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ANALYTICAL AND EXPERIMENTAL STUDY OF HIGH PHASE ORDER INDUCTION MOTORS

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OF HIGH PHASE ORDER INDUCTION MOTORS

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SUMMARY

Induction motors having more than three phases have been investigated to determine their suitability for electric vehicle applications. The objective is to have a motor with a current rating lower than that of a three-phase motor. The name chosen for these is high phase order (HPO) motors. Motors having six phases and nine phases were given the most attention.

It was found that HPO motors are quite suitable for electric vehicles, and for many other applications as well. They have characteristics which are as good as or better than three-phase motors for practically all applications where polyphase induction motors are appropriate. Unlike three-phase motors, they can be used with one phase open. In comparison to three-phase motors, they have lower I^2R loss and smaller torque pulsations due to harmonic currents in the rotor when the source is not sinusoidal.

Some of the analysis methods are presented, and several of the equivalent circuits which facilitate the determination of harmonic currents and losses, or currents with unbalanced sources, are included. The means for finding the appropriate winding inductances needed for calculations are explained, and the effects of these inductances on performance are indicated.

The sometimes large stator currents due to harmonics in the source voltages are pointed out. Filters which can limit these currents were developed as part of this project. An analysis and description of these

filters is included. When filters are used, it is possible to have these HPO motors supplied by non-sinusoidal voltage sources such as inverters and the motors can perform practically as well a they would on a sinusoidal source with comparable efficiency and smoothness of operation. These filters are selective in limiting certain harmonic or sequence currents without affecting the fundamental frequency, positive sequence currents which produce useful torque.

Experimental results which confirm and illustrate much of the theory are also included. These include locked rotor test results and full-load performance with an open phase. Also shown are oscillograms which display the reduction in harmonic currents when a filter is used with the experimental motor supplied by a non-sinusoidal source.

CHAPTER 1. INTRODUCTION

1-1 Purpose of This Investigation

In the process of designing and building inverters and motors for electric vehicles, the use of relatively low voltage imposes the need for high current when three-phase motors are used. This sometimes requires that multiple power transistors or thyristors be used in parallel in the inverter. An even more serious problem is that the motor stator winding requires conductors of large cross section. This makes it difficult to wind the stator, and the large cross section also results in increased stator I^2R loss due to skin effect (eddy currents) in the large conductors. This is especially the case with the harmonic frequency currents produced by inverter sources.

It is apparent that the current per phase is lower if more phases are used. This reduces the number of parallel power devices required, and also reduces the required conductor cross section. Based on this obvious advantage, the investigation on characteristics of motors having more than three phases was undertaken. The name suggested for these is high phase order (HPO) motors.

Early in the investigation, several advantages of HPO motors over the usual three-phase motor were recognized. Some of these advantages are:

-Lower current per phase for a given voltage rating.

-Able to start and run with one line open.

-Elimination of more of the space harmonics from the air gap flux.

-Reduction in rotor harmonic I^2R loss with inverter sources.

-Reduction of torque pulsations with inverter sources.

-Filters can be designed to reduce stator harmonic currents and their I^2R losses with inverter sources, and these filters do not affect fundamental torque-producing currents.

1-2 Literature Review

Several papers have been published which present some of the characteristics of HPO motors. In reference 1, a comparison of the harmonic losses is made for three-phase, nine-phase, and 27-phase motors supplied by an inverter. That study shows reduced rotor I^2R loss, and an increase in stator I^2R loss when more phases are used.

Nelson and Krause (ref. 2) derived a model for motors having multiple sets of three-phase windings. They then did a simulation on six-phase motors supplied by a 6-step voltage source inverter. Their study reveals the elimination of the sixth harmonic torque pulsation when the six-phase motor with 30 degree winding displacement is used. Also, the reduction in rotor harmonic current and the increase in stator harmonic current is shown.

A study of motors having many phases was carried out by Jahns (ref. 3). The ability of these motors to start and to run with one phase open or shorted out was brought out. The improved reliability of inverter supplied HPO motors was demonstrated.

An analysis of a six-phase motor supplied by a current source inverter was made by Andresen and Bieniek (ref. 4). Their work brings out the reduction in torque pulsations and the reductions in rotor I^2R loss.

The early analytical work and experimental results of this project have been reported in references 5 and 6. These include a description of the machines, the relationship between symmetrical components and time harmonics, the effect of coil pitch on inductances, the results of speed-torque tests, and the currents with non-sinusoidal voltage sources.

All of the above references pertain to induction motors. There is also considerable activity in the design of synchronous motors having six phases. These are generally used with current source inverters. Their lower torque pulsations and lower I^2R losses make them attractive, especially for very large variable speed motors.

1-3 General Analysis Methods

Methods for analyzing HPO motors are generally extensions of the methods used for the common three-phase motors. The methods include some which transform the machine voltage and current variables (called phase variables) to an equal number of variables in another reference frame. Two of the most well-known of these are the transformation to the symmetrical component reference frame, and the transformation to the direct-quadrature, or d-q reference frame. When the reference frame is stationary, the d-q variables reduce to the so-called two-phase, or α-β variables. For motors having more than two phases, there must, of course, be additional variables for the d-q set or the α-β set to meet the need for a sufficient number of variables.

The textbook authored by White and Woodson [ref. 7] has an entire chapter on the general machine having n stator phases, and m rotor phases where the above transformation methods are presented. Most of the theoretical work included in this report is based on the White-Woodson book.

In this report, the symmetrical component method is used extensively. Other methods are briefly presented only for comparison, or are mentioned as techniques for the analysis of transient type problems not attempted in this investigation.

1-4 Description of HPO Motors

High Phase Order (HPO) motors may be defined as ac motors having more than three phases. These may be induction motors, synchronous motors, or reluctance-synchronous motors. This report presents the analysis and behavior of induction motors only.

Induction motors may be built with any number of phases, but the number of slots in the stator laminations must be such that a symmetrical winding can be accommodated. To give an example, a four-pole motor with 12 phases requires at least 48 stator slots. For motors in the 10 to 20 HP range, using more than about 48 slots becomes somewhat impractical.

1-5 Connections and Winding Terminology

The winding terminology is based on the phase belt angle β, and the number of phase belts per pole which is given the symbol q. The relationship between these is

$$q = 180/\beta$$

where β is the phase belt angle in electrical degrees.

The basic machine stator has 2q coil groups symmetrically arranged with β degrees separating them as in figure 1-1.

If the machine has n phases, and if $n = 2q$, the name for the connection is an n-phase or a 2q-phase connection (for example, a 12-phase connection). Then, if q is a whole number, there can be $n = q$ phases, and this comes about when one of the two coils on each magnetic axis is deleted. The name given to this connection is the semi 2q-phase connection (for example, a semi 12-phase connection which has six phases).

If q is an odd number, the semi 2q-phase arrangement is derived from the 2q phase type by deleting alternate coils around the stator (coils b, d, ..., n-1 in figure 1-1). The semi 18-phase (S18ϕ) arrangement is an example of this, and it has nine coils 40° apart.

If q is an even number and a multiple of three, the connection uses the first q/3 coils of the 2q-phase arrangement, deletes the next q/3, includes the next q/3, etc. The semi 12-phase (S12ϕ) is an example. Finally, if q is an odd number and not a multiple of 3, the connection uses the first set of q coils taken sequentially (a, b, c, .., q), and omits all others.

In this report, semi 12-phase and semi 18-phase will be abbreviated S12ϕ and S18ϕ. Figures 1-2 and 1-3 show some of the arrangements and illustrate both star and mesh connections. In Figure 1-2, five-phase or semi 10-phase connections are shown for star and for mesh.

1-6 Use of Three-Phase Groups

The most practical arrangement should consist of several phase groups, each of which is either a wye or a delta connection of three phases. These three-phase groups, if isolated from each other, cannot carry third harmonic currents and this is a distinct advantage. In most of the analyt-

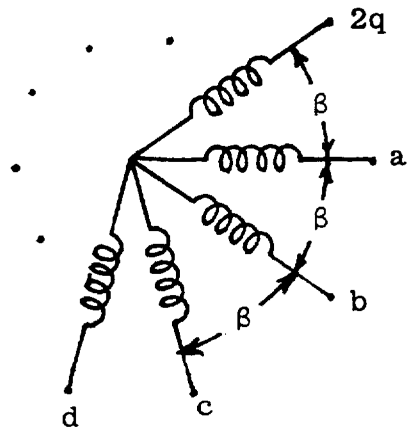


Figure 1-1. Schematic diagram of a 2q-phase machine

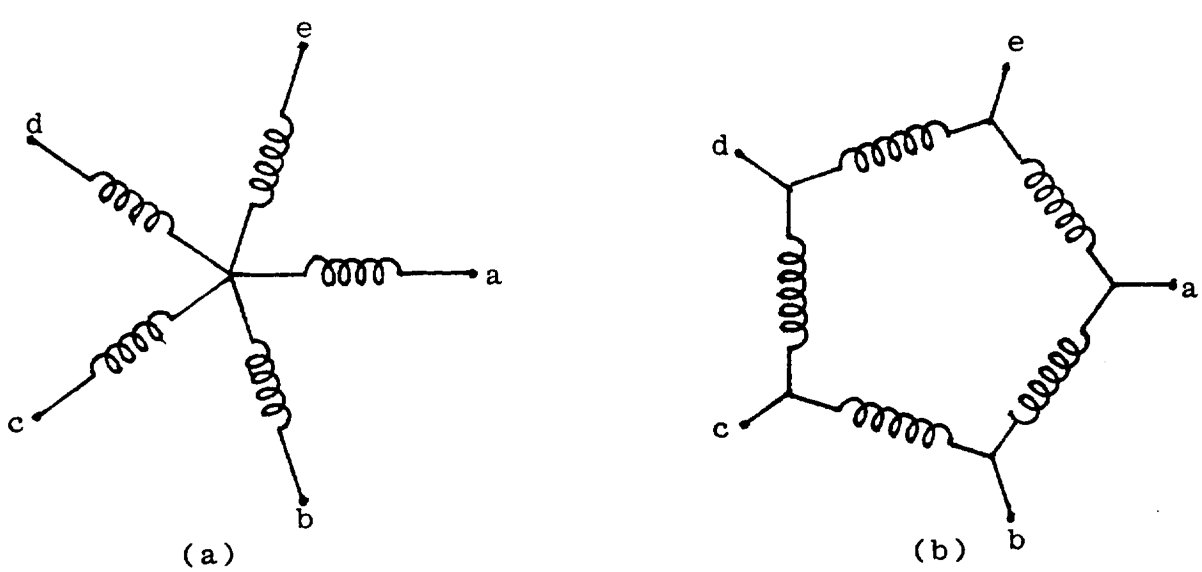
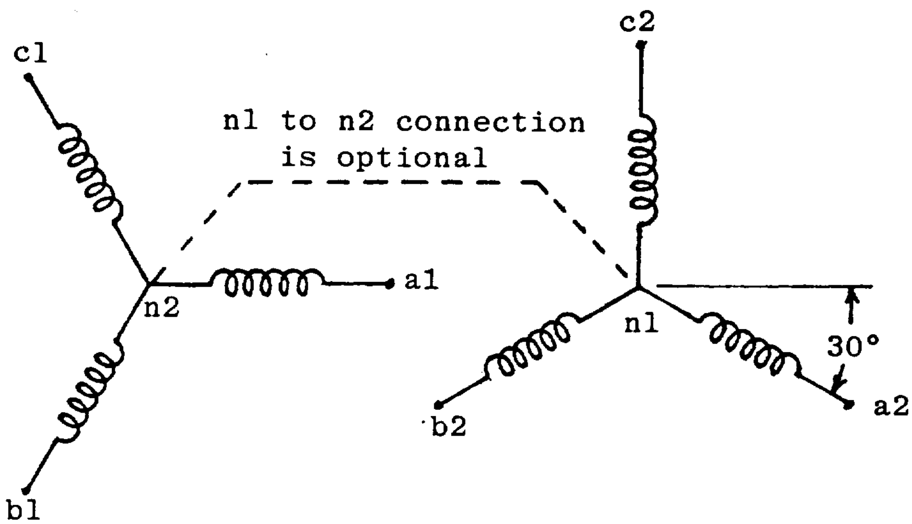


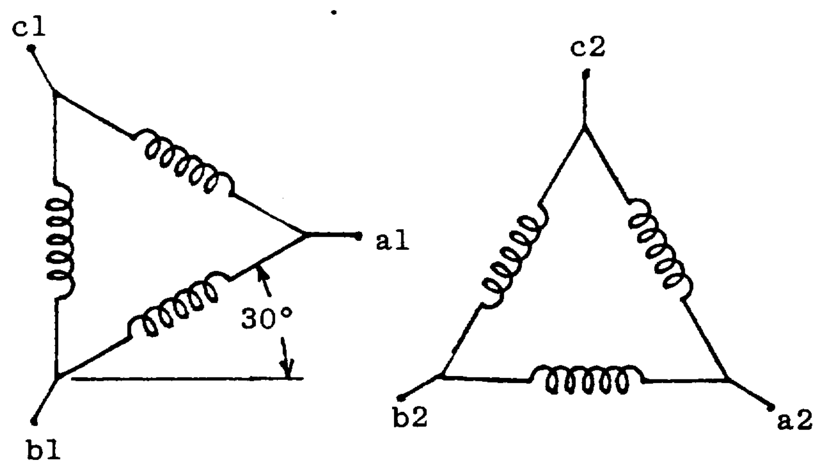
Figure 1-2. Five-Phase or Semi 10-Phase Stator.
(a) Star-connected (b) Mesh connected

ical work and all of the experimental work carried out in this project, only the three-phase, the S12ϕ, and the S18ϕ connections were used. Figure 1-3 shows the S12ϕ arrangements.

For low voltage motors where the number of turns per coil tends to be small and the conductor cross-section tends to be large making winding difficult, the delta connected groups, shown in Figure 1-3(b), would be better since, for a given voltage and current rating, when compared to a wye connection, they would use $\sqrt{3}$ times as many turns of conductor with $1/\sqrt{3}$ times the cross section.



(a)



(b)

Figure 1-3. Semi 12-Phase Stator Connections
(a) Star-connected phase groups
(b) Mesh-connected phase groups

CHAPTER 2. SYMMETRICAL COMPONENT METHOD AND CIRCUITS

2-1 Introduction

In a great many applications, motors operate at practically constant speed and with a periodic applied voltage or current. These steady state situations lend themselves to an analysis based on the use of phasor forms for voltages and currents. The motor may then be represented by equivalent circuits using resistances and reactances. At a given speed, the slip is known and the equivalent circuit parameters are all known. Then, for a given applied voltage (or current), the circuit solution gives the currents (or voltages), the torque, the I^2R losses, and the power input and output. This is the conventional, well-known method for determining the performance of three-phase motors.

For HPO motors, this steady state phasor approach is an attractive method. The symmetrical component method provides the basis for constructing the set of simple equivalent circuits which are applicable to these motors operating from balanced or unbalanced, sinusoidal or non-sinusoidal sources. In this last case, the Fourier series method is used to separate the non-sinusoidal applied voltage or current into its fundamental and harmonic frequency components. Then, the phasor approach applies to each of these separately, and results can be combined according to the principle of superposition.

2-2 Inductance Values for Equivalent Circuits

The symmetrical component method requires that the machine variables (phase voltages and currents) be transformed to the sequence variables using the appropriate matrix transformation. The matrix required is the inverse of matrix [A] given in Appendix A. In this transformation process, the machine resistances and inductances are transformed to the sequence resistance and inductances. It is assumed that the machine has a balanced n-phase stator winding as described in Appendix B. The relationship between the stator resistance and inductance matrices and their symmetrical component forms is described in Appendix B. The stator resistance matrix is a diagonal matrix and it is unchanged when it is transformed to the symmetrical component resistance matrix.

The stator inductances include the self inductance of each phase and the mutual inductances between phases. Appendix B gives the details for finding the sequence inductance values from the stator inductances. The result is the diagonal stator sequence matrix $[L_S]$ given by the following:

$$[L_s] = \begin{array}{c|c|c|c|c|c|c|}
 & 0 & \overset{P}{1} & 2 & \cdot \ \cdot \ \cdot \ \cdot \ \cdot & n-2 & \overset{N}{n-1} \\ \hline
0 & L_o & & & & & \\ \hline
1 & & L_1+L_m & & & & \\ \hline
3 & & & L_2 & & & \\ \hline
\cdot & & & & \ddots & & \\
\cdot & & & & & & \\
\cdot & & & & & & \\ \hline
n-2 & & & & & L_2 & \\ \hline
n-1 & & & & & & L_1+L_m \\ \hline
\end{array}$$

In the above matrix, the various inductances are:

- L_o zero sequence inductance
- L_1 leakage inductance for positive sequence (also for negative sequence)
- L_m magnetizing inductance
- L_2 inductance for sequence two (also for sequence n-2)
- L_3 inductance for sequence three (also for sequence n-3)

$\cdot$
$\cdot$
$\cdot$

- L_i inductance for sequence i (also for sequence n-i)

2-3 Sequence Circuits

The symmetrical component method leads to a set of equivalent circuits; one circuit for each of the n sequences of an n-phase motor. Since the sequence resistance matrix and the sequence inductance matrix are both diagonal matrices, the sequence circuits have no coupling between them and can therefore be solved as independent circuits.

No matter how many phases a polyphase motor might have, it is only the positive and the negative sequence circuits which have coupling between the stator and the rotor portion of each of these circuits. Therefore, only these two sequence circuits include rotor elements. The rotor circuit configurations applicable to the positive and the negative sequence circuits of three-phase motors apply to the following HPO motor sequence circuits as well. Since these sequence circuits for three-phase motors are well known, they can be presented here without derivation. These circuits for the positive and negative sequence are shown in Figure 2-1 on page 17.

2-4 Simplification of the Positive and Negative Sequence Circuits

It is convenient to simplify the sequence circuit of figure 2-1(a) by reducing the parallel combination of magnetizing impedance $j\omega L_m$ and rotor impedance $(r_r/s + j\omega L_r)$ to the equivalent impedance $(R_f + jX_f)$. Details of this are given in reference 8, pages 547-564. The subscript "f" here means "forward," indicating the forward revolving positive sequence field which produces forward torque. The simplified positive sequence circuit is shown in figure 2-2(a). The same kind of a parallel circuit reduction can be made for the negative sequence circuit of Figure 2-1(b). The result is shown in Figure 2-2(b). The subscript "b" means "backward".

2-5 Effect of Rotor Skin Effect on Rotor Parameters

The effective rotor resistance and inductance are dependent on the frequency of rotor currents, and on the rotor bar geometry and material.

Reference 9 gives details on pages 265-271. This should be taken into account when the values of r_r and L_r are used to calculate R_b and X_b for the negative sequence circuit. Also, in a later section, harmonic frequencies will be applied and this rotor frequency effect should be taken into account in calculating R_f, X_f, R_b, and X_b.

2-6 Sequence Circuits for HPO Motors

In general, the number of independent sequence circuits is equal to the number of phases. A two-phase motor requires only the positive and the negative sequence circuit discussed in Section 2-3. A three-phase motor requires one additional circuit called the zero sequence circuit. It consists of the resistance and zero sequence reactance of the stator only. Then, for more than three phases, additional circuits similar to the zero sequence ciruit are required. Table 2-1 shows all of the circuits for an n-phase motor.

2-7 Solution of Sequence Circuits

The symmetrical component method is generally used to analyze motors operating at constant speed and with periodic applied voltages. Phasor forms of voltages and currents are used. The phase voltages and currents for a general n-phase motor stator are shown in Figure 2-3. The solution procedure to be used for a situation where applied voltages are specified is to first transform them to the sequence voltages by using the appropriate symmetrical component transformation matrix $[A]^{-1}$ given in Appendix A. This is expressed as

$$[V_S] = [A]^{-1}[V_{n\phi}] \tag{2-1}$$

or written out, with the bar above each voltage symbol to indicate that it is a phasor quantity,

$$\begin{bmatrix} \overline{V}_o \\ \overline{V}_P \\ \overline{V}_2 \\ \overline{V}_3 \\ \cdot \\ \overline{V}_i \\ \cdot \\ \cdot \\ \cdot \\ \overline{V}_N \end{bmatrix} = [A]^{-1} \begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \overline{V}_n \end{bmatrix} \tag{2-2}$$

The subscripts are o for the zero sequence,

P for the positive sequence (sequence 1),

2, 3, ...i for sequences two, three, ...i,

and N for the negative sequence (sequence n-1).

These sequence voltages are then used with the appropriate sequence circuits shown in Table 2-1 to solve for the sequence currents. If the actual phase currents are wanted, they can be found using matrix [A] to transform from sequence currents, giving $[I_{n\phi}] = [A][I_S]$ (2-3) or, written out and again using the bar above each symbol for current to identify it as a phasor,

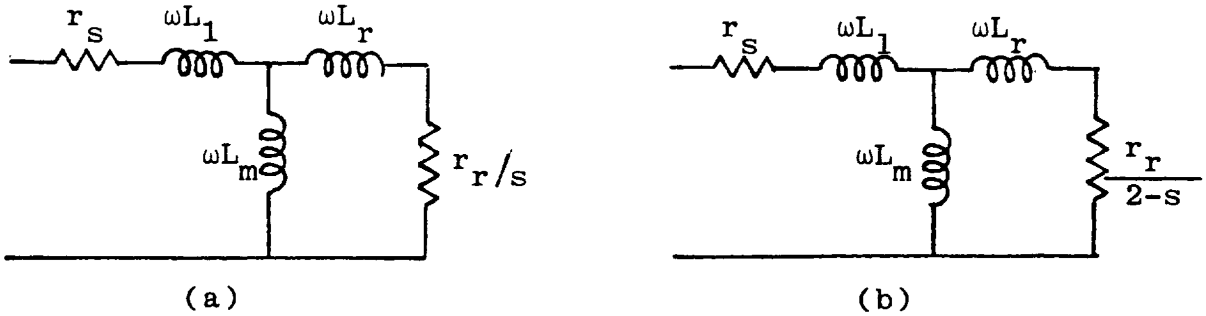


Figure 2-1. Sequence Circuits for a Polyphase Motor, (s is the slip).
(a) Positive sequence
(b) Negative sequence

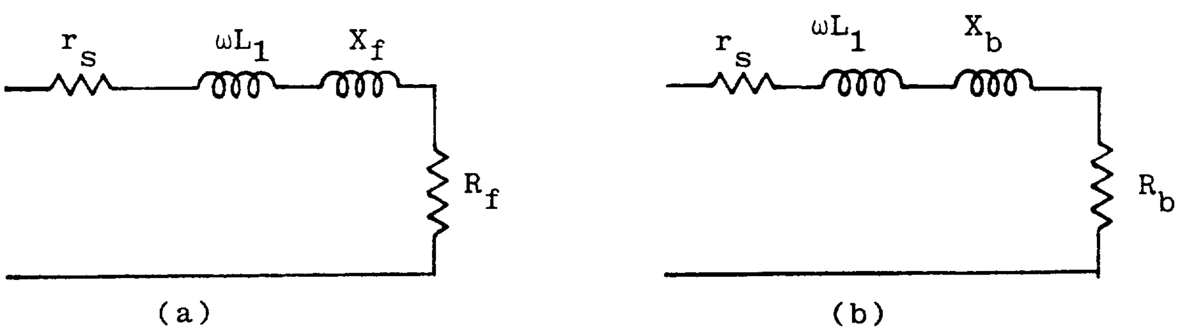


Figure 2-2. Simplified Sequence Circuits
(a) Positive sequence
(b) Negative sequence

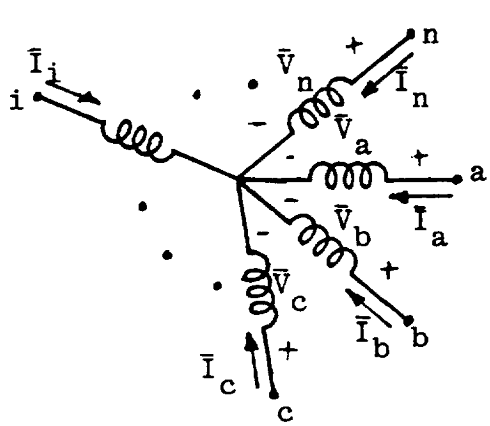
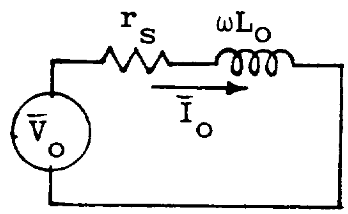
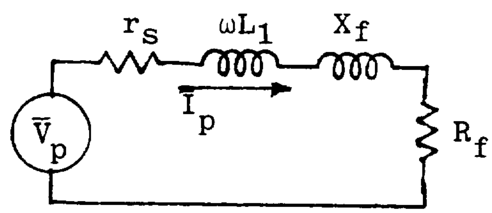
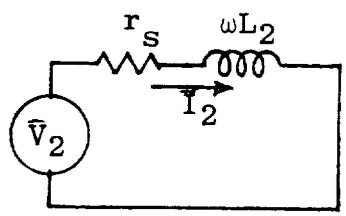
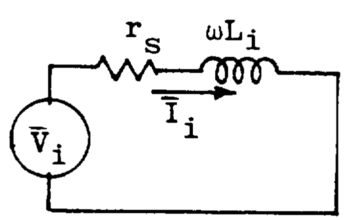
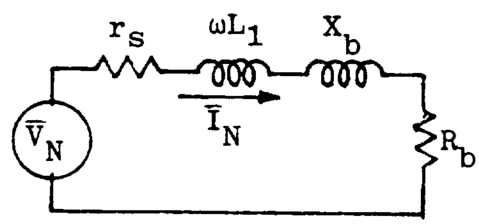


Figure 2-3. Stator Connections for an n-Phase Motor

TABLE 2-1. SYMMETRICAL COMPONENT SEQUENCE CIRCUITS FOR n-PHASE MOTOR

(Constant speed is assumed, and phasor voltages and currents are used)

Sequence Number	Sequence Circuit	Torque	Stator I^2R Loss	Rotor I^2R Loss
0		0	$I_0{}^2 r_s$	0
1 or P, Positive Sequence		$\dfrac{I_p{}^2 R_f}{\omega_{syn}}$ $= T_f$	$I_p{}^2 r_s$	$sI_p{}^2 R$
2		0	$I_2{}^2 r_s$	0
. . .		. . .	. . .	. . .
i		0	$I_i{}^2 r_s$	0
. . .		. . .	. . .	. . .
(n-1) or N, Negative Sequence		$I_N{}^2 R_b$ $\doteq T_b$	$I_N{}^2 r_s$	$(2-s)I_N{}^2 R_b$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \bar{I}_n \end{bmatrix} = [A] \begin{bmatrix} \bar{I}_o \\ \bar{I}_p \\ \bar{I}_2 \\ \bar{I}_3 \\ \cdot \\ \bar{I}_i \\ \cdot \\ \cdot \\ \cdot \\ \bar{I}_N \end{bmatrix} \qquad (2\text{-}4)$$

2-8 Torque and Losses from Sequence Circuits

If torque or losses are wanted, it is not necessary to solve for the actual phase currents. Equations for torque and the stator and rotor I^2R losses are given in Table 2-1 in terms of the sequence currents. The total motor torque is then $T_f - T_b$. The total stator I^2R loss is the sum of the individual sequence circuit losses as found in Table 2-1. The same is true for total rotor I^2R loss. This is true because the transformation matrix [A] given in Appendix A is "power invariant," meaning that the total loss using all sequence variables is equal to the total loss using machine variables.

2-9 Analysis of S12ϕ Motors, Steady State Case

It is expected that the S12ϕ version of HPO motors will be the most popular. These will be the subject of the next several sections. The S12ϕ motor is derived from a 12-phase motor by deleting alternate pairs of stator windings from the 12-phase motor. See Appendix A. In the symmetrical component circuits, this eliminates all even numbered sequences from the 12-phase sequence sets. The sequence circuits for the S12ϕ motor are shown in Table 2-2. The torque and the I^2R losses are found in the same manner as in Table 2-1. The matrix [A] for transforming between phase variables and sequence variables for a S12ϕ motor is given in Appendix A.

2-10 Sequence Circuits for Certain Common Operating Modes

There are several modes of operation of S12ϕ motors which are amenable to simple solutions when the symmetrical component method is applied. Among these are:

Case I - Balanced S12ϕ operation on sinusoidal source.

Case II - Sinusoidal S12ϕ source consisting of two balanced three-phase sources, but the two sources either:

(Case II-a) are displaced $\pi/6$ radians from each other, but with unequal magnitudes (magnitude unbalance),

or

(Case II-b) have equal magnitudes, but are displaced $\pi/6 + \varepsilon$ radians from each other (angle unbalance).

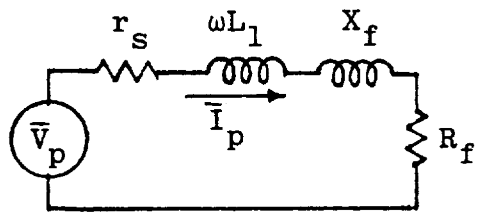
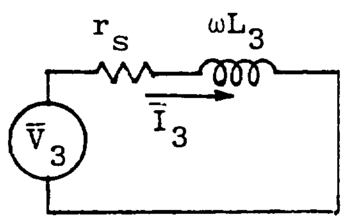
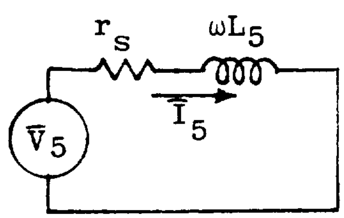
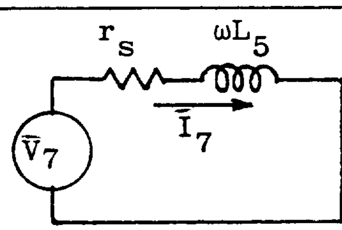
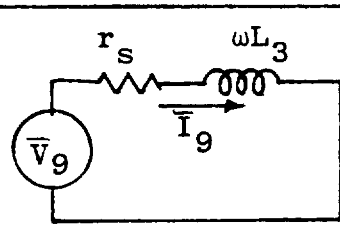
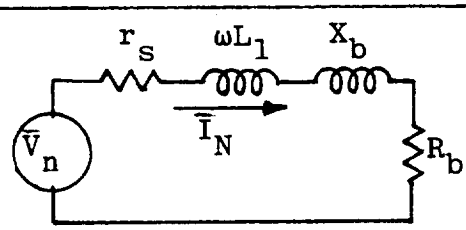
Case III - Sinusoidal S12ϕ source, and with one line open.

Case IV - Non-sinusoidal, balanced S12ϕ source.

For case IV, the Fourier series expansion for the source voltages or currents give the fundamental and harmonic frequency components, each of which forms a balanced S12ϕ sinusoidal source. For each of these, one of the several sequence circuits is applicable with an appropriate frequency and value for slip.

TABLE 2-2. SYMMETRICAL COMPONENT SEQUENCE CIRCUITS FOR S12ϕ MOTOR

(Constant speed is assumed, and phasor quantities are used.)

Sequence Number	Sequence Circuit	Torque	Stator I^2R Loss	Rotor I^2R Loss
1 or P, positive		$\frac{I_p^2 R_f}{\omega_{syn}} = T_f$	$I_p^2 r_s$	$sI_p^2 R_f$
3		0	$I_3^2 r_s$	0
5		0	$I_5^2 r_s$	0
7		0	$I_7^2 r_s$	0
9		0	$I_9^2 r_s$	0
11 or N, Negative		$\frac{I_N^2 R_b}{\omega_{syn}} = T_b$	$I_N^2 r_s$	$(2-s)I_N^2 R_b$

These special cases are covered in the following sections.

2-11 Case I. S12ϕ Motor with a Balanced Sinusoidal Source

For this case, the stator voltage and current variables are the phasor quantities shown on the stator connection diagram for the S12ϕ motor in Figure 2-4. The phasor voltages for this balanced situation are shown in Figure 2-5. In that figure, the set of phase voltages $\bar{V}_{a1}$-$\bar{V}_{b1}$-$\bar{V}_{c1}$ form a balanced three-phase set with abc sequence. This will be called phase group number 1. The phasor set $\bar{V}_{a2}$-$\bar{V}_{b2}$-$\bar{V}_{c2}$ is also a balanced three-phase set, called phase group 2, and it is displaced 30° from phase group 1 as shown in Figure 2-5.

For case I, these voltages must form a balanced S12ϕ set and, therefore, all six voltage magnitudes are equal so that

$$V_\phi = |\bar{V}_{a1}| = |\bar{V}_{a2}| = |\bar{V}_{b1}| = |\bar{V}_{b2}| = |\bar{V}_{c1}| = |\bar{V}_{c2}|$$

When these voltages are transformed to sequence voltages according to Section 2-7, they are as follows

$$\bar{V}_P = \sqrt{6}\, \bar{V}_\phi$$

and all other sequence voltages are zero. Therefore, the positive sequence circuit alone is required, and it is shown in Figure 2-6. The equations listed for the positive sequence circuit in Table 2-2 give the total torque and I^2R losses. Also, the phase current in line a1 may be found as

$$\bar{I}_{a1} = \bar{I}_P/\sqrt{6}$$

and the set of phase currents will be balanced S12ϕ with phasor relationships like those shown for voltages in Figure 2-5.

2-12 Case II. S12ϕ Motor with Intergroup Unbalance

This case is similar to Case I, with each of the two three-phase groups forming a balanced three-phase group. However, balance between the groups is lacking. In Case II-a, voltage magnitudes are

$$V_{\phi 1} = |\overline{V}_{a1}| = |\overline{V}_{b1}| = |\overline{V}_{c1}|$$

$$V_{\phi 2} = |\overline{V}_{a2}| = |\overline{V}_{b2}| = |\overline{V}_{c2}|$$

$$V_{\phi 1} \neq V_{\phi 2}$$

This is shown in Figure 2-7(a), and is called intergroup magnitude unbalance. Then, for Case II-b, the voltage magnitudes are all equal, but the phase group 2 set of voltages is ε radians more than $\pi/6$ radians (30°) behind phase group 1. This is shown in the phasor diagram of Figure 2-7(b), and is called intergroup angle unbalance.

The equations giving sequence voltages for Cases II-a and II-b are as follows:

For Case II-a: $\overline{V}_p = \sqrt{6}\, V_{ave}$, where $V_{ave} = 1/2(V_{\phi 1} + V_{\phi 2})$

$\overline{V}_7 = \sqrt{6}\, \Delta V$, where $\Delta V = 1/2(V_{\phi 1} - V_{\phi 2})$

All other sequence voltages are zero.

For Case II-b: $\overline{V}_p = \sqrt{6}\, V_{\phi} \underline{/-\varepsilon/2}$

$\overline{V}_7 = \sqrt{6}\, V_{\phi}\varepsilon/2 \underline{/\pi/2-\varepsilon/2}$

All other sequence voltages are zero.

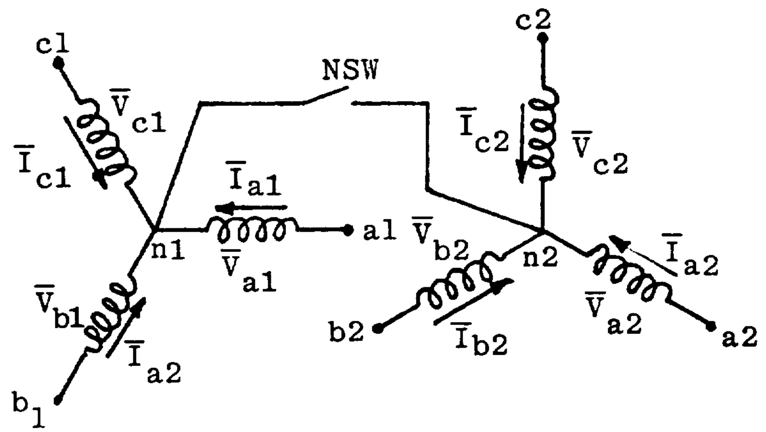


Figure 2-4. Stator Connections for a S12ϕ Motor.

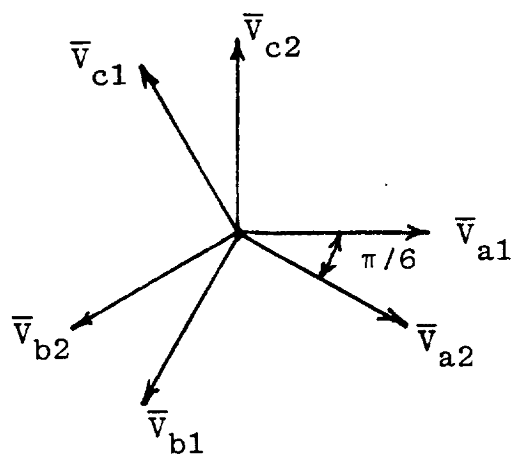


Figure 2-5. Phasor Voltages for S12ϕ Motor.

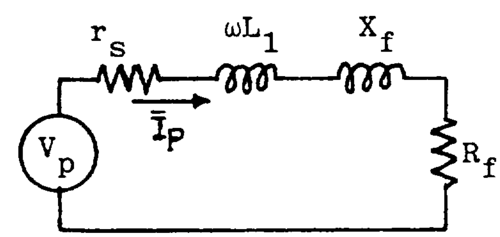


Figure 2-6. Positive Sequence Circuit.

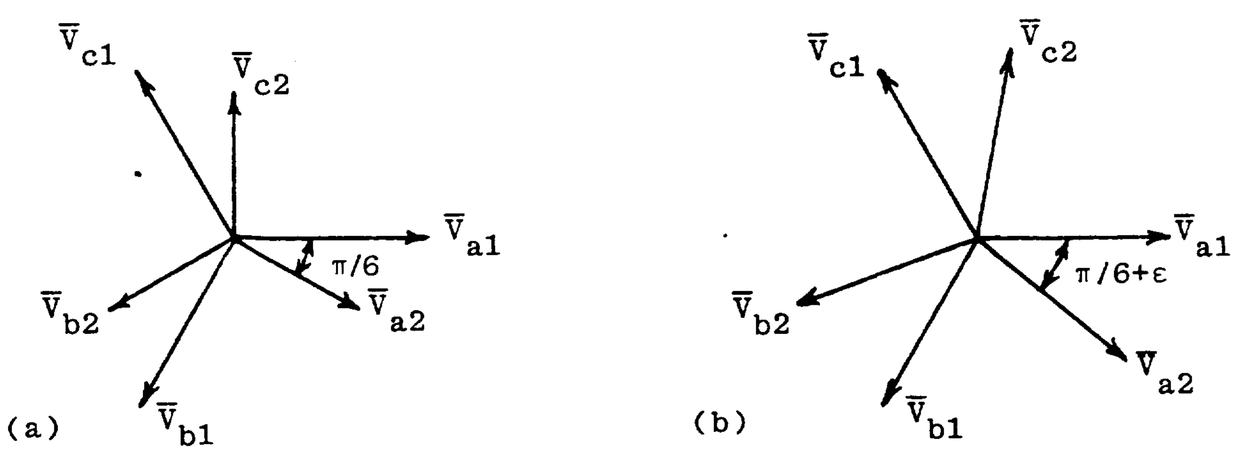


Figure 2-7. S12ϕ Voltages with Intergroup Unbalance.
(a) Magnitude unbalance (b) Angle unbalance

The equations for Case II-b involve approximations, but are quite accurate for ε small (less than about 1/10 radian).

As indicated above, only the positive sequence and the 7th sequence circuits of Table 2-2 are used for Case II. The equation listed there for positive sequence torque is the total torque. The I^2R losses consist of the positive sequence losses as for Case I, plus the 7th sequence losses. If, for Case II, the positive sequence voltage is assumed to be the same as in Case I, then the torque will remain the same. But the I^2R losses will increase by the amount of 7th sequences losses. This additional loss is the result of the unbalance, and causes additional heat to be generated in the stator winding.

For Case II, the actual phase currents may be found from the sequence currents according to:

$\bar{I}_{a1} = 1/\sqrt{6}(\bar{I}_p + \bar{I}_7)$, and $\bar{I}_{a1}$, $\bar{I}_{b1}$, $\bar{I}_{c1}$, form a balanced 3-phase set with $I_{\phi 1} = |\bar{I}_{a1}|$

$\bar{I}_{a2} = 1/\sqrt{6}(\bar{I}_p - \bar{I}_7)$, and $\bar{I}_{a2}$, $\bar{I}_{b2}$, $\bar{I}_{c2}$ also form a balanced 3-phase set with $I_{\phi 2} = |\bar{I}_{a2}|$

2-13 Case III. S12ϕ Motor with One Line Open

For Case III, the source is a balanced, sinusoidal S12ϕ voltage source. Line a1 is open, simulated by opening switch SW in Figure 2-8. The equivalent circuit in Figure 2-9 is based on the symmetrical component transformation. In this circuit, the voltage V_ϕ is the line to neutral voltage magnitude for the balanced S12ϕ voltage source. The circuit may be

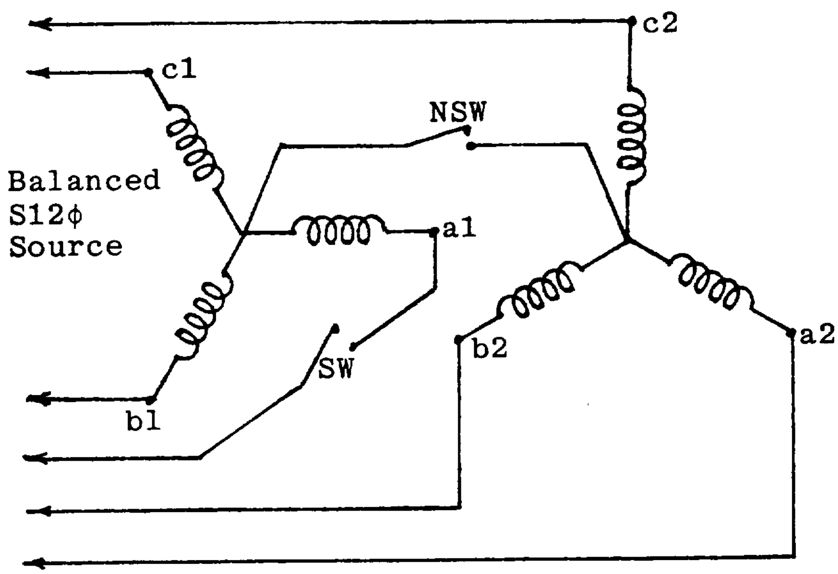


Figure 2-8. Stator Connections for S12φ Motor: showing switch SW for operation with phase a1 open, and switch NSW for neutral connections. The source provides V_ϕ volts per phase.

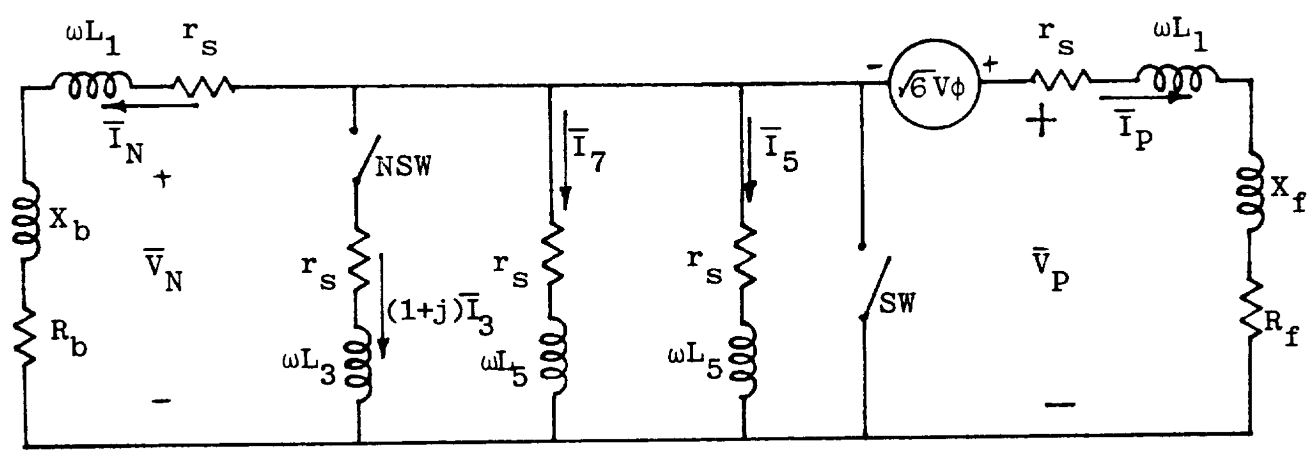


Figure 2-9. Equivalent Circuit for S12φ Motor with One Line Open. Switches SW and NSW correspond to those in Figure 2-8.

solved for all the sequence currents, which are indicated on the circuit. (Note that $\bar{I}_9=j\bar{I}_3$). The total torque is (T_f-T_b) where T_f and T_b are as given in Table 2-2. The I^2R losses are also as given in that table.

If SW is closed as for balanced operation, this circuit reduces to that of Case I. But when the switch is open to simulate one open phase the negative sequence circuit is introduced along with the other sequence circuits in parallel with it. The effect of these parallel circuits for sequences 5, 7, (and also sequences 3 with 9 if neutral switch NSW is closed) is to shunt current away from the negative sequence circuit, thus reducing I_N and the negative sequence torque T_b. The overall effect is that, for this motor, torque reduction at a given slip can be small, even though one phase is open. For machines having small values for the 5th and 3rd sequence inductances (L_5 and L_3), this reduction in torque can be quite small. Experimental results are given in Chapter 6.

2-14 Case IV Motors with Periodic, Non-Sinusoidal Source

For Case IV, the voltage (or current) source is non-sinusoidal, typically an inverter source. But, based on the assumption of constant speed, the phasor voltages and currents and equivalent circuits based on slip and impedance may be used separately for the fundamental component, and for each harmonic of the Fourier series.

Each harmonic component, as well as the fundamental, requires one of the sequence circuits to represent the motor. The slip and the reactances, and the rotor frequency which affects rotor bar skin effect, must be based on the rotor speed and on the frequency of the particular Fourier series component. After each significant harmonic has been used, the principle of superposition can be used to arrive at final results for currents, torque, and losses for the non-sinusoidal case. Reference 10 gives details of this method as applied to three-phase motors.

Table 2-3 has been prepared for S12ϕ motors to summarize the details of the appropriate equivalent circuit for each harmonic component, and Table 2-4 is similar, but for 9-phase and S18ϕ motors.

2-15 Discussion of Non Sinusoidal Performance

The equivalent circuits presented in Tables 2-3 and 2-4 make it possible to compute the individual harmonic current magnitudes if the harmonic voltages are first found for any given voltage waveform produced, for example, by a voltage source inverter (VSI). If the source is a current source inverter (CSI), then the harmonic currents are found directly from a Fourier series analysis of the inverter current waveform. In either case, the currents produce I^2R losses as given in the two tables. The inductances in the equivalent circuits are the individual sequence inductances defined and used for the symmetrical component method. The following are important observations.

-Only the fundamental component of current produces torque, which is equal to

$$T = \frac{p}{2} \frac{I_1{}^2 R_f}{\omega}$$

where p is the number of poles on the motor.

TABLE 2-3. HARMONIC CIRCUITS FOR S12ϕ MOTOR

(j is any integer, and $\omega = 2\pi f_1$)

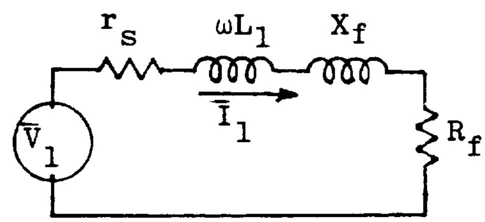
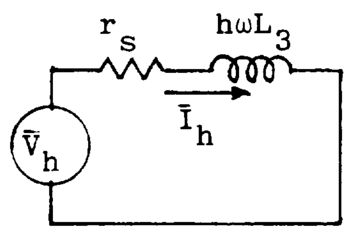
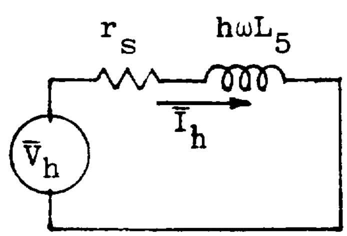
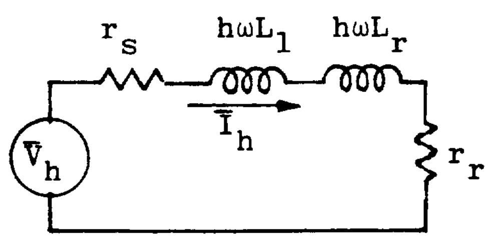
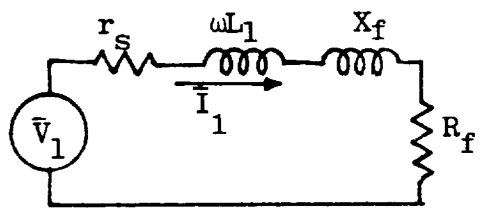
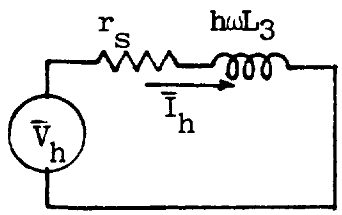
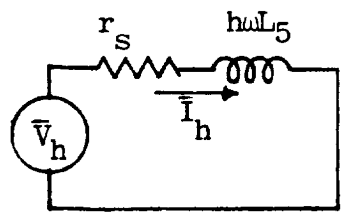
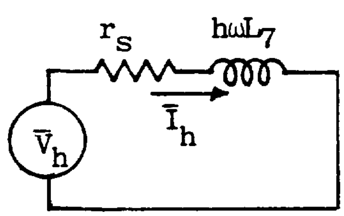
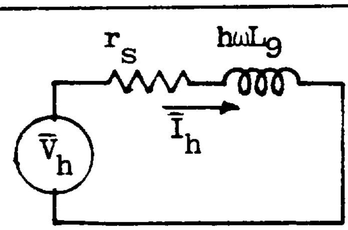
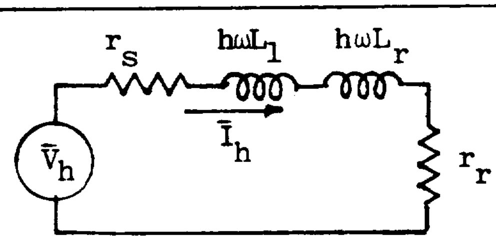
Harmonic Order h	Circuit Diagram	Stator I^2R	Rotor I^2R	Comment
1, or funda-mental		$I_1{}^2 r_s$	$sI_1{}^2 R_f$	s is the slip, $\bar{I}_1$ is the same as the positive sequence current $\bar{I}_p$.
3,9,15, 21,...., (6j + 3)		$I_h{}^2 r_s$	0	I_h=0 for delta phase groups, or for wye with neutrals separate.
5,7,17, 19,..., (12j-6±1)		$I_h{}^2 r_s$	0	
11, 13, 23,..., (12j±1)		$I_h{}^2 r_s$	$I_h{}^2 r_r$	Correct L_r and r_r to account for skin effect using rotor frequency $f_r = 12jf_1$

TABLE 2-4. HARMONIC CIRCUITS FOR S18ϕ MOTOR

(j is any integer, and $\omega = 2\pi f_1$)

Harmonic Order h	Circuit Diagram	Stator I^2R	Rotor I^2R	Comment
1		$I_1{}^2 r_s$	$sI_1{}^2 R_f$	s = slip
3,15,21, 33,...,(9j±6)		$I_h{}^2 r_s$	0	I_h=0 for delta phase groups, or for wye with neutrals separate.
5,13,23, 31,...,(18j-9±4)		$I_h{}^2 r_s$	0	
7,11,25, 29,...,(18j-9±2)		$I_h{}^2 r_s$	0	
9,27, 45,...,(18j±9)		$I_h{}^2 r_s$	0	I_9=0 for delta, or for wye with neutrals separate.
17,19, 35,...,(18j±1)		$I_h{}^2 r_s$	$I_h{}^2 r_r$	Current L_r and r_r to account for skin effect using rotor frequency f_r=18jf_1.

-There are harmonic currents in the rotor only for those harmonics listed in the last line of each table. In general, these are harmonic order numbers given by

$$h = 2qj \pm 1$$

where q is number of phase belts per pole, and j is any integer.

-Only the harmonics which produce rotor currents can produce the pulsating components of torque. These pulsating torque components are at a frequency

$$f_p = 2qfj \quad \text{Hz}$$

where f is fundamental frequency and j is any integer.

-The average torque for the harmonics which produce no rotor currents is zero, and it is very nearly zero for those producing rotor current.

-Certain harmonics are limited by stator impedance alone. The inductance, which is part of that impedance, is influenced by stator coil pitch. Some of these inductances can have a very small value. The result is that some stator harmonic currents can be quite large, but an external filter can be added to greatly reduce these currents. (For details see Chapter 4.)

CHAPTER 3. OTHER TRANSFORMATION METHODS

3-1 Introduction

Motors operating with rapidly varying speed, or with sudden changes in input voltage are examples of transient operation. Cases such as these lead to a set of differential equations with time-varying coefficients. The most common method used to simplify these equations is to transform the machine voltage and current variables to the direct-quadrature-zero (d-q-γ) reference frame. This subject has been fully developed and the literature contains many good explanations of the method applied to three-phase motors (Ref. 7 and 11). This general theory can be extended to HPO motors and Chapter 10 in Reference 7 covers this. The theory therefore need not be repeated here. The transformation matrices involved are given in Appendix A. A brief discussion of the methods begins in the next section, and equivalent circuits are included.

The presentation of these general circuits is included mainly for background purposes. The general transient solutions were not pursued in this investigation. However, these general circuits will be reduced to their sinusoidal steady state forms. They are useful as alternatives to the symmetrical component circuits presented in Chapter 2, and are closely related to them as will be shown.

3-2 General α-b-γ and d-q-γ Circuits

Several versions of methods using the machine variables tranformed to the d-q-γ variables are found in literature. In particular, those dealing with HPO motors are found in references 2, 3, and 7. Use of a stationary reference frame is most often the best choice for induction motors. In that case, the d-q-γ variables for the stator are identical to what are generally called the two-phase, or α-β-γ variables. White-Woodson (ref. 7) uses this in the analysis of motors having any number of stator phases, and calls them the "generalized two-phase" components. A discussion of these for HPO motors follows, with a simplification of the White-Woodson notation.

For a two-phase motor, the stator variables are i_α and i_β, and v_α and v_β. For three-phase motors the zero order or γ variable is also needed. Then, for more than three phases, the α and β variables will occur as multiple pairs. These, along with one or two of the γ variables for some cases, must give a total of n variables for an n-phase motor. The variables will be identified as α_1 and β_1 forming pair 1, α_2 and β_2 forming pair 2, etc; plus γ_0 and $\gamma_{n/2}$ if required.

3-3 Equivalent Circuits for α-β-γ Components

The general d-q-γ theory leads to equivalent circuits as a pictorial way of interpreting the method. An example of this is shown in Figure 3-1 for a S12ϕ motor. Circuits for a S18ϕ motor would be similar, but also require a pair of circuits for α_7 and β_7 and one γ circuit consisting of r_s and L_0 in series. The inductance values used in these circuits are the same as those used in the symmetrical component circuits of Chapter 2. Only the α_1 and β_1 circuits include rotor currents i_{dr} and i_{qr}. The other circuits have stator currents only. Therefore, only the α_1 and β_1 circuits can produce torque. Also, the α_1 and β_1 circuits must include the

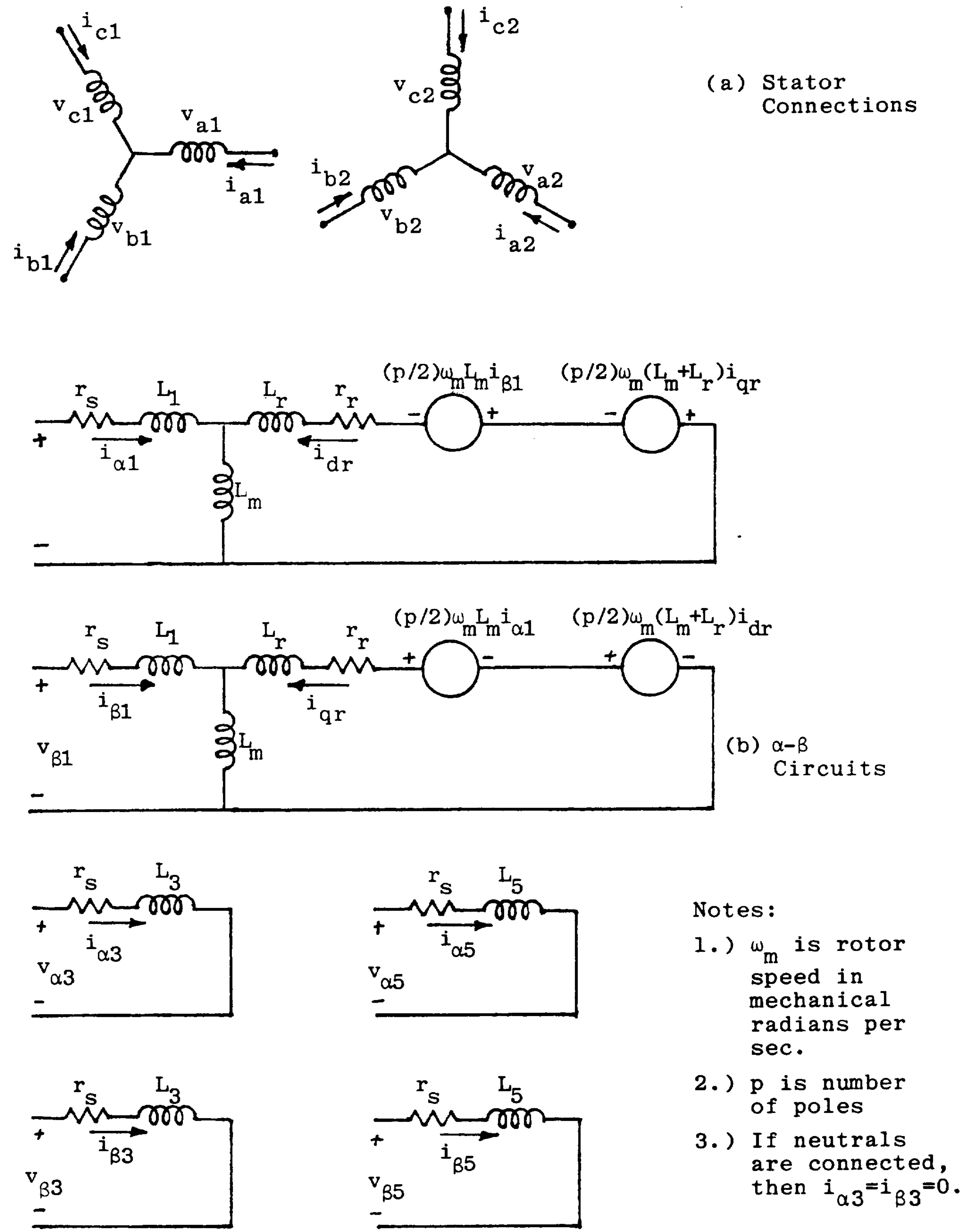


Figure 3-1. Circuits for S12ϕ Motor with α-β Variables.

speed voltages shown as part of their rotor circuits. These result in cross coupling between the two circuits in the general transient solution.

The close similarity between the α_1 and β_1 circuits on one hand, and the positive and negative sequence circuits on the other is apparent. Likewise, the similarity between other α-β pairs and the symmetrical component circuits is noted.

3-4 Transient Solution Using α-β-γ Variables

The matrix [T] given as in reference 7, page 567 is used to transform between the actual phase variables and the general two-phase or α-β-γ variables. This is expressed by the transformation equations

$$[v_{\alpha\beta\gamma}] = [T]^{-1}[v_{abc}] \tag{3-1}$$

$$\text{and } [i_{abc}] = [T][i_{\alpha\beta\gamma}] \tag{3-2}$$

For the S12ϕ motor, and using the machine model and variables shown in Figure 3-1(a), equation 3-1 becomes

$$\begin{bmatrix} v_{\alpha 1} \\ v_{\alpha 3} \\ v_{\alpha 5} \\ v_{\beta 5} \\ v_{\beta 3} \\ v_{\beta 1} \end{bmatrix} = [T_{S12\phi}]^{-1} \begin{bmatrix} v_{a1} \\ v_{a2} \\ v_{b1} \\ v_{b2} \\ v_{c1} \\ v_{c2} \end{bmatrix} \tag{3-3}$$

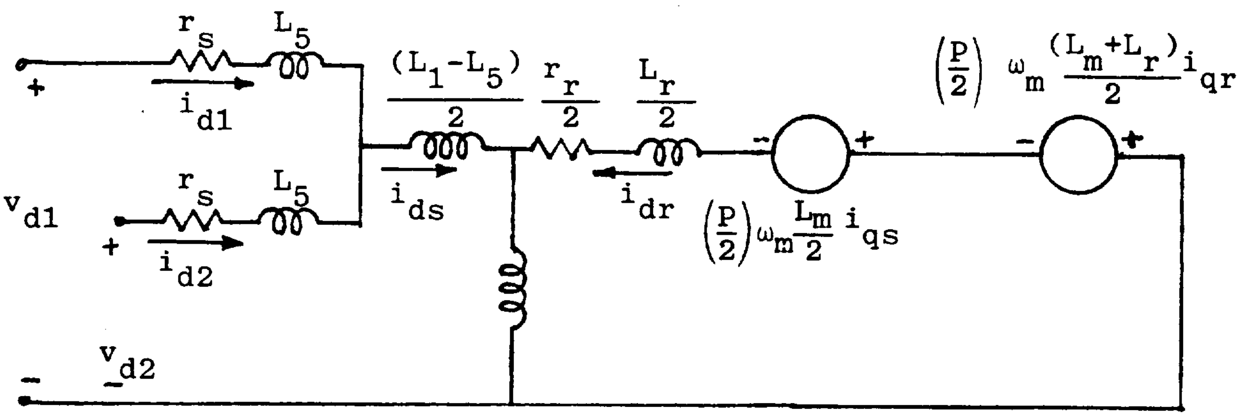
where $[T_{S12\phi}]^{-1}$ is as given in Appendix A.

The α-β voltages found from equation 3-3 are applied to the separate circuits shown in Fig. 3-1(b). The currents in these circuits may then be found by solving the set of simultaneous differential equations. In the general transient solution where speed is a variable, a torque equation is also required. After the currents are found, they may be transformed to the actual phase currents by means of equation 3-2. For the S12ϕ example, the expanded form of that equation is

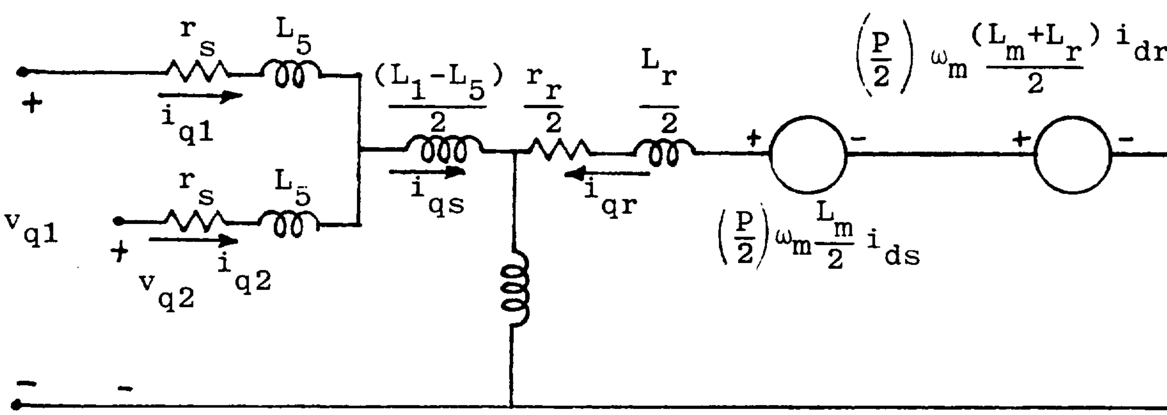
$$\begin{bmatrix} i_{a1} \\ i_{a2} \\ i_{b1} \\ i_{b2} \\ i_{c1} \\ i_{c2} \end{bmatrix} = [T_{S12\phi}] \begin{bmatrix} i_{\alpha 1} \\ i_{\alpha 3} \\ i_{\alpha 5} \\ i_{\beta 5} \\ i_{\beta 3} \\ i_{\beta 1} \end{bmatrix} \qquad (3\text{-}4)$$

3-5 Equivalent Circuits Based on Multiple d-q-γ Components

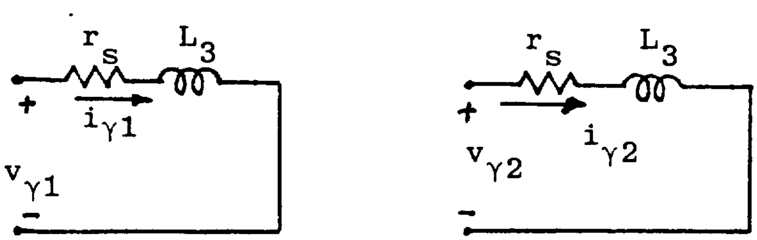
In Appendix A, the matrix [D] is given. It is used to transform from the phase variables of a machine having multiple three-phase windings to multiple sets of d-q-γ variables. While the principle involved applies to motors with 6, 9, 12, 15...3k phases, the equivalent circuit turns out to be simple only for the S12ϕ case. Therefore, only the S12ϕ case is presented here. The equivalent circuits shown in Figure 3-2 summarize the method. The voltage variables may be transformed from the a-b-c reference frame to the d-q-γ reference frame. Then, with these voltages applied to the circuits in Figure 3-2, the solution for currents is carried out. Note that in the general transient case, this involves the numerical solution of a set of non-linear differential equations, including also a torque equation not listed here. After solving for the currents in the d-q-γ refer-



(a)



(b)



(c)

Figure 3-2. Equivalent Circuits for S12ϕ Motor Based on Multiple d-q-γ Circuits
(a) d-axis circuit
(b) q-axis circuit
(c) γ-axis circuits

ence frame, they are transformed to the a-b-c currents to complete the solution. These transformations are summarized as:

$$[v_{dq\gamma}] = [D]^{-1}[v_{abc}] \text{ and } [i_{abc}] = [D][i_{dq\gamma}]$$

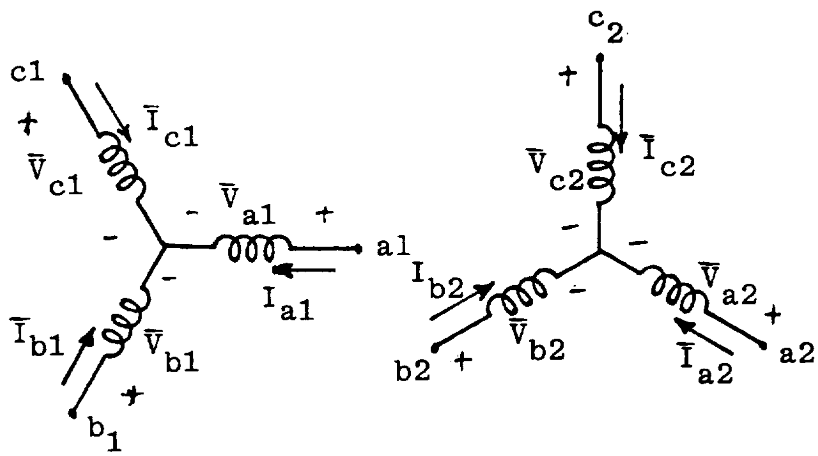
or

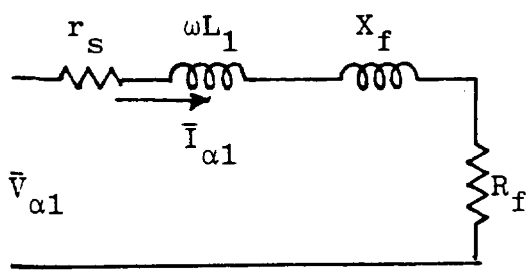
$$\begin{bmatrix} v_{d1} \\ v_{q1} \\ v_{\gamma 1} \\ v_{d2} \\ v_{q2} \\ v_{\gamma 2} \end{bmatrix} = [D]^{-1} \begin{bmatrix} v_{a1} \\ v_{b1} \\ v_{c1} \\ v_{a2} \\ v_{b2} \\ v_{c2} \end{bmatrix} \text{ and } \begin{bmatrix} i_{a1} \\ i_{b1} \\ i_{c1} \\ i_{a2} \\ i_{b2} \\ i_{c2} \end{bmatrix} = [D] \begin{bmatrix} i_{d1} \\ i_{q1} \\ i_{\gamma 1} \\ i_{d2} \\ i_{q2} \\ i_{\gamma 2} \end{bmatrix}$$

3-6 <u>α-β Equivalent Circuits for Sine Wave Case</u>

When the voltages and currents are sinusoidal and the steady state has been reached with speed practically constant, phasor forms of the variables may be used for the equivalent circuits. The equivalent circuit of Figure 3-1 retains the same form except that instantaneous voltages and currents are replaced by phasors, and also the inductive reactance associated with each inductance may be found. The circuits can then be solved as complex number algebraic equations rather than differential equations.

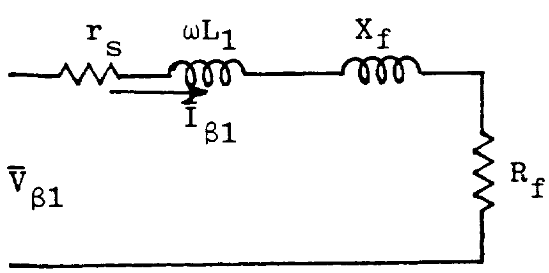
For certain quasi-balanced voltage sources such as those specified in Section 2-10, Case I, Case II, and Case IV, the phasor equivalent circuits can be simplified and appear as shown in Figure 3-3. Note that for these quasi-balanced cases, the speed voltages do not appear in the equivalent circuits. Speed is implicitly included when the value of slip s is specified, and values for R_f and X_f are computed according to Section 2-4.





Note: These circuits apply to the quasi-balanced cases I, II, and IV only.

Currents $\bar{I}_{\alpha 3}$ and $\bar{I}_{\beta 3}$ are zero if neutrals are not connected.



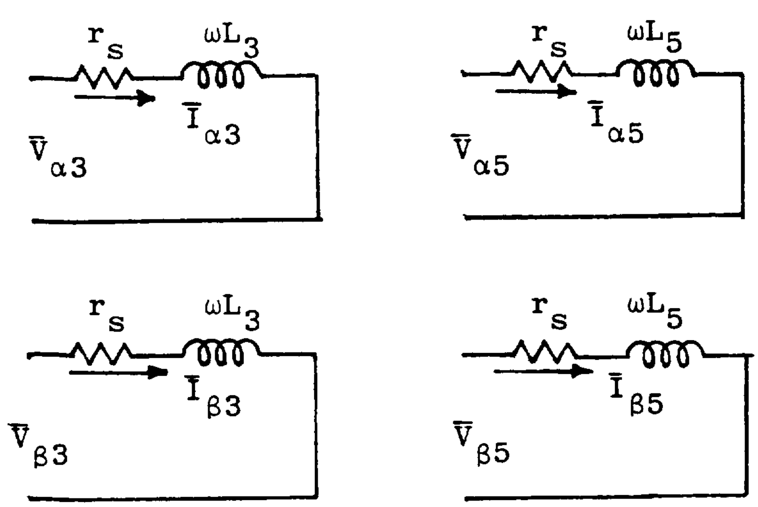


Figure 3-3. α-β Circuits for S12ϕ Motor, Phasor Form

3-7 α-β Circuits for Harmonics

Case IV presented in Section 2-10 is an important case to consider. It applies to motors supplied by inverters. The harmonic currents resulting when a motor is supplied by a voltage source inverter are of concern since they result in I^2R loss, but produce no useful torque.

When the non-sinusoidal source is balanced, the relationship between currents in the α-circuits and the β-circuits of Fig. 3-3 is as follows

$$\begin{aligned} \bar{I}_{\beta 1} &= -j\bar{I}_{\alpha 1} \\ \bar{I}_{\beta 3} &= -j\bar{I}_{\alpha 3} \\ \bar{I}_{\beta 5} &= -j\bar{I}_{\alpha 5} \end{aligned} \tag{3-5}$$

Therefore, all currents are known if the α-currents are found.

For the S12ϕ motor for which Fig. 3-3 is applicable, the fundamental and harmonics 11, 13, 23, 25, ... produce current in the $\alpha 1$ and $\beta 1$ circuits, and these produce rotor currents and torque. The harmonics 3, 9, 15, 21, ... produce currents in the $\alpha 3$ and $\beta 3$ circuits, and harmonics 5, 7, 17, 19, 29, 31, ... produce currents in the $\alpha 5$ and $\beta 5$ circuits, and none of these produce rotor currents or torque. These observations are consistent with those which can be made from Table 2-3 where the symmetrical component circuit for each harmonic is shown.

3-8 Multiple d-q-γ Circuits for Sine Wave Case

In a manner exactly the same as explained in Section 3-6, the equivalent circuits shown in Fig. 3-2 may be modified to accommodate phasor voltages and currents for the sinusoidal steady state cases. Again, for special cases I, II, and IV given in Section 2-10, the circuits become quite simple and are shown in Fig. 3-4.

In order to use the principle of Superposition, the circuits may be represented by those shown in Fig. 3-5. The voltages shown there are based on voltages $\bar{V}_{d1}$, $\bar{V}_{d2}$, $\bar{V}_{q1}$, and $\bar{V}_{q2}$ of Fig. 3-4, and are given by

$$\begin{aligned} \bar{V}_d' &= 1/2(\bar{V}_{d1} + \bar{V}_{d2}) \\ \bar{V}_d'' &= 1/2(\bar{V}_{d1} - \bar{V}_{d2}) \\ \bar{V}_q' &= 1/2(\bar{V}_{q1} + \bar{V}_{q2}) \\ \bar{V}_q'' &= 1/2(\bar{V}_{q1} - \bar{V}_{q2}) \end{aligned} \tag{3-6}$$

Then, in Fig. 3-6, the separate circuits for voltage components $\bar{V}_d'$ and $\bar{V}_d''$ are shown. It is obvious from these circuits that $\bar{V}_d'$ produces rotor current and torque, while $\bar{V}_d''$ produces current circulating through the stator impedance elements only, with no rotor current or torque. When balanced non-sinusoidal voltages are applied to a S12ϕ motor, voltage component $\bar{V}_d'$ results from the fundamental and harmonics 11, 13, 23, 25, ..., and $\bar{V}_d''$ is zero for these. But harmonics 5, 7, 17, 19, ... result in voltage component $\bar{V}_d''$, and give a zero value for $\bar{V}_d'$. This again illustrates the same observations made in Section 3-7.

3-9 Summary of Equivalent Circuits

While the symmetrical component circuits presented in Chapter 2 are the most versatile for the steady state cases, other methods such as those based on α-β-γ variables or multiple d-q-γ variables may also be used for the quasi-balanced cases, including the important non-sinusoidal case. These latter methods are also useful for the general transient solutions.

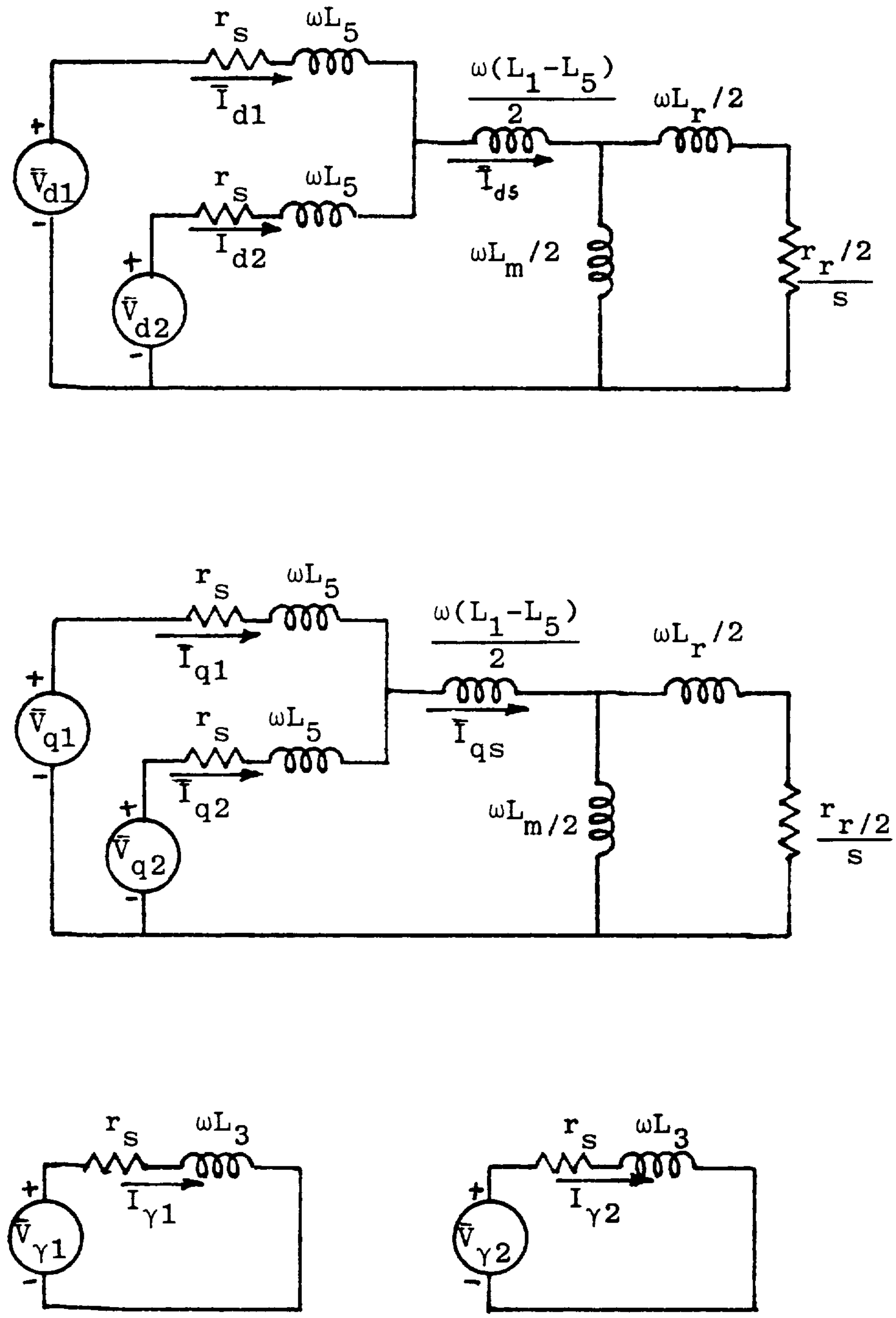


Figure 3-4. Phasor Form of Multiple d-q-γ Circuit for S12ϕ Motor, Cases I, II, or IV.

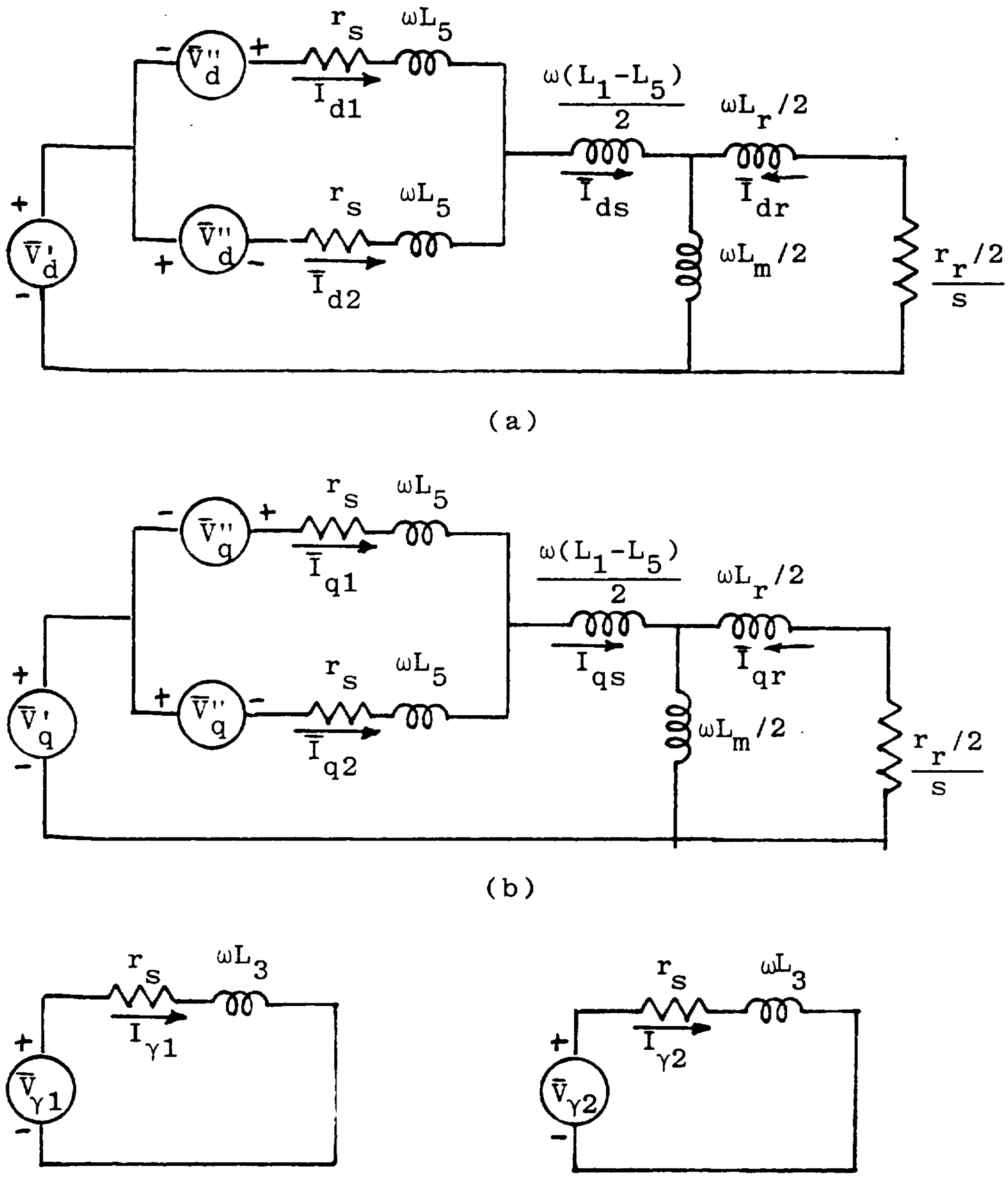
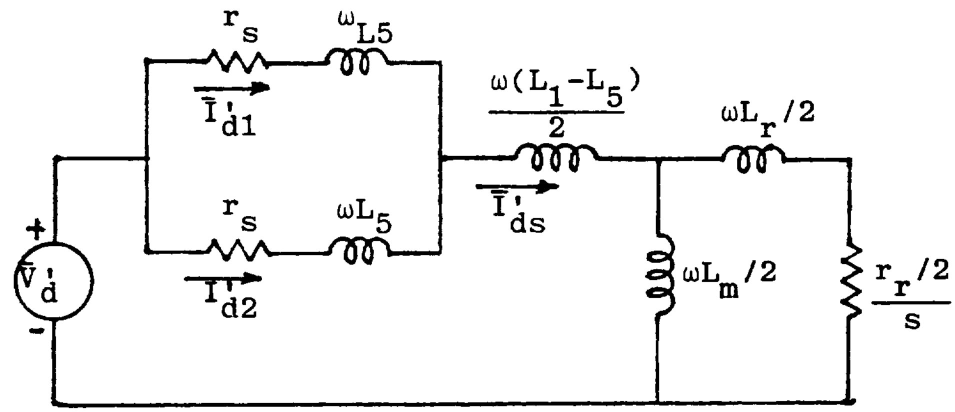
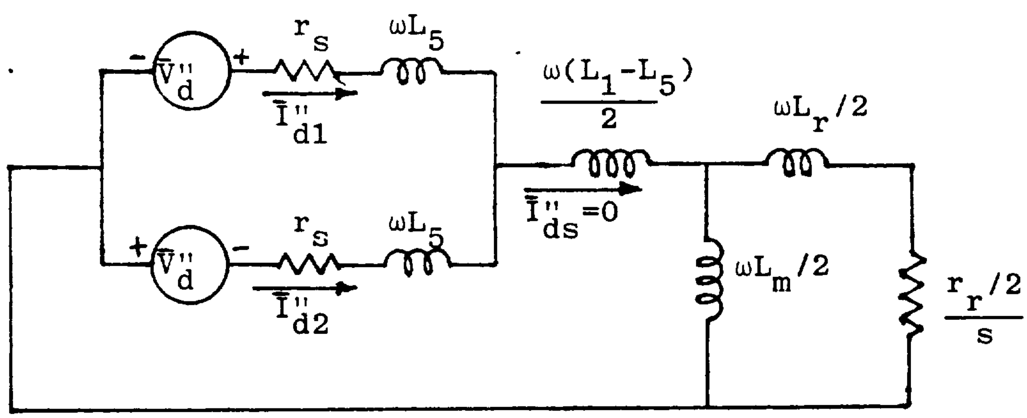


Figure 3-5. Modified Voltages for Multiple d-q-γ Circuit of S12ϕ Motor.
(a) d-axis
(b) q-axis
(c) γ-axis



(a)



(b)

Figure 3-6. Component Circuits for d-axis Based on Superposition Method.

(a) $\bar{V}''_d = 0$

(b) $\bar{V}'_d = 0$

CHAPTER 4. FILTERS DESIGNED FOR HPO MOTORS

4-1 Stator I^2R Losses Due to Unbalance or Harmonics

In Section 2-2, the various stator sequence inductances are defined. They are involved in the stator impedances in the sequence circuits as shown in Tables 2-1 and 2-2. The positive and negative sequence circuits have the positive sequence inductance L_1 for the stator. For all of the other sequence circuits, the stator inductance is one of the other sequence inductances L_i, where $i \neq 1$. Also, unlike the positive and negative sequence circuits which have both stator and rotor impedances to limit the current, these other sequence circuits have only the stator impedance to limit the current. As a consequence, any kind of unbalance that produces sequence voltages of the non-positive and non-negative sequence kind can result in relatively large sequence currents. The only effect these currents have on motor performance is the detrimental effect of added I^2R heating losses in the stator.

A very similar situation exists when motors are supplied from non-sinusoidal voltage sources, such as voltage source inverters. Tables 2-3 and 2-4 show the equivalent circuits for S12ϕ and S18ϕ motors, respectively, for the various time harmonics. These circuits are similar to the sequence circuits of Table 2-2. The positive sequence inductance L_1 is used for the fundamental frequency circuit in Tables 2-3 or 2-4, and this is the essential torque-producing circuit. Inductance L_1 must also be used in the circuit at the bottom of those tables, and that circuit applies to certain time harmonics as listed with it in each table. These harmonics produce stator and rotor I^2R losses, but the torque they produce is generally very small.

The equivalent circuits shown in Tables 2-3 and 2-4 for all the other harmonics involve only the stator impedance which consists of the stator resistance and a sequence inductance as shown in the tables. Since currents for these harmonics are limited by stator impedance alone, they can be large. Reference 5 explains how the coil pitch used for the stator winding affects the values of sequence inductances. For certain pitch values, one or more of the sequence inductances can be very small so that the impedance for certain harmonics can be small, and large harmonic currents can result. This causes excessive heating of the stator winding by these non torque-producing harmonics.

4-2 Filters to Reduce Stator I^2R Losses

From the description in Section 4-1 above, it is clear that, except for positive sequence inductance L_1, all sequence inductances should be large so that the non torque-producing sequence or harmonic currents can be kept small to reduce their I^2R losses. In this way, less heating and higher motor efficiency can be achieved. With HPO motors, it is easy to increase certain sequence inductances without increasing the positive sequence inductance. The devices used for this purpose will be called harmonic filters. They are described in detail in Appendix C where connection diagrams and tables which list the connections for the various filter types are shown. Also included there are the inductance values for those filters.

4-3 A Filter Example for a S12ϕ Motor

A S12ϕ motor with a filter can be used as an example. If no neutral connections are used, the currents for sequence numbers and time harmonics which are multiples of three will be zero, and therefore need not be included. As Figure C-7 in the Appendix indicates, one version of a practical filter for a S12ϕ motor has for its sequence inductances:

for positive and negative sequence, $L_{1F} = 0$

for sequence 3 and sequence 9, $L_{3F} = 3N^2P$ (4-1)

for sequence 5 and sequence 7, $L_{5F} = 12N^2P$

where N is the number of turns on a filter coil and P is the permeance of its magnetic core.

When the filter is included with the motor, each of the stator sequence inductances shown in Table 2-2 or 2-3 is increased by the corresponding sequence inductance for the filter so that L_3 is replaced by (L_3+L_{3F}), L_5 is replaced by (L_5+L_{5F}), etc. Thus, for this S12ϕ motor, the inductance applicable to harmonics 5, 7, 17, 19,...in Table 2-3 will be L_5+L_{5F}, and this can be made as large as desired by choosing values of N and P for the filter. It follows that currents for these harmonics can be made as small as desired. But the positive sequence inductance is not affected by the filter and therefore, torque will not be affected. Also, the currents for harmonics 11, 13, 23, 25,...will not be affected.

The overall effect of the filter can be a substantial reduction in stator I^2R losses. For a S18ϕ motor-operating from a non-sinusoidal source, all harmonics below the 17th can be greatly reduced when a filter is used. In contrast to this, a filter for a three-phase motor could not reduce any of the non-triplen harmonics without reducing torque-producing current as well.

4-4 Other Versions of Filters for S12ϕ Motors

Messrs. Tad W. Macie and Darius Irani designed a filter for a S12ϕ motor in about 1964. It used exactly the same principle as the one used in the example of the previous section. Their design with three transformers can produce the same filtering effect as the six transformer version used in the previous section. It is described in Figure C-8 in Appendix C. For the same total per phase inductance value, it requires a value of the quantity N^2P which is twice as large as the six transformer version. Its advantage is that fewer transformers and interconnections are required, although each transformer is somewhat larger.

There is another possible arrangement of small transformers which can be used as a filter for the S12ϕ motor. It uses six transformers, each with two coils. The six primary coils are in series with the six stator leads, one coil for each lead. The secondaries are interconnected with three in delta and three in wye. The connection diagram and equations are given in Appendix C. This type of filter has relatively small inductances for a given value of N^2P and is therefore inferior compared to the three-coil versions described.

4-5 Example of Filter to Reduce Harmonics in S12ϕ Motor

When inverter-supplied motors are to be operated at variable speed by varying frequency, it is necessary to maintain a constant ratio between fundamental voltage and frequency to produce constant air gap flux. The pulse width modulation (PWM) scheme has been widely used with three-phase motors to achieve this constant volts per hertz relationship. Various strategies have been used to achieve low harmonic currents and small torque pulsations with three-phase motors. These strategies usually require high switching rates and the associated switching losses in the inverter.

It would be advantageous to use a PWM waveform which requires a low switching rate. The lowest possible switching rate results if the PWM wave has only one pulse per half cycle, such as that shown in figure 4-1(a). The amplitudes for the fundamental, and for a few of the harmonics of this waveform are plotted in figure 4-1(b) as a function of the pulse width δ. The fundamental voltage is controlled by varying δ so that constant volts per hertz can be maintained over a wide frequency range.

If this simple waveform is used with a three-phase motor, the relatively large 5th and 7th harmonic components of voltage produce 5th and 7th harmonic currents. These currents produce a sometimes troublesome 6th harmonic torque pulsation, and also produce excessively large I^2R losses in the stator and rotor.

HPO motors inherently suppress some of the torque pulsations which are usual for three-phase inverter supplied motors, including some of the largest of these. For example, a S12ϕ motor has no 6th harmonic torque pulsation. The 12th harmonic torque pulsation which is due to the 11th and 13th harmonic remains, but is often small enough in amplitude and high enough in frequency so that it is not troublesome.

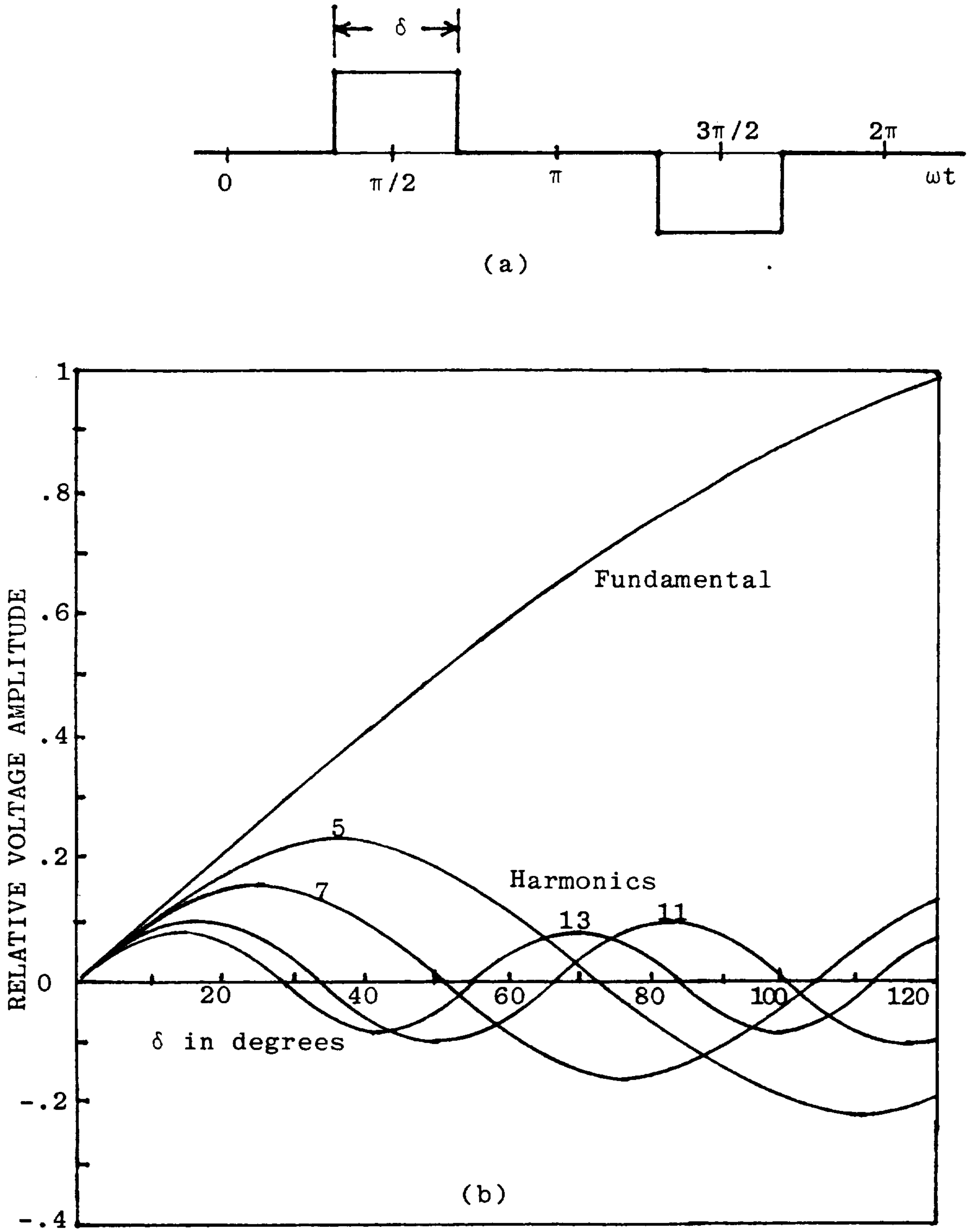


Figure 4-1. Single Pulse PWM Waveform and its Harmonics
(a) Line to line voltage waveform
(b) Harmonic amplitudes

But the 5th and 7th harmonic stator currents to a S12ϕ motor would be extremely large for this single pulse PWM waveform since voltage amplitudes are large for most values of δ, and also stator impedance limiting these currents can be quite low as explained in section 4-1. However, these harmonic currents can be greatly reduced by using a filter as described in section 4-2, and very good performance can be achieved with this simple PWM waveform. To show the very low I^2R losses which can be achieved using the single pulse PWM waveform for a HPO motor with filter, the following example is presented.

A typical 20 HP, 60 Hz, squirrel cage induction motor is chosen to illustrate the performance attainable when the S12ϕ arrangement is used. The voltage source inverter is assumed to consist of two separate three-phase inverters with their outputs 30 degrees out of phase as required for the S12ϕ source. Each inverter is assumed to produce a single pulse PWM waveform whose line-to-line voltage is as shown in Figure 4-1(a). For this waveform, the fundamental voltage amplitude and some of the harmonic voltage amplitudes are as shown in figure 4-1(b), showing amplitudes expressed as a fraction of the 60 Hz fundamental voltage. The I^2R losses were computed for this motor for frequencies from about 5 to 60 Hz. It was assumed that δ was adjusted to maintain a constant volts per hertz value throughout the range. A filter of the type described for S12ϕ motors in Appendix C was included for the computations. The fifth sequence inductance of this filter was taken to be ten times the stator leakage inductance of the motor; that is, $L_{5F} = 10L_1$.

Results of this analysis are plotted in Fig. 4-2. Since the heating and efficiency of three-phase motors operating from a 6-step variable voltage inverter source have been widely publicized and are well-known, curves for them have been included in Figure 4-2 for comparison.

4-6 Rotor I^2R Losses for the Above Example

As shown in Figure 4-2(a), the rotor loss for the S12ϕ motor is very small, remaining less than 10% of the fundamental rotor loss at full load for any frequency above 15 Hz. This frequency corresponds to 1/4 of base speed. For low speed, this harmonic loss increases and it reaches 75% of full load rotor loss at 5 Hz. This rotor loss characteristic is typical of HPO motors, and is not affected by the filter.

For comparison, the dashed curve shows the rotor I^2R loss for harmonics when the a 3-phase motor is used with a 6-step waveform. The S12ϕ motor on single pulse PWM has less rotor loss than the 3-phase motor on its 6-step waveform, down to about 13 Hz.

4-7 Stator I^2R Loss

Figure 4-2(b) shows the stator harmonic loss for the S12ϕ motor using the filter, and using the single pulse waveform. This loss remains less than 10% of the fundamental stator I^2R loss at full load for frequency above 12 Hz, and is 50% of full load loss at 5 Hz. Again, the dashed curve shows the 3-phase motor loss on a 6-step waveform for comparison.

4-8 Total Harmonic Losses

When the stator and rotor I^2R losses are combined as in Figure 4-2(c), it is evident that harmonic losses are very low for speeds above 1/4 of

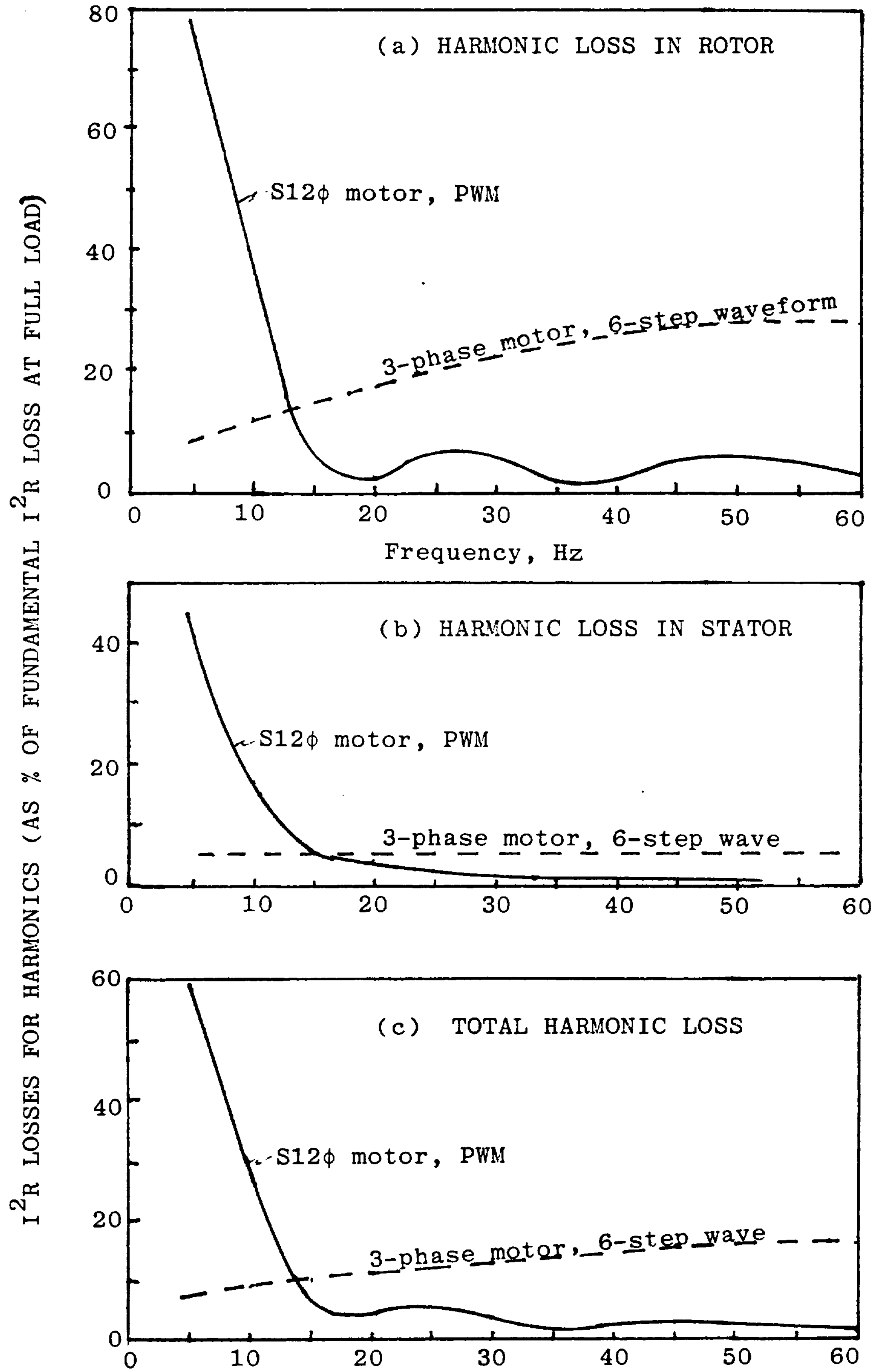


Figure 4-2. Harmonic I^2R Losses with Single Pulse PWM

base speed if the S12ϕ motor is used on the single pulse PWM inverter source. This is with a filter having the sequence inductance L_{5F} ten times the stator leakage inductance. A lower filter inductance requiring a filter of reduced size could be used with only a modest increase in harmonic losses. According to the analysis above, the motor operates quite well on a simple waveform which allows adjustment of voltage amplitude as frequency is varied. The inverter would have switching losses comparable to those in a 6-step inverter. Above about 1/4 base speed, motor efficiency and heating can be practically the same as when the motor operates from a sine wave source.

By using two to four pulses per half cycle for the PWM waveform when speed is low, these excellent characteristics could be extended down to 1/10 base speed or below, and switching losses would remain low.

To summarize, HPO motors with filter are superior to three-phase motors using PWM waveforms, or using 6-step waveforms. They can easily be made to operate practically as well as on a sinusoidal source. Experimental results for a S18ϕ motor with a filter are presented in Chapter 8.

CHAPTER 5. EXPERIMENTAL EQUIPMENT

5-1 The Motor Used

The experimental results were obtained using a squirrel cage induction motor built by G.E. for use in educational laboratories. Its rating as a 4-pole motor on 60 Hz. is 2 H.P. All of its 36 stator coils have their leads brought out to a 72-terminal panel. By externally interconnecting these coils in different ways, it was possible to get the following combination of phases and poles:

3-phase, 4 or 6 poles

Semi 12-phase, 4 or 6 poles

Semi 18-phase, 4 poles

The current rating was assumed to be 12 amps for the three-phase connections with parallel wye, and 6 amps for the S12ϕ and S18ϕ connections. Voltage ratings depend on connections, and are listed in the following table.

The motor was coupled to a dc machine used as a cradled dynamometer. This made it possible to measure the torque produced by the motor. Some of the impedances of the motor are included in Table 5-2 on page 61.

TABLE 5-1 Motor Voltage and Current Ratings

Connection	Rated Current Amps	Voltage Rating, Volts	
		Line to neutral	Line to line for each 3-phase group
3ϕ, 4P, parallel Y	12	63.5	110
3ϕ, 4P, series Y	6	126	220
$S12\phi$, 4P, Y-Y	6	63.5	110
$S18\phi$, 4P, Y-Y-Y	6	43.3	75
3ϕ, 6P, parallel Y	12	63.5	110
3ϕ, 6P, series Y	6	126	220
$S12\phi$, 6P, Y-Y	6	66	114

5-2 60 Hz Power Source

Many of the tests were made using a 60 Hz sinusoidal source. A special set of power transformers were used to supply either $S12\phi$ or 9ϕ ($S18\phi$) voltages from the 3-phase 60 Hz lines in the laboratory. There were three of these transformers, each rated 3 KVA. The $S12\phi$ motor was supplied by connecting one of its three-phase winding sets directly to the 3-phase source, with its other three-phase winding set supplied by the transformers connected delta to wye to give the necessary 30 degree phase displacement.

For $S18\phi$ operation, the three transformers were connected as shown in Figure 5-1. They were designed with the proper taps on their windings to give the nine equally spaced points on the circular locus as shown in that figure.

5-3 Measurement of Current and Power

The line currents in the six or nine leads to the S12ϕ or S18ϕ motor were measured using conventional iron-vane type ammeters. They were accurate to about ± 1/10 ampere. This accuracy was considered adequate to demonstrate the effects of loading, and particularly the effect of operation with an unbalanced source or with one line open. Comparative readings were taken with and without the filters, and results are given in Chapter 6.

The power input to the motor was determined by measuring the power taken from the three-phase source using two wattmeters connected between the source and the phase conversion transformer primary. The power as measured was therefore the power input to the motor plus transformer losses. Transformer core loss was determined by measuring transformer power input with the motor disconnected. The transformer I^2R losses were measured by a short circuit test. Then, for each motor test, the motor power input was found by subtracting these known transformer losses from the indicated wattmeter power taken from the three-phase source.

5-4 Motor Temperature Measurements

The temperature of the stator winding was monitored by a thermocouple fixed to the end-turn on one stator coil. Temperature was read using a "Digitemp" digital meter. For all tests involving full load power input, slip, or torque; the temperature was held nearly constant as follows:

For 6-pole load tests, temperature was 60° C ± 3°

For 4-pole load tests, temperature was 58° C ± 3°

For locked rotor tests, temperature was 30° C ± 4°

5-5 Motor Slip Measurements

The motor slip speed was measured during load tests by visually observing the apparent shaft speed when it was illuminated by a stroboscopic light source. This strobe light was synchronized with the same 60 Hz source which supplied the motor, given 30 flashes per second for 4-pole motors and 20 flashes per second for 6-pole motors. It was possible to actually count the apparent number of shaft revolutions (slip revolutions) in an interval accurately timed with a digital stop watch.

5-6 Motor Tests on Non-Sine Wave Source

A rotary switch mechanism was constructed using an 18-bar commutator driven at variable speed by a d-c motor. Two segments are formed by joining eight adjacent bars for each segment. These were supplied from a dc source through slip rings. Then, nine equally spaced brushes in contact with the commutator provided a 6-step waveform for the S18ϕ motor. The waveform of motor voltage and current were recorded by means of a dual beam oscilloscope with a camera attached.

5-7 Experimental Filters

Chapter 4 describes various types of filters which can limit some of the unwanted sequence or harmonic currents. Two of the filters described there were built and tested with the experimental motor. Details of these are presented here.

The filter requires a number of magnetic cores, each having two or three coils. Each laminated core has dimensions shown in Figure 5-2. The permeance (P) of each core is approximately 17 microhenries at low flux

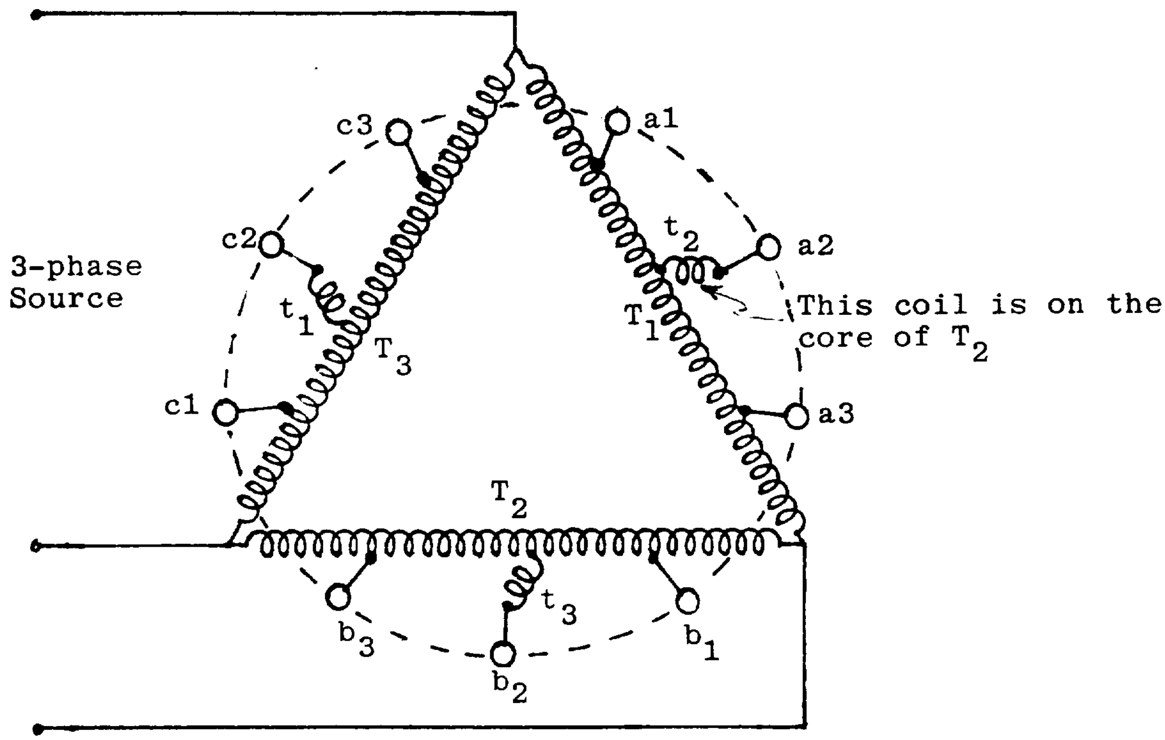


Figure 5-1. Power Transformer Connection for S18ϕ Source

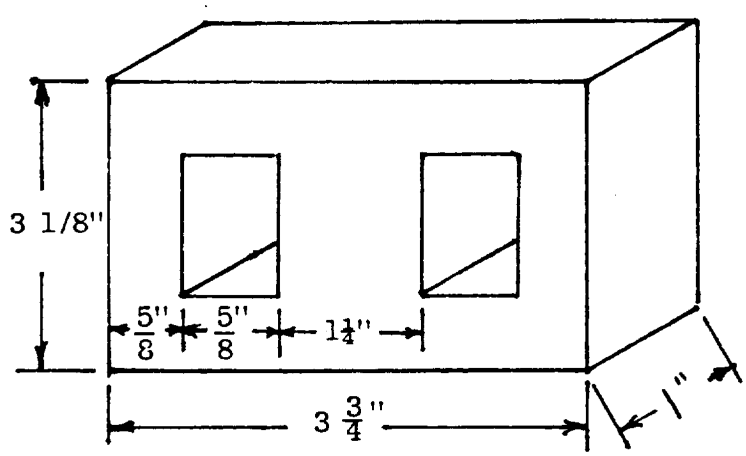


Figure 5-2. Steel Core Used for Harmonic Filter Elements

density, and this value decreases as core flux increases because of magnetic saturation.

For the S18ϕ filter, nine of these cores are required. Each has three coils as shown in Figure C-1 in Appendix C. The number of turns was chosen to be N = 8 for each auxiliary coil. Then, for the filter based on the phase opposite scheme and described in Figure C-5, the value of k had to be 1.879. Based on this, the main coil required kN or 1.879 X 8 = 15.032 turns. Therefore, a coil having 15 turns was used. The nine transformers were connected according to the Connection Table in Figure C-5 to form the S18ϕ filter.

For the S12ϕ filter, the wye-delta type of filter shown in Figure C-9 was built. It used six of the magnetic cores already described. The primary on each filter transformer had N = 16 turns. The value chosen for the turns ratio was a = 11/16. Three of the transformer secondaries therefore had 11 turns and these were wye-connected as shown in Figure C-9. The other three transformers required $11\sqrt{3}$ = 19.05 turns, and 19 turns were used for the delta-connected secondaries.

5-8 Impedances of Experimental Filters and Motor

The sequence impedances of the motor with its various pole and phase combinations, and also the sequence impedances of the two filters were found. To measure these impedances, the filter was connected between the motor and the phase-transforming power source described in Section 5-2. By interchanging the appropriate leads at the S12ϕ or S18ϕ source, the voltages applied to the filter-motor combination were fifth sequence for the S12ϕ motor, and fifth and then seventh sequence for the S18ϕ motor. It was found necessary to keep currents below about 2.5 amperes to avoid saturation in the filter cores. These low currents were obtained by using a low source voltage. Measurements were made for each phase current, the voltage across the filter in series with each motor lead and the voltage from each motor phase to neutral. Then voltage divided by current gives the impedance in ohms, from which reactance can be calculated. Results are tabulated in Table 5-2.

TABLE 5-2 Sequence Reactances for Filters and Motors

Motor Phases/Poles	Sequence Number	Reactance at 60 Hz ohms per phase		Apparent Values for Filter Core, μH	
		Motor	Filter	N^2P	P
S12ϕ/4P	5 and 7	1.79	1.69	4500*	17.5
S12ϕ/6P	5 and 7	0.45	1.56	4100*	16.2
S18ϕ/4P	5 and 13	1.51	1.96	1050**	16.4
	7 and 11	small	3.53	804**	12.6$^+$

$^+$saturation may be cause of lower value
*N = 16 turns
**N = 8 turns

The motor reactances recorded in Table 5-2 vary for the different sequences. One reason for this variation is the influence of coil pitch as explained in reference 5. Attention is called to the low value of reactance for sequences 5 and 7 in the 6 pole S12ϕ motor, and also for sequence 7 and 11 in the S18ϕ motor. In each of these cases, sequence inductance is very small.

CHAPTER 6. MOTOR OPERATION WITH ONE LINE OPEN

6-1 Introduction

It is useful to consider motors which are sometimes required to operate, at least temporarily, with one phase unable to carry current. This might be due to a blown fuse or a faulty inverter source. It is well known that three-phase motors can continue to run with one phase open, but they will generally overheat in a short time. They have no starting torque with one phase open.

HPO motors can be stated and can be run with one phase open, and their running characteristics can be nearly as good as with all phases carrying current. The equivalent circuit applicable to this situation is given for the S12ϕ motor in Figure 2-9, and the analytical solution for currents, torque, losses, and efficiency can be based on that symmetrical component circuit. The 2HP experimental motor was tested with one line open and results are reported in the following sections, and also in Reference 6.

6-2 Locked Rotor Tests

The motor was tested as a S12ϕ type with its rotor locked. Torque measurements were made with one-half rated voltage applied. The sequence filter described in Chapter 4 was used in some tests to determine its effect on torque. Results of these tests are shown in the bar graphs of Figure 6-1(a) and (b) where locked rotor torque is expressed as a percentage of that produced when all lines carry current and no filter is used. The locked rotor torque for the motor connected for three-phase bal-

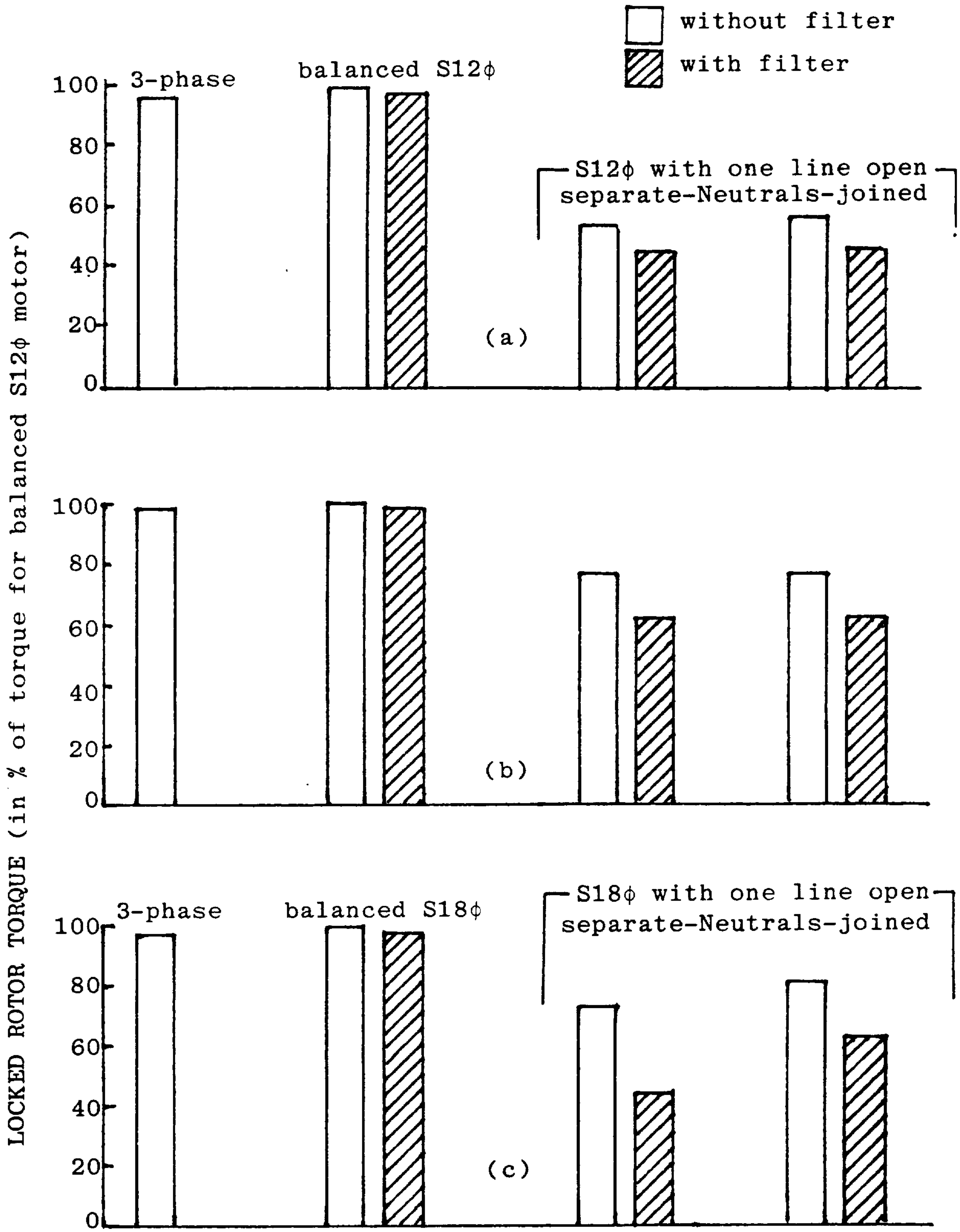


Figure 6-1. Locked Rotor Torque with One Line Open
(a) 4-pole, S12ϕ motor
(b) 6-pole, S12ϕ motor
(c) 4-pole, S18ϕ motor

anced operation is also shown in the figures for comparison. Figure 6-1(c) has a similar set of bar graphs for the S18ϕ connection of the motor.

These results show that, for balanced S12ϕ or S18ϕ operation, locked rotor torque is slightly greater than for the three-phase connection. Also, torque with a balanced source is very slightly reduced when the filter is included. This reduction would be due to the small resistance and leakage reactance of the non-ideal filter transformers. It is obvious from these tests that the filter has almost no effect on torque when the source is balanced, as predicted by the theory.

The bar graphs in Figure 6-1 which pertain to the motor with one line open also conform to what the theory predicts. In Chapter 2, section 2-13, it was pointed out that, for HPO motors, the reduction in torque when one line was open would be small, and that the amount of reduction would depend on the value of sequence inductance L_5 and L_3 when neutrals are joined, or L_5 only if neutrals are separate. This reduction will be smallest when these inductances are small. The bar graphs verify this. The 6-pole S12ϕ motor has a value of L_5 much smaller than that of the 4-pole S12ϕ motor. With one line open and without the filter, the 6-pole motor with its very small L_5 has torque reduced from 100% to 76.6% with neutrals separate. In contrast to this, the 4-pole motor with its larger value of L_5 gives a corresponding reduction of 100% to 55.5%. These reductions are only slightly less when the neutrals are joined.

When the filter is added to these motors, the values of L_5 and L_3 increase considerably. The effect is to cause a greater reduction in locked rotor torque when one line is open; from 100% to 62% for the 6-pole motor, and from 100% to 46% for the 4-pole motor. The filter is therefore detrimental to torque production when one line is open, as would be expected from the theory.

Similar results are shown in Figure 6-1(c) for the S18$_\phi$ motor which has a very small value for L_7.

6-3 Full Load Tests with One Line Open

The motor was tested from a sinusoidal, 60 Hz source at full load for the several phase and pole number combinations, with and without filter, on a balanced source, then with one line open. For the 4-pole motors, torque was maintained constant at 8.33 N-n for all tests. For the 6-pole motors, the torque value was a constant 12 N-m. The bar graphs in Figures 6-2 and 6-3 show results for efficiency and slip, respectively. As with locked rotor tests in the previous section, the performance is compared with the 3-phase motor version.

6-4 Efficiency Measurements

According to these experimental results shown in Figure 6-2, the efficiency as a HPO motor on balanced source is slightly lower than the 3-phase version. This unexpected result may be due to the I^2R losses in the additional wiring required, or to unaccounted for losses in the power transformers needed for the HPO motors, or to the fact that the HPO sources were not perfectly balanced. It is known that any voltage unbalance results in additional I^2R losses due to the sequence currents.

When one line is open, the bar graphs in Figure 6-2 indicate that efficiency drops. But this drop in efficiency is very small for the 6 pole

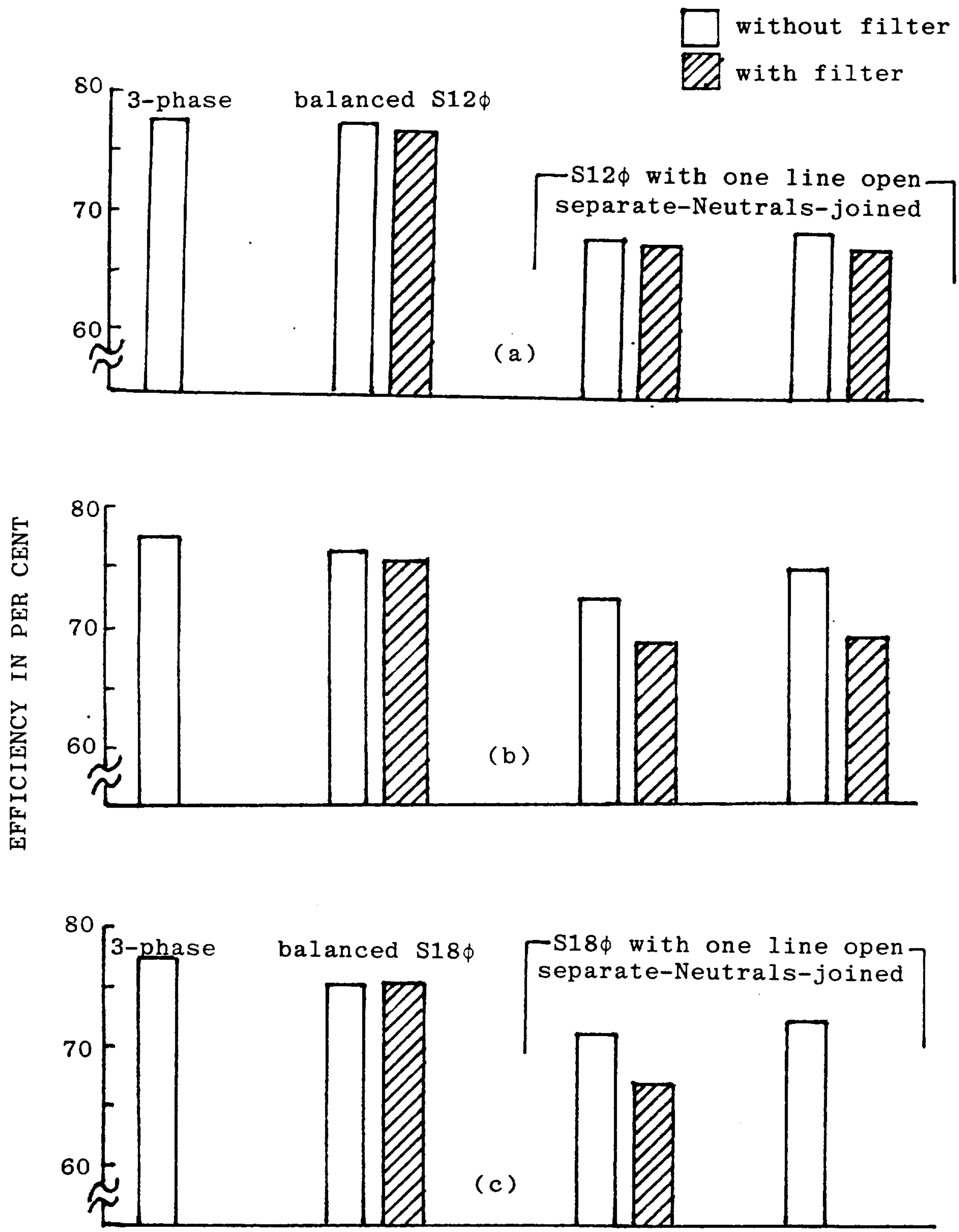


Figure 6-2. Full Load Efficiency with One Line Open
(a) 4-pole, S12ϕ
(b) 6-pole, S12ϕ
(c) 4-pole, S18ϕ

version (Figure b) and less than that for the 4-pole version (Figure a). This is again due to the shunting effect of the sequence impedance branches as seen in the applicable equivalent circuit in Figure 2-9. The current shunted away from the negative sequence circuit flows through the sequence paths. The lower the sequence inductance values, the more current is diverted to the sequence branches. The negative sequence I^2R loss becomes much less while I^2R loss due to this diverted current is only slightly more. The net effect is a reduction in loss.

When a filter is added, sequence inductances other than the positive and negative sequence inductances are increased, less current is diverted through the sequence branches so that negative sequence current and its associated I^2R losses are greater. The efficiency with one line open therefore is less when the filter is included. The bar graphs of Figure 6-2 verify this.

6-5 Full Load Slip Measurements

Full load tests are described in Section 6-3. The slip was measured and Figure 6-3 shows the slip in percent for each case. Except for the S18ϕ motor [Figure 6-3(c)], slip on a balanced source without a filter is slightly less for the HPO motors tested as compared to the 3-phase case. Then, the inclusion of the filter causes a slight increase in slip because the filter is not quite ideal.

Without the filter, opening one line causes a significant increase in slip for the 4-pole motor which has a rather large value for L_5, but a much smaller increase in slip for the 6-pole motor which has a very small value of L_5. Again, this is explained by the diversion of current from the nega-

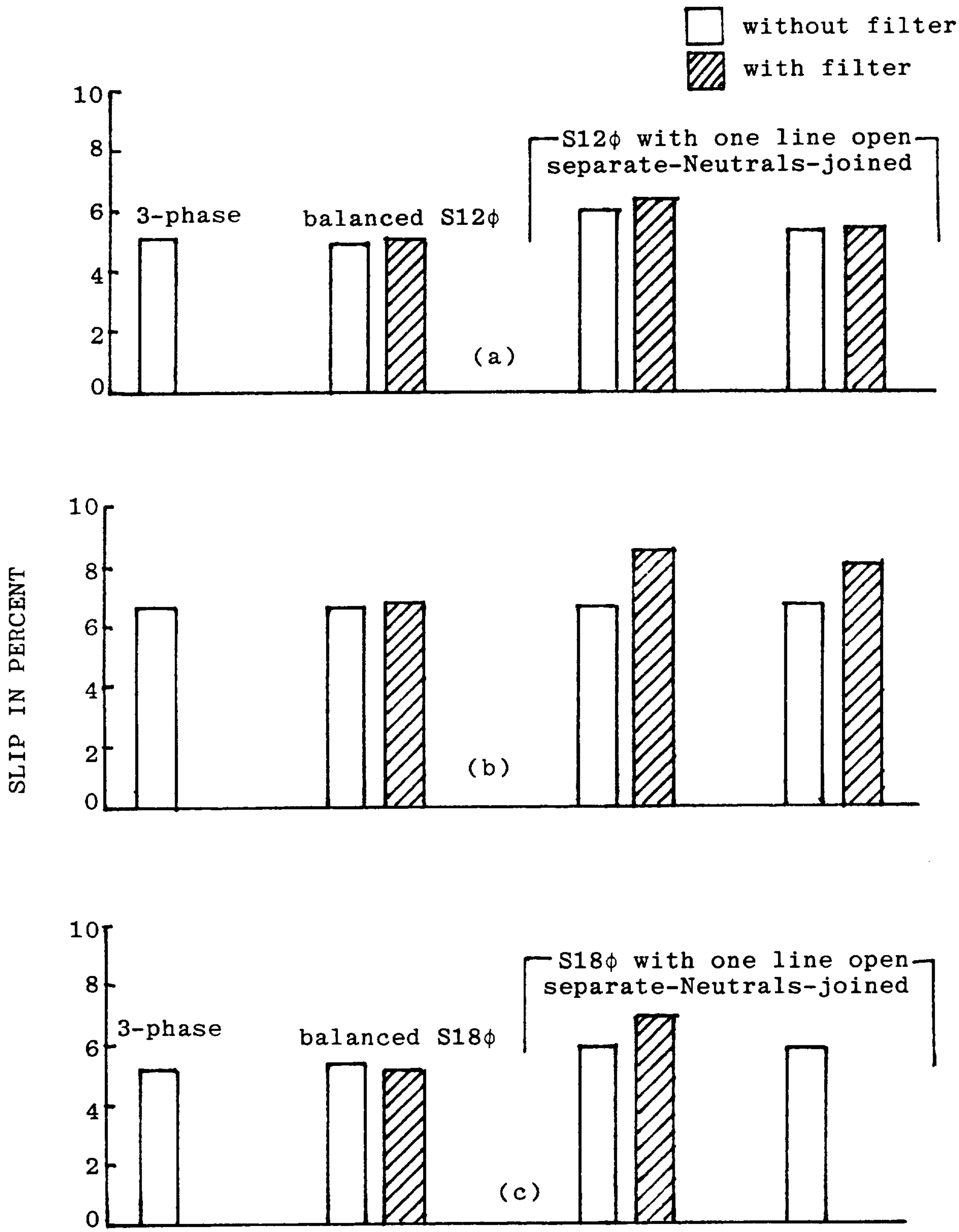


Figure 6-3. Full Load Slip with One Line Open
(a) 4-pole, S12ϕ
(b) 6-pole, S12ϕ
(c) 4-pole, S18ϕ

tive sequence branch, thus reducing negative sequence torque and its effect on slip. When the filter is added, L_5 is increased and results show the predictable increase in full load slip with one line open.

6-6 Summary of Full Load Tests

The important observations which theory predicts and experimental tests verify are: 1) The filter has almost no effect on operation of the motor when the source is balanced, 2) a very low sequence inductance value can result in a very small deterioration in full load performance (slip and efficiency) when one line is open, and 3) the filter has a detrimental effect on full load performance when one line is open.

Locked rotor tests show similar results, with the least reduction in torque when a line is opened if sequence inductance is small.

HPO motors can be designed to take advantage of these excellent characteristics with one line open. The effects noted in the experimental motor can be even more dramatic if the motor is larger and therefore has its sequence impedances less influenced by the resistive component. Then sequence impedance can be made very low by the proper choice of coil pitch, and this will enhance the performance with one line open.

CHAPTER 7. CURRENTS IN EXPERIMENTAL MOTOR WITH SOURCE UNBALANCE

7-1 Introduction

It is common to have at least a small amount of unbalance among the phase voltages of a polyphase system. Induction motors can be sensitive to such unbalance which produce sometimes large current unbalance and additional I^2R losses. HPO motors can be especially sensitive to unbalance in the voltage source when one or more of their sequence inductances are small. A sequence/harmonic filter can be used to reduce the amount of current unbalance, thereby reducing the stator I^2R loss somewhat. However, the filter might not give much improvement in overall efficiency, as experimental tests will show.

7-2 Effect of Intergroup Voltage Unbalance

Section 2-12 in Chapter 2 defines intergroup voltage unbalance and the quantities ΔV and ε associated with it. The experimental motor was subjected to voltages having different amounts of voltage magnitude unbalance ΔV, and also different amounts of angle unbalance ε. Results are summarized in Table 7-1 for the S12ϕ, 6-pole motor. The 4-pole motor was also tested, but because of its higher value for sequence inductance L_5, current unbalance was less than for the 6-pole version with its small value of L_5.

The test results reported in Table 7-1 are for the motor at full load. In the table voltages $V_{\phi 1}$ and $V_{\phi 2}$ are the magnitudes of the line to neutral source voltages for motor phase group 1 and 2, respectively. The angle δ

TABLE 7-1 Results of Voltage Unbalance on S12ϕ, 6P Motor at Full Load

Case No.	Filter Used	Volts to Neutral $V_{\phi}1$	$V_{\phi}2$	ΔV Volts	%*	δ deg.	ε deg.	Phase Current, A $I_{\phi}1$	$I_{\phi}2$	ΔI %**
1	No	63.2	62.9	.15	.23	29.2	-.8	6.53	5.30	10.0
2	Yes	63.4	62.9	.25	.40	29.2	-.8	6.10	5.86	2.0
3	No	63.3	63.8	-.25	-.40	30.6	.6	5.63	6.17	-5.0
4	No	61.5	63.2	-.85	-1.40	30.7	.7	4.85	7.13	-19.0
5	Yes	61.0	62.8	-.90	-1.45	30.7	.7	5.75	6.36	-5.0

*Expressed as percent of V_{ave}, where $V_{ave} = 1/2(V_{\phi}1 + V_{\phi}2)$

**Expressed as percent of I_{ave}, where $I_{ave} = 1/2(I_{\phi}1 + I_{\phi}2)$

is equal to (30° + ε). Within phase group 1, voltages are substantially balanced three-phase, and the same is true for phase group 2. As the theory in Section 2-12 shows, the kinds of unbalances used here result in 7th sequence currents in addition to positive sequence.

For case 1 and 2 in Table 7-1, the source is simply the delta-wye power transformers used to give 30 degrees displacement between phase groups 1 and 2, with one phase group supplied directly from the three-phase line. The small magnitude unbalance in voltages, and the phase shift of 29.2 instead of 30 degrees are the result of impedance drops in the non-ideal power transformers. The considerable unbalance in currents seen for case 1, where $I_{\phi 1}$ and $I_{\phi 2}$ are 10% above and below the average current, respectively, is a consequence of the unbalance in voltages and the small sequence inductance for 7th sequence (L_5). For case 2, the filter described in Figure C-9 in Appendix C, and having reactance values tabulated for the S12ϕ/6P motor in Table 5-2, was added. As Table 7-1 shows, the filter reduced the current unbalance from 10% to only 2%. However, the power input actually increased by about 5 watts (out of about 1800 watts total) when the filter was added. Reducing the 7th sequence current with the filter must have decreased motor power input slightly. Apparently, the I^2R losss in the added filter was greater than the reduction in motor I^2R loss.

For case 3 in Table 7-1, the power transformers were modified by the addition of a one-turn coil on each, and this coil connected in series with the appropriate transformer secondary gave a phase shift δ = 30.6° instead of 29.2°. Source voltage $V_{\phi 2}$ was also slightly increased. The combination of these small voltage changes gives a current unbalance with $I_{\phi 1}$ less than the average by 5%. Finally, with the one-turn coil reconnected, the source voltage $V_{\phi 1}$ was reduced for cases 4 and 5. With this larger value of ΔV, and with δ = 30.7°, current unbalance was -19% without the filter, and reduced to -5% with the filter. Again, the power input was practically the same whether the filter was used or not.

7-3 Conclusions Regarding Current Unbalance

For the intergroup unbalance studied here for the experimental motor, the following observations are made:

1. For motors having small values for sequence inductances, large current unbalance can occur for voltage magnitude unbalance over about 1%, or for angle unbalance more than about 1 degree.
2. Filters are effective in reducing the current unbalance.

3. The current unbalance caused by 7th sequence current in the S12ϕ motor does not affect torque and causes only a slight increase in stator I^2R loss.

4. Including a filter can actually increase rather than decrease the total I^2R loss.

In summary, HPO motors can be operated successfully on somewhat unbalanced sources, even though the current unbalance might be rather large. It is possible to design the motor with coil pitch chosen to give a sufficiently large value for sequence inductances to control current unbalance.

CHAPTER 8. EXPERIMENTAL RESULTS USING FILTER WITH 6-STEP WAVEFORM OF VOLTAGE

8-1 Test Setup

The 2HP motor connected as a S18ϕ, 4-pole motor was supplied from the rotary switch described in Section 5-6. This provided a balanced 6-step voltage to the motor with a fundamental frequency of 17 Hz. A camera mounted on a dual-beam oscilloscope was used to record the waveform of motor line-to-neutral voltage, and also the line current.

A filter of the phase-opposite type described in Figure C-5 of Appendix C was included between the source and the motor for one of the tests. Based on the reactances of this filter recorded in Table 5-2, its fifth sequence inductance is 5.2 mH, and this same inductance value applies to the fifth harmonic. Its seventh sequence inductance is 9.4 mH, and this value applies to the seventh and eleventh harmonics. Based on this information, the filter reactances for the various harmonics were as given in Table 8-1. The motor reactances are also included.

TABLE 8-1. Reactances for S18ϕ Filter

Harmonic	Frequency Hz	Reactance, Ohms	
		Motor	Filter
1	17	---	0
5	85	2.2	2.8
7	119	Small	7.0
11	187	Small	11.0

8-2 Experimental Results

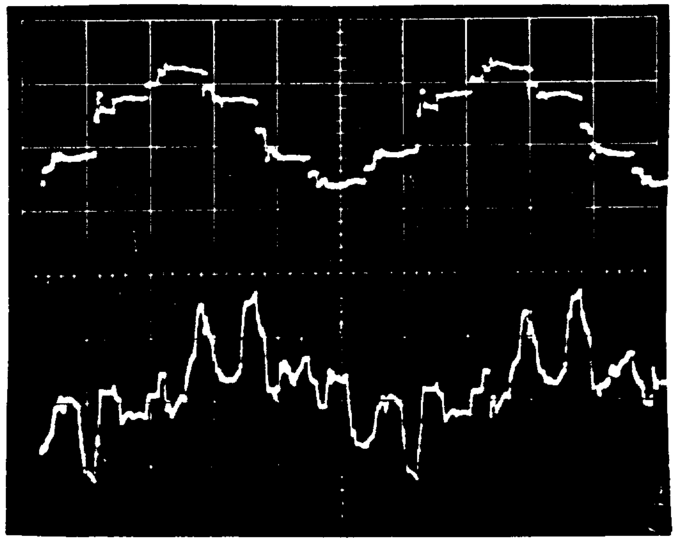
The S18ϕ motor was run at no load using the 17 Hz 6-step voltage waveform. The oscillograms of line to neutral voltage and line current are shown in Figure 8-1. Figure 8-1(a) shows these without the filter, and the large harmonic content of the current is evident. Then Figure 8-1(b) shows results when the filter is included between the same 6-step voltage source and the motor. The voltage waveform in Figure 8-1(b) is taken at the motor terminals, and is more nearly sinusoidal than the 6-step source voltage because of the filter. The current waveform with the filter clearly shows that the harmonics have been suppressed.

The current waveforms for Figure 8-1 were analyzed to determine the amplitude of each frequency component, and results are given in Table 8-2.

TABLE 8-2. Harmonic Currents

Harmonic	Current; Amps RMS	
	Without Filter	With Filter
1	1.73	1.76
5	0.68	0.40
7	1.20	0.12
11	0.70	0.08

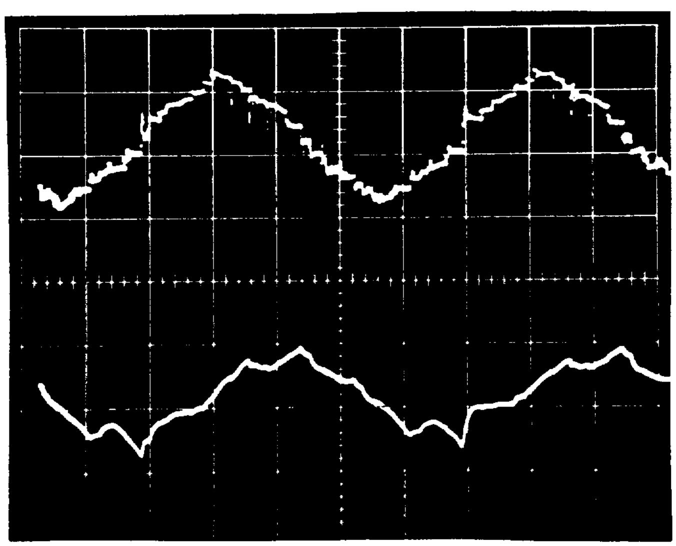
As Table 8-2 indicates, the fundamental current is not significantly affected by the filter. The fifth harmonic is reduced somewhat by the filter. The seventh and eleventh harmonic currents are rather large without the filter because motor reactance for these harmonics is small. But when the filter is added, these currents are reduced by a factor of



6-step source voltage, and also motor voltage line to neutral

Line current to motor, no load

(a)



Motor voltage, line to neutral

Line current to motor, no load

(b)

Figure 8-1. Oscillograms of Voltage and Current for S18ϕ Motor with 6-Step Source. Frequency is 17 Hz.
(a) Without filter
(b) With harmonic filter of Figure C-5

about ten due to the substantial increase in reactances produced by the filter.

The results here clearly demonstrate that by using a filter with an HPO motor, harmonic currents can be made small.

CHAPTER 9. CONCLUSIONS

9-1 Results of the Investigation

It has been demonstrated that induction motors having more than three phases are entirely practical. They meet the need for electric vehicles where the high current per phase for three-phase motors has been a problem. They have several other advantages which can be important for electric vehicles and for many other applications as well.

They can be more reliable than three-phase motors because of their ability to start and run with one phase open (or shorted). It has been shown that by choosing coil pitch so that one or more of the sequence inductances is small, the starting and running characteristics of these HPO motors with one phase open can be almost as good as for balanced operation. However, when these inductances are small, large current unbalance results if source voltage is unbalanced.

An important application of HPO motors is to systems where the source is an electronic inverter, either the current source or the voltage source type. When supplied by a current source inverter and in comparison to three-phase motors, HPO motors will have certain of the lower frequency torque pulsations eliminated, and will also have lower I^2R loss produced by harmonic currents in the rotor because currents for certain harmonics cannot flow in the rotor.

When supplied by a voltage source inverter, these same characteristics of small pulsating torque and small rotor I^2R losses apply. However, those harmonics eliminated from the rotor will produce larger currents in the stator, causing an increase in stator I^2R loss. This can be easily overcome by using one of the harmonic filters developed as part of this investigation. The result can then be a motor which, when supplied by a non-sinusoidal voltage source, can be practically as efficient and smooth running as a motor supplied by a sinusoidal source. This use of harmonic filters is considered to be an especially important result of the project.

While motors having any large number of phases would perform quite well, practical considerations limit the number to six or nine. Choosing multiples of three is important because the well-known advantages of three-phase wye or delta connections can be retained. With more than about nine phases, the number of stator slots and the number of stator leads becomes excessive, filters if used require too many components, and the improvement over six or nine phase types is slight.

9-2 Suggestions for Further Investigation

A complete evaluation of a system consisting of a voltage source inverter, a harmonic filter, and a motor having six or nine phases would be the next logical step. This should include the design of the components of the system, and the building and testing of prototypes. One area of concern is the size, the weight, and the cost of a filter to be included. Since the filter reduces the harmonic losses in the motor, the motor itself could be somewhat smaller than without a filter. Therefore, the overall weight of the system need not be increased much by the inclusion of a filter. Also, the use of a filter could reduce the complexity as well as the losses in the inverter so that the overall cost of the system including a filter could be comparable to that of a conventional system.

REFERENCES

1. McLean, G.W., Nix, G.F., and Alwash, S.R., Performance and Design of Induction Motors with Square-Wave Excitation, Proceedings of IEEE, Vol. 116, No. 8, August, 1969, pp. 1405-1411.
2. Nelson, R.H. and Krause, P.C., Induction Machine Analysis for Arbitrary Displacement Between Multiple Winding Sets, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-93, May/June, 1974, pp. 841-848.
3. Jahns, T.M., Improved Reliability in Solid-State AC Drives by Means of Multiple Independent Phase Drive Units, IEEE Transactions on Industry Applications, Vol. IA-16, May/June, 1980, pp. 321-331.
4. Andresen, E. and Bieniek, K., 6-Phase Induction Motors for Current-Source Inverter Drives, Conference Record from Industry Applications Society Meeting in Philadelphia, October, 1981, pp. 607-618.
5. Klingshirn, E.A., High Phase Order Induction Motors, Part I - Description and Theoretical Considerations, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, January, 1983, pp. 47-53.
6. Klingshirn, E.A., High Phase Order Induction Motors, Part II - Experimental Results, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-102, January, 1983, pp. 54-59.
7. White, D.C. and Woodson, H.H., Electromechanical Energy Conversion, John Wiley & Sons, New York, 1959.
8. Fitzgerald, A.E., Kingsley, Charles Jr., and Kusko, Alexander, Electric Machinery, Third Edition, McGraw-Hill, New York, 1971.

9. Alger, P.L., Induction Machines, 2nd edition, Gordon and Breach, New York, 1970.

10. Klingshirn, E.A. and Jordan, H.E., Polyphase Induction Motor Performance and Losses on Non-Sinusoidal Voltage Sources, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-87, March, 1968, pp. 624-631.

11. Krause, P.C. and Thomas, C.H., Simulation of Symmetrical Induction Machine, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-84, November, 1965, pp. 1038-1053.

APPENDIX A

TRANSFORMATION MATRICES

A-1 Symmetrical Component Transformation Matrix for General n-phase Case

The symmetrical component transformation matrix for n phases is:

$$[A] = \frac{1}{\sqrt{n}} \quad \begin{array}{c|c|c|c|c|c|}
 & 0 & & 2 & & n-1 \\ \hline
a & 1 & 1 & 1 & \cdot \quad \cdot & 1 \\ \hline
b & 1 & \alpha^{-1} & \alpha^{-2} & & \alpha^{-(n-1)} \\ \hline
c & 1 & \alpha^{-2} & \alpha^{-4} & \cdot \quad \cdot & \alpha^{-2(n-1)} \\ \hline
\vdots & \vdots & & & \ddots & \vdots \\ \hline
n & 1 & \alpha^{-(n-1)} & \alpha^{-2(n-1)} & \cdot \quad \cdot & \alpha^{-1} \\ \hline
\end{array}$$

where $\alpha = \varepsilon^{j2\pi/n}$

The inverse of matrix [A] is equal to its conjugate transpose giving

$$[A]^{-1} = \frac{1}{\sqrt{n}} \quad \begin{array}{c|c|c|c|c|c|}
 & a & b & c & & n \\ \hline
0 & 1 & 1 & 1 & \cdot \quad \cdot & 1 \\ \hline
1 & 1 & \alpha & \alpha^{2} & & \alpha^{(n-1)} \\ \hline
2 & 1 & \alpha^{2} & \alpha^{4} & & \alpha^{2(n-1)} \\ \hline
\vdots & \vdots & & & \ddots & \vdots \\ \hline
n-1 & 1 & \alpha^{(n-1)} & \alpha^{2(n-1)} & \cdot \quad \cdot & \alpha \\ \hline
\end{array}$$

Note that sequence 1 is the positive (P) sequence, and sequence (n-1) is the negative (N) sequence.

A-2 Symmetrical Component Transformation, 12-Phase

To illustrate the symmetrical component transformation, the matrix [A] is given for 12-phases, with the phases labeled as four sets of three phases each.

$[A12\phi] = \frac{1}{\sqrt{12}}$

	0	1	2	3	4	5	6	7	8	9	10	11
a1	1	1	1	1	1	1	1	1	1	1	1	1
a2	1	α^{-1}	α^{-2}	α^{-3}	α^{-4}	α^{-5}	-1	α^{5}	α^{4}	α^{3}	α^{2}	α
a3	1	α^{-2}	α^{-4}	-1	α^{4}	α^{2}	1	α^{-2}	α^{-4}	-1	α^{4}	α^{2}
a4	1	α^{-3}	-1	α^{3}	1	α^{-3}	-1	α^{3}	1	α^{-3}	-1	α^{3}
b1	1	α^{-4}	α^{4}	1	α^{-4}	α^{4}	1	α^{-4}	α^{4}	1	α^{-4}	α^{4}
b2	1	α^{-5}	α^{2}	α^{-3}	α^{4}	α^{-1}	-1	α	α^{-4}	α^{3}	α^{-2}	α^{5}
b3	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
b4	1	α^{5}	α^{-2}	α^{3}	α^{-4}	α	-1	α^{-1}	α^{4}	α^{-3}	α^{2}	α^{-5}
c1	1	α^{4}	α^{-4}	1	α^{4}	α^{-4}	1	α^{4}	α^{-4}	1	α^{4}	α^{-4}
c2	1	α^{3}	-1	α^{-3}	1	α^{3}	-1	α^{-3}	1	α^{3}	-1	α^{-3}
c3	1	α^{2}	α^{4}	-1	α^{-4}	α^{-2}	1	α^{2}	α^{4}	-1	α^{-4}	α^{-2}
c4	1	α	α^{2}	α^{3}	α^{4}	α^{5}	-1	α^{-5}	α^{-4}	α^{-3}	α^{-2}	α^{-1}

where $\alpha = \varepsilon^{j\pi/6} = 1 \underline{/\ 30^\circ}$

A-3 The Symmetrical Component Transformation, S12ϕ

The S12ϕ stator is derived from the 12-phase machine by deleting phase sets 3 and 4 from the 12-phase version. The corresponding rows of its symmetrical component matrix are therefore deleted. Also, all even numbered columns must be deleted from the 12-phase matrix. Finally, the resulting matrix must be multiplied by $\sqrt{2}$ to preserve power invariance in the transformation. The resulting symmetrical component matrix for the S12ϕ machine is,

$$[A_{S12\phi}] = \frac{1}{\sqrt{6}} \begin{array}{c|c|c|c|c|c|c|}
 & 1 & 3 & 5 & 7 & 9 & 11 \\ \cline{2-7}
a1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \cline{2-7}
a2 & \alpha^{-1} & \alpha^{-3} & \alpha^{-5} & \alpha^{5} & \alpha^{3} & \alpha \\ \cline{2-7}
b1 & \alpha^{-4} & 1 & \alpha^{4} & \alpha^{-4} & \alpha^{-4} & \alpha^{4} \\ \cline{2-7}
b2 & \alpha^{-5} & \alpha^{-3} & \alpha^{-1} & \alpha & \alpha^{3} & \alpha^{5} \\ \cline{2-7}
c1 & \alpha^{4} & 1 & \alpha^{-4} & \alpha^{4} & 1 & \alpha^{-4} \\ \cline{2-7}
c2 & \alpha^{3} & \alpha^{-3} & \alpha^{3} & \alpha^{-3} & \alpha^{3} & \alpha^{-3} \\ \cline{2-7}
\end{array}$$

where $\alpha = \varepsilon^{j\pi/6} = 1\underline{/30^\circ}$

The inverse of this matrix is its conjugate transpose given by

$[A_{S12\phi}]^{-1} = \frac{1}{\sqrt{6}}$

	a1	a2	b1	b2	c1	c2
1	1	α	α^4	α^5	α^{-4}	α^{-3}
3	1	α^3	1	α^3	1	α^3
5	1	α^5	α^{-4}	α	α^4	α^{-3}
7	1	α^{-5}	α^4	α^{-1}	α^{-4}	α^3
9	1	α^{-3}	1	α^{-3}	1	α^{-3}
11	1	α^{-1}	α^{-4}	α^{-5}	α^4	α^3

A-4 The Two-Phase ($\alpha\beta\gamma$) Transformation Matrix

The general matrix for transforming from phase variables to general two-phase or $\alpha\beta\gamma$ variables can be found in reference 7 on pages 565–568. When applied to the S12ϕ motor, the matrix is

$[T_{S12\phi}] = \frac{1}{\sqrt{3}}$

	α_1	α_3	α_5	β_5	β_3	β_1
a_1	1	1	1	0	0	0
a_2	$\sqrt{3}/2$	0	$-\sqrt{3}/2$	1/2	1	1/2
b_1	-1/2	1	-1/2	$-\sqrt{3}/2$	0	$\sqrt{3}/2$
b_2	$-\sqrt{3}/2$	0	$\sqrt{3}/2$	1/2	1	1/2
c_1	-1/2	1	-1/2	$\sqrt{3}/2$	0	$-\sqrt{3}/2$
c_2	0	0	0	-1	1	-1

Its inverse is equal to its transpose.

$$[T_{S12\phi}]^{-1} = \frac{1}{\sqrt{3}} \begin{array}{c|cccccc} & a_1 & a_2 & b_1 & b_2 & c_1 & c_2 \\ \hline \alpha_1 & 1 & \sqrt{3}/2 & -1/2 & -\sqrt{3}/2 & -1/2 & 0 \\ \alpha_3 & 1 & 0 & 1 & 0 & 1 & 0 \\ \alpha_5 & 1 & -\sqrt{3}/2 & -1/2 & \sqrt{3}/2. & -1/2 & 0 \\ \beta_5 & 0 & 1/2 & -\sqrt{3}/2 & 1/2 & \sqrt{3}/2 & -1 \\ \beta_3 & 0 & 1 & 0 & 1 & 0 & 1 \\ \beta_1 & 0 & 1/2 & \sqrt{3}/2 & 1/2 & -\sqrt{3}/2 & -1 \end{array}$$

A-5 The dq Transformation for Machines with Multiple Three-Phase Winding Groups

Reference 2 presents a transformation to represent the multiple sets of three-phase windings by multiple sets of direct-quadrature (d-q-γ) windings. This is an extension of the d-q-γ method commonly used for three phase machines.

The S12ϕ machine will be used as an example. Its stator currents are represented as two three-phase sets in Figure A-2. Set number 1 (a_1-b_1-c_1) is resolved into components d_1-q_1-γ_1, where d_1 and q_1 are shown along the d and q-axis directions in Figure A-2. Likewise, set number 2 (a_2-b_2-c_2) is resolved into d_2-q_2-γ_2. The γ-components are defined in exactly the same way as in the standard dqγ representation of 3-phase machines.

This graphical construction leads to the transformation matrix

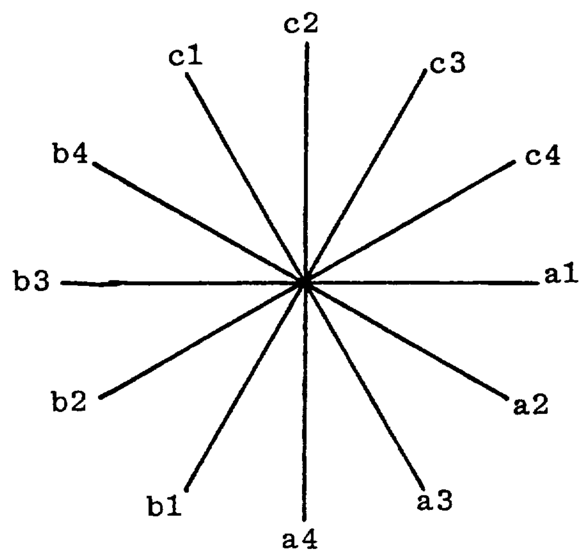


Figure A-1. Phasor Arrangement for 12-Phase System

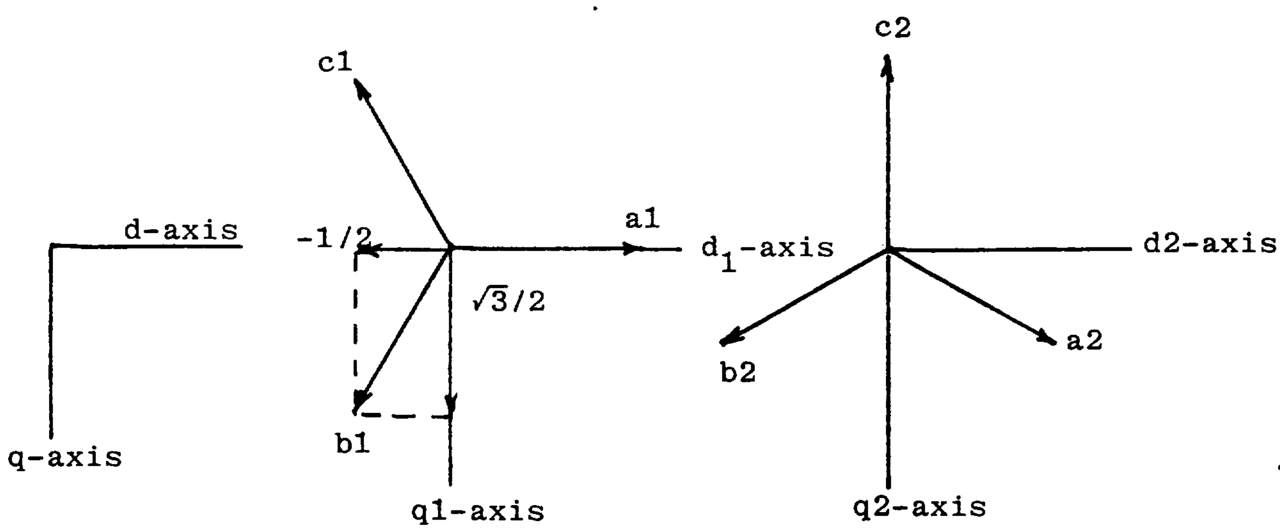


Figure A-2. Geometric Arrangement for Multiple d-q-γ Sets

$$[D] = \sqrt{2/3}\;\begin{array}{c|cccccc|}
 & d_1 & q_1 & \gamma_1 & d_2 & q_2 & \gamma_2 \\
\hline
a_1 & 1 & 0 & 1/\sqrt{2} & 0 & 0 & 0 \\
b_1 & -1/2 & \sqrt{3}/2 & 1/\sqrt{2} & 0 & 0 & 0 \\
c_1 & -1/2 & -\sqrt{3}/2 & 1/\sqrt{2} & 0 & 0 & 0 \\
a_2 & 0 & 0 & 0 & \sqrt{3}/2 & 1/2 & 1/\sqrt{2} \\
b_2 & 0 & 0 & 0 & -\sqrt{3}/2 & 1/2 & 1/\sqrt{2} \\
c_2 & 0 & 0 & 0 & 0 & -1 & 1/\sqrt{2} \\
\hline
\end{array}$$

The $\sqrt{2/3}$ factor makes this a power invariant transformation. The inverse of [D] is its transpose.

$$[D]^{-1} = \sqrt{2/3}\;\begin{array}{c|cccccc|}
 & a_1 & b_1 & c_1 & a_2 & b_2 & c_2 \\
\hline
d_1 & 1 & -1/2 & -1/2 & 0 & 0 & 0 \\
q_1 & 0 & \sqrt{3}/2 & -\sqrt{3}/2 & 0 & 0 & 0 \\
\gamma_1 & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 & 0 \\
d_2 & 0 & 0 & 0 & \sqrt{3}/2 & -\sqrt{3}/2 & 0 \\
q_2 & 0 & 0 & 0 & 1/2 & 1/2 & -1 \\
\gamma_2 & 0 & 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\
\hline
\end{array}$$

APPENDIX B

SYMMETRICAL COMPONENT RESISTANCES AND INDUCTANCES

B-1 Stator Windings and Parameters

The basic machine considered here has a uniform air gap (is non-salient pole type) and the stator has a balanced n-phase winding. Figure B-1 shows the schematic diagram for the spatial arrangement of windings. The stator phases are identified as a, b, c, ..., n. These are spaced with an angle $\delta = 2\pi/n$.

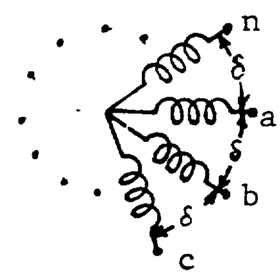


Figure B-1. Diagram of an n-phase stator.

Each phase has a resistance r_s ohms. This leads to the stator resistance matrix, which is a diagonal matrix:

$$[R_\phi] = \begin{array}{c|ccccc|} & a & b & c & & n \\ \hline a & r_s & & & & \\ b & & r_s & & & \\ c & & & r_s & & \\ \vdots & & & & \ddots & \\ n & & & & & r_s \\ \hline \end{array}$$

Also, each phase has a self inductance; these are all equal for a balanced n-phase winding such that

$$L_{aa} = L_{bb} = L_{cc} = \ldots\ldots = L_{nn}$$

There is a mutual inductance between each pair of phases, such as L_{ab}, L_{ac}, L_{bc}, etc. Because of symmetry, all pairs of phases separated by the same angle have the same mutual inductance. Thus $L_{ab} = L_{bc}$, $L_{ac} = L_{bd}$, etc.

All of the stator inductances may be displayed in matrix form as

$$[L_\phi] = \begin{array}{c} \\ a \\ b \\ c \\ \cdot \\ \cdot \\ \cdot \\ (n-1) \\ n \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & & (n-1) & n \end{array} \\ \left[\begin{array}{cccccc} L_{aa} & L_{ab} & L_{ac} & \cdots\cdots & L_{ac} & L_{ab} \\ L_{ab} & L_{aa} & L_{ab} & & L_{ad} & L_{ac} \\ L_{ac} & L_{ab} & L_{aa} & & & L_{ad} \\ \cdot & & & \ddots & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ L_{ac} & L_{ad} & & & L_{aa} & L_{ab} \\ L_{ab} & L_{ac} & L_{ad} & \cdots\cdots & L_{ab} & L_{aa} \end{array}\right] \end{array}$$

Note that there are several characteristics of this highly symmetric matrix:

- It is symmetric about both diagonals
- Each successive row is the same as the row above, but shifted one column to the right. This is called "circulant symmetry".
- Any one row includes all of the inductance values involved. Thus, the entire matix is specified if any one row is given.

B-2 Transformation to Symmetrical Component Inductances

The equations for transforming the stator inductance values to the symmetrical component sequence inductances are given in some detail in reference 7, page 582. The equations take advantage of the symmetry observed for the stator inductance matrix. They lead to values for the sequence inductances L_{00}, L_{11}, L_{22}, ..., $L_{(n-1)(n-1)}$. It will be convenient to simplify the notation by using a single subscript instead of the double. Also, the equations given there for sequence inductances may be reduced to either of the following:

The value for the inductance of the ith sequence is:

If number of phases n is an odd number,

$$L_i = L_{aa} + 2\{L_{ab} \cos i\delta + L_{ac} \cos 2i\delta + \dots + L_{a(n/2+1/2)} \cos(n/2-1/2)\delta\} \tag{B-1}$$

and if n is even,

$$L_i = L_{aa} + 2\{L_{ab} \cos i\delta + L_{ac} \cos 2i\delta + \dots + L_{(a)(n/2)} \cos(n/2-1/2)\delta\} + L_{a(n/2+1)} \tag{B-2}$$

In these equations, $\delta = 2\pi/n$, and $i = 0, 1, 2, \dots, (n-1)$.

There must be n sequence inductances for an n-phase stator. However, equation B-1 or B-2 will verify that $L_{(n-i)} = L_i$ for any of the n possible values for i and therefore, there can be only n/2 different numerical values for these inductances if n is even, and (n+1)/2 different values if n is odd. The remaining values are repeated such that $L_{(n-1)} = L_1$, $L_{(n-2)} = L_2$, etc.

The sequence inductance matrix displays these inductance values in a diagonal matrix which is, for the general n-phase case.

$$
[L_s] = \begin{array}{c|c|c|c|c|c|c|c|c|}
 & 0 & 1 & 2 & 3 & \bullet\ \bullet & (n-3) & (n-2) & (n-1) \\
\hline
0 & L_0 & & & & & & & \\
\hline
1 & & L_1 & & & & & & \\
\hline
2 & & & L_2 & & & & & \\
\hline
3 & & & & L_3 & & & & \\
\hline
\bullet & & & & & \bullet & & & \\
\bullet & & & & & \quad\bullet & & & \\
\hline
n-3 & & & & & & L_3 & & \\
\hline
n-2 & & & & & & & L_2 & \\
\hline
n-1 & & & & & & & & L_1 \\
\hline
\end{array}
$$

B-3 <u>Magnetizing and Leakage Inductances</u>

Each of the self and mutual inductances may be divided into two components; that due to the air gap flux, and that due to leakage flux. This makes it possible to write the stator inductance matrix as

$$[L_\phi] = [L_\phi'] + [L_\phi''] \tag{B-3}$$

where $[L_\phi']$ is the air gap inductance matrix consisting of the inductances due to air gap flux only, and $[L_\phi'']$ is the leakage inductance matrix due to leakage fluxes only.

For the symmetric, uniform air gap machine, and assuming a sinusoidal distribution of air gap flux, the first row of the air gap inductance matrix is

$$
[L'_{\phi(a)}] = \begin{array}{|c|c|c|c|c|c|}
\multicolumn{1}{c}{a} & \multicolumn{1}{c}{b} & \multicolumn{1}{c}{c} & \multicolumn{1}{c}{} & \multicolumn{1}{c}{(n-1)} & \multicolumn{1}{c}{n} \\
\hline
L'_{aa} & L'_{aa}\cos\delta & L'_{aa}\cos 2\delta & \cdots\cdots & L'_{aa}\cos 2\delta & L'_{aa}\cos\delta \\
\hline
\end{array}
$$

or slightly simplified

$$[L'_{\phi(a)}] = L'_{aa} \begin{array}{cccccc} a & b & c & & (n-1) & n \\ \end{array}$$

$$[L'_{\phi(a)}] = L'_{aa}\begin{bmatrix} 1 & \cos\delta & \cos 2\delta & \cdots\cdots & \cos 2\delta & \cos\delta \end{bmatrix}$$

When equation B-1 or B-2 is applied to these air gap inductances to find the sequence components, the result gives for the sequence inductance matrix, air gap component;

$$[L'_s] = \begin{array}{c|cccccc} & 0 & 1 & 2 & 3 & & (n-1) \\ \hline 0 & 0 & & & & & \\ 1 & & n/2\,L'_{aa} & & & & \\ 2 & & & 0 & & & \\ 3 & & & & 0 & & \\ \vdots & & & & & \ddots & \\ (n-1) & & & & & & n/2\,L'_{aa} \end{array} \qquad \text{(B-4)}$$

Or simply, $L'_i = \begin{cases} n/2\ L'_{aa} & \text{for } i = 1 \text{ and } (n-1) \\ 0 & \text{for } i \neq 1 \text{ or } (n-1) \end{cases}$

The inductance $L'_1 = n/2\ L'_{aa}$ is called the magnetizing inductance,

$$L'_1 = n/2\ L'_{aa}, \qquad \text{(B-5)}$$

When equation B-1 or B-2 is applied to the leakage inductance matrix $[L''_\phi]$, the resulting sequence leakage inductance matrix is

$$[L''_s] = \begin{array}{c} 0 \\ 1 \\ 2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ (n-2) \\ (n-1) \end{array} \left[\begin{array}{cccccc} L_0'' & & & & & \\ & L_1'' & & & & \\ & & L_2'' & & & \\ & & & \ddots & & \\ & & & & L_2'' & \\ & & & & & L_1'' \end{array} \right] \qquad \text{(B-6)}$$

The sequence inductance matrix is then found by adding the air gap and leakage components from equations B-4 and B-6, or

$$[L_s] = [L'_s] + [L_s'']$$

$$[L_s] = \begin{array}{c|ccccccc} & 0 & 1 & 2 & 3 & \cdot\;\cdot\;\cdot & (n-2) & (n-1) \\ \hline 0 & L_0'' & & & & & & \\ 1 & & L_m+L_1'' & & & & & \\ 2 & & & L_2'' & & & & \\ 3 & & & & L_3'' & & & \\ \cdot & & & & & \ddots & & \\ \cdot & & & & & & & \\ (n-2) & & & & & & L_2'' & \\ (n-1) & & & & & & & L_m+L_1'' \end{array} \qquad \text{(B-7)}$$

The primes and double primes may now be deleted without ambiguity giving the stator sequence inductance matrix as

$$
[Ls] = \begin{array}{c|cccccccc|}
 & 0 & 1 & 2 & 3 & \cdot\;\cdot\;\cdot & (n-3) & (n-2) & (n-1) \\
\hline
0 & L_0 & & & & & & & \\
1 & & L_m+L_1 & & & & & & \\
2 & & & L_2 & & & & & \\
3 & & & & L_3 & & & & \\
\vdots & & & & & \ddots & & & \\
(n-3) & & & & & & L_3 & & \\
(n-2) & & & & & & & L_2 & \\
(n-1) & & & & & & & & L_m+L_1 \\
\hline
\end{array}
$$

where L_0 is the zero sequence inductance

L_m is the magnetizing inductance

L_1 is the positive and negative sequence leakage inductance

L_2 is the sequence 2 and sequence (n-2) leakage inductance

$\vdots$

L_i is the sequence i and sequence (n-i) leakage inductance.

APPENDIX C

FILTER DESCRIPTION AND ANALYSIS

C-1 Filter Description

The filters for HPO motors consist of small transformer type devices connected in series with the stator leads. The general version which can be applied to any motor having more than four phases uses n magnetic cores for an n-phase motor. Each core is wound with three coils. These coils are interconnected so that each of the stator leads has three of these coils from three separate cores in series with it. The selection of which specific line currents pass through the three coils on each core and the turns ratios for these coils are based on the requirement that the filter should not increase the positive sequence inductance. The analysis of these filters is given in the next section.

C-2 Analysis of the General Filter Type

Figure C-1 shows one of the n identical transformers used for the general type of filter. As the figure shows, each core has two identical coils with N turns on each. Let these be called the auxiliary coils. Each core also has one coil with k times N, or kN turns, and this will be called the main coil. The core has a permeance value P which is dependent on core dimensions and permeability of the core steel. This is expressed as

$$P = \frac{\mu A}{\ell} \tag{C-1}$$

where μ is permeability

A is cross sectional area of the core

ℓ is length of the core

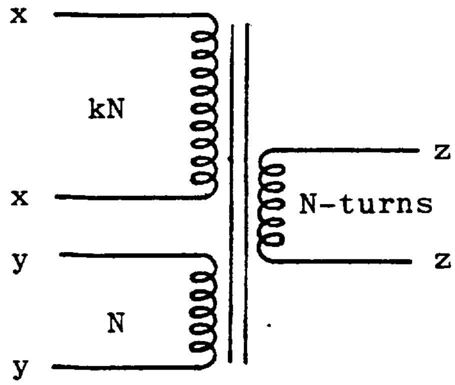
Based on magnetic circuit theory, the self and mutual inductances for the various coils are dependent on the number of turns on the coils and the core permeance. The inductances of the transformer in Figure C-1 are given with the figure.

A filter for an n-phase motor is made up by combining n of these transformers. Since the objective is to first of all have a zero value for the positive sequence inductance, three of the phase currents and a particular value of the turns ratio k must be selected so that the magnetomotive force (MMF) will be zero for each core when only positive sequence currents are used. Further explanation is based on the following five phase example.

C-3 Five-Phase Filter Example

The phasor diagram for the positive sequence currents of a five-phase system is shown in Figure C-2. If the current of phase a passes through the main coil on the transformer of Figure C-1, its MMF can be cancelled by having either currents I_c and I_d or I_b and I_e through the two auxiliary coils. The former case will be called the phase-opposite scheme because currents I_c and I_d are nearly opposite I_a in the phasor diagram of Figure C-2. The latter case will be called the phase-adjacent scheme since I_b and I_e are adjacent to I_a.

Figure C-3 shows the connection diagram for the entire filter for a five-phase application using the phase-opposite scheme. As this diagram



Coil x-x has kN turns and is the main coil.

Coils y-y and z-z have N turns and are the Auxiliary coils.

The winding inductances are:

<u>Self inductances</u>

Main coil, $L_{xx}=k^2N^2P$

Aux. coils, $L_{yy}=L_{zz}=N^2P$

<u>Mutual inductances</u>

Main to Aux., $L_{xy}=L_{xz}=kN^2P$

Aux. to Aux., $L_{yz}=N^2P$

where P=permeance of the core.

Figure C-1. Diagram and Inductances of Filter Transformers

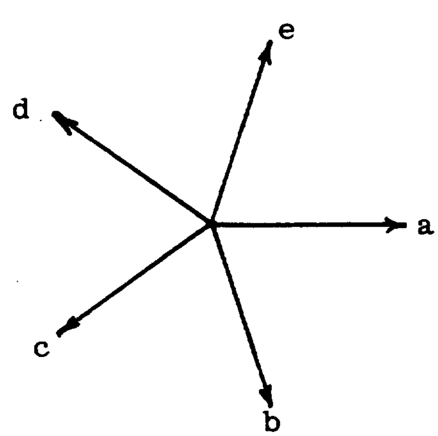
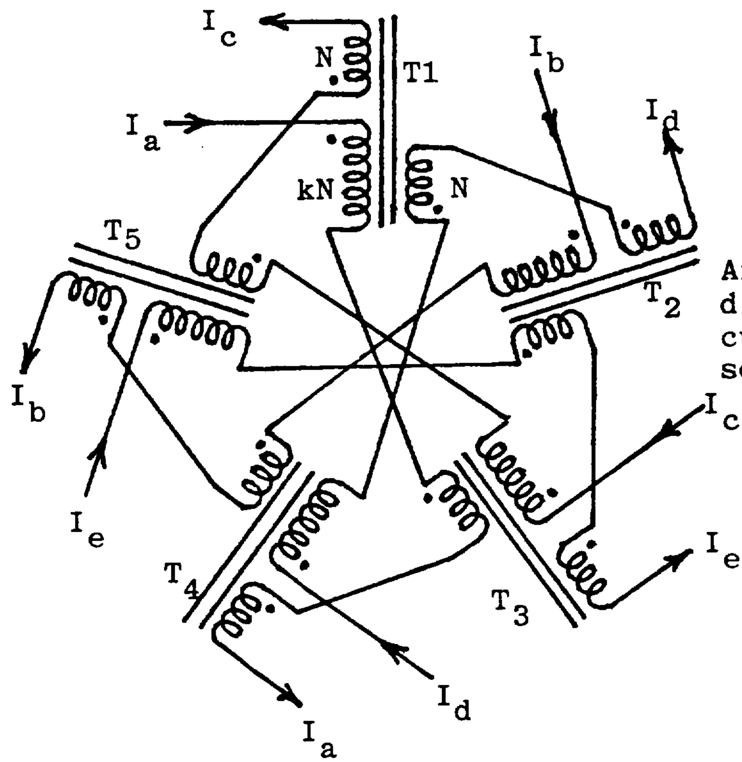


Figure C-2. Phasor Diagram for Positive Sequence Currents of a Five-Phase Motor



Arrows show direction of currents from source to motor.

CONNECTION TABLE
For 5-Phase Filter
(Phase opposite type)

Transf.	Phase Current		
	Main	Auxiliary	
	kN	N	N
T1	a	c	d
T2	b	d	e
T3	c	e	a
T4	d	a	b
T5	e	b	c

Row "a" of normalized phase inductance matrix is

$$\frac{[L_{\phi F(a)}]}{N^2 P} = \begin{array}{ccccc} a & b & c & d & e \\ \hline k^2+2 & 1 & 2k & 2k & 1 \end{array}$$

The normalized sequence inductance matrix with k=1.618 is

$$\frac{[L_{SF}]}{N^2 P} = \text{diag} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \hline 13.1 & 0 & 4 & 4 & 0 \end{array}$$

Figure C-3. Five-Phase Filter, Phase Opposite Type shows, for transformer T_1, the total MMF around the core is

$$F_1 = kN\bar{I}_a + N\bar{I}_c + N\bar{I}_d \qquad (C-2)$$

This MMF must be zero if the positive sequence inductance for the filter is to be zero. For this five-phase case, it is zero if $F_1 = 0$, and this is satisfied if $k = 1.618$.

Figure C-4 shows the five-phase filter connected to use the phase-adjacent scheme. For this case, MMF is

$$F_1 = kN\bar{I}_a - N\bar{I}_b - N\bar{I}_c \qquad (C-3)$$

Setting this equal to zero requires that $k = .618$.

In either of the two schemes, the connections for the other four transformers are similar to transformer T_1, as Figures C-3 and C-4 show. All of the information shown in these connection diagrams can be displayed in a Connection Table such as those given with Figures C-3 and C-4. For the filter of Figure C-3, the Connection Table indicates by the entries in the first line that transformer T_1 has $\bar{I}_a$ through its main (kN-turn) coil, and $\bar{I}_c$ and $\bar{I}_d$ through the two auxiliary (N-turn) coils, all with additive polarity. The second line gives similar information for T_2, etc.

With these connections, the matrix of inductances for the complete filter can be written using the various self and mutual inductance values given with Figure C-1. The result is the normalized filter inductance matrix given by

$$\frac{1}{N^2P}[L_{\phi F}] = \begin{array}{c|c|c|c|c|c|} & a & b & c & d & e \\ \hline a & k^2+2 & 1 & 2k & 2k & 1 \\ \hline b & 1 & k^2+2 & 1 & 2k & 2k \\ \hline c & 2k & 1 & k^2+2 & 1 & 2k \\ \hline d & 2k & 2k & 1 & k^2+2 & 1 \\ \hline e & 1 & 2k & 2k & 1 & k^2+2 \\ \hline \end{array} \qquad \text{(C-4)}$$

Since this matrix has circulant symmetry, row a alone contains all of the information. The matrix may therefore be simplified by writing its first row (row a) only as

$$\frac{1}{N^2P}[L_{\phi F(a)}] = \begin{array}{|c|c|c|c|c|} a & b & c & d & e \\ \hline k^2+2 & 1 & 2k & 2k & 1 \\ \hline \end{array} \qquad \text{(C-5)}$$

To find the sequence inductance values for the filter, equation B-1 from Appendix B may now be used. It is convenient to use it as an alternate to the method given in Section C-3 for finding k. The method is based on the fact that the positive sequence inductance L_{1F}, found by using i = 1 in equation B-1, must be zero. For the five-phase case, equation B-1 uses δ = 360/5 = 72°, and values for L_{aa}, L_{ab}, and L_{ac} come from equation C-4. Thus, $L_{1f} = N^2P\{(k^2+2) + 2(\cos 72° + 2k\cos 144°)\} = 0$, and this gives k = 1.618 as expected. Then, the use of equation B-1 with i = 0 and i = 2 gives the remaining sequence inductances L_{0F} and L_{2F}. (Recall that L_{4F} = L_{1F} and $L_{3F} = L_{2F}$ since, in general, $L_{(n-1)F} = L_{1F}$). The resulting normalized sequence inductance matrix for the five-phase filter using the phase-opposite scheme is

$$\frac{1}{N^2P}[L_{SF}] = \begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 13.1 & & & & \\ 1 & & 0 & & & \\ 2 & & & 4 & & \\ 3 & & & & 4 & \\ 4 & & & & & 0 \end{array}$$

This diagonal matrix may be written more compactly as

$$\frac{1}{N^2P}[L_{SF}] = \text{diag} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \hline 13.1 & 0 & 4 & 4 & 0 \end{array} \qquad \text{(C-5)}$$

To summarize, for the five-phase example, the filter using the phase opposite scheme gives sequence inductance values:

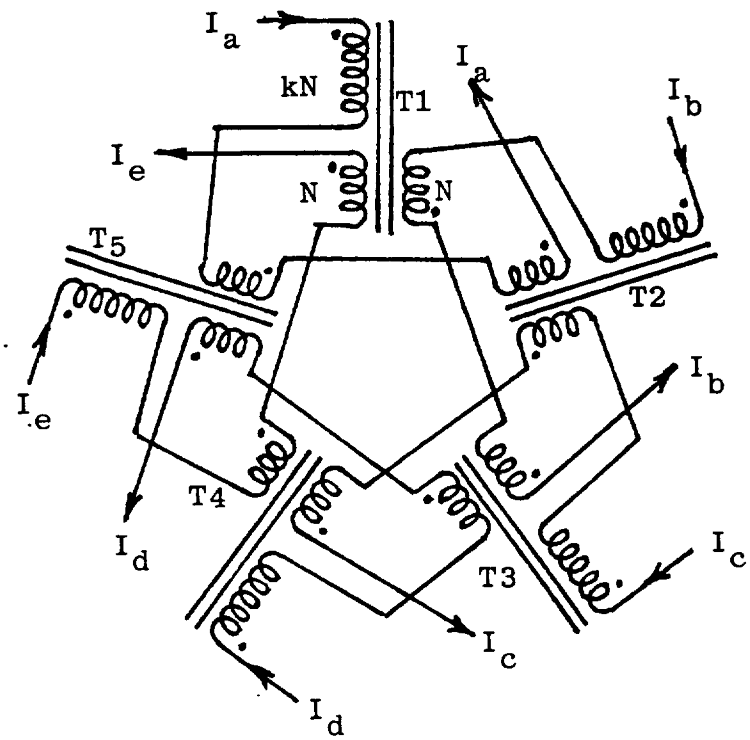
$L_0 = 13.1\ N^2P$ H.

$L_1 = L_4 = 0$

$L_2 = L_3 = 4N^2P$ H.

C-4 <u>Phase Adjacent Scheme</u>

Figure C-4 shows connections for the five-phase filter example using the phase adjacent scheme described in Section C-4. The Connection Table is given with Figure C-4. The negative signs in the last two columns of that table indicate that, if current through the main coil is into the dot-



Arrows show directions of currents from source toward motor.

CONNECTION TABLE
for 5-Phase Filter
(Phase adjacent type)

Transf.	Phase Current		
	Main	Auxiliary	
	kN	N	N
T_1	a	-b	-e
T_2	b	-c	-a
T_3	c	-d	-b
T_4	d	-3	-c
T_5	e	-a	-d

Row "a" of the normalized phase inductance matrix is

$$\frac{[L_{\phi F(a)}]}{N^2 P} = \begin{array}{ccccc} a & b & c & d & e \\ \hline k^2+2 & -2k & 1 & 1 & -2k \end{array}$$

The normalized sequence inductance matrix with k=0.618 is

$$\frac{[L_{SF}]}{N^2 P} = \text{diag} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \hline 1.91 & 0 & 5 & 5 & 0 \end{array}$$

Figure C-4. Five-Phase Filter, Phase Adjacent Type ted terminal, then the currents through both auxiliary coils must be out of the dotted terminals. An equivalent statement is that the mutual inductance between the main coil and either auxiliary coil is negative, and this explains the negative terms in the inductance matrix $[L_{\phi F(a)}]$ given with Figure C-4.

The sequence inductance matrix $[L_{SF}]$ is determined using $[L_{\phi F(a)}]$ and equation B-1 from Appendix B in the same way as explained for the example in Section C-3. The result is the sequence inductance matrix given with Figure C-4. In this case, with k = 0.618, sequence inductances are:

$L_{0F} = 1.91\ N^2P$

$L_{1F} = L_{4F} = 0$

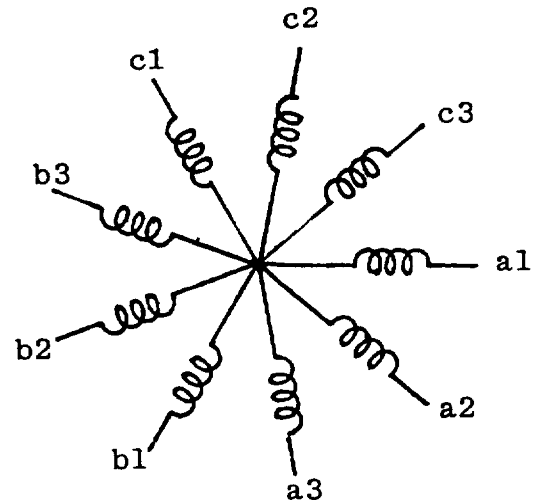
$L_{2F} = L_{3F} = 5\ N^2P$

This phase-adjacent scheme gives a somewhat larger value of inductance for sequences 2 and 3 than does the filter using the phase-opposite scheme of Section C-3.

C-5 Filters for S12ϕ and S18ϕ Motors

The extension of the filter design method to more phases is obvious. Results are summarized for the S18ϕ case in Figures C-5 and C-6.

For S12ϕ motors, a variation of the method is used. First, the filter is set up for a 12-phase case where 12 transformers would be used. Its sequence inductances can be found in the usual manner. But then, just as the S12ϕ motor results when certain phases of the 12ϕ design are deleted, the six transformers associated with the deleted phases are removed from the filter. The previously determined sequence inductances for the odd-



Schematic Diagram of
a 9-Phase or S18ϕ Stator

CONNECTION TABLE
For 9ϕ or S18ϕ Filter
(Phase opposite type)

Transf.	Phase Current		
	Main	Auxiliary	
	kN	N	N
T1	a1	b2	b3
T2	a2	b3	c1
T3	a3	c1	c2
T4	b1	c2	c3
T5	b2	c3	a1
T6	b3	a1	a2
T7	c1	a2	a3
T8	c2	a3	b1
T9	c3	b1	b2

Row "a1" of the normalized phase inductance matrix is

$$\frac{[L_{\phi F(a1)}]}{N^2 P} = \begin{array}{ccccccccc} a1 & a2 & a3 & b1 & b2 & b3 & c1 & c2 & c3 \\ \hline 2+k^2 & 1 & 0 & 0 & 2k & 2k & 0 & 0 & 1 \end{array}$$

The normalized sequence inductance matrix for k = 1.879 is

$$\frac{[L_{SF}]}{N^2 P} = \text{diag} \begin{array}{ccccccccc} 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 \\ \hline 0 & 0.77 & 4.95 & 11.64 & 15.04 & 11.64 & 4.95 & 0.77 & 0 \end{array}$$

Figure C-5. Nine-Phase or S18ϕ Filter, Phase Opposite Type

CONNECTION TABLE
For 9ϕ or $S18\phi$ Filter
(Phase Adjacent Type)

Transf.	Phase Current		
	Main	Auxiliary	
	kN	N	N
T1	a1	-a2	-c3
T2	a2	-a3	-a1
T3	a3	-b1	-a2
T4	b1	-b2	-a3
T5	b2	-b3	-b1
T6	b3	-c1	-b2
T7	c1	-c2	-b3
T8	c2	-c3	-c1
T9	c3	-a1	-c2

Row "a1" of the normalized phase inductance matrix is

$$\frac{[L_{\phi F(a1)}]}{N^2 P} = \begin{array}{ccccccccc} a1 & a2 & a3 & b1 & b2 & b3 & c1 & c2 & c3 \\ \hline \multicolumn{1}{|c|}{2+k^2} & \multicolumn{1}{c|}{-2k} & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{0} & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{-2k} \\ \hline \end{array}$$

The normalized sequence inductance matrix with k=1.532 is

$$\frac{[L_{SF}]}{N^2 P} = \text{diag} \begin{array}{ccccccccc} 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 & 17 \\ \hline \multicolumn{1}{|c|}{0} & \multicolumn{1}{c|}{6.41} & \multicolumn{1}{c|}{11.64} & \multicolumn{1}{c|}{1.40} & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{1.40} & \multicolumn{1}{c|}{11.64} & \multicolumn{1}{c|}{6.41} & \multicolumn{1}{c|}{0} \\ \hline \end{array}$$

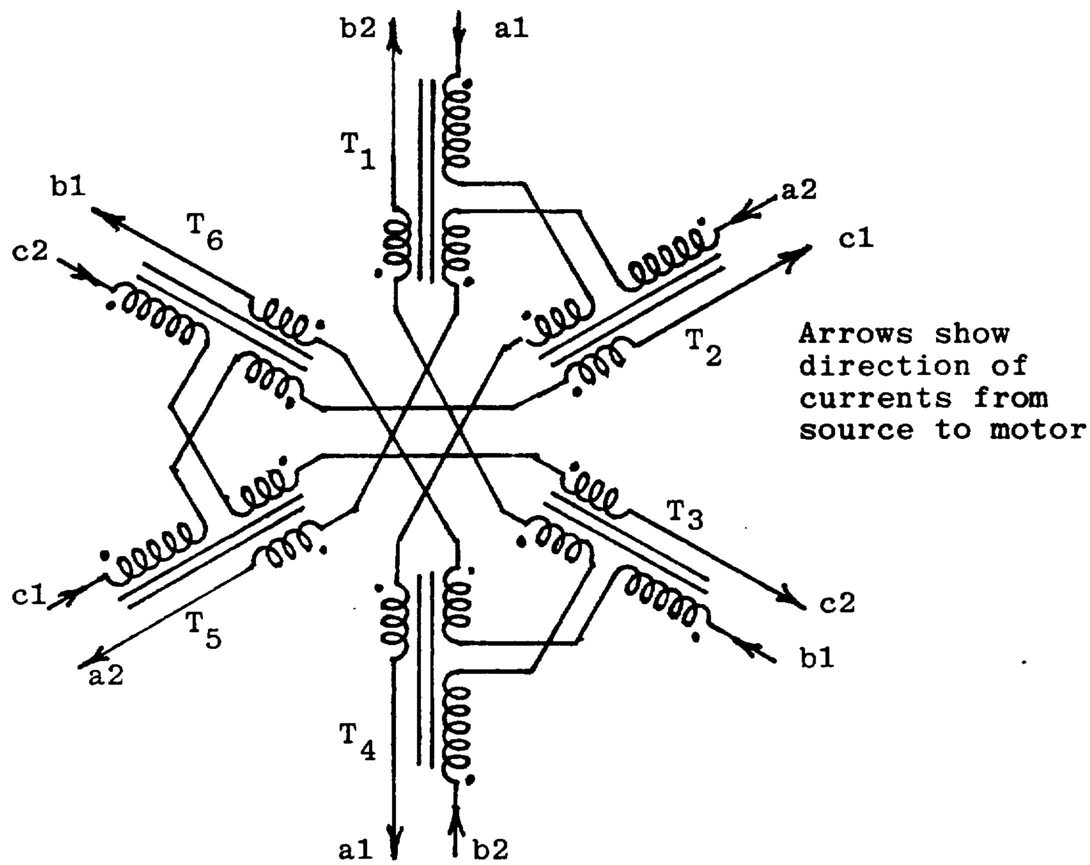
Figure C-6. Nine-Phase or $S18\phi$ Filter, Phase Adjacent Type numbered sequences are unchanged in this process and become the values for the S12ϕ motor. The results are summarized in Figure C-7.

In about 1965, Messrs. Tad Macie and Darius Irani developed a filter for a S12ϕ motor which is based on the same principle as presented in this appendix. Their version requires only three transformers. The connection diagram and inductances are shown in Figure C-8.

C-6 The Wye-Delta Filter for S12ϕ Motors

Another type of filter which can be used for S12ϕ motors only was investigated. It is based on the phase shift inherent in a wye-delta transformer connection. It uses six transformers, each having two windings. The connection diagram is shown in Figure C-9. The primary winding of each transformer has N turns, and one of these windings is in series with each stator lead. The secondaries of three of the transformers each have aN turns, where a is any convenient number. These three secondaries are connected in wye. Then, the secondaries on the other three transformers each have $\sqrt{3}$ aN turns, and they are connected in delta. Finally, the wye and delta groups are connected together to complete the connection. With connections as shown in Figure C-9, the positive sequence inductance is practically zero, and the inductance for sequence five is $L_{5F}=N^2P$.

For the same total amount of copper for windings and steel for the core, L_{5F} for this filter is only about 1/3 that of the filter using three coils per transformer as described in Figure C-7. The three-coil version is obviously the better one to use.



CONNECTION TABLE
For S12ϕ Filter
(6-Transformer Type)

Transf.	Phase Current		
	Main	Auxiliary	
	kN	N	N
T1	a1	b2	-a2
T2	a2	c1	-a1
T3	b1	c2	-b2
T4	b2	a1	-b1
T5	c1	a2	-c2
T6	c2	b1	-c1

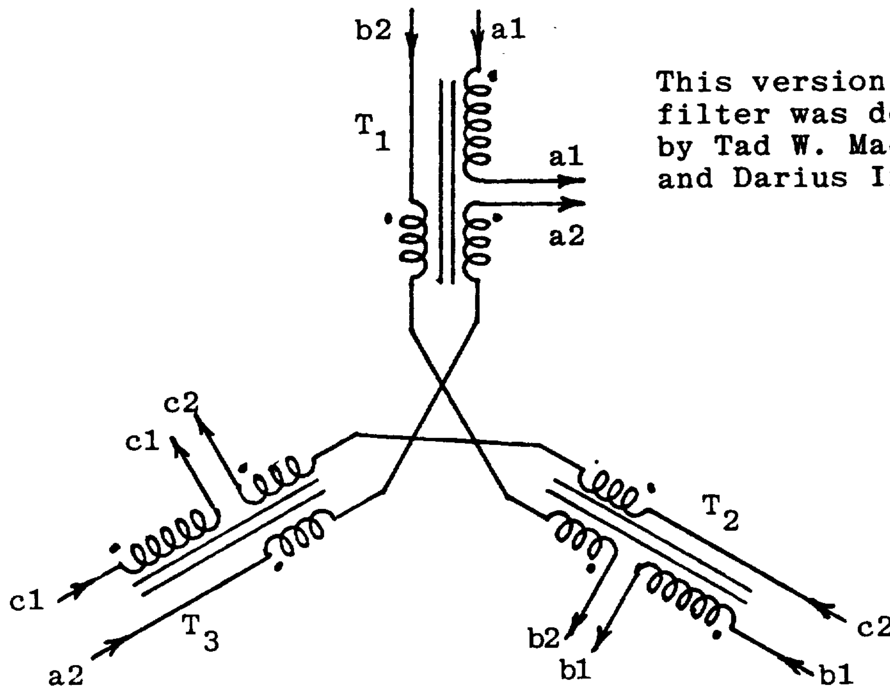
Row "a1" of the normalized phase inductance matrix is

$$\frac{[L_{\phi F(a1)}]}{N^2 P} = \begin{array}{c} \begin{array}{cccccc} a1 & a2 & b1 & b2 & c1 & c2 \end{array} \\ \begin{bmatrix} k^2+2 & -2k & -1 & 2k & -1 & 0 \end{bmatrix} \end{array}$$

The normalized sequence inductance matrix with k-1.732 is

$$\frac{[L_{SF}]}{N^2 P} = \text{diag} \begin{array}{c} \begin{array}{cccccc} 1 & 3 & 5 & 7 & 9 & 11 \end{array} \\ \begin{bmatrix} 0 & 3 & 12 & 12 & 3 & 0 \end{bmatrix} \end{array}$$

Figure C-7. S12ϕ Filter, 6-Transformer Type



This version of a filter was devised by Tad W. Macie and Darius Irani.

CONNECTION TABLE
For S12ϕ Filter
(3-Transformer Type)

Transf	Phase Current		
	Main	Auxiliary	
	kN	N	N
T1	a1	b2	-a2
T2	b1	c2	-b2
T3	c1	a2	-c2

Row "a1" of the normalized phase inductance matrix is

$$\frac{[L_{\phi F(a1)}]}{N^2P} = \begin{array}{cccccc} a1 & a2 & b1 & b2 & c1 & c2 \\ \hline k^2 & -k & 0 & 1 & 0 & 0 \end{array}$$

The normalized sequence inductance matrix for k=1.732 is

$$\frac{[L_{SF}]}{N^2P} = \text{diag} \begin{array}{cccccc} 1 & 3 & 5 & 7 & 9 & 11 \\ \hline 0 & 3 & 6 & 6 & 3 & 0 \end{array}$$

Figure C-8. S12ϕ Filter, 3-Transformer Type

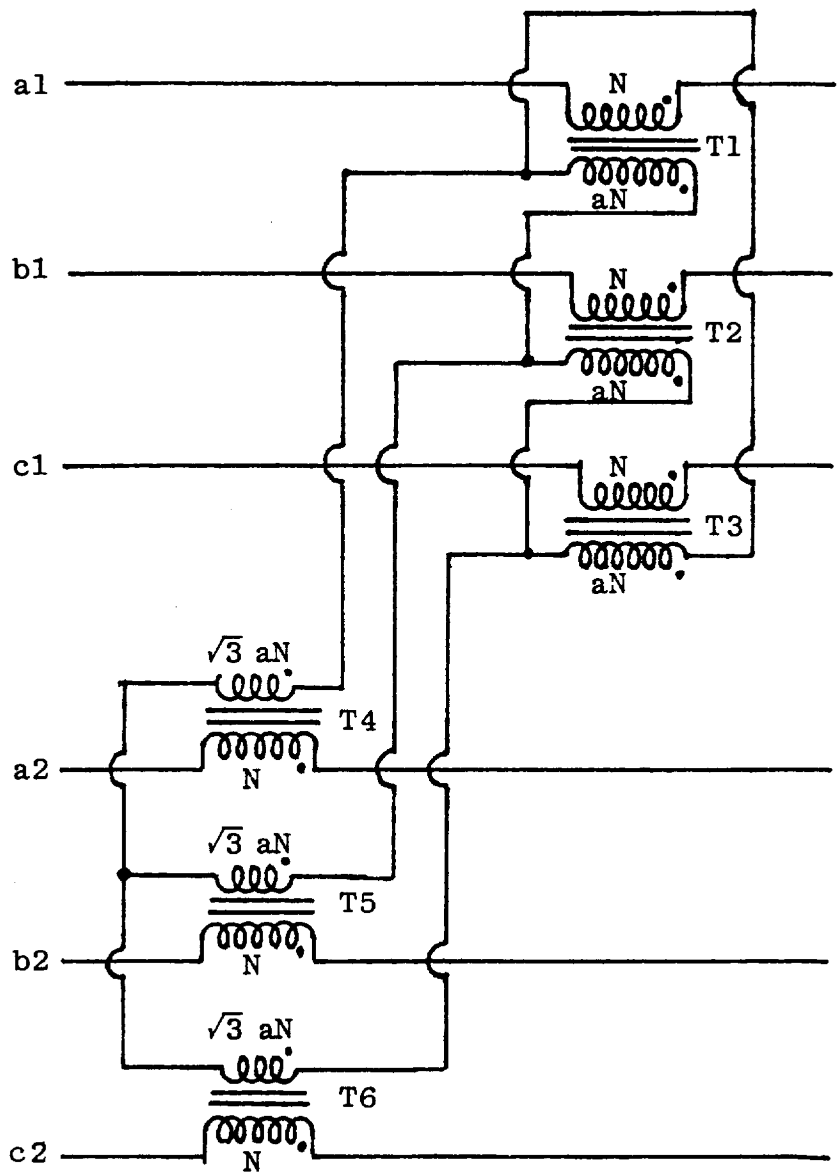


Figure C-9. Wye-Delta Filter for S12ϕ Motors