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Semiconductor Measurement Technology:

A Collection of Computer Programs for Two-Probe Resistance (Spreading Resistance) and Four-Probe Resistance Calculations, RESPAC

John Albers and Harry L. Berkowitz

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John Albers

Semiconductor Electronics Division
Electronics and Electrical Engineering Laboratory
National Institute of Standards and Technology
Gaithersburg, MD 20899

and

Harry L. Berkowitz

Electronics Technology and Devices Laboratory
Fort Monmouth, NJ 07703

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Table of Contents

List of Figures

List of Tables

List of Tables, Cont'd.

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A Collection of Computer Programs for
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and Four-Probe Resistance Calculations, **RESPAC**

John Albers
Semiconductor Electronics Division
National Institute of Standards and Technology
Gaithersburg, MD 20899

Harry L. Berkowitz†
Electronics Technology and Devices Laboratory
Fort Monmouth, NJ 07703

Abstract

This report presents and describes a number of FORTRAN programs which may be used to perform two-probe resistance (spreading resistance) and four-probe resistance calculations for vertically nonuniform resistivity structures. These programs fall into three general categories. They are: 1) programs for calculating the two-probe resistance (spreading resistance) from the resistivity profile, 2) programs for calculating the resistivity profile from the two-probe resistance (the inverse of 1), and 3) programs for calculating the four-probe resistance from the resistivity profile. Programs in the first and third category are useful for understanding the effects of resistivity variations on the two-probe resistance (spreading resistance) and the four-probe resistance. Programs in the second category are useful for extracting the resistivity profile from spreading resistance data (either measured or calculated).

All of the programs are based upon the Schumann and Gardner solution of the multilayer Laplace equation. As such, local charge neutrality is assumed. The limitations of this assumption are described in the text.

The first part of this report consists of an outline of the derivation of the Schumann and Gardner multilayer Laplace equations. In addition, there is a discussion of the evolution and simplification of this model which has taken place over the past 2 decades. This part of the report is intended to provide the reader with a background so as to make optimal use of the computer programs. The second part of the report contains a discussion of the structure and inner workings of each of the programs. Special attention is paid to the aspects which make the individual programs different from others in the same category. In addition, sample input data used in the programs and the corresponding output data calculated by the programs are also presented. The final parts of the report (the appendices) contain annotated, internally documented listings of the FORTRAN source codes.

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† Present Address: Solid State Measurements, 110 Technology Drive, Pittsburgh, PA 15275

In all, there are ten programs contained in the RESPAC package. The FORTRAN source code (total of about 123 kbytes) and sample input and output data files are available in ASCII format using a number of transfer vehicles. These include: standard 8-track magnetic tape (ASCII, density = 1600, record = 80, block = 1600), 5.25-in. (360-kbyte and 1.2-Mbyte) DOS-formatted floppy disks, and using electronic mail over the Internet. This package is self-contained and is straightforward to run once the FORTRAN is compiled and linked by the user-supplied software. The sample input and output data files are included so that the user can check the programs for proper operation as well as to become acquainted with the setup and use of the codes.

Key words: correction factor; FORTRAN; four-point resistance; Laplace's equation; multilayer analysis; Poisson's equation; probe radius; probe separation; profiles; resistivity; semiconductor devices; semiconductor materials; sheet resistance; spreading resistance; two-point resistance.

INTRODUCTION

The last 2 decades have witnessed the growth and development of a number of atomic and electrical techniques which have been used in the profiling of semiconductor device structures. These developments have tried to keep cadence with the trend in semiconductor technology towards submicrometer devices. The push made by VLSI technology into the submicrometer regime has placed increased demands upon the ability to adequately measure and interpret dopant profiles. The acquisition and analysis of one-dimensional (vertical) profiles form the basis for present research and commercial development. The two-dimensional extension of these techniques has proven to be a formidable problem.

The Secondary Ion Mass Spectrometry (SIMS) technique [1] has become a forefront tool for micrometer and submicrometer chemical analysis, in general, and for the depth-dependent atomic profiling of semiconductor structures, in particular. Other atomic techniques used for submicrometer profiling include the Rutherford Back Scattering (RBS) [2] and Neutron Depth Profiling (NDP) [3]. All of these techniques provide for the indirect measure of the atomic profile. The primary importance of dopants in semiconductors is through the enhancement or diminishing of the electrical conduction. Consequently, a complete picture of the processing of a semiconductor structure requires that the atomic profile be supplemented with the corresponding electrical profile.

In the area of electrical profiling, two-probe spreading resistance [4] and capacitance-voltage [5] are probably the most widely used techniques. Capacitance-voltage is limited by a total carrier concentration above which avalanche breakdown takes place. In addition, four-probe resistance is a useful tool. However, it is limited by the fact that vertical profiling is cumbersome and difficult with this method. Hall measurements [6] also provide useful information, but are also hindered as a vertical profiling technique.

The intelligent use of the spreading resistance technique for micrometer and submicrometer structures requires an understanding of the models which have been developed over the

past two decades for the interpretation of the data. This is important as the technique does not provide the carrier profile directly. Rather, the resistivity profile is indirectly obtained from the analysis of the depth-dependent spreading resistance data. This requires a physical model of the measurement in order to connect the data and the underlying resistivity structure. The conversion of the resistivity profile to a carrier density profile takes place through a resistivity-carrier density relation. In addition, the carrier density profile which is obtained from the resistivity profile need not be in one-to-one correspondence with the atomic density. Consequently, it is important to understand the basic model of spreading resistance insofar as its assumptions and possible limitations are concerned. This is especially the case as a number of analysis schemes or algorithms have appeared over the past decade. While they may appear to be quite different, many of them are related to the same basic model. Their differences are mostly due to ways of handling probe current densities, calibration data, and the evaluation of integrals which appear in the relation between the spreading resistance and the resistivity. The purpose of this report is to discuss a number of these algorithms and to present FORTRAN programs based upon them. The intent is to provide a common basis for the interpretation of spreading resistance data. This is motivated in part by the experience of the first author in providing "model" spreading resistance data for an ASTM F.1 effort to compare spreading resistance profiles. The public availability of the RESPAC codes should lead to a common ground for the discussion and interpretation of spreading resistance data.

This report is divided into three parts. The first contains a history of the multilayer Laplace equation model which relates the two- and four-probe resistance to the underlying resistivity structure. This history is not exhaustive but is intended to provide the reader with an indication of the changes and improvements which have taken place over the years since the original calculations of Schumann and Gardner. In addition, the incorporation of several forms of the probe current density, the possible determination of the probe radius and its relation to calibration data, and the effects of the assumption of finite thickness, uniform resistivity "layers" are also considered. The second part of this report contains discussions of the FORTRAN codes developed from the various models discussed in the first part. These codes are contained in the appendices which make up the third portion of this report.

Certain commercial equipment, instruments, or materials are identified in this report in order to specify the procedure adequately. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose. In spite of the authors' experiences that the programs perform correctly on every set of data which has been tried, there can be no assurance that the program will perform equally well on all (possibly anomalous) data. Therefore, both the authors and NIST assume no liability for possible losses resulting from the use of these programs.

HISTORY OF THE SCHUMANN-GARDNER MULTILAYER LAPLACE EQUATION

The primary purpose of the theory of two- and four-probe resistance is to provide a simple

and yet complete model which relates the resistance to the details of the resistivity structure. As is seen in this section, the present theory provides for a well-defined procedure for obtaining the spreading resistance from the resistivity. The reverse or inverse process of obtaining the resistivity from the spreading resistance requires the solution of what is known mathematically as the inverse problem. Simply stated, it is necessary to determine part of an integrand of an integral from the numerical value of the integral. In general, there is no mathematical method for uniquely determining the solution to this problem. The solution to this problem in the case of spreading resistance analysis has tested the ingenuity of numerical analysis.

Over the past two decades, there have been a number of advances which have taken place in the theoretical analysis of two- and four-probe resistance. The starting point of this history is the multilayer Laplace equation analysis of Schumann and Gardner [7,8]. The reader is referred to the book by Koefoed [9] for a comprehensive discussion of Laplace's equation, its solution in cylindrical coordinates, and the multilayer approach applied to geoelectric resistance measurements. The derivation of the recursion relation (discussed later in this report) is also presented in considerable detail by Koefoed. It should be noted that the mathematical description of spreading resistance and geoelectric resistance are exactly the same even though the length scales are vastly different.

The multilayer solution of the Laplace equation provides for the calculation of the resistance on the top surface of a nonuniform resistivity structure. The structure is viewed as being planar and made up of a number of "layers," each of uniform resistivity. For the case of two-probe measurements made on a beveled sample, the results of the planar calculations are used under the assumption that the bevel does not alter the results. As the probes move down the bevel, the definition of the surface changes to that which is presently in contact with the probes. The material which is uphill, so to speak, is neglected. The fundamental assumption of the multilayer analysis is that Laplace's equation is satisfied in each of the "layers" in the material.

There are a number of assumptions which are clearly spelled out and discussed in [7]. It is important to emphasize that Schumann and Gardner were very careful to point out what they thought to be limitations of the model.

First, consider the case of a single uniform layer. The solution to this problem provides a solution which is assumed to describe the potential in each of the "layers." The problem is set up in cylindrical coordinates to emulate the symmetry of a single circular contact on the top surface of the material. The potential is assumed to be independent of the angular variable in this coordinate system. The Laplace equation may then be written as

$$\nabla^2 V(r,z) = \frac{\partial^2}{\partial r^2} V(r,z) + \frac{1}{r}\frac{\partial}{\partial r} V(r,z) + \frac{\partial^2}{\partial z^2} V(r,z) = 0, \tag{1}$$

where $V(r,z)$ is the potential, r is the radial coordinate, and z is the depth coordinate. This equation may be solved by means of separation of variables, with the result that a

particular solution is

$$V(r,z) = \exp(-\lambda z)J_0(\lambda r) + \exp(+\lambda z)J_0(\lambda r), \tag{2}$$

where $J_0(\lambda r)$ is the Bessel function and λ^2 is the separation of variables constant. The boundary condition on the r part of the solution is that $V(r,z)$ approaches zero as r tends to infinity. This is satisfied by the above for all values of λ. Then, the general solution is an integral of the particular solution with a weighting factor and is of the form

$$V(r,z) = \int_0^\infty \Big\{ \big(1+\theta(\lambda)\big)\exp(-\lambda z) + \psi(\lambda)\exp(+\lambda z) \Big\} J_0(\lambda r) d\lambda, \tag{3}$$

where the weighting functions, $\theta(\lambda)$ and $\psi(\lambda)$, are determined from the z-dependent boundary conditions. The above is the general solution of Laplace's equation in cylindrical coordinates for a single layer.

For the case of an n-layer structure (pictured in fig. 1), Laplace's equation is assumed to be valid in each of the layers. The single-layer solution as given by eq (3) then provides the basis for the solution in each of the layers. The solution in the i-th layer may then be written as

$$V_i(r,z) = \int_0^\infty \Big\{ \big(1+\theta_i(\lambda)\big)\exp(-\lambda z) + \psi_i(\lambda)\exp(+\lambda z) \Big\} J_0(\lambda r) d\lambda. \tag{4}$$

The boundary conditions used to solve the system of equations (determine $\{\theta_i(\lambda)\}$ and $\{\psi_i(\lambda)\}$, $i = 1,...,n$) are provided by conditions on the top surface, the intermediate interfaces, and the bottom surface. On the top surface, current flow takes place only through the probe which is modeled as a circular plate of radius a. Then the top surface boundary condition is expressed as

$$-\frac{1}{\rho_n}\frac{\partial V_n(r,z)}{\partial z} = J(r), \tag{5}$$

where $J(r)$ is the current density. The original Schumann and Gardner calculation makes use of the current density for the case of a semi-infinite slab in the form

$$J(r) = \begin{cases} I/2\pi a(a^2 - r^2)^{1/2} & \text{if } r \le a \text{ and } z = 0 \\ 0 & \text{if } r > a \text{ and } z = 0 \end{cases}. \tag{6}$$

Several other forms of the current density can be used. In particular, the uniform current density in the form

$$J(r) = \begin{cases} I/\pi a^2 & \text{if } r \le a \text{ and } z = 0 \\ 0 & \text{if } r > a \text{ and } z = 0 \end{cases}, \tag{7}$$

would describe the current flow for the case of a layer over a highly conductive layer. On the other hand, the ring delta function form of the current density,

$$J(r) = (I/2\pi r)\delta(r - a), \tag{8}$$

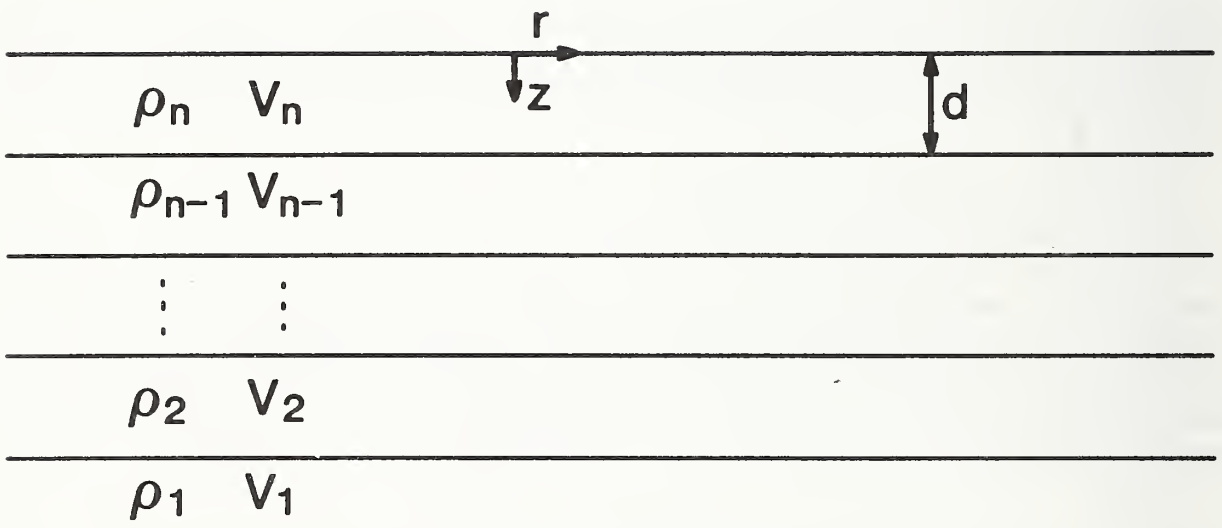

Figure 1. Schematic representation of the multilayer geometry used in the Schumann and Gardner multilayer analysis. In this figure, r and z are the radial and depth coordinate in the cylindrical coordinate system. d is the thickness of the layers and ρ_i and V_i are the resistivity and potential in the i-th layer.

where $\delta(r-a)$ is the Dirac delta function, would describe the current flow for a conducting layer over a highly resistive layer. These three forms of the probe current density are pictured in figure 2.

Berkowitz and Lux [10] have discussed how the above forms of the probe current density enter into the final correction factor equations. It should be noted that the above current densities have different patterns but all have the same total current, i.e.,

$$\int_0^{2\pi}\int_0^{\infty} J(r)d\theta r dr = I. \tag{9}$$

It might be argued that the "real" current density would be some linear combination of those given by eqs (6 through 8). This is the basis of the variational calculation which is discussed later in this section. For the present time, the probe current density given by eq (6) is used, and the changes introduced by the use of other current densities are presented.

Returning to the boundary conditions, on the bottom surface, the potential is assumed to be well behaved and, more specifically, is assumed to approach zero, i.e.,

$$\lim_{z\to\infty} V_1(r,z) = 0. \tag{10}$$

At the interfaces between the layers, the potentials and the current densities are assumed to be continuous. These are expressed as

$$V_i(r,z) = V_{i-1}(r,z), \tag{11}$$

and

$$\frac{1}{\rho_i}\frac{\partial V_i(r,z)}{\partial z} = \frac{1}{\rho_{i-1}}\frac{\partial V_{i-1}(r,z)}{\partial z}, \tag{12}$$

where the functions and their derivatives are to be evaluated at the interfacial boundaries.

For the case of an n-layer structure, the substitution of eq (4) into the boundary conditions given by eqs (5), (10), (11), and (12) gives rise to a set of $2n$ equations in $2n$ unknowns ($\{\theta_i(\lambda)\}$, $\{\psi_i(\lambda)\}$, $i = 1,...,n$). This simplifies by one equation as $\theta_n(\lambda) = \psi_n(\lambda)$. The analytic solution of this system of equations requires the use of the Cramer's rule of linear algebra. Clearly, this can become rather tedious especially since the expansion coefficients are functions of the continuous variable, λ. Schumann and Gardner worked out the system of equations for the cases up to $i = 3$. However, it is possible to show that the potential on the top surface of the n-layer structure may be written as

$$V_n(r,0) = \frac{I\rho_n}{2\pi a}\int_0^{\infty} \frac{A_n(\lambda)\sin(\lambda a)J_0(\lambda r)}{\lambda} d\lambda, \tag{13}$$

where the kernel function, $A_n(\lambda) = 1 + 2\theta_n(\lambda)$, depends upon the resistivities and thicknesses of the "layers" in the multilayer structure (through the solution of the above system

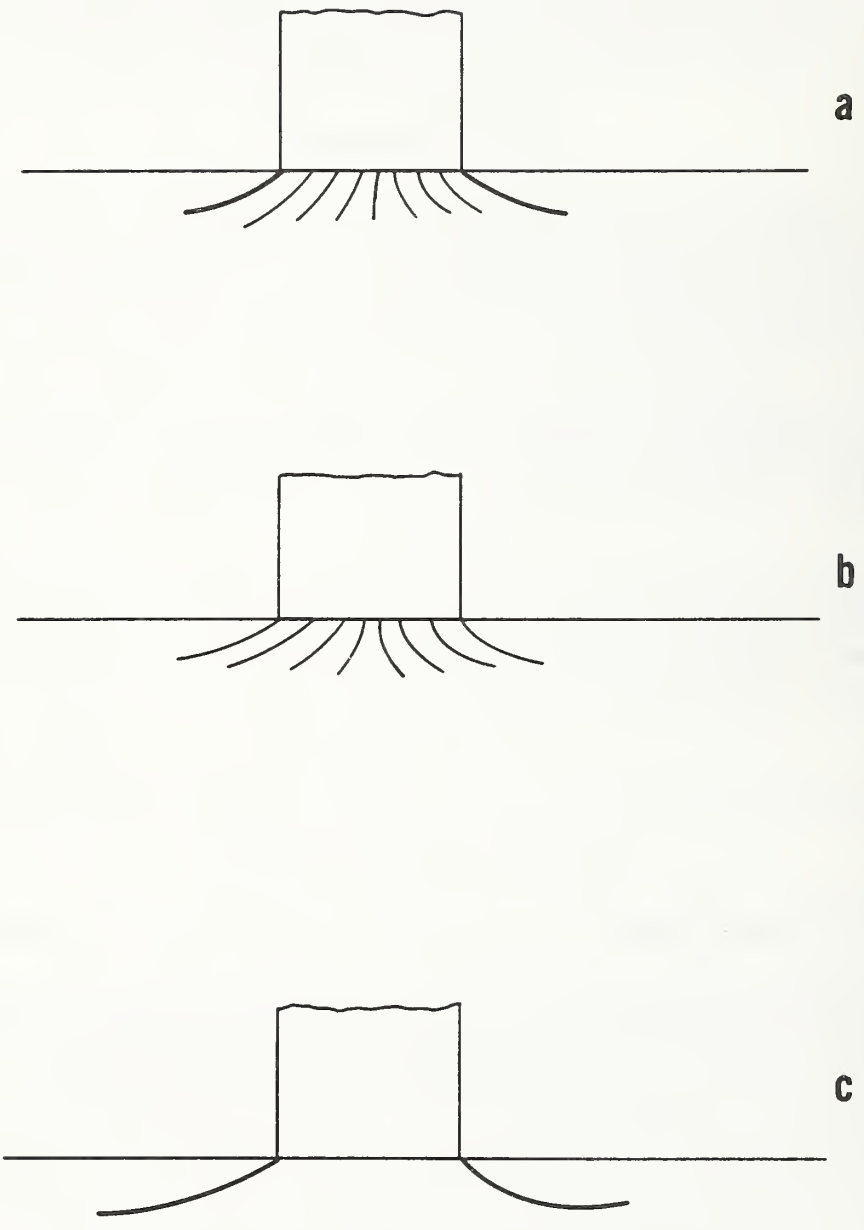

Figure 2. Graphical representation of the three forms of the probe current density. The Schumann and Gardner density is represented in (a), the uniform density in (b) and the ring delta function density in (c).

of simultaneous equations). It is this potential which is of prime importance as the measurement is made on the top surface of the material. Equation (13) represents the potential at a distance r from a single probe. The spreading resistance for a two-probe configuration is defined as the voltage difference between the probes divided by the current, $\Delta V/I$. In order to obtain an expression for the spreading resistance, two things must be done. First, the potential must be averaged over the area of the probe since the probe is not sensitive to the details of the potential, but rather the average potential. Second, the voltage difference between the two probes is calculated assuming that the second probe does not perturb the potential due to the first probe. This is known as superposition and leads to the result that the two-probe spreading resistance on the top surface of an n-layer structure may be written as

$$R_n = \frac{\Delta V}{I} = \frac{\rho_n}{2a} C_n, \tag{14}$$

where C_n is the correction factor given by

$$C_n = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda}, \tag{15}$$

where S is the separation between the probes. The Bessel function term, $J_1(\lambda a)$, arises from an averaging of the potential over the area of the probe.

There are three distinct terms in the integrand of the correction factor integral, with each representing different parts of the problem. First, there is the kernel function, $A_n(\lambda)$, which depends upon the resistivities and thicknesses of the layers in the multilayer structure. The second part of the integral, $(J_1(\lambda a)/\lambda a - J_0(\lambda S)/2)$ relates to the two-probe configuration with the probes separated by a distance S. Finally, the term $\sin(\lambda a)/\lambda$ arises from the probe current density (from eq (6)) passing through the probes. In terms of the uniform current density (eq (7)) and the ring delta function current density (eq (8)), the corresponding correction factors would be given by

$$C_n^{un} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} 2J_1(\lambda a) \frac{d\lambda}{\lambda}, \tag{16}$$

and

$$C_n^{rd} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} a J_0(\lambda a) d\lambda, \tag{17}$$

respectively. These forms of the correction factor integral have been discussed by Berkowitz and Lux [10].

Within the confines of the assumptions presented by Schumann and Gardner, eqs (14) and (15) represent a relation between the spreading resistance and the resistivity. This relation is not a simple one as the determination of the resistivity from the spreading resistance, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, requires the inversion of the correction factor integral. For the sake of simplicity and brevity, the notation $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ is used to indicate the calculation of the n spreading resistance values from the n values of the resistivity,

while the notation ($\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n$) is used to indicate the inverse process, i.e., the determination of the n values of the resistivity from the n spreading resistance values. The former requires single evaluations of the correction factor integral, whereas the latter require inversions of the correction factor integral. There were, in addition, two difficulties associated with the implementation of these equations. First, the determination of the kernel function for a large number of layers required the use of a mainframe computer [11] to carry out the numerical matrix algebra (determination of the set of functions $\{\theta_i(\lambda)\}$, $\{\psi_i(\lambda)\}$, $i = 1, ..., n$ by numerical evaluation of Cramer's rule). Power-law interpolation [12] techniques were of limited utility due to the large requirement of CPU (still on a mainframe). Also, the partitioning of the ($2n$ by $2n$) matrix problem into a smaller set of (2 by 2) matrices had been presented but has not been carried beyond this point [13]. The second problem in the implementation of the correction factor equation has to do with the evaluation of the integral. The last two parts of the integral described above are oscillatory, and a direct evaluation of the integral requires the use of at least a minicomputer.

The first breakthrough in the evaluation of eq (15) effectively removed the numerical difficulty associated with matrix inversion. This was hinted at in the geophysical literature [14] and worked out in detail using the theory of determinants in Koefoed's book [9]. The application of these techniques to the spreading resistance problem culminated with the introduction of a recursion relation by Choo *et al.* [15]. The utility of a recursion relation is that the kernel for an n-layer structure can be easily generated from the kernel of an $(n-1)$ layer structure by means of an algebraic relation. In particular, if the kernel is known for the $(n-1)$ layer case, then the kernel for the n layer case is given by

$$A_n(\lambda) = \frac{\omega(\lambda)\rho_n + \rho_{n-1}A_{n-1}(\lambda)}{\rho_n + \omega(\lambda)\rho_{n-1}A_{n-1}(\lambda)}, \tag{18}$$

where

$$\omega(\lambda) = \frac{1 - \exp(-2\lambda d)}{1 + \exp(-2\lambda d)}, \tag{19}$$

and d is the layer thickness. In practice, the recursion relation is begun with the one-layer case from which the two-layer kernel is generated. This sequence is repeated until the n-layer kernel is determined. This method was used in reference [15] with a great reduction in computation time.

At this particular time, the one major problem which remained was the evaluation of the integral in the correction factor equation. D'Avanzo *et al.* [16] made use of a prestored partial integral technique which usually required the use of a minicomputer in its implementation. The focal point of this technique was to use a cubic spline approximation for the kernel of the correction factor integral, i.e.,

$$A(\lambda) = \alpha_i\lambda^3 + \beta_i\lambda^2 + \gamma_i\lambda + A(\lambda_i). \tag{20}$$

Then the correction factor integral with the Schumann and Gardner infinite slab current density is given by

$$C = \frac{4}{\pi}\sum_{i=1}^{m-1}\left\{\alpha_i\int_{\lambda_i}^{\lambda_{i+1}}\lambda^3 f(\lambda)d\lambda + \beta_i\int_{\lambda_i}^{\lambda_{i+1}}\lambda^2 f(\lambda)d\lambda\right.$$

$$+\gamma_i \int_{\lambda_i}^{\lambda_{i+1}} \lambda f(\lambda) d\lambda + \delta_i \int_{\lambda_i}^{\lambda_{i+1}} f(\lambda) d\lambda \Big\}, \tag{21}$$

where the upper integration limit, λ_m, is chosen to ensure the convergence of the integrals, and the function, $f(\lambda)$, is the remaining part of the correction factor integral, i.e.,

$$f(\lambda) = \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \frac{\sin(\lambda a)}{\lambda}. \tag{22}$$

In practice, the integrand of the correction factor goes to zero when λ is at most on the order of 100. Then, the $\{\lambda_i\}$ values are chosen as logarithmically equispaced on the interval, $0 \leq \lambda \leq 100$. The integrals in eq (21) are evaluated for specific choices of the probe radius and probe spacing by means of a program called SPINT, and the results of the partial integrals are stored in array form. These arrays need be calculated only once for each value of a and S. The evaluation of the correction factor integral and the extraction of the resistivity from the spreading resistance required the evaluation of the set of 280 (4 by 70) spline coefficients $(\{\alpha_i\}, \{\beta_i\}, \{\gamma_i\}, \{\delta_i\})$. This is carried out in the program called SPRED. Both of these programs are described and listed in reference [16]. They require the use of a minicomputer to carry out the calculation.

At about the same time, Choo *et al.* [17] introduced an integration scheme based upon the Gauss-Laguerre quadrature. They use the uniform current density form of the correction factor integral as given by eq (16). The general philosophy behind quadrature techniques of evaluating integrals is the search for a set of integration points and weights, $\{x_k\}, \{w_k\}, (k = 1, ..., N)$ such that an integral may be evaluated as a finite sum in the form

$$\int_a^b f(x) dx = \sum_{k=1}^{N} w_k f(x_k). \tag{23}$$

It should be clear that knowing the set of integration points and weights vastly simplifies the evaluation of the integral. This simplification may render the integration amenable to evaluation on a microcomputer system. For the case at hand, the Gauss-Laguerre quadrature makes use of the first 33 roots and weights obtained from the 128th order Laguerre polynomial. For the case of the uniform current density, the correction factor may be evaluated as the weighted finite sum,

$$C_n = \frac{8}{\pi} \sum_{k=1}^{33} w_k^{gl} \left\{ \frac{J_1(\lambda_k a)}{\lambda_k a} - \frac{J_0(\lambda_k S)}{2} \right\} \frac{J_1(\lambda_k a)}{\lambda_k} A_n(\lambda_k), \tag{24}$$

where the weights (w_k^{gl}) and integration points $(a\lambda_k)$ are listed in reference [17].

While the above is formulated for the case of the uniform current density, the Schumann and Gardner current density as well as the ring delta function current densities may also be used. It is important to note that the Gauss-Laguerre technique can be used to carry out the calculation $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$. The inverse calculation, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, requires a convergence or bounding criterion.

From a completely different point of view, Dickey [18] presented a technique known as the local slope method for the calculation $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$. This method does not require the evaluation of the correction factor integral but rather makes use of a heuristically derived algebraic relation between the correction factor and the local depth derivative of the spreading resistance.

Starting with the trapezoidal Rhomberg integration technique employed by D'Avanzo *et al.* [16], Albers [19] generated model spreading resistance data, $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$, for a number of resistivity structures. In these calculations, the Schumann and Gardner current density was used with given values of the probe radius and the probe spacing. This technique may be used to carry out this calculation for all three forms of the probe current density. These model spreading resistance data were then used in the D'Avanzo *et al.* SPINT-SPRED codes as well as the Dickey local slope method to determine the $(\{Ri\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ results for each algorithm. It was found that the SPINT-SPRED codes solved the inversion problem to within 1 percent. The local slope method gave very reasonable semiquantitative results. These are discussed in detail in reference [19]. While it has been shown that the local slope equations cannot be derived from the multilayer equations [20], the speed and quality of the results make it a reasonable program to use.

A typical resistivity structure and the model spreading resistance data generated according to the work of reference [19] are contained in figure 3. There are several things to note from the figure. First, the spreading resistance does arise from a simple scaling of the resistivity. The mapping from the resistivity to the spreading resistance is complex. For example, from the surface down to about 0.2 μm, the resistivity undergoes about a two-order-of-magnitude decrease, whereas the spreading resistance shows only a small decrease. This arises from the fact that the spreading resistance senses the entire resistivity structure. The second point of interest is that the total variation of the resistivity is often large compared with the corresponding variation of the spreading resistance. In practice, small changes in the spreading resistance mirror large changes in the resistivity. This makes the interpretation of spreading resistance particularly sensitive to the presence of noise in the data.

Berkowitz and Lux [21] have carried out an investigation which has shown that it is possible to evaluate the spreading resistance correction factor using a 22-point scheme. The central point of this analysis is the replacement of the correction factor integral by an approximate form as

$$C_n = \frac{8}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} J_1(\lambda a) \frac{d\lambda}{\lambda}$$

$$= \frac{8}{\pi} \int_L^\infty A_n(\lambda) \left(\frac{J_1(\lambda a)}{\lambda a} \right)^2 d\lambda, \tag{25}$$

where $L = 1.12292/S$. The choice of the truncated lower limit of integration is dictated by the form of the functions for the case of both insulating and conducting boundaries. The key point is that the approximate integral can be divided into four domains in each of which the Newton-Cotes integration method may be used. The evaluation of the approximate

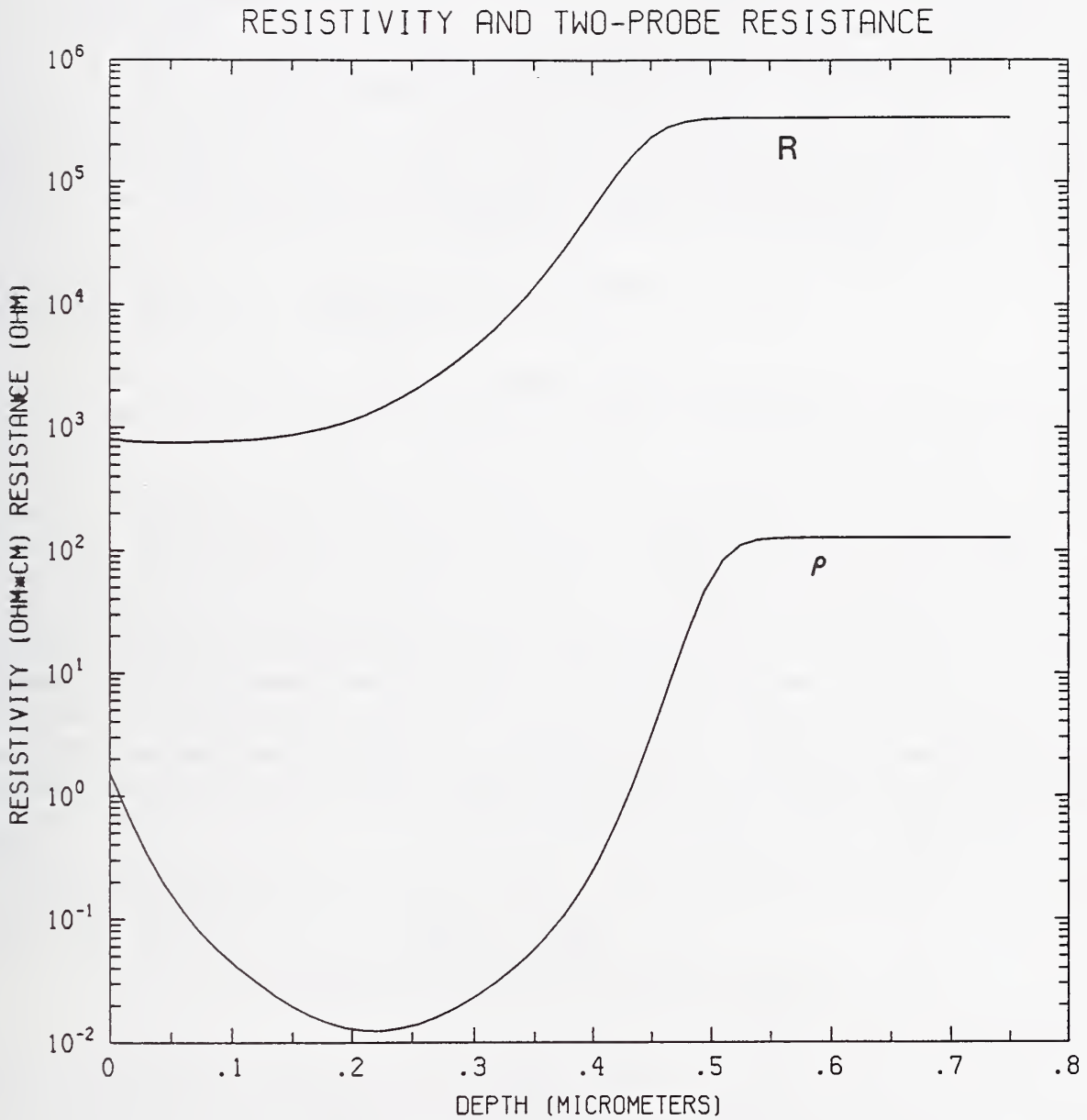

Figure 3. The resistivity profile and the calculated spreading resistance for a Gaussian implant type structure. This figure is intended to illustrate two important features. First, the variation of the resistivity is always larger than that of the spreading resistance. Second, the mapping from resistivity to spreading resistance is more than just a scaling.

integral in eq (25) is then performed by means of a finite sum as

$$C_n = \sum_{k=1}^{22} w_k^{bl} A_n(\lambda_k), \tag{26}$$

where the weights, w_k^{bl}, are different from those used in the Gauss-Laguerre technique. Comparison of the correction factor calculated by means of eq (26) and the more extensive technique discussed by Albers [19] shows that the two agree to within better than 1 percent. Equation (26) provides a quick way of carrying out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. Berkowitz and Lux have used the above Newton-Cotes technique to carry out the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation. This makes use of a bounding procedure on the kernel of the integral. This bounding procedure reduces the time required to accurately invert the correction factor integral and is based upon the realization that the recursion relation for the kernel function could be expressed as

$$A_n(\lambda) = \frac{\tanh(\lambda d)\rho_n + \rho_{n-1} A_{n-1}(\lambda)}{\rho_n + \rho_{n-1} A_{n-1}(\lambda) \tanh(\lambda d)}. \tag{27}$$

For the cases of insulating or conducting boundaries, the kernel function is bounded by the hyperbolic functions, coth and tanh, respectively. This provides for two bounding envelopes between which the kernel must lie. Calculation of $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ using eq (26) followed by the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation using eq (26) in conjunction with the bounding procedure shows that the original resistivities can be obtained to within less than 1 percent. This bounding procedure is extremely general. It may be used in conjunction with the Gauss-Laguerre technique for the $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation.

All of the above techniques make use of a single form of the probe current density. Choo *et al.* [22] have presented a calculation of the spreading resistance correction factor based on a variational technique. This makes use of the uniform current density and the Schumann and Gardner semi-infinite slab current densities as a "basis set," and the amount of each current is varied according to a variational principle. The spreading resistance obtained from this variational method is

$$R_n^v = \frac{\rho_n}{2} C_n^v, \tag{28}$$

where

$$C_n^v = \frac{4}{\pi} \frac{K_1 I_{3s} + K_2 I_{2s}}{K_1 + K_2/2}, \tag{29}$$

where K_1 and K_2 are given by

$$K_1 = \frac{I_2 - I_3/2}{I_1 I_2 - I_3^2}, \tag{30}$$

$$K_2 = \frac{I_1/2 - I_3}{I_1 I_2 - I_3^2}. \tag{31}$$

The integrals I_1, I_2, I_3 are

$$I_1 = \int_0^\infty A_n(\lambda) \left\{ \frac{\sin(\lambda a)}{\lambda a} \right\}^2 a d\lambda, \tag{32}$$

$$I_2 = \int_0^\infty A_n(\lambda)\left\{\frac{J_1(\lambda a)}{\lambda a}\right\}^2 a d\lambda, \tag{33}$$

$$I_3 = \int_0^\infty A_n(\lambda)\left\{\frac{\sin(\lambda a)J_1(\lambda a)}{(\lambda a)^2}\right\} a d\lambda. \tag{34}$$

The integrals I_{3S} and I_{2S} are

$$I_{3S} = \int_0^\infty A_n(\lambda)\left(\frac{\sin(\lambda a)}{\lambda a}\right)\left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda, \tag{35}$$

and

$$I_{2S} = \int_0^\infty A_n(\lambda)\left(\frac{J_1(\lambda a)}{\lambda a}\right)\left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right) d\lambda. \tag{36}$$

Choo *et al.* [23] have made use of the Berkowitz-Lux approximation [21] to simplify the integrals in eqs (35) and (36) as

$$I_{3S} = \int_L^\infty A_n(\lambda)\left(\frac{\sin(\lambda a)}{\lambda a}\right)\left(\frac{J_1(\lambda a)}{\lambda a}\right) d\lambda, \tag{37}$$

$$I_{2S} = \int_L^\infty A_n(\lambda)\left(\frac{J_1(\lambda a)}{\lambda a}\right)^2 d\lambda. \tag{38}$$

It should be noted that the integrals given by eqs (32), (33), and (34) contain all of the information which is needed to evaluate eqs (28) and (29). This follows from the fact that the approximate integrals given by eqs (37) and (38) are contained in the corresponding I_3 and I_2 integrals. The various integrals (I_1, I_2, and I_3) may be evaluated by means of a segmented Newton-Cotes method similar to that used by Berkowitz and Lux. The coordinates and weights are listed in reference [23]. These equations can be implemented in order to carry out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. The inverse procedure, $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$, can be carried out by means of a generalization of the Berkowitz-Lux bounding technique where the bounding by the hyperbolic functions is needed for all three of the integrals involved. This analysis shows that the original resistivities can be recovered to within 1 percent.

EFFECTS OF PROBE CURRENT DENSITY

As can be seen from the above discussion, there are three primary current densities which can be used in the calculation of the spreading resistance correction factor. The Schumann and Gardner form seems appropriate to the case of a semi-infinite slab and the uniform current density should be adequate for the case of a layer over a highly conductive layer, while the ring delta function seems appropriate for the case of a layer over a highly resistive layer. The model data calculations presented by Albers [19] (using the trapezoidal Romberg integration scheme) can easily be generalized in order to use any one of the three current densities. The Gauss-Laguerre quadrature [17] may also be used with any one of the above current densities. The Berkowitz-Lux calculations [21] (originally specialized to the

uniform current density but, in principle, extendable to the other two current densities) as well as the variational technique employing the Berkowitz-Lux approximate integral [23] can also be used to calculate the spreading resistance from the resistivity. For several structures, the various techniques were used to carry out the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation. The calculated spreading resistance data obtained from typical diffusion and implant structures show that the differences are typically on the order of 10 to 20 percent. This is in keeping with the results discussed in reference [10].

CALIBRATION DATA AND THE PROBE RADIUS

Having looked at the effects of the probe current density on the results of spreading resistance calculations, it is important to understand that there is a parameter in eqs (14) and (15) which must be measured or at least estimated. This is the probe radius, a. The results of the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation and the inverse $(\{R_i\} \rightarrow \{\rho_i\}, i = 1, ..., n)$ calculation are sensitive to the choice of a. One possible way of measuring the probe radius was originally proposed by Dickey [18] as part of a conjecture relating spreading resistance and sheet resistance. In particular, an image calculation of the spreading resistance on the top of an isolated layer of thickness, t, and resistivity, ρ, yields the result

$$R = (\rho/\pi t)\ln(S/a) = (\rho/\pi t)\{\ln(S) - \ln(a)\}. \tag{39}$$

As the sheet resistance, $\mathcal{R}$, of this layer is given by $\mathcal{R} = \rho/t$, this would suggest that the spreading resistance may be linear in the logarithm of the probe spacing with a slope related to the sheet resistance and an intercept related to the probe radius. For junction-isolated boron diffusions with junction depths up to 4.3 μm, Dickey showed that there was good agreement between the sheet resistance measured by the standard four-probe technique and determined from the R versus $\ln(S)$ analysis. This agreement provided justification for the proposed relation between the spreading resistance and the sheet resistance but did not give any indication of the validity of the relation of the intercept of the R versus $\ln(S)$ analysis with the probe radius. The values obtained by Dickey were certainly of the expected size.

In a series of investigations to study the validity of the proposed relation between the spreading resistance and the sheet resistance (as well as the possibility of determining the probe radius), Albers [24,25] used the $(\{\rho_i\} \rightarrow \{R_i\}, i = 1, ..., n)$ calculation for a number of resistivity structures, probe current densities, a fixed value of the probe radius, and a series of values of the probe separation. The resistivity structures could be used to calculate the sheet resistance directly, and the probe radius was a known input quantity. The model spreading resistance data were used in linear regression analysis in $\ln(S)$ according to eq (39) and the slope and intercept correlated with the sheet resistance and the probe radius. The regression analysis of the model spreading resistance data indicated that the relation between the spreading resistance and the sheet resistance was of general validity, but that the probe radius was not generally obtainable from the regression analysis. Hence, some other technique seemed to be required to determine this quantity.

One possibility was to adopt the point of view that the probe radius is a function of the resistivity. This is hinted at by the form of the calibration data obtained on various

resistivity bulk samples. This point of view was incorporated into the Dickey local slope analysis of boron implant data used in the comparison of spreading resistance, SIMS, and CV profiles by Ehrstein *et al.* [26]. The local slope analysis is ideally suited to this as the updating of the probe radius takes place in the algebraic equation relating the correction factor and the local slope. All of the other algorithms and integration schemes would be hampered by the updating of the probe radius which would necessitate the reevaluation of the sampling points and the integration weights.

Piessens *et al.* [27] have discussed a technique based upon Chebyshev polynomial methods for the evaluation of the Schumann and Gardner equations in the case of a resistivity-dependent probe radius. This technique requires a minicomputer system in the evaluation of the resistivity from the spreading resistance. The basis of the assumption of a resistivity-dependent probe radius is the level of agreement obtained by Ehrstein *et al.* [26] in comparing profiles obtained from spreading resistance, SIMS, and CV. As is seen later in this part of the report, there are a number of reasons why carrier and atomic profiles do not have to agree. Indeed, the use of a resistivity-dependent probe radius appears to allow for one more degree of freedom for every data point taken.

The point of view adopted by Albers and Berkowitz [28,29] is that the probe radius is a single parameter which can best be evaluated by optimizing the agreement between the four-probe resistance obtained from the resistivity profile and measured on the top surface of the structure. The multilayer expression for the in-line four-probe resistance, $Z_n(S)$, has been derived in the form [28]

$$Z_n(S) = 2\rho_n \int_0^\infty A_n(\lambda)\Big\{J_0(\lambda S) - J_0(2\lambda S)\Big\}\mathcal{I}_\nu(\lambda a)d\lambda, \tag{40}$$

where $\mathcal{I}_\nu(\lambda a)$ is the Hankel transform of the generalized probe-current density normalized to unit current [10], and ν is an index which refers to the specific form of the current density. It was shown that the four-probe resistance could always be obtained from the spreading resistance

$$Z_n(S) = R_n(2S) - R_n(S). \tag{41}$$

The four-probe resistance was shown to be: (i) independent of the probe radius and the probe-current density, and (ii) simply related to the sheet resistance, $\mathcal{R}_n$, when the distance to an insulating boundary is small compared with the probe spacing. The former observation provided the basis for a calibration procedure for determining the value of the probe radius to be used in spreading resistance profile analysis. This is discussed in detail in reference [28]. The evaluation of the four-probe resistance integral was found to be simplified by the alternative form given by [29]

$$Z_n(S) = \frac{\rho_n}{\pi}\int_{1/2S}^{1/S} A_n(\lambda)d\lambda. \tag{42}$$

In the above form, the four-probe resistance can easily be calculated using the Newton-Cotes method of evaluation,

$$Z_n(S) = \frac{\rho_n}{\pi}\ln(2)\sum_{i=0}^{8} w_{9,i}\lambda_i A_n(\lambda_i), \tag{43}$$

where $\lambda_i = 2^{i/8}/2S(0 \leq i \leq 8), w_{9,i} = C_i/8$, and where C_i are the Newton-Cotes weighting factors. This procedure allows for the variation of the resistivity profile under the requirement that the measured and calculated four-probe resistances agree within given limits.

The general relation between the four-probe resistance and the two-probe resistance as given by eq (41) was derived from the general definitions of these functions. The simplified form for the four-probe resistance integral contained in eq (42) arose from a consideration of the integral contained in eq (40). These two equations have been verified experimentally [30] by means of data obtained from lithographically deposited contacts as well as conventional pressure contacts.

EFFECTS OF FINITE LAYER THICKNESS

The original Schumann and Gardner derivation assumed that the material may be viewed as a series of uniform resistivity layers. These authors realized the possible limitation of this assumption as they showed how the correction factor depended upon the number of layers used in the calculation. This same observation was studied in more detail by Choo *et al.* [15] by means of the recursion relation presented in eq (18). In order to offset some of the possible difficulties associated with the finite thickness of the "layers," Choo *et al.* [31] introduced an exponential model in which each of the layers was assumed to have an exponentially varying resistivity. This piecewise exponential was to be a more realistic representation of the continuously varying resistivity. The differences between the correction factor calculated by means of a finite layer picture and a continuum picture were also considered by Albers [32]. Beginning with the recursion relation, he showed how a nonlinear, inhomogeneous differential equation could be rigorously derived for the kernel of the spreading resistance correction factor integral in the limit as the layer thicknesses approach zero. This equation may be transformed to a Riccatti equation which can be solved analytically for an exponentially varying resistivity. When the correction factor calculated from this technique was compared with that obtained from the finite layer picture, it was shown that finite-layer analysis will underestimate the correction factor for the case of a low-resistivity layer over a high-resistivity layer. Over the past 10 years, the beveling techniques which have evolved have allowed for the preparation of very shallow bevels. These shallow bevels have pushed the measurements into the region where the finite layer thickness approximation is reasonably good.

ATOMIC DENSITIES AND CARRIER DENSITIES

In this section of the report, attention is focused on the calculation of carrier densities from atomic densities for planar and beveled structures. The purpose is to provide insight into the validity of the use of the multilayer Laplace equation which assumes charge neutrality in each of the "layers" in the material. In terms of the difference between carrier densities and atomic densities along the bevel cut into the material for spreading resistance profile analysis, Hu [33] has presented a calculation based upon the solution of Poisson's equation for a number of profile types into substrates of the opposite conductivity type,

i.e., junction-isolated structures. Using a finite difference solution method, he finds that electrical junctions may be much shallower than metallurgical junctions for structures of the Gaussian and exponential type. As the deviation between carrier densities and atomic densities has also been observed for same conductivity-type substrates, Albers *et al.* [34] have approached the problem from the point of view of the solution of the semiconductor equations using an adaptive finite-element method discussed by Blue and Wilson [35,36]. They find that the deviation of the carrier densities from the atomic densities occurs for both same- and opposite-conductivity type substrates.

Consider the structure presented in figure 4. This consists of an n-type region and a p-type region. A portion of the material is removed producing the beveled surface. The variables of interest to the problem are the electrostatic potential and the electron and hole densities which arise from the atomic donor and acceptor concentrations. From a computational point of view, it is more convenient to formulate the problem in terms of the electrostatic potential and the electron and hole quasi-Fermi levels which are related to the corresponding densities. The model which shall be used to describe the system is based on the basic semiconductor equations [37,38] (steady-state, time-independent):

$$\nabla\cdot(\kappa\,\nabla\,V) = -\frac{q}{\epsilon_0}(p - n + N), \tag{44}$$

$$\nabla\cdot(\mu_n\, n\,\nabla\,\phi_n) = R, \tag{45}$$

$$\nabla\cdot(\mu_p\, p\,\nabla\,\phi_p) = -R, \tag{46}$$

where κ is the dielectric constant, V is the electrostatic potential, q is the electronic charge, ϵ_0 is the permittivity of free space, N is the net ionized impurity density, $N = |N_D - N_A|$, N_D is the atomic donor density, N_A is the atomic acceptor density, μ_n and μ_p are the electron and hole mobilities, respectively, and ϕ_n and ϕ_p are the electron and hole quasi-Fermi levels, respectively. The Shockley-Read-Hall recombination is given by:

$$R = \frac{(pn - n_i^2)}{\tau_{n0}(n + n_1) + \tau_{p0}(p + p_1)}, \tag{47}$$

where n_i is the intrinsic carrier density. The electron density, n, and the hole density, p, are related to the electrostatic potential and the corresponding quasi-Fermi levels (for Boltzmann statistics) by:

$$n = n_i \exp(q(V - \phi_n)/kT), \tag{48}$$

$$p = n_i \exp(q(\phi_p - V)/kT). \tag{49}$$

The solutions of eqs (44), (45), and (46) are obtained by means of an adaptive finite-element technique. The reader is referred to references [35] and [36] for a more detailed discussion of the solution method and strategies.

For the purpose of investigating the validity of the use of the multilayer Laplace equation, figures 5 through 7 contain the results of the calculations for the cases of Gaussian diffusion type structures ranging in junction depths from 15 μm down to 0.25 μm. These

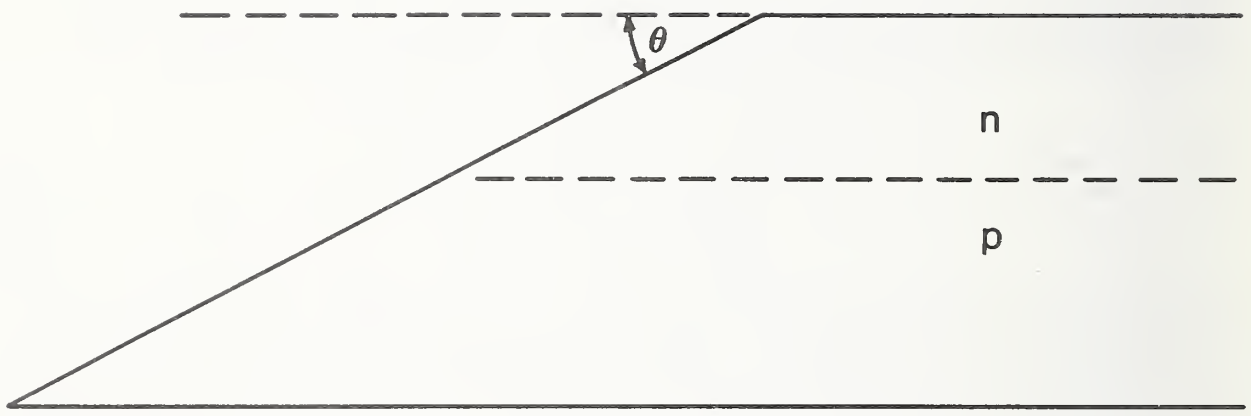

Figure 4. Geometry of the beveled material in which the semiconductor equations are solved by means of a finite element method.

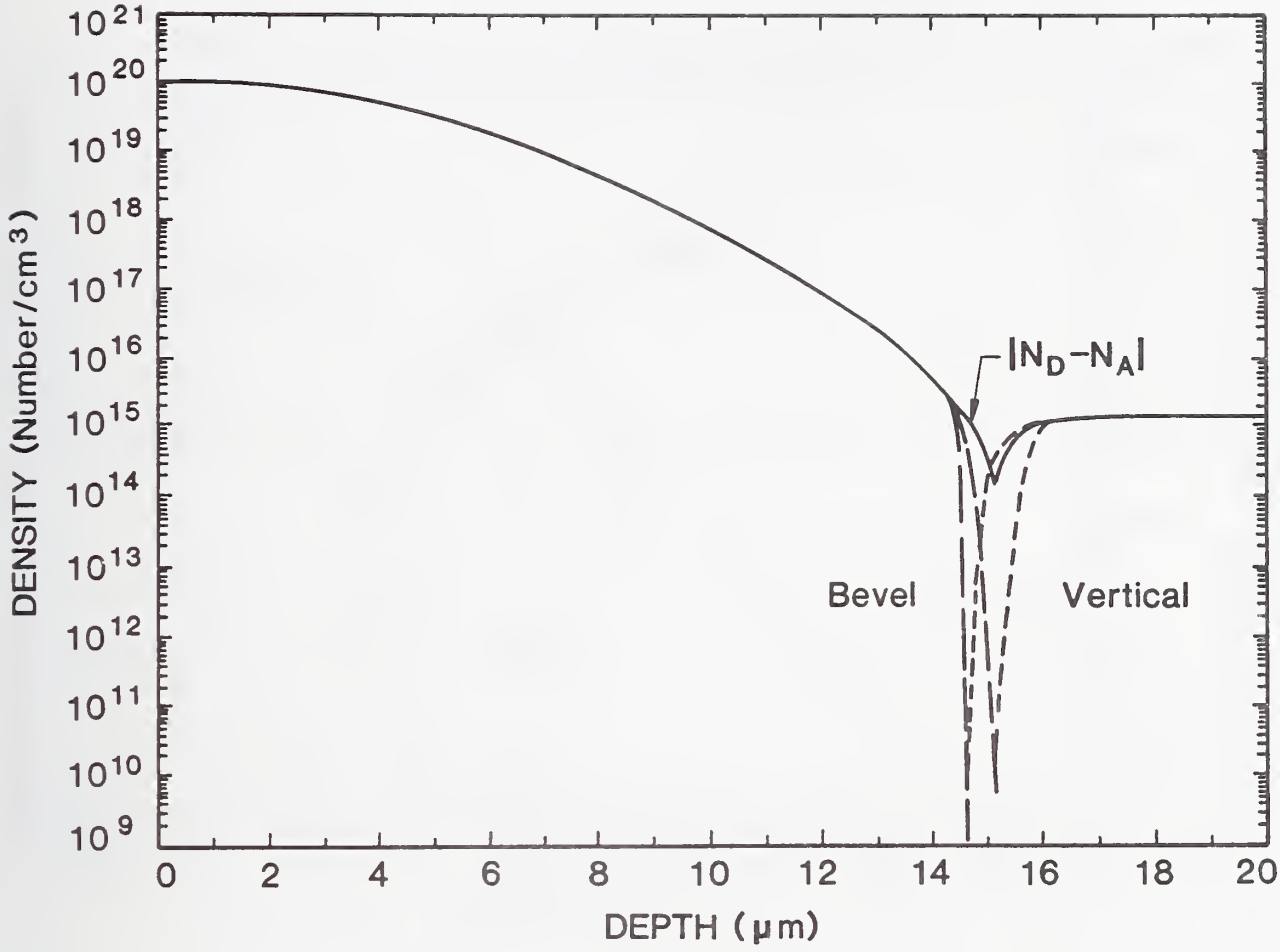

Figure 5. Carrier and atomic distributions for a 15-μm Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{20}\text{cm}^{-3}$, and the background concentration is $1.5 \times 10^{15}\text{cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.

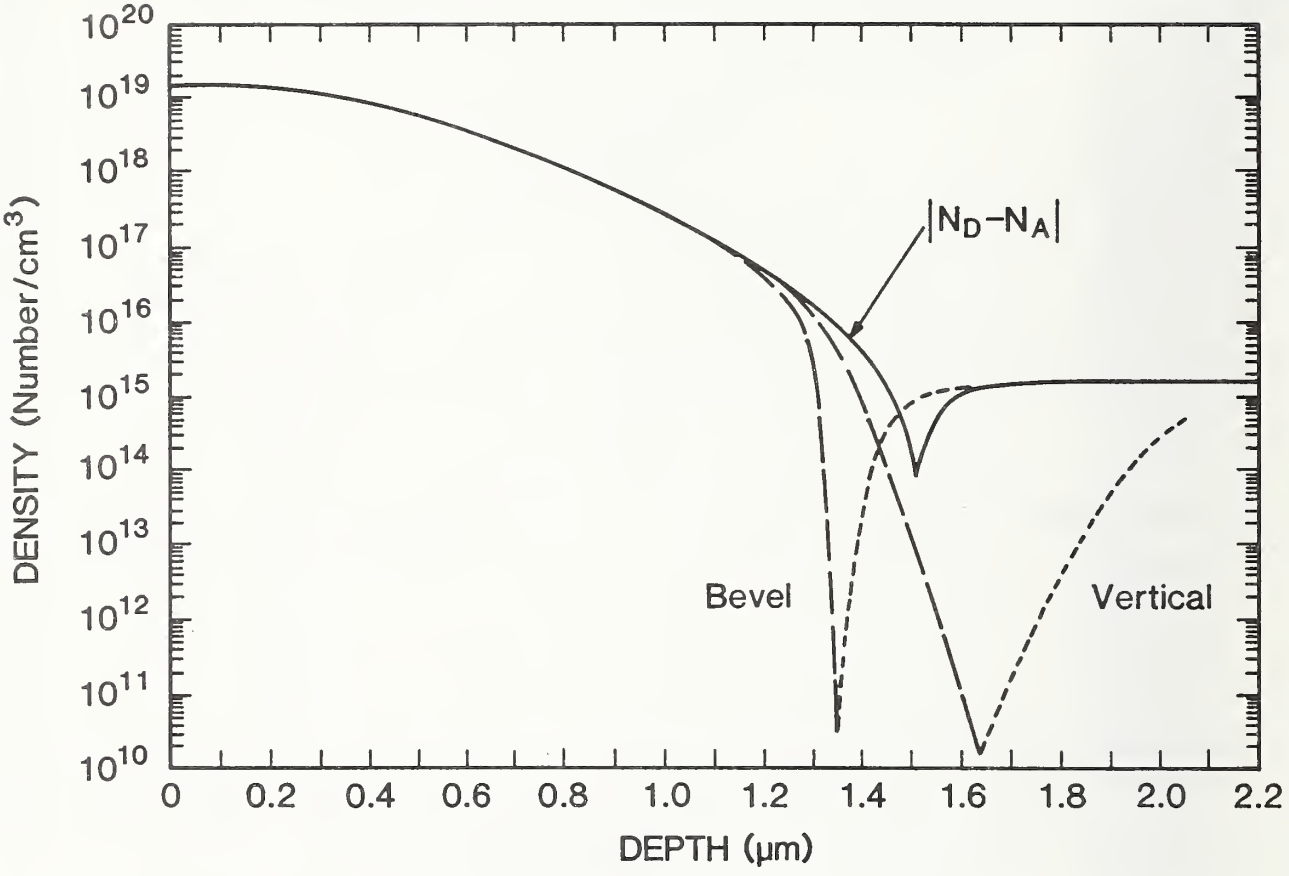

Figure 6. Carrier and atomic distributions for a 1.5-μm Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{19}\text{cm}^{-3}$, and the background concentration is $1.5 \times 10^{15}\text{cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.

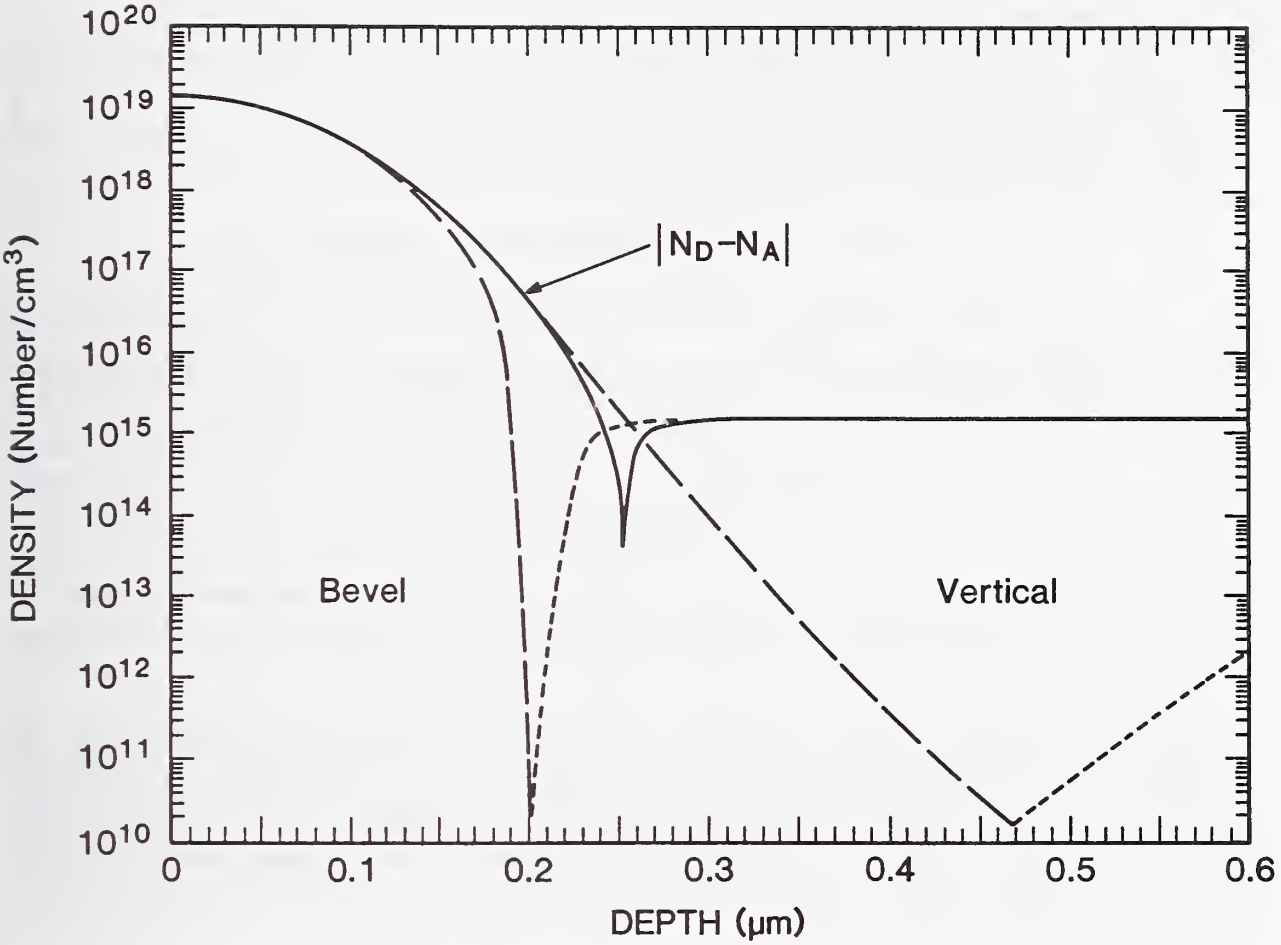

Figure 7. Carrier and atomic distributions for a 0.25-μm Gaussian structure with an opposite conductivity-type substrate. The surface concentration is $1.5 \times 10^{19}\text{cm}^{-3}$, and the background concentration is $1.5 \times 10^{15}\text{cm}^{-3}$. The net impurity concentration is denoted by the solid curve, the long dashed curve represents the electron density, and the short dashed curve represents the hole density.

figures contain the net ionized impurity (atomic) density, the electron and hole densities calculated for a planar material, and the electron and hole densities calculated along the bevel. Figures 5, 6, and 7 contain the results for opposite-conductivity type substrates for a 15-μm, 1.5-μm, and 0.25-μm Gaussian, respectively. In all three cases, the surface concentration is 1.5×10^{20} cm^{-3}, and the substrate concentration is 1.5×10^{15} cm^{-3}. These deviations represent regions where there is a nonzero charge. This charge separation is due to the repulsion of the mobile carriers in the restraining field due to the immobile ionized impurities. In the regions where there is no deviation, the net local charge is zero. It is extremely important to keep in mind that these represent a best-case scenario where damage, defects, incomplete activation, etc., have not come into play.

STATUS OF SPREADING RESISTANCE ANALYSIS

The calculation of the spreading resistance correction factor using the Schumann and Gardner multilayer Laplace equation has progressed to the point where the calculation of the spreading resistance from the resistivity and the inverse calculation can be performed on microcomputer systems. The interpretation of the probe radius can be done by using a resistivity-dependent probe radius model or by comparing the interpreted profiles in terms of the four-probe resistance. The FORTRAN codes presented in this report provide for the practical ability to perform these calculations. They represent the fruits of the labor of many researchers over the past twenty years. However, there is a question of the validity of the use of the multilayer Laplace equations for the case of shallow layers where the net local charge is nonzero. The user should always be aware of this.

Spreading resistance data can certainly be measured in submicrometer regions, but the interpretation of the data in terms of the multilayer analysis Laplace needs to be very carefully investigated. From a conceptual point of view, it might be argued that the measured data should be related to the physical description provided by the Poisson equation. However, Poisson's equation has not yielded to the simple mathematical structure which the Laplace equation has.

The resolution of this problem for the interpretation of spreading resistance data on shallow VLSI structures requires the investigation of the solution of the Laplace and Poisson equations. The solutions of these two equations should provide an estimate of the differences between the actual carrier distribution and that determined from the Schumann and Gardner analysis. The answer to this problem is actively being pursued by researchers all over the globe.

DISCUSSION OF THE PROGRAMS

The purpose of this portion of the report is to discuss some of the aspects of the various programs and to present examples of the results of the calculations performed by these programs. The reader should keep in mind that the programs listed in the appendices have extensive internal documentation in regard to the background information, input data, and the running logic of the programs. The reader who is familiar with FORTRAN should find

these programs easy to follow and use. This section is intended to supplement the internal documentation by presenting the information outside of the framework of the FORTRAN source code and to discuss typical input and output data.

There are three major subsections of this portion of the report. These are concerned with: 1) programs for calculating the two-probe resistance from the resistivity, 2) programs for calculating the resistivity from the two-probe resistance, and 3) programs for calculating the four-probe resistance from the resistivity. From the discussion of the history of the Schumann and Gardner solution of the multilayer Laplace equation, it should be clear that a conceptually and mathematically efficient representation of the kernel of the integral, $A_n(\lambda)$, is given by eqs (18) and (19) or (27). This same kernel is involved in all two-probe and four-probe integrals contained in this report. Hence, the discussion of the various programs primarily focuses upon various techniques for numerically evaluating the integrals involved or for carrying out efficient inversions of these integrals.

A special note is in order concerning the units used in the calculations. The resistances (two-probe and four-probe) are in ohms and the resistivities are in ohm·cm. All lengths (probe radius, probe spacing, and depths) are in micrometers.

It is also important to note the numerical format of the various output data files. In particular, the format used for the construction of the output data is of a general nature. It has been written to accommodate a wide range of numerical values without loss of generality or data. This was motivated by the fact that datum may be lost on output if it exceeds the format of the write statement. In order to circumvent this, some data may appear to have more significant figures than might be reasonable or needed. This will be invariably the case if the user decides to use the free-field write option of FORTRAN77. This circumvents the loss of data which is always possible when writing in a fixed-field format. The user should keep this in mind when looking at the output. The user should round the output following accepted rules for this purpose.

CALCULATION OF THE TWO-PROBE RESISTANCE FROM THE RESISTIVITY

The first group of programs is used to calculate the spreading resistance from the resistivity. Hence, the programs presented in this section represent various methods for evaluating the correction factor integral. In order to familiarize the reader with each of these four programs, they are discussed individually along with sample input and output data. The four programs included in this section are: RGENTR, RGENBL, RGENGL, and RGENVAR.

RGENTR

The RGENTR (Resistance GENeration Trapezoidal Romberg) program calculates the two-probe spreading resistance from eqs (14) to (17).

$$R_n = \frac{\Delta V}{I} = \frac{\rho_n}{2a} C_n,$$

where C_n is the correction factor for the Schumann and Gardner form of the current density given by

$$C_n = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda}.$$

In terms of the uniform current density and the ring delta function current density, the corresponding correction factors would be given by

$$C_n^{un} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} 2J_1(\lambda a) \frac{d\lambda}{\lambda},$$

and

$$C_n^{rd} = \frac{4}{\pi} \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} a J_0(\lambda a) d\lambda.$$

For the purposes of illustration, the discussion of the code will focus upon the use of the Schumann and Gardner current density. The same program and logic flow follow directly for the uniform current and the ring delta current densities.

The integral is approximated to be over a finite interval as

$$\int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda} \approx \int_{xl}^{xu} A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right\} \sin(\lambda a) \frac{d\lambda}{\lambda}.$$

The correction factor integral is calculated between the limits of xl and xu (both input data) by means of a trapezoidal Romberg integration technique. This particular integration scheme was used previously by D'Avanzo *et al.* [16] in order to calculate the array of partial integrals used in their SPINT-SPRED programs. It was also used by Albers [19] to generate model date for the comparison of correction factor algorithms.

RGENTR reads the input data from logical device 10 (for010) which can be referenced to a data file through an assign statement in the run sequence (see the examples in the tape files). These data must include the following:

npt	number of data points in the structure. Total number of points is num=npt+1 (include substrate point)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)
rho	array of resistivity values (Ω·cm)
icur	form of the current density (Schumann Gardner=1, Choo uniform=2, ring delta=3)
xl	lower limit of correction factor integral (usually zero)
xu	upper limit of correction factor integral

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The trapezoidal Romberg integration scheme is carried out at each depth increment until convergence is achieved to within 0.005 percent. This convergence criterion has been found to give good results with a nominal use of CPU time on a minicomputer system.

The parts of the program are as follows:

rgentr	the main program
intgrl(f,a,b,atemp,iflag)	the subroutine which carries out the integration of the function f from the limits of a to b until the predefined convergence is attained
tpint(x)	a double precision function which represents the integrand of the two-probe resistance integral and includes the kernel and the choice of a probe current density
alam(x)	a double precision function which constructs the kernel function according to the recursion relation
dbj0(x)	a double precision function which uses a polynomial approximation to generate the J_0 Bessel function
dbj1(x)	a double precision function which uses a polynomial approximation to generate the J_1 Bessel function

The input data is read from for010. The calculation is performed and the output data are written to for011 and for006. In the example presented here, rgentr.in contains the data to be read in for010, rgentr.out contains the data written to for011, and rgentr.log contains the data written to for006. The rgentr.log file also contains information as to the CPU time used on a VAX 11/785 system. Many workstations, such as SUN and SPARK2, are much faster. Many PCs, especially those operating with 386 or 486 CPU chips and with coprocessor chips, will probably run at about VAX speeds or faster.

Table 1. Annotated input data file (rgentr.in) for RGENTR program

```
50              NPT-number of points in structure. Total is NPT + 1
2.0             AA-probe radius in micrometers
50.0            SEP- probe separation in micrometers
0.015           DELTH-depth increment in micrometers
1               IBBC - back boundary condition (none=1, cond=2, ins=3)
126.5           SUBRHO-substrate resistivity in ohm*cm =RHO(1)
0.015           SUBTH-substrate thickness in micrometers
1.542           RHO(NPT+1) surface resistivity in ohm*cm
0.6933          RHO(NPT)
0.3442          RHO(NPT-1)
0.19                .
0.1159              .
0.0769              .
0.0543              .
0.0401              .
0.0307              .
0.0242              .
0.0196              .
0.0165              .
0.0144              .
0.0131              .
0.0125              .
0.0124              .
0.013               .
0.0141              .
0.0161              .
0.019               .
0.0233              .
0.0294              .
0.0382              .
0.0514              .
0.0721              .
0.1074              .
0.1732              .
0.3081              .
0.6095              .
1.334               .
3.18                .
8.036               .
20.4                .
46.62               .
82.75               .
109.7               .
121.4               .
125.1               .
126.1               .
126.4               .
126.4               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5           RHO(2)
2               ICUR-probe current density (S&G=1,unif=2,ring=3)
0.0             XL-lower limit of integral (usually zero)
10.0            XU-upper limit of integral (usually 20/AA)
```

Table 2. Annotated output data file (rgentr.out) for RGENTR program

```
        51                          total number of points calculated
  0.0000E+00   8.1209E+02           depth and spreading resistance
  1.5000E-02   7.7709E+02                        .
  3.0000E-02   7.6227E+02                        .
  4.5000E-02   7.5662E+02                        .
  6.0000E-02   7.5634E+02                        .
  7.5000E-02   7.6041E+02                        .
  9.0000E-02   7.6898E+02                        .
  1.0500E-01   7.8279E+02                        .
  1.2000E-01   8.0318E+02                        .
  1.3500E-01   8.3205E+02                        .
  1.5000E-01   8.7224E+02                        .
  1.6500E-01   9.2789E+02                        .
  1.8000E-01   1.0042E+03                        .
  1.9500E-01   1.1090E+03                        .
  2.1000E-01   1.2528E+03                        .
  2.2500E-01   1.4501E+03                        .
  2.4000E-01   1.7238E+03                        .
  2.5500E-01   2.1024E+03                        .
  2.7000E-01   2.6367E+03                        .
  2.8500E-01   3.3921E+03                        .
  3.0000E-01   4.4808E+03                        .
  3.1500E-01   6.0718E+03                        .
  3.3000E-01   8.4566E+03                        .
  3.4500E-01   1.2140E+04                        .
  3.6000E-01   1.8008E+04                        .
  3.7500E-01   2.7658E+04                        .
  3.9000E-01   4.3787E+04                        .
  4.0500E-01   7.0412E+04                        .
  4.2000E-01   1.1173E+05                        .
  4.3500E-01   1.6792E+05                        .
  4.5000E-01   2.2925E+05                        .
  4.6500E-01   2.7909E+05                        .
  4.8000E-01   3.0907E+05                        .
  4.9500E-01   3.2358E+05                        .
  5.1000E-01   3.2984E+05                        .
  5.2500E-01   3.3227E+05                        .
  5.4000E-01   3.3308E+05                        .
  5.5500E-01   3.3331E+05                        .
  5.7000E-01   3.3338E+05                        .
  5.8500E-01   3.3340E+05                        .
  6.0000E-01   3.3340E+05                        .
  6.1500E-01   3.3340E+05                        .
  6.3000E-01   3.3340E+05                        .
  6.4500E-01   3.3340E+05                        .
  6.6000E-01   3.3340E+05                        .
  6.7500E-01   3.3340E+05                        .
  6.9000E-01   3.3340E+05                        .
  7.0500E-01   3.3340E+05                        .
  7.2000E-01   3.3340E+05                        .
  7.3500E-01   3.3340E+05                        .
  7.5000E-01   3.3340E+05                        .
  2.000000000000000           probe radius in micrometers
  50.00000000000000           probe separation in micrometers
1.5000000000000000E-02        depth increment in micrometers
```

Table 3. Annotated log file (rgentr.log) for RGENTR program

```
 CALCULATION OF SPREADING RESISTANCE FROM THE  RESISTIVITY
 USING THE TRAPEZOIDAL ROMBERG INTEGRATION METHOD
 CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X
 NO BOUNDARY (INFINITE SLAB) CONDITION USED IN CALCULATION

    XL          XU
    0.000000    10.000000
  RADIUS        SEP         DELTH
    2.000000    50.000000     0.015000
 SUBRHO=1.26500E+02; SUBTHICK=1.50000E-02
 DEPTH(U)          RHO       CORR-FAC          RSP         VOLTAGE
0.00000E+00 1.54200E+00 2.10660E-01 8.12093E+02 2.90961E+00
1.50000E-02 6.93300E-01 4.48340E-01 7.77086E+02 2.89047E+00
3.00000E-02 3.44200E-01 8.85841E-01 7.62266E+02 2.88211E+00
4.50000E-02 1.90000E-01 1.59287E+00 7.56615E+02 2.87887E+00
6.00000E-02 1.15900E-01 2.61030E+00 7.56335E+02 2.87871E+00
7.50000E-02 7.69000E-02 3.95534E+00 7.60413E+02 2.88105E+00
9.00000E-02 5.43000E-02 5.66467E+00 7.68979E+02 2.88591E+00
1.05000E-01 4.01000E-02 7.80842E+00 7.82794E+02 2.89365E+00
1.20000E-01 3.07000E-02 1.04648E+01 8.03177E+02 2.90481E+00
1.35000E-01 2.42000E-02 1.37529E+01 8.32049E+02 2.92015E+00
1.50000E-01 1.96000E-02 1.78009E+01 8.72242E+02 2.94064E+00
1.65000E-01 1.65000E-02 2.24942E+01 9.27885E+02 2.96749E+00
1.80000E-01 1.44000E-02 2.78957E+01 1.00424E+03 3.00184E+00
1.95000E-01 1.31000E-02 3.38629E+01 1.10901E+03 3.04493E+00
2.10000E-01 1.25000E-02 4.00908E+01 1.25284E+03 3.09789E+00
2.25000E-01 1.24000E-02 4.67762E+01 1.45006E+03 3.16139E+00
2.40000E-01 1.30000E-02 5.30387E+01 1.72376E+03 3.23648E+00
2.55000E-01 1.41000E-02 5.96436E+01 2.10244E+03 3.32272E+00
2.70000E-01 1.61000E-02 6.55089E+01 2.63673E+03 3.42107E+00
2.85000E-01 1.90000E-02 7.14131E+01 3.39212E+03 3.53047E+00
3.00000E-01 2.33000E-02 7.69232E+01 4.48078E+03 3.65135E+00
3.15000E-01 2.94000E-02 8.26096E+01 6.07181E+03 3.78332E+00
3.30000E-01 3.82000E-02 8.85510E+01 8.45662E+03 3.92720E+00
3.45000E-01 5.14000E-02 9.44741E+01 1.21399E+04 4.08422E+00
3.60000E-01 7.21000E-02 9.99057E+01 1.80080E+04 4.25547E+00
3.75000E-01 1.07400E-01 1.03009E+02 2.76578E+04 4.44182E+00
3.90000E-01 1.73200E-01 1.01125E+02 4.37873E+04 4.64135E+00
4.05000E-01 3.08100E-01 9.14151E+01 7.04124E+04 4.84765E+00
4.20000E-01 6.09500E-01 7.33241E+01 1.11728E+05 5.04816E+00
4.35000E-01 1.33400E+00 5.03497E+01 1.67916E+05 5.22509E+00
4.50000E-01 3.18000E+00 2.88370E+01 2.29255E+05 5.36032E+00
4.65000E-01 8.03600E+00 1.38921E+01 2.79093E+05 5.44575E+00
4.80000E-01 2.04000E+01 6.06019E+00 3.09070E+05 5.49006E+00
4.95000E-01 4.66200E+01 2.77631E+00 3.23579E+05 5.50998E+00
5.10000E-01 8.27500E+01 1.59440E+00 3.29841E+05 5.51830E+00
5.25000E-01 1.09700E+02 1.21158E+00 3.32274E+05 5.52150E+00
5.40000E-01 1.21400E+02 1.09747E+00 3.33081E+05 5.52255E+00
5.55000E-01 1.25100E+02 1.06575E+00 3.33314E+05 5.52285E+00
5.70000E-01 1.26100E+02 1.05750E+00 3.33377E+05 5.52294E+00
5.85000E-01 1.26400E+02 1.05505E+00 3.33395E+05 5.52296E+00
6.00000E-01 1.26400E+02 1.05506E+00 3.33400E+05 5.52296E+00
6.15000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
6.30000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
6.45000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
6.60000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
6.75000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
6.90000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
7.05000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
7.20000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
7.35000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00
7.50000E-01 1.26500E+02 1.05424E+00 3.33404E+05 5.52297E+00

  CPU time: 00:05:07.72 - this is CPU time for VAX 11/785
```

RGENGL

The RGENGL (Resistance GENeration Gauss Laguerre) program calculates the two-probe spreading resistance from eqs (14) to (17). These are presented in the text as well as in the previous section where RGENTR was discussed. The original work of Choo *et al.* [17] makes use of the uniform current density form of the correction factor integral. The Schumann and Gardner current density as well as the ring delta function forms may also be treated by this method. The discussion presented here makes use of the uniform current density. The central point of the method is to evaluate the correction factor integral by means of an integration scheme based upon the Gauss-Laguerre quadrature. As noted in the text, quadrature techniques for evaluating integrals are based on knowing a set of integration points and weights, $\{x_k\},\{w_k\},(k = 1,...,N)$ such that an integral may be evaluated as a finite sum in the form

$$\int_a^b f(x)dx = \sum_{k=1}^{N} w_k f(x_k).$$

The calculation of the correction factor integral is carried out by using a 33-point Gauss-Laguerre method,

$$\frac{4}{\pi}\int_0^\infty A_n(\lambda)\left\{\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right\}2J_1(\lambda a)\frac{d\lambda}{\lambda}$$

$$\approx \frac{8}{\pi}\sum_{k=1}^{33} w_k^{gl}\left\{\frac{J_1(\lambda_k a)}{\lambda_k a} - \frac{J_0(\lambda_k S)}{2}\right\}\frac{J_1(\lambda_k a)}{\lambda_k}A_n(\lambda_k),$$

where the weights (w_k^{gl}) and integration points ($\lambda_k a$) are listed in reference [17] and are used in RGENGL. These are fixed weights and points which are entered through DATA statements. As the points are fixed by the roots of the polynomial and not by the behavior of the integrand, this technique is of limited value as it may not accurately reproduce the probe-spacing dependence of the integrand. It (and the inverse program, GL) is presented here as it represents one of the first attempts to evaluate the integrals and provides the user with the opportunity to run the programs and test the particular quadrature.

RGENGL reads in the input data from logical device 10 (for010). These data include the following:

npt	number of data points in the structure. Total number of points is num=npt+1 (include substrate point)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)

rho	array of resistivity values (Ω·cm)
icur	form of the current density (Schumann Gardner=1, Choo uniform=2, ring delta=3)

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The parts of the program are as follows:

rgengl	the main program which carries out the sums for the particular structure (the kernel function is constructed by the recursion relation inside the main program)
dbj0(x)	a double precision function which uses a polynomial approximation to construct the J_0 Bessel function
dbj1(x)	a double precision function which uses a polynomial approximation to construct the J_1 Bessel function

After the calculation is finished, the output is written to logical devices for006 and for011. The output may then be assigned to files for storage. Typical results are presented in the subsequent listings of the two files rgengl.log and rgengl.out. The CPU required is typically several seconds on a VAX 11/785. This is down from minutes for RGENTR. Herein lies the very attractive feature of quadrature techniques.

Table 4. Annotated input data file (rgengl.in) for RGENGL program

```
50              NPT-number of points in structure. Total is NPT + 1
2.0             AA-probe radius in micrometers
50.0            SEP- probe separation in micrometers
0.015           DELTH-depth increment in micrometers
1               IBBC - back boundary condition (none=1, cond=2, ins=3)
126.5           SUBRHO-substrate resistivity in ohm*cm =RHO(1)
0.015           SUBTH-substrate thickness in micrometers
1.542           RHO(NPT+1) surface resistivity in ohm*cm
0.6933          RHO(NPT)
0.3442          RHO(NPT-1)
0.19                .
0.1159              .
0.0769              .
0.0543              .
0.0401              .
0.0307              .
0.0242              .
0.0196              .
0.0165              .
0.0144              .
0.0131              .
0.0125              .
0.0124              .
0.013               .
0.0141              .
0.0161              .
0.019               .
0.0233              .
0.0294              .
0.0382              .
0.0514              .
0.0721              .
0.1074              .
0.1732              .
0.3081              .
0.6095              .
1.334               .
3.18                .
8.036               .
20.4                .
46.62               .
82.75               .
109.7               .
121.4               .
125.1               .
126.1               .
126.4               .
126.4               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5           RHO(2)
2               ICUR-probe current density (S&G=1,unif=2,ring=3)
```

Table 5. Annotated output data file (rgengl.out) for RGENGL program

```
        51                          total number of points calculated
     0.000    8.4549E+02            depth and spreading resistance
     0.015    8.0781E+02                          .
     0.030    7.9183E+02                          .
     0.045    7.8567E+02                          .
     0.060    7.8522E+02                          .
     0.075    7.8936E+02                          .
     0.090    7.9819E+02                          .
     0.105    8.1249E+02                          .
     0.120    8.3362E+02                          .
     0.135    8.6356E+02                          .
     0.150    9.0527E+02                          .
     0.165    9.6303E+02                          .
     0.180    1.0423E+03                          .
     0.195    1.1510E+03                          .
     0.210    1.3003E+03                          .
     0.225    1.5051E+03                          .
     0.240    1.7893E+03                          .
     0.255    2.1825E+03                          .
     0.270    2.7374E+03                          .
     0.285    3.5223E+03                          .
     0.300    4.6539E+03                          .
     0.315    6.3090E+03                          .
     0.330    8.7925E+03                          .
     0.345    1.2634E+04                          .
     0.360    1.8769E+04                          .
     0.375    2.8891E+04                          .
     0.390    4.5895E+04                          .
     0.405    7.4171E+04                          .
     0.420    1.1844E+05                          .
     0.435    1.7905E+05                          .
     0.450    2.4531E+05                          .
     0.465    2.9900E+05                          .
     0.480    3.3121E+05                          .
     0.495    3.4679E+05                          .
     0.510    3.5352E+05                          .
     0.525    3.5613E+05                          .
     0.540    3.5700E+05                          .
     0.555    3.5725E+05                          .
     0.570    3.5732E+05                          .
     0.585    3.5734E+05                          .
     0.600    3.5734E+05                          .
     0.615    3.5735E+05                          .
     0.630    3.5735E+05                          .
     0.645    3.5735E+05                          .
     0.660    3.5735E+05                          .
     0.675    3.5735E+05                          .
     0.690    3.5735E+05                          .
     0.705    3.5735E+05                          .
     0.720    3.5735E+05                          .
     0.735    3.5735E+05                          .
     0.750    3.5735E+05                          .
  2.000000000000000              probe radius in micrometers
  50.00000000000000              probe separation in micrometers
1.5000000000000000E-02           depth increment in micrometers
          2                      current density (SG=1,unif=2,ring=3)
```

Table 6. Annotated log file (rgengl.log) for RGENGL program

```
 CALCULATION OF THE SPREADING RESISTANCE FROM THE RESISTIVITY
 USING THE 33-POINT GAUSS-LAGUERRE SCHEME
 NO BOUNDARY AT     0.765 UM. SUBSTRATE AT   0.750UM
 CHOO UNIFORM CURRENT DENSITY
 AA= 2.000 SEP= 50.0 DELTH= 0.015 SUBTH=  1.500E-02 SUBRHO=  1.265E+02 #PT= 50

  DEPTH      RHO       INTEGRAL    CORR.FAC      RSP         VOL
  0.000  1.5420E+00  8.6128E-02  2.1932E-01  8.4549E+02  2.9271E+00
  0.015  6.9330E-01  1.8302E-01  4.6607E-01  8.0781E+02  2.9073E+00
  0.030  3.4420E-01  3.6136E-01  9.2020E-01  7.9183E+02  2.8986E+00
  0.045  1.9000E-01  6.4954E-01  1.6540E+00  7.8567E+02  2.8952E+00
  0.060  1.1590E-01  1.0642E+00  2.7100E+00  7.8522E+02  2.8950E+00
  0.075  7.6900E-02  1.6124E+00  4.1059E+00  7.8936E+02  2.8973E+00
  0.090  5.4300E-02  2.3090E+00  5.8798E+00  7.9819E+02  2.9021E+00
  0.105  4.0100E-02  3.1827E+00  8.1046E+00  8.1249E+02  2.9098E+00
  0.120  3.0700E-02  4.2653E+00  1.0861E+01  8.3362E+02  2.9210E+00
  0.135  2.4200E-02  5.6053E+00  1.4274E+01  8.6356E+02  2.9363E+00
  0.150  1.9600E-02  7.2551E+00  1.8475E+01  9.0527E+02  2.9568E+00
  0.165  1.6500E-02  9.1680E+00  2.3346E+01  9.6303E+02  2.9836E+00
  0.180  1.4400E-02  1.1370E+01  2.8952E+01  1.0423E+03  3.0180E+00
  0.195  1.3100E-02  1.3802E+01  3.5146E+01  1.1510E+03  3.0611E+00
  0.210  1.2500E-02  1.6341E+01  4.1611E+01  1.3003E+03  3.1141E+00
  0.225  1.2400E-02  1.9066E+01  4.8551E+01  1.5051E+03  3.1776E+00
  0.240  1.3000E-02  2.1620E+01  5.5054E+01  1.7893E+03  3.2527E+00
  0.255  1.4100E-02  2.4314E+01  6.1915E+01  2.1825E+03  3.3390E+00
  0.270  1.6100E-02  2.6708E+01  6.8011E+01  2.7374E+03  3.4373E+00
  0.285  1.9000E-02  2.9120E+01  7.4153E+01  3.5223E+03  3.5468E+00
  0.300  2.3300E-02  3.1375E+01  7.9895E+01  4.6539E+03  3.6678E+00
  0.315  2.9400E-02  3.3708E+01  8.5836E+01  6.3090E+03  3.8000E+00
  0.330  3.8200E-02  3.6155E+01  9.2068E+01  8.7925E+03  3.9441E+00
  0.345  5.1400E-02  3.8611E+01  9.8321E+01  1.2634E+04  4.1015E+00
  0.360  7.2100E-02  4.0891E+01  1.0413E+02  1.8769E+04  4.2734E+00
  0.375  1.0740E-01  4.2255E+01  1.0760E+02  2.8891E+04  4.4608E+00
  0.390  1.7320E-01  4.1624E+01  1.0599E+02  4.5895E+04  4.6618E+00
  0.405  3.0810E-01  3.7815E+01  9.6295E+01  7.4171E+04  4.8702E+00
  0.420  6.0950E-01  3.0523E+01  7.7727E+01  1.1844E+05  5.0735E+00
  0.435  1.3340E+00  2.1084E+01  5.3689E+01  1.7905E+05  5.2530E+00
  0.450  3.1800E+00  1.2117E+01  3.0856E+01  2.4531E+05  5.3897E+00
  0.465  8.0360E+00  5.8445E+00  1.4883E+01  2.9900E+05  5.4757E+00
  0.480  2.0400E+01  2.5503E+00  6.4943E+00  3.3121E+05  5.5201E+00
  0.495  4.6620E+01  1.1685E+00  2.9755E+00  3.4679E+05  5.5401E+00
  0.510  8.2750E+01  6.7106E-01  1.7088E+00  3.5352E+05  5.5484E+00
  0.525  1.0970E+02  5.0995E-01  1.2986E+00  3.5613E+05  5.5516E+00
  0.540  1.2140E+02  4.6192E-01  1.1763E+00  3.5700E+05  5.5527E+00
  0.555  1.2510E+02  4.4857E-01  1.1423E+00  3.5725E+05  5.5530E+00
  0.570  1.2610E+02  4.4510E-01  1.1334E+00  3.5732E+05  5.5531E+00
  0.585  1.2640E+02  4.4407E-01  1.1308E+00  3.5734E+05  5.5531E+00
  0.600  1.2640E+02  4.4408E-01  1.1308E+00  3.5734E+05  5.5531E+00
  0.615  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.630  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.645  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.660  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.675  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.690  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.705  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.720  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.735  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
  0.750  1.2650E+02  4.4373E-01  1.1299E+00  3.5735E+05  5.5531E+00
 CPU time: 00:00:06.89 - this CPU time is for VAX 11/785
```

RGENBL

The RGENBL (Resistance GENeration Berkowitz Lux) program calculates the two-probe spreading resistance starting with eqs (14) to (17). Whereas RGENTR carried out the integration by successive splitting of the subintervals and RGENGL carried out the integration by using fixed sampling points, the RGENBL program makes use of two essential features. The first is to approximate the correction factor integral by removing the probe-spacing-dependent Bessel function and to replace its effect by a truncated S-dependent lower limit of integration. In particular, this approximation of the correction factor integral takes the form as

$$C_n = \frac{8}{\pi}\int_0^\infty A_n(\lambda)\left\{\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2}\right\}J_1(\lambda a)\frac{d\lambda}{\lambda}$$

$$= \frac{8}{\pi}\int_L^\infty A_n(\lambda)\left(\frac{J_1(\lambda a)}{\lambda a}\right)^2 d\lambda,$$

where $L = 1.12292/S$. The choice of the truncated lower limit of integration is dictated by the form of the functions for the case of both insulating and conducting boundaries. The second feature of the technique is that the approximate integral can be divided into four domains in each of which the Newton-Cotes integration method may be used. The evaluation of the above approximate integral is then performed by means of a finite sum as

$$C_n = \sum_{k=1}^{22} w_k^{bl} A_n(\lambda_k),$$

where the weights, w_k^{bl}, are different from those used in the Gauss-Laguerre technique. The first eight sampling points for this method are dependent upon the probe spacing. The remaining points are fixed. It is the dependence of the first eight sampling points on the probe spacing that allows the technique to have numerical simplicity (of the RGENGL code) and yet reproduce the results of the RGENTR code (with the uniform current density) to within a percent.

RGENBL reads the input data from logical device for010. This can be referenced to a data file through an assign statement. For the purposes of illustration, the input data file, rgenbl.in, is used here. The structure of the input data file is as follows:

npt	number of data points in structure (excluding substrate)
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)
rho	array of resistivity values (Ω·cm)

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The probe current density is not entered as input data as the analysis is performed with the uniform current density. Application of the technique for the Schumann and Gardner or the ring density requires the analysis of the S-dependence of the integrand. Such analysis (not performed here) should provide the correct representation of the appropriate weights and sampling points.

The RGENBL code consists of just the main program. This contains the fixed points and weights through DATA statements and calculates the S-dependent points from the input. Upon calculating the integrals and the spreading resistance, output is directed to the logical devices for006 and for011. The file rgenbl.in contains the input for the sample run, whereas rgenbl.log and rgenbl.out contain the corresponding output. Typical CPU times are on the order of seconds on a VAX 11/785.

Table 7. Annotated input data file (rgenbl.in) for RGENBL program

```
50              NPT-number of points in structure. Total is NPT + 1
2.0             AA-probe radius in micrometers
50.0            SEP- probe separation in micrometers
0.015           DELTH-depth increment in micrometers
1               IBBC - back boundary condition (none=1, cond=2, ins=3)
126.5           SUBRHO-substrate resistivity in ohm*cm =RHO(1)
0.015           SUBTH-substrate thickness in micrometers
1.542           RHO(NPT+1) surface resistivity in ohm*cm
0.6933          RHO(NPT)
0.3442          RHO(NPT-1)
0.19                .
0.1159              .
0.0769              .
0.0543              .
0.0401              .
0.0307              .
0.0242              .
0.0196              .
0.0165              .
0.0144              .
0.0131              .
0.0125              .
0.0124              .
0.013               .
0.0141              .
0.0161              .
0.019               .
0.0233              .
0.0294              .
0.0382              .
0.0514              .
0.0721              .
0.1074              .
0.1732              .
0.3081              .
0.6095              .
1.334               .
3.18                .
8.036               .
20.4                .
46.62               .
82.75               .
109.7               .
121.4               .
125.1               .
126.1               .
126.4               .
126.4               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5           RHO(2)
```

Table 8. Annotated output data file (rgenbl.out) for RGENBL program

```
        51                          total number of points calculated
   0.000    8.1512E+02              depth and spreading resistance
   0.015    7.7918E+02                          .
   0.030    7.6396E+02                          .
   0.045    7.5813E+02                          .
   0.060    7.5776E+02                          .
   0.075    7.6180E+02                          .
   0.090    7.7036E+02                          .
   0.105    7.8419E+02                          .
   0.120    8.0460E+02                          .
   0.135    8.3353E+02                          .
   0.150    8.7382E+02                          .
   0.165    9.2961E+02                          .
   0.180    1.0062E+03                          .
   0.195    1.1112E+03                          .
   0.210    1.2554E+03                          .
   0.225    1.4532E+03                          .
   0.240    1.7278E+03                          .
   0.255    2.1078E+03                          .
   0.270    2.6440E+03                          .
   0.285    3.4024E+03                          .
   0.300    4.4955E+03                          .
   0.315    6.0932E+03                          .
   0.330    8.4870E+03                          .
   0.345    1.2180E+04                          .
   0.360    1.8053E+04                          .
   0.375    2.7680E+04                          .
   0.390    4.3721E+04                          .
   0.405    7.0159E+04                          .
   0.420    1.1122E+05                          .
   0.435    1.6716E+05                          .
   0.450    2.2830E+05                          .
   0.465    2.7804E+05                          .
   0.480    3.0802E+05                          .
   0.495    3.2259E+05                          .
   0.510    3.2891E+05                          .
   0.525    3.3139E+05                          .
   0.540    3.3221E+05                          .
   0.555    3.3244E+05                          .
   0.570    3.3251E+05                          .
   0.585    3.3253E+05                          .
   0.600    3.3253E+05                          .
   0.615    3.3254E+05                          .
   0.630    3.3254E+05                          .
   0.645    3.3254E+05                          .
   0.660    3.3254E+05                          .
   0.675    3.3254E+05                          .
   0.690    3.3254E+05                          .
   0.705    3.3254E+05                          .
   0.720    3.3254E+05                          .
   0.735    3.3254E+05                          .
   0.750    3.3254E+05                          .
 2.000000000000000             probe radius in micrometers
 50.00000000000000             probe separation in micrometers
1.5000000000000000E-02         depth incrrement in micrometers
```

Table 9. Annotated log file (rgenbl.log) for RGENBL program

```
 CALCULATION OF SPREADING RESISTANCE FROM THE  RESISTIVITY
 BERKOWITZ-LUX APPROXIMATE INTEGRAL WITH THE UNIFORM CURRENT DENSITY
 NO BOUNDARY AT     0.765 UM.  SUBSTRATE AT   0.750 UM
 RADIUS=2.0000 SEP= 50.0 DELTH= 0.015 SUBTH=1.500E-02 SUBRHO=1.265E+02 #PTS=50

  DEPTH       RHO       INTEGRAL    CORR.FAC      RSP       VOL
  0.000  1.5420E+00  8.3034E-02  2.1144E-01  8.1512E+02  2.9112E+00
  0.015  6.9330E-01  1.7654E-01  4.4955E-01  7.7918E+02  2.8916E+00
  0.030  3.4420E-01  3.4864E-01  8.8781E-01  7.6396E+02  2.8831E+00
  0.045  1.9000E-01  6.2677E-01  1.5961E+00  7.5813E+02  2.8797E+00
  0.060  1.1590E-01  1.0270E+00  2.6152E+00  7.5776E+02  2.8795E+00
  0.075  7.6900E-02  1.5561E+00  3.9626E+00  7.6180E+02  2.8818E+00
  0.090  5.4300E-02  2.2285E+00  5.6748E+00  7.7036E+02  2.8867E+00
  0.105  4.0100E-02  3.0718E+00  7.8223E+00  7.8419E+02  2.8944E+00
  0.120  3.0700E-02  4.1168E+00  1.0483E+01  8.0460E+02  2.9056E+00
  0.135  2.4200E-02  5.4104E+00  1.3777E+01  8.3353E+02  2.9209E+00
  0.150  1.9600E-02  7.0030E+00  1.7833E+01  8.7382E+02  2.9414E+00
  0.165  1.6500E-02  8.8498E+00  2.2536E+01  9.2961E+02  2.9683E+00
  0.180  1.4400E-02  1.0975E+01  2.7949E+01  1.0062E+03  3.0027E+00
  0.195  1.3100E-02  1.3324E+01  3.3930E+01  1.1112E+03  3.0458E+00
  0.210  1.2500E-02  1.5776E+01  4.0174E+01  1.2554E+03  3.0988E+00
  0.225  1.2400E-02  1.8409E+01  4.6878E+01  1.4532E+03  3.1623E+00
  0.240  1.3000E-02  2.0877E+01  5.3163E+01  1.7278E+03  3.2375E+00
  0.255  1.4100E-02  2.3481E+01  5.9795E+01  2.1078E+03  3.3238E+00
  0.270  1.6100E-02  2.5796E+01  6.5690E+01  2.6440E+03  3.4223E+00
  0.285  1.9000E-02  2.8129E+01  7.1629E+01  3.4024E+03  3.5318E+00
  0.300  2.3300E-02  3.0307E+01  7.7176E+01  4.4955E+03  3.6528E+00
  0.315  2.9400E-02  3.2555E+01  8.2900E+01  6.0932E+03  3.7848E+00
  0.330  3.8200E-02  3.4899E+01  8.8869E+01  8.4870E+03  3.9288E+00
  0.345  5.1400E-02  3.7223E+01  9.4788E+01  1.2180E+04  4.0857E+00
  0.360  7.2100E-02  3.9330E+01  1.0015E+02  1.8053E+04  4.2565E+00
  0.375  1.0740E-01  4.0484E+01  1.0309E+02  2.7680E+04  4.4422E+00
  0.390  1.7320E-01  3.9652E+01  1.0097E+02  4.3721E+04  4.6407E+00
  0.405  3.0810E-01  3.5769E+01  9.1085E+01  7.0159E+04  4.8461E+00
  0.420  6.0950E-01  2.8664E+01  7.2992E+01  1.1122E+05  5.0462E+00
  0.435  1.3340E+00  1.9684E+01  5.0124E+01  1.6716E+05  5.2231E+00
  0.450  3.1800E+00  1.1277E+01  2.8717E+01  2.2830E+05  5.3585E+00
  0.465  8.0360E+00  5.4349E+00  1.3840E+01  2.7804E+05  5.4441E+00
  0.480  2.0400E+01  2.3718E+00  6.0396E+00  3.0802E+05  5.4886E+00
  0.495  4.6620E+01  1.0869E+00  2.7678E+00  3.2259E+05  5.5086E+00
  0.510  8.2750E+01  6.2436E-01  1.5899E+00  3.2891E+05  5.5171E+00
  0.525  1.0970E+02  4.7451E-01  1.2083E+00  3.3139E+05  5.5203E+00
  0.540  1.2140E+02  4.2984E-01  1.0946E+00  3.3221E+05  5.5214E+00
  0.555  1.2510E+02  4.1743E-01  1.0630E+00  3.3244E+05  5.5217E+00
  0.570  1.2610E+02  4.1420E-01  1.0547E+00  3.3251E+05  5.5218E+00
  0.585  1.2640E+02  4.1324E-01  1.0523E+00  3.3253E+05  5.5218E+00
  0.600  1.2640E+02  4.1324E-01  1.0523E+00  3.3253E+05  5.5218E+00
  0.615  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.630  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.645  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.660  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.675  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.690  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.705  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.720  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.735  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
  0.750  1.2650E+02  4.1292E-01  1.0515E+00  3.3254E+05  5.5218E+00
 CPU time: 00:00:07.21 - this is CPU time for VAX 11/785
```

RGENVAR

The program RGENVAR (Resistance GENeration VARiational) is based upon the simplified implementation of the variation scheme discussed by Choo *et al.* [22,23]. There are three aspects of this calculation and the code which are of interest. First, the calculation does not make use of a single-probe current density, but rather makes use of a "mix" of the uniform current density and the Schumann and Gardner semi-infinite slab current densities. These provide a "basis set," where the amount of each current is varied according to a variational principle. The spreading resistance obtained from this variational method is

$$R_n^v = \frac{\rho_n}{2} C_n^v,$$

where

$$C_n^v = \frac{4}{\pi} \frac{K_1 I_{3s} + K_2 I_{2s}}{K_1 + K_2/2},$$

where K_1 and K_2 are given by

$$K_1 = \frac{I_2 - I_3/2}{I_1 I_2 - I_3^2},$$

$$K_2 = \frac{I_1/2 - I_3}{I_1 I_2 - I_3^2}.$$

The integrals I_1, I_2, I_3 are

$$I_1 = \int_0^\infty A_n(\lambda) \left\{ \frac{\sin(\lambda a)}{\lambda a} \right\}^2 a d\lambda,$$

$$I_2 = \int_0^\infty A_n(\lambda) \left\{ \frac{J_1(\lambda a)}{\lambda a} \right\}^2 a d\lambda,$$

$$I_3 = \int_0^\infty A_n(\lambda) \left\{ \frac{\sin(\lambda a) J_1(\lambda a)}{(\lambda a)^2} \right\} a d\lambda.$$

Finally, the integrals I_{3s} and I_{2s} are

$$I_{3s} = \int_0^\infty A_n(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a} \right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right) d\lambda,$$

and

$$I_{2s} = \int_0^\infty A_n(\lambda) \left(\frac{J_1(\lambda a)}{\lambda a} \right) \left(\frac{J_1(\lambda a)}{\lambda a} - \frac{J_0(\lambda S)}{2} \right) d\lambda.$$

The second and third aspects of this calculation are appropriate generalizations of the RGENBL analysis to this problem. In particular, the Berkowitz-Lux approximation [21] is used to simplify the integrals (and to remove the S-dependent Bessel function with an S-dependent lower limit of integration) as

$$I_{3s} = \int_L^\infty A_n(\lambda) \left(\frac{\sin(\lambda a)}{\lambda a} \right) \left(\frac{J_1(\lambda a)}{\lambda a} \right) d\lambda,$$

$$I_{2s} = \int_L^\infty A_n(\lambda)\left(\frac{J_1(\lambda a)}{\lambda a}\right)^2 d\lambda.$$

It should be noted that the integrals, I_1, I_2, and I_3, contain all of the information which is needed to evaluate the correction factor and the spreading resistance. This follows from the fact that the approximate integrals, I_{3s} and I_{2s} are contained in the corresponding I_3 and I_2 integrals. The third aspect of the calculation is the evaluation of the various integrals (I_1, I_2, and I_3) by means of a segmented Newton-Cotes method similar to that used by Berkowitz and Lux. The coordinates and weights of the fixed points as well as the form of the S-dependent points are listed and discussed in reference [23]. These are incorporated in RGENVAR through DATA statements and the appropriate computations.

RGENVAR reads the input data from the logical device for010. These include:

n	number of resistivity, spreading resistance values
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
ibbc	boundary condition (none=1, conducting=2, insulation=3)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)
rho(i)	*i-th* resistivity relative to the surface (Ω·cm)

As originally formulated, the integration scheme is not able to handle an insulating boundary. In the present implementation, this has been circumvented by starting the integration one point away from the singularity at the origin when there is an insulating boundary. This has been found to lead to very good results with no loss of generality.

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The file rgenvar.in contains the input data to be read in for010. The files rgenvar.log and rgenvar.out contain the output data written to for006 and for011. The computation also takes several CPU seconds much like RGENGL and RGENBL.

Table 10. Annotated input data file (rgenvar.in) for RGENVAR program

```
50              NPT-number of points in structure. Total is NPT + 1
2.0             AA-probe radius in micrometers
50.0            SEP- probe separation in micrometers
0.015           DELTH-depth increment in micrometers
1               IBBC - back boundary condition (none=1, cond=2, ins=3)
126.5           SUBRHO-substrate resistivity in ohm*cm =RHO(1)
0.015           SUBTH-substrate thickness in micrometers
1.542           RHO(NPT+1) surface resistivity in ohm*cm
0.6933          RHO(NPT)
0.3442          RHO(NPT-1)
0.19                .
0.1159              .
0.0769              .
0.0543              .
0.0401              .
0.0307              .
0.0242              .
0.0196              .
0.0165              .
0.0144              .
0.0131              .
0.0125              .
0.0124              .
0.013               .
0.0141              .
0.0161              .
0.019               .
0.0233              .
0.0294              .
0.0382              .
0.0514              .
0.0721              .
0.1074              .
0.1732              .
0.3081              .
0.6095              .
1.334               .
3.18                .
8.036               .
20.4                .
46.62               .
82.75               .
109.7               .
121.4               .
125.1               .
126.1               .
126.4               .
126.4               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5               .
126.5           RHO(2)
```

Table 11. Annotated output data file (rgenvar.out) for RGENVAR program

51		total number of points calculated
0.000	8.0688E+02	depth and spreading resistance
0.015	7.6620E+02	.
0.030	7.4523E+02	.
0.045	7.3371E+02	.
0.060	7.2839E+02	.
0.075	7.2830E+02	.
0.090	7.3334E+02	.
0.105	7.4399E+02	.
0.120	7.6136E+02	.
0.135	7.8711E+02	.
0.150	8.2382E+02	.
0.165	8.7534E+02	.
0.180	9.4651E+02	.
0.195	1.0445E+03	.
0.210	1.1794E+03	.
0.225	1.3646E+03	.
0.240	1.6217E+03	.
0.255	1.9775E+03	.
0.270	2.4795E+03	.
0.285	3.1891E+03	.
0.300	4.2110E+03	.
0.315	5.7027E+03	.
0.330	7.9339E+03	.
0.345	1.1367E+04	.
0.360	1.6804E+04	.
0.375	2.5663E+04	.
0.390	4.0285E+04	.
0.405	6.4054E+04	.
0.420	1.0034E+05	.
0.435	1.4903E+05	.
0.450	2.0237E+05	.
0.465	2.4762E+05	.
0.480	2.7753E+05	.
0.495	2.9401E+05	.
0.510	3.0209E+05	.
0.525	3.0550E+05	.
0.540	3.0667E+05	.
0.555	3.0701E+05	.
0.570	3.0710E+05	.
0.585	3.0713E+05	.
0.600	3.0713E+05	.
0.615	3.0714E+05	.
0.630	3.0714E+05	.
0.645	3.0714E+05	.
0.660	3.0714E+05	.
0.675	3.0714E+05	.
0.690	3.0714E+05	.
0.705	3.0714E+05	.
0.720	3.0714E+05	.
0.735	3.0714E+05	.
0.750	3.0714E+05	.
2.000000000000000		probe radius in micrometers
50.00000000000000		probe separation in micrometers
1.5000000000000000E-02		depth increment in micrometers

Table 12. Annotated log file (rgenvar.log) for RGENVAR program

```
 CALCULATION OF SPREADING RESISTANCE FROM THE  RESISTIVITY
 USING CHOO ET AL VARIATIONAL TECHNIQUE-BL
 NO BOUNDARY AT     0.765 UM.  SUBSTRATE AT   0.750 UM
 RADIUS=2.0000 SEP= 50.0 DELTH= 0.015 SUBTH=1.500E-02 SUBRHO=1.265E+02 #PTS=50

  DEPTH       RHO       CORR.FAC       RSP       VOL
  0.000  1.5420E+00  2.0931E-01  8.0688E+02  2.9068E+00
  0.015  6.9330E-01  4.4206E-01  7.6620E+02  2.8843E+00
  0.030  3.4420E-01  8.6604E-01  7.4523E+02  2.8723E+00
  0.045  1.9000E-01  1.5446E+00  7.3371E+02  2.8655E+00
  0.060  1.1590E-01  2.5139E+00  7.2839E+02  2.8624E+00
  0.075  7.6900E-02  3.7883E+00  7.2830E+02  2.8623E+00
  0.090  5.4300E-02  5.4021E+00  7.3334E+02  2.8653E+00
  0.105  4.0100E-02  7.4214E+00  7.4399E+02  2.8716E+00
  0.120  3.0700E-02  9.9200E+00  7.6136E+02  2.8816E+00
  0.135  2.4200E-02  1.3010E+01  7.8711E+02  2.8960E+00
  0.150  1.9600E-02  1.6813E+01  8.2382E+02  2.9158E+00
  0.165  1.6500E-02  2.1220E+01  8.7534E+02  2.9422E+00
  0.180  1.4400E-02  2.6292E+01  9.4651E+02  2.9761E+00
  0.195  1.3100E-02  3.1894E+01  1.0445E+03  3.0189E+00
  0.210  1.2500E-02  3.7741E+01  1.1794E+03  3.0717E+00
  0.225  1.2400E-02  4.4018E+01  1.3646E+03  3.1350E+00
  0.240  1.3000E-02  4.9898E+01  1.6217E+03  3.2100E+00
  0.255  1.4100E-02  5.6098E+01  1.9775E+03  3.2961E+00
  0.270  1.6100E-02  6.1602E+01  2.4795E+03  3.3944E+00
  0.285  1.9000E-02  6.7138E+01  3.1891E+03  3.5037E+00
  0.300  2.3300E-02  7.2292E+01  4.2110E+03  3.6244E+00
  0.315  2.9400E-02  7.7588E+01  5.7027E+03  3.7561E+00
  0.330  3.8200E-02  8.3077E+01  7.9339E+03  3.8995E+00
  0.345  5.1400E-02  8.8460E+01  1.1367E+04  4.0556E+00
  0.360  7.2100E-02  9.3225E+01  1.6804E+04  4.2254E+00
  0.375  1.0740E-01  9.5578E+01  2.5663E+04  4.4093E+00
  0.390  1.7320E-01  9.3036E+01  4.0285E+04  4.6051E+00
  0.405  3.0810E-01  8.3161E+01  6.4054E+04  4.8065E+00
  0.420  6.0950E-01  6.5849E+01  1.0034E+05  5.0015E+00
  0.435  1.3340E+00  4.4686E+01  1.4903E+05  5.1733E+00
  0.450  3.1800E+00  2.5455E+01  2.0237E+05  5.3061E+00
  0.465  8.0360E+00  1.2326E+01  2.4762E+05  5.3938E+00
  0.480  2.0400E+01  5.4417E+00  2.7753E+05  5.4433E+00
  0.495  4.6620E+01  2.5226E+00  2.9401E+05  5.4684E+00
  0.510  8.2750E+01  1.4603E+00  3.0209E+05  5.4801E+00
  0.525  1.0970E+02  1.1139E+00  3.0550E+05  5.4850E+00
  0.540  1.2140E+02  1.0104E+00  3.0667E+05  5.4867E+00
  0.555  1.2510E+02  9.8165E-01  3.0701E+05  5.4872E+00
  0.570  1.2610E+02  9.7415E-01  3.0710E+05  5.4873E+00
  0.585  1.2640E+02  9.7192E-01  3.0713E+05  5.4873E+00
  0.600  1.2640E+02  9.7194E-01  3.0713E+05  5.4873E+00
  0.615  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.630  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.645  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.660  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.675  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.690  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.705  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.720  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.735  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
  0.750  1.2650E+02  9.7120E-01  3.0714E+05  5.4873E+00
CPU time: 00:00:07.49 - this is CPU time for VAX 11/785
```

CALCULATION OF THE RESISTIVITY FROM THE TWO-PROBE RESISTANCE

The second group of programs is used to calculate the resistivity from the spreading resistance. The three programs included in this section are: BL, GL, and VAR. These are related to the programs RGENBL, RGENGL, and RGENVAR which were discussed in the previous section. For example, the calculation of the spreading resistance from the resistivity is carried out with RGENBL, then the inversion or calculation of the resistivity from the spreading resistance is accomplished with the program BL. All three inversion programs have very similar architecture and program logic. The program BL is discussed first because of historical reasons. It was the first program to make use of a bounding procedure in order to carry out the spreading resistance correction factor integral inversion. The procedure is used until the calculated spreading resistance agrees to within given limits with the input spreading resistance. This bounding procedure is extremely general and is not dependent upon the quadrature method for evaluating the correction factor integral. Hence, it is easily used in a straightforward way in the Gauss-Laguerre program, GL, and is readily generalized in the variational program, VAR, where several forms of the current density necessitate the application of the bounding procedure to several integrals.

It is of interest to note that there are a number of other programs which can be used to calculate the resistivity from the spreading resistance. These include the SPINT-SPRED programs presented by D'Avanzo *et al.* [16] and the LOCAL SLOPE program presented by Dickey [18]. Both of the programs have been discussed and documented in the appropriate references. The SPINT-SPRED programs represent the inversion of the spreading resistance following the calculation of the integrals involved in the RGENTR program. The use of a earlier version of RGENTR in the discussion of the SPINT-SPRED and the LOCAL SLOPE programs has been discussed elsewhere [19].

BL

The BL (Berkowitz-Lux) program is based upon the Berkowitz-Lux approximation for the correction factor integral. This is discussed and used in the RGENBL program. The program calculates the resistivity by varying the resistivity needed to obtain a predetermined level of agreement between the input spreading resistance and the calculated spreading resistance. In this, the bounding procedure is very important. This is based upon the observation that the hyperbolic functions, tanh and coth, provide the limits of the form of the kernel of the correction factor integral for the cases of conducting and insulating boundaries. The correct kernel must then lie between these two extremes which then provide bounding envelopes for the kernel function. This assists the code in the seeking of the best value of the resistivity for a given value of the spreading resistance.

The BL program reads the input data from for011 (bl.in is the example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	*i-th* value of depth, in micrometers (relative to surface) and *i-th* value of two-probe resistance (relative to surface)
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)

The BL program uses the approximate correction factor, the 22-point Newton-Cotes quadrature, and the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in bl.out and bl.log. Notice that the CPU needed to carry out the inversion is on the order of seconds. This puts it into the time frame of real-time analysis which is possible on a PC-based system.

Table 13. Annotated input data file (bl.in) for BL program
This is the output data file (rgenbl.out) from RGENBL program.

51		NPT-number of spreading resistance values
0.000	8.1512E+02	DEPTH(1) RSP(1)
0.015	7.7918E+02	DEPTH(2) RSP(2)
0.030	7.6396E+02	DEPTH(3) RSP(3)
0.045	7.5813E+02	. .
0.060	7.5776E+02	. .
0.075	7.6180E+02	. .
0.090	7.7036E+02	. .
0.105	7.8419E+02	. .
0.120	8.0460E+02	. .
0.135	8.3353E+02	. .
0.150	8.7382E+02	. .
0.165	9.2961E+02	. .
0.180	1.0062E+03	. .
0.195	1.1112E+03	. .
0.210	1.2554E+03	. .
0.225	1.4532E+03	. .
0.240	1.7278E+03	. .
0.255	2.1078E+03	. .
0.270	2.6440E+03	. .
0.285	3.4024E+03	. .
0.300	4.4955E+03	. .
0.315	6.0932E+03	. .
0.330	8.4870E+03	. .
0.345	1.2180E+04	. .
0.360	1.8053E+04	. .
0.375	2.7680E+04	. .
0.390	4.3721E+04	. .
0.405	7.0159E+04	. .
0.420	1.1122E+05	. .
0.435	1.6716E+05	. .
0.450	2.2830E+05	. .
0.465	2.7804E+05	. .
0.480	3.0802E+05	. .
0.495	3.2259E+05	. .
0.510	3.2891E+05	. .
0.525	3.3139E+05	. .
0.540	3.3221E+05	. .
0.555	3.3244E+05	. .
0.570	3.3251E+05	. .
0.585	3.3253E+05	. .
0.600	3.3253E+05	. .
0.615	3.3254E+05	. .
0.630	3.3254E+05	. .
0.645	3.3254E+05	. .
0.660	3.3254E+05	. .
0.675	3.3254E+05	. .
0.690	3.3254E+05	. .
0.705	3.3254E+05	. .
0.720	3.3254E+05	. .
0.735	3.3254E+05	. .
0.750	3.3254E+05	DEPTH(NPT) RSP(NPT)
2.000000000000000		AA-probe radius in micrometers
50.00000000000000		SEP-probe separation in micrometers
1.5000000000000000E-02		DELTH-depth increment (layer thickness)

Table 14. Annotated output data file (bl.out) for BL program

```
      51                    number of corrected points
 0.000    1.545E+00         depth (micrometers) and resistivity (ohm*cm)
 0.015    6.951E-01                       .
 0.030    3.449E-01                       .
 0.045    1.904E-01                       .
 0.060    1.160E-01                       .
 0.075    7.691E-02                       .
 0.090    5.431E-02                       .
 0.105    4.012E-02                       .
 0.120    3.070E-02                       .
 0.135    2.420E-02                       .
 0.150    1.960E-02                       .
 0.165    1.649E-02                       .
 0.180    1.441E-02                       .
 0.195    1.310E-02                       .
 0.210    1.250E-02                       .
 0.225    1.240E-02                       .
 0.240    1.300E-02                       .
 0.255    1.410E-02                       .
 0.270    1.610E-02                       .
 0.285    1.900E-02                       .
 0.300    2.330E-02                       .
 0.315    2.940E-02                       .
 0.330    3.821E-02                       .
 0.345    5.141E-02                       .
 0.360    7.213E-02                       .
 0.375    1.074E-01                       .
 0.390    1.732E-01                       .
 0.405    3.081E-01                       .
 0.420    6.091E-01                       .
 0.435    1.336E+00                       .
 0.450    3.180E+00                       .
 0.465    8.035E+00                       .
 0.480    2.039E+01                       .
 0.495    4.542E+01                       .
 0.510    8.225E+01                       .
 0.525    1.098E+02                       .
 0.540    1.263E+02                       .
 0.555    1.264E+02                       .
 0.570    1.265E+02                       .
 0.585    1.265E+02                       .
 0.600    1.265E+02                       .
 0.615    1.265E+02                       .
 0.630    1.265E+02                       .
 0.645    1.265E+02                       .
 0.660    1.265E+02                       .
 0.675    1.265E+02                       .
 0.690    1.265E+02                       .
 0.705    1.265E+02                       .
 0.720    1.265E+02                       .
 0.735    1.265E+02                       .
 0.750    1.265E+02                       .
```

Table 15. Annotated log file (bl.log) for BL program

```
 CALCULATION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE
 USING THE BERKOWITZ-LUX APPROXIMATE INTEGRAL
 RADIUS=   2.0  SEP=  50.0  DELTH=  0.015   # PTS= 51
 R5=   0.412922 R6=   0.003736 R7= 115.745620

  NPT   DEPTH     RMEAS       1-RCAL/R    RHO         SENS        ITER

   1    0.000   8.151E+02 -9.882E-05  1.545E+00  2.182E+01       2
   2    0.015   7.792E+02 -5.763E-05  6.951E-01  4.398E+01       2
   3    0.030   7.640E+02 -3.414E-05  3.449E-01  7.286E+01       2
   4    0.045   7.581E+02 -2.080E-05  1.904E-01  8.813E+01       2
   5    0.060   7.578E+02 -1.301E-05  1.160E-01  8.021E+01       2
   6    0.075   7.618E+02 -8.485E-06  7.691E-02  6.329E+01       2
   7    0.090   7.704E+02 -5.816E-06  5.431E-02  4.790E+01       2
   8    0.105   7.842E+02 -4.177E-06  4.012E-02  3.611E+01       2
   9    0.120   8.046E+02 -3.173E-06  3.070E-02  2.746E+01       2
  10    0.135   8.335E+02 -2.580E-06  2.420E-02  2.110E+01       2
  11    0.150   8.738E+02 -2.189E-06  1.960E-02  1.640E+01       2
  12    0.165   9.296E+02 -2.044E-06  1.649E-02  1.301E+01       2
  13    0.180   1.006E+03 -2.087E-06  1.441E-02  1.052E+01       2
  14    0.195   1.111E+03 -2.250E-06  1.310E-02  8.671E+00       2
  15    0.210   1.255E+03 -2.762E-06  1.250E-02  7.327E+00       2
  16    0.225   1.453E+03 -3.235E-06  1.240E-02  6.280E+00       2
  17    0.240   1.728E+03 -4.600E-06  1.300E-02  5.539E+00       2
  18    0.255   2.108E+03 -5.931E-06  1.410E-02  4.926E+00       2
  19    0.270   2.644E+03 -9.154E-06  1.610E-02  4.483E+00       2
  20    0.285   3.402E+03 -1.389E-05  1.900E-02  4.111E+00       2
  21    0.300   4.496E+03 -2.436E-05  2.330E-02  3.813E+00       2
  22    0.315   6.093E+03 -4.523E-05  2.940E-02  3.547E+00       2
  23    0.330   8.487E+03 -8.382E-05  3.821E-02  3.303E+00       2
  24    0.345   1.218E+04 -1.473E-04  5.141E-02  3.084E+00       2
  25    0.360   1.805E+04 -1.694E-04  7.213E-02  2.896E+00       2
  26    0.375   2.768E+04  2.557E-04  1.074E-01  2.763E+00       2
  27    0.390   4.372E+04  4.929E-06  1.732E-01  2.725E+00       3
  28    0.405   7.016E+04  6.893E-05  3.081E-01  2.830E+00       3
  29    0.420   1.112E+05  4.325E-04  6.091E-01  3.174E+00       3
  30    0.435   1.672E+05 -6.103E-04  1.336E+00  4.010E+00       4
  31    0.450   2.283E+05  8.160E-07  3.180E+00  5.946E+00       5
  32    0.465   2.780E+05  1.110E-06  8.035E+00  1.065E+01       5
  33    0.480   3.080E+05  2.862E-06  2.039E+01  2.149E+01       5
  34    0.495   3.226E+05  7.309E-04  4.542E+01  3.881E+01       3
  35    0.510   3.289E+05  1.206E-04  8.225E+01  5.284E+01       3
  36    0.525   3.314E+05  1.709E-05  1.098E+02  5.628E+01       3
  37    0.540   3.322E+05 -6.751E-04  1.263E+02  5.660E+01       1
  38    0.555   3.324E+05 -2.054E-04  1.264E+02  5.660E+01       1
  39    0.570   3.325E+05 -5.860E-05  1.265E+02  5.660E+01       1
  40    0.585   3.325E+05  5.080E-07  1.265E+02  5.660E+01       1
  41    0.600   3.325E+05 -2.954E-05  1.265E+02  5.660E+01       1
  42    0.615   3.325E+05  0.000E+00  1.265E+02  5.660E+01       1
  43    0.630   3.325E+05  2.776E-17  1.265E+02  5.660E+01       1
  44    0.645   3.325E+05  0.000E+00  1.265E+02  5.660E+01       1
  45    0.660   3.325E+05  2.776E-17  1.265E+02  5.660E+01       1
  46    0.675   3.325E+05  0.000E+00  1.265E+02  5.660E+01       1
  47    0.690   3.325E+05  2.776E-17  1.265E+02  5.660E+01       1
  48    0.705   3.325E+05  0.000E+00  1.265E+02  5.660E+01       1
  49    0.720   3.325E+05  2.776E-17  1.265E+02  5.660E+01       1
  50    0.735   3.325E+05  0.000E+00  1.265E+02  5.660E+01       1
  51    0.750   3.325E+05  0.000E+00  1.265E+02  0.000E+00       0
 CPU time: 00:00:06.69 - this is CPU time for VAX 11/785
```

GL

The GL (Gauss-Laguerre) program is based upon the Gauss-Laguerre quadrature for calculating the correction factor integral. This is discussed and used in the RGENGL program. The present implementation of this program makes use of the same bounding procedure used in the BL code.

The GL program reads the input data from for011 (gl.in is an example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	*i-th* value of depth relative to surface and *i-th* value of two-probe resistance relative to surface
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
icur	current density (Schumann Gardner=1, Choo uniform=2, ring delta=3)

The code, like the RGENGL code, consists of the main program, GL, along with the two double precision functions, dbj0(x) and dbj1(x), needed to construct the Bessel functions involved in the calculations.

The GL program uses the Gauss-Laguerre quadrature and the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in gl.out and gl.log. Like BL, the CPU needed to carry out the inversion is on the order of seconds. The limitation of the paired set of programs, RGENGL and GL, is that the use of the fixed sampling points may not adequately represent the S-dependence of the integral and may consequently give results which are inaccurate relative to RGENTR.

Table 16. Annotated input data file (gl.in) for GL program
This is output data file (rgengl.out) from RGENGL program.

```
      51                    NPT-number of spreading resistance values
   0.000    8.4549E+02      DEPTH(1)        RSP(1)
   0.015    8.0781E+02      DEPTH(2)        RSP(2)
   0.030    7.9183E+02      DEPTH(3)        RSP(3)
   0.045    7.8567E+02        .               .
   0.060    7.8522E+02        .               .
   0.075    7.8936E+02        .               .
   0.090    7.9819E+02        .               .
   0.105    8.1249E+02        .               .
   0.120    8.3362E+02        .               .
   0.135    8.6356E+02        .               .
   0.150    9.0527E+02        .               .
   0.165    9.6303E+02        .               .
   0.180    1.0423E+03        .               .
   0.195    1.1510E+03        .               .
   0.210    1.3003E+03        .               .
   0.225    1.5051E+03        .               .
   0.240    1.7893E+03        .               .
   0.255    2.1825E+03        .               .
   0.270    2.7374E+03        .               .
   0.285    3.5223E+03        .               .
   0.300    4.6539E+03        .               .
   0.315    6.3090E+03        .               .
   0.330    8.7925E+03        .               .
   0.345    1.2634E+04        .               .
   0.360    1.8769E+04        .               .
   0.375    2.8891E+04        .               .
   0.390    4.5895E+04        .               .
   0.405    7.4171E+04        .               .
   0.420    1.1844E+05        .               .
   0.435    1.7905E+05        .               .
   0.450    2.4531E+05        .               .
   0.465    2.9900E+05        .               .
   0.480    3.3121E+05        .               .
   0.495    3.4679E+05        .               .
   0.510    3.5352E+05        .               .
   0.525    3.5613E+05        .               .
   0.540    3.5700E+05        .               .
   0.555    3.5725E+05        .               .
   0.570    3.5732E+05        .               .
   0.585    3.5734E+05        .               .
   0.600    3.5734E+05        .               .
   0.615    3.5735E+05        .               .
   0.630    3.5735E+05        .               .
   0.645    3.5735E+05        .               .
   0.660    3.5735E+05        .               .
   0.675    3.5735E+05        .               .
   0.690    3.5735E+05        .               .
   0.705    3.5735E+05        .               .
   0.720    3.5735E+05        .               .
   0.735    3.5735E+05        .               .
   0.750    3.5735E+05      DEPTH(NPT)      RSP(NPT)
 2.000000000000000         AA-probe radius in micrometers
 50.00000000000000         SEP-probe separation in micrometers
1.5000000000000000E-02     DELTH-depth increment (layer thickness)
         2                 ICUR-probe current density (S&G=1,unif=2,ring=3)
```

Table 17. Annotated output data file (gl.out) for GL program

51		number of points
0.000	1.543E+00	depth (micrometers) and resistivity (ohm*cm)
0.015	6.940E-01	.
0.030	3.446E-01	.
0.045	1.902E-01	.
0.060	1.159E-01	.
0.075	7.692E-02	.
0.090	5.431E-02	.
0.105	4.010E-02	.
0.120	3.071E-02	.
0.135	2.420E-02	.
0.150	1.960E-02	.
0.165	1.650E-02	.
0.180	1.441E-02	.
0.195	1.310E-02	.
0.210	1.250E-02	.
0.225	1.240E-02	.
0.240	1.300E-02	.
0.255	1.410E-02	.
0.270	1.610E-02	.
0.285	1.900E-02	.
0.300	2.330E-02	.
0.315	2.941E-02	.
0.330	3.822E-02	.
0.345	5.142E-02	.
0.360	7.213E-02	.
0.375	1.074E-01	.
0.390	1.732E-01	.
0.405	3.081E-01	.
0.420	6.092E-01	.
0.435	1.336E+00	.
0.450	3.180E+00	.
0.465	8.038E+00	.
0.480	2.040E+01	.
0.495	4.560E+01	.
0.510	8.249E+01	.
0.525	1.097E+02	.
0.540	1.263E+02	.
0.555	1.264E+02	.
0.570	1.265E+02	.
0.585	1.265E+02	.
0.600	1.265E+02	.
0.615	1.265E+02	.
0.630	1.265E+02	.
0.645	1.265E+02	.
0.660	1.265E+02	.
0.675	1.265E+02	.
0.690	1.265E+02	.
0.705	1.265E+02	.
0.720	1.265E+02	.
0.735	1.265E+02	.
0.750	1.265E+02	.

Table 18. Annotated log file (gl.log) for GL program

```
CALCULATION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE
USING THE CHOO ET AL GAUSS-LAGUERRE QUADRATURE
CHOO UNIFORM CURRENT DENSITY
RADIUS=   2.0  SEP=  50.0  DELTH=  0.015   # PTS= 51
R5=   0.443730 R6=   0.003903 R7= 119.978679
```

NPT	DEPTH	RMEAS	1-RCAL/R	RHO	SENS	ITER
1	0.000	8.455E+02	-3.794E-05	1.543E+00	2.172E+01	2
2	0.015	8.078E+02	-2.407E-05	6.940E-01	4.380E+01	2
3	0.030	7.918E+02	-1.552E-05	3.446E-01	7.259E+01	2
4	0.045	7.857E+02	-1.022E-05	1.902E-01	8.792E+01	2
5	0.060	7.852E+02	-6.917E-06	1.159E-01	8.010E+01	2
6	0.075	7.894E+02	-4.963E-06	7.692E-02	6.328E+01	2
7	0.090	7.982E+02	-3.854E-06	5.431E-02	4.789E+01	2
8	0.105	8.125E+02	-3.258E-06	4.010E-02	3.610E+01	2
9	0.120	8.336E+02	-3.070E-06	3.071E-02	2.747E+01	2
10	0.135	8.636E+02	-3.183E-06	2.420E-02	2.111E+01	2
11	0.150	9.053E+02	-3.440E-06	1.960E-02	1.641E+01	2
12	0.165	9.630E+02	-3.988E-06	1.650E-02	1.302E+01	2
13	0.180	1.042E+03	-4.799E-06	1.441E-02	1.052E+01	2
14	0.195	1.151E+03	-5.773E-06	1.310E-02	8.674E+00	2
15	0.210	1.300E+03	-7.528E-06	1.250E-02	7.329E+00	2
16	0.225	1.505E+03	-9.049E-06	1.240E-02	6.284E+00	2
17	0.240	1.789E+03	-1.286E-05	1.300E-02	5.543E+00	2
18	0.255	2.183E+03	-1.610E-05	1.410E-02	4.929E+00	2
19	0.270	2.737E+03	-2.367E-05	1.610E-02	4.485E+00	2
20	0.285	3.522E+03	-3.348E-05	1.900E-02	4.112E+00	2
21	0.300	4.654E+03	-5.371E-05	2.330E-02	3.813E+00	2
22	0.315	6.309E+03	-8.934E-05	2.941E-02	3.544E+00	2
23	0.330	8.793E+03	-1.461E-04	3.822E-02	3.295E+00	2
24	0.345	1.263E+04	-2.275E-04	5.142E-02	3.071E+00	2
25	0.360	1.877E+04	-2.447E-04	7.213E-02	2.875E+00	2
26	0.375	2.889E+04	2.097E-04	1.074E-01	2.734E+00	2
27	0.390	4.590E+04	4.046E-06	1.732E-01	2.688E+00	3
28	0.405	7.417E+04	5.862E-05	3.081E-01	2.787E+00	3
29	0.420	1.184E+05	3.782E-04	6.092E-01	3.127E+00	3
30	0.435	1.791E+05	-5.083E-04	1.336E+00	3.959E+00	4
31	0.450	2.453E+05	5.649E-07	3.180E+00	5.905E+00	5
32	0.465	2.990E+05	7.286E-07	8.038E+00	1.064E+01	5
33	0.480	3.312E+05	1.448E-06	2.040E+01	2.157E+01	5
34	0.495	3.468E+05	5.899E-04	4.560E+01	3.931E+01	3
35	0.510	3.535E+05	9.708E-05	8.249E+01	5.381E+01	3
36	0.525	3.561E+05	1.454E-05	1.097E+02	5.746E+01	3
37	0.540	3.570E+05	-6.836E-04	1.263E+02	5.787E+01	1
38	0.555	3.573E+05	-1.912E-04	1.264E+02	5.787E+01	1
39	0.570	3.573E+05	-5.455E-05	1.265E+02	5.787E+01	1
40	0.585	3.573E+05	4.669E-07	1.265E+02	5.787E+01	1
41	0.600	3.573E+05	-2.750E-05	1.265E+02	5.787E+01	1
42	0.615	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
43	0.630	3.574E+05	-2.776E-17	1.265E+02	5.787E+01	1
44	0.645	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
45	0.660	3.574E+05	-2.776E-17	1.265E+02	5.787E+01	1
46	0.675	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
47	0.690	3.574E+05	-2.776E-17	1.265E+02	5.787E+01	1
48	0.705	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
49	0.720	3.574E+05	-2.776E-17	1.265E+02	5.787E+01	1
50	0.735	3.574E+05	0.000E+00	1.265E+02	5.787E+01	1
51	0.750	3.574E+05	0.000E+00	1.265E+02	0.000E+00	0

```
CPU time: 00:00:07.14 - this is CPU time for VAX 11/785
```

VAR

The VAR (VARiational) program is based upon the variational technique for calculating the correction factor integral. This is discussed and used in the RGENVAR program. The present implementation of this program makes use of the bounding procedure used in the BL code. As there are several integrals involved (coming from the mixing of the two current forms), the bounding procedure must be generalized to include all of the integrals.

The VAR program reads the input data from for011 (var.in is an example). The data in this file are in the following form.

npt	number of two-probe resistance and resistivity values
depth(i) rsp(i)	*i-th* value of depth (relative to surface)
	and *i-th* value of two-probe resistance (relative to surface)
aa	probe radius (micrometers)
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)

The code, like the RGENVAR code, consists of just the main program, VAR. This program uses the variational generalization of the Berkowitz-Lux approximation for the integrals, the extended Newton-Cotes quadrature, and the above-mentioned generalization of the bounding procedure to calculate the resistivity from the spreading resistance. Once finished, it writes onto for012 and for006. Examples of these files are contained in var.out and var.log. Like BL, the CPU needed to carry out the inversion is on the order of seconds.

Table 19. Annotated input data file (var.in) for VAR program
This is output data file (rgenvar.out) from RGENVAR program.

```
        51                      NPT-number of spreading resistance values
     0.000    8.0688E+02        DEPTH(1)        RSP(1)
     0.015    7.6620E+02        DEPTH(2)        RSP(2)
     0.030    7.4523E+02        DEPTH(3)        RSP(3)
     0.045    7.3371E+02          .               .
     0.060    7.2839E+02          .               .
     0.075    7.2830E+02          .               .
     0.090    7.3334E+02          .               .
     0.105    7.4399E+02          .               .
     0.120    7.6136E+02          .               .
     0.135    7.8711E+02          .               .
     0.150    8.2382E+02          .               .
     0.165    8.7534E+02          .               .
     0.180    9.4651E+02          .               .
     0.195    1.0445E+03          .               .
     0.210    1.1794E+03          .               .
     0.225    1.3646E+03          .               .
     0.240    1.6217E+03          .               .
     0.255    1.9775E+03          .               .
     0.270    2.4795E+03          .               .
     0.285    3.1891E+03          .               .
     0.300    4.2110E+03          .               .
     0.315    5.7027E+03          .               .
     0.330    7.9339E+03          .               .
     0.345    1.1367E+04          .               .
     0.360    1.6804E+04          .               .
     0.375    2.5663E+04          .               .
     0.390    4.0285E+04          .               .
     0.405    6.4054E+04          .               .
     0.420    1.0034E+05          .               .
     0.435    1.4903E+05          .               .
     0.450    2.0237E+05          .               .
     0.465    2.4762E+05          .               .
     0.480    2.7753E+05          .               .
     0.495    2.9401E+05          .               .
     0.510    3.0209E+05          .               .
     0.525    3.0550E+05          .               .
     0.540    3.0667E+05          .               .
     0.555    3.0701E+05          .               .
     0.570    3.0710E+05          .               .
     0.585    3.0713E+05          .               .
     0.600    3.0713E+05          .               .
     0.615    3.0714E+05          .               .
     0.630    3.0714E+05          .               .
     0.645    3.0714E+05          .               .
     0.660    3.0714E+05          .               .
     0.675    3.0714E+05          .               .
     0.690    3.0714E+05          .               .
     0.705    3.0714E+05          .               .
     0.720    3.0714E+05          .               .
     0.735    3.0714E+05          .               .
     0.750    3.0714E+05        DEPTH(NPT)      RSP(NPT)
  2.000000000000000            AA-probe radius in micrometers
  50.00000000000000            SEP-probe separation in micrometers
1.5000000000000000E-02         DELTH-depth increment (layer thickness)
```

Table 20. Annotated output data file (var.out) for VAR program

51		number of points
0.000E+00	1.536E+00	depth (micrometers) and resistivity (ohm*cm)
1.500E-02	6.835E-01	.
3.000E-02	3.563E-01	.
4.500E-02	1.873E-01	.
6.000E-02	1.110E-01	.
7.500E-02	7.565E-02	.
9.000E-02	5.400E-02	.
1.050E-01	4.000E-02	.
1.200E-01	3.067E-02	.
1.350E-01	2.419E-02	.
1.500E-01	1.960E-02	.
1.650E-01	1.650E-02	.
1.800E-01	1.440E-02	.
1.950E-01	1.310E-02	.
2.100E-01	1.250E-02	.
2.250E-01	1.240E-02	.
2.400E-01	1.300E-02	.
2.550E-01	1.410E-02	.
2.700E-01	1.610E-02	.
2.850E-01	1.900E-02	.
3.000E-01	2.330E-02	.
3.150E-01	2.940E-02	.
3.300E-01	3.820E-02	.
3.450E-01	5.140E-02	.
3.600E-01	7.220E-02	.
3.750E-01	1.072E-01	.
3.900E-01	1.732E-01	.
4.050E-01	3.079E-01	.
4.200E-01	6.084E-01	.
4.350E-01	1.333E+00	.
4.500E-01	3.177E+00	.
4.650E-01	8.018E+00	.
4.800E-01	2.014E+01	.
4.950E-01	4.650E+01	.
5.100E-01	8.210E+01	.
5.250E-01	1.088E+02	.
5.400E-01	1.216E+02	.
5.550E-01	1.264E+02	.
5.700E-01	1.265E+02	.
5.850E-01	1.265E+02	.
6.000E-01	1.265E+02	.
6.150E-01	1.265E+02	.
6.300E-01	1.265E+02	.
6.450E-01	1.265E+02	.
6.600E-01	1.265E+02	.
6.750E-01	1.265E+02	.
6.900E-01	1.265E+02	.
7.050E-01	1.265E+02	.
7.200E-01	1.265E+02	.
7.350E-01	1.265E+02	.
7.500E-01	1.265E+02	.

Table 21. Annotated log file (var.log) for VAR program

```
 CALCULATION OF THE RESISTIVITY FROM SPREADING RESISTANCE
 USING CHOO ET AL VARIATIONAL TECHNIQUE-BL
 AA=     0.000 SEP=  25.000 DELTH= 0.00750 NPT=   51
 R5= 0.7628E+00 R6= 0.7473E-02 R7= 0.2314E+03

 NPT      DEPTH      RMEAS      1-RCAL/R      RHO         ITER

  1     0.000E+00  8.069E+02   1.472E-04   1.536E+00      3
  2     1.500E-02  7.662E+02   5.933E-04   6.835E-01      3
  3     3.000E-02  7.452E+02  -9.894E-04   3.563E-01      4
  4     4.500E-02  7.337E+02   5.037E-05   1.873E-01      5
  5     6.000E-02  7.284E+02   8.414E-04   1.110E-01      3
  6     7.500E-02  7.283E+02   3.534E-04   7.565E-02      3
  7     9.000E-02  7.333E+02   1.571E-04   5.400E-02      3
  8     1.050E-01  7.440E+02   7.414E-05   4.000E-02      3
  9     1.200E-01  7.614E+02   3.881E-05   3.067E-02      3
 10     1.350E-01  7.871E+02   2.187E-05   2.419E-02      3
 11     1.500E-01  8.238E+02   1.193E-05   1.960E-02      3
 12     1.650E-01  8.753E+02   7.087E-06   1.650E-02      3
 13     1.800E-01  9.465E+02   4.265E-06   1.440E-02      3
 14     1.950E-01  1.045E+03   2.452E-06   1.310E-02      3
 15     2.100E-01  1.179E+03   1.613E-06   1.250E-02      3
 16     2.250E-01  1.365E+03   8.666E-07   1.240E-02      3
 17     2.400E-01  1.622E+03   6.332E-07   1.300E-02      3
 18     2.550E-01  1.978E+03   3.445E-07   1.410E-02      3
 19     2.700E-01  2.480E+03   2.576E-07   1.610E-02      3
 20     2.850E-01  3.189E+03   1.759E-07   1.900E-02      3
 21     3.000E-01  4.211E+03   1.600E-07   2.330E-02      3
 22     3.150E-01  5.703E+03   1.669E-07   2.940E-02      3
 23     3.300E-01  7.934E+03   1.904E-07   3.820E-02      3
 24     3.450E-01  1.137E+04   2.112E-07   5.140E-02      3
 25     3.600E-01  1.680E+04  -7.855E-04   7.220E-02      2
 26     3.750E-01  2.566E+04   9.868E-04   1.072E-01      2
 27     3.900E-01  4.029E+04   1.204E-05   1.732E-01      3
 28     4.050E-01  6.405E+04   1.208E-04   3.079E-01      3
 29     4.200E-01  1.003E+05   6.416E-04   6.084E-01      3
 30     4.350E-01  1.490E+05   5.601E-07   1.333E+00      5
 31     4.500E-01  2.024E+05   4.395E-06   3.177E+00      5
 32     4.650E-01  2.476E+05   4.609E-05   8.018E+00      5
 33     4.800E-01  2.775E+05   7.224E-04   2.014E+01      5
 34     4.950E-01  2.940E+05   5.300E-05   4.650E+01      7
 35     5.100E-01  3.021E+05   2.038E-04   8.210E+01      5
 36     5.250E-01  3.055E+05   2.889E-04   1.088E+02      3
 37     5.400E-01  3.067E+05   3.746E-05   1.216E+02      3
 38     5.550E-01  3.070E+05  -2.818E-04   1.264E+02      1
 39     5.700E-01  3.071E+05  -9.420E-05   1.265E+02      1
 40     5.850E-01  3.071E+05   7.280E-07   1.265E+02      1
 41     6.000E-01  3.071E+05  -3.161E-05   1.265E+02      1
 42     6.150E-01  3.071E+05  -2.776E-17   1.265E+02      1
 43     6.300E-01  3.071E+05  -2.776E-17   1.265E+02      1
 44     6.450E-01  3.071E+05  -2.776E-17   1.265E+02      1
 45     6.600E-01  3.071E+05  -2.776E-17   1.265E+02      1
 46     6.750E-01  3.071E+05  -2.776E-17   1.265E+02      1
 47     6.900E-01  3.071E+05  -2.776E-17   1.265E+02      1
 48     7.050E-01  3.071E+05  -2.776E-17   1.265E+02      1
 49     7.200E-01  3.071E+05  -2.776E-17   1.265E+02      1
 50     7.350E-01  3.071E+05  -2.776E-17   1.265E+02      1
 51     7.500E-01  3.071E+05   0.000E+00   1.265E+02      0
CPU time: 00:00:07.12 - this is CPU time for VAX 11/785
```

CALCULATION OF THE FOUR-PROBE RESISTANCE FROM THE RESISTIVITY

The third and final group of programs is used to calculate the four-probe resistance from the resistivity. The three programs included in this section are: PROBE24, FOURCAL, and SHAZAM. PROBE24 and FOURCAL are included for the calculation of the four-probe resistance by means of fixed point and trapezoidal Romberg techniques, respectively. On the other hand, SHAZAM is based upon an approximate form of the four-probe resistance integral which can be fairly easily calculated using a Newton-Cotes quadrature.

PROBE24

The multilayer expression for the in-line four-probe resistance, $Z_n(S)$, has been derived in the form [28]

$$Z_n(S) = 2\rho_n \int_0^\infty A_n(\lambda)\Big\{J_0(\lambda S) - J_0(2\lambda S)\Big\}\mathcal{I}_\nu(\lambda a)d\lambda,$$

where $\mathcal{I}_\nu(\lambda a)$ is the Hankel transform of the generalized probe-current density normalized to unit current [10], and ν is an index which refers to the specific form of the current density.

The PROBE24 code calculates the four-probe resistance from the resistivity using a 1250- or 10000-point sum to replace the above integral. In its present form, PROBE24 makes use of the uniform current density. As discussed in the FORTRAN source code, this can be changed with no difficulty. However, the results of the previous work [28,29] indicate that the four-probe resistance is independent of both the current density and the probe radius. Hence, the current density need not be altered.

The program reads the input data from for005. Probe24.in contains an example of the data file in the form containing the following information.

npt	number of data points
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
aa	probe radius (micrometers)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)
ibbc	backsurface boundary condition (none=1, conducting=2, insulating=3)
rho	array of resistivity values (Ω·cm)
nmany	number of integration points to be used (either 1250 or 10000)

The PROBE24 code consists of the main program along with the double precision functions dbj0(x) and dbj1(x) which are used to construct the Bessel functions which appear in the integral. The kernel function is evaluated directly inside the program. Once the calculation has been completed, the output data are written to for006, for011, for012, for013. Not all of the information contained in these files may be of interest. The user may wish to comment out the lines involving writing to logical devices 11, 12, and 13.

Table 22. Annotated input data file (probe24.in) for PROBE24 program

```
2.0                     AA - probe radius in micrometers
50.0                    SEP - probe separation in micrometers
0.015                   DELTH - depth increment in micrometers
50                      NPT - number of points
0.015                   SUBT - substrate thickness in micrometers
126.5                   SUTRHO - substrate resistivity in ohm*cm
1                       IBBC - substrate boundary condition (none=1,con=2,ins=3)
1.542                   RHO(NPT+1)
0.6933                  RHO(NPT)
0.3442                      .
0.19                        .
0.1159                      .
0.0769                      .
0.0543                      .
0.0401                      .
0.0307                      .
0.0242                      .
0.0196                      .
0.0165                      .
0.0144                      .
0.0131                      .
0.0125                      .
0.0124                      .
0.013                       .
0.0141                      .
0.0161                      .
0.019                       .
0.0233                      .
0.0294                      .
0.0382                      .
0.0514                      .
0.0721                      .
0.1074                      .
0.1732                      .
0.3081                      .
0.6095                      .
1.334                       .
3.18                        .
8.036                       .
20.4                        .
46.62                       .
82.75                       .
109.7                       .
121.4                       .
125.1                       .
126.1                       .
126.4                       .
126.4                       .
126.5                       .
126.5                       .
126.5                       .
126.5                       .
126.5                       .
126.5                       .
126.5                       .
126.5                       .
1250                    NMANY - number of integration points
```

Table 23. Annotated log file (probe24.log) for PROBE24 program

```
 FOUR-PROBE RESISTANCE CALCULATIONS USING THE
 BERKOWITZ NMANY=   1250 POINT INTEGRATOR
 STRUCTURE OVER UNIFORM LAYER (SUBRHO,SUBTH)
  ALL OVER (NONE=1,CON=2,INS=3)    1

   AA        SEP        DELTH      SUBTH         SUBRHO
  2.00        50.0     0.015      1.50E-02      1.27E+02

 DEPTH(UM)       RHO        FOUR-PROBE  DERIVATIVE     SHEET
 0.0000E+00  1.5420E+00  1.4397E+02  6.6443E+02  6.7505E+02
 1.5000E-02  6.9330E-01  1.4406E+02  6.6206E+02  6.7549E+02
 3.0000E-02  3.4420E-01  1.4426E+02  6.6176E+02  6.7648E+02
 4.5000E-02  1.9000E-01  1.4468E+02  6.6307E+02  6.7848E+02
 6.0000E-02  1.1590E-01  1.4543E+02  6.6623E+02  6.8213E+02
 7.5000E-02  7.6900E-02  1.4668E+02  6.7183E+02  6.8821E+02
 9.0000E-02  5.4300E-02  1.4861E+02  6.8063E+02  6.9757E+02
 1.0500E-01  4.0100E-02  1.5143E+02  6.9358E+02  7.1128E+02
 1.2000E-01  3.0700E-02  1.5542E+02  7.1199E+02  7.3072E+02
 1.3500E-01  2.4200E-02  1.6096E+02  7.3761E+02  7.5778E+02
 1.5000E-01  1.9600E-02  1.6858E+02  7.7291E+02  7.9512E+02
 1.6500E-01  1.6500E-02  1.7906E+02  8.2148E+02  8.4664E+02
 1.8000E-01  1.4400E-02  1.9333E+02  8.8776E+02  9.1724E+02
 1.9500E-01  1.3100E-02  2.1276E+02  9.7822E+02  1.0141E+03
 2.1000E-01  1.2500E-02  2.3919E+02  1.1016E+03  1.1474E+03
 2.2500E-01  1.2400E-02  2.7499E+02  1.2695E+03  1.3306E+03
 2.4000E-01  1.3000E-02  3.2388E+02  1.5000E+03  1.5858E+03
 2.5500E-01  1.4100E-02  3.9004E+02  1.8143E+03  1.9410E+03
 2.7000E-01  1.6100E-02  4.8062E+02  2.2491E+03  2.4460E+03
 2.8500E-01  1.9000E-02  6.0349E+02  2.8469E+03  3.1680E+03
 3.0000E-01  2.3300E-02  7.7069E+02  3.6759E+03  4.2246E+03
 3.1500E-01  2.9400E-02  9.9631E+02  4.8234E+03  5.8028E+03
 3.3000E-01  3.8200E-02  1.2986E+03  6.4145E+03  8.2434E+03
 3.4500E-01  5.1400E-02  1.6960E+03  8.6069E+03  1.2189E+04
 3.6000E-01  7.2100E-02  2.1961E+03  1.1548E+04  1.8918E+04
 3.7500E-01  1.0740E-01  2.7738E+03  1.5248E+04  3.1196E+04
 3.9000E-01  1.7320E-01  3.3372E+03  1.9285E+04  5.5282E+04
 4.0500E-01  3.0810E-01  3.7498E+03  2.2663E+04  1.0606E+05
 4.2000E-01  6.0950E-01  3.9502E+03  2.4545E+04  2.1930E+05
 4.3500E-01  1.3340E+00  4.0121E+03  2.5212E+04  4.7645E+05
 4.5000E-01  3.1800E+00  4.0255E+03  2.5442E+04  1.0263E+06
 4.6500E-01  8.0360E+00  4.0281E+03  2.5663E+04  1.9892E+06
 4.8000E-01  2.0400E+01  4.0289E+03  2.6010E+04  3.1640E+06
 4.9500E-01  4.6620E+01  4.0294E+03  2.6410E+04  4.1232E+06
 5.1000E-01  8.2750E+01  4.0298E+03  2.6704E+04  4.7539E+06
 5.2500E-01  1.0970E+02  4.0300E+03  2.6849E+04  5.2022E+06
 5.4000E-01  1.2140E+02  4.0301E+03  2.6902E+04  5.6006E+06
 5.5500E-01  1.2510E+02  4.0301E+03  2.6917E+04  6.0170E+06
 5.7000E-01  1.2610E+02  4.0301E+03  2.6921E+04  6.4848E+06
 5.8500E-01  1.2640E+02  4.0301E+03  2.6922E+04  7.0269E+06
 6.0000E-01  1.2640E+02  4.0301E+03  2.6923E+04  7.6661E+06
 6.1500E-01  1.2650E+02  4.0301E+03  2.6923E+04  8.4333E+06
 6.3000E-01  1.2650E+02  4.0301E+03  2.6923E+04  9.3704E+06
 6.4500E-01  1.2650E+02  4.0301E+03  2.6923E+04  1.0542E+07
 6.6000E-01  1.2650E+02  4.0301E+03  2.6923E+04  1.2048E+07
 6.7500E-01  1.2650E+02  4.0301E+03  2.6923E+04  1.4056E+07
 6.9000E-01  1.2650E+02  4.0301E+03  2.6923E+04  1.6867E+07
 7.0500E-01  1.2650E+02  4.0301E+03  2.6923E+04  2.1083E+07
 7.2000E-01  1.2650E+02  4.0301E+03  2.6923E+04  2.8111E+07
 7.3500E-01  1.2650E+02  4.0301E+03  2.6923E+04  4.2167E+07
 7.5000E-01  1.2650E+02  4.0301E+03  2.6923E+04  8.4333E+07

 CPU time: 00:00:11.87 - this is CPU time for VAX 11/785
```

FOURCAL

The FOURCAL code makes use of the trapezoidal Romberg numerical integration to evaluate the integral in the four-probe resistance,

$$Z_n(S) = 2\rho_n \int_0^\infty A_n(\lambda)\Big\{ J_0(\lambda S) - J_0(2\lambda S)\Big\}\mathcal{I}_\nu(\lambda a)d\lambda.$$

This technique has been discussed previously in the RGENTR program.

FOURCAL performs the calculation for any of the three forms of the probe current density. Calculations using FOURCAL were originally employed to show that the four-probe resistance is independent of the probe current density and the probe radius. The program reads the input data from for005. Fourcal.in contains an example of the data file in the form containing the following information.

npt	number of data points
aa	probe radius (micrometers)
sep	separation between the probes (micrometers)
delth	depth increment (micrometers)
ibbc	backsurface boundary condition (none =1, conducting =2, insulating =3)
subrho	substrate resistivity (Ω·cm)
subth	substrate thickness (micrometers)
rho	array of resistivity values (Ω·cm)
icur	current density (Schumann Gardner=1, Choo uniform=2, ring delta=3)
xl	lower limit of integration (usually zero)
xu	upper limit of integration (from 10 on up)

To run the program for a structure directly on a backsurface boundary, take the last point in the structure as the substrate and set subth=delth and subrho=rho(1).

The FORTRAN source code consists of the following parts:

fourcal	the main program
intgrl(f,a,b,atemp,iflag)	the subroutine which carries out the numerical integration
fpint(x)	the double precision function which constructs the integrand of the four-probe resistance integral
deint(x)	the double precision function which constructs the integrand of the logarithmic derivative function (related to the Dickey conjecture)
alam(x)	the double precision function which constructs the kernel of the integral by means of the recursion relation
dbj0(x) and dbj1(x)	the two double precision functions generate the two Bessel functions, J_0 and J_1

Once the calculation has been completed, the output data are written to for006, for011, for012, for013. Not all of the information contained in these files may be of interest. The user may wish to comment out the lines involving writing to logical devices 11, 12, and 13.

Table 24. Annotated input data file (fourcal.in) for FOURCAL program

```
50                  NPT - number of points in the structure
2.0                 AA - probe radius in micrometers
50.0                SEP - probe separation in micrometers
0.015               DELTH - depth increment in micrometers
1                   IBBC - boundary condition (none=1,cond=2,ins=3)
126.5               SUBRHO - substrate resistivity in ohm*cm
0.015               SUBTH - substrate thickness in micrometers
1.542               RHO(NPT+1)
0.6933              RHO(NPT)
0.3442              RHO(NPT-1)
0.1900                   .
0.1159                   .
0.0769                   .
0.0543                   .
0.0401                   .
0.0307                   .
0.0242                   .
0.0196                   .
0.0165                   .
0.0144                   .
0.0131                   .
0.0125                   .
0.0124                   .
0.013                    .
0.0141                   .
0.0161                   .
0.019                    .
0.0233                   .
0.0294                   .
0.0382                   .
0.0514                   .
0.0721                   .
0.1074                   .
0.1732                   .
0.3081                   .
0.6095                   .
1.334                    .
3.18                     .
8.036                    .
20.4                     .
46.62                    .
82.75                    .
109.7                    .
121.4                    .
125.1                    .
126.1                    .
126.4                    .
126.4                    .
126.5                    .
126.5                    .
126.5                    .
126.5                    .
126.5                    .
126.5                    .
126.5                    .
126.5                    .
2                   ICUR - current density (S&G=1, unif=2, ring=3)
0.0                 XL - lower limit of integral (usually zero)
10.0                XU - upper limit of integral
```

Table 25. Annotated log file (fourcal.log) for FOURCAL program

```
CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X
NO BOUNDARY CONDITION USED IN CALCULATION

XL=   0.000000; XU=  10.000000
RADIUS=   2.000000; SEP=  50.000000; DELTH=   0.015000
SUBRHO=1.26500E+02; SUBTHICK=1.50000E-02
```

DEPTH(U)	RHO	FOUR-PROBE	DERIV	SHEET
0.000E+00	1.542E+00	1.437E+02	6.686E+02	6.750E+02
1.500E-02	6.933E-01	1.438E+02	6.638E+02	6.755E+02
3.000E-02	3.442E-01	1.440E+02	6.625E+02	6.765E+02
4.500E-02	1.900E-01	1.444E+02	6.633E+02	6.785E+02
6.000E-02	1.159E-01	1.451E+02	6.662E+02	6.821E+02
7.500E-02	7.690E-02	1.464E+02	6.716E+02	6.882E+02
9.000E-02	5.430E-02	1.483E+02	6.803E+02	6.976E+02
1.050E-01	4.010E-02	1.511E+02	6.932E+02	7.113E+02
1.200E-01	3.070E-02	1.551E+02	7.115E+02	7.307E+02
1.350E-01	2.420E-02	1.606E+02	7.370E+02	7.578E+02
1.500E-01	1.960E-02	1.682E+02	7.723E+02	7.951E+02
1.650E-01	1.650E-02	1.786E+02	8.207E+02	8.466E+02
1.800E-01	1.440E-02	1.928E+02	8.869E+02	9.172E+02
1.950E-01	1.310E-02	2.122E+02	9.772E+02	1.014E+03
2.100E-01	1.250E-02	2.385E+02	1.100E+03	1.147E+03
2.250E-01	1.240E-02	2.742E+02	1.268E+03	1.331E+03
2.400E-01	1.300E-02	3.228E+02	1.498E+03	1.586E+03
2.550E-01	1.410E-02	3.887E+02	1.812E+03	1.941E+03
2.700E-01	1.610E-02	4.789E+02	2.246E+03	2.446E+03
2.850E-01	1.900E-02	6.013E+02	2.843E+03	3.168E+03
3.000E-01	2.330E-02	7.680E+02	3.671E+03	4.225E+03
3.150E-01	2.940E-02	9.933E+02	4.818E+03	5.803E+03
3.300E-01	3.820E-02	1.295E+03	6.409E+03	8.243E+03
3.450E-01	5.140E-02	1.693E+03	8.602E+03	1.219E+04
3.600E-01	7.210E-02	2.195E+03	1.155E+04	1.892E+04
3.750E-01	1.074E-01	2.773E+03	1.525E+04	3.120E+04
3.900E-01	1.732E-01	3.338E+03	1.929E+04	5.528E+04
4.050E-01	3.081E-01	3.750E+03	2.268E+04	1.061E+05
4.200E-01	6.095E-01	3.950E+03	2.457E+04	2.193E+05
4.350E-01	1.334E+00	4.012E+03	2.527E+04	4.765E+05
4.500E-01	3.180E+00	4.025E+03	2.558E+04	1.026E+06
4.650E-01	8.036E+00	4.028E+03	2.596E+04	1.989E+06
4.800E-01	2.040E+01	4.028E+03	2.660E+04	3.164E+06
4.950E-01	4.662E+01	4.028E+03	2.734E+04	4.123E+06
5.100E-01	8.275E+01	4.028E+03	2.789E+04	4.754E+06
5.250E-01	1.097E+02	4.028E+03	2.815E+04	5.202E+06
5.400E-01	1.214E+02	4.028E+03	2.825E+04	5.601E+06
5.550E-01	1.251E+02	4.028E+03	2.828E+04	6.017E+06
5.700E-01	1.261E+02	4.028E+03	2.829E+04	6.485E+06
5.850E-01	1.264E+02	4.028E+03	2.829E+04	7.027E+06
6.000E-01	1.264E+02	4.028E+03	2.829E+04	7.666E+06
6.150E-01	1.265E+02	4.028E+03	2.829E+04	8.433E+06
6.300E-01	1.265E+02	4.028E+03	2.829E+04	9.370E+06
6.450E-01	1.265E+02	4.028E+03	2.829E+04	1.054E+07
6.600E-01	1.265E+02	4.028E+03	2.829E+04	1.205E+07
6.750E-01	1.265E+02	4.028E+03	2.829E+04	1.406E+07
6.900E-01	1.265E+02	4.028E+03	2.829E+04	1.687E+07
7.050E-01	1.265E+02	4.028E+03	2.829E+04	2.108E+07
7.200E-01	1.265E+02	4.028E+03	2.829E+04	2.811E+07
7.350E-01	1.265E+02	4.028E+03	2.829E+04	4.217E+07
7.500E-01	1.265E+02	4.028E+03	2.829E+04	8.433E+07

```
CPU time: 00:31:25.85 - this is CPU time for VAX 11/785
```

SHAZAM

Using the PROBE24 and FOURCAL programs, extensive numerical investigations have shown that the four-probe resistance is: (i) independent of the probe radius and the probe-current density, and (ii) simply related to the sheet resistance, $\mathcal{R}_n$, when the distance to an insulating boundary is small compared with the probe spacing. The evaluation of the four-probe resistance integral was found to be simplified by the alternative form given by

$$Z_n(S) = \frac{\rho_n}{\pi} \int_{1/2S}^{1/S} A_n(\lambda) d\lambda.$$

In the above form, the four-probe resistance can easily be calculated using the Newton-Cotes method of evaluation,

$$Z_n(S) = \frac{\rho_n}{\pi} \ln(2) \sum_{i=0}^{8} w_{9,i} \lambda_i A_n(\lambda_i),$$

where $\lambda_i = 2^{i/8}/2S (0 \leq i \leq 8), w_{9,i} = C_i/8$, and where C_i are the Newton-Cotes weighting factors.

The SHAZAM code carries out the above calculation of the four-probe resistance using the approximate four-probe integral and the nine-point Newton-Cotes quadrature. The program reads the input data from for009. Shazam.in contains an example of the data file in the form containing the following information.

n	number of resistivity, sheet resistance values
sep	probe separation (micrometers)
delth	sublayer thickness (micrometers)
ibbc	boundary condition (none=0; conducting=1; insulating=2)
subrho	substrate resistivity (Ω·cm)
subt	substrate thickness (micrometers)
rho(i)	*i-th* resistivity relative to the surface

Once the calculation has been completed, the output data are written to for006, for011, for012. Not all of the information contained in these files may be of interest.

Table 26. Annotated input data file (shazam.in) for SHAZAM program

```
50.0                    SEP - probe separation in micrometers
0.015                   DELTH - depth increment in micrometers
50                      N - number of points
126.5                   SUBRHO - substrate resistivity in ohm*cm
0.015                   SUBT - substrate thickness in micrometers
1                       IBBC - boundary condition (none-1,con-2,ins-3)
1.542                   RHO(NPT+1)
0.6933                  RHO(NPT)
0.3442                  RHO(NPT-1)
0.19                          .
0.1159                        .
0.0769                        .
0.0543                        .
0.0401                        .
0.0307                        .
0.0242                        .
0.0196                        .
0.0165                        .
0.0144                        .
0.0131                        .
0.0125                        .
0.0124                        .
0.013                         .
0.0141                        .
0.0161                        .
0.019                         .
0.0233                        .
0.0294                        .
0.0382                        .
0.0514                        .
0.0721                        .
0.1074                        .
0.1732                        .
0.3081                        .
0.6095                        .
1.334                         .
3.18                          .
8.036                         .
20.4                          .
46.62                         .
82.75                         .
109.7                         .
121.4                         .
125.1                         .
126.1                         .
126.4                         .
126.4                         .
126.5                         .
126.5                         .
126.5                         .
126.5                         .
126.5                         .
126.5                         .
126.5                         .
126.5                         .
```

Table 27. Annotated log file (shazam.log) for SHAZAM program

```
 FOUR-PROBE RESISTANCE CALCULATED BY SIMPLIFIED  INTEGRAL
 ALSO CALCULATED IS THE EDUCATED GUESS OF THE LN(S) DERIVATIVE FORM
 NO BOUNDARY AT     0.765 UM.  SUBSTRATE AT   0.750 UM
 SEP=   50.  DELTH=  0.015  SUBTH=1.500E-02  SUBRHO=1.265E+02

 LOWER LIMIT= 0.01000000  UPPER LIMIT= 0.02000000

  DEPTH      RHO          FOUR-PROBE    DERIVATIVE

  0.000    1.5420E+00     1.4347E+02    4.2657E-02
  0.015    6.9330E-01     1.4355E+02    9.4924E-02
  0.030    3.4420E-01     1.4374E+02    1.9146E-01
  0.045    1.9000E-01     1.4415E+02    3.4783E-01
  0.060    1.1590E-01     1.4490E+02    5.7320E-01
  0.075    7.6900E-02     1.4614E+02    8.7138E-01
  0.090    5.4300E-02     1.4805E+02    1.2504E+00
  0.105    4.0100E-02     1.5085E+02    1.7255E+00
  0.120    3.0700E-02     1.5481E+02    2.3137E+00
  0.135    2.4200E-02     1.6030E+02    3.0407E+00
  0.150    1.9600E-02     1.6786E+02    3.9338E+00
  0.165    1.6500E-02     1.7824E+02    4.9659E+00
  0.180    1.4400E-02     1.9237E+02    6.1480E+00
  0.195    1.3100E-02     2.1159E+02    7.4446E+00
  0.210    1.2500E-02     2.3769E+02    8.7827E+00
  0.225    1.2400E-02     2.7299E+02    1.0197E+01
  0.240    1.3000E-02     3.2105E+02    1.1483E+01
  0.255    1.4100E-02     3.8586E+02    1.2790E+01
  0.270    1.6100E-02     4.7411E+02    1.3860E+01
  0.285    1.9000E-02     5.9290E+02    1.4827E+01
  0.300    2.3300E-02     7.5279E+02    1.5548E+01
  0.315    2.9400E-02     9.6519E+02    1.6070E+01
  0.330    3.8200E-02     1.2436E+03    1.6297E+01
  0.345    5.1400E-02     1.5994E+03    1.6030E+01
  0.360    7.2100E-02     2.0331E+03    1.5046E+01
  0.375    1.0740E-01     2.5231E+03    1.3045E+01
  0.390    1.7320E-01     3.0135E+03    1.0057E+01
  0.405    3.0810E-01     3.4296E+03    6.6577E+00
  0.420    6.0950E-01     3.7199E+03    3.7388E+00
  0.435    1.3340E+00     3.8870E+03    1.8097E+00
  0.450    3.1800E+00     3.9685E+03    7.8036E-01
  0.465    8.0360E+00     4.0038E+03    3.1246E-01
  0.480    2.0400E+01     4.0179E+03    1.2366E-01
  0.495    4.6620E+01     4.0233E+03    5.4210E-02
  0.510    8.2750E+01     4.0255E+03    3.0562E-02
  0.525    1.0970E+02     4.0263E+03    2.3060E-02
  0.540    1.2140E+02     4.0265E+03    2.0839E-02
  0.555    1.2510E+02     4.0266E+03    2.0224E-02
  0.570    1.2610E+02     4.0266E+03    2.0063E-02
  0.585    1.2640E+02     4.0266E+03    2.0016E-02
  0.600    1.2640E+02     4.0266E+03    2.0016E-02
  0.615    1.2650E+02     4.0266E+03    2.0000E-02
  0.630    1.2650E+02     4.0266E+03    2.0000E-02
  0.645    1.2650E+02     4.0266E+03    2.0000E-02
  0.660    1.2650E+02     4.0266E+03    2.0000E-02
  0.675    1.2650E+02     4.0266E+03    2.0000E-02
  0.690    1.2650E+02     4.0266E+03    2.0000E-02
  0.705    1.2650E+02     4.0266E+03    2.0000E-02
  0.720    1.2650E+02     4.0266E+03    2.0000E-02
  0.735    1.2650E+02     4.0266E+03    2.0000E-02
  0.750    1.2650E+02     4.0266E+03    2.0000E-02

 CPU time: 00:00:06.69 - this is CPU time for VAX 11/785
```

AVAILABILITY OF RESPAC SOFTWARE PACKAGE

The source codes for the RESPAC software package are written in FORTRAN77 and have been run on a VAX 11/785 minicomputer system. These codes should run on other systems with little, if any, need for modification.

In all, there are ten programs contained in the RESPAC software package. The FORTRAN77 source code (total of about 123 kbytes) and sample input and output data files are available in ASCII format using a number of transfer vehicles. These include: standard 8-track magnetic tape (ASCII, density = 1600, Record = 80, block = 1600), 5.25-in. (360-kbyte and 1.2-Mbyte) DOS-formatted floppy disks, and using electronic mail over the Internet. This package is self-contained and is straightforward to run once the FORTRAN is compiled and linked by the user-supplied software. The sample input and output data files are included so that the user can check the programs for proper operation as well as to become acquainted with the setup and use of the codes.

For more information or to receive a copy of the code and the report, please contact:

John Albers
NIST, Bldg. 225, Room B-310
Gaithersburg, MD USA 20899
Telephone: 1-301-975-2075
FAX Number: 1-301-948-4081
Netmail (Internet) address: Albers@sed.eeel.nist.gov

In requesting a copy of the RESPAC software package, please specify the transfer mode and include an 8-track tape or a DOS-formatted floppy disk. Requests over Internet will be answered by return e-mail in ASCII mode.

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REFERENCES

[1] Zinner, E., Sputter Depth Profiling of Microelectronic Structures, *Journal of the Electrochemical Society* **130**, 199-222 (1983).

[2] Chu, W. K., Mayer, J. W., and Nicolet, M. A., *Backscattering Spectroscopy* (Academic Press, 1978).

[3] Downing, R. G., Maki, J. T., and Fleming, R. F., Application of Neutron Depth Profiling to Microelectronic Materials Processing, pp. 163-180, in ACS Symposium Series 295, *Microelectronics Processing: Inorganic Materials Characterization*, Casper, L. A., Ed., American Chemical Society, Washington, DC, 1986.

[4] Ehrstein, J. R., Two Probe (Spreading Resistance) Measurements for Evaluation of Semiconductor Materials and Devices, in *Nondestructive Evaluation of Semiconductor Materials and Devices*, J. N. Zemel, Ed. (Plenum Press, New York, 1979), pp. 1-66.

[5] Wilson, R. G., and Jamba, D. M., *Semiconductor Measurement Technology:* Differential Capacitance-Voltage Profiling of Schottky Barrier Diodes for Measuring Implanted Depth Distributions in Silicon, NBS Special Publication 400-71 (February 1982).

[6] Putley, E. H., *The Hall Effect and Related Phenomena* (Butterworth & Co., Ltd., London, 1960).

[7] Schumann, P. A., and Gardner, E. E., Application of Multilayer Potential Distribution to Spreading Resistance Correction Factors, *Journal of the Electrochemical Society* **116**, 87-91 (1969).

[8] Gardner, E. E., and Schumann, P. A., Spreading Resistance Correction Factors, *Solid-State Electronics* **12**, 371-375 (1969).

[9] Koefoed, O., *The Application of the Kernel Function in Interpretation of Geoelectrical Resistivity Measurements* (Gebrüder Borntraeger, Berlin-Stuttgart, 1968).

[10] Berkowitz, H. L., and Lux, R. A., Errors in Resistivities Calculated by Multilayer Analysis of Spreading Resistance, *Journal of the Electrochemical Society* **126**, 1479-1482 (1979).

[11] Yeh, T. H., and Kokhani, K. H., Multilayer Theory of Correction Factors for Spreading-Resistance Measurements, *Journal of the Electrochemical Society* **116**, 1461-1464 (1969).

[12] Hu, S. M., Calculation of Spreading Resistance Correction Factors, *Solid-State Electronics* **15**, 809-817 (1972).

[13] Iida, Y., Abe, H., and Kondo, M., Impurity Profile Measurements of Thin Epitaxial

Silicon Wafer by Multilayer Spreading Resistance Analysis, *Journal of the Electrochemical Society* **124**, 1118-1122 (1977).

[14] Sunde, E. D., *Earth Conduction Effects in Transmission Systems* (Dover, New York, 1968).

[15] Choo, S. C., Leong, M. S., and Kuan, K. L., On the Calculation of Spreading Resistance Correction Factors, *Solid-State Electronics* **19**, 561-565 (1976).

[16] D'Avanzo, D. C., Rung, R. D., Gat, A., and Dutton, R. W., High Speed Implementation and Experimental Evaluation of Multilayer Spreading Resistance Analysis, *Journal of the Electrochemical Society* **125**, 1170-1176 (1978).

[17] Choo, S. C., Leong, M. S., Hong, H. L., Li, L., and Tan, L. S., Spreading Resistance Calculations by the Use of Gauss-Laguerre Quadrature, *Solid-State Electronics* **21**, 769-774 (1978).

[18] Dickey, D. H., and Ehrstein, J. R., *Semiconductor Measurement Technology*: Spreading Resistance Analysis for Silicon Layers with Nonuniform Resistivity, NBS Special Publication 400-48 (May 1979).

[19] Albers, J., Comparison of Spreading Resistance Correction Factor Algorithms Using Model Data, *Solid-State Electronics* **23**, 1197-1205 (1980).

[20] Albers, J., The Relation Between the Correction Factor and the Local Slope in Spreading Resistance, *Journal of the Electrochemical Society* **130**, 2076-2080 (1983).

[21] Berkowitz, H. L., and Lux, R. A., An Efficient Integration Technique for Use in the Multilayer Analysis of Spreading Resistance Profiles, *Journal of the Electrochemical Society* **128**, 1137-1141 (1981).

[22] Choo, S. C., Leong, M. S., and Tan, L. S., Spreading Resistance Calculations by the Variational Method, *Solid-State Electronics* **24**, 557-562 (1981).

[23] Choo, S. C., Leong, M. S., and Sim, J. H., An Efficient Numerical Scheme for Spreading Resistance Calculations Based on the Variational Method, *Solid-State Electronics* **26**, 723-730 (1983).

[24] Albers, J., Probe Spacing Experiment Simulation and the Relation Between Spreading Resistance and Sheet Resistance, *Journal of the Electrochemical Society* **129**, 599-605 (1982).

[25] Albers, J., Spreading Resistance Probe Spacing Experiment Simulations: Effects of Probe Current Density and Layer Thickness, *Journal of the Electrochemical Society* **129**, 2788-2795 (1982).

[26] Ehrstein, J. R., Albers, J., Wilson, R. G., and Comas, J., Comparison of Spreading

Resistance with C-V and SIMS Profiles for Submicron Layers in Silicon, *Extended Abstracts* The Electrochemical Society, **80-1**, 496-498 (1980).

[27] Piessens, R., Vandervorst, W. B., and Maes, H. E., Incorporation of a Resistivity Dependent Contact Radius in an Accurate Integration Algorithm for Spreading Resistance Calculations, *Journal of the Electrochemical Society* **130**, 468-474 (1983).

[28] Albers, J., and Berkowitz, H. L., The Relation Between Two-Probe and Four-Probe Resistances on Nonuniform Structures, *Journal of the Electrochemical Society* **131**, 392-398 (1984).

[29] Albers, J., and Berkowitz, H. L., An Alternative Approach to the Calculation of Four-Probe Resistances on Nonuniform Structures, *Journal of the Electrochemical Society* **132**, 2453-2456 (1985).

[30] Kopanski, J. J., Albers, J. Carver, G. P., and Ehrstein, J. R., Verification of the Relation Between Two-Probe and Four-Probe Resistances as Measured on Silicon Wafers, *Journal of the Electrochemical Society* **137**, 3935-3941 (1990).

[31] Choo, S. C., and Leong, M. S., A Multilayer Exponential Model for Spreading Resistance Calculations, *Solid-State Electronics* **22**, 405-415 (1979).

[32] Albers, J., Continuum Formulation of Spreading Resistance Correction Factors, *Journal of the Electrochemical Society* **127**, 2259-2263 (1980).

[33] Hu, S. M., Between Carrier Distributions and Dopant Atomic Distribution in Beveled Silicon Substrates, *Journal of Applied Physics* **53**, 1499-1510 (1982).

[34] Albers, J., Wilson, C. L., and Blue, J. L., Effect of Surface Beveling on Carrier Profiles, *Extended Abstracts of the Electrochemical Society* **83-1**, 641-642 (1983).

[35] Blue, J. L., and Wilson, C. L., Two-Dimensional Analysis of Semiconductor Devices Using General-Purpose Interactive PDE Software, *IEEE Transactions on Electron Devices* **ED-30**, 1056-1070 (1983).

[36] Blue, J. L., and Wilson, C. L., Two-Dimensional Analysis of Semiconductor Devices Using General-Purpose Interactive PDE Software, *SIAM Journal Scientific Statistical Computing* **4**, 462-484 (1983).

[37] Kurata, M., *Numerical Analysis for Semiconductor Devices* (Lexington Books, Lexington, Mass., 1982).

[38] Selberherr, S., *Analysis and Simulation of Semiconductor Devices* (Springer-Verlag, New York, 1984).

```
C*********************************************************************
C     RGENTR VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT TWO-PROBE (SPREADING)
C     RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C     THE CORRECTION FACTOR INTEGRAL IS CALCULATED OVER THE INTERVAL FROM
C     XL TO XU BY MEANS OF A TRAPEZOIDAL ROMBERG METHOD USED ORIGINALLY BY
C     D'AVANZO ET AL. IN THEIR "SPINT" PROGRAM FOR CALCULATING THE ARRAY OF
C     PARTIAL INTEGRALS.
C
C     REFERENCES: THE TRAPEZOIDAL ROMBERG METHOD WAS USED BY D'AVANZO ET AL.
C                 FOR CALCULATING THE CORRECTION FACTOR INTEGRAL FOR THE
C                 PURPOSE OF SPREADING RESISTANCE PROFILE ANALYSIS.  THIS
C                 IS DESCRIBED IN THE REPORT "SPREADING RESISTANCE FOR
C                 IMPURITY PROFILES" BY DONALD C. D'AVANZO, ROBERT D. RUNG,
C                 AND ROBERT W. DUTTON, TECHNICAL REPORT NO. 5013-2 (FEBRUARY
C                 1977), STANFORD UNIVERSITY, STANFORD, CALIFORNIA.  THIS IS
C                 ALSO DESCRIBED IN THE PAPER "HIGH SPEED IMPLEMENTATION AND
C                 EXPERIMENTAL EVALUATION OF MULTILAYER SPREADING RESISTANCE
C                 ANALYSIS" BY DONALD C. D'AVANZO, ROBERT D. RUNG, ARNON GAT,
C                 AND ROBERT W. DUTTON, JOURNAL OF THE ELECTROCHEMICAL SOCIETY,
C                 VOL. 125, PP.1170-1176 (1978).
C                 THE CALCULATION OF THE SPREADING RESISTANCE FROM THE
C                 RESISTIVITY USING THE TRAPEZOIDAL ROMBERG MEHTOD IS DISCUSSED
C                 IN THE PAPER "COMPARISON OF SPREADING RESISTANCE CORRECTION
C                 FACTOR ALGORITHMS USING MODEL DATA" BY JOHN ALBERS,
C                 SOLID-STATE ELECTRONICS, VOL. 23, PP. 1197-1205 (1980).
C
C     THE CALCULATION MAY BE CARRIED OUT FOR THE CHOO ET AL. UNIFORM CURRENT
C     DENSITY, THE SCHUMANN AND GARDNER CURRENT DENSITY, OR THE RING DELTA
C     FUNCTION CURRENT DENSITY.
C
C     THE PROGRAM READS (FROM FOR010) THE NUMBER OF POINTS IN THE STRUCTURE,
C     THE PROBE RADIUS, THE PROBE SEPARATION, DEPTH INCREMENT (IN MICROMETERS),
C     THE SUBSTRATE RESISTIVITY, SUBSTRATE THICKNESS, THE BACKSURFACE BOUNDARY
C     CONDITION, THE RESISTIVIES OF THE STRUCTURE (FROM THE TOP SURFACE), THE
C     PROBE CURRENT DENSITY, AND THE LOWER AND UPPER LIMITS OF INTEGRATION.
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING, OR FURTHER ANALYSIS.
C
C*********************************************************************
```

```
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL TPINT
      DIMENSION RHO(300),CF(300),RSP(300),DEPTH(300),VOL(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      DATA PI/3.141592654D0/
C*********************************************************************
C
C                ----INPUT SECTION---
C     THE FOLLOWING VARIABLES ARE READ BY THE PROGRAM FROM FORO10
C         NPT----NUMBER OF DATA POINTS IN THE STRUCTURE.  TOTAL NUMBER OF
C                POINTS IS NUM=NPT+1 (INCLUDE SUBSTRATE POINT)
C         AA-----PROBE RADIUS (MICROMETERS)
C         SEP----SEPARATION BETWEEN THE PROBES (MICROMETERS)
C         DELTH--DEPTH INCREMENT (MICROMETERS)
C         IBBC---BACKSURFACE BOUNDARY CONDITION
C                (NONE=1, COND=2, INS=3)
C         SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM)
C         SUBTH--SUBSTRATE THICKNESS (MICROMETERS)
C         RHO----ARRAY OF RESISTIVITY VALUES (OHM*CM)
C         ICUR---FORM OF THE CURRENT DENSITY
C                (SCHUMANN GARDNER=1, CHOO UNIFORM=2, RING DELTA =3)
C         XL-----LOWER LIMIT OF CORRECTION FACTOR INTEGRAL (USUALLY ZERO)
C         XU-----UPPER LIMIT OF CORRECTION FACTOR INTEGRAL
C
C     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY,
C     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
C     SUBTH=DELTH AND SUBRHO=RHO(1).
C
C*********************************************************************
C     READ THE TOTAL NUMBER OF POINTS OR SUBLAYERS IN STRUCTURE.
      READ (10,*) NPT
      NUM=NPT+1
C     READ THE EFFECTIVE ELECTRICAL PROBE RADIUS
      READ (10,*) AA
C     READ THE SEPARATION BETWEEN THE PROBES
      READ (10,*) SEP
C     READ THE DISTANCE BETWEEN POINTS OR THE SUBLAYER THICKNESS.
      READ (10,*) DELTH
C     READ THE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ (10,*) IBBC
C     READ THE SUBSTRATE RESISTIVITY
      READ (10,*) SUBRHO
C     READ THE SUBSTRATE THICKNESS
      READ (10,*) SUBTH
```

```
C     NEXT LOOP READS THE RESISTIVITIES FROM THE
C     SURFACE TO THE SUBSTRATE.
      DO 10 I=NUM,2,-1
      READ (10,*) RHO(I)
   10 CONTINUE
      RHO(1)=SUBRHO
C     READ THE FORM OF THE CURRENT DENSITY
C     (SCHUMANN GARDNER=1, CHOO UNIF=2, RING=3)
      READ (10,*) ICUR
C     READ THE LOWER LIMIT OF INTEGRATION (USUALLY ZERO)
      READ (10,*) XL
C     READ THE UPPER LIMIT OF INTEGRATION (20/AA USUALLY GOOD FOR XU)
      READ (10,*) XU
C*********************************************************************
C
C     THE 1200 DO LOOP IS THE MAJOR LOOP WHICH CALCULATES THE CORRECTION
C     FACTOR INTEGRALS (OVER THE INTERVAL FROM XL TO XU) FROM THE SUBSTRATE
C     (NUM1=NUM) TO THE SURFACE (NUM1=1) BY INCREMENTING NUM1 BY -1.
C     THE RELEVANT INFORMATION IS PASSED THROUGH AND CONTAINED IN THE
C     COMMON STATEMENT IN THE MAIN PROGRAM AND THE SUBROUTINES.
C     IFLAG IS SET EQUAL TO ZERO AS XL IS USUALLY ZERO AND THE INTEGRAND
C     IS ZERO AT XL=0.  HOWEVER, THE CURRENT DENSITY FUNCTIONS MAY GIVE
C     ARTIFICIAL SINGULARITIES AT THE ORIGIN.  FOR EXAMPLE, THE SCHUMANN
C     AND GARDNER CURRENT DENSITY FUNCTION, SIN(X)/X, GOES TO ZERO AS X
C     GOES TO ZERO BUT THE COMPUTER WOULD SIGNAL AN OVERFLOW DUE TO THE
C     1/X.  THIS IS REMOVED BY THE SETTING OF IFLAG=0.
C
C*********************************************************************
      DO 1200 NUM1=1,NUM
      IFLAG=0
      CALL INTGRL(TPINT,XL,XU,CF(NUM1),IFLAG)
      CF(NUM1)=4.0D0*CF(NUM1)/PI
      RSP(NUM1)=CF(NUM1)*RHO(NUM1)/(2.0D-4*AA)
      DEPTH(NUM1)=DELTH*(NUM-NUM1)
      VOL(NUM1)=DLOG10(RSP(NUM1))
 1200 CONTINUE
```

```
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE (6,1300)
 1300 FORMAT(2X,'CALCULATION OF SPREADING RESISTANCE FROM THE
     1 RESISTIVITY',/,2X,
     2 'USING THE TRAPEZOIDAL ROMBERG INTEGRATION METHOD')
      GO TO (1400,1420,1440),ICUR
 1400 WRITE(6,1410)
 1410 FORMAT(2X,'SCHUMANN AND GARDNER CURRENT DENSITY=SIN(AX)/X')
      GO TO 1453
 1420 WRITE (6,1430)
 1430 FORMAT(2X,'CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X')
      GO TO 1453
 1440 WRITE (6,1450)
 1450 FORMAT(2X,'RING CURRENT DENSITY=A*DBJ0(AX)')
 1453 GO TO (1455,1465,1475), IBBC
 1455 WRITE (6,1460)
 1460 FORMAT (2X,'NO BOUNDARY (INFINITE SLAB) CONDITION USED
     1 IN CALCULATION',/)
      GO TO 1500
 1465 WRITE (6,1470)
 1470 FORMAT (2X,'PERFECTLY CONDUCTING BOUNDARY A1=TANH(LD)',/)
      GO TO 1500
 1475 WRITE (6,1480)
 1480 FORMAT (2X,'PERFECTLY INSULATING BOUNDARY A1=COTH(LD)',/)
 1500 WRITE (6,1550)
 1550 FORMAT(2X,'   XL         XU')
      WRITE (6,1600) XL,XU
 1600 FORMAT(2X,2(F11.6,1X))
      WRITE (6,1700)
 1700 FORMAT(2X,' RADIUS      SEP      DELTH')
      WRITE (6,2200) AA,SEP,DELTH
 2200 FORMAT (1X,3(F11.6,1X))
      WRITE (6,2250) SUBRHO,SUBTH
 2250 FORMAT(2X,'SUBRHO=',1PE11.5,'; SUBTHICK=',1PE11.5)
      WRITE (6,2300)
 2300 FORMAT(2X,'DEPTH(U)',8X,'RHO',5X,'CORR-FAC',8X,
     1    'RSP',7X,'VOLTAGE')
      WRITE (11,*)NUM
      DO 2500 I=NUM,1,-1
      WRITE (6,2400)  DEPTH(I),RHO(I),CF(I),RSP(I),VOL(I)
 2400 FORMAT(1X,6(1PE11.5,1X))
      WRITE (11,2405)DEPTH(I),RSP(I)
 2405 FORMAT(1X,2(1PE12.4,1X))
 2500 CONTINUE
      WRITE (11,*)AA
      WRITE (11,*)SEP
      WRITE (11,*)DELTH
      STOP
      END
C*****END OF THE MAIN PROGRAM PART OF RGENTR*******************************
```

```
      SUBROUTINE INTGRL(F,A,B,ATEMP,IFLAG)
C*************************************************************************
C
C     D'AVANZO'S ET AL. TRAPEZOIDAL ROMBERG INTEGRATION SUBROUTINE.
C     THIS ROUTINE NUMERICALLY INTEGRATES F BETWEEN THE LIMITS A AND B
C     USING THE TRAPEZOIDAL AND ROMBERG METHODS AND RETURNS THE INTEGRAL
C     FROM A TO B IN ATEMP AND ACCUMULATES THE RESULTS OF SEVERAL
C     SUCCESSIVE CALLS IN ACC.
C     IF IFLAG=0 THEN F(0) IS SET TO 0 (THIS OBVIATES OVERFLOW PROBLEMS
C     DUE TO FUNCTIONS LIKE SIN(X)/X
C
C*************************************************************************
c      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL F
      DOUBLE PRECISION AI(30,30),MI,A,B,H,ACC,ATEMP,F0,F,CONV
c      DIMENSION AI(30,30)
      CONV=5.0D-5
      J=30
      M=1
      K=1
      F0=0.D0
      IF ((A.NE.0.D0).OR.(IFLAG.EQ.1)) F0=F(A)
      AI(M,K)=.5D0*(B-A)*(F0+F(B))
      DO 30 I=1,J
      MI=0.0D0
      H=(B-A)/(2.0D0**(M-1))
      LPCNT=2**(M-1)
      DO 10 N=1,LPCNT
      MI=MI+F(A+(N*H-.5D0*H))
10    CONTINUE
      MI=MI*H
      AI(M+1,K)=.5D0*(AI(M,K)+MI)
      MPONE = M+1
      DO 20 K=1,MPONE
      AI(M+1,K+1)=((4.0D0**K)*AI(M+1,K)-AI(M,K))/(4.0D0**K-1.D0)
20    CONTINUE
      ATEMP=AI(M+1,M+1)
      IF (I.LE.6) GO TO 60
C     STOP INTEGRATING WHEN INTEGRAL CONVERGES TO WITHIN CONV (.005%)
      IF (DABS(ATEMP-AI(M,M))/DABS(ATEMP).GT.CONV) GO TO 60
53    ACC=ACC+ATEMP
      RETURN
60    M=M+1
      K=1
30    CONTINUE
      RETURN
      END
C*****END OF INTGRL SUBROUTINE*****************************************
```

```
      DOUBLE PRECISION FUNCTION TPINT(X)
C*********************************************************************
C
C     TPINT IS THE INTEGRAND OF THE SCHUMANN AND GARDNER MULTILAYER
C     LAPLACE EXPRESSION FOR TWO-PROBE RESISTANCE INTEGRAL WITH A CHOICE
C     OF ONE OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY.
C     LINE 10 CONTAINS THE PROBE-TO-PROBE CONTRIBUTION WHILE THE CHOICE
C     OF THE PROBE CURRENT DENSITY IS PROVIDED ON THE FOLLOWING LINES:
C        100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY
C        200-THE CHOO ET AL. UNIFORM CURRENT DENSITY
C        300-THE RING DELTA FUNCTION CURRENT DENSITY
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHO(300)
C      DOUBLE PRECISION X,DBJ0,DBJ1,AA,SEP,ALAM,DELTH,RHO(300),XL,XU
C      DOUBLE PRECISION BESINT,SUBRHO,SUBTH
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
 10   PRTPR=(DBJ1(AA*X)/(AA*X)-DBJ0(SEP*X)/2.D0)
      GO TO (100,200,300),ICUR
 100  TPINT=ALAM(X)*PRTPR*DSIN(X*AA)/X
      RETURN
 200  TPINT=ALAM(X)*PRTPR*2.D0*DBJ1(AA*X)/X
      RETURN
 300  TPINT=ALAM(X)*PRTPR*AA*DBJ0(AA*X)
      RETURN
      END
C*****END OF FTPINT DOUBLE PRECISION FUNCTION*****************************
      DOUBLE PRECISION FUNCTION ALAM(X)
C*********************************************************************
C
C     THIS FUNCTION CALCULATES THE VALUE OF THE RESISTIVITY DEPENDENT
C     PORTION OF THE VARIOUS INTEGRALS USING THE RECURSION RELATION OF
C     CHOO ET AL. AND EXTENDED BY BERKOWITZ AND LUX.
C
C     REFERENCES: "ON THE CALCULATION OF SPREADING RESISTANCE CORRECTION
C                 FACTORS" BY S. C. CHOO, M. S. LEONG AND K. L. KUAN,
C                 SOLID-STATE ELECTRONICS, VOL. 19, PP. 561-565 (1976).
C                 "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE
C                 MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES"
C                 BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE
C                 ELECTROCHEMICAL SOCIETY, VOL. 128, PP. 1137-1141 (1981).
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHO(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      DTXSB=DTANH(X*SUBTH)
      DTXDL=DTANH(X*DELTH)
      ALAM=1.0D0
      IF (IBBC.EQ.2) ALAM=DTXSB
      IF (IBBC.EQ.3) ALAM=1.0D0/DTXSB
      IF (NUM1.EQ.1) GO TO 110
      DO 100 I=2,NUM1
      ALAM=(DTXDL*RHO(I)+RHO(I-1)*ALAM)/(RHO(I)+RHO(I-1)*ALAM*DTXDL)
```

```
 100  CONTINUE
 110  RETURN
      END
C*****END OF ALAM DOUBLE PRECISION FUNCTION*********************************
      DOUBLE PRECISION FUNCTION DBJ0(X)
C***********************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ0
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.1 IS USED FOR X < 3
C                AND EQUATION 9.4.3 IS USED FOR X > 3
C
C***********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F0,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/1.0000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/
      DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
      DATA A7/0.0002100D0/
      DATA B1/.79788456D0/,B2/-.00000077D0/,B3/-.00552740D0/
      DATA B4/-.00009512D0/,B5/.00137237D0/,B6/-.00072805D0/
      DATA B7/.00014476D0/
      DATA C1/-.78539816D0/,C2/-.04166397D0/,C3/-.00003954D0/
      DATA C4/.00262573D0/,C5/-.00054125D0/,C6/-.00029333D0/
      DATA C7/.00013558D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.0D0
      DBJ0=((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
      RETURN
10    CONTINUE
      X1=3.D0/X
      F0=((((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ0=F0*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DBJ0 DOUBLE PRECISION FUNCTION*********************************
```

```
      DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.4 IS USED FOR X < 3
C                AND EQUATION 9.4.6 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F1,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/
      DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/
      DATA A7/.00001109D0/
      DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/
      DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/
      DATA B7/-.00020033D0/
      DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/
      DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/
      DATA C7/-.00029166D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.D0
      DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
      RETURN
   10 X1=3.0D0/X
      F1=((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ1=F1*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF RGENTR V1.0 PROGRAM****
```

```
C*********************************************************************
C     RGENGL  VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING)
C     RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C     THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE CHOO ET AL.
C     33-POINT INTEGRATION METHOD BASED UPON THE GAUSS-LAGUERRE QUADRATURE.
C     THE 33 SAMPLING POINTS AND WEIGHTS ARE INITIALIZED BY DATA
C     STATEMENTS.
C     THE FINAL SET OF 33 WEIGHTS USED IN THE CALCULATIONS ARE CALCULATED
C     FROM THE INITIAL SET OF WEIGHTS AND THE PROBE CURRENT DENSITY.
C     THE DISCUSSION OF THE 33-POINT GAUSS-LAGUERRE QUADRATURE MAY BE
C     FOUND IN THE REFERENCE.
C
C     REFERENCE: "SPREADING RESISTANCE CALCULATIONS BY THE GAUSS-LAGUERRE
C                 QUADRATURE," BY S. C. CHOO, M. S. LEONG, H. L. HONG,
C                 L. LE, AND L. S. TAN,
C                 SOLID STATE ELECTRONICS VOL 21, PP. 769-774, 1978.
C
C     THE CURRENT DENSITIES WHICH MAY BE USED ARE THE SCHUMANN AND GARDNER,
C     THE CHOO ET AL. UNIFORM CURRENT DENSITY, AND THE RING DELTA FUNCTION
C     CURRENT DENSITY.
C
C     THE PROGRAM READS (FROM FOR010) THE PROBE RADIUS, PROBE SEPARATION,
C     DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE
C     RESISTIVITY, SUBSTRATE THICKNESS, THE RESISTIVITY ARRAY (FROM THE
C     TOP SURFACE), AND THE PROBE CURRENT DENSITY.
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING, OR FURTHER ANALYSIS.
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION T(33),W(33),A(2,33),RHO(500),RSP(500),DEPTH(500)
      DIMENSION CF(500),VOL(500)
      DOUBLE PRECISION KA(33),L(33),INTGRL(500)
```

```
C*************************************************************************
C
C     THE SET OF 33 FIXED SAMPLING POINTS, KA(I), AND THE INITIAL SET OF
C     WEIGHTS, W(I), I=1,33, ARE ENTERED THROUGH THE DATA STATEMENTS
C
C*************************************************************************
      DATA KA(1)/.011251388D0/,KA(2)/.059284741D0/
      DATA KA(3)/.145707967D0/,KA(4)/.270553179D0/
      DATA KA(5)/.433841408D0/,KA(6)/.635597666D0/
      DATA KA(7)/.875852385D0/,KA(8)/1.154641702D0/
      DATA KA(9)/1.472007563D0/,KA(10)/1.827997778D0/
      DATA KA(11)/2.222666072D0/,KA(12)/2.65607213D0/
      DATA KA(13)/3.128281655D0/,KA(14)/3.63936642D0/
      DATA KA(15)/4.18940433D0/,KA(16)/4.778479488D0/
      DATA KA(17)/5.406682272D0/,KA(18)/6.074109403D0/
      DATA KA(19)/6.78086404D0/,KA(20)/7.527055861D0/
      DATA KA(21)/8.312801165D0/,KA(22)/9.138222971D0/
      DATA KA(23)/10.003451129D0/,KA(24)/10.908622439D0/
      DATA KA(25)/11.853880769D0/,KA(26)/12.839377192D0/
      DATA KA(27)/13.865270123D0/,KA(28)/14.931725465D0/
      DATA KA(29)/16.038916768D0/,KA(30)/17.187025392D0/
      DATA KA(31)/18.376240681D0/,KA(32)/19.606760148D0/
      DATA KA(33)/20.878789667D0/
      DATA W(1)/.028874906D0/,W(2)/.067219473D0/
      DATA W(3)/.10563085D0/,W(4)/.14406297D0/
      DATA W(5)/.18251763D0/,W(6)/.22099994D0/
      DATA W(7)/.25951551D0/,W(8)/.2980701D0/
      DATA W(9)/.33666957D0/,W(10)/.3753198D0/
      DATA W(11)/.41402672D0/,W(12)/.45279633D0/
      DATA W(13)/.49163467D0/,W(14)/.53054783D0/
      DATA W(15)/.56954199D0/,W(16)/.60862338D0/
      DATA W(17)/.6477983D0/,W(18)/.69707314D0/
      DATA W(19)/.7264544D0/,W(20)/.7659486D0/
      DATA W(21)/.80556248D0/,W(22)/.84530276D0/
      DATA W(23)/.88517632D0/,W(24)/.9251902D0/
      DATA W(25)/.96535161D0/,W(26)/1.00566765D0/
      DATA W(27)/1.04614585D0/,W(28)/1.08679375D0/
      DATA W(29)/1.12761908D0/,W(30)/1.16862972D0/
      DATA W(31)/1.20983377D0/,W(32)/1.25123946D0/
      DATA W(33)/1.2928553D0/,PI/3.14159265D0/
```

```
C*************************************************************************
C
C     INPUT SECTION--THE FOLLOWING VARIABLES ARE READ FROM FOR010
C           N------NUMBER OF POINTS (TOTAL NUMBER IS N+1)
C           AA-----PROBE RADIUS (MICROMETERS)
C           SEP----PROBE SEPARATION (MICROMETERS)
C           DELTH--SUBLAYER THICKNESS (MICROMETERS)
C           IBBC---BOUNDARY CONDITION (NONE=1, CONDUCTING=2, INSULATING=3)
C           SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM)
C           SUBTH--SUBSTRATE THICKNESS (MICROMETERS)
C           RHO( )-RESISTIVITY ARRAY (OHM*CM)
C           ICUR---CURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3)
C
C     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY,
C     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
C     SUBTH=DELTH AND SUBRHO=RHO(1).
C
C*************************************************************************
C     READ THE NUMBER OF DATA POINTS IN THE STRUCTURE (EXCLUSIVE OF SUBSTRATE)
      READ (10,*) N
C     READ THE PROBE RADIUS (MICROMETERS)
      READ (10,*) AA
C     READ THE PROBE SEPARATION (MICROMETERS)
      READ (10,*) SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ (10,*) DELTH
C     READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ (10,*) IBBC
C     READ THE SUBSTRATE RESISTIVITY (OHM*CM)
      READ (10,*) SUBRHO
C     READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ (10,*) SUBTH
C     READ THE RESISTIVITIES (OHM*CM) OF THE STRUCTURE
      DO 10 I=1,N
      READ (10,*) RHO(I)
  10  CONTINUE
      RHO(N+1)=SUBRHO
C     READ THE PROBE CURRENT DENSITY
C          (SCHUMANN AND GARDNER=1, CHOO UNIFORM=2, RING DELTA=3)
      READ (10,*) ICUR
      DO 11 I=1,N+1
      DEPTH(I)=DELTH*(I-1)
 11   CONTINUE
```

```
      WRITE(6,7777)
7777  FORMAT (1H1)
      WRITE(6,12)
12    FORMAT(2X,'CALCULATION OF THE SPREADING RESISTANCE FROM THE
     1 RESISTIVITY',/,2X,'USING THE 33-POINT GAUSS-LAGUERRE SCHEME')
      GO TO (15,20,25), IBBC
15    WRITE(6,17)SUBTH+N*DELTH,N*DELTH
17    FORMAT(2X,'NO BOUNDARY AT',F10.3,' UM. SUBSTRATE AT',F8.3,'UM')
      GO TO 29
20    WRITE(6,21)SUBTH+N*DELTH,N*DELTH
21    FORMAT(2X,'CONDUCTING BOUNDARY AT',F10.3,' UM. SUBSTRATE AT',
     1 F8.3,' UM')
      GO TO 29
25    WRITE(6,26)SUBTH+N*DELTH,N*DELTH
26    FORMAT(2X,'INSULATING BOUNDARY AT',F10.3,' UM. SUBSTRATE AT',
     1 F8.3,' UM')
29    CONTINUE
      GO TO (30,40,50), ICUR
30    WRITE(6,35)
35    FORMAT(2X,'SCHUMANN AND GARDNER CURRENT DENSITY')
      GO TO 60
40    WRITE(6,45)
45    FORMAT(2X,'CHOO UNIFORM CURRENT DENSITY')
      GO TO 60
50    WRITE(6,55)
55    FORMAT(2X,'RING DELTA FUNCTION CURRENT DENSITY')
60    WRITE(6,65)AA,SEP,DELTH,SUBTH,SUBRHO,N
65    FORMAT(2X,'AA=',F6.3,' SEP=',F7.1,' DELTH=',F6.3,' SUBTH=',
     1 1PE11.3,' SUBRHO=',1PE11.3,' #PT=',I3)
      DO 70 I=1,33
      L(I)=KA(I)/AA
      T(I)=DTANH(L(I)*DELTH)
70    CONTINUE
```

```
C**********************************************************************
C
C     THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3) IS
C     INCORPORATED IN THE CALCULATION OF THE KERNEL FUNCTION FOR THE FIRST
C     POINT IN THE CALCULATION, THE SUBSTRATE POINT.  FOR THE INFINITE
C     SLAB BOUNDARY CONDITION, THE KERNEL FUNCTION IS UNITY.  FOR THE
C     CONDUCTING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE TANH FUNCTION.
C     FOR THE INSULATING BOUNDARY CONDITION, THE KERNEL IS THE COTH FUNCTION.
C     THESE THREE FORMS FOLLOW FROM THE FORM OF THE KERNEL FUNCTION PRESENTED
C     IN THE REFERENCE.
C
C**********************************************************************
      DO 80 I=1,33
      R1=L(I)*SUBTH
      A(1,I)=1.0D0
      A(2,I)=1.0D0
      IF (IBBC.EQ.2) A(1,I)=DTANH(R1)
      IF (IBBC.EQ.3) A(1,I)=1.0D0/DTANH(R1)
  80  CONTINUE
C**********************************************************************
C
C     THE FINAL SET OF WEIGHTS, W(I) (I=1,33) ARE NOW CONSTRUCTED FROM THE
C     INITIAL SET OF WEIGHTS AND THE TWO-PROBE CONFIGURATION FUNCTION (IN
C     LINE 90) AND THE FORM OF THE CURRENT DENSITY FUNCTION (IN THE
C     APPROPRIATE NEXT LINE DEPENDING UPON THE VALUE OF ICUR).
C
C**********************************************************************
      DO 100 I=1,33
  90  W(I)=W(I)*(DBJ1(KA(I))/KA(I)**2-DBJ0(KA(I)*SEP/AA)/(2.D0*KA(I)))
      IF (ICUR.EQ.1) W(I)=W(I)*DSIN(KA(I))/2.D0
      IF (ICUR.EQ.2) W(I)=W(I)*DBJ1(KA(I))
      IF (ICUR.EQ.3) W(I)=W(I)*KA(I)*DBJ0(KA(I))/2.D0
 100  CONTINUE
C**********************************************************************
C
C     THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION
C     FACTOR INTEGRAL FOR THE SUBSTRATE POINT.  THE CORRECTION FACTOR, THE
C     SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C
C**********************************************************************
      R2=0.0D0
      DO 200 I=1,33
      R2=R2+W(I)*A(1,I)
 200  CONTINUE
      INTGRL(N+1)=R2
      CF(N+1)=(8.0D0*R2)/PI
      RSP(N+1)=RHO(N+1)*CF(N+1)*1D4/(2.0D0*AA)
      VOL(N+1)=DLOG10(RSP(N+1))
```

```
C*************************************************************************
C
C     THE 500 LOOP IS THE MAJOR LOOP WHICH CONTAINS THE CALCULATION OF THE
C     CORRECTION FACTOR INTEGRAL FOR THE N-POINT LAYERED STRUCTURE STARTING
C     FROM THE DEEPEST POINT TO THE TOP SURFACE.  THE CORRECTION FACTOR,
C     THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C
C*************************************************************************
      DO 500 KK=N,1,-1
      R2=0.0D0
      R3=RHO(KK+1)/RHO(KK)
      DO 250 I=1,33
      A(2,I)=(R3*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R3)
 250  CONTINUE
      DO 275 I=1,33
      R2=R2+W(I)*A(2,I)
 275  CONTINUE
      INTGRL(KK)=R2
      CF(KK)=(8.0D0*INTGRL(KK))/PI
      RSP(KK)=RHO(KK)*CF(KK)*1D4/(2.0D0*AA)
      VOL(KK)=DLOG10(RSP(KK))
      DO 350 I=1,33
      A(1,I)=A(2,I)
 350  CONTINUE
 500  CONTINUE
C*************************************************************************
C
C     END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C
C*************************************************************************
      WRITE(6,550)
 550  FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X,
     1 'RSP',8X,'VOL')
      WRITE(11,*)N+1
      DO 600 I=1,N+1
      WRITE(6,650)DEPTH(I),RHO(I),INTGRL(I),CF(I),RSP(I),VOL(I)
 650  FORMAT(F8.3,5(1PE12.4))
      WRITE(11,675)DEPTH(I),RSP(I)
 675  FORMAT(2X,F7.3,2X,1PE12.4)
 600  CONTINUE
      WRITE(11,*)AA
      WRITE(11,*)SEP
      WRITE(11,*)DELTH
      WRITE(11,*)ICUR
      STOP
      END
C*****END OF MAIN PROGRAM PART OF RGENGL**********************************
```

```
      DOUBLE PRECISION FUNCTION DBJO(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJO
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.1 IS USED FOR X < 3
C                EQUATION 9.4.3 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,FO,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/1.0000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/
      DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
      DATA A7/0.0002100D0/
      DATA B1/.79788456D0/,B2/-.00000077D0/,B3/-.00552740D0/
      DATA B4/-.00009512D0/,B5/.00137237D0/,B6/-.00072805D0/
      DATA B7/.00014476D0/
      DATA C1/-.78539816D0/,C2/-.04166397D0/,C3/-.00003954D0/
      DATA C4/.00262573D0/,C5/-.00054125D0/,C6/-.00029333D0/
      DATA C7/.00013558D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.0D0
      DBJO=(((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
      RETURN
10    CONTINUE
      X1=3.D0/X
      FO=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJO=FO*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DBJO DOUBLE PRECISION FUNCTION*********************************
```

```
      DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.4 IS USED FOR X < 3
C                AND EQUATION 9.4.6 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F1,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/
      DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/
      DATA A7/.00001109D0/
      DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/
      DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/
      DATA B7/-.00020033D0/
      DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/
      DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/
      DATA C7/-.00029166D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.D0
      DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
      RETURN
   10 X1=3.0D0/X
      F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+((((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ1=F1*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF RGENGL V1.0 PROGRAM*****
```

```
C*********************************************************************
C     RGENBL VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING)
C     RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C     THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE BERKOWITZ AND
C     LUX APPROXIMATE INTEGRAL AND THEIR 22-POINT INTEGRATION METHOD BASED
C     UPON THE NEWTON-COTES QUADRATURE.
C
C     REFERENCE: "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE
C                 MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES",
C                 BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE
C                 ELECTROCHEMICAL SOCIETY, VOL 128, PP. 1137-1141 (1981).
C
C     THE SET OF SAMPLING POINTS AND WEIGHTS ARE SPECIALIZED TO THE CHOO
C     ET AL. UNIFORM CURRENT DENSITY (ALTHOUGH THE SETS OF SAMPLING POINTS
C     AND WEIGHTS MAY BE CONSTRUCTED FOR THE SCHUMANN AND GARDENER CURRENT
C     DENSITY OR THE RING DELTA FUNCTION CURRENT DENSITY BY FOLLOWING THE
C     REASONING PRESENTED IN THE REFERENCE).
C
C     THE PROGRAM READS (FROM FOR010) THE NUMBER OF POINTS IN THE STRUCTURE,
C     THE PROBE RADIUS, THE PROBE SEPARATION, THE DEPTH INCREMENT, THE
C     BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE RESISTIVITY, THE
C     SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY (FROM THE TOP SURFACE).
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING OR FURTHER ANALYSIS.
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RSP(500),RHO(500),DEPTH(500),CF(500),VOL(500)
      DIMENSION A(2,22),C(9),W(22),T(22)
      DOUBLE PRECISION LAMDA(22),KA(22),INGRL(500)
```

```
C*************************************************************************
C
C     THE SET OF FIXED (PROBE-SPACING INDEPENDENT) SAMPLING POINTS, KA(I),
C     AND WEIGHTS, W(I), (I=9,...,22) ARE ENTERED BY THE DATA STATEMENTS.
C     THE 9 POINT NEWTON-COTES COEFFICIENTS ARE ENTERED BY DATA STATEMENTS.
C
C*************************************************************************
      DATA KA(9)/0.1D0/,KA(10)/0.177828D0/,KA(11)/0.316228D0/
      DATA KA(12)/0.562341D0/,KA(13)/1.0D0/,KA(14)/1.70793D0/
      DATA KA(15)/2.41586D0/,KA(16)/3.12378D0/,KA(17)/5.23853D0/
      DATA KA(18)/8.48093D0/,KA(19)/11.6681D0/,KA(20)/14.829D0/
      DATA KA(21)/17.9861D0/,KA(22)/39.3D0/,PI/3.14159265D0/
      DATA W(9)/0.00446607D0/,W(10)/0.03611D0/,W(11)/0.0236709D0/
      DATA W(12)/0.106292D0/,W(13)/0.0773292D0/,W(14)/0.115397D0/
      DATA W(15)/0.0172741D0/,W(16)/0.0087751D0/,W(17)/0.00668606D0/
      DATA W(18)/0.00163407D0/,W(19)/0.000628701D0/,W(20)/0.0003062D0/
      DATA W(21)/0.0001717D0/,W(22)/0.000412D0/
      DATA C(1)/0.0348854D0/,C(2)/0.20769D0/,C(3)/-0.0327337D0/
      DATA C(4)/0.370229D0/,C(5)/-0.160141D0/
      C(6)=C(4)
      C(7)=C(3)
      C(8)=C(2)
      C(9)=C(1)
C*************************************************************************
C
C     INPUT SECTION--THE FOLLOWING VARIABLES ARE READ BY THE PROGRAM
C         NPT----NUMBER OF DATA POINTS IN STRUCTURE (EXCLUDING SUBSTRATE)
C         AA-----PROBE RADIUS (MICROMETERS)
C         SEP----SEPARATION BETWEEN THE PROBES (MICROMETERS)
C         DELTH--DEPTH INCREMENT (MICROMETERS)
C         IBBC---BACKSURFACE BOUNDARY CONDITION (NONE=1,COND=2,INS=3)
C         SUBRHO-SUBSTRATE RESISTIVITY (OHM*CM)
C         SUBTH--SUBSTRATE THICKNESS (MICROMETERS)
C         RHO----ARRAY OF RESISTIVITY VALUES (OHM*CM)
C
C     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY,
C     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
C     SUBTH=DELTH AND SUBRHO=RHO(1).
C
C*************************************************************************
C     READ THE NUMBER OF POINTS IN THE STRUCTURE (EXCLUSIVE OF SUBSTRATE)
      READ (10,*) N
C     READ THE PROBE RADIUS (MICROMETERS)
      READ (10,*) AA
C     READ THE PROBE SPACING (MICROMETERS)
      READ (10,*) SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ (10,*) DELTH
C     READ THE BACK SURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ (10,*) IBBC
C     READ THE SUBSTRATE RESITIVITY (OHM*CM)
      READ (10,*) SUBRHO
C     READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ (10,*) SUBTH
C     READ THE N DEPTH-DEPENDENT VALUES OF THE STRUCTURE RESISTIVITY (OHM*CM)
```

```
      DO 70 I=1,N
      READ (10,*) RHO(I)
 70   CONTINUE
      RHO(N+1)=SUBRHO
      WRITE(6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
 80   FORMAT(2X,'CALCULATION OF SPREADING RESISTANCE FROM THE
     1 RESISTIVITY',/,2X,'BERKOWITZ-LUX APPROXIMATE INTEGRAL WITH
     2 THE UNIFORM CURRENT DENSITY')
      GO TO (90, 100, 110), IBBC
 90   WRITE(6,95)SUBTH+N*DELTH,N*DELTH
 95   FORMAT(2X,'NO BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',F8.3,' UM')
      GO TO 120
 100  WRITE(6,105)SUBTH+N*DELTH,N*DELTH
 105  FORMAT(2X,'CONDUCTING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
      GO TO 120
 110  WRITE(6,115)SUBTH+N*DELTH,N*DELTH
 115  FORMAT(2X,'INSULATING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
 120  WRITE(6,130)AA,SEP,DELTH,SUBTH,SUBRHO,N
 130  FORMAT(2X,'RADIUS=',F6.4,2X,'SEP=',F6.1,2X,'DELTH=',F7.3,2X,
     1 'SUBTH=',1PE9.3,2X,'SUBRHO=',1PE9.3,' # PTS= ',I3)
      DO 140 I=1,N+1
      DEPTH(I)=DELTH*(I-1)
 140  CONTINUE
      Q=1.12292D0*(AA/SEP)
      QQ=DLOG(0.1/Q)
C*************************************************************************
C
C     HERE THE SAMPLING POINTS, KA (I), AND WEIGHTS, W(I), FOR THE FIRST
C     NINE POINTS (WHICH DEPEND UPON THE PROBE SPACING) ARE CALCULATED.
C
C*************************************************************************
      DO 150 I=1,8
      KA(I)=Q*(0.1D0/Q)**((I-1.0)/8.0)
 150  CONTINUE
      DO 155 I=1,22
      LAMDA(I)=KA(I)/AA
 155  CONTINUE
      DO 160 I=1,8
      W(I)=0.25D0*QQ*KA(I)*C(I)
 160  CONTINUE
      W(9)=W(9)+0.249376D0*QQ*KA(9)*C(9)
      DO 180 I=1,22
      R1=LAMDA(I)*DELTH
      T(I)=DTANH(R1)
 180  CONTINUE
```

```
C*************************************************************************
C
C     THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3) IS
C     INCORPORATED IN THE CALCULATION OF THE KERNEL FUNCTION FOR THE FIRST
C     POINT IN THE CALCULATION, THE SUBSTRATE POINT.
C     FOR THE INFINITE SLAB BOUNDARY CONDITION, THE KERNEL FUNCTION IS UNITY.
C     FOR THE CONDUCTING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE TANH
C     FUNCTION.
C     FOR THE INSULATING BOUNDARY CONDITION, THE KERNEL FUNCTION IS THE COTH
C     FUNCTION.
C     THESE THREE FORMS FOLLOW FROM THE FORM OF THE KERNEL FUNCTION PRESENTED
C     IN THE REFERENCE.
C
C*************************************************************************
      DO 190 I=1,22
      R1=LAMDA(I)*SUBTH
      A(1,I)=1.0D0
      A(2,I)=1.0D0
      IF(IBBC.EQ.2) A(1,I)=DTANH(R1)
      IF(IBBC.EQ.3) A(1,I)=1.0D0/DTANH(R1)
 190  CONTINUE
C*************************************************************************
C
C     THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION
C     FACTOR INTEGRAL FOR THE SUBSTRATE POINT.  THE CORRECTION FACTOR, THE
C     SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C
C*************************************************************************
      R2=0.0D0
      DO 200 I=1,22
      R2=R2+W(I)*A(1,I)
 200  CONTINUE
      INGRL(N+1)=R2
      CF(N+1)=(8.0D0*R2)/PI
      RSP(N+1)=RHO(N+1)*CF(N+1)*1.0D4/(2.0D0*AA)
      VOL(N+1)=DLOG10(RSP(N+1))
```

```
C*************************************************************************
C
C     THE 500 LOOP IS THE MAJOR LOOP WHICH CONTAINS THE CALCULATION OF THE
C     CORRECTION FACTOR INTEGRAL FOR THE N-POINT LAYERED STRUCTURE STARTING
C     FROM THE DEEPEST POINT TO THE TOP SURFACE.  THE CORRECTION FACTOR,
C     THE SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C
C*************************************************************************
      DO 500 KK=N,1,-1
      R2=0.0D0
      R3=RHO(KK+1)/RHO(KK)
      DO 250 I=1,22
      A(2,I)=(R3*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R3)
 250  CONTINUE
      DO 275 I=1,22
      R2=R2+W(I)*A(2,I)
 275  CONTINUE
      INGRL(KK)=R2
      CF(KK)=(8.0*INGRL(KK))/PI
      RSP(KK)=RHO(KK)*CF(KK)*1.D4/(2.D0*AA)
      VOL(KK)=DLOG10(RSP(KK))
      DO 350 I=1,22
      A(1,I)=A(2,I)
 350  CONTINUE
 500  CONTINUE
C*************************************************************************
C
C     END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C
C*************************************************************************
      WRITE(6,550)
 550  FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'INTEGRAL',4X,'CORR.FAC',6X,
     1 'RSP',6X,'VOL')
      WRITE(11,*)N+1
      WRITE(13,*)N+1
      DO 600 I=1,N+1
      WRITE(6,650)DEPTH(I),RHO(I),INGRL(I),CF(I),RSP(I),VOL(I)
 650  FORMAT(F8.3,5(1PE12.4))
      WRITE(11,675)DEPTH(I),RSP(I)
      WRITE(13,675)DEPTH(I),RHO(I)
 675  FORMAT(2X,F7.3,2X,1PE12.4)
 600  CONTINUE
      WRITE(11,*)AA
      WRITE(11,*)SEP
      WRITE(11,*)DELTH
      STOP
      END
```

```
C*************************************************************************
C     RGENVAR VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, TWO-PROBE (SPREADING)
C     RESISTANCE FROM THE DEPTH-DEPENDENT RESISTIVITY USING THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C
C     THE CALCULATIONS ARE BASED UPON THE VARIATIONAL METHOD DEVELOPED BY
C     CHOO AND COWORKERS.  THIS MAKES USE OF A VARIATIONALLY DETERMINED
C     MIXING OF THE SCHUMANN AND GARDNER CURRENT DENSITY AND THE CHOO ET AL.
C     UNIFORM CURRENT DENSITY.
C
C     THE CALCULATION OF THE VARIOUS INTEGRALS IS CARRIED OUT BY AN
C     ADAPTATION OF THE BERKOWITZ-LUX APPROXIMATE INTERGAL CONCEPT AS WELL
C     AS A SEGMENTED NEWTON-COTES QUADRATURE.
C
C     REFERENCES: "SPREADING RESISTANCE CALCULATIONS BY THE VARIATIONAL
C                  METHOD," BY S. C. CHOO, M. S. LEONG, AND L. S. TAN,
C                  SOLID STATE ELECTRONICS, VOL. 24, PP. 557-562, 1981.
C                 "AN EFFICIENT NUMERICAL SCHEME FOR SPREADING RESISTANCE
C                  CALCULATIONS BASED ON THE VARIATIONAL METHOD," BY
C                  S. C. CHOO, M. S. LEONG, AND J. H. SIM,
C                  SOLID STATE ELECTRONICS, VOL. 26, PP. 723-730, 1983.
C
C     THE PROGRAM READS (FROM FOR010) THE PROBE RADIUS, PROBE SEPARATION,
C     DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE
C     RESISTIVITY, SUBSTRATE THICKNESS, AND  THE RESISTIVITY ARRAY (FROM
C     THE TOP SURFACE).
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING OR FURTHER ANALYSIS.
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RSP(500),RHO(500),DEPTH(500),CF(500),VOL(500)
      DIMENSION A(2,55),CN9(9),W(3,55),T(55)
      DOUBLE PRECISION KA(55),L,K1,K2
      DOUBLE PRECISION I1(500),I2(500),I3(500),I2S(500),I3S(500)
C*************************************************************************
C
C     THE SET OF PROBE-SPACING INDEPENDENT SAMPLING POINTS, KA(I), (I=27,55)
C     AND WEIGHTS, W(J,I), (J=1,3 I=27,55) ARE ENTERED HERE BY MEANS OF
C     DATA STATEMENTS.  THE NEWTON-COTES COEFFICIENTS ARE ALSO ENTERED
C     HERE IN DATA STATEMENTS.
C
C*************************************************************************
```

```
DATA KA(27)/0.12D0/,KA(28)/0.203885D0/,KA(29)/0.346410D0/
DATA KA(30)/0.588566D0/,KA(31)/1.0D0/,KA(32)/1.53540D0/
DATA KA(33)/2.07080D0/,KA(34)/2.60619D0/,KA(35)/3.14159D0/
DATA KA(36)/3.31412D0/,KA(37)/3.48665D0/,KA(38)/3.65917D0/
DATA KA(39)/3.83171D0/,KA(40)/6.28319D0/,KA(41)/7.01558D0/
DATA KA(42)/9.42478D0/,KA(43)/10.1735D0/,KA(44)/12.5664D0/
DATA KA(45)/13.3237D0/,KA(46)/15.7080D0/,KA(47)/16.4701D0/
DATA KA(48)/18.8496D0/,KA(49)/19.6159D0/,KA(50)/39.24D0/
DATA KA(51)/49.87D0/,KA(52)/56.32D0/,KA(53)/311.0D0/
DATA KA(54)/983.5D0/,KA(55)/1100.0D0/,W(1,27)/1.96943D-2/
DATA W(2,27)/4.92949D-3/,W(3,27)/9.85305D-3/
DATA W(1,28)/1.51585D-1/,W(2,28)/3.80282D-2/
DATA W(3,28)/7.59243D-2/,W(1,29)/9.40755D-2/
DATA W(2,29)/2.37573D-2/,W(3,29)/4.72756D-2/
DATA W(1,30)/3.94777D-1/,W(2,30)/1.01659D-1/
DATA W(3,30)/2.00331D-1/,W(1,31)/2.34711D-1/
DATA W(2,31)/6.41889D-2/,W(3,31)/1.22743D-1/
DATA W(1,32)/3.22596D-1/,W(2,32)/1.02248D-1/
DATA W(3,32)/1.81618D-1/,W(1,33)/5.12837D-2/
DATA W(2,33)/2.17231D-2/,W(3,33)/3.33773D-2/
DATA W(1,34)/2.91802D-2/,W(2,34)/2.46688D-2/
DATA W(3,34)/2.68298D-2/,W(1,35)/4.03186D-2/
DATA W(2,35)/1.80765D-3/,W(3,35)/-9.61334D-4/
DATA W(1,36)/0.0D0/,W(2,36)/1.03120D-3/
DATA W(3,36)/0.0D0/,W(1,37)/0.0D0/
DATA W(2,37)/1.54721D-4/,W(3,37)/0.0D0/
DATA W(1,38)/0.0D0/,W(2,38)/9.17871D-5/
DATA W(3,38)/0.0D0/,W(1,39)/0.0D0/
DATA W(2,39)/3.84395D-3/,W(3,39)/9.19809D-3/
DATA W(1,40)/4.73102D-2/,W(2,40)/0.0D0/
DATA W(3,40)/8.01205D-3/,W(1,41)/0.0D0/
DATA W(2,41)/3.91858D-3/,W(3,41)/2.52148D-3/
DATA W(1,42)/1.90479D-2/,W(2,42)/0.0D0/
DATA W(3,42)/2.32728D-3/,W(1,43)/0.0D0/
DATA W(2,43)/1.08897D-3/,W(3,43)/1.08692D-3/
DATA W(1,44)/1.03669D-2/,W(2,44)/0.0D0/
DATA W(3,44)/1.02754D-3/,W(1,45)/0.0D0/
DATA W(2,45)/4.57291D-4/,W(3,45)/5.80787D-4/
DATA W(1,46)/6.53592D-3/,W(2,46)/0.0D0/
DATA W(3,46)/5.56264D-4/,W(1,47)/0.0D0/
DATA W(2,47)/2.25498D-4/,W(3,47)/3.52250D-4/
DATA W(1,48)/8.71084D-3/,W(2,48)/0.0D0/
DATA W(3,48)/3.40149D-4/,W(1,49)/0.0D0/
DATA W(2,49)/8.28909D-5/,W(3,49)/3.09148D-4/
DATA W(1,50)/0.0D0/,W(2,50)/4.12225D-4/
DATA W(3,50)/0.0D0/,W(1,51)/0.0D0/,PI/3.14159265D0/
DATA W(2,51)/0.0D0/,W(3,51)/1.92433D-3/
DATA W(1,52)/1.78441D-2/,W(2,52)/0.0D0/,W(3,52)/0.0D0/
DATA W(1,53)/9.02512D-4/,W(2,53)/0.0D0/,W(3,53)/0.0D0/
DATA W(1,54)/1.52804D-3/,W(2,54)/0.0D0/,W(3,54)/0.0D0/
DATA W(1,55)/0.0D0/,W(2,55)/0.0D0/,W(3,55)/9.40768D-6/
```

```
      C09=4.0/14175.0D0
      CN9(1)=C09*989.0D0
      CN9(2)=C09*5888.0D0
      CN9(3)=-C09*928.0D0
      CN9(4)=C09*10496.0D0
      CN9(5)=-C09*4540.0D0
      CN9(6)=CN9(4)
      CN9(7)=CN9(3)
      CN9(8)=CN9(2)
      CN9(9)=CN9(1)
C*************************************************************************
C
C     INPUT SECTION--DEFINITION OF SYMBOLS
C           N = NUMBER OF RESISTIVITY, SPREADING RESISTANCE VALUES
C           AA = PROBE RADIUS (MICROMETERS)
C           SEP = PROBE SEPARATION (MICROMETERS)
C           DELTH = SUBLAYER THICKNESS (MICROMETERS)
C           IBBC = BOUNDARY CONDITION (NONE=1, CONDUCTING=2, INSULATION=3)
C           SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM)
C           SUBTH = SUBSTRATE THICKNESS (MICROMETERS)
C           RHO(I) = I-TH RESISTIVITY RELATIVE TO THE SURFACE
C
C     AS ORIGINALLY FORMULATED THE INTEGRATION SCHEME OF CHOO ET AL. IS NOT
C     ABLE TO HANDLE AN INSULATING BOUNDARY.  WE HAVE CIRCUMVENTED
C     THIS PROBLEM BY STARTING THE INTEGRATION ONE POINT AWAY FROM THE
C     SINGULARITY AT THE ORIGIN WHEN THERE IS AN INSULATING BOUNDARY.
C     WE HAVE FOUND THAT THIS LEADS TO VERY GOOD RESULTS WITH NO LOSS
C     OF GENERALITY.
C
C     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY
C     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
C     SUBTH=DELTH AND SUBRHO=RHO(1).
C
C*************************************************************************
C     READ THE NUMBER OF DATA POINTS IN THE STRUCTURE (EXCLUSIVE OF SUBSTRATE)
      READ(10,*) N
C     READ THE PROBE RADIUS (MICROMETERS)
      READ(10,*) AA
C     READ THE PROBE SPACING (MICROMETERS)
      READ(10,*) SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ(10,*) DELTH
C     READ THE BACK SURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ(10,*) IBBC
C     READ THE SUBSTRATE RESISTIVITY (OHM*CM)
      READ(10,*) SUBRHO
C     READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ(10,*) SUBTH
C     READ THE RESISTIVITIES (OHM*CM) OF THE STRUCTURE
      DO 70 I=1,N
      READ(10,*) RHO(I)
 70   CONTINUE
```

```
      RHO(N+1)=SUBRHO
      L=1.122918967D0*(AA/SEP)
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
 80   FORMAT(2X,'CALCULATION OF SPREADING RESISTANCE FROM THE
     1 RESISTIVITY',/,2X,'USING CHOO ET AL VARIATIONAL TECHNIQUE-BL')
      GO TO (90,100,110), IBBC
 90   WRITE(6,95)SUBTH+N*DELTH,N*DELTH
 95   FORMAT(2X,'NO BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',F8.3,' UM')
      GO TO 120
 100  WRITE(6,105)SUBTH+N*DELTH,N*DELTH
 105  FORMAT(2X,'CONDUCTING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
      GO TO 120
 110  WRITE(6,115)SUBTH+N*DELTH,N*DELTH
 115  FORMAT(2X,'INSULATING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
 120  WRITE(6,130)AA,SEP,DELTH,SUBTH,SUBRHO,N
 130  FORMAT(2X,'RADIUS=',F6.4,2X,'SEP=',F6.1,2X,'DELTH=',F7.3,2X,
     1 'SUBTH=',1PE9.3,2X,'SUBRHO=',1PE9.3,' # PTS= ',I3)
      DO 140 I=1,N+1
      DEPTH(I)=DELTH*(I-1)
 140  CONTINUE
C*************************************************************************
C
C     BEGIN CONSTRUCTION OF SAMPLING POINTS AND WEIGHTS
C     REMEMBER TO ADD WEIGHTS AT BOUNDARY OF SAMPLING REGIONS.
C     THE REGION DESIGNATIONS FOLLOW THE NOTATION OF THE SECOND REFERENCE.
C
C*************************************************************************
C
C     REGION A (0<=KA<=1D-5) (I=1,9)
C
C*************************************************************************
      DO 142 I=1,9
      KA(I)=1.D-5*(I-1)/8.D0
      W(1,I)=1.D-5*CN9(I)/8.D0
      W(2,I)=W(1,I)/4.D0
      W(3,I)=W(1,I)/2.D0
 142  CONTINUE
      WEND1=W(1,9)
      WEND2=W(2,9)
      WEND3=W(3,9)
```

```
C*************************************************************************
C
C     REGION B (1D-5<=KA<=L) (I=9,17)
C
C*************************************************************************
      DO 144 I=9,17
      KA(I)=1.D-5*(L*1.D5)**((I-9.)/8.)
      W(1,I)=CN9(I-8)*KA(I)*DLOG(L*1.D5)/8.
      W(2,I)=W(1,I)/4.DO
      W(3,I)=W(1,I)/2.DO
 144  CONTINUE
      W(1,9)=W(1,9)+WEND1
      W(2,9)=W(2,9)+WEND2
      W(3,9)=W(3,9)+WEND3
C*************************************************************************
C
C     REGION C (L<=KA<=.12) (I=18,26)
C
C*************************************************************************
      DO 146 I=18,26
      KA(I)=L*(0.12DO/L)**((I-18.)/8.)
      W(1,I)=CN9(I-17)*KA(I)*DLOG(0.12/L)/8.
      W(2,I)=W(1,I)/4.DO
      W(3,I)=W(1,I)/2.DO
 146  CONTINUE
C*************************************************************************
C
C     REGION D, E, AND F
C     THE VALUES OF THE KA(I) AND W(I) HAVE BEEN ENTERED BY THE
C     DATA STATEMENTS AT THE BEGINNING OF THE PROGRAM
C
C*************************************************************************
      II=1
      IF(IBBC.EQ.2) II=2
      DO 180 I=II,55
      R1=KA(I)*DELTH/AA
      T(I)=DTANH(R1)
 180  CONTINUE
      DO 190 I=II,55
      R1=KA(I)*SUBTH/AA
      A(1,I)=1.0DO
      A(2,I)=1.0DO
      IF(IBBC.EQ.2) A(1,I)=DTANH(R1)
      IF(IBBC.EQ.3) A(1,I)=1.0DO/DTANH(R1)
 190  CONTINUE
      RI1=0.0DO
      RI2=0.0DO
      RI3=0.0DO
      DO 200 I=18,55
      RI1=RI1+W(1,I)*A(1,I)
      RI2=RI2+W(2,I)*A(1,I)
      RI3=RI3+W(3,I)*A(1,I)
```

```
 200  CONTINUE
      I2S(N+1)=RI2
      I3S(N+1)=RI3
      DO 210 I=II,17
      RI1=RI1+W(1,I)*A(1,I)
      RI2=RI2+W(2,I)*A(1,I)
      RI3=RI3+W(3,I)*A(1,I)
 210  CONTINUE
      I1(N+1)=RI1
      I2(N+1)=RI2
      I3(N+1)=RI3
      DEN=(I1(N+1)*I2(N+1)-I3(N+1)*I3(N+1))
      K1=(I2(N+1)-0.5D0*I3(N+1))/DEN
      K2=(0.5D0*I1(N+1)-I3(N+1))/DEN
      CFDEN=4.0D0/(PI*(K1+0.5D0*K2))
      CF(N+1)=CFDEN*(K1*I3S(N+1)+K2*I2S(N+1))
      RSP(N+1)=RHO(N+1)*CF(N+1)*1.D4/(2.0D0*AA)
      VOL(N+1)=DLOG10(RSP(N+1))
C*************************************************************************
C
C     LAYERED STRUCTURE CALCULATION
C
C*************************************************************************
      DO 500 KK=N,1,-1
      RI1=0.0D0
      RI2=0.0D0
      RI3=0.0D0
      R3=RHO(KK+1)/RHO(KK)
      DO 250 I=II,55
      A(2,I)=(R3*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R3)
 250  CONTINUE
      DO 275 I=18,55
      RI1=RI1+W(1,I)*A(2,I)
      RI2=RI2+W(2,I)*A(2,I)
      RI3=RI3+W(3,I)*A(2,I)
 275  CONTINUE
      I2S(KK)=RI2
      I3S(KK)=RI3
      DO 280 I=II,17
      RI1=RI1+W(1,I)*A(2,I)
      RI2=RI2+W(2,I)*A(2,I)
      RI3=RI3+W(3,I)*A(2,I)
 280  CONTINUE
      I1(KK)=RI1
      I2(KK)=RI2
      I3(KK)=RI3
      DEN=(I1(KK)*I2(KK)-I3(KK)*I3(KK))
      K1=(I2(KK)-0.5D0*I3(KK))/DEN
      K2=(0.5D0*I1(KK)-I3(KK))/DEN
      CFDEN=4.0D0/(PI*(K1+0.5D0*K2))
      CF(KK)=CFDEN*(K1*I3S(KK)+K2*I2S(KK))
      RSP(KK)=RHO(KK)*CF(KK)*1.D4/(2.0D0*AA)
      VOL(KK)=DLOG10(RSP(KK))
      DO 350 I=II,55
      A(1,I)=A(2,I)
```

```
 350  CONTINUE
 500  CONTINUE
C*************************************************************************
C     END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C*************************************************************************
      WRITE(6,550)
 550  FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'CORR.FAC',6X,'RSP',6X,'VOL')
      WRITE(11,*)N+1
      DO 600 I=1,N+1
      WRITE(6,650)DEPTH(I),RHO(I),CF(I),RSP(I),VOL(I)
 650  FORMAT(F8.3,5(1PE12.4))
      WRITE(11,675)DEPTH(I),RSP(I)
 675  FORMAT(2X,F7.3,2X,1PE12.4)
 600  CONTINUE
      WRITE(11,*)AA
      WRITE(11,*)SEP
      WRITE(11,*)DELTH
      STOP
      END
C*****END OF RGENVAR V1.0 PROGRAM*************************************
```

```
C*********************************************************************
C     BL VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE RESISTIVITY PROFILE FROM THE
C     SPREADING RESISTANCE BY USING: 1) THE BERKOWITZ-LUX APPROXIMATE
C     INTEGRAL, 2) THE 22-POINT INTEGRATION SCHEME BASED UPON THE
C     NEWTON-COTES TECHNIQUE, AND 3) A BOUNDING PROCEDURE WHICH
C     SIMPLIFIES THE INVERSION OF THE CORRECTION FACTOR INTEGRAL.
C     THIS PROGRAM INVERTS THE CALCULATION PERFORMED IN RGENBL VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT RESISTIVITY FROM THE
C     DEPTH-DEPENDENT TWO-PROBE (SPREADING) RESISTANCE USING THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C     THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE BERKOWITZ AND
C     LUX APPROXIMATE INTEGRAL AND THEIR 22-POINT INTEGRATION METHOD BASED
C     UPON THE NEWTON-COTES QUADRATURE.
C     AS THE EXTRACTION OF THE RESISTIVITY FROM THE SPREADING RESISTANCE
C     INVOLVES THE SOLUTION OF THE INVERSION PROBLEM, THE CODE MAKES USE
C     OF AN ITERATION METHOD BASED UPON A BOUNDING PROCEDURE FIRST USED BY
C     BERKOWITZ AND LUX.
C
C     THE SET OF SAMPLING POINTS AND WEIGHTS ARE SPECIALIZED TO THE CHOO
C     ET AL. UNIFORM CURRENT DENSITY (ALTHOUGH THE SETS OF SAMPLING POINTS
C     AND WEIGHTS MAY BE CONSTRUCTED FOR THE SCHUMANN AND GARDENER CURRENT
C     DENSITY OR THE RING DELTA FUNCTION CURRENT DENSITY BY FOLLOWING THE
C     REASONING PRESENTED IN THE REFERENCE).
C
C     REFERENCE: "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE
C                 MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES",
C                 BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE
C                 ELECTROCHEMICAL SOCIETY, VOL 128, PP. 1137-1141 (1981).
C
C     THE PROGRAM READS (FROM FOR010) THE PROBE RADIUS, PROBE SEPARATION,
C     DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE
C     RESISTIVITY, THE SUBSTRATE THICKNESS, AND THE RESISTIVITY ARRAY
C     (FROM THE TOP SURFACE)
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING, OR FURTHER ANALYSIS.
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RSP(500),RHO(500),DEPTH(500),CF(500),VOL(500)
      DIMENSION A(2,22),COND(500),B(22),SDIF(500),SENS(500)
      DIMENSION C(9),W(22),T(22),ITER(500)
      DOUBLE PRECISION KA(22),INGRL(500)
```

```
C*************************************************************************
C
C     HERE THE FIXED VALUES OF THE SAMPLING POINTS (KA) AND WEIGHTS (W)
C     ARE ENTERED BY DATA STATEMENTS.  THE REMAINING SAMPLING POINTS
C     AND WEIGHTS (THE FIRST 8) ARE CALCULATED FROM AA AND SEP.
C     THE COEFFICIENTS FOR THE NINE-POINT NEWTON COTES SCHEME ARE ALSO
C     ENTERED HERE BY MEANS OF DATA STATEMENTS.
C
C*************************************************************************
      DATA KA(9)/0.1D0/,KA(10)/0.177828D0/,KA(11)/0.316228D0/
      DATA KA(12)/0.562341D0/,KA(13)/1.0D0/,KA(14)/1.70793D0/
      DATA KA(15)/2.41586D0/,KA(16)/3.12378D0/,KA(17)/5.23853D0/
      DATA KA(18)/8.48093D0/,KA(19)/11.6681D0/,KA(20)/14.829D0/
      DATA KA(21)/17.9861D0/,KA(22)/39.3D0/,PI/3.14159265D0/
      DATA W(9)/0.00446607D0/,W(10)/0.03611D0/,W(11)/0.0236709D0/
      DATA W(12)/0.106292D0/,W(13)/0.0773292D0/,W(14)/0.115397D0/
      DATA W(15)/0.0172741D0/,W(16)/0.0087751D0/,W(17)/0.00668606D0/
      DATA W(18)/0.00163407D0/,W(19)/0.000628701D0/,W(20)/0.0003062D0/
      DATA W(21)/0.0001717D0/,W(22)/0.000412D0/
      DATA C(1)/0.0348854D0/C(2)/0.20769D0/,C(3)/-0.0327337D0/
      DATA C(4)/0.370229D0/,C(5)/-0.160141D0/
      C(6)=C(4)
      C(7)=C(3)
      C(8)=C(2)
      C(9)=C(1)
C*************************************************************************
C
C     BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
C
C     NPT      = NUMBER OF TWO-PROBE RESISTANCE & RESISTIVITY VALUES
C     DEPTH(I) = I-TH VALUE OF DEPTH (RELATIVE TO SURFACE)
C     RSP(I)   = I-TH VALUE OF TWO-PROBE RESISTANCE (RELATIVE TO SURFACE)
C     AA       = PROBE RADIUS (MICROMETERS)
C     SEP      = PROBE SEPARATION (MICROMETERS)
C     DELTH    = SUBLAYER THICKNESS (MICROMETERS)
C
C*************************************************************************
C     READ THE NUMBER OF TWO-PROBE RESISTANCE VALUES
      READ(11,*)NPT
C     READ THE DEPTH, TWO-PROBE RESISTANCES FROM THE SURFACE
      DO 70 I=1,NPT
      READ(11,*)DEPTH(I),RSP(I)
 70   CONTINUE
C     READ THE PROBE RADIUS (MICROMETERS)
      READ(11,*)AA
C     READ THE PROBE SEPARATION (MICROMETERS)
      READ(11,*)SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ(11,*)DELTH
      SUBRSP=RSP(NPT)
```

```
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
 80   FORMAT(2X,'CALCULATION OF THE RESITIVITY FROM THE SPREADING
     1 RESISTANCE',/,' USING THE BERKOWITZ-LUX APPROXIMATE INTEGRAL')
 120  WRITE(6,130)AA,SEP,DELTH,NPT
 130  FORMAT(2X,'RADIUS=',F6.1,2X,'SEP=',F6.1,2X,'DELTH=',F7.3,2X,
     1 ' # PTS=',I3)
      AA=AA*1D-4
      SEP=SEP*1D-4
      DELTH=DELTH*1D-4
      SEP=SEP/AA
      DELTH=DELTH/AA
      R1=1.12292D0*(1.D0/SEP)
      R2=DLOG(0.1/R1)
      R3=DEXP(R2/8.)
C*************************************************************************
C
C     HERE THE SAMPLING POINTS, KA (I), AND WEIGHTS, W(I), FOR THE FIRST
C     NINE POINTS (WHICH DEPEND UPON THE PROBE SPACING) ARE CALCULATED.
C     THESE DEPEND UPON THE PROBE SPACING THROUGH THE BERKOWITZ-LUX
C     APPROXIMATE FORM OF THE CORRECTION FACTOR INTEGRAL
C
C*************************************************************************
      DO 140 I=8,1,-1
      KA(I)=KA(I+1)/R3
 140  CONTINUE
      DO 150 I=1,8
      W(I)=.25D0*R2*KA(I)*C(I)
 150  CONTINUE
      W(9)=W(9)+.249376D0*R2*KA(9)*C(9)
      DO 155 I=1,22
      R4=KA(I)*DELTH
      T(I)=DTANH(R4)
 155  CONTINUE
      R5=0.0D0
      R6=0.0D0
      R7=0.0D0
```

```
C*************************************************************************
C
C     THE INFINITE SLAB, CONDUCTING BOUNDARY, AND INSULATING BOUNDARY
C     FORMS ARE CONSTRUCTED HERE FOR USE IN THE BOUNDING PROCEDURE
C     USED TO INVERT THE CORRECTION FACTOR INTEGRAL.
C
C*************************************************************************
      DO 160 I=1,22
      R5=R5+W(I)
      R6=R6+W(I)*T(I)
      R7=R7+W(I)/T(I)
 160  CONTINUE
      WRITE(6,165)R5,R6,R7
 165  FORMAT(2X,'R5=',F11.6,' R6=',F11.6,' R7=',F11.6,/)
      R8=1.27324D0/AA
      R9=R8/SUBRSP*R5
      SDIF(NPT)=0.0D0
      COND(NPT)=R9
      RHO(NPT)=1.0D0/R9
      ITER(NPT)=0
      SENS(NPT)=0.0D0
      DO 170 I=1,22
      B(I)=1.0D0
 170  CONTINUE
      DO 190 I=1,2
      DO 180 K=1,22
      A(I,K)=1.0D0
 180  CONTINUE
 190  CONTINUE
      DO 1000 K=NPT-1,1,-1
      ITER(K)=0
      R10=0.D0
      R11=R8/RSP(K)/COND(K+1)
      R12=RSP(K+1)/RSP(K)
      R13=R12
      GOTO 1450
 1060 CONV=DABS(1.0D0-R14*R11/R12)
      IF (CONV.LT.1D-3) GO TO 1200
      IF (R10.EQ.1.) R15=0.0D0
      R16=R15
      R15=R12
      R17=(R14-R6)/(R7-R14)/R12
      R18=R7*R17*R11-1.0
      R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
      IF (R18.GT.0.0) R13=R13/2.0/R17
      IF (R18.LT.0.0) R13=2.0*R11*R6/R13
      VAR1=(R15-R13)*(R15-R16)
      IF (VAR1.LE.0.0) GO TO 1170
      VAR2=R10-2.*JIDINT(R10/2.)
      IF (VAR2.EQ.0.0) R13=(R15*R15-R13*R16)/(2*R15-R13-R16)
 1170 R12=R13
      GO TO 1450
 1200 R19=R11*R14*COND(K+1)
      R20=0.0
```

```
      DO 200 I=1,22
      SEP=A(1,I)*(1.0-T(I)*T(I))
      DELTH=(R12*T(I)*A(1,I)+1.0)**2
      R20=W(I)*SEP/DELTH+R20
      A(1,I)=A(2,I)
 200  CONTINUE
      SENS(K)=1.0/(1.0-R20*R13/R14)
      COND(K)=R19
      RHO(K)=1.0/R19
      SDIF(K)=R14*R8/R13/COND(K+1)
      SDIF(K)=1.0-SDIF(K)/RSP(K)
      ITER(K)=R10
 1000 CONTINUE
 1001 FORMAT(2X,I3,2X,F7.3,2X,4(1PE11.3),3X,I5)
 1002 FORMAT(4X,'NPT',4X,'DEPTH',5X,'RMEAS',6X,'1-RCAL/R',4X,'RHO',8X
     1 ,'SENS',6X,'ITER',/)
 1003 FORMAT(2X,F7.3,2X,1PE11.3)
 1004 FORMAT(2X,1PE11.4)
      WRITE(6,1002)
      WRITE(12,*)NPT
      DO 1020 I=1,NPT
      WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),SENS(I),ITER(I)
      WRITE(12,1003)DEPTH(I),RHO(I)
 1020 CONTINUE
      STOP
C*************************************************************************
C
C     THE LINES BELOW ARE USED AS PART OF THE INVERSION PROCEDURE BASED
C     UPON THE BOUNDING METHOD.
C
C*************************************************************************
 1450 DO 1475 I=1,22
      A(2,I)=(R12*A(1,I)+T(I))/(1.0+T(I)*A(1,I)*R12)
      B(I)=A(2,I)
 1475 CONTINUE
      R14=0.0
      DO 1480 I=1,22
      R14=R14+W(I)*B(I)
 1480 CONTINUE
      R10=R10+1.
      GO TO 1060
      END
C*****END OF THE BL VERSION 1.0 PROGRAM************************************
```

```
C*************************************************************************
C     GL VERSION 1.0
C
C     THIS PROGRAM CALCULATES RESISTIVITY FROM SPREADING RESISTANCE.
C     THE CALCULATION MAKES USE OF THE 33-POINT GAUSS-LAGUERRE
C     EVALUATION OF THE CORRECTION FACTOR INTEGRAL ALONG WITH THE
C     BERKOWITZ-LUX BOUNDING METHOD.
C     THIS PROGRAM IS THE INVERSE OF THE RGENGL VERSION 1.0 PROGRAM
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT RESISTIVITY FROM THE
C     TWO-PROBE (SPREADING) RESISTANCE USING THE SCHUMANN AND GARDNER
C     SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C
C     THE CORRECTION FACTOR INTEGRAL IS CALCULATED BY USING THE CHOO ET AL.
C     33-POINT INTEGRATION METHOD BASED UPON THE GAUSS-LAGUERRE QUADRATURE.
C     THE BERKOWITZ AND LUX BOUNDING METHOD IS USED TO CARRY OUT THE
C     INVERSION OF THE CORRECTION FACTOR INTEGRAL.
C     THE 33 SAMPLING POINTS AND WEIGHTS ARE INITIALIZED BY DATA
C     STATEMENTS.
C     THE FINAL SET OF 33 WEIGHTS USED IN THE CALCULATIONS IS CALCULATED
C     FROM THE INITIAL SET OF WEIGHTS AND THE PROBE CURRENT DENSITY.
C
C     THE CURRENT DENSITIES WHICH MAY BE USED ARE THE SCHUMANN AND GARDNER,
C     THE CHOO ET AL. UNIFORM CURRENT DENSITY, AND THE RING DELTA FUNCTION
C     CURRENT DENSITY.
C     THE DISCUSSION OF THE 33-POINT GAUSS-LAGUERRE QUADRATURE MAY BE
C     FOUND IN THE REFERENCE.
C
C     REFERENCE: "SPREADING RESISTANCE CALCULATIONS BY THE GAUSS-LAGUERRE
C                 QUADRATURE," BY S. C. CHOO, M. S. LEONG, H. L. HONG,
C                 L. LE, AND L. S. TAN,
C                 SOLID STATE ELECTRONICS VOL 21, PP. 769-774, 1978.
C
C     THE PROGRAM READS (FROM FOR010) THE PROBE RADIUS, PROBE SEPARATION,
C     DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE
C     RESISTIVITY, SUBSTRATE THICKNESS, THE RESISTIVITY ARRAY (FROM THE
C     TOP SURFACE), AND THE PROBE CURRENT DENSITY.
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING, OR FURTHER ANALYSIS.
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION T(33),W(33),A(2,33),RHO(500),RSP(500),DEPTH(500)
      DIMENSION CF(500),VOL(500),COND(500),B(33),SDIF(500),SENS(500)
      DIMENSION ITER(500)
      DOUBLE PRECISION KA(33)
```

```
C*************************************************************************
C
C     THE SET OF 33 FIXED SAMPLING POINTS, KA(I), AND THE INITIAL SET OF
C     WEIGHTS, W(I), I=1,33, ARE ENTERED THROUGH THE DATA STATEMENTS.
C     THESE ARE THE FIRST 33 POINTS AND WEIGHTS OF THE GAUSS-LAGUERRE
C     METHOD OBTAINED FROM A 128-ORDER LAGUERRE POLYNOMIAL
C
C*************************************************************************
      DATA KA(1)/.011251388D0/,KA(2)/.059284741D0/
      DATA KA(3)/.145707967D0/,KA(4)/.270553179D0/
      DATA KA(5)/.433841408D0/,KA(6)/.635597666D0/
      DATA KA(7)/.875852385D0/,KA(8)/1.154641702D0/
      DATA KA(9)/1.472007563D0/,KA(10)/1.827997778D0/
      DATA KA(11)/2.222666072D0/,KA(12)/2.65607213D0/
      DATA KA(13)/3.128281655D0/,KA(14)/3.63936642D0/
      DATA KA(15)/4.18940433D0/,KA(16)/4.778479488D0/
      DATA KA(17)/5.406682272D0/,KA(18)/6.074109403D0/
      DATA KA(19)/6.78086404D0/,KA(20)/7.527055861D0/
      DATA KA(21)/8.312801165D0/,KA(22)/9.138222971D0/
      DATA KA(23)/10.003451129D0/,KA(24)/10.908622439D0/
      DATA KA(25)/11.853880769D0/,KA(26)/12.839377192D0/
      DATA KA(27)/13.865270123D0/,KA(28)/14.931725465D0/
      DATA KA(29)/16.038916768D0/,KA(30)/17.187025392D0/
      DATA KA(31)/18.376240681D0/,KA(32)/19.606760148D0/
      DATA KA(33)/20.878789667D0/
      DATA W(1)/.028874906D0/,W(2)/.067219473D0/
      DATA W(3)/.10563085D0/,W(4)/.14406297D0/
      DATA W(5)/.18251763D0/,W(6)/.22099994D0/
      DATA W(7)/.25951551D0/,W(8)/.2980701D0/
      DATA W(9)/.33666957D0/,W(10)/.3753198D0/
      DATA W(11)/.41402672D0/,W(12)/.45279633D0/
      DATA W(13)/.49163467D0/,W(14)/.53054783D0/
      DATA W(15)/.56954199D0/,W(16)/.60862338D0/
      DATA W(17)/.6477983D0/,W(18)/.69707314D0/
      DATA W(19)/.7264544D0/,W(20)/.7659486D0/
      DATA W(21)/.80556248D0/,W(22)/.84530276D0/
      DATA W(23)/.88517632D0/,W(24)/.9251902D0/
      DATA W(25)/.96535161D0/,W(26)/1.00566765D0/
      DATA W(27)/1.04614585D0/,W(28)/1.08679375D0/
      DATA W(29)/1.12761908D0/,W(30)/1.16862972D0/
      DATA W(31)/1.20983377D0/,W(32)/1.25123946D0/
      DATA W(33)/1.2928553D0/,PI/3.14159265D0/
```

```
C*************************************************************************
C
C     BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
C
C     NPT      = NUMBER OF RESISTIVITY, SPREADING RESISTANCE VALUES
C     DEPTH(I) = DEPTH RELATIVE TO SURFACE (MICROMETERS)
C     RSP(I)   = TWO-PROBE RESISTANCE RELATIVE TO SURFACE (OHMS)
C     AA       = PROBE RADIUS (MICROMETERS)
C     SEP      = PROBE SEPARATION (MICROMETERS)
C     DELTH    = SUBLAYER THICKNESS (MICROMETERS)
C     ICUR     = CURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3)
C
C*************************************************************************
C     READ THE NUMBER OF POINTS IN THE STRUCTURE
      READ(11,*)NPT
C     READ THE DEPTH AND TWO-PROBE RESISTANCE VALUES RELATIVE TO SURFACE
      DO 10 I=1,NPT
      READ(11,*)DEPTH(I),RSP(I)
 10   CONTINUE
C     READ THE ELECTRICAL PROBE RADIUS (MICROMETERS)
      READ(11,*)AA
C     READ THE SEPARATION BETWEEN THE PROBES (MICROMETERS)
      READ(11,*)SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ(11,*)DELTH
C     READ THE PROBE CURRENT DENSITY (S&G=1, CHOO UNIFORM=2, RING DELTA=3)
      READ(11,*)ICUR
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE(6,20)
 20   FORMAT(2X,'CALCULATION OF THE RESITIVITY FROM THE SPREADING
     1 RESISTANCE',/,2X,
     2 'USING THE CHOO ET AL GAUSS-LAGUERRE QUADRATURE')
      GO TO (30, 40, 50), ICUR
 30   WRITE(6,35)
 35   FORMAT(2X,'SCHUMANN AND GARDNER CURRENT DENSITY')
      GO TO 60
 40   WRITE(6,45)
 45   FORMAT(2X,'CHOO UNIFORM CURRENT DENSITY')
      GO TO 60
 50   WRITE(6,55)
 55   FORMAT(2X,'RING DELTA FUNCTION CURRENT DENSITY')
 60   WRITE(6,65)AA,SEP,DELTH,NPT
 65   FORMAT(2X,'RADIUS=',F6.1,2X,'SEP=',F6.1,2X,'DELTH=',F7.3,2X,
     1 ' # PTS=',I3)
```

```
C*************************************************************************
C
C     THE FINAL SET OF WEIGHTS, W(I) (I=1,33) ARE NOW CONSTRUCTED FROM THE
C     INITIAL SET OF WEIGHTS AND THE TWO-PROBE CONFIGURATION FUNCTION (IN
C     LINE 90) AND THE FORM OF THE CURRENT DENSITY FUNCTION (IN THE
C     APPROPRIATE NEXT LINE DEPENDING UPON THE VALUE OF ICUR).
C
C*************************************************************************
      DO 70 I=1,33
      T(I)=DTANH(DELTH*KA(I)/AA)
      A(1,I)=1.0D0
      A(2,I)=1.0D0
 90   W(I)=W(I)*(DBJ1(KA(I))/KA(I)**2-DBJ0(KA(I)*SEP/AA)/(2.D0*KA(I)))
      IF (ICUR.EQ.1) W(I)=W(I)*DSIN(KA(I))/2.D0
      IF (ICUR.EQ.2) W(I)=W(I)*DBJ1(KA(I))
      IF (ICUR.EQ.3) W(I)=W(I)*KA(I)*DBJ0(KA(I))/2.D0
 70   CONTINUE
C*************************************************************************
C
C     THE NEXT GROUP OF LINES CONTAIN THE CALCULATION OF THE CORRECTION
C     FACTOR INTEGRAL FOR THE SUBSTRATE POINT.  THE CORRECTION FACTOR, THE
C     SPREADING RESISTANCE, AND THE VOLTAGE ARE ALSO CALCULATED.
C
C*************************************************************************
      R5=0.0
      R6=0.0
      R7=0.0
      DO 80 I=1,33
      R5=R5+W(I)
      R6=R6+W(I)*T(I)
      R7=R7+W(I)/T(I)
 80   CONTINUE
      WRITE(6,165)R5,R6,R7
 165  FORMAT(2X,'R5=',F11.6,' R6=',F11.6,' R7=',F11.6,/)
      R8=1.27324D4/AA
      R9=R8/RSP(NPT)*R5
      SDIF(NPT)=0.0
      COND(NPT)=R9
      RHO(NPT)=1.0/R9
      ITER(NPT)=0
      SENS(NPT)=0.0
      DO 170 I=1,33
      B(I)=1.0
 170  CONTINUE
```

```
      DO 1000 K=NPT-1,1,-1
      ITER(K)=0
      R10=0.
      R11=R8/RSP(K)/COND(K+1)
      R12=RSP(K+1)/RSP(K)
      R13=R12
      GOTO 1450
1060  CONV=DABS(1.0-R14*R11/R12)
      IF (CONV.LT.1D-3) GO TO 1200
      IF (R10.EQ.1.) R15=0.0
      R16=R15
      R15=R12
      R17=(R14-R6)/(R7-R14)/R12
      R18=R7*R17*R11-1.0
      R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
      IF (R18.GT.0.0) R13=R13/2.0/R17
      IF (R18.LT.0.0) R13=2.0*R11*R6/R13
      VAR1=(R15-R13)*(R15-R16)
      IF (VAR1.LE.0.0) GO TO 1170
      VAR2=R10-2.*JIDINT(R10/2.)
      IF (VAR2.EQ.0.0) R13=(R15*R15-R13*R16)/(2*R15-R13-R16)
1170  R12=R13
      GO TO 1450
1200  R19=R11*R14*COND(K+1)
      R20=0.0
      DO 200 I=1,33
      SEP=A(1,I)*(1.0-T(I)*T(I))
      DELTH=(R12*T(I)*A(1,I)+1.0)**2
      R20=W(I)*SEP/DELTH+R20
      A(1,I)=A(2,I)
200   CONTINUE
      SENS(K)=1.0/(1.0-R20*R13/R14)
      COND(K)=R19
      RHO(K)=1.0/R19
      SDIF(K)=R14*R8/R13/COND(K+1)
      SDIF(K)=1.0-SDIF(K)/RSP(K)
      ITER(K)=R10
1000  CONTINUE
1001  FORMAT(2X,I3,2X,F7.3,2X,4(1PE11.3),3X,I5)
1002  FORMAT(4X,'NPT',4X,'DEPTH',5X,'RMEAS',6X,'1-RCAL/R',4X,'RHO',8X
     1 ,'SENS',6X,'ITER',/)
1003  FORMAT(2X,F7.3,2X,1PE11.3)
1004  FORMAT(2X,1PE11.4)
      WRITE(6,1002)
      WRITE(12,*)NPT
      DO 1020 I=1,NPT
      WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),SENS(I),ITER(I)
      WRITE(12,1003)DEPTH(I),RHO(I)
1020  CONTINUE
      STOP
```

```
C*************************************************************************
C
C     THE REMAINING LINES ARE PART OF THE INVERSION METHOD BASED UPON THE
C     BERKOWITZ AND LUX BOUNDING TECHNIQUE.
C
C*************************************************************************
 1450 DO 1475 I=1,33
      A(2,I)=(R12*A(1,I)+T(I))/(1.0+T(I)*A(1,I)*R12)
      B(I)=A(2,I)
 1475 CONTINUE
      R14=0.0
      DO 1480 I=1,33
      R14=R14+W(I)*B(I)
 1480 CONTINUE
      R10=R10+1.
      GO TO 1060
      END
C*****END OF MAIN PART OF THE GL VERSION 1.0 PROGRAM**********************
```

```
      DOUBLE PRECISION FUNCTION DBJ0(X)
C*********************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ0
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.1 IS USED FOR X < 3
C                EQUATION 9.4.3 IS USED FOR X > 3
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F0,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/1.0000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/
      DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
      DATA A7/0.0002100D0/
      DATA B1/.79788456D0/,B2/-.00000077D0/,B3/-.00552740D0/
      DATA B4/-.00009512D0/,B5/.00137237D0/,B6/-.00072805D0/
      DATA B7/.00014476D0/
      DATA C1/-.78539816D0/,C2/-.04166397D0/,C3/-.00003954D0/
      DATA C4/.00262573D0/,C5/-.00054125D0/,C6/-.00029333D0/
      DATA C7/.00013558D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.0D0
      DBJ0=(((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
      RETURN
10    CONTINUE
      X1=3.D0/X
      F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ0=F0*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF THE DBJ0 DOUBLE PRECISION FUNCTION**************************
```

```
      DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.4 IS USED FOR X < 3
C                AND EQUATION 9.4.6 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F1,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/
      DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/
      DATA A7/.00001109D0/
      DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/
      DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/
      DATA B7/-.00020033D0/
      DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/
      DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/
      DATA C7/-.00029166D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.D0
      DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
      RETURN
   10 X1=3.0D0/X
      F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ1=F1*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DBJ1 DOUBLE PRECISION FUNCTION & END OF THE GL VERSION 1.0****
```

```
C*************************************************************************
C     VAR VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE RESISTIVITY PROFILE FROM THE
C     SPREADING RESISTANCE USING THE VARIATIONAL TECHNIQUE OF
C     CHOO ET AL. AND A GENERALIZATION TO THE VARIATIONAL TECHNIQUES
C     OF THE BERKOWITZ AND LUX BOUNDING METHOD FOR INVERTING THE CORRECTION
C     FACTOR INTEGRAL.
C     THIS PROGRAM IS THE INVERSE OF THE RGENVAR VERSION 1.0 PROGRAM
C
C     THE CALCULATION OF THE VARIOUS INTEGRALS IS CARRIED OUT BY AN
C     ADAPTATION OF THE BERKOWITZ-LUX APPROXIMATE INTERGAL AS WELL AS THE
C     SEGMENTED NEWTON-COTES QUADRATURE.
C
C     REFERENCES: "SPREADING RESISTANCE CALCULATIONS BY THE VARIATIONAL
C                  METHOD," BY S. C. CHOO, M. S. LEONG, AND L. S. TAN,
C                  SOLID STATE ELECTRONICS, VOL. 24, PP. 557-562, 1981.
C                 "AN EFFICIENT NUMERICAL SCHEME FOR SPREADING RESISTANCE
C                  CALCULATIONS BASED ON THE VARIATIONAL METHOD," BY
C                  S. C. CHOO, M. S. LEONG, AND J. H. SIM,
C                  SOLID STATE ELECTRONICS, VOL. 26, PP. 723-730, 1983.
C
C     THE PROGRAM READS (FROM FOR010) THE PROBE RADIUS, PROBE SEPARATION,
C     DEPTH INCREMENT, THE BACKSURFACE BOUNDARY CONDITION, THE SUBSTRATE
C     RESISTIVITY, SUBSTRATE THICKNESS, AND  THE RESISTIVITY ARRAY (FROM
C     THE TOP SURFACE).
C
C     THE CORRECTION FACTOR AND THE TWO-PROBE (SPREADING) RESISTANCE
C     ARE CALCULATED FROM THE CALCULATED CORRECTION FACTOR INTEGRAL.
C
C     THE OUTPUT LIST INCLUDES THE VALUES OF ALL OF THE PERTINENT VARIABLES
C     AS WELL AS THE RESULTS OF THE CALCULATION.
C
C     IN ADDITION, SELECTED DATA MAY SENT TO OTHER FILES FOR STORAGE,
C     PLOTTING, OR FURTHER ANALYSIS.
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RSP(1000),RHO(1000),DEPTH(1000),CF(1000)
      DIMENSION A(2,55),CN9(9),W(3,55),T(55),VOL(1000)
      DIMENSION COND(1000),B(55),SDIF(1000),SENS(1000),ITER(1000)
      DOUBLE PRECISION KA(55),L,K1,K2,I1(1000),I2(1000)
      DOUBLE PRECISION I3(1000),I2S(1000),I3S(1000)
C*************************************************************************
C
C     THE SET OF PROBE-SPACING INDEPENDENT SAMPLING POINTS, KA(I), (I=27,55)
C     AND WEIGHTS, W(J,I), (J=1,3 I=27,55) ARE ENTERED HERE BY MEANS OF
C     DATA STATEMENTS.  THE NEWTON-COTES COEFFICIENTS ARE ALSO ENTERED
C     HERE IN DATA STATEMENTS.
C
C*************************************************************************
```

```
DATA KA(27)/0.12D0/,KA(28)/0.203885D0/,KA(29)/0.346410D0/
DATA KA(30)/0.588566D0/,KA(31)/1.0D0/,KA(32)/1.53540D0/
DATA KA(33)/2.07080D0/,KA(34)/2.60619D0/,KA(35)/3.14159D0/
DATA KA(36)/3.31412D0/,KA(37)/3.48665D0/,KA(38)/3.65917D0/
DATA KA(39)/3.83171D0/,KA(40)/6.28319D0/,KA(41)/7.01558D0/
DATA KA(42)/9.42478D0/,KA(43)/10.1735D0/,KA(44)/12.5664D0/
DATA KA(45)/13.3237D0/,KA(46)/15.7080D0/,KA(47)/16.4701D0/
DATA KA(48)/18.8496D0/,KA(49)/19.6159D0/,KA(50)/39.24D0/
DATA KA(51)/49.87D0/,KA(52)/56.32D0/,KA(53)/311.0D0/
DATA KA(54)/983.5D0/,KA(55)/1100.0D0/,W(1,27)/1.96943D-2/
DATA W(2,27)/4.92949D-3/,W(3,27)/9.85305D-3/
DATA W(1,28)/1.51585D-1/,W(2,28)/3.80282D-2/
DATA W(3,28)/7.59243D-2/,W(1,29)/9.40755D-2/
DATA W(2,29)/2.37573D-2/,W(3,29)/4.72756D-2/
DATA W(1,30)/3.94777D-1/,W(2,30)/1.01659D-1/
DATA W(3,30)/2.00331D-1/,W(1,31)/2.34711D-1/
DATA W(2,31)/6.41889D-2/,W(3,31)/1.22743D-1/
DATA W(1,32)/3.22596D-1/,W(2,32)/1.02248D-1/
DATA W(3,32)/1.81618D-1/,W(1,33)/5.12837D-2/
DATA W(2,33)/2.17231D-2/,W(3,33)/3.33773D-2/
DATA W(1,34)/2.91802D-2/,W(2,34)/2.46688D-2/
DATA W(3,34)/2.68298D-2/,W(1,35)/4.03186D-2/
DATA W(2,35)/1.80765D-3/,W(3,35)/-9.61334D-4/
DATA W(1,36)/0.0D0/,W(2,36)/1.03120D-3/
DATA W(3,36)/0.0D0/,W(1,37)/0.0D0/
DATA W(2,37)/1.54721D-4/,W(3,37)/0.0D0/
DATA W(1,38)/0.0D0/,W(2,38)/9.17871D-5/
DATA W(3,38)/0.0D0/,W(1,39)/0.0D0/
DATA W(2,39)/3.84395D-3/,W(3,39)/9.19809D-3/
DATA W(1,40)/4.73102D-2/,W(2,40)/0.0D0/
DATA W(3,40)/8.01205D-3/,W(1,41)/0.0D0/
DATA W(2,41)/3.91858D-3/,W(3,41)/2.52148D-3/
DATA W(1,42)/1.90479D-2/,W(2,42)/0.0D0/
DATA W(3,42)/2.32728D-3/,W(1,43)/0.0D0/
DATA W(2,43)/1.08897D-3/,W(3,43)/1.08692D-3/
DATA W(1,44)/1.03669D-2/,W(2,44)/0.0D0/
DATA W(3,44)/1.02754D-3/,W(1,45)/0.0D0/
DATA W(2,45)/4.57291D-4/,W(3,45)/5.80787D-4/
DATA W(1,46)/6.53592D-3/,W(2,46)/0.0D0/
DATA W(3,46)/5.56264D-4/,W(1,47)/0.0D0/
DATA W(2,47)/2.25498D-4/,W(3,47)/3.52250D-4/
DATA W(1,48)/8.71084D-3/,W(2,48)/0.0D0/
DATA W(3,48)/3.40149D-4/,W(1,49)/0.0D0/
DATA W(2,49)/8.28909D-5/,W(3,49)/3.09148D-4/
DATA W(1,50)/0.0D0/,W(2,50)/4.12225D-4/
DATA W(3,50)/0.0D0/,W(1,51)/0.0D0/,PI/3.14159265D0/
DATA W(2,51)/0.0D0/,W(3,51)/1.92433D-3/
DATA W(1,52)/1.78441D-2/,W(2,52)/0.0D0/,W(3,52)/0.0D0/
DATA W(1,53)/9.02512D-4/,W(2,53)/0.0D0/,W(3,53)/0.0D0/
DATA W(1,54)/1.52804D-3/,W(2,54)/0.0D0/,W(3,54)/0.0D0/
DATA W(1,55)/0.0D0/,W(2,55)/0.0D0/,W(3,55)/9.40768D-6/
```

```
      C09=4.0D0/14175.0D0
      CN9(1)=C09*989.0D0
      CN9(2)=C09*5888.0D0
      CN9(3)=-C09*928.0D0
      CN9(4)=C09*10496.0D0
      CN9(5)=-C09*4540.0D0
      CN9(6)=CN9(4)
      CN9(7)=CN9(3)
      CN9(8)=CN9(2)
      CN9(9)=CN9(1)
C*************************************************************************
C
C     BEGINNING OF INPUT SECTION: DEFINITION OF SYMBOLS
C
C     NPT       = NUMBER OF RESISTIVITY & SPREADING RESISTANCE VALUES
C     AA        = PROBE RADIUS (MICROMETERS)
C     SEP       = PROBE SEPARATION (MICROMETERS)
C     DELTH     = SUBLAYER THICKNESS (MICROMETERS)
C     DEPTH(I)  = DEPTH (MICROMETERS)
C     RSP(I)    = SPREADING RESISTANCE (OHMS)
C
C*************************************************************************
C     READ THE NUMBER OF POINTS IN THE STRUCTURE
      READ(11,*)NPT
C     READ THE DEPTH AND TWO-PROBE RESISTANCE FROM THE SURFACE
      DO 70 I=1,NPT
      READ(11,*)DEPTH(I),RSP(I)
 70   CONTINUE
C     READ THE ELECTRICAL PROBE RADIUS (MICROMETERS)
      READ(11,*)AA
C     READ THE PROBE SEPARATION (MICROMETERS)
      READ(11,*)SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ(11,*)DELTH
      SUBRSP=RSP(NPT)
 75   FORMAT(8X,'AA=',F10.3,' SEP=',F8.3,' DELTH=',F8.5,' NPT=',I5)
      AA=AA*1D-4
      SEP=SEP*1D-4
      DELTH=DELTH*1D-4
      SEP=SEP/AA
      DELTH=DELTH/AA
      L=1.122918967D0*(1.0/SEP)
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
 80   FORMAT(8X,'CALCULATION OF THE RESISTIVITY FROM SPREADING
     1 RESISTANCE')
      WRITE(6,81)
 81   FORMAT(8X,'USING CHOO ET AL VARIATIONAL TECHNIQUE-BL')
      WRITE(6,75)AA,SEP,DELTH,NPT
```

```
C*********************************************************************
C
C     BEGIN CONSTRUCTION OF SAMPLING POINTS AND WEIGHTS
C     REMEMBER TO ADD WEIGHTS AT BOUNDARY OF SAMPLING REGIONS.
C     THE REGION DESIGNATIONS FOLLOW THE NOTATION OF THE SECOND REFERENCE.
C
C*********************************************************************
C
C     REGION A (0<=KA<=1D-5) (I=1,9)
C
C*********************************************************************
      DO 142 I=1,9
      KA(I)=1.D-5*(I-1)/8.DO
      W(1,I)=1.D-5*CN9(I)/8.DO
      W(2,I)=W(1,I)/4.DO
      W(3,I)=W(1,I)/2.DO
 142  CONTINUE
      WEND1=W(1,9)
      WEND2=W(2,9)
      WEND3=W(3,9)
C*********************************************************************
C
C     REGION B (1D-5<=KA<=L) (I=9,17)
C
C*********************************************************************
      DO 144 I=9,17
      KA(I)=1.D-5*(L*1.D5)**((I-9.)/8.)
      W(1,I)=CN9(I-8)*KA(I)*DLOG(L*1.D5)/8.DO
      W(2,I)=W(1,I)/4.DO
      W(3,I)=W(1,I)/2.DO
 144  CONTINUE
      W(1,9)=W(1,9)+WEND1
      W(2,9)=W(2,9)+WEND2
      W(3,9)=W(3,9)+WEND3
C*********************************************************************
C
C     REGION C (L<=KA<=.12) (I=18,26)
C
C*********************************************************************
      DO 146 I=18,26
      KA(I)=L*(0.12DO/L)**((I-18.)/8.)
      W(1,I)=CN9(I-17)*KA(I)*DLOG(0.12/L)/8.DO
      W(2,I)=W(1,I)/4.DO
      W(3,I)=W(1,I)/2.DO
 146  CONTINUE
```

```
C*************************************************************************
C
C     REGION D, E, AND F
C     THE VALUES OF THE KA(I) AND W(I) HAVE BEEN ENTERED BY THE
C     DATA STATEMENTS AT THE BEGINNING OF THE PROGRAM
C
C*************************************************************************
      DO 155 I=1,55
      R4=KA(I)*DELTH
      T(I)=DTANH(R4)
      A(1,I)=1.0D0
      A(2,I)=1.0D0
      B(I)=1.0D0
 155  CONTINUE
      R5I1=0.0D0
      R6I1=0.0D0
      R7I1=0.0D0
      R5I2=0.0D0
      R6I2=0.0D0
      R7I2=0.0D0
      R5I3=0.0D0
      R6I3=0.0D0
      R7I3=0.0D0
      DO 200 I=18,55
      R5I1=R5I1+W(1,I)
      R5I2=R5I2+W(2,I)
      R5I3=R5I3+W(3,I)
      R6I1=R6I1+W(1,I)*T(I)
      R6I2=R6I2+W(2,I)*T(I)
      R6I3=R6I3+W(3,I)*T(I)
      R7I1=R7I1+W(1,I)/T(I)
      R7I2=R7I2+W(2,I)/T(I)
      R7I3=R7I3+W(3,I)/T(I)
 200  CONTINUE
      R5I2S=R5I2
      R5I3S=R5I3
      R6I2S=R6I2
      R6I3S=R6I3
      R7I2S=R7I2
      R7I3S=R7I3
      DO 210 I=2,17
      R5I1=R5I1+W(1,I)
      R5I2=R5I2+W(2,I)
      R5I3=R5I3+W(3,I)
      R6I1=R6I1+W(1,I)*T(I)
      R6I2=R6I2+W(2,I)*T(I)
      R6I3=R6I3+W(3,I)*T(I)
      R7I1=R7I1+W(1,I)/T(I)
      R7I2=R7I2+W(2,I)/T(I)
      R7I3=R7I3+W(3,I)/T(I)
 210  CONTINUE
```

```
      R5I1=R5I1+W(1,1)
      R5I2=R5I2+W(2,1)
      R5I3=R5I3+W(3,1)
      R6I1=R6I1+W(1,1)*T(1)
      R6I2=R6I2+W(2,1)*T(1)
      R6I3=R6I3+W(3,1)*T(1)
      DEN5=(R5I1*R5I2-R5I3*R5I3)
      R5K1=(R5I2-0.5D0*R5I3)/DEN5
      R5K2=(0.5D0*R5I1-R5I3)/DEN5
      CFDEN5=1.0D0/(R5K1+0.5D0*R5K2)
      CFINT5=CFDEN5*(R5K1*R5I3S+R5K2*R5I2S)
      R5=CFINT5
      DEN6=(R6I1*R6I2-R6I3*R6I3)
      R6K1=(R6I2-0.5D0*R6I3)/DEN6
      R6K2=(0.5D0*R6I1-R6I3)/DEN6
      CFDEN6=1.0D0/(R6K1+0.5D0*R6K2)
      CFINT6=CFDEN6*(R6K1*R6I3S+R6K2*R6I2S)
      R6=CFINT6
      DEN7=(R7I1*R7I2-R7I3*R7I3)
      R7K1=(R7I2-0.5D0*R7I3)/DEN7
      R7K2=(0.7D0*R7I1-R7I3)/DEN7
      CFDEN7=1.0D0/(R7K1+0.5*R7K2)
      CFINT7=CFDEN7*(R7K1*R7I3S+R7K2*R7I2S)
      R7=CFINT7
      WRITE(6,165)R5,R6,R7
165   FORMAT(8X,'R5=',E11.4,' R6=',E11.4,' R7=',E11.4)
      R8=1.27324/(2.0*AA)
      R9=R8/SUBRSP*R5
      SDIF(NPT)=0.0D0
      COND(NPT)=R9
      RHO(NPT)=1.0D0/R9
      ITER(NPT)=0
      SENS(NPT)=0.0D0
```

```
C*************************************************************************
C
C     LAYERED STRUCTURE CALCULATION
C
C*************************************************************************
      DO 1000 KK=NPT-1,1,-1
      ITER(KK)=0
      IR10=0
      R11=R8/RSP(KK)/COND(KK+1)
      R12=RSP(KK+1)/RSP(KK)
      R13=R12
      GOTO 1450
 1060 CONV=DABS(1.0D0-R14*R11/R12)
      IF (CONV.LT.1D-3) GO TO 1200
      IF (IR10.EQ.1) R15=0.0D0
      R16=R15
      R15=R12
      R17=(R14-R6)/(R7-R14)/R12
      R18=R7*R17*R11-1.0D0
      R13=DABS(R18)+DSQRT(R18*R18+4*R17*R11*R6)
      IF (R18.GT.0.0) R13=R13/2.0D0/R17
      IF (R18.LT.0.0) R13=2.0D0*R11*R6/R13
      VAR1=(R15-R13)*(R15-R16)
      IF (VAR1.LE.0.0D0) GO TO 1170
      VAR2=IR10-2.D0*INT(IR10/2.D0)
      IF (VAR2.EQ.0.0) R13=(R15*R15-R13*R16)/(2*R15-R13-R16)
 1170 R12=R13
      GO TO 1450
 1200 R19=R11*R14*COND(KK+1)
      R20=0.0D0
      DO 1059 I=1,55
      SEP=A(1,I)*(1.0-T(I)*T(I))
      DELTH=(R12*T(I)*A(1,I)+1.0)**2
      R20=W(1,I)*SEP/DELTH+R20
      A(1,I)=A(2,I)
 1059 CONTINUE
      SENS(KK)=1.0D0/(1.0D0-R20*R13/R14)
      COND(KK)=R19
      RHO(KK)=1.0D0/R19
      SDIF(KK)=R14*R8/R13/COND(KK+1)
      SDIF(KK)=1.0D0-SDIF(KK)/RSP(KK)
      ITER(KK)=IR10
 1000 CONTINUE
 1001 FORMAT(8X,I5,2X,4(1PE12.3),3X,I5)
 1002 FORMAT(12X,'NPT',6X,'DEPTH',5X,'RMEAS',8X,'1-RCAL/R',4X,'RHO ',
     1  8X,'ITER',/)
 1003 FORMAT(8X,1PE11.3,2X,1PE11.3)
 1004 FORMAT(8X,1PE11.4)
      WRITE(6,1002)
      WRITE(12,*)NPT
      DO 1020 I=1,NPT
      WRITE(6,1001)I,DEPTH(I),RSP(I),SDIF(I),RHO(I),ITER(I)
      WRITE(12,1003)DEPTH(I),RHO(I)
 1020 CONTINUE
      STOP
```

```
C*************************************************************************
C
C     THE REMAINING LINES ARE PART OF THE GENERALIZATION TO THE VARIATIONAL
C     TECHNIQUE OF THE BERKOWITZ AND LUX BOUNDING METHOD FOR INVERSION OF
C     THE CORRECTION FACTOR INTEGRAL (FOR WHICH THERE ARE SEVERAL INTEGRALS
C     IN THE PRESENT CASE).
C
C*************************************************************************
 1450 DO 1475 I=1,55
      A(2,I)=(R12*A(1,I)+T(I))/(1.0D0+T(I)*A(1,I)*R12)
      B(I)=A(2,I)
 1475 CONTINUE
      R14=0.0D0
      R5I1=0.0D0
      R5I2=0.0D0
      R5I3=0.0D0
      DO 1480 I=18,55
      R5I1=R5I1+W(1,I)*B(I)
      R5I2=R5I2+W(2,I)*B(I)
      R5I3=R5I3+W(3,I)*B(I)
 1480 CONTINUE
      R5I2S=R5I2
      R5I3S=R5I3
      DO 1485 I=1,17
      R5I1=R5I1+W(1,I)*B(I)
      R5I2=R5I2+W(2,I)*B(I)
      R5I3=R5I3+W(3,I)*B(I)
 1485 CONTINUE
      DEN=(R5I1*R5I2-R5I3*R5I3)
      R5K1=(R5I2-0.5D0*R5I3)/DEN
      R5K2=(0.5D0*R5I1-R5I3)/DEN
      CFDEN=1.0D0/(R5K1+0.5D0*R5K2)
      CFINT=CFDEN*(R5K1*R5I3S+R5K2*R5I2S)
      R14=CFINT
      IR10=IR10+1
      GO TO 1060
      END
C*****END OF THE VAR VERSION 1.0 PROGRAM*************************************
```

```
C*********************************************************************
C     PROBE24 VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE DEPTH-DEPENDENT, FOUR-PROBE RESISTANCE,
C     Z(X,S), THE DERIVATIVE OF THE TWO-PROBE (SPREADING) RESISTANCE WITH
C     RESPECT TO THE PROBE SPACING, D(X,S), AS WELL AS THE FOUR-PROBE SHEET
C     RESISTANCE, SH(X).
C
C     THE FOUR-PROBE RESISTANCE AND THE DERIVATIVE FUNCTION ARE CALCULATED
C     FROM THE MULTILAYER SOLUTION OF LAPLACE'S EQUATION (SEE REFERENCE).
C     THE FOUR-PROBE SHEET RESISTANCE IS CALCULATED FROM THE TRADITIONAL
C     LAMINAR FLOW MODEL.
C
C     THE INTEGRALS INVOLVED IN Z(X,S) AND D(X,S) ARE EVALUATED BY MEANS
C     OF A 1250- (OR 10000-) POINT INTEGRATION METHOD USED BY BERKOWITZ.
C
C     REFERENCES: "THE RELATION BETWEEN TWO-PROBE AND FOUR-PROBE RESISTANCES
C                  ON NONUNIFORM STRUCTURES",
C                  BY JOHN ALBERS AND H. L. BERKOWITZ, JOURNAL OF THE
C                  ELECTROCHEMICAL SOCIETY, VOL. 131, PP. 392-398 (1984).
C
C     THIS PROGRAM WAS ORIGINALLY WRITTEN
C     FOR THE CALCULATION OF THE FOUR-PROBE FUNCTION Z(X).  WE HAVE
C     USED THE INTEGRATION SCHEME TO ALSO CALCULATE THE FUNCTION S(X),
C     THE DERIVATIVE OF SPREADING RESISTANCE WITH RESPECT TO LN(S).  IN
C     ADDITION, THE SHEET RESISTANCE IS ALSO CALCULATED BY MEANS OF THE
C     STANDARD LAMINAR FLOW MODEL.  THESE THREE FUNCTION ARE COMPARED,
C     AND EACH IS STORED IN APPROPRIATE DATA FILES.
C
C     NOTE-THE EQUATIONS USED MAKE USE OF THE CHOO UNIFORM CURRENT
C          DENSITY.  THIS CAN BE CHANGED IF DESIRED BY ALTERING THE
C          LINES WHERE W(1,I) AND W(2,I) ARE DEFINED.
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION COND(300), OUT(6,300), ROL(300), W5(5)
      DOUBLE PRECISION HTAN(10000), F(10000), W(2,10000)
C*********************************************************************
C     DESCRIPTION OF VARIABLES AND ARRAYS
C
C         NPT    = NUMBER OF DATA POINTS
C         SEP    = SEPARATION BETWEEN THE PROBES (MICROMETERS)
C         DELTH  = DEPTH INCREMENT (MICROMETERS)
C         AA     = PROBE RADIUS (MICROMETERS)
C         SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM)
C         SUBTH  = SUBSTRATE THICKNESS (MICROMETERS)
C         IBBC   = BACKSURFACE BOUNDARY CONDITION
C                  (NONE = 1, COND = 2, INS = 3)
C         RHO    = ARRAY OF RESISTIVITY VALUES (OHM*CM)
C         NMANY  = NUMBER OF INTEGRATION POINTS TO BE USE
C                  (EITHER 1250 OR 10000 (ALL BETS OFF FOR OTHERS))
C
C*********************************************************************
```

```
C     READ THE ELECTRICAL PROBE RADIUS (MICROMETERS)
      READ (5,*) AA
C     READ THE PROBE SEPARATION (MICROMETERS)
      READ (5,*) SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ (5,*) DELTH
C     READ THE NUMBER OF POINTS IN THE STRUCTURE
      READ (5,*) NPT
C     READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ (5,*) SUBT
C     READ THE SUBSTRATE RESSISTIVITY (OHM*CM)
      READ (5,*) SUBRHO
C     READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1,COND=2,INS=3)
      READ (5,*) IBBC
      NUM=NPT
C     READ THE RESISTIVITIES OF THE STRUCTURE (OHM*CM) FROM SURFACE
      DO 7 I=1,NUM
      READ (5,*) ROL(I)
      COND(I)=1.0D0/ROL(I)
  7   CONTINUE
C     READ THE NUMBER OF INTEGRATION POINTS TO BE USED
C     THIS MAY BE EITHER 1250 OR 10000 (ALL BETS OFF FOR OTHERS)
      READ (5,*) NMANY
C     IF(NMANY.NE.1250.OR.NMANY.NE.10000) GOTO 666
      DO 5 I=1,2
      DO 2 J=1,NMANY
      W(I,J)=0.0D0
  2   CONTINUE
  5   CONTINUE
      W5(1)=7.D0
      W5(2)=32.D0
      W5(3)=12.D0
      W5(4)=32.D0
      W5(5)=7.D0
      TEMP=0.0
      TEMP1=0.0
      TEMP2=0.0
      TEMP3=0.0
      DX=500.03/NMANY
      DO 10 I=1,5
 10   W5(I)=W5(I)*2.0D0*DX/45
      PI=3.141592654D0
      DO 20 I=2,5
      II=I-1
      W(1,II)=W5(I)
 20   W(2,II)=W5(I)
      A=AA/SEP
      T=DELTH/SEP
      SUB=SUBT/SEP
      S1=SEP*1.D-4
```

```
      DO 30 I=4,1246,4
      DO 30 J=0,4
      IJ=I+J
      J1=J+1
      W(1,IJ)=W(1,IJ)+W5(J1)
30    W(2,IJ)=W(2,IJ)+W5(J1)
      DO 11 I=1,NMANY
      X=DX*I
      W(1,I)=W(1,I)*(DBJ0(X)-DBJ0(2*X))*2*DBJ1(A*X)/(A*X)
 11   W(2,I)=W(2,I)*2*DBJ1(X)*DBJ1(A*X)
      DO 13 I=1,NMANY
      X=DX*I
      XT=X*T
      IF (XT.GT.40) HTAN(I)=1.0D0
      IF (XT.GT.40) GO TO 17
      HTAN(I)=(EXP(X*T)-EXP(-X*T))/(EXP(X*T)+EXP(-X*T))
17    XSUB=X*SUB
      IF (XSUB.GT.40) F(I)=1.0D0
      IF (XSUB.GT.40) GO TO 375
      F(I)=(EXP(X*SUB)+EXP(-X*SUB))/(EXP(X*SUB)-EXP(-X*SUB))
375   GO TO (400,500,600),IBBC
400   F(I)=1.0D0
      GO TO 600
500   F(I)=1.0D0/F(I)
600   TEMP1=TEMP1+W(1,I)
      TEMP=TEMP+W(1,I)*F(I)
      TEMP2=TEMP2+W(2,I)
      TEMP3=TEMP3+W(2,I)*F(I)
 13   CONTINUE
      CORR=0.5D0*(1+7./64.*A*A)-TEMP1
      IF (A.GT..01) CORR=0.0
      NUM1=NUM+1
      COND(NUM1)=1.0D0/SUBRH0
      OUT(1,NUM1)=NUM*T*SEP
      OUT(2,NUM1)=SUBRH0
      OUT(3,NUM1)=COND(NUM1)
      OUT(4,NUM1)=(TEMP+CORR)/PI/S1*SUBRH0
      OUT(5,NUM1)=COND(NUM1)*SUB*S1
      OUT(6,NUM1)=(TEMP3+CORR)/PI/AA/1.0D-4*SUBRH0
      DO 14 I=NUM,1,-1
      OUT(1,I)=(I-1)*T*SEP
      OUT(2,I)=ROL(I)
      OUT(3,I)=COND(I)
      TEMP=0.0
      TEMP3=0.0
      I1=I+1
      OUT(5,I)=OUT(5,I1)+COND(I)*T*S1
```

```
      DO 15 J=1,NMANY
      F(J)=(COND(I)*F(J)+HTAN(J)*COND(I1))
     1 /(COND(I1)+COND(I)*F(J)*HTAN(J))
      TEMP=TEMP+W(1,J)*F(J)
   15 TEMP3=TEMP3+W(2,J)*F(J)
      OUT(4,I)=(TEMP+CORR)/(PI*S1*COND(I))
      OUT(6,I)=(TEMP3+CORR)/(PI*AA*1.0D-4*COND(I))
   14 CONTINUE
      DO 16 I=1,NUM1
      OUT(5,I)=1.0D0/OUT(5,I)
C      OUT(4,I)=OUT(4,I)*PI/DLOG(2.0D0)
   16 OUT(6,I)=OUT(6,I)*PI
      WRITE (6,7777)
 7777 FORMAT(1H1)
      WRITE (6,100)
  100 FORMAT (2X,'FOUR-PROBE RESISTANCE CALCULATIONS USING THE')
      WRITE (6,125) NMANY
  125 FORMAT (2X,'BERKOWITZ NMANY=',I7,' POINT INTEGRATOR ')
      WRITE (6,150)
  150 FORMAT (2X,'STRUCTURE OVER UNIFORM LAYER (SUBRHO,SUBTH)')
      WRITE (6,155) IBBC
  155 FORMAT (2X,' ALL OVER (NONE=1,CON=2,INS=3)',I4,/)
      WRITE (6,200)
  200 FORMAT (4X,'AA',7X,'SEP',7X,'DELTH',5X,'SUBTH',8X,'SUBRHO')
      WRITE (6,250) AA,SEP,DELTH,SUBT,SUBRHO
  250 FORMAT (2X,F5.2,2X,F9.1,2X,F7.3,2(2X,1PE11.2))
      WRITE (6,275)
  275 FORMAT(/,2X,'DEPTH(UM)',6X,'RHO',7X,'FOUR-PRB',4X,'DERIVATI',5X
     1 ,'SHEET')
      DO 300 I=1,NUM1
      WRITE (6,350) OUT(1,I),OUT(2,I),OUT(4,I),OUT(6,I),OUT(5,I)
  350 FORMAT (1X,5(1PE11.4,1X))
      WRITE (11,800) OUT(1,I),OUT(4,I)
      WRITE (12,800) OUT(1,I),OUT(6,I)
      WRITE (13,800) OUT(1,I),OUT(5,I)
  800 FORMAT(1X,2(1PE11.3,1X))
  300 CONTINUE
      WRITE (11,900)
      WRITE (12,950)
      WRITE (13,975)
  900 FORMAT(2X,'RESULTS FOR DIFFERENCE FUNCTION USING BL NMANY=',I7,
     1' INT')
  950 FORMAT(2X,'RESULTS FOR DERIVATIVE FUNCTION USING BL NMANY=',I7,
     1' INT')
  975 FORMAT(2X,'CALCULATED SHEET RESISTANCE FROM LAMINAR MODEL')
      STOP
      END
C*****END OF MAIN PORTION OF PROBE24 VERSION 1.0**************************
```

```
      DOUBLE PRECISION FUNCTION DBJ0(X)
C*********************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ0
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.1 IS USED FOR X < 3
C                EQUATION 9.4.3 IS USED FOR X > 3
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F0,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/1.0000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/
      DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
      DATA A7/0.0002100D0/
      DATA B1/.79788456D0/,B2/-.00000077D0/,B3/-.00552740D0/
      DATA B4/-.00009512D0/,B5/.00137237D0/,B6/-.00072805D0/
      DATA B7/.00014476D0/
      DATA C1/-.78539816D0/,C2/-.04166397D0/,C3/-.00003954D0/
      DATA C4/.00262573D0/,C5/-.00054125D0/,C6/-.00029333D0/
      DATA C7/.00013558D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.0D0
      DBJ0=(((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
      RETURN
10    CONTINUE
      X1=3.D0/X
      F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ0=F0*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DOUBLE PRECISION FUNCTION DBJ0*******************************
```

```
      DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.4 IS USED FOR X < 3
C                AND EQUATION 9.4.6 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F1,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/
      DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/
      DATA A7/.00001109D0/
      DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/
      DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/
      DATA B7/-.00020033D0/
      DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/
      DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/
      DATA C7/-.00029166D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.D0
      DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
      RETURN
   10 X1=3.0D0/X
      F1=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ1=F1*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DOUBLE PRECISION FUNCTION DBJ1 END OF PROBE24 VERSION 1.0****
```

```
C***********************************************************************
C     FOURCAL VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE FOLLOWING DEPTH-DEPENDENT QUANTITIES:
C          1) FOUR-PROBE RESISTANCE, Z(X,S),
C          2) THE DERIVATIVE OF THE TWO-PROBE (SPREADING) RESISTANCE WITH
C             RESPECT TO THE LOGARITHM OF THE PROBE SPACING, D(X,S), AND
C          3) THE FOUR-PROBE SHEET RESISTANCE, SH(X).
C
C     THE FOUR-PROBE RESISTANCE AND THE DERIVATIVE FUNCTION ARE CALCULATED
C     FROM THE SCHUMANN AND GARDNER MULTILAYER SOLUTION OF LAPLACE'S
C     EQUATION (SEE REFERENCES) WHILE THE FOUR-PROBE SHEET RESISTANCE IS
C     CALCULATED FROM THE TRADITIONAL LAMINAR FLOW MODEL.
C
C     THE INTEGRALS INVOLVED IN Z(X,S) AND D(X,S) ARE EVALUATED OVER THE
C     INTERVAL FROM XL TO XU BY MEANS OF A TRAPEZOIDAL ROMBERG METHOD USED
C     ORIGINALLY BY D'AVANZO ET AL. IN THEIR SPINT PROGRAM FOR CALCULATING
C     THE ARRAY OF PARTIAL INTEGRALS.
C
C     REFERENCES: THE TRAPEZOIDAL ROMBERG METHOD WAS USED BY D'AVANZO ET AL.
C                 FOR CALCULATING THE CORRECTION FACTOR INTEGRAL FOR THE
C                 PURPOSE OF SPREADING RESISTANCE PROFILE ANALYSIS.  THIS
C                 IS DESCRIBED IN THE REPORT "SPREADING RESISTANCE FOR
C                 IMPURITY PROFILES" BY DONALD C. D'AVANZO, ROBERT D. RUNG,
C                 AND ROBERT W. DUTTON, TECHNICAL REPORT NO. 5013-2 (FEBRUARY
C                 1977), STANFORD UNIVERSITY, STANFORD, CALIFORNIA.  THIS IS
C                 ALSO DESCRIBED IN THE PAPER "HIGH SPEED IMPLEMENTATION AND
C                 EXPERIMENTAL EVALUATION OF MULTILAYER SPREADING RESISTANCE
C                 ANALYSIS" BY DONALD C. D'AVANZO, ROBERT D. RUNG, ARNON GAT,
C                 AND ROBERT W. DUTTON, JOURNAL OF THE ELECTROCHEMICAL SOCIETY,
C                 VOL. 125, PP.1170-1176 (1978).
C
C                 THE CALCULATION OF THE SPREADING RESISTANCE FROM THE
C                 RESISTIVITY USING THE TRAPEZOIDAL ROMBERG MEHTOD IS DISCUSSED
C                 IN THE PAPER "COMPARISON OF SPREADING RESISTANCE CORRECTION
C                 FACTOR ALGORITHMS USING MODEL DATA" BY JOHN ALBERS,
C                 SOLID-STATE ELECTRONICS, VOL. 23, PP. 1197-1205 (1980).
C
C                 THE PRESENT CALCULATION IS DESCRIBE IN THE PAPER "THE
C                 RELATION BETWEEN TWO-PROBE AND FOUR-PROBE RESISTANCES ON
C                 NONUNIFORM STRUCTURES", BY JOHN ALBERS AND H. L. BERKOWITZ,
C                 JOURNAL OF THE ELECTROCHEMICAL SOCIETY, VOL. 131,
C                 PP. 392-398 (1984).
C
C***********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      EXTERNAL FPINT
      EXTERNAL DEINT
      DIMENSION RHO(300), DEPTH(300), COND(300), ZEX(300), DEX(300)
      DIMENSION SHEET(300),SHECON(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      DATA PI/3.141592654DO/
```

```
C*************************************************************************
C
C     DESCRIPTION OF THE VARIABLES AND ARRAYS
C
C          NPT    = NUMBER OF DATA POINTS
C          AA     = PROBE RADIUS (MICROMETERS)
C          SEP    = SEPARATION BETWEEN THE PROBES (MICROMETERS)
C          DELTH  = DEPTH INCREMENT (MICROMETERS)
C          IBBC   = BACKSURFACE BOUNDARY CONDITION
C                   (NONE = 1,COND = 2,INS = 3)
C          SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM)
C          SUBTH  = SUBSTRATE THICKNESS (MICROMETERS)
C          RHO    = ARRAY OF RESISTIVITY VALUES (OHM*CM)
C          ICUR   = CURRENT DENSITY
C                   (SCHUMANN & GARDNER = 1, CHOO UNIFORM = 2, RING DELTA = 3)
C          XL = LOWER LIMIT OF INTEGRATION (USUALLY ZERO)
C          XU = UPPER LIMIT OF INTEGRATION (FROM 10 ON UP)
C
C     TO RUN THE PROGRAM FOR A STRUCTURE DIRECTLY ON A BACKSURFACE BOUNDARY,
C     TAKE THE LAST POINT IN THE STRUCTURE AS THE SUBSTRATE AND SET
C     SUBTH=DELTH AND SUBRHO=RHO(1).
C
C*************************************************************************
C     READ THE TOTAL NUMBER OF POINTS OR SUBLAYERS IN STRUCTURE.
      READ (5,*) NPT
      NUM=NPT+1
C     READ THE EFFECTIVE ELECTRICAL PROBE RADIUS
      READ (5,*) AA
C     READ THE SEPARATION BETWEEN THE PROBES
      READ (5,*) SEP
C     READ THE DISTANCE BETWEEN POINTS OR THE SUBLAYER THICKNESS.
      READ (5,*) DELTH
C     READ THE BOUNDARY CONDITION (NONE=1,COND=2,INS=3)
      READ (5,*) IBBC
C     READ THE SUBSTRATE RESISTIVITY
      READ (5,*) SUBRHO
C     READ THE SUBSTRATE THICKNESS
      READ (5,*) SUBTH
C     RESISTIVITY ARRAY READ FROM SURFACE TO SUBSTRATE.
      DO 10 I=NUM,2,-1
      READ (5,*) RHO(I)
      COND(I)=1.0D0/RHO(I)
  10  CONTINUE
C     READ THE FORM OF THE CURRENT DENSITY
      READ(5,*) ICUR
C     READ THE LOWER LIMIT OF INTEGRATION
      READ (5,*) XL
C     READ THE UPPER LIMIT OF INTEGRATION
      READ (5,*) XU
      RHO(1)=SUBRHO
      COND(1)=1.0D0/SUBRHO
```

```
C*************************************************************************
C
C     THE 1200 DO LOOP IS THE MAJOR LOOP WHICH CALCULATES THE VARIOUS
C     INTEGRALS (OVER THE INTERVAL FROM XL TO XU) FROM THE SUBSTRATE
C     (NUM1=NUM) TO THE SURFACE (NUM1=1) BY INCREMENTING NUM1 BY -1.
C     THE RELEVANT INFORMATION IS PASSED THROUGH AND CONTAINED IN THE
C     COMMON STATEMENT IN THE MAIN PROGRAM AND THE SUBROUTINES.
C     IFLAG IS SET EQUAL TO ZERO AS XL IS USUALLY ZERO AND THE INTEGRAND
C     IS  ZERO AT XL=0.  HOWEVER, THE CURRENT DENSITY FUNCTIONS MAY GIVE
C     ARTIFICIAL SINGULARITIES AT THE ORIGIN.  FOR EXAMPLE, THE SCHUMANN
C     AND GRADNER CURRENT DENSITY FUNCTION, SIN(X)/X, GOES TO ZERO AS X
C     GOES TO ZERO BUT THE COMPUTER WOULD SIGNAL AN OVERFLOW DUE TO THE
C     1/X.  THIS IS REMOVED BY THE SETTING OF IFLAG=0.
C
C*************************************************************************
      DO 1200 NUM1=1,NUM
      IFLAG=0
      CALL INTGRL(FPINT,XL,XU,ZEX(NUM1),IFLAG)
      ZEX(NUM1)=ZEX(NUM1)*RHO(NUM1)/(PI*AA*1.0D-4)
      CALL INTGRL(DEINT,XL,XU,DEX(NUM1),IFLAG)
      DEX(NUM1)=DEX(NUM1)*RHO(NUM1)/(AA*1.0D-4)
      DEPTH(NUM1)=DELTH*(NUM-NUM1)
 1200 CONTINUE
      SHEET(1)=COND(1)*SUBTH*1.0D-4
      SHEET(1)=1.0D0/SHEET(1)
      DO 1300 NUM1=2,NUM
      SHEET(NUM1)=1.0D0/SHEET(NUM1-1)+COND(NUM1)*DELTH*1.0D-4
      SHEET(NUM1)=1.0D0/SHEET(NUM1)
 1300 CONTINUE
      WRITE (6,7777)
 7777 FORMAT(1H1)
      GO TO (1400,1420,1440),ICUR
 1400 WRITE(6,1410)
 1410 FORMAT(2X,'SCHUMANN AND GARDNER CURRENT DENSITY=SIN(AX)/X')
      GO TO 1453
 1420 WRITE(6,1430)
 1430 FORMAT(2X,'CHOO UNIFORM CURRENT DENSITY=2*DBJ1(AX)/X')
      GO TO 1453
 1440 WRITE (6,1450)
 1450 FORMAT(2X,'RING CURRENT DENSITY=A*DBJ0(AX)')
 1453 GO TO (1455,1465,1475), IBBC
 1455 WRITE (6,1460)
 1460 FORMAT (2X,'NO BOUNDARY CONDITION USED IN CALCULATION',/)
      GO TO 1500
 1465 WRITE (6,1470)
 1470 FORMAT (2X,'PERFECTLY CONDUCTING BOUNDARY A1=TANH(LD)',/)
      GO TO 1500
 1475 WRITE (6,1480)
 1480 FORMAT (2X,'PERFECTLY INSULATING BOUNDARY A1=COTH(LD)',/)
```

```
 1500 WRITE (6,1550) XL,XU
 1550 FORMAT(2X,'XL=',F11.6,'; XU=',F11.6)
      WRITE (6,1700) AA,SEP,DELTH
 1700 FORMAT(2X,'RADIUS=',F11.6,'; SEP=',F11.6,'; DELTH=',F11.6)
      WRITE (6,2250) SUBRHO,SUBTH
 2250 FORMAT(2X,'SUBRHO=',1PE11.5,'; SUBTHICK=',1PE11.5)
      WRITE (6,2300)
 2300 FORMAT(2X,'DEPTH(U)',8X,'RHO',5X,'FOUR-PROBE',6X,'DERIV',8X,
     1    'SHEET')
      WRITE(11,*)NUM
      WRITE(12,*)NUM
      WRITE(13,*)NUM
      DO 2500 I=NUM,1,-1
      WRITE (6,2400)  DEPTH(I),RHO(I),ZEX(I),DEX(I),SHEET(I)
 2400 FORMAT(1X,5(1PE11.3,1X))
      WRITE (11,2405) DEPTH(I),ZEX(I)
      WRITE (12,2405) DEPTH(I),DEX(I)
      WRITE (13,2405) DEPTH(I),SHEET(I)
 2405 FORMAT(1X,2(1PE11.3,1X))
 2500 CONTINUE
      WRITE (11,1550) XL,XU
      WRITE (11,1700) AA,SEP,DELTH
      WRITE (11,2250) SUBRHO,SUBTH
      WRITE (12,1550) XL,XU
      WRITE (12,1700) AA,SEP,DELTH
      WRITE (12,2250) SUBRHO,SUBTH
      STOP
      END
C*****END OF MAIN PORTION OF FOURCAL VERSION 1.0 PROGRAM*******************
```

```
      SUBROUTINE INTGRL(F,A,B,ATEMP,IFLAG)
C*************************************************************************
C
C     D'AVANZO'S ET AL. TRAPEZOIDAL ROMBERG INTEGRATION SUBROUTINE.
C     THIS ROUTINE NUMERICALLY INTEGRATES F BETWEEN THE LIMITS A AND B
C     USING THE TRAPEZOIDAL AND ROMBERG METHODS AND RETURNS THE INTEGRAL
C     FROM A TO B IN ATEMP AND ACCUMULATES THE RESULTS OF SEVERAL
C     SUCCESSIVE CALLS IN ACC.
C     IF IFLAG=0 THEN F(0) IS SET TO 0 (THIS OBVIATES OVERFLOW PROBLEMS
C     DUE TO FUNCTIONS LIKE SIN(X)/X
C
C*************************************************************************
      EXTERNAL F
      DOUBLE PRECISION AI(30,30),MI,A,B,H,ACC,ATEMP,F0,F,CONV
C     CALCULATE THE TRAPEZOIDAL APPROXIMATION TO THE AREA
      CONV=5.0D-5
      J=30
      M=1
      K=1
C     SET F(0)=0
      F0=0.D0
      IF ((A.NE.0.D0).OR.(IFLAG.EQ.1)) F0=F(A)
      AI(M,K)=.5D0*(B-A)*(F0+F(B))
      DO 30 I=1,J
      MI=0.0D0
      H=(B-A)/(2.0D0**(M-1))
      LPCNT=2**(M-1)
      DO 10 N=1,LPCNT
      MI=MI+F(A+(N*H-.5D0*H))
10    CONTINUE
      MI=MI*H
      AI(M+1,K)=.5D0*(AI(M,K)+MI)
C     INCREASE THE ACCURACY OF THE TRAPEZOIDAL APPROXIMATIONS
C     BY USING THE ROMBERG METHOD
      MPONE = M+1
      DO 20 K=1,MPONE
      AI(M+1,K+1)=((4.0D0**K)*AI(M+1,K)-AI(M,K))/(4.0D0**K-1.D0)
20    CONTINUE
      ATEMP=AI(M+1,M+1)
      IF (I.LE.6) GO TO 60
C     STOP INTEGRATING WHEN INTEGRAL CONVERGES TO WITHIN .005%
C     NOTE-CONVERGENCE CRITERION IS ON NEXT PROGRAM LINE
      IF (DABS(ATEMP-AI(M,M))/DABS(ATEMP).GT.CONV) GO TO 60
53    ACC=ACC+ATEMP
      RETURN
60    M=M+1
      K=1
30    CONTINUE
      RETURN
      END
C*****END OF SUBROUTINE INTGRL*****************************************
```

```
      DOUBLE PRECISION FUNCTION FPINT(X)
C*************************************************************************
C
C     FPINT IS THE INTEGRAND OF THE INTEGRAL INVOLVED IN THE FOUR-PROBE
C     RESISTANCE.  THE EXPRESSION MAY BE DERIVED FROM THE SCHUMANN AND
C     GARDNER SOLUTION OF THE MULTILAYER LAPLACE EQUATION.
C     THE IS AN OPTION OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY:
C          100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY
C          200-THE CHOO ET AL. UNIFORM CURRENT DENSITY
C          300-THE RING DELTA FUNCTION CURRENT DENSITY
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHO(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      PRTPR=DBJ0(SEP*X)-DBJ0(SEP*X*2.D0)
      GO TO (100,200,300),ICUR
  100 FPINT=ALAM(X)*PRTPR*DSIN(X*AA)/X
      RETURN
  200 FPINT=ALAM(X)*PRTPR*2.D0*DBJ1(AA*X)/X
      RETURN
  300 FPINT=ALAM(X)*PRTPR*AA*DBJ0(AA*X)
      RETURN
      END
C*****END OF DOUBLE PRECISION FUNCTION FPINT*******************************
      DOUBLE PRECISION FUNCTION DEINT(X)
C*************************************************************************
C
C     DEINT IS THE INTEGRAND OF THE INTEGRAL INVOLVED IN THE DERIVATIVE
C     OF THE TWO-PROBE (SPREADING) RESISTANCE WITH RESPECT TO THE NATURAL
C     LOGARITHM OF THE PROBE SPACING, D(X,S) = d R(X,S)/ d lnS.
C     THE EXPRESSION MAY BE DERIVED FROM THE SCHUMANN AND GARDNER SOLUTION
C     OF THE MULTILAYER LAPLACE EQUATION.
C     THERE IS AN OPTION OF SEVERAL FORMS OF THE PROBE CURRENT DENSITY:
C          100-THE SCHUMANN AND GARDNER (INFINITE SLAB) CURRENT DENSITY
C          200-THE CHOO ET AL. UNIFORM CURRENT DENSITY
C          300-THE RING DELTA FUNCTION CURRENT DENSITY
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHO(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      PRTPR=X*SEP*DBJ1(SEP*X)
      GO TO (100,200,300),ICUR
  100 DEINT=ALAM(X)*PRTPR*DSIN(X*AA)/X
      RETURN
  200 DEINT=ALAM(X)*PRTPR*2.D0*DBJ1(AA*X)/X
      RETURN
  300 DEINT=ALAM(X)*PRTPR*AA*DBJ0(AA*X)
      RETURN
      END
C*****END OF DOUBLE PRECISION FUNCTION DEINT*******************************
```

```
      DOUBLE PRECISION FUNCTION ALAM(X)
C*************************************************************************
C
C     THIS FUNCTION CALCULATES THE VALUE OF THE RESISTIVITY DEPENDENT
C     PORTION OF THE VARIOUS INTEGRALS USING THE RECURSION RELATION OF
C     CHOO ET AL. AND EXTENDED BY BERKOWITZ AND LUX.
C
C     REFERENCES: "ON THE CALCULATION OF SPREADING RESISTANCE CORRECTION
C                 FACTORS" BY S. C. CHOO, M. S. LEONG AND K. L. KUAN,
C                 SOLID-STATE ELECTRONICS, VOL. 19, PP. 561-565 (1976).
C
C                 "AN EFFICIENT INTEGRATION TECHNIQUE FOR USE IN THE
C                 MULTILAYER ANALYSIS OF SPREADING RESISTANCE PROFILES"
C                 BY H. L. BERKOWITZ AND R. A. LUX, JOURNAL OF THE
C                 ELECTROCHEMICAL SOCIETY, VOL. 128, PP. 1137-1141 (1981).
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION RHO(300)
      COMMON DELTH,AA,SEP,XL,XU,RHO,NUM1,ICUR,IBBC,SUBRHO,SUBTH
      DTXSB=DTANH(X*SUBTH)
      DTXDL=DTANH(X*DELTH)
      ALAM=1.0D0
      IF (IBBC.EQ.2) ALAM=ALAM*DTXSB
      IF (IBBC.EQ.3) ALAM=ALAM/DTXSB
      IF (NUM1.EQ.1) GO TO 110
      DO 100 I=2,NUM1
      ALAM=(DTXDL*RHO(I)+RHO(I-1)*ALAM)/(RHO(I)+RHO(I-1)*ALAM*DTXDL)
  100 CONTINUE
  110 RETURN
      END
C*****DOUBLE PRECISION FUNCTION ALAM*************************************
```

```
      DOUBLE PRECISION FUNCTION DBJ0(X)
C*********************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ0
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.1 IS USED FOR X < 3
C                AND EQUATION 9.4.3 IS USED FOR X > 3
C
C*********************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F0,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/1.0000000D0/,A2/-2.2499997D0/,A3/1.2656208D0/
      DATA A4/-0.3163866D0/,A5/0.0444479D0/,A6/-0.0039444D0/
      DATA A7/0.0002100D0/
      DATA B1/.79788456D0/,B2/-.00000077D0/,B3/-.00552740D0/
      DATA B4/-.00009512D0/,B5/.00137237D0/,B6/-.00072805D0/
      DATA B7/.00014476D0/
      DATA C1/-.78539816D0/,C2/-.04166397D0/,C3/-.00003954D0/
      DATA C4/.00262573D0/,C5/-.00054125D0/,C6/-.00029333D0/
      DATA C7/.00013558D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.0D0
      DBJ0=(((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1
      RETURN
10    CONTINUE
      X1=3.D0/X
      F0=(((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ0=F0*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****DOUBLE PRECISION FUNCTION DBJ0*************************************
```

```
      DOUBLE PRECISION FUNCTION DBJ1(X)
C*************************************************************************
C
C     POLYNOMIAL APPROXIMATION TO THE BESSEL FUNCTION BJ1
C
C     REFERENCE: "HANDBOOK OF MATHEMATICAL FUNCTIONS", EDITED BY ABRAMOWITZ
C                AND STEGUN, NATIONAL BUREAU OF STANDARDS APPLIED
C                MATHEMATICS SERIES 55.  EQUATION 9.4.4 IS USED FOR X < 3
C                AND EQUATION 9.4.6 IS USED FOR X > 3
C
C*************************************************************************
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION X,X1,X2,F1,THETA
C      DOUBLE PRECISION A1,A2,A3,A4,A5,A6,A7
C      DOUBLE PRECISION B1,B2,B3,B4,B5,B6,B7
C      DOUBLE PRECISION C1,C2,C3,C4,C5,C6,C7
      DATA A1/0.500000D0/,A2/-.56249985D0/,A3/.21093573D0/
      DATA A4/-.03954289D0/,A5/.00443319D0/,A6/-.00031761D0/
      DATA A7/.00001109D0/
      DATA B1/.79788456D0/,B2/.00000156D0/,B3/.01659667D0/
      DATA B4/.00017105D0/,B5/-.00249511D0/,B6/.00113653D0/
      DATA B7/-.00020033D0/
      DATA C1/-2.35619449D0/,C2/.12499612D0/,C3/.00005650D0/
      DATA C4/-.00637879D0/,C5/.00074348D0/,C6/.00079824D0/
      DATA C7/-.00029166D0/
      IF (X.GT.3.0D0) GO TO 10
      X2=X*X/9.D0
      DBJ1=X*((((((A7*X2+A6)*X2+A5)*X2+A4)*X2+A3)*X2+A2)*X2+A1)
      RETURN
   10 X1=3.0D0/X
      F1=((((B7*X1+B6)*X1+B5)*X1+B4)*X1+B3)*X1+B2)*X1+B1
      THETA=X+(((((C7*X1+C6)*X1+C5)*X1+C4)*X1+C3)*X1+C2)*X1+C1
      DBJ1=F1*DCOS(THETA)/DSQRT(X)
      RETURN
      END
C*****END OF DOUBLE PRECISION FUNCTION DBJ1 & END OF FOURCAL VERSION 1.0****
```

```
C*************************************************************************
C     SHAZAM VERSION 1.0
C
C     THIS PROGRAM CALCULATES THE FOUR-PROBE RESISTANCE (FOR IN-LINE
C     CONFIGURATION) FROM THE RESISTIVITY BY USING THE
C     1/2S AND 1/S LIMITS IN THE SIMPLIFIED INTEGRAL DISCUSSED BY ALBERS
C     AND BERKOWITZ.  ALSO CALCULATED IS THE BEST EDUCATED GUESS AS TO THE
C     APPROXIMATE FORM FOR THE LN(S) SLOPE.
C
C     REFERENCE: "AN ALTERNATIVE APPROACH TO THE CALCULATION OF FOUR-PROBE
C                 RESISTANCES ON NONUNIFORM STRUCTURES",
C                 BY JOHN ALBERS AND H. L. BERKOWITZ, JOURNAL OF THE
C                 ELECTROCHEMICAL SOCIETY, VOL. 132, PP. 2453-2456, (1985).
C
C*************************************************************************
      DIMENSION RHO(500),DEPTH(500),ZAP(500),DAP(500)
      DIMENSION A(2,9),C(9),W(9),T(9)
      REAL L(9)
C*************************************************************************
C
C     THE NINE NEWTON-COTES COEFFICIENTS ARE ENTERED IN THE
C     FOLLOWING DATA STATEMENT
C
C*************************************************************************
      DATA C(1)/0.0348854/,C(2)/0.20769/,C(3)/-0.0327337/
      DATA C(4)/0.370229/,C(5)/-0.160141/,PI/3.14159265/
      C(6)=C(4)
      C(7)=C(3)
      C(8)=C(2)
      C(9)=C(1)
C*************************************************************************
C
C     BEGINNING OF INPUT SECTION: DEFINITION OF SYSBOLS
C
C     N      = NUMBER OF RESISTIVITY, SHEET RESISTANCE VALUES
C     SEP    = PROBE SEPARATION (MICROMETERS)
C     DELTH  = SUBLAYER THICKNESS (MICROMETERS)
C     IBBC   = BOUNDARY CONDITION (NONE = 0;CONDUCTING = 1;INSULATING = 2)
C     SUBRHO = SUBSTRATE RESISTIVITY (OHM*CM)
C     SUBT   = SUBSTRATE THICKNESS (MICROMETERS)
C     RHO(I) = I-TH RESISTIVITY RELATIVE TO THE SURFACE
C
C*************************************************************************
```

```
C     READ THE PROBE SEPARATION (MICROMETERS)
      READ(9,*)SEP
C     READ THE DEPTH INCREMENT (MICROMETERS)
      READ(9,*)DELTH
C     READ THE NUMBER OF POINTS IN THE STRUCTURE (EXCLUDING SUBSTRATE POINT)
      READ(9,*)N
C     READ THE SUBSTRATE RESISTIVITY (OHM*CM)
      READ(9,*)SUBRHO
C     READ THE SUBSTRATE THICKNESS (MICROMETERS)
      READ(9,*)SUBT
C     READ THE BACKSURFACE BOUNDARY CONDITION (NONE=1, COND=2, INS=3)
      READ(9,*)IBBC
C     READ THE SUBSTRATE RESISTIVITIES (OHM*CM)
      DO 70 I=1,N
      READ(9,*)RHO(I)
 70   CONTINUE
      WRITE(6,7777)
 7777 FORMAT(1H1)
      WRITE(6,80)
 80   FORMAT(2X,'FOUR-PROBE RESISTANCE CALCULATED BY SIMPLIFIED
     1 INTEGRAL')
      WRITE(6,81)
 81   FORMAT(2X,'ALSO CALCULATED IS THE EDUCATED GUESS OF THE
     1 LN(S) FORM')
      GO TO (90, 100, 110) IBBC
 90   WRITE(6,95)SUBT+N*DELTH,N*DELTH
 95   FORMAT(2X,'NO BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',F8.3,' UM')
      GO TO 120
 100  WRITE(6,105)SUBT+N*DELTH,N*DELTH
 105  FORMAT(2X,'CONDUCTING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
      GO TO 120
 110  WRITE(6,115)SUBT+N*DELTH,N*DELTH
 115  FORMAT(2X,'INSULATING BOUNDARY AT',F10.3,' UM.  SUBSTRATE AT',
     1 F8.3,' UM')
 120  WRITE(6,130)SEP,DELTH,SUBT,SUBRHO
 130  FORMAT(2X,'SEP=',F6.0,2X,'DELTH=',F7.3,2X,'SUBTH=',1PE9.3,2X,
     1 'SUBRHO=',1PE9.3,/)
      DO 140 I=1,N+1
      DEPTH(I)=DELTH*(I-1)
 140  CONTINUE
      Q=1.0/(2.0*SEP)
      DO 150 I=1,9
      L(I)=Q*2.0**((I-1.0)/8.0)
 150  CONTINUE
      DO 160 I=1,9
      W(I)=L(I)*C(I)
 160  CONTINUE
      WRITE(6,170)L(1),L(9)
 170  FORMAT(2X,'LOWER LIMIT= ',F10.8,2X,'UPPER LIMIT= ',F10.8)
      DO 180 I=1,9
      R1=L(I)*DELTH
      T(I)=TANH(R1)
 180  CONTINUE
```

```
C*************************************************************************
C
C     CALCULATION FOR THE SUBSTRATE POINT
C
C*************************************************************************
      DO 190 I=1,9
      R1=L(I)*SUBT
      A(1,I)=1.0
      A(2,I)=1.0
      IF(IBBC.EQ.2) A(1,I)=TANH(R1)
      IF(IBBC.EQ.3) A(1,I)=1.0/TANH(R1)
 190  CONTINUE
      RHO(N+1)=SUBRHO
      R2=0.0
      R3=1.0
      WW=SUBT/SUBRHO
      DO 200 I=1,9
      R2=R2+W(I)*A(1,I)
 200  CONTINUE
      ZAP(N+1)=R2*ALOG(2.0)*RHO(N+1)/PI
      DAP(N+1)=L(9)*A(1,9)
      R5=SUBT
C*************************************************************************
C
C     CALCULATION FOR THE N-LAYER STRUCTURE
C
C*************************************************************************
      DO 500 K=N,1,-1
      R4=0.0
      R3=RHO(K+1)/RHO(K)
      DO 250 I=1,9
      A(2,I)=(R3*A(1,I)+T(I))/(1.0+T(I)*A(1,I)*R3)
 250  CONTINUE
      R2=0.0
      DO 275 I=1,9
      R2=R2+W(I)*A(2,I)
 275  CONTINUE
      DO 300 II=N,K,-1
      R4=R4+1.0/RHO(II)
 300  CONTINUE
      ZAP(K)=R2*ALOG(2.0)*RHO(K)/PI
      DAP(K)=L(9)*A(2,9)
      DO 350 I=1,9
      A(1,I)=A(2,I)
 350  CONTINUE
 500  CONTINUE
```

```
C*************************************************************************
C
C     END OF CALCULATIONS AND BEGINNING OF OUTPUT SECTION
C
C*************************************************************************
      WRITE(6,550)
 550  FORMAT(/,3X,'DEPTH',6X,'RHO',6X,'FOUR-PROBE',4X,'DERIVAT',/)
      WRITE(11,*)N+1
      WRITE(12,*)N+1
      DO 600 I=1,N+1
      WRITE(6,650)DEPTH(I),RHO(I),1e4*ZAP(I),DAP(I)
 650  FORMAT(F8.3,3(2X,1PE12.4))
      WRITE(11,700)DEPTH(I),ZAP(I)
      WRITE(12,700)DEPTH(I),DAP(I)
 700  FORMAT(1X,2(1PE11.3,1X))
 600  CONTINUE
      STOP
      END
C*****END OF SHAZAM VERSION 1.0 PROGRAM*********************************
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National Institute of Standards and Technology
Gaithersburg, MD 20899