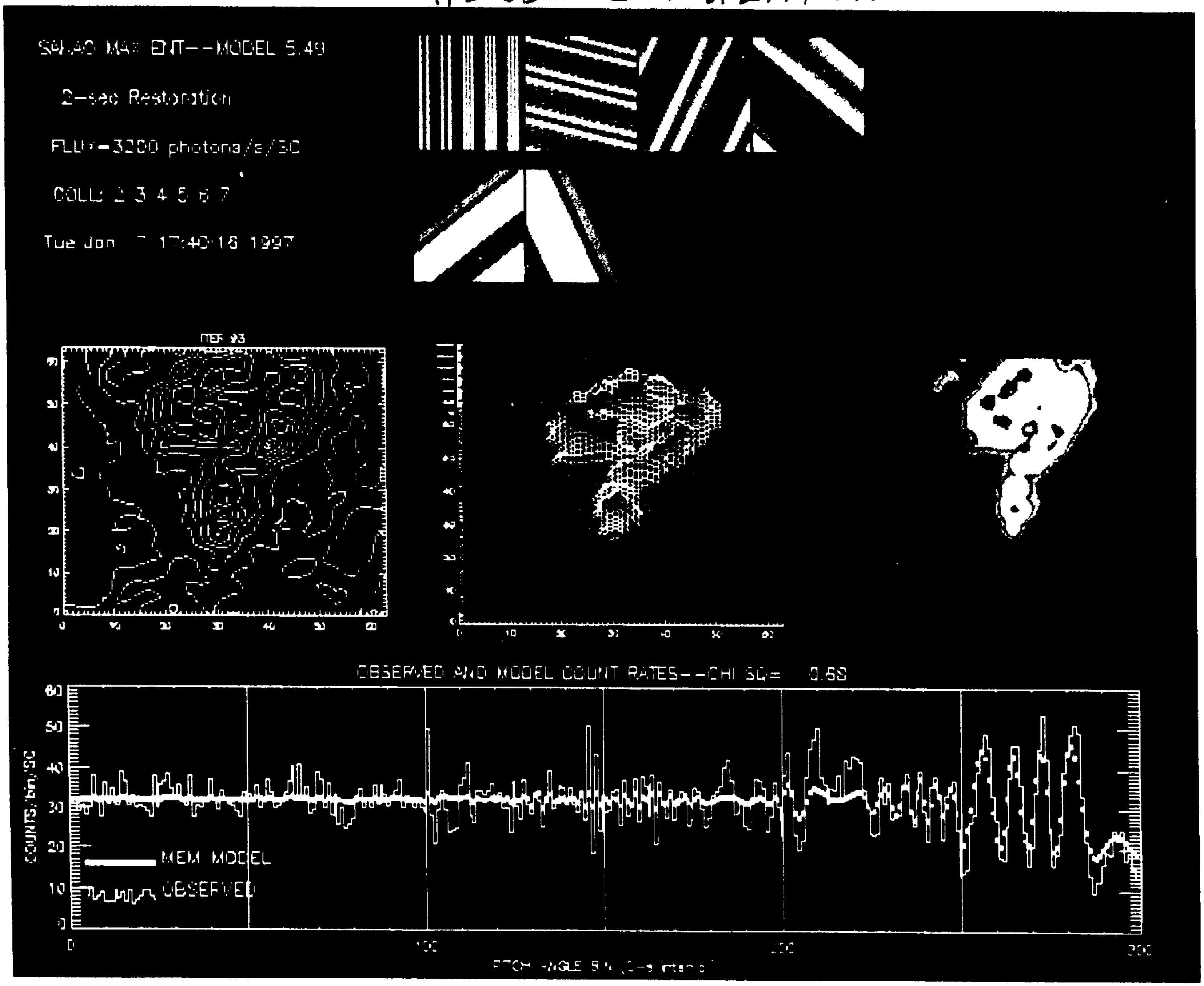
HESSI 2-S SIMULATIONS

2-seconds

HESSI SIMULATION



HESSI SIMULATION



HXT cts/s/sc ≈ 200

HESSI SIMULATION