Measurement of Mixing Characteristics of the Missouri River Between Sioux City, Iowa, and Plattsmouth, Nebraska

---

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1899–G

Measurement of Mixing Characteristics of the Missouri River Between Sioux City, Iowa, and Plattsmouth, Nebraska

By NOBUHIRO YOTSUKURA, HUGO B. FISCHER, *and* WILLIAM W. SAYRE

CONTRIBUTIONS TO THE HYDROLOGY OF THE UNITED STATES

GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1899-G

Measurement of longitudinal dispersion, transverse mixing, channel geometry, and transverse velocity distribution

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1970

UNITED STATES DEPARTMENT OF THE INTERIOR

WALTER J. HICKEL, *Secretary*

GEOLOGICAL SURVEY

William T. Pecora, *Director*

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price 25 cents (paper cover)

CONTENTS

ILLUSTRATIONS

TABLES

SYMBOLS

A	Cross-sectional flow area
a_s	Dividing area between adjacent stream tubes
C	Cross-sectional mean tracer concentration
c	Local tracer concentration
D	Longitudinal dispersion coefficient
d	Mean flow depth
E_z	Transverse mixing coefficient
g	Acceleration of gravity
k	Constant of transverse mixing coefficient
l	Characteristic length
M	Mass flux of tracer per unit time
q	Discharge in a stream tube
Q	Discharge
RR	Recovery ratio
s	Mean energy slope
T	Lagrangian time scale
t	Time measured from the instant of injection
t'	Dimensionless time scale
$\bar{t}$	Mean time of passage for tracer cloud
$\bar{t}'$	Dimensionless mean time of passage for tracer cloud
$\bar{U}$	Mean tracer velocity (mean flow velocity)
$U*$	Shear velocity
u	Local velocity
V_o	Volume
W	Surface width
x	Longitudinal distance measured from injection site
Δx	Increment of longitudinal distance
z	Transverse distance from a boundary
Δz	Transverse distance between centroids of adjacent stream tubes
$\sigma^2{}_t$	Variance of longitudinal tracer distribution
$\sigma^2{}_z$	Variance of transverse tracer distribution

CONTRIBUTIONS TO THE HYDROLOGY OF THE UNITED STATES

MIXING CHARACTERISTICS OF THE MISSOURI RIVER BETWEEN SIOUX CITY, IOWA, AND PLATTSMOUTH, NEBRASKA

By NOBUHIRO YOTSUKURA, HUGO B. FISCHER, and WIILLIAM W. SAYRE

ABSTRACT

Measurements of longitudinal dispersion, transverse mixing, channel geometry, and transverse velocity distribution were made in the Missouri River at a flow of about 33,000 cubic feet per second. The results show that the longitudinal dispersion coefficient for the 141-mile reach from Sioux City, Iowa, to Plattsmouth, Nebr., is about 16,000 square feet per second (approximately 5,600 U^*d, where U^* is the shear velocity and d is mean depth). The transverse mixing coefficient, E_z, for a 6-mile reach immediately downstream from Blair, Nebr., is about 1.3 square feet per second (approximately 0.6 U^*d). The value of the longitudinal dispersion coefficient is one of the largest ever measured, and the value of the ratio E_z/U^*d is approximately three times as large as that frequently reported for small straight channels.

INTRODUCTION

Interest in dispersion processes in open channels has accelerated during recent years because of mounting concern over water pollution. A knowledge of the dispersion capacities of streams is necessary in order to control pollution, and important improvements in tracer technology and theories for predicting dispersion have made possible for the first time significant headway in observing and understanding dispersion phenomena in large natural streams.

The application of tracer technology to large streams became feasible following the introduction of radioactive and fluorometric tracing techniques, both of which are capable of measuring extremely small concentrations of tracer materials. Godfrey and Frederick (1963) have reported on the use of radioactive-tracer techniques in natural streams. The application of fluorescent-dye methods in large streams has been described by Wilson and Forrest (1965), Thackston and Krenkel (1966, 1967), and Bowie and Petri (1968).

An important step in the development of a method for predicting longitudinal dispersion in natural streams has been the adaptation by Fischer (1966b, 1967b) of Taylor's (1954) theory of convective dispersion in axisymmetric pipe flow. According to Fischer's formulation, transverse mixing combined with the variation of velocity with respect to lateral position in the channel is the dominant mechanism contributing to longitudinal dispersion in most natural streams. One important aspect of Fischer's result is that, other factors being equal, the longitudinal dispersion coefficient is proportional to the square of the channel width. This accounts for the extremely large values of the longitudinal dispersion coefficient that have been observed in large natural streams. For example, Yotsukura (1967) has reported values on the order of 10,000 sq ft per sec (square feet per second) for the Missouri River. For open channels, earlier applications of Taylor's theory, which assume that vertical mixing combined with the variation of velocity with respect to depth is the dominant mechanism, typically predict longitudinal dispersion coefficients for natural streams that are two or more orders of magnitude too small.

Prediction of the longitudinal dispersion coefficient by Fischer's method requires that the geometry, the transverse velocity distribution, and the value of the transverse mixing coefficient be known for a typical cross section. The necessary velocity and cross-sectional geometry information can be readily obtained by conventional streamgaging procedure. However, the value of the transverse mixing coefficient for large natural streams is not so easily determined.

The transverse mixing coefficient, in addition to being one of the important factors controlling the rate of longitudinal dispersion, is important in its own right because it controls the rate at which cross-channel mixing occurs. This is often an important consideration in the design of pollutant-outfall and water-intake systems. As of 1968, no theoretical basis for predicting the value of the transverse mixing coefficient had yet been formulated. On dimensional grounds and by analogy with the form derived from the logarithmic velocity profile for the vertical mixing coefficient, the transverse mixing coefficient may be assumed to have the form

$$E_z = kU^*d \qquad (1)$$

where d is depth of flow, U^* is shear velocity, and k is a numerical constant, which in straight uniform channels has a value of approximately 0.2. Elder (1959), for flows on the order of 1 centimeter deep in a laboratory flume, obtained $k=0.23$; Sayre and Chang (1968), in flows up to 1 foot deep in an 8-foot-wide flume, obtained $k=0.17$; and Fischer (1967a), in a flow 2 feet deep in a 60-foot-wide sand-bed canal, found $k=0.24$. Equation 1, with $k=0.2$, is probably a valid representation of the transverse mixing coefficient in straight uniform channels where turbulent mass transfer is the main contributing mechanism. In natural streams, additional mechanisms such as secondary currents induced by bends may significantly increase the rate of transverse mixing. For example, Glover (1964) has reported $k=0.72$ for the Columbia River near Richland, Wash. Therefore, it is of considerable practical importance to obtain additional experimental data on transverse mixing in large natural streams.

Ideally, the objectives of the experimental phase of the investigation reported in this paper would have been to obtain in a large natural stream sufficient data to define simultaneously: (1) the longitudinal dispersion coefficient, (2) the transverse mixing coefficient, and (3) the typical cross-sectional gometry and transverse velocity distribution. This information, if obtained for a long reach, would be sufficient for a complete evaluation of Fischer's theory. Lack of time and funds, however, restricted the transverse mixing phase of the experiment to one relatively short reach, and the cross-sectional geometry and velocity observations to just a few cross sections.

ACKNOWLEDGMENTS

The authors would like to express their sincere gratitude to Charles L. Hipp, Chief, Engineering Division, and to other members of the U.S. Army Corps of Engineers, Omaha District, for the generous cooperation and support which they rendered this investigation.

The tests were made possible by the voluntary participation of the following members of the research staff of the Water Resources Division: F. A. Kilpatrick, E. V. Richardson, R. S. McQuivey, and W. E. Gaskill. J. K. Okoye, a graduate student at the California Institute of Technology, also participated. Assistance in equipment and manpower was supplied by the Council Bluffs Subdistrict Office, Iowa District of the Water Resources Division, and also by the Nebraska and Missouri Districts. The water agencies of the municipalities of Omaha and Council Bluffs conducted the dye sampling at the water intakes. All these contributions are gratefully acknowledged.

LONGITUDINAL DISPERSION

DESCRIPTION OF THE TEST REACH

The longitudinal dispersion test was conducted in a 141-mile reach of the Missouri River between Sioux City, Iowa, and Plattsmouth, Nebr. (fig. 1). Discharge in this section is controlled by a system of upstream reservoirs, including the Gavins Point Dam near Yankton, South Dakota, releases from which are coordinated with downstream

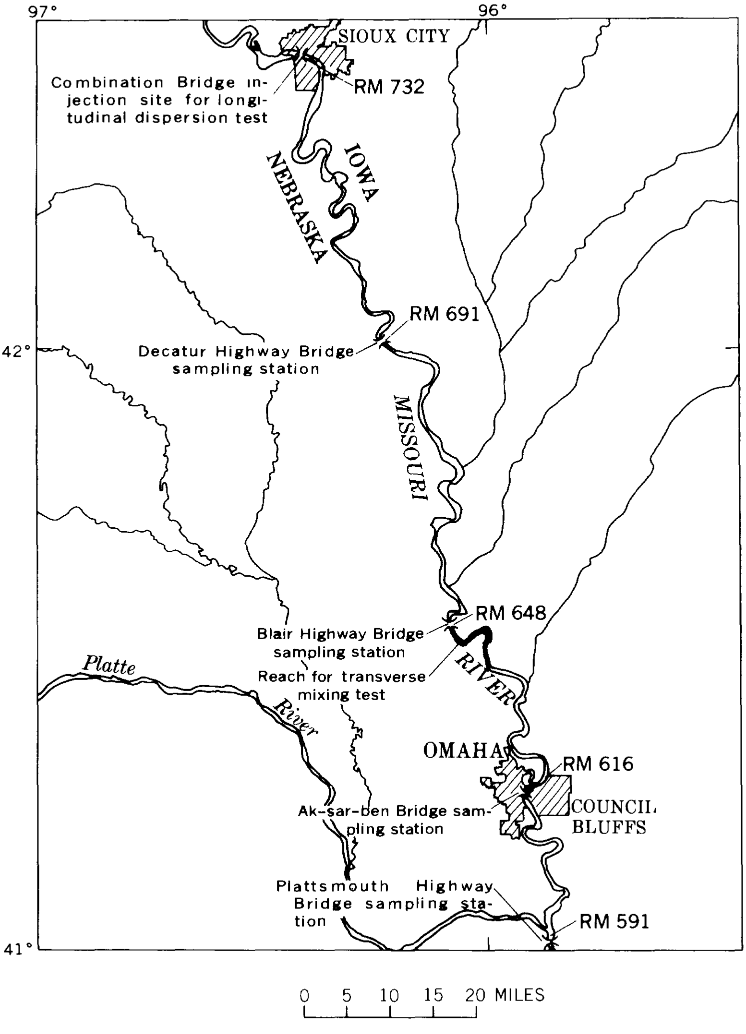

FIGURE 1.—Study reach, Missouri River between Sioux City, Iowa, and Plattsmouth, Nebr.

tributary inflows to maintain a minimum flow of 30,000 to 35,000 cfs (cubic feet per second) during the navigation season. Within the study reach the only significant tributary is the Platte River, which joins the Missouri a few miles upstream from Plattsmouth. The channel is maintained at a width of 500 to 800 feet by a system of dikes and jetties constructed to stabilize it. The depth of the thalweg is generally less than 25 feet.

An extensive survey conducted by the U.S. Army Corps of Engineers, Omaha District (1967) has disclosed the following characteristics of sediments and flow regimes of the reach: The channel bottom consists predominantly of sand, 90 percent of which is coarser than 0.15 mm in size. The total suspended sediment load shows seasonal variation and, in the late fall, reaches a maximum of between 700 and 1,000 ppm (parts per million). During the late fall, as the temperature drops about 20° to 30°F, the channel bed changes from a dune-bed to a plane-bed configuration. Consequently the water surface falls 1 to 2 feet, with a correspondent increase in average velocity, even though the discharge remains steady. The channel degradation has been known to be minimal. The change in resistance due to change in bed form is reflected in the Manning n value which ranges from 0.020 for a dune bed to about 0.015 for a plane bed.

TEST PROCEDURE

A total of 600 pounds of rhodamine WT 20-percent solution was injected downstream from the Combination Bridge at Sioux City, at about 1700 hours, November 13, 1967, in a line source extending across the middle half of the channel. The mode of injection was similar to that for the 1966 time-of-travel test reported by Bowie and Petri (1968). Rhodamine BA dye was employed in the 1966 test. The amount of pure dye used in both tests was about the same (120 pounds).

Samples for dye-concentration analysis were obtained at four downstream locations: Decatur Highway Bridge, Blair Highway Bridge, Ak-sar-ben Bridge in Omaha, and Plattsmouth Highway Bridge. The distance from the injection point to the respective bridges is 40.8, 83.5, 116.0, and 141.3 river miles. The samples were collected from near the surface by lowering 25-cc glass bottles, attached to the end of a weighted handline, into the water. At Decatur Highway Bridge, five sampling stations were maintained across the channel, whereas at the other bridges enough transverse mixing had occurred for three stations to suffice. The sampling stations were located so as to represent increments of cross-sectional area carrying approximately equal discharges. The time interval between samples was varied from 15 minutes to 2 hours depending on the predicted rate of change of concentration.

All test samples were taken to the Hydraulic Laboratory at Colorado State University and analyzed for dye concentration with a Turner Model 111 fluorometer. Before readings were taken, sample temperatures were brought to 25.1 ± 0.1°C by means of a constant-temperature water bath to eliminate the need for temperature corrections. The fluorometer was calibrated by obtaining readings for standard solutions of known dye concentrations. The standards were prepared by successive dulitions of rhodamine WT 20-percent solution in distilled water. The calibration was linear except for concentrations less than 0.1 ppb (part per billion).

Hydraulic data were obtained at a number of cross sections during the dispersion experiment. Discharge measurements by the current-meter method were obtained from bridges at Sioux City, Decatur, and Omaha. Discharge measurements were made at four additional cross sections by the current-meter method from a boat anchored to a tag line. Data from these measurements were used to obtain the geometric characteristics of the cross section and the lateral velocity distribution as well as the discharge.

DATA

The longitudinal dispersion data consist of cross-sectional average dye concentration as a function of time at each sampling cross section. An average concentration was obtained as an arithmetic average of all concentrations obtained at the cross section during each sampling period. The sampling time was taken as that at the middle of the sampling period, which was normally less than 8 minutes. The data are presented in table 1. The concentrations in table 1 are net values

TABLE 1.—*Distribution of cross-sectional average dye concentration with time, longitudinal dispersion test on the Missouri River, November 1967*

Time after injection, in hours and minutes	Dye concentration, in parts per billion	Time after injection, in hours and minutes	Dye concentration, in parts per billion	Time after injection, in hours and minutes	Dye concentration, in parts per billion	Time after injection, in hours and minutes	Dye concentration, in parts per billion
Decatur Bridge ***RR*=0.882**		**Blair Bridge** ***RR*=0.780**		**Ak-sar-ben Bridge** ***RR*=0.775**		**Plattsmouth Bridge** ***RR*=0.775**	
10:30	0	20:59	0.01	28:33	0	34:02	0
10:45	.25	21:27	.02	29:06	.04	35·02	.03
11:00	.80	21:51	.22	29:32	.11	36·02	.15
11:25	1.89	22:10	.35	30:03	.29	37:01	.49
11:36	2.32	22:30	.60	30:33	.53	38 02	1.00
11:49	2.84	22:47	.88	31:03	.88	39.00	1.37
11:58	3.21	23:01	1.09	31:32	1.16	40.00	1.64
16:16	3.55	23:16	1.38	32:02	1.52	41:02	1.57
12:35	3.92	23:52	1.88	32:33	1.79	42.02	1.39
12:47	4.05	24:12	2.23	33:02	2.01	43 00	1.07
13:03	4.03	24:34	2.45	34:02	2.09	44:30	.62
13:17	3.88	24:55	2.50	34:35	1.84	46·11	.29
13:33	3.64	25:27	2.52	35:05	1.73	47:58	.15
13:47	3.36	25:55	2.39	36:05	1.28	50 03	.08
14:03	3.10	26:24	2.10	37:03	.88	52.05	.06
14:18	2.70	26:53	1.78	38:00	.54	55·12	.03
14:33	2.38	27:46	1.20	39:01	.32	59.06	.02
14:47	2.07	28:02	1.01	40:01	.20	63 10	.01
15:02	1.64	28:33	.74	41:01	.13	66·58	.01
15:17	1.39	29:02	.61	42:01	.09		
15:32	1.12	29:35	.42	43:02	.07		
15:48	.86	30:02	.30	45:00	.05		
16:03	.72	30:32	.21	47:10	.03		
16:23	.49	31:02	.14				
16:43	.37	32:03	.09				
17:03	.28	33:01	.09				
17:22	.19	34:04	.06				
17:42	.14	35:06	.07				
18:04	.11	36:03	.05				
18:37	.09	38:03	.03				
19:02	.08	40:02	.02				
19:35	.06	43:05	.02				
20:02	.06						

(background subtracted) and have not been compensated for dye loss that occurred in the stream. The data are also presented graphically in figure 2. Corresponding data from the 1966 test are included in figure 2 for comparison. Two striking features are evident in this comparison: the differences in the time displacements of the curves and the areas under them. Since the description of these aspects is not the major topic of this report, it is sufficient to say that the shortened travel time in the present test was caused by the reduction of bottom resistance due to a shift from dune-bed to plane-bed. The 1966 test was done under the condition of a dune bed. The difference in the areas under the curves indicates the difference in the observed dye recoveries and is attributed to the difference in the adsorption characteristics between WT dye (present test) and BA dye (1966 test).

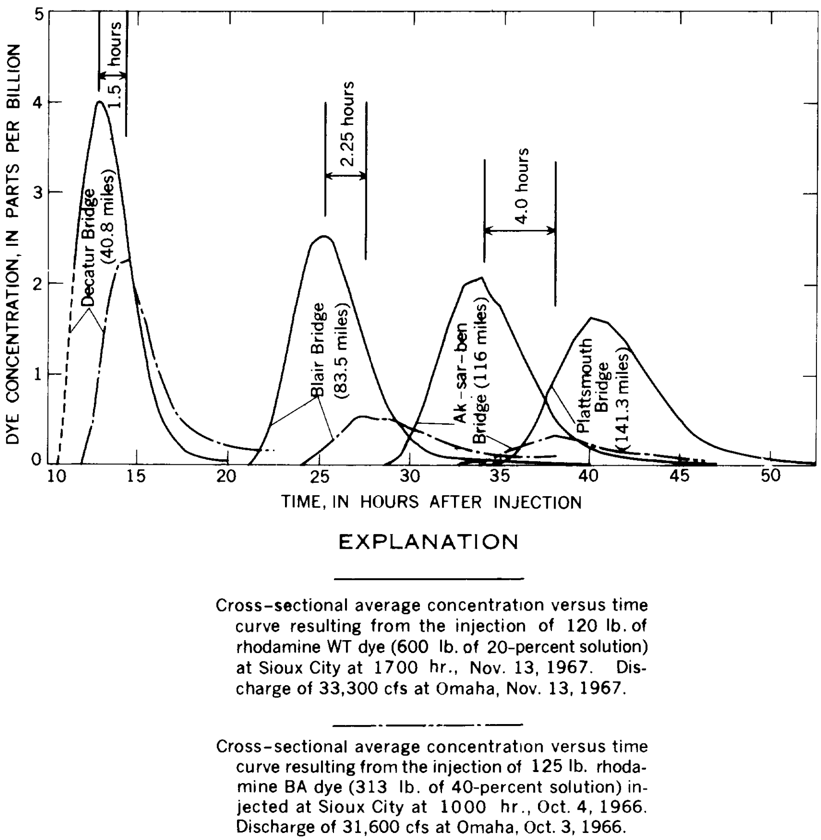

FIGURE 2.—Longitudinal dispersion on the Missouri River—comparison of the November 1967 data with the October 1966 data.

Values of the recovery ratio, RR, for the present test are also shown in table 1. It is the ratio of the amount of dye actually recovered at the cross section to the amount that was injected initially and is computed by a formula

$$RR=\frac{Q\int_0^\infty Cdt}{V_0C_0}$$

where C is the cross-sectional average concentration observed at time t, Q is the discharge, and C_0 and V_0 are respectively the concentration and the volume of the injected solution. It was concluded from the high percentage of recovery that the effect of dye loss on the analysis of dispersion characteristics is negligible in the present test.

The hydraulic data are presented in table 2. The average depth, d, was computed by dividing the area, A, by the surface width, W, while the shear velocity, U^*, was computed as the square root of the product gds, where g is the acceleration of gravity and s is the energy slope which was assumed to be a constant, 0.0002, for the entire reach.

TABLE 2.—*Hydraulic data, longitudinal dispersion test on the Missouri River, November 1967*

Location of cross section	Date November—	Discharge Q (cfs)	Channel width W (feet)	Cross-sectional area A (sq ft)	Average velocity $\overline{U}$ (feet per sec)	Average depth d (feet)	Average slear velocity U^* (ft per sec)
Combination Bridge, Sioux City	13	33, 500	750	8, 220	4. 08	11. 0	0. 27
	15	34, 100	755	8, 780	3. 88	11. 6	. 28
Decatur Highway Bridge	13	31, 200	610	7, 650	4. 08	12. 5	. 28
Near the mouth of the Little Sioux River	14	32, 600	600	6, 390	5. 10	10. 7	. 26
1,000 ft upstream from Blair Highway Bridge	14	34, 500	600	6, 010	5. 74	10. 0	. 25
River mile 645, 17,500 ft downstream of Blair Highway Bridge	15	33, 700	740	5, 870	5. 74	7. 9	. 23
Ak-sar-ben Bridge in Omaha	13	33, 300	577	6, 340	5. 25	11. 0	. 27
	16	33, 500	577	6, 340	5. 28	11. 0	. 27
Near Plattsmouth Highway Bridge	15	34, 000	585	5, 634	6. 04	9. 6	. 25

COMPUTATION OF THE LONGITUDINAL DISPERSION COEFFICIENT

ROUTING PROCEDURE

Although several other methods for computing the longitudinal dispersion coefficient are available, the routing procedure presented by Fischer (1968) appears to be the least sensitive to either human judgment or data scatter and produces a coefficient which matches the data as closely as possible. Fischer (1967b) has divided dispersion into two time periods according to a dimensionless system in which dimensionless time, t', is given by

$$t'=\frac{t}{T} \tag{2}$$

In equation 2, t is a time measured from the instant of injection, and T is a time scale given by the relation

$$T=\frac{l^2}{14.8E_z} \tag{3}$$

in which l is a characteristic length (distance between the maximum velocity thread and the more distant bank) and E_z is the transverse mixing coefficient. As shown in a later section, the measured value of the transverse mixing coefficient for the study reach of the Missouri River is approximately $E_z=0.6\ U^*d$. Fischer (1967b) states that if $\bar{t}'$, the dimensionless mean time of passage of the tracer cloud past the measuring station, is less than 6, the cloud is in the "convective period," in which the dispersion theory is not applicable. If possible, the routing procedure should be applied using data obtained at measuring stations sufficiently far downstream that the dimensionless mean time of passage past each station is greater than 6.

Table 3 shows the distance downstream from injection, mean time of passage of the tracer cloud, and dimensionless mean time of passage for each of the measurement stations of the present experiment. The reach from Blair to Plattsmouth is entirely within the "diffusive period" ($\bar{t}'>6$) and is sufficiently long to permit an accurate calculation of the dispersion coefficient. The reach beginning at Decatur is partly within the "convective period," so it yields a somewhat erroneous coefficient; the remainder of the reaches are probably too short to permit accurate calculations.

Table 4 shows the result of applying the routing procedure in turn from each measured section to each subsequent measured section.

TABLE 3.—*Time-of-travel data, longitudinal dispersion test on the Missouri River, November 1967*

[$\bar{t}'$ is calculated on the basis of the average values l=450 ft, d=9.68 ft, U^*=0.250 ft per sec, and T=167 min]

Measurement station	Distance from injection x, in feet	Mean time of passage $\bar{t}$, in minutes	Dimensionless mean time of passage $\bar{t}'$
Decatur	216,000	800	4.8
Blair	441,000	1,570	9.5
Omaha	612,000	2,080	12.5
Plattsmouth	796,000	2,520	15.0

TABLE 4.—*Dispersion coefficients by the routing procedure, longitudinal dispersion test on the Missouri River, November 1967*

Initial Station	Dispersion coefficients, in square feet per second, at final stations		
	Blair	Omaha	Plattsmouth
Reported data:			
Decatur	7,420	8,600	14,300
Blair		9,160	16,100
Omaha			26,600
Data with reduced tails: [1]			
Decatur	6,540	8,660	11,900
Blair		10,400	16,700
Omaha			23,400

[1] Reductions; 0.05 ppb at Decatur and Blair; 0.03 ppb at Omaha; 0.02 ppb at Plattsmouth.

For the first set of values, the data are used exactly as reported in the previous tables; for the second, a portion of the trailing end of the tracer distribution has been subtracted out by the procedure outlined and for the reasons given by Fischer (1968). As expected, the results of the routing procedure are insensitive to whether or not the tails are included, as is seen by comparing the two parts of the table. The value of the computed dispersion coefficient increases steadily for points successively farther downstream in the measurement reach. This may be in part because the velocity of flow at Omaha and Plattsmouth is slightly higher than at Decatur and Blair, or it may be because of the low values of dimensionless time at the upstream stations. For this reach of the river and flow condition, the value obtained by routing from Blair to Plattsmouth,

$$D \approx 16{,}000 \text{ sq ft per sec},$$

is probably the best approximation to the true dispersion coefficient obtainable from the present data.

METHOD OF MOMENTS

For comparison, longitudinal dispersion coefficients as calculated by the method of moments (Fischer, 1966a; Sayre and Chang, 1968) are also presented here. In this method, the longitudinal dispersion coefficient can be defined as

$$D = \frac{\bar{U}^3}{2} \frac{d\sigma_t^2}{dx}. \tag{4}$$

In equation 4, $\bar{U}$ is the mean rate of longitudinal displacement of the dye cloud which can be expressed as

$$\bar{U} = \frac{dx}{d\bar{t}} \tag{5}$$

where $\bar{t}$ is the mean time of passage of the dye cloud from the injection station to a point at distance x downstream from the injection station,

$$\bar{t}=\frac{\int_0^\infty tCdt}{\int_0^\infty Cdt}. \tag{6}$$

Another term in equation 4, σ_t^2, is the variance of the concentration versus time curve at x,

$$\sigma_t^2=\frac{\int_0^\infty t^2Cdt}{\int_0^\infty Cdt}-[\bar{t}]^2. \tag{7}$$

The main difficulty in evaluating D by the method of moments is the large contribution to the variance by the tails of the concentration distributions. Even a small amount of tracer that is temporarily trapped in slow-moving flow near the banks and subsequently released to the main channel, where it shows up as a tail on the concentration distribution, can greatly inflate the value of D. In anticipation of this problem the curves were truncated at times when concentrations on the recession limbs of the curves reached 3 percent and 1 percent of the peak concentration. The truncations were performed before calculating $\bar{t}$ and σ_t^2 by equations 6 and 7. The results are shown in figure 3 and table 5. If the variances corresponding to the 1- and 3-percent truncation levels are considered as maximum and minimum estimates respectively, a reasonable estimate of an average value of D for the entire reach is 15,000 sq ft per sec as shown in figure 3. The values of D given in table 5, although somewhat larger than the values for the corresponding subreaches given in table 4, exhibit the same general trend and variation within the reach. On the whole, the values of D calculated by the method of moments compare reasonably well with those determined by the routing procedure, evidently because the channel of the Missouri River in the study reach is sufficiently well maintained that there are few dead zones near the banks capable of significantly affecting the longitudinal dispersion process.

TRANSVERSE MIXING

DESCRIPTION OF THE TEST REACH

The transverse mixing test was performed in a reach immediately downstream from Blair Highway Bridge, about 85 miles downstream from Sioux City and 56 miles upstream from Plattsmouth. It was

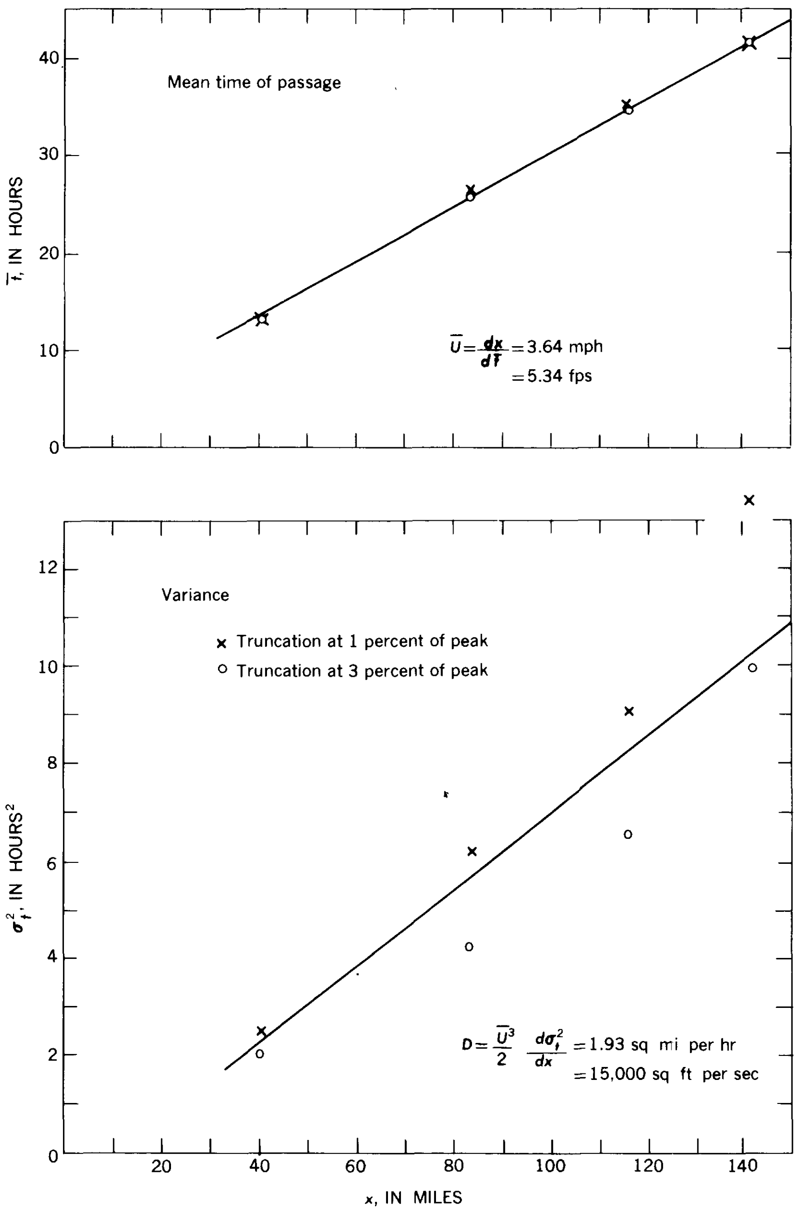

FIGURE 3.—Measured mean time of passage and variance as functions of distance.

TABLE 5.—*Dispersion coefficients by the method of moments, longitudinal dispersion test on the Missouri River, November 1967*

Initial station	Dispersion coefficients, in square feet per second, at final stations		
	Blair	Omaha	Platts-mouth
Truncation at 1 percent of peak concentration:			
Decatur	16,300	16,300	20,600
Blair		16,300	24,000
Omaha			33,600
Truncation at 3 percent of peak concentration:			
Decatur	9,730	11,200	14,800
Blair		13,500	18,300
Omaha			24,900

selected for the test because of fairly gentle meandering for a 33,000-foot long stretch, availability of a bridge for installing the injection equipment, and not much river traffic. Channel geometry and hydraulic characteristics are as described in the section on the longitudinal dispersion test. Looking downstream from Blair Highway Bridge, the river meanders gently to the left for about 3.5 miles with the thalweg running near the right bank (see fig. 4), after which the direction of the meander reverses, and the thalweg crosses to the left side. The channel width ranges from 500 to 700 feet, and the average depth is about 9 feet. The variation in water depth and mean velocity across the channel is illustrated in figure 5; one cross section is at 1,000 feet upstream from Blair Highway Bridge, and another is at River Mile 645 or 17,500 feet downstream from the same bridge.

TEST PROCEDURE

A mixture consisting of 150 pounds of 40-percent rhodamine BA solution, 20 gallons of tapwater, and 10 gallons of methyl cellusolve was prepared in a 50-gallon mariotte vessel installed at Blair Highway Bridge on the morning of November 17, 1967. The vessel, a constant-head tank designed for maintaining a uniform rate of discharge, was set 200 feet from the pier on the west bank, so that the river discharge was about equally divided on either side. A 50-foot-long garden hose was used to carry the mixture, coming out of a 0.1065-inch orifice, down to a level about 5 feet above the water surface. The mixture dripped freely from the end of the hose into the water. The injection started at 1040 hours and continued until 1420 hours emptying most of the 48-gallon mixture in the vessel. The overall

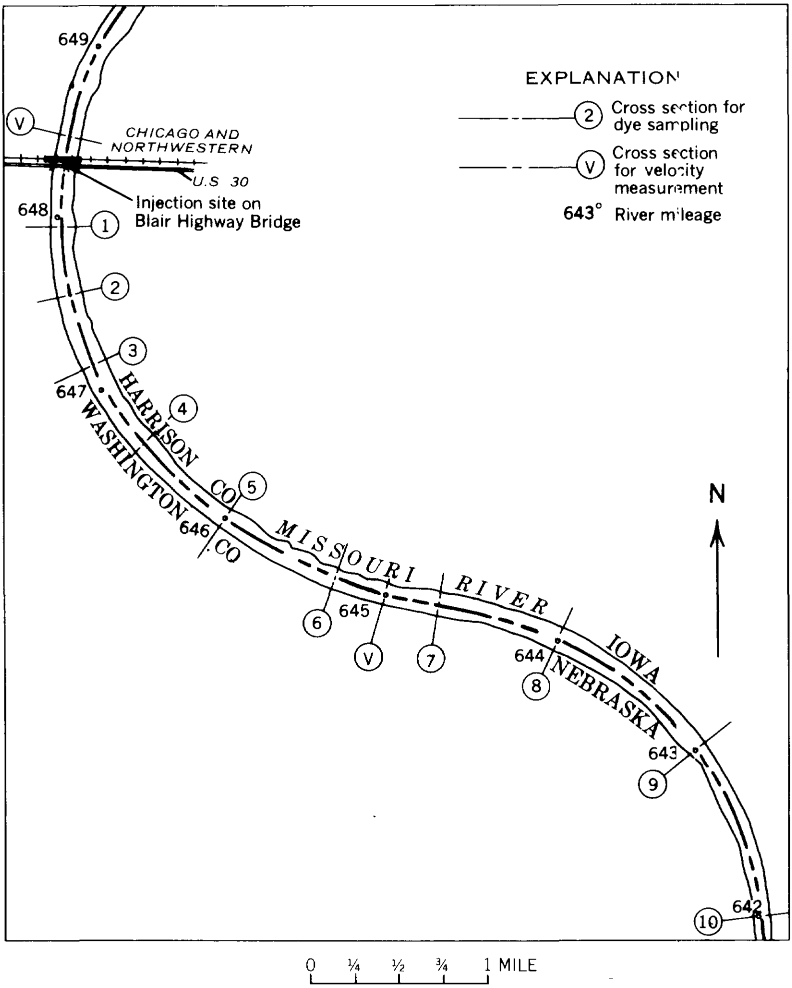

FIGURE 4.—Reach of the Missouri River for the transverse mixing test.

rate of injection was 13.05 ml per sec, and the dye concentration in the mixture was 1.51×10^8 ppb or 15.1 percent. The pattern of mixing immediately downstream from the injection site is shown in figure 6. The boat ramp on the right bank is about 1,200 feet downstream from the injection site.

Samples for determining the transverse distribution of dye concentration were collected at 10 cross sections at distances between 1,700 and 33,000 feet downstream from the injection site. The sampling started at the 1,700-foot cross section at 1100 hours and continued until 1509 hours when the sampling at the 33,000-foot cross section was completed. At each cross section, a sampling boat traversed the channel in a zigzag path along the cross-sectional line marked by three range poles as shown in figure 7. Samples were collected by dipping glass bottles into the water at reasonably uniform intervals so that a minimum of 20 samples was obtained during each traverse. Referring again to the sketch in figure 7, since the length of the base line AB and the angle β had been measured previously, the transverse sampling positions could be determined by reading the angle α with a transit at the instant each sample was obtained. A sample was taken immediately upon receiving a signal given by a control party at B when the sampler was in line with the three range poles. Since two transits were available, considerable time was saved by having an advance party set up a transit at the next section downstream. A set

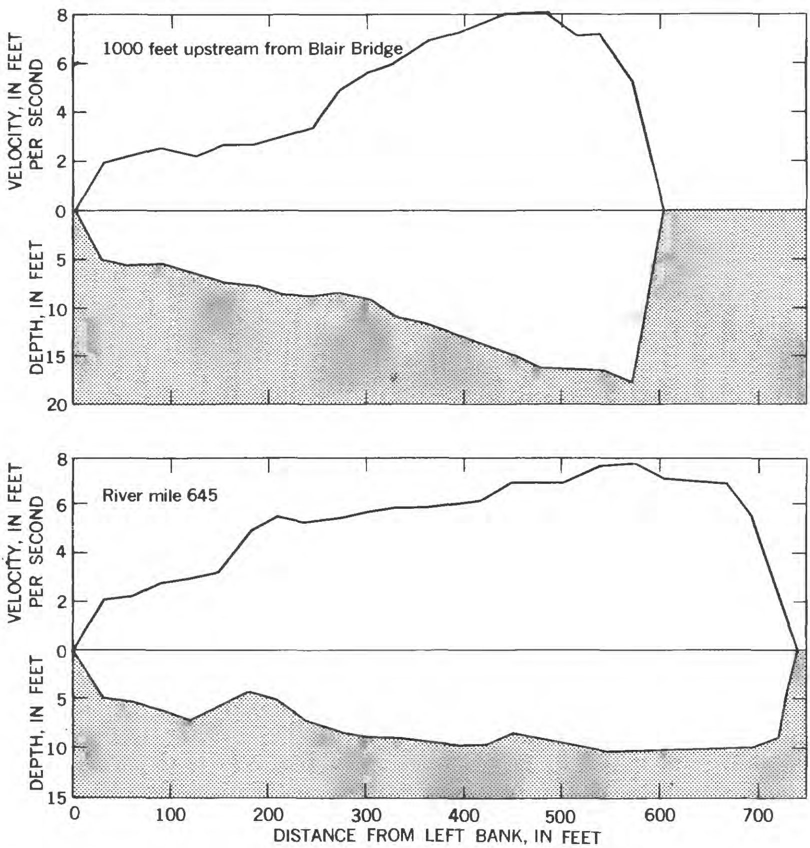

FIGURE 5.—Observed transverse profiles of velocity and depth at cross sections not affected by bridge abutments.

FIGURE 6.—Mixing of dye immediately downstream from injection site at Blair Highway Bridge.

of walkie-talkie radios was used for communication between the sampling, transit, and control parties.

The samples were analyzed for dye concentration with a Turner Model 111 fluorometer in the same manner as in the longitudinal

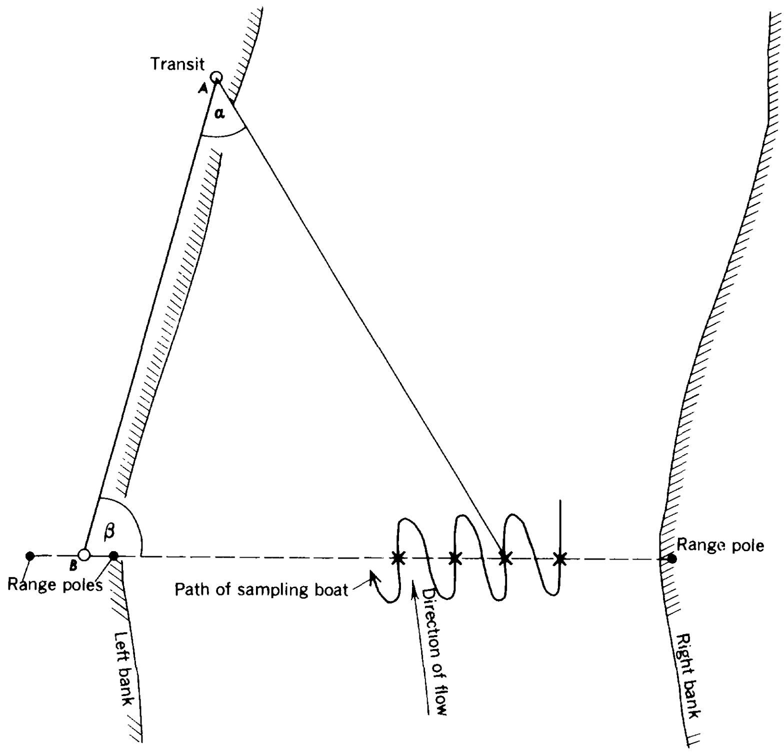

FIGURE 7.—Method for locating lateral sampling positions.

dispersion experiment. Neither the calibration nor the sample readings were as satisfactory for the BA dye as for the WT dye used in the longitudinal dispersion test. Because of absorption of BA dye on the walls of sampling bottles and perhaps also because of chemical reactions between the BA dye and impurities in the river water, the calibration shifted with time. The resulting calibration error was probably on the order of ± 5 percent. The calibration errors together with the sampling scheme, which allowed only one traverse at each cross section, probably caused many of the observed instataneous point concentrations to differ from the time-averaged point concentrations by considerably more than 5 percent.

Velocity measurements by current meter were obtained prior to the test at two cross sections, one 1,000 feet up stream from Blair Highway Bridge on November 14 and the other 17,500 feet downstream from the same bridge on November 15 (see fig. 5). Several available staff gages were read before and after the test to determine the water-surface slope for this reach.

DATA

The transverse mixing data presented in table 6 consist of dye concentrations and lateral sampling positions at each cross section. Each concentration determined from the surface sampling is assumed to be uniform over the depth and also to be representative, at the particular location, of the time-averaged concentration resulting from the continuous uniform-rate injection of the solute. Values of the recovery ratio, RR, are also shown for each cross section. It is computed by the formula

$$RR=\frac{Q\int_0^W c\,dz}{W c_0 q_0},$$

where W is the width of the cross section, and c_0 and q_0 are respectively the concentration and volumetric discharge rate of the injected dye solution.

TABLE 6.—*Distribution of dye concentration in transverse direction, transverse mixing test on the Missouri River, November 17, 1967*

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
			Cross section 1 **X=1,692 ft** **W=665 ft** **RR=1.34**					
1	649.1	0.00	10	528.7	.00	19	286.4	.03
2	635.6	.13	11	498.1	.00	20	263.8	.00
3	628.5	.00	12	478.1	.00	21	208.9	.00
4	621.2	.05	13	452.8	.00	22	168.6	.00
5	609.5	.00	14	420.6	.79	23	146.7	.02
6	587.9	.11	15	397.8	51.8	24	109.4	.00
7	568.2	.13	16	365.4	11.4	25	68.4	.00
8	569.6	.03	17	336.4	1.2	26	30.3	.00
9	544.3	.00	18	315.5	.08			
			Cross section 2 **X=3,692 ft** **W=735 ft** **RR=0.58**					
1	706.6	0.00	9	580.7	.00	17	449.5	3.09
2	687.3	.00	10	562.2	.00	18	409.4	2.67
3	682.9	.00	11	552.7	1.55	20	355.1	.13
4	668.4	.02	12	547.1	.15	22	288.1	.00
5	647.8	.00	13	533.6	4.3	23	235.1	.00
6	622.4	.00	14	493.7	9.4	24	157.5	.00
7	608.4	.00	15	470.4	9.6	25	79.3	.00
8	587.1	.00	16	452.2	5.8			
			Cross section 3 **X=5,730 ft** **W=689 ft** **RR=0.58**					
1	655.9	0.03	10	498.8	.00	18	325.8	1.41
2	645.1	.00	11	480.7	.81	19	290.9	.37
3	642.1	.00	12	445.0	6.37	20	268.1	1.20
4	609.9	.00	13	409.8	6.56	21	237.3	.44
5	593.0	.00	14	400.7	7.00	22	207.2	.00
6	577.4	.00	15	388.3	6.12	23	161.9	.00
7	561.6	.00	16	373.2	5.10	24	109.9	.00
8	536.1	.00	17	343.1	2.70	25	44.9	.00
9	512.3	.00						

TABLE 6.—*Distribution of dye concentration in transverse direction, transverse mixing test on the Missouri River, November 17, 1967*—Continued

Cross section 4
X=8,730 ft
W=644 ft
RR=0.49

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
1	620.4	0.00	10	438.9	5.25	18	308.2	.73
3	579.9	.02	11	418.7	5.78	19	275.3	.13
4	576.7	.00	12	418.2	5.68	20	225.1	.00
5	556.3	.00	13	411.8	5.59	21	200.6	.05
6	537.3	.02	14	392.9	4.27	22	157.3	.03
7	511.2	.02	15	379.7	2.65	23	112.3	.03
8	480.4	2.20	16	356.2	3.05	24	91.7	.00
9	474.3	1.10	17	335.4	3.19			

Cross section 5
X=11,850 ft
W=627 ft
RR=0.65

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
1	605.4	0.00	10	446.4	1.43	19	311.0	4.21
2	591.9	.00	11	411.7	1.87	20	300.3	4.31
3	580.8	.00	12	389.8	3.98	21	261.7	2.80
4	564.7	.00	13	382.1	3.29	22	222.2	1.10
5	549.1	.09	14	369.4	3.82	23	178.7	.54
6	533.2	.00	15	342.1	3.58	24	136.0	.23
7	518.9	.13	16	320.3	4.12	25	64.1	.11
8	497.6	.44	17	327.8	4.02			
9	481.5	.40	18	330.2	5.10			

Cross section 6
X=15,490 ft
W=733 ft
RR=0.56

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
1	731.7	0.05	10	523.8	2.39	18	309.5	1.10
3	702.2	.15	11	485.8	3.63	19	264.3	.37
4	669.8	.23	12	438.1	4.27	20	218.5	.27
5	663.7	.15	13	445.4	4.16	21	162.1	.35
6	637.0	.33	14	451.6	3.00	22	121.3	.25
7	610.2	.75	15	405.2	3.34	23	48.2	.17
8	586.1	.69	16	406.6	3.44			
9	559.6	.77	17	364.1	2.49			

Cross section 7
X=18,720 ft
W=657 ft
RR=0.66

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
1	627.0	0.33	9	494.5	.83	17	334.1	3.19
2	607.2	.23	10	471.2	1.22	18	283.9	3.82
3	590.6	.29	11	449.0	2.05	19	258.7	----------
4	568.2	.23	12	419.6	1.64	20	226.5	1.85
5	549.3	.29	13	449.0	1.89	21	191.9	1.10
6	516.0	.87	14	439.0	2.36	22	146.0	.38
7	560.1	.50	15	441.8	1.51	23	105.7	.37
8	524.2	.58	16	387.0	2.76	24	76.2	.50

Cross section 8
X=22,410 ft
W=657 ft
RR=0.64

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
1	583.0	0.43	10	474.7	.50	19	225.2	3.10
2	570.9	.27	11	442.2	.54	20	195.7	2.26
3	566.2	.25	12	424.6	.91	21	188.9	3.25
4	545.0	.28	13	398.4	.72	22	146.1	3.20
5	534.4	.25	14	366.5	2.10	23	98.6	.81
6	522.6	.31	15	338.5	1.55	24	60.2	.58
7	500.7	.34	16	307.8	2.24	25	35.8	.44
8	490.1	.38	17	291.4	2.68			
9	475.1	.44	18	272.3	2.51			

TABLE 6.—*Distribution of dye concentration in transverse direction, transverse mixing test on the Missouri River, November 17, 1967*—Continued

Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion	Station	Distance from left bank, in feet	Dye concentration, in parts per billion
				Cross section 9 **X=27,690 ft** **W=605 ft** **RR=0.60**				
1	586. 6	0. 27	10	467. 8	. 31	19	222. 1	2. 83
2	576. 0	. 48	11	453. 1	. 31	20	189. 1	2. 39
3	561. 9	. 19	12	440. 0	. 34	21	190. 0	2. 63
4	555. 1	. 27	13	414. 3	. 62	22	171. 7	2. 74
5	540. 3	. 23	14	371. 4	. 93	23	168. 8	2. 35
6	529. 9	. 25	15	364. 8	. 94	24	141. 4	2. 05
7	514. 2	. 23	16	337. 3	1. 62	25	112. 8	1. 51
8	496. 4	. 33	17	308. 3	1. 80	26	72. 9	1. 08
9	483. 7	. 23	18	255. 6	2. 47	27	22. 7	. 88
				Cross section 10 **X=32,970 ft** **W=509 ft** **RR=0.58**				
8	490. 6	0. 27	17	354. 6	. 44	26	172. 9	2. 03
9	470. 4	. 27	18	339. 6	. 46	27	163. 6	2. 08
10	442. 5	. 31	19	320. 3	. 48	28	135 6	2. 13
11	427. 3	. 27	20	295. 6	1. 12	29	93 0	2. 14
12	411. 1	----------	21	263. 1	1. 58	30	74 1	1. 47
13	399. 5	. 33	22	252. 7	1. 82	31	60. 9	1. 64
14	381. 8	. 40	23	205. 8	2. 01	32	23 5	1. 35
15	367. 1	. 43	24	180. 7	2. 13			
16	362. 8	. 44	25	172. 0	2. 26			

COMPUTATION OF THE TRANSVERSE MIXING COEFFICIENT

METHOD OF MOMENTS

For a flow in an infinitely wide, open channel, flowing at a uniform depth and uniform velocity throughout its width, Sayre and Chang (1968) have shown that the transverse mixing coefficient may be calculated by the relationship

$$E_z = \frac{\overline{U}}{2} \frac{d\sigma_z^2}{dx}, \tag{8}$$

in which E_z is the transverse mixing coefficient, σ_z^2 is the variance of the transverse distribution of the tracer, x is distance downstream from the point of injection of the tracer, and $\overline{U}$ is the mean flow velocity. In a nonuniform stream, such as the Missouri, equation 8, which is derived assuming an equal convective velocity at every point, is not strictly applicable because of the changing pattern of convective velocity. Nevertheless, equation 8 can be applied to the present data to provide a useful first approximation to the correct mixing coefficient, at least so long as the tracer distribution is not significantly affected by the presence of the boundaries.

Figure 8 shows a plot of variance of the tracer cloud at each of the first seven measuring stations versus distance downstream from the injection point. Downstream from station 7 significant quantities of tracer had reached the boundaries. The variances were calculated directly from the transverse concentration distributions, without discharge weighting. Figure 8 shows a reasonably linear increase of the tracer variance for measuring sections 1 through 5; the slope of the straight line of best fit yields a transverse mixing coefficient $E_z=1.56$ sq ft per sec. On the basis of mean shear velocity $U^*=0.24$ ft per sec and a mean depth $d=9.1$ ft, this yields a dimensionless transverse mixing coefficient $E_z/U^*d=0.71$.

COMPUTER SIMULATION METHOD

The mixing coefficient calculated by equation 8 is only an approximation; because the transverse mixing experiment was conducted over a meandering reach of a river where the velocity is nonuniform across the width and along the length. A more valid method of calculating a mixing coefficient must take into account the pattern of convective velocities. In view of the complex pattern of velocity distribution, however, such calculation is only possible through a step-by-step simulation of convective diffusion process by a numerical method. This leads to a trial-and-error method of evaluating the mixing

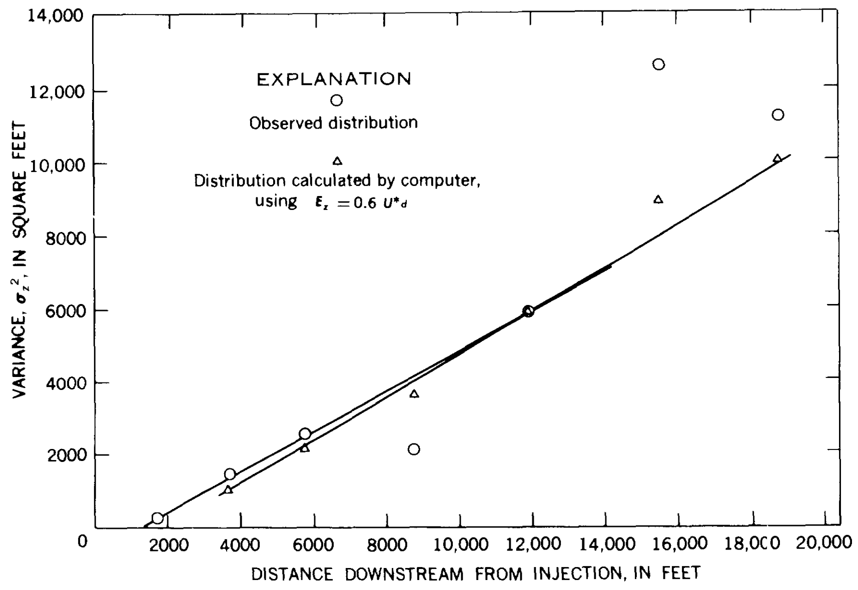

FIGURE 8.—Variance of transverse tracer distribution, observed and calculated.

coefficient in which the coefficient that yields the calculated concentration distribution closest to the observed data is considered as the correct value. To establish accurately the pattern of convective velocities and to deduce the effect of diffusive mixing, one would need point velocity measurements at every point within the stream. Although this is not possible, it would be useful for future experiments to have at least a complete transverse velocity profile at each measuring station. In the present experiment only two velocity profiles were measured within the reach of the transverse mixing experiment, one 1,000 feet upstream from the injection point and the other halfway between cross sections six and seven at mile 645. These velocity profiles are shown in figure 5. The first velocity measurement is in the upstream section of a long curve to the left, and the second is somewhat downstream from the end of the same curve, where the flow is beginning to shift from the right to the left bank.

The computer program which is to be described requires that the cross-sectional geometry and velocity distribution be known at the beginning of every distance step. On the basis of the two measured sections, the following velocity distributions were assumed: For the reach from the injection point to measuring station 5 (at the end of the bend to the left), the velocity distribution measured upstream from the injection point was assumed to apply to all sections. At mile 645 the measured velocity distribution was used, except that the 70-foot width of shallow water near the left bank was omitted as being, from visual observations, not representative of the reach. At a point halfway from section 8 to section 9, the velocity distribution was assumed to be the inverse of the distribution at mile 645; that is, the same velocities and depths were used as at mile 645, except that distances measured from the right bank were assumed to be measured from the left bank. At station 10 (in the middle of a bend to the right), the velocity distribution was assumed to be the mirror image of the velocity distribution upstream from the injection point, except that at this point the width of the stream is only 510 feet, so that all transverse distances were reduced by the factor 510/600 and all depths were increased by the inverse of the same factor. For cross sections between the above sections, depths, widths, and velocities were interplated linearly from the nearest assumed distributions.

The computer program divides the stream longitudinally into distance steps, Δx, 200 feet long, and transversely into 20 stream tubes of equal discharge. The velocity within each stream tube and the transverse location of its end point are found from the measured or assumed distribution of velocity within the cross section. Diffusion is assumed to be occurring between the stream tubes at a rate given by the Fickian equation

$$M=E_z a_s \frac{\partial c}{\partial z} \tag{9}$$

in which M is the mass transport through the dividing surface between stream tubes per unit time, a_s is the dividing area between the stream tubes, and z is the transverse distance. A steady-state equation for the conservation of mass within each stream tube for each distance step is

$$qc_{i+1,j}=qc_{i,j}+M_{j-1,j}-M_{j,j+1} \tag{10}$$

in which q is the discharge per stream tube (constant for all stream tubes), $c_{i,j}$ is the concentration in the jth stream tube at the start of the ith distance step, and $M_{j,j+1}$ is the rate of mass transfer between streams j and $j+1$. The mass transfer between stream tubes is assumed to be entirely diffusive (that is, any effects of secondary currents are absorbed in the diffusive term), and is calculated by the formula

$$M_{j,j+1}=E_z d_{j,j+1}\Delta x\left(\frac{c_{i,j}-c_{i,j+1}}{\Delta z_{j,j+1}}\right) \tag{11}$$

in which $d_{j,j+1}$ is the depth at the interface between stream tubes, j and $j+1$; and $\Delta z_{j,j+1}$ is the distance between the centroids of adjacent stream tubes. The derivation of this formula assumes that for computing mass transfer the concentration in each of the stream tubes is constant throughout the distance step. The boundary condition of no transport through either bank is satisfied by allowing stream tubes 1 and 20 to exchange only with stream tubes 2 and 19, respectively.

The variation in stream-tube velocity is not shown explicitly in equation 10, but the functional dependence can be seen by substituting equation 11 into equation 10 and dividing the result by q. Dropping the stream-tube subscripts, taking $q=u\Delta zd$, where u is the local velocity, and, for simplicity, replacing the finite difference form of the transverse concentration gradient by the differential form yield

$$C_{i+1}=C_i+E_z\frac{\Delta x}{u}\frac{\partial^2 c}{\partial z^2}.$$

This shows that for a constant mixing coefficient and a constant distance step, the change of concentration within a stream tube caused by transverse mixing is inversely proportional to the local velocity, u.

To complete one distance step, the computer carries out the computation given in equation 10 for each of the 20 stream tubes using the concentration values at the beginning of the distance step, replaces each concentration value by its new value at the end of the distance step, and prints the 20 concentrations. Between distance steps the positions of the midpoints of each of the stream tubes and the depths of the stream tubes may be changed, according to the pattern of convective velocities assumed at the outset. The program is designed to read velocity-distribution data normally recorded in the Geological Survey stream-gaging operations. From these data the computer constructs a set of stream tubes of equal discharge and interpolates stream-tube velocities and dimensions between measuring stations.

Before the program was used in the present experiment, it was verified by assuming a flow having a width of 500 feet, a depth of 10 feet, a uniform transverse velocity distribution, a mean velocity of 5 ft per sec, and a shear velocity of 0.25 ft per sec. The distance step was taken to be 500 feet and the mixing coefficient $0.23U^*d$, or (0.575 sq ft per sec). The variance of the tracer cloud produced by the computer program increased exactly linearly with distance, the rate of increase giving a mixing coefficient of 0.56 sq ft per sec by equation 8. The difference between the given mixing coefficient and the computer result is 2.6 percent, well within the level of other experimental errors.

The program was next applied to predict the transverse mixing pattern in the experimental reach by introducing the velocity distributions previously assumed. The initial tracer distribution was taken to be that measured at measuring section 1, because upstream from section 1 the tracer probably was not fully mixed over the depth. The program then generated tracer distributions at all downstream sections. The computer program is written to accept up to 10 values of k in the mixing coefficient, kU^*d, and to generate tracer distributions simultaneously for each k value; for this experiment k values of 0.4, 0.5, 0.6, 0.7, and 0.8 were tried. Figure 9 shows the observed tracer distributions and those which were generated by the program using the value $k=0.6$. This value produced what appeared to be the best match between observed and computer-predicted concentration distributions; much of the error probably results from incorrect assumption of the velocity distribution. However, the comparison is reasonably good at all sections and shows that the equation

$$E_z=0.6\ U^*d \tag{12}$$

gives an adequate prediction of the mixing pattern for the reach in question.

The sensitivity of the mixing pattern to changes in the value of k can be seen in figure 10, which shows the transverse concentration

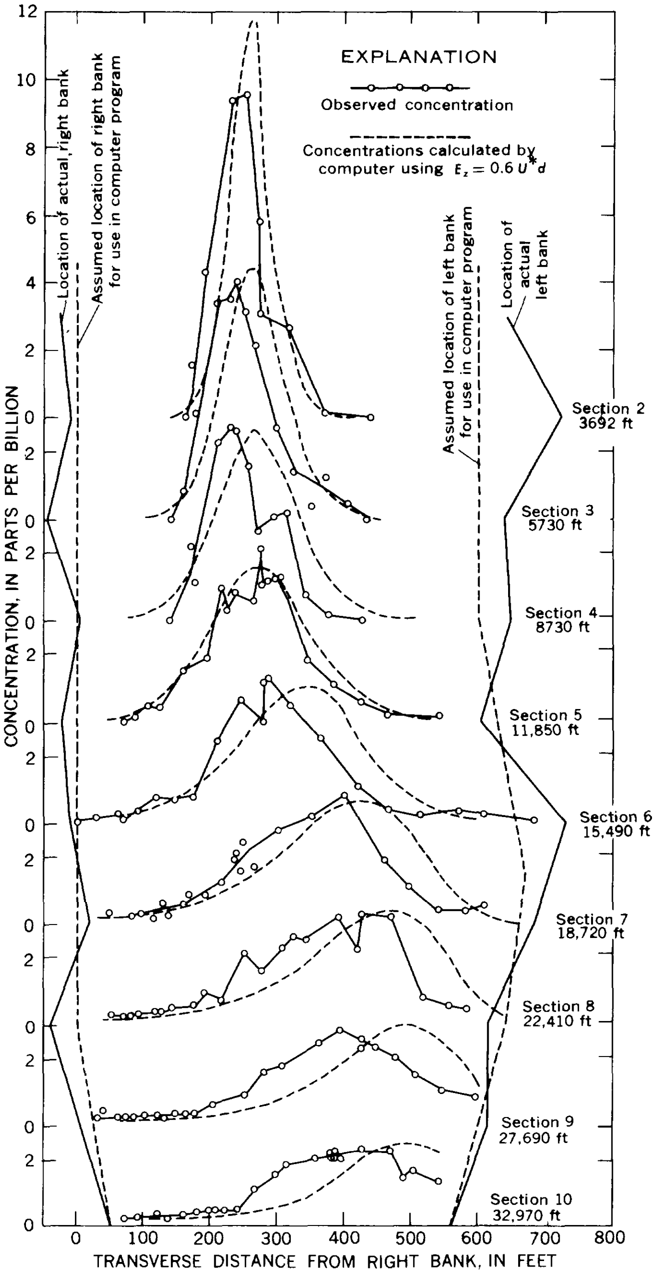

FIGURE 9.—Comparison of observed and calculated transverse distribution of dye concentration at nine cross sections

distribution at sections 5 and 9 for k equal to 0.4 and 0.8. The wide range of k produces only a modest change in the distribution; at section 5, 0.4 appears to give the better match, but at section 9, 0.8 appears to be the better choice. Thus the value 0.6 can be regarded only as an adequate approximation, probably accurate to within ± 20 percent. Unfortunately there appears to be no more accurate method of measuring the value of the coefficient. The variances of the tracer

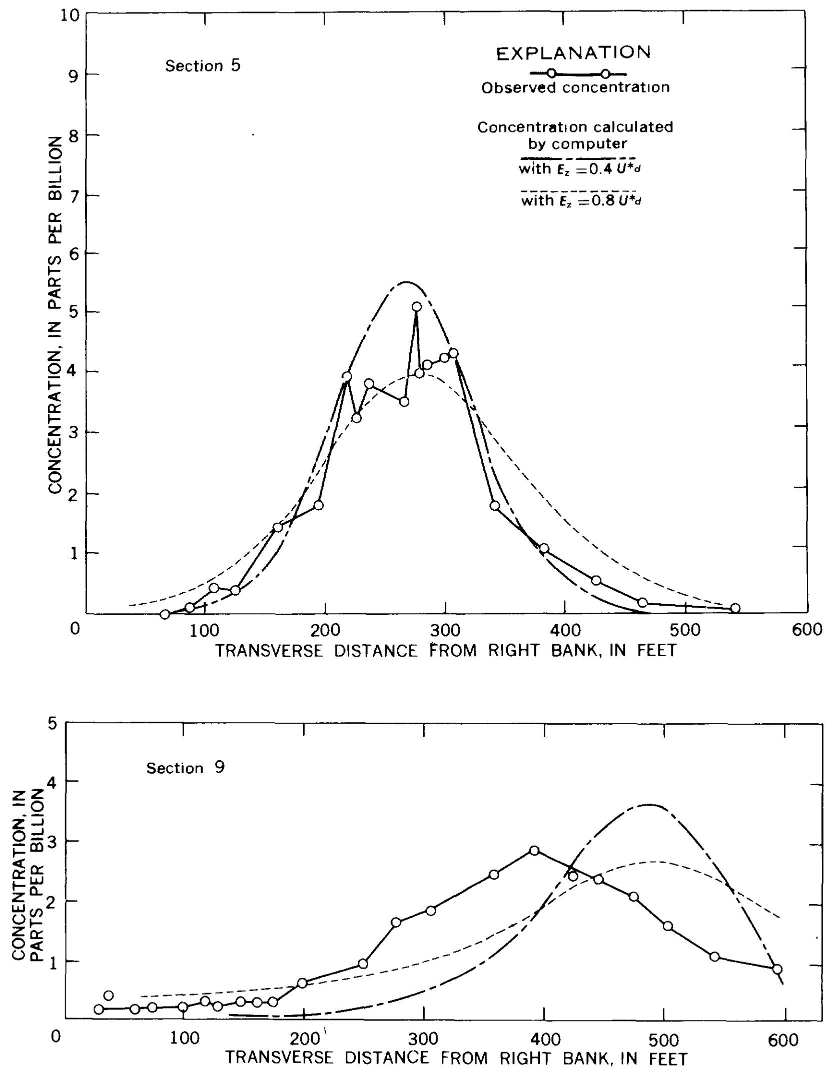

FIGURE 10.—Influence of E_z on transverse distribution of dye concentration.

cloud predicted using the value $k=0.6$ have been plotted in figure 8; the excellent agreement with the variance calculated from the observed data lends support to the value $k=0.6$.

SUMMARY AND CONCLUSIONS

In November 1967 a longitudinal dispersion experiment was conducted in a reach of the Missouri River by introducing a concentrated slug of 600 pounds of Rhodamine WT dye at Sioux City, Iowa. Observations of dye concentration as a function of time were made at four downstream locations, ranging from 41 to 141 miles below Sioux City, over a period of 3 days. Longitudinal dispersion coefficients were calculated from these data, using both the routing and change-of-moment methods. During the experiment the channel geometry and transverse distribution of velocity were measured at seven cross sections distributed along the reach. Channel widths varied from about 500 to 750 feet, average depths from about 8 to 12 feet, average velocities from about 3.9 to 6.0 ft per sec, and discharges from about 31,000 to 34,000 cfs.

On the day following the completion of the longitudinal dispersion experiment, a transverse mixing experiment was performed in a 6-mile reach immediately downstream from Blair, Nebr., somewhat more than halfway down the longitudinal dispersion study reach. In this experiment, a continuous stream of Rhodamine BA dye was injected near the center of the river at Blair, and sets of samples defining the transverse distribution of the dye plume were obtained at 10 cross sections ranging from 1,700 to 33,000 feet downstream. Values of the transverse mixing coefficient were calculated by both the method of moments and a computer simulation method which takes into account the longitudinal variations of channel geometry and the transverse distribution of velocity.

Analysis of the experimental data leads to the following conclusions:

1. The best estimate of the longitudinal dispersion coefficient for the entire study reach at the time of the experiment is

$$D \approx 16{,}000 \text{ sq ft per sec.}$$

   Estimates for individual subreaches range from 10,000 to about 30,000 sq ft per sec. Estimates based on the routing and moment methods are in reasonably good agreement. However, it is concluded that in general the routing method gives more consistent results.

2. The average rate of movement of the dye cloud through the 141-mile study reach was 5.34 ft per sec. This compares well with average velocities ranging from about 3.9 to 6.0 ft per sec

measured by current meter at various cross sections through the reach.

3. The best estimates of the transverse mixing coefficient are, by the moment method,

$$E_z \approx 0.7\ U^*d,$$

and, by the computer simulation method,

$$E_z \approx 0.6\ U^*d.$$

The data suggest that E_z varied, perhaps by as much as 100 percent within the 6-mile study reach. It is concluded that the ratio E_z/U^*d is apt to be significantly larger and more variable in large meandering channels than in small straight channels. This underlines the need for further study of transverse mixing in large natural streams.

4. Rhodamine WT is far superior to Rhodamine BA as a water tracer, both with respect to dye loss in the channel and adsorption of dye on glassware used for storing experimental and calibration samples. Whereas the longitudinal dispersion experiment with Rhodamine WT gave a dye loss of less than 25 percent for the 141-mile study reach, the transverse mixing experiment with Rhodamine BA gave a dye loss on the order of 40 percent for a 6-mile reach. The relative amounts due to dye loss in the river and to adsorption on glassware are not known, but both are probably important.

REFERENCES

Bowie, J. E., and Petri, L. R., 1968, Travel of solutes in lower Missouri River: U.S. Geol. Survey Hydrol. Inv. Atlas HA-332.

Elder, J. W., 1959, The dispersion of marked fluid in turbulent shear flow: Jour. Fluid Mechanics, v. 5, no. 4, p. 544-560.

Fischer, H. B., 1966a, A note on the one dimensional dispersion model: Air and Water Pollution, an Internat. Jour., v. 10, p. 443-452.

Fischer, H. B., 1966b, Longitudinal dispersion in laboratory and natural streams: California Inst. Tech., W. M. Keck Lab. Hydraulic and Water Resources, Rept. KH-R-12, 250 p.

Fischer, H. B., 1967a, Transverse mixing in a sand-bed channel: U.S. Geol. Survey Prof. Paper 575-D, p. D277-D282.

Fischer, H. B., 1967b, The mechanics of dispersion in natural streams: Jour. Hydraulics Div., Am. Soc. Civil Engineers, v. 93, no. HY6, p. 187-216.

Fischer, H. B., 1968, Dispersion predictions in natural streams: Jour. Sanitary Engineering Div., Am. Soc. Civil Engineers, v. 94, no. SA5, p. 927-943.

Glover, R. E., 1964, Dispersion of dissolved or suspended materials in flowing streams: U.S. Geol. Survey Prof. Paper 433-B, 32 p.

Godfrey, R. G., and Frederick, B. J., 1963, Dispersion in natural streams: U.S. Geol. Survey open-file report, 75 p.

Sayre, W. W., and Chang, F. M., 1968, A laboratory investigation of open-channel dispersion processes for dissolved, suspended, and floating dispersants, U.S. Geological Survey Prof. Paper 433-E, 71 p.

Taylor, G. E., 1954, The dispersion of matter in turbulent flow through a pipe: Royal Soc. London Proc., v. 223A, p. 446-468.

Thackston, E. L., and Krenkel, P. A., 1966, Longitudinal mixing and reaeration in natural streams: Vanderbilt Univ. Sanitary and Water Resources Eng., Tech. Rept. 7,212 p.

Thackston, E. L., and Krenkel, P. A., 1967, Longitudinal mixing in natural streams: Jour. Sanitary Engineering Div., Am. Soc. Civil Engineers, v. 93, no. SA5, p. 67-190.

U.S. Army Corps of Engineers, Omaha District, 1967, Missouri River Channel regime studies: M.R.D. (Missouri River Division) Sediment Series 13, 13 p.

Wilson, J. F., and Forrest, W. E., 1965, Potomac River time-of-travel measurements: Lamont Geol. Observatory Symposium on Diffusion in Oceans and Fresh Waters, Palisades, N.Y., 1964, Proc., p. 1-18.

Yotsukura, Nobuhiro, 1967, General discussion in subject D—microturbulent diffusion and dispersion: Internat. Assoc. Hydraulic Research Cong., 12th, Fort Collins, Colo., 1967, Proc., v. 5, p. 542-551.

U.S. GOVERNMENT PRINTING OFFICE: 1970 O—372-385