NISTIR 7115

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May 2004

U.S. DEPARTMENT OF COMMERCE
Donald L. Evans, Secretary

TECHNOLOGY ADMINISTRATION
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DELBRÜCK SCATTERING: A BIBLIOGRAPHY, AND SOME NUMERICAL COMPARISONS WITH OTHER PHOTON INTERACTION PROCESSES[#†]

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Abstract

The most-probable interaction processes by which photons in the x-ray, gamma-ray and higher-energy regions are scattered by atoms are (1) incoherent (Compton) scattering by individual atomic electrons and (2) coherent (Rayleigh) scattering from the atomic electron cloud as a whole. In current NIST and other compilations of the widely-used photon mass attenuation coefficients, only these two processes are considered, to account for the scattering part of the attenuation. Some details of other, less-probable scattering processes are known, one of them being the deflection of photons in the Coulomb field of atomic nuclei via virtual electron-positron pairs. This consequence of vacuum polarization and non-linear effects of quantum electrodynamics is often referred to as "scattering of light by light," and was first mentioned by Delbrück in 1933. Although a number of studies of Delbrück scattering appear in the literature, no quantitative evaluations of the total integrated cross sections, except for the high energy limit asymptotic case (Milstein and Strakhovenko 1983), analogous to real pair and triplet production, were found to be available, to compare with Compton and Raleigh total cross sections at intermediate and low photon energies. Among the works examined, a 1992 work by Falkenberg et al. does provide Delbrück-amplitude differential results in four different approximations, with some overlap of applicable energy regions. For this report, the necessary numerical integrations have been performed for selected elements in the range $Z = 1$ to 92 and energies between 0.255 MeV and 100 GeV. These integrated Delbrück cross sections are compared with the currently-included scattering cross sections, also with total cross sections, now provided in the NIST XCOM data base.

Key words: Attenuation coefficient, Coulomb field, Delbrück scattering, gamma rays, photons, x rays

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[#]Work supported by the Office of Standard Reference Data of the National Institute of Standards and Technology.

[†]J. H. Hubbell contribution to this work carried out for the National Institute of Standards and Technology in fulfillment of Service Order NA1341-02-W-1374.

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1. Introduction.

In the theoretical computations of photon (x-ray, gamma-ray, bremsstrahlung) attenuation coefficients for the widely-used tabulations of these data for medical diagnostic and therapy, reactor shielding, industrial radiation processing and other applications, only two processes are considered, by which photons are scattered by atoms in the target materials. One of these processes is coherent (Rayleigh) scattering, σ_{coh}, of the photon by the atomic electron cloud as a whole, from which the deflected photon emerges with no significant loss of energy (see, e.g., Hubbell and Øverbø 1979, Schaupp et al. 1983). The other included process is incoherent (Compton) scattering, σ_{incoh}, of the photon by an individual atomic electron which is then ejected from the atom, sharing the photon's incident energy with that of the deflected reduced-energy photon (see, e.g., Hubbell et al. 1975, Kahane 1998).

There exist other, less-probable scattering processes, including nuclear resonance scattering, nuclear Thomson scattering, and Delbrück scattering. The focus of this report will be on Delbrück scattering, σ_D, sometimes referred to as "scattering of light by light," to examine whether this process, in which a photon is scattered in the electrostatic (Coulomb) field of the atomic nucleus, becomes sufficiently significant in some region of Z and energy to merit inclusion in future tabulations of photon cross sections and attenuation coefficients. Actually, "scattering of light by light" is a separate but related process, also involving virtual electron-positron pair creation without the presence of a target nucleus, first studied by Halpern (1933), by Euler and Kockel (1935) and by Kemmer and Weisskopf (1936).

Delbrück scattering, closely related to electron-positron pair production (see, e.g., Hubbell et al. 1980) has attracted continuing interest ever since its prediction by Delbrück (1933), as one of the few nonlinear quantum electrodynamic processes amenable to experimental testing (Milstein and Schumacher 1994), and because it can be considered a manifestation of the existence of vacuum polarization (Jackson and Wetzel 1969). Differentially in angle, the cross section for Delbrück scattering can be expressed in terms of a complex scattering amplitude. The imaginary part of this complex amplitude describes scattering by the real electron-positron pairs produced in the nuclear Coulomb field and also gives information about the corresponding inelastic process, namely real pair production. The real part of this amplitude corresponds to scattering by virtual pairs produced in the nuclear field and is therefore related to the quantum-electrodynamically-predicted vacuum polarization as mentioned.

2. Experimental and Theoretical Literature on Delbrück Scattering:

Several review and summary papers on Delbrück scattering are available and have been helpful in assembling this report. To name a few, information was obtained from Constantini et al. (1971), Papatzacos and Mork (1975a,b), Kane et al. (1986), Milstein and Schumacher (1994), Akhmadaliev et al (1998), Canetta et al. (1999) and Schumacher (1999). No attempt will be made to individually cite in the text of this report all the experimental studies aimed at this effect. However, in addition to the **Text References** listing, an extensive further sampling of **measurement** papers will be found in the **Extended Bibliography**, in which full titles are provided, for identifying the content of these papers. Similarly, an extensive sampling of further **theoretical** studies of Delbrück scattering is also provided and identified (by titles) in the **Extended Bibliography** at the end of this report.

Quantitative expressions for the Delbrück scattering cross section were presented by Kemmer (1937) and Kemmer and Ludwig (1937) including the Z^4 dependence of the process, and by Achieser and Pomerantschuk (1937) who presented expressions for the total Delbrück scattering cross section for the low- and high-energy limiting cases, but including an unknown multiplicative constant.

3. Aim of this Report:

In order to assess in some compact form the relative importance of Delbrück scattering on the photon cross section and mass attenuation coefficient compilations, we decided to obtain the total integrated Delbrück scattering cross sections, σ_D, to compare with the now-tabulated coherent (Rayleigh) scattering, incoherent (Compton) scattering integrated cross sections, σ_{coh} and σ_{incoh}, and with the total interaction cross sections σ_{tot} including also the atomic photoeffect τ_{pe} and pair and triplet production cross sections κ_n and κ_e.

A complicating factor: The Delbrück scattered photons have a coherence with the Rayleigh scattered photons, resulting in the possibility of destructive as well as constructive interference. In some of the Delbrück measurement papers examined, the comparisons with theory suggest both plus and minus contributions to coherent (Rayleigh) scattering, with a crossover at some intermediate deflection angle. However, we have been advised by Schumacher (2003) that such interferences should not significantly affect the aims of this study, in which comparisons will be in terms of orders of magnitude.

4. General Features of Delbrück Total Scattering. Dependence on Z and E:

According to the literature sources examined, the Delbrück total scattering cross section σ_D varies with Z as Z^4, which can be compared to its related process of nuclear-field real pair production κ_n which varies with Z^2 and with triplet production (pair production on the field of an atomic electron) κ_e which varies with Z, also with Rayleigh scattering σ_{coh} which varies with Z^2 and with Compton scattering σ_{incoh} which varies with Z. The only photon-atom interaction process with a Z-dependence comparable to Delbrück scattering is the atomic photoelectric cross section τ_{pe} which varies approximately as Z^4 at low energies and as Z^5 at high energies. For these Z-dependencies of the various processes, see, e.g., Hubbell (1969), Table 2.-1.

Not surprising, considering the connection between σ_D and κ_n (pair production), σ_D has a somewhat similar dependence on E to that of κ_n. However, the Delbrück process can take place at photon energies below the pair production nominal threshold of 1.022 MeV, with σ_D

data available for photon energies as low as 0.2555 MeV. As will be seen in the tables and discussions following, σ_D increases with photon energy up to the region of tens of MeV, where it flattens out to a constant asymptotic value for higher energies, adhering to the approximate Z^4 dependence on the nuclear charge.

5. High-Energy Asymptotic Integrated $\sigma_D(\infty)$ Values:

In our examination of the literature, the only already-integrated (over angle) total σ_D values available appeared to be the high-energy-limit values of Bethe and Rohrlich (1952) and of Milstein and Strakhovenko (1983a,b) as quoted by Milstein and Schumacher (1994) in their review paper. However, it was pointed out by Cheng and Wu (1969), and mentioned also by Milstein and Strakhovenko (1983a,b) and by Milstein and Schumacher (1994) that the Born approximation results of Bethe and Rohrlich (1952) were based on an incorrect approach, that would make them too low by as much as a factor of two. Hence, we will here consider only the Milstein and Strakhovenko (1983a,b) high energy asymptotic energy-independent total $\sigma_D(\infty)$ results, based on the integrable expressions for the amplitudes which they developed.

Milstein and Strakhovenko (1983a,b) present their results graphically in the form of the dimensionless ratio $\sigma_D(\infty)/\sigma_o$ as a function of Z, in which σ_o is defined as:

$$\sigma_o = (r_o^2/16\pi)(Z/137)^4 = 1.58 \text{ mb} \cdot Z^4/137^4 = Z^4 \cdot 0.00158 \text{ b}/3.522 \cdot 10^8$$

$$= Z^4 \cdot 4.485 \cdot 10^{-12} \text{ b} \qquad (1)$$

which graph and information are reproduced in the Milstein and Schumacher (1994) review paper (Figure 5) [in which here and elsewhere b (= 10^{28} m^2) is the atomic cross section unit in "barns"], from which we read values to obtain the following Table 1.

Table 1. Extreme high-energy asymptotic (flat) Delbrück integrated cross sections $\sigma_D(\infty)$ obtained from graph in Milstein and Schumacher (1994) due to Milstein and Strakhovenko (1983a,b).

Z	$\sigma_D(\infty)/\sigma_o$(graph)	σ_o(b)	$\sigma_D(\infty)$(b) $=(\sigma_D(\infty)/\sigma_o)\cdot\sigma_o$(b)
1	54.2	4.485E-12	2.431E-10
5	53.9	2.803E-09	1.511E-07
10	53.5	4.485E-08	2.399E-06
20	52.1	7.176E-07	3.739E-05
40	46.3	1.148E-05	5.316E-04
50	42.8	2.803E-05	1.200E-03
82	30.0	2.028E-04	6.084E-03
92	26.4	3.213E-04	8.482E-03
94	25.8	3.502E-04	9.035E-03

These extreme high energy asymptotic (constant) values of the integrated Delbrück scattering cross section $\sigma_D(\infty)$ can then be compared with the coherent (Rayleigh) scattering cross section σ_{coh}, and the total cross section σ_{tot}, including the pair and triplet cross sections κ_n and κ_e (also constant at high energies) from Hubbell, Gimm and Øverbø (1980):

Table 2. Comparison of $\sigma_D(\infty)$ from Table 1 with σ_{coh} and σ_{tot} from Hubbell et al. (1980) at intermediate and high energies.

Z	$\sigma_D(\infty)$ $= (\sigma_D(\infty)/\sigma_o)\cdot\sigma_o$ (b)	σ_{coh}(100 MeV) (b)	σ_{coh}(100 GeV) (b)	σ_{tot}(100 GeV) (b)
1	2.431E-10	4.620E-10	4.620E-16	1.110E-02
5	1.511E-07	4.821E-08	4.821E-14	2.612E-01
10	2.399E-06	3.194E-07	3.193E-13	8.799E-01
20	3.739E-05	2.016E-06	2.015E-12	3.194E+00
40	5.316E-04	1.344E-05	1.344E-11	1.144E+01
50	1.200E-03	2.499E-05	2.499E-11	1.732E+01
82	6.084E-03	1.059E-04	1.059E-10	4.118E+01
92	8.482E-03	1.527E-04	1.527E-10	5.016E+01
94	9.035E-03	1.638E-04	1.638E-10	5.147E+01

Here, it can be seen that at 100 MeV high-energy Delbrück scattering is comparable with σ_{coh} for Z = 1, but for high Z elements dominates by an order of magnitude. Since σ_{coh} is decreasing with energy as E^{-2} and σ_D is remaining constant, σ_{coh} at 100 GeV becomes smaller than σ_D by several orders of magnitude, making σ_D the dominant coherent (or elastic) scattering process at high photon energies. However, the Delbrück scattering contribution at 100 GeV is also seen to be 4 to 6 orders of magnitude less than the total photon interaction cross section σ_{tot}, consisting primarily of the cross section κ_n for the production of real e^+e^- electron-positron pairs.

6. Delbrück Scattering Cross Sections σ_D below the Asymptotic (Constant) Region:

In our examination of the literature, no total Delbrück scattering cross sections σ_D for finite specified photon energies were found. However, tables of Delbrück amplitudes have been computed and tabulated, for example by Kahane (1992) and by Falkenberg et al. (1992), from which the scattering angular distributions can be computed, and in turn integrated over angle to provide values of the total Delbrück scattering cross sections σ_D for a range of photon incident energies, as well as for a range of elemental Z values.

The Kahane (1992) first-order Delbrück amplitudes over the angular range 0.001° to 120° were estimated to have an "accuracy of as good as 1%," but were provided only for the photon energy range 7.92 MeV to 28 MeV. The Falkenberg et al. (1992) computations and compilation, intended to "remain state-of-the art for the foreseeable future," mentions estimated errors of the order of 1% to 5%. The amplitudes in the latter work, as discussed below, are presented piecewise in four different low-, medium- and high-energy approximations. The first two approximations encompass the photon energy ranges 0.2555 MeV to 2.7539 MeV and 3 MeV to 100 MeV, respectively. The second two are high-energy approximations for large-angle and small-angle respectively, allowing complete coverage of the remaining photon energy range, from 100 MeV up to the extreme high-energy region discussed in Section 5., where the integrated Delbrück cross sections become flat asymptotically at the values $\sigma_D(\infty)$ listed in Table 1.

Where it was possible to compare the Kahane (1992) results with those compiled by Falkenberg et al. (1992), differences ranging from -7% to $+8\%$ were found, with $\pm 3\%$ being more typical. Given the more comprehensive energy range of the Falkenberg et al. (1992) compilation, and the material agreement with Kahane (1992) sufficient for our rough-estimate purposes, we elected to use the Falkenberg et al. (1992) compilation for this part of our study. We are aware that there could be interference effects, both constructive and destructive, with the amplitudes of other coherent scattering processes such as atomic Rayleigh scattering and with scattering involving the different modes of nuclear excitation. However, as mentioned earlier (in Section 3.), such interferences are not expected to have a significant effect on our study which is aimed at orders-of-magnitude comparisons.

In the Falkenberg et al. (1992) tables, real (Re) and imaginary (Im) parts of the Delbrück scattering amplitudes are given either in terms of linear polarization states ($A_\parallel$ and $A_\perp$, where the incoming photon is polarized parallel and perpendicular to the scattering plane, respectively) or in terms of helicity states (A^{++} non-helicity-flip amplitude, A^{+-} helicity-flip amplitude). These amplitudes are related according to

$$A^{++} = \tfrac{1}{2}(A_\parallel + A_\perp), \text{ and } A^{+-} = \tfrac{1}{2}(A_\parallel - A_\perp). \qquad (2)$$

Thus, for unpolarized incident photons, the differential cross section for Delbrück scattering, as a function of deflection angle, is given by

$$d\sigma_D/d\Omega = (Z\alpha)^4 r_o^2 \tfrac{1}{2}\{|A_\parallel|^2 + |A_\perp|^2\} \qquad (3)$$

or by

$$d\sigma_D/d\Omega = (Z\alpha)^4 r_o^2 \tfrac{1}{2}\{|A^{++}|^2 + |A^{+-}|^2\} \qquad (4)$$

Falkenberg et al. (1992) provide four sets of tables (I, II, III, and IV) of Delbrück amplitudes, computed using four different methods and approaches, for the different energy (and angular) regions of applicability. Tables III and IV for high energies are complementary, applicable for large-angle and small-angle regions, respectively, hence our results from these two amplitude sets could be and were added together following integration, as described below, to give a total of three sets of results, rather than four.

Method I: Lowest Born Approximation. Photon Energies below 3 MeV: This method, for photon energies below 3 MeV, is based on the work of Papatzacos and Mork (1975a,b) and employs the lowest-order Born approximation as discussed also by De Tollis and

Luminari (1984), Bar-Noy and Kahane (1977), Cheng et al. (1982), Rullhusen et al. (1987) and by Turrini et al. (1989). For low Z atoms the predicted angular distributions are expected to be accurate within a few percentage points, but for high Z elements the results are less reliable due to neglect of Coulomb corrections. Falkenberg et al. in their Table I present real (Re) Delbrück amplitudes, Re $A_{\perp}$ and Re $A_{\parallel}$, computed according to Method I, as a function of angle from 1° to 150° for a selection of photon energies from 0.2555 MeV to 2.7539 MeV. Above the 1.022 MeV threshold for real electron-positron pair production, beginning with 1.1205 MeV, some imaginary (Im) amplitudes have also been computed and listed.

For the present study, we inserted the real (Re) amplitudes from Falkenberg et al. (1992) into eqn (3) and integrated these angular distributions over angle, extrapolating to 180°, to obtain the following Table 3:

Table 3. σ_D(I) derived from Table I, Falkenberg et al. (1992).

	σ_D(I)(b)			
E(MeV)	Z=1	Z=20	Z=40	Z=92
0.2555	3.485E-14	5.576E-09	8.921E-08	2.496E-06
0.511	3.045E-13	4.872E-08	7.795E-07	2.181E-05
0.6616	6.602E-13	1.056E-07	1.690E-06	4.730E-05
0.7665	1.026E-12	1.642E-07	2.627E-06	7.350E-05
0.8892	1.602E-12	2.564E-07	4.102E-06	1.148E-04
1.0220	2.451E-12	3.922E-07	6.275E-06	1.756E-04
1.1150	3.206E-12	5.130E-07	8.208E-06	2.297E-04
1.1205	3.205E-12	5.128E-07	8.205E-06	2.296E-04
1.170	3.772E-12	6.035E-06	9.655E-06	2.702E-04
1.330	5.762E-12	9.219E-07	1.475E-05	4.128E-04
1.70	1.145E-11	1.833E-06	2.932E-05	8.206E-04
2.09	1.813E-11	2.900E-06	4.641E-05	1.299E-03
2.7539	2.454E-11	3.927E-06	6.283E-05	1.758E-03

Method II: Lowest Born Approximation. Photon Energies 3 MeV to 100 MeV: This method extends the general principles of Method I up to 100 MeV, with the application of a two-dimensional cubic spline incorporating data also from Papatzacos and Mork (1975a,b), De Tollis and Luminari (1984). We used the Falkenberg et al. Table II amplitudes to perform the integrations over eqn.(4) to obtain the following Table 4:

Table 4. Lowest Born approximation, integrated from Table II, Falkenberg et al. (1992). The apparent decrease in the cross section above 17 MeV we believe to be fictitious, due to the sharp forward peaking of the angular distribution, for which the angular grid is too coarse to provide a realistic peak-shape over which to integrate. At 17 MeV, the extreme high-energy asymptotic (constant) values $\sigma_D(\infty)$ from Table 2 are inserted for comparison.

	σ_D(II)(b)				
E(MeV)	Z=1	Z=20	Z=40	Z=92	
3	3.527E-11	5.644E-06	9.030E-05	2.527E-03	
5	6.912E-11	1.106E-05	1.769E-04	4.952E-03	
7	8.931E-11	1.429E-05	2.286E-04	6.398E-03	
9	1.072E-10	1.715E-05	2.743E-04	7.677E-03	
11	1.184E-10	1.894E-05	3.031E-04	8.482E-03	
13	1.251E-10	2.002E-05	3.203E-04	8.963E-03	
15	1.285E-10	2.056E-05	3.289E-04	9.205E-03	
17	1.291E-10	2.066E-05	3.306E-04	9.252E-03	= σ_D(II) max.
	2.431E-10	3.739E-05	5.316E-04	8.482E-03	= $\sigma_D(\infty)$, M.&S.(1994)
19	1.288E-10	2.060E-05	3.296E-04	9.225E-03	
21	1.271E-10	2.033E-05	3.253E-04	9.104E-03	
23	1.252E-10	2.004E-05	3.206E-04	8.971E-03	
25	1.221E-10	1.954E-05)	3.126E-04	8.749E-03	
30	1.154E-10	1.847E-05	2.955E-04	8.271E-03	
35	1.082E-10	1.731E-05	2.770E-04	7.752E-03	
40	1.009E-10	1.615E-05	2.583E-04	7.229E-03	
45	9.392E-11	1.503E-05	2.404E-04	6.728E-03	
50	8.724E-11	1.396E-05	2.233E-04	6.250E-03	
55	8.102E-11	1.296E-05	2.074E-04	5.804E-03	
60	7.494E-11	1.199E-05	1.919E-04	5.369E-03	
65	6.942E-11	1.111E-05	1.777E-04	4.973E-03	
70	6.443E-11	1.031E-05	1.649E-04	4.616E-03	
80	5.504E-11	8.806E-06	1.409E-04	3.943E-03	
90	4.744E-11	7.591E-06	1.215E-04	3.399E-03	
100	4.105E-11	6.567E-06	1.051E-04	2.940E-03	

Method III: Delbrück Amplitudes Calculated in the High-Energy Large-Angle Approximation, 5° to 180°: This method uses the approach developed by Milstein and Shaisultanov (1988) to all orders in $Z\alpha$ for the limit $E \gg mc^2$ and for large angles, $\theta(Q \gg mc)$, from which in their Table III, Falkenberg et al. (1992) present values of F^{++} and F^{+-}. These tabulated values must then be multiplied by the factor $(2\omega/4\pi\Delta^2)$, in which $\omega = E/mc^2$, $\Delta = Q/mc = 2\omega \sin(\theta/2)$ is the momentum transfer in units of mc, to obtain, respectively, the values of A^{++} and A^{+-} to insert in eqn.(4), above, for the required integrations, over the angular range of θ from 5° to 180°.

Method IV: Delbrück Amplitudes Calculated in the High-Energy Small-Angle Approximation, 0° to 5°: This method uses the approach of Cheng and Wu (1969, 1970, 1972) and by Milstein and Strakhovenko (1983a,b) in which the calculation is performed by summing in a definite approximation the Feynman diagrams with an arbitrary number of photons exchanged with a Coulomb center, hence including the Coulomb correction. For our study, we used Table IV in Falkenberg et al. (1992), in which $\mathrm{Im}A^{++}/\omega$ and $\mathrm{Im}A^{+-}/\omega$ are tabulated for several values of Z, for values of the energy-angle momentum transfer parameter Δ, as defined above, from Δ = 1.879E-03 to Δ = 1.498E+04. Multiplying by ω, we obtained the values of A^{++} and A^{+-} to insert in eqn.(4) for the integrations over the angular range 0° to 5° where this small-angle approximation is applicable.

Combining the results, after integrating, from the two high-energy approximations in their respective large-angle and small-angle angular ranges, we obtain the high-energy total integrated Delbrück scattering cross sections σ_D(III,IV), presented in Table 5.

Table 5. Intermediate- to high-energy Delbrück total integrated cross sections σ_D(III,IV) obtained from differential data in Tables III and IV, Falkenberg et al. (1992), for a selection of Z and E values.

	σ_D(III,IV)(b)			
E(MeV)	Z=1	Z=20	Z=40	Z=92
3	3.301E-09	5.049E-04	7.421E-03	1.750E-01
30	2.551E-10	4.045E-05	5.686E-04	1.012E-02
100	2.514E-10	3.881E-05	5.511E-04	9.007E-03
100,000	2.543E-10	3.919E-05	5.554E-04	8.908E-03

7. Comparisons of Delbrück Scattering Integrated Cross Sections σ_D with Coherent (Rayleigh) and Incoherent (Compton) Scattering Cross Sections σ_{coh} and σ_{incoh} and with Total Interaction Cross Sections (for Attenuation Coefficients) σ_{tot}:

For this comparison, we used cross section values from the NIST "XCOM" data base and calling program as described by Berger and Hubbell (1987). This program provides cross section values for the individual photon-atom interaction processes including coherent (Rayleigh) scattering, σ_{coh}, incoherent (Compton) scattering, σ_{incoh}, atomic photoeffect, τ_{pe}, nuclear-field pair production, κ_n, and atomic-electron-field pair production ("triplet"), κ_e, and the sum over these individual cross sections, σ_{tot}(b) from which the mass attenuation coefficient μ/ρ(cm^2/g) is obtained. XCOM neglects the contribution from the nuclear photoeffect, σ_{pn} which can add as much as 5% to σ_{tot} at the peak of the giant dipole resonance (GDR) occurring between 10 MeV and 30 MeV, and neglects other, less-probable processes such as nuclear resonance and nuclear Compton scattering. XCOM provides the above dominant individual, and total cross sections for arbitrary energies from 1 keV to 100 GeV, for arbitrary Z from 1 to 100, and for arbitrary combinations of Z. Finally, XCOM provided the following Table 6 for purposes of our comparison:

Table 6. Cross sections for the dominant photon-atom interaction processes, obtained from XCOM (Berger and Hubbell 1987 and updates), for selected values of Z and E. The total cross section σ_{tot} is "with coherent," that is, it is the sum over the previous five columns.

H, Z=1: Cross sections in b, from XCOM:

E(MeV)	σ_{coh}	σ_{incoh}	τ_{pe}	κ_n	κ_e	σ_{tot}
2.550E-01	7.112E-05	3.746E-01	7.353E-08	0.000E+00	0.000E+00	3.747E-01
1.022E+00	4.428E-06	2.092E-01	1.880E-09	0.000E+00	0.000E+00	2.092E-01
2.754E+00	6.100E-07	1.215E-01	3.948E-10	4.232E-04	2.059E-05	1.219E-01
3.000E+00	5.139E-07	1.153E-01	3.524E-10	5.048E-04	4.035E-05	1.158E-01
3.000E+01	5.139E-09	2.212E-02	2.662E-11	3.998E-03	3.143E-03	2.926E-02
1.000E+02	4.620E-10	8.276E-03	7.811E-12	6.116E-03	5.593E-03	1.998E-02
1.000E+05	4.620E-16	1.785E-05	7.736E-15	9.601E-03	1.110E-02	2.072E-02

Ca, Z=20: Cross sections in b, from XCOM:

E(MeV)	σ_{coh}	σ_{incoh}	τ_{pe}	κ_n	κ_e	σ_{tot}
2.550E-01	3.005E-01	7.408E+00	3.241E-01	0.000E+00	0.000E+00	8.032E+00
1.022E+00	1.926E-02	4.178E+00	8.931E-03	0.000E+00	0.000E+00	4.206E+00
2.754E+00	2.657E-03	2.429E+00	1.674E-03	1.731E-01	4.117E-04	2.607E+00
3.000E+00	2.239E-03	2.306E+00	1.482E-03	2.053E-01	8.070E-04	2.516E+00
3.000E+01	2.240E-05	4.425E-01	1.007E-04	1.531E+00	6.202E-02	2.036E+00
1.000E+02	2.016E-06	1.655E-01	2.922E-05	2.199E+00	1.060E-01	2.471E+00
1.000E+05	2.015E-12	3.571E-04	2.880E-08	3.015E+00	1.789E-01	3.194E+00

Zr, Z=40: Cross sections in b, from XCOM:

E(MeV)	σ_{coh}	σ_{incoh}	τ_{pe}	κ_n	κ_e	σ_{tot}
2.550E-01	1.933E+00	1.459E+01	7.484E+00	0.000E+00	0.000E+00	2.401E+01
1.022E+00	1.278E-01	8.339E+00	2.288E-01	0.000E+00	0.000E+00	8.696E+00
2.754E+00	1.771E-02	4.857E+00	4.120E-02	7.278E-01	8.230E-04	5.644E+00
3.000E+00	1.492E-02	4.610E+00	3.632E-02	8.574E-01	1.613E-03	5.520E+00
3.000E+01	1.493E-04	8.849E-01	2.291E-03	5.891E+00	1.222E-01	6.901E+00
1.000E+02	1.344E-05	3.311E-01	6.589E-04	8.349E+00	2.044E-01	8.885E+00
1.000E+05	1.344E-11	7.142E-04	6.468E-07	1.111E+01	3.265E-01	1.144E+01

U, Z=92: Cross sections in b, from XCOM:

E(MeV)	σ_{coh}	σ_{incoh}	τ_{pe}	κ_n	κ_e	σ_{tot}
2.550E-01	1.923E+01	3.232E+01	2.412E+02	0.000E+00	0.000E+00	2.928E+02
1.022E+00	1.405E+00	1.905E+01	1.005E+01	0.000E+00	0.000E+00	3.051E+01
2.754E+00	1.998E-01	1.115E+01	1.735E+00	4.698E+00	1.888E-03	1.778E+01
3.000E+00	1.685E-01	1.059E+01	1.520E+00	5.294E+00	3.698E-03	1.758E+01
3.000E+01	1.697E-03	2.035E+00	8.482E-02	2.682E+01	2.718E-01	2.921E+01
1.000E+02	1.527E-04	7.614E-01	2.399E-02	3.763E+01	4.429E-01	3.886E+01
1.000E+05	1.527E-10	1.643E-03	2.336E-05	4.948E+01	6.740E-01	5.016E+01

Comparison of Near-Threshold Delbrück Cross Sections with Scattering and Total XCOM Cross Sections: Below 3 MeV, in Table 7, comparing values taken from Tables 3 and 6, we see that for high Z elements, such as for Z = 92, the Delbrück integrated cross section σ_D comes within two orders of magnitude of the coherent scattering (Rayleigh) cross section, but is less than σ_{incoh} and σ_{tot} by four or more orders of magnitude.

Table 7. Integrated Delbrück cross sections σ_D(I) compared with coherent (Rayleigh), incoherent (Compton) scattering, and total cross sections, σ_{coh}, σ_{incoh} and σ_{tot}, from XCOM.

E = 0.255 MeV, cross sections in b

Z	σ_D(I)	σ_{coh}	σ_{incoh}	σ_{tot}
1	3.485E-14	7.112E-05	3.746E-01	3.747E-01
20	5.576E-09	3.005E-01	7.408E+00	8.032E+00
40	8.921E-08	1.933E+00	1.459E+01	2.401E+01
92	2.496E-06	1.923E+01	3.232E+01	2.928E+02

E = 1.0220 MeV, cross sections in b

Z	σ_D(I)	σ_{coh}	σ_{incoh}	σ_{tot}
1	2.451E-12	4.428E-06	2.092E-01	2.092E-01
20	3.922E-07	1.926E-02	4.178E+00	4.206E+00
40	6.275E-06	1.278E-01	8.339E+00	8.696E+00
92	1.756E-04	1.405E+00	1.905E+01	3.051E+01

E = 2.7539 MeV, cross sections in b

Z	σ_D(I)	σ_{coh}	σ_{incoh}	σ_{tot}
1	2.454E-11	6.100E-07	1.215E-01	1.219E-01
20	3.927E-06	2.657E-03	2.429E+00	2.607E+00
40	6.283E-05	1.771E-02	4.857E+00	5.644E+00
92	1.758E-03	1.998E-01	1.115E+01	1.778E+01

Comparison of Intermediate, High-Energy, and Extreme-High-Energy Delbrück Scattering Cross Sections σ_D with XCOM Scattering and Total Cross Sections: In Table 8, comparing values of σ_D(II) from Table 4 and values of σ_D(III,IV) from Table 5 with values of σ_{coh}, σ_{incoh} and σ_{tot} taken from Table 7, we see that for photon energies above 30 MeV and for medium and high Z elements σ_D exceeds σ_{coh}, although remaining four orders of magnitude below σ_{tot} for all combinations of Z and E. For 100,000 MeV (100 GeV) the extreme high-energy asymptotic values $\sigma_D(\infty)$ from Table 2 are included in the comparison in Table 8, and are seen to be consistent with the values of σ_D(III,IV) at this high energy.

Table 8. Integrated Delbrück cross sections σ_D(II) and σ_D(III,IV) from Falkenberg et al. (1992) and $\sigma_D(\infty)$ from Milstein and Strakhovenko (1983) compared with coherent (Rayleigh), incoherent (Compton) scattering, and total cross sections, σ_{coh}, σ_{incoh} and σ_{tot}, from XCOM.

E = 3.0 MeV, cross sections in b

Z	σ_D(II)	σ_D(III,IV)	σ_{coh}	σ_{incoh}	σ_{tot}
1	3.527E-11	3.301E-09	5.139E-07	1.153E-01	1.158E-01
20	5.644E-06	5.049E-04	2.239E-03	2.306E+00	2.516E+00
40	9.030E-05	7.421E-04	1.492E-02	4.610E+00	5.520E+00
92	2.527E-03	1.750E-02	1.685E-01	1.059E+01	1.758E+01

E = 30.0 MeV, cross sections in b

Z	σ_D(II)	σ_D(III,IV)	σ_{coh}	σ_{incoh}	σ_{tot}
1	1.154E-10	2.551E-10	5.139E-09	2.212E-02	2.926E-02
20	1.847E-05	4.045E-05	2.240E-05	4.425E-01	2.036E+00
40	2.955E-04	5.686E-04	1.493E-04	8.849E-01	6.901E+00
92	8.271E-03	1.012E-02	1.697E-03	2.035E+00	2.921E+01

E = 100.0 MeV, cross sections in b

Z	σ_D(II)	σ_D(III,IV)	σ_{coh}	σ_{incoh}	σ_{tot}
1	4.105E-11	2.514E-10	4.620E-10	8.276E-03	1.998E-02
20	6.567E-06	3.881E-05	2.016E-06	1.655E-01	2.471E+00
40	1.051E-03	5.511E-04	1.344E-05	3.311E-01	8.885E+00
92	2.940E-03	9.007E-03	1.527E-04	7.614E-01	3.886E+01

E = 100,000 MeV (100 GeV), cross sections in b

Z	σ_D(III,IV)	$\sigma_D(\infty)$	σ_{coh}	σ_{incoh}	σ_{tot}
1	2.543E-10	2.431E-10	4.620E-16	1.785E-05	2.072E-02
20	3.919E-05	3.739E-05	2.015E-12	3.571E-04	3.194E+00
40	5.554E-04	5.316E-04	1.344E-11	7.142E-04	1.144E+01
92	8.908E-03	8.482E-03	1.527E-10	1.643E-03	5.016E+01

8. Observations and Conclusions:

From this study, we note that the integrated Delbrück scattering cross section σ_D, related to real pair production κ_n in its underlying physical origin, somewhat mimics the photon energy dependence of real electron-positron pair production in threshold ($\approx$.25 MeV) and shape (flat at high energy), but remains less than σ_{tot} (primarily κ_n) by between 10 orders of magnitude (low energy, low Z) and 4 orders of magnitude (high energy, high Z). Hence, since the mass attenuation coefficient is known experimentally to $\pm$0.1% at best, and in general to in excess of $\pm$1.0% (Cullen et al. 1997), the effort to obtain the 0.01% (maximum) contribution of σ_D to σ_{tot} for a wide range of energies and atoms would not be justified.

However, if the user is particularly interested in the contributions of photon scattering to the total interaction cross section, it can be seen, if the above source theoretical data (e.g., amplitudes from Falkenberg et al. 1992) drawn from can be considered credible, at photon energies as low as 30 MeV and Z as low as 20, σ_D has overtaken σ_{coh} which is decreasing with increasing energy approximately as E^{-2} while σ_D is increasing to a constant high-energy value. For incoherent scattering, which is also decreasing with increasing energy but closer to E^{-1}, σ_D does not overtake σ_{incoh} until E approaches $\approx$100 GeV, for high Z elements.

Conclusion: Given current experimental and theoretical uncertainties in μ/ρ, addition of σ_D to σ_{tot} does not appear to be warranted. Also, where σ_D is significant at high photon energies, it is also peaked sharply forward, so that in transport calculations it behaves almost as a "noninteraction." However, it is indeed seen to be a strong competitor to σ_{coh} for medium- to high-Z elements, for photon energies of tens of MeV and higher, and could be a significant effect for applications where high-energy elastic photon scattering is important.

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