Fluvial Sediment of the Mississippi River at St. Louis, Missouri

By PAUL R. JORDAN

FLUVIAL SEDIMENT
MISSISSIPPI RIVER
ST. LOUIS, MISSOURI

The Mississippi River, part of St. Louis, and East St. Louis. Fluvial sediment and streamflow were measured at the bridge nearest the bottom of the photograph. Photograph by Surdex Corp., St. Louis, Mo.

UNITED STATES DEPARTMENT OF THE INTERIOR

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DEFINITIONS

Bedload discharge.   Discharge of sediment that moves in essentially continuous contact with the streambed.

Bed-material discharge.   That part of the total sediment discharge consisting of particles whose sizes are the same as those present in significant quantities in the bed.

Discharge (or streamflow).   Rate of flow of a stream, expressed as a volume or weight per unit time, or the quantity of flow that passes in a given time. It includes the sediment and dissolved solids that are in the water. "Discharge" is the term generally used in this report; streamflow is used where necessary to avoid confusion with various types of sediment discharge.

Measured bed-material concentration. Ratio of the measured bed-material discharge to the streamflow when both are expressed in weight per unit time.

Measured bed-material discharge. That part of the measured suspended-sediment discharge consisting of particles whose sizes are the same as those present in significant quantities in the bed.

Measured suspended-sediment discharge. Computed as the product of the concentration of depth-integrated samples, the total discharge, and a constant for converting the units to a weight per unit time, generally tons per day.

Station. A particular location in the cross section at which sediment samples are obtained or hydraulic measurements are made; it is designated by the distance, in feet, from an initial point on the west bank at the MacArthur Bridge. The initial point has remained the same throughout the investigation.

Streamflow. See **Discharge.**

Total sediment discharge. Weight of all the sediment passing a section in a unit time.

Unmeasured sediment discharge. Difference between the total sediment discharge and the measured suspended-sediment discharge.

SYMBOLS

a A specified distance above the streambed.

B Mathematical abbreviation for a group of parameters that depend on the particle size.

b Width of flow.

C A constant relating the rate of change of stage to its equivalent effect in the reach.

c Concentration of sediment particles in a size range.

c_a Sediment concentration at a distance a above the streambed.

c_y Sediment concentration at a distance y above the streambed.

d Grain size of bed material.

d_e Effective diameter of bed material, equal to $\Sigma\, d_i\, i_b$ where d_i is the average size of particles in a size range and i_b is the fraction of material by weight in that size range.

d_i Geometric mean size of a bed-material size range.

dh/dt Rate of change of gage height.

$d_{25}, d_{35}, d_{50}, d_{65}, d_{75}, d_{90}$ Particle size at which 25, 35, 50, 65, 75, or 90 percent of the bed material by weight is finer.

g Acceleration due to gravity.

g_b "Bed load" according to the definition of Bagnold (1956).

h_b Mean bed elevation between stations 275 and 1,800.

j Coefficient in $b = jQ^m$.

k Von Karman coefficient for turbulent exchange.

k_r Metric coefficient of flow resistance of particles, equal to $26/d_{90}^{1/6}$ where d_{90} is in meters.

k_s A linear measure of the bed roughness.

k_t Metric coefficient of total flow resistance, equal to $\bar{u}/\bar{y}_0^{2/3}S^{1/2}$ where $\bar{u}$ is in meters per second and $\bar{y}_0$ is in meters.

L Length between sections at which Q_m and Q_s are determined.

$\log$ Logarithm to the base 10.

m Exponent in $b=jQ^m$.

n Manning resistance factor, equal to 1.49 $R^{2/3}\ S_e^{1/2}/\bar{u}$ where R is in feet and $\bar{u}$ is in feet per second.

n_d Manning resistance factor during decreasing discharge.

n_i Manning resistance factor during increasing discharge.

p Coefficient in $\bar{y}_0=pQ^r$.

p_i Fraction of bed area occupied by the particles in a size range, equal to 0.35 $(i_b/d_i)/\Sigma(i_b/d_i)$ where i_b is the fraction of bed material in a size range and d_i is the geometric mean of the size range.

Q Discharge (volumetric streamflow rate).

Q_c Discharge under conditions of constant discharge.

Q_g Discharge at the gage.

Q_m Measured changing discharge.

q Discharge (volumetric streamflow rate) per unit width.

q_{BM} Bed-material discharge per unit width.

q_b Bedload discharge per unit width.

R Hydraulic radius.

r Exponent in $\bar{y}_0=pQ^r$.

S Slope of the water surface for a 6.5-mile reach.

S_c Slope of the water surface under conditions of constant discharge.

S_e Energy gradient.

s Coefficient in $\bar{u}=sQ^t$.

t Exponent in $\bar{u}=sQ^t$.

U Velocity of flood wave.

$\bar{U}_g/\bar{U}$ Ratio of the mean speed of movement of grains to mean fluid velocity at the grain level.

$\bar{u}$ Mean velocity of flow in cross section.

$\bar{u}_c$ Mean velocity under conditions of constant discharge.

$\bar{u}_m$ Measured mean velocity under conditions of changing discharge.

$\bar{u}_s$ Mean velocity at a station.

u_y Time-averaged velocity of flow at a point a distance y above the streambed.

$u*$ Shear velocity, equal to $\sqrt{gRS_e}$.

$u*'$ Shear velocity with respect to the grain.

$u*''$ Shear velocity for channel irregularities such as ripples and dunes.

w Average fall velocity of sediment particles in a size range.

x A parameter to cover the transition from hydraulically smooth to hydraulically rough boundaries.

y Distance above the streambed.

y_0 Depth of flow at a station.

$\bar{y}_0$ Mean depth of flow in cross section.

z Theoretical exponent for vertical distribution of suspended sediment.

z_1 Exponent for the actual vertical distribution of suspended sediment.

γ Specific weight of water.

γ_s Specific weight of sediment particles.

θ Sediment characteristic equivalent to Straub's Ψ

τ_c The particular transporting force required to start movement of the bed material.

Ψ' Shear function (intensity of shear on representative particle).

ρ Density of water.

ρ_s Density of sediment particles.

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ABSTRACT

An investigation of the fluvial sediment of the Mississippi River at St. Louis, Mo., was begun in 1948. Most data have been obtained only to determine the daily suspended-sediment discharge and the particle-size distribution of suspended sediment and bed material, but a few data have been obtained to study the flow resistance, the vertical distribution of sediment and velocity, and the bed-material discharge.

The flow of the Mississippi River at St. Louis is made up of the flows from the Missouri River, which had an average flow of 79,860 cubic feet per second for 1897–1958 at Hermann, Mo., and from the upper Mississippi River, which had an average flow of 91,890 cubic feet per second for 1928–58 at Alton, Ill. The Missouri River is partly controlled by reservoirs that had a total capacity of 90,300,000 acre-feet in 1956, and the upper Mississippi River is partly controlled by lakes and reservoirs that had a total capacity of 4,890,000 acre-feet in 1956.

The flows of the Missouri and upper Mississippi Rivers have not become mixed at St. Louis; so the river has a lateral gradient of suspended-sediment concentration. The concentration near the west bank has been as much as 2,400 parts per million greater than the concentration near the east bank.

Suspended-sediment discharges from April 1948 to September 1958 ranged from 4,250 to 7,010,000 tons per day and averaged 496,000 tons per day. Mean concentrations for water years decreased steadily from 1,690 parts per million in 1949 to 403 parts per million in 1956, but they increased to 756 parts per million in 1958. Effects of new reservoirs in the Missouri River basin on the concentration have been obscured by the close relation of concentration to streamflow.

Measured suspended-sediment discharge through September 1958 averaged 47 percent clay, 38 percent silt, and 15 percent sand. Variations of particle size were due mainly to differences in the source areas of the sediment.

Most of the bed material in the main flow was between 0.125 and 1.000 millimeter in diameter. The average of median diameters was related to the discharge for periods of 1 year and longer. Geometric quartile deviations of the bed material ranged from 1.1 to 2.5 and averaged 1.5.

The mean elevation of the bed had a range of almost 10 feet and was related to the median diameter of bed material by the regression equation $h_b = 363.0 - 7.8\, d_{50}$ for which the standard error of estimate was 0.91 foot.

The resistance to flow as measured by Manning's n ranged from 0.024 to 0.041 and was related to the discharge and mean velocity but not to the shear velocity. Normal dune height is 2–8 feet, and average dune length is about 270 feet. When the resistance to flow was low, much of the bed was fairly flat; a few dunes were

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present, but they were much longer than the average. For a given discharge during individual rises in stage, the gage height was lower for increasing discharge than for decreasing discharge even though the bed elevation was higher. The changes in gage height were not caused by changes in energy gradient due to changing discharge, by channel storage between the gage and the measuring section, nor by return of overbank flow; but they were probably caused by a combination of changes in roughness due to changing bed configuration and of changes in turbulence constant due to changing sediment concentration.

Turbulence constants (Von Karman's k) computed from velocity measurements at 5–10 points in the vertical and from routine velocity measurements at 2 points in the vertical averaged 0.35 and 0.33, respectively.

The exponent z_1 of the vertical distribution of concentration for different size ranges varied with about the 0.77 power of the fall velocity. Except for the difference between the theoretical variation and the actual variation of z_1 with changing fall velocity, the theoretical equation for the vertical distribution of sediment concentration seems to apply reasonably well for the Mississippi River at St. Louis.

The accuracy of methods for computing bedload discharge and bed-material discharge could not be evaluated directly but was evaluated from the relations of computed bedload discharge or bed-material discharge to the measured part of the bed-material discharge and to mean velocity. These relations indicated that bedload discharges can be computed most accurately by using the modified Einstein, Schoklitsch, and Meyer-Peter and Müller methods as compared with the Kalinske and Bagnold methods. The relations indicated that bed-material discharges can be computed most accurately by adding the unmeasured sediment discharge from Colby's procedure to the measured part of the bed-material discharge when data on velocity, concentration, and particle-size distribution are available, or by using Straub's equation when such data are not available. Measured suspended-sediment discharge averaged about 95 percent of the total sediment discharge, and bed-material discharge averaged about 14 percent.

INTRODUCTION

The Mississippi River is constantly changing its channel by cutting into its banks, building bars, and aggrading or degrading its bed. Any major changes, either natural or artificial, in the shape or alinement of the channel involve the transportation and redistribution of billions of tons of sediment. The greatest part of the sediment that is being transported by the river will be carried to the deltas beyond the mouth; but a large amount of sediment will be deposited in bars, some of which will eventually become part of the flood plain as the course of the river shifts. Knowledge of the amount and behavior of the sediment is necessary for the planning, construction, and maintenance of works for control and use of the river or its flood plain such as for navigation, waste disposal, water supplies, and flood control.

The investigation to provide the necessary knowledge of fluvial sediment at St. Louis began in April 1948 with the collection of data on suspended-sediment concentration and discharge. Data on particle size of suspended sediment and bed material have been

collected since 1950 and 1951, respectively. Data on water temperature have been obtained since 1951. Although most of the data were obtained only to determine the daily suspended-sediment discharge and the particle-size distribution of suspended sediment and bed material rather than to study the mechanics of sediment transportation, a few data were obtained to study the vertical distribution of sediment and velocity, the bed-material discharge, and the flow resistance caused by bed-material grains and by dunes and other irregularities on the bed. The data on sediment concentrations and discharges, particle size, and water temperature are published in the annual series of U.S. Geological Survey water-supply papers entitled, "Quality of Surface Waters of the United States."

The purposes of this report are to evaluate and interpret the sediment data, to show the effect of upstream developments on sediment concentration and discharge, and to present some theoretical aspects of sediment transportation as applied to the river. This report includes evaluation of the data obtained through September 1958 and the results of some special studies in 1959 and 1960.

This investigation was made by the U.S. Geological Survey under the successive supervision of P. C. Benedict, regional engineer, and D. M. Culbertson, district engineer. It was made in cooperation with the U.S. Army Corps of Engineers, St. Louis District, who did much of the laboratory analysis of concentration and particle size, provided gage readings for determination of water-surface slopes, and provided results of sonic-fathometer measurements of the configuration of the bed.

The only previously published report that includes information on the fluvial sediment of the Mississippi River in the vicinity of St. Louis is a study of the water resources of the St. Louis area (Searcy, Baker, and Durum, 1952), which gives results of turbidity measurements and of some sediment analyses.

The symbols used in this report conform, where practicable, to those established by the American Standards Association (1958).

GENERAL FEATURES OF THE RIVER REACH

The flow of the Mississippi River at St. Louis is a composite of the flows from many tributaries in a drainage area of about 701,000 square miles. An average of about 45 percent of the flow at St. Louis is from the Missouri River, which is confluent with the Mississippi River about 15 miles upstream from St. Louis.

The sediment is sampled and streamflow is measured at the MacArthur Bridge, 178.9 miles upstream from the mouth of the Ohio River and 1.1 miles downstream from the water-stage recorder. Because the flow is confined between artificially stabilized banks and

levees, the channel in the vicinity of MacArthur Bridge is not typical of the channel between the Missouri and Ohio Rivers. At normal river stages, the channel from about 4½ miles upstream to about 4 miles downstream from the bridge is slightly curved and is nearly uniform in width. (See frontispiece.) The width of flow at the bridge varies from about 1,500 feet at low stage to about 1,800 feet at the highest stage that is confined within the banks. When the flow is over the banks, the width increases, sometimes to more than 2,800 feet. The mean depth of the flow ranges from about 15 to 50 feet.

In the vicinity of the sampling section, the west bank is riprapped, the east bank is subject to scour, and the flow is affected by two bridge piers, each 27 feet wide. The distribution of sediment and the elevation of the streambed are locally affected by barges that move through the section and that are sometimes moored just upstream from the bridge near either bank.

At St. Louis, the flows of the Missouri and upper Mississippi Rivers are not homogeneously mixed, and, because the Missouri River usually has a higher concentration of suspended sediment than does the upper Mississippi, each flow can generally be identified by the appearance of the water. The difference in concentration has evoked the comments of many observers. Chambers (1910, p. 189) wrote that he "saw the coffee-coloured flood of the Missouri add itself to the clear water of the Mississippi. As far as Saint Louis, the western third of the broad river remained *café au lait* in hue." Figure 1 shows the appearance of the Missouri River water as it enters the Mississippi.

The discharge and the sediment concentration and particle size are major determinants of the shape of the cross section of a channel. According to Leopold and Maddock (1953), up to the bank-full stage in a natural river section, the relations of width, depth, and velocity to discharge are in the mathematical form of simple power functions. The relative rates of increase of width, depth, and velocity are determined by the shape of the channel, the slope of the water surface, and the roughness of the wetted perimeter. The relations shown by Leopold and Maddock for natural river cross sections are, in the notation of this report,

$$b = jQ^m$$
$$\bar{y}_0 = pQ^r$$
$$\bar{u} = sQ^t$$

where

b is width of flow
Q is discharge
$\bar{y}_0$ is mean depth of flow in the cross section
$\bar{u}$ is mean velocity of flow in the cross section
$j, m, p, r, s,$ and t are constants for a particular cross section

FIGURE 1.—Confluence of Missouri and Mississippi Rivers. Photograph by Massie-Missouri Resources Division.

Widths, mean depths, and mean velocities of the Mississippi River at St. Louis were plotted against the corresponding discharges on logarithmic graph paper (fig. 2). Straight lines were drawn to fit the plotted points; and slopes $m=0.07$, $r=0.43$, and $t=0.50$ were measured directly from the graphs. The value of m (0.07) is much less than the average (0.26) given by Leopold and Maddock (1953), because the banks of the Mississippi at St. Louis are artificially stabilized to a rather steep slope.

Leopold and Maddock asserted that because of the identity

$$Q = b\bar{y}_0\bar{u}$$

then

$$Q = jQ^m \, pQ^r \, sQ^t$$

and it follows that

$$m+r+t=1.0$$

and

$$j \, p \, s = 1.0$$

For the average lines that were drawn to fit the Mississippi River data,

$$m+r+t = 0.07+0.43+0.50$$
$$= 1.00$$

and

$$j \, p \, s = (680)(0.16)(0.0092)$$
$$= 1.00$$
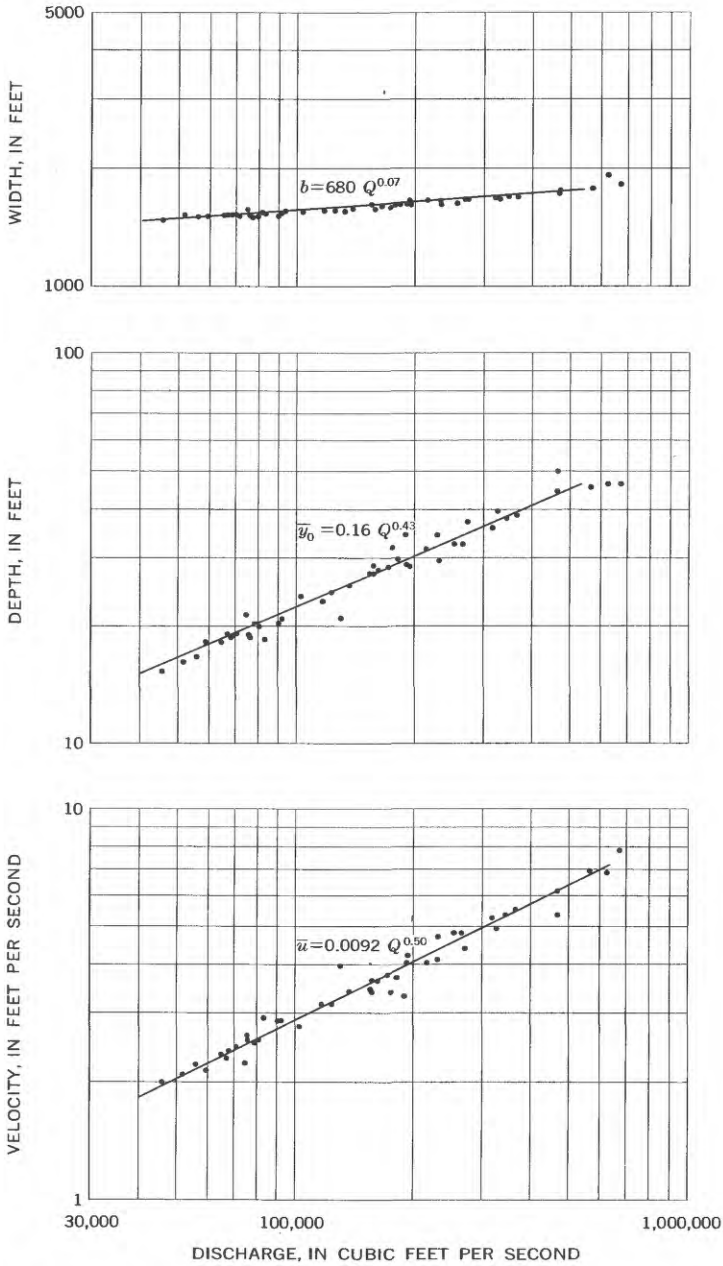
FIGURE 2.—Relation of width, depth, and velocity to discharge.

DISCHARGE CHARACTERISTICS

The discharge of the Missouri River near its mouth, at Hermann, Mo., averaged 79,860 cfs (cubic feet per second) for 1897–1958. As of 1956, the Missouri River was partly controlled by reservoirs having capacities totaling about 90,300,000 acre-feet, of which about 51,300,000 acre-feet was on the main stem of the Missouri River upstream from Yankton, S. Dak. (Storage-capacity figures were computed from Thomas and Harbeck, 1956.) Thus, the major part of the control of the river is in the upper half of the basin, where floods are caused mainly by mountain and plains snowmelt.

Mississippi River at Alton, Ill., 7.7 miles upstream from the Missouri River, had a mean discharge of 91,890 cfs for 1928–58. The river is partly regulated by a series of 26 dams that provide slack-water navigation to St. Paul, Minn., and by several reservoirs and lakes on the main stem and the tributaries. Total capacity of the reservoirs and lakes in 1956 was about 4,890,000 acre-feet.

Comparison of the flow-duration curves for Mississippi River at Alton and for Missouri River at Hermann (fig. 3) indicates that the flow of the Missouri River is more variable than the flow of the Mississippi and that the low flow, particularly in 1929–58, was not well sustained. The low flow of the Missouri River was better sustained in 1949–58 than in 1929–58 partly because the 1953–56 drought was not so severe as the 1931–35 drought and mainly because a large volume of stored water was released in 1953–56 (Pafford, 1959), particularly from Fort Peck Reservoir, which began storage in 1938. Comparison of all the curves shows the effects of the flows of the Missouri River and upper Mississippi River on the flow at St. Louis. The similarity of the two curves for each station indicates that the flow for the period of sediment record is representative of the flow for the past 25–30 years. However, although the mean discharge at St. Louis may be about the same in the future as in the past (174,800 cfs for 1861–1958), the high and low flows will probably be less extreme because of the reservoir storage that has been added in recent years.

The discharge at St. Louis follows a definite monthly pattern. (See fig. 4.) Discharges are usually high during March–July because of runoff from snowmelt in the mountains and plains and from spring and early summer rains.

Extremely high flows carry the most sediment, both in suspension and along the bed, and they cause the greatest changes in channel patterns by cutting the banks, depositing sediment in bars and overflow areas, and scouring the bed, particularly around bridge piers. Because high sediment concentrations are associated with high flows,

EXPLANATION

——— ——— (1934–58)
– – – – – – – (1949–58)
Mississippi River at Alton, Ill.

————————(1929–58)
—— —— —— ——(1949–58)
Missouri River at Hermann, Mo.

—— – – —— (1934–58)
———— ———— (1949–58)
Mississippi River at St. Louis, Mo.
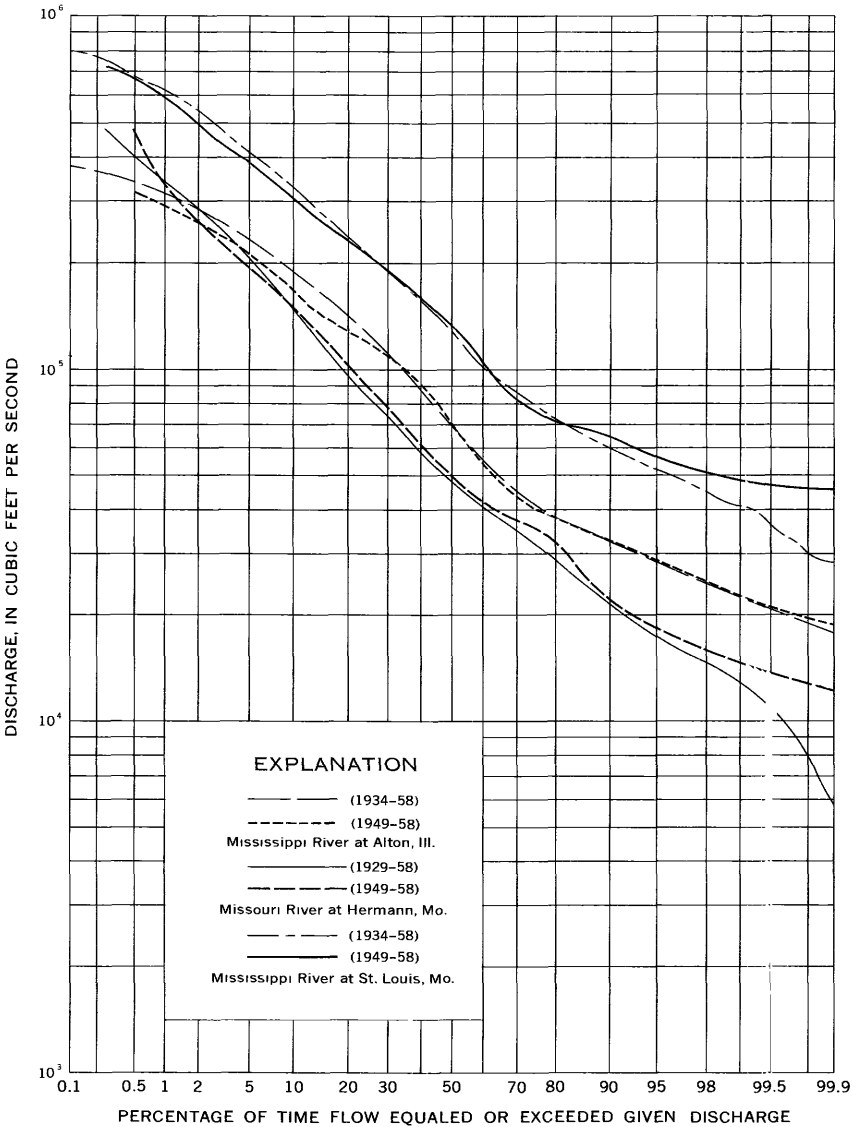
FIGURE 3.—Duration curves of daily flows, Missouri River and Mississippi River.

the proportion of sediment discharge during a given period of high
flow to sediment discharge for the year is larger than the proportion
of streamflow during the period to streamflow for the year. In some
small rivers, as much as 60 percent of the water and 90 percent of
the sediment for the year have been discharged during a few days
of high flow; and in the Mississippi River, as much as 17 percent of
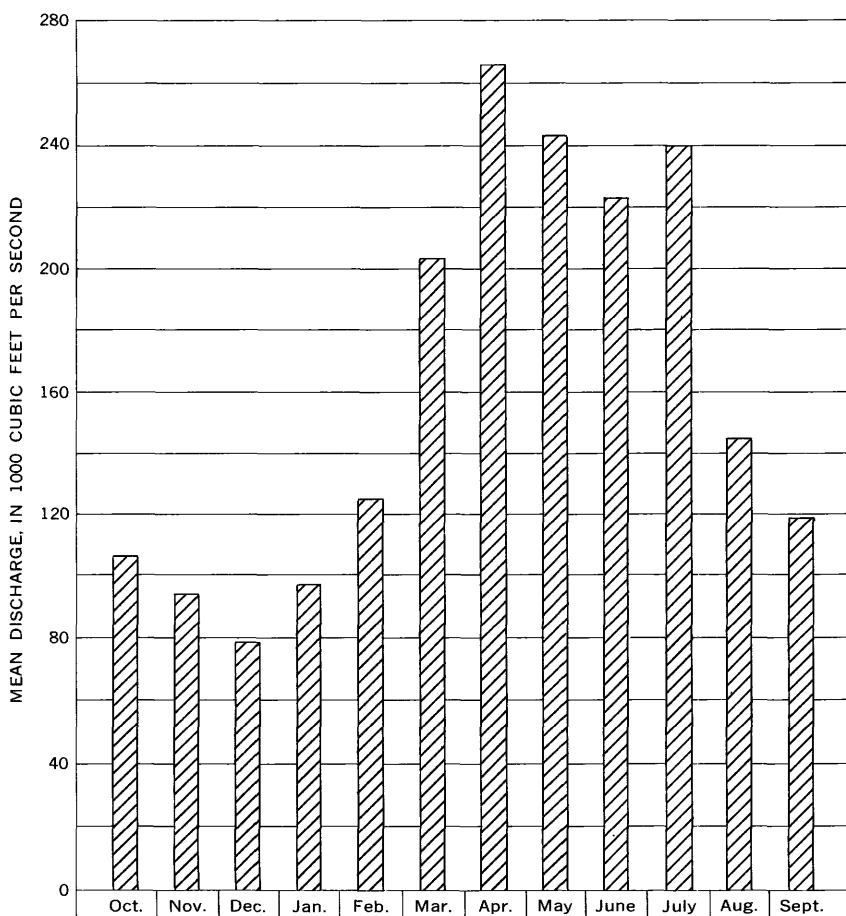
FIGURE 4.—Mean monthly discharges, 1934–58.

the water and 41 percent of the sediment for the year have been discharged during a 30-day period.

Because of the significance of high flows in transporting sediment, a graph that shows the average recurrence interval of annual peak discharges is presented (fig. 5). The points that are plotted in figure 5 represent the annual peak discharges from 1861 to 1958, and the curve of figure 5 was derived from graphs of peak discharges presented by Searcy (1955, figs. 11, 12). The curve of figure 5 shows that the peak discharge in July 1951 of 782,000 cfs would have an average recurrence interval of about 9 years and that the peak discharge in April 1952 of 684,000 cfs would have an average recurrence interval of about 5 years. According to the curve, a discharge of 990,000 cfs or more would occur once in 100 years; actually, discharges of more
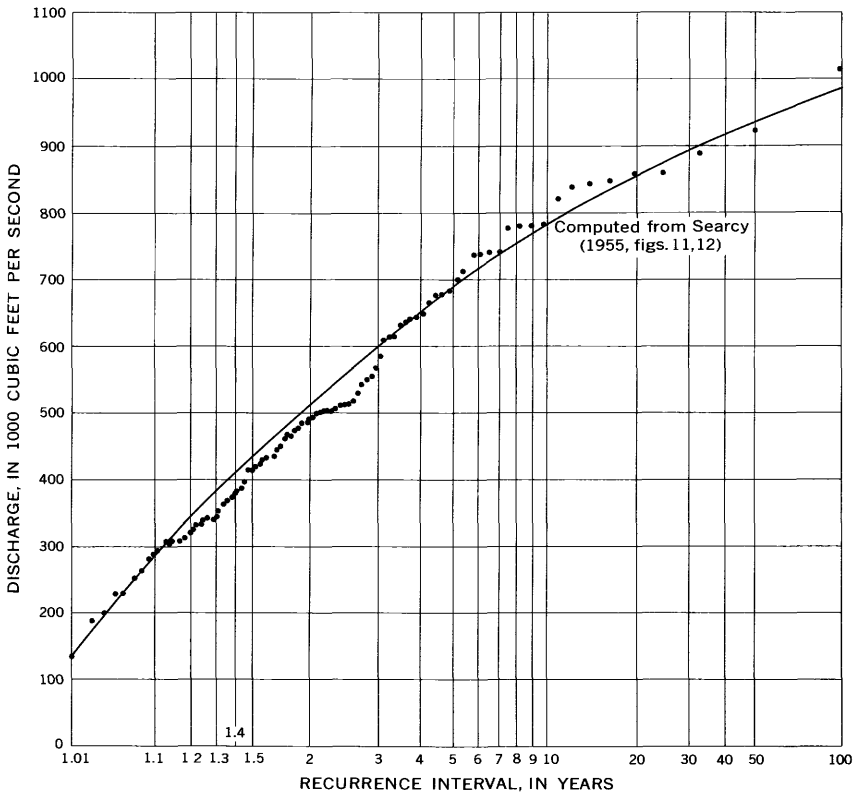
FIGURE 5.—Recurrence intervals of annual peak discharges, 1861–1958.

than 990,000 cfs have occurred twice in 184 years (in 1844 and in 1903).

Because of the increased storage capacity that has been added in recent years, particularly in the Missouri River basin, the flood-frequency curve that has been developed from past records probably does not represent accurately the frequencies of future floods. Most of the floods from the upper Missouri River drainage, such as the one that resulted from snowmelt in April 1952, will be greatly reduced by the reservoirs on the main stem of the Missouri River. Some floods from the lower Missouri River drainage, such as the one that resulted from heavy rains in July 1951, will be reduced by storage behind dams that have been recently completed or are under construction on many of the tributaries. Also, peak sediment discharges will be reduced by the reservoirs. Most of the annual peak water discharges have occurred in April, May, and June; and many have occurred in July. (See fig. 6.)
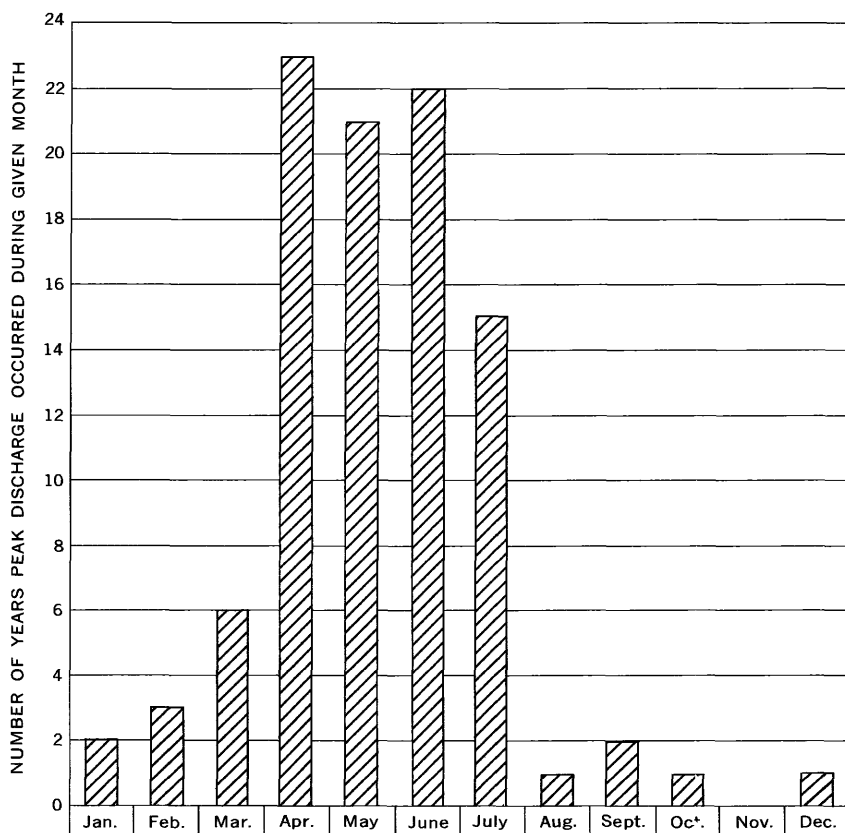
FIGURE 6.—Occurrence of peak discharges, 1861–1958.

The Mississippi River is subject to periods of low flow, particularly in November–January. Bars built up from the large amounts of sediment that were transported during high flows become obstructions to river traffic during extremely low flows; a 9-foot channel cannot be maintained at all places, and barges cannot be loaded as heavily as usual. Figure 7 shows the magnitude and duration of low flows for the period 1861–1958. Low flows are likely to be less extreme in the future than in the past because of the storage that has recently become available in the Missouri River basin and that can be used to sustain the flow.

SUSPENDED-SEDIMENT DISCHARGE AND PARTICLE SIZE

Samples of suspended sediment were collected by standard procedures of the Geological Survey. All point-integrated samples and most depth-integrated samples were obtained with a U.S. P–46 sampler suspended from a power-operated reel. At times of high
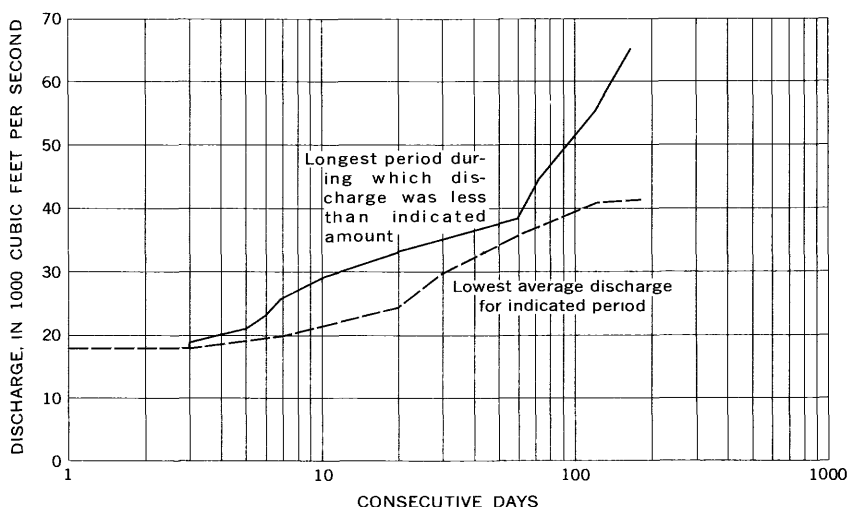
FIGURE 7.—Minimum discharges, 1861–1958.

flow, a 100- or 200-pound weight was attached below the sampler to stabilize it. When heavy floating ice prevented the use of the P–46 sampler, the upper 10 percent of the flow was sampled with a U.S. D–43 sampler. Depth-integrated samples were obtained at 2, 8, or 10 stations in the cross section to define stream concentrations. Usually, stations were chosen at centroids of equal portions of flow as defined by streamflow measurements. On a few occasions the stations were chosen at points of equal spacing relative to stream width, and at these times equal transit rates of the sampler were used.

All suspended-sediment samples were analyzed for concentration, and the results are expressed in parts per million by weight. All the point-integrated samples and some of the depth-integrated samples were analyzed for particle size. Particle-size analyses of depth-integrated samples generally were for composites of samples taken at 8–10 stations. Before October 1952, the bottom-withdrawal-tube method was used for all particle-size analyses of suspended sediment. From October 1952 to May 1955, for most analyses, a bottom-withdrawal tube was used for particles smaller than 0.062 mm and sieves were used for particles larger than 0.062 mm; for a few analyses, a bottom-withdrawal tube was used alone. Subsequent to May 1955, either a bottom-withdrawal tube or a pipet was used for particles smaller than 0.062 mm, and either sieves or a visual-accumulation tube was used for particles larger than 0.062 mm. Results from the bottom-withdrawal-tube, pipet, and visual-accumulation-tube methods are expressed in terms of fall diameter, which is defined (Inter-Agency Committee on Water Resources, 1958) as the diameter of a

sphere that has a specific gravity of 2.65 and has the same standard fall velocity as the particle. Results from the sieve analyses are expressed in terms of sieve diameter, which is the length of the side of the smallest square opening through which the given particle will pass.

Suspended-sediment samplers currently available cannot obtain samples that are representative of the total sediment discharge. The P–46 sampler can obtain samples from the water surface to about 0.5 foot above the bed or, when the 100- or 200-pound weight is attached, from the water surface to about 1.8 feet above the bed. Therefore, the part of the total sediment discharge that is suspended within 0.5 foot (sometimes 1.8 ft) of the bed or that is sliding or rolling in almost continuous contact with the bed is not sampled.

The sediment concentration obtained from depth-integrated samples for a cross section is the discharge-weighted mean concentration in the sampled zone. This concentration is multiplied by the discharge for the entire depth and by a units-conversion factor, and the result is called the measured suspended-sediment discharge. If the concentration in the sampled zone is equal to the suspended-sediment concentration for the entire depth, the measured suspended-sediment discharge is equal to the total suspended-sediment discharge; but because the concentration, especially of sand, increases toward the bed, the measured suspended-sediment discharge normally is less than the total suspended-sediment discharge.

In the Mississippi River, the unsampled zone is only 1–5 percent of the total depth; and even though the concentration of sand increases substantially toward the bed, most of the material in suspension is finer than sand and is nearly uniformly distributed in the vertical direction. Therefore, the measured suspended-sediment discharge is nearly equal to the total suspended-sediment discharge, and the particle-size distribution of the depth-integrated samples is nearly the same as that for the entire depth. In the following discussion, the term "suspended-sediment discharge" is used to refer to the measured suspended-sediment discharge, which is a close approximation of the total suspended-sediment discharge; and the term "mean concentration" is used to refer to the discharge-weighted mean concentration, either for an individual sample, for the cross section, or for a time period.

Because the flows of the Missouri and Mississippi Rivers have not become mixed at the sampling section, the river has a lateral gradient of suspended-sediment concentration. The concentration near the east bank is usually fairly low, and the concentration increases progressively toward the west bank. Differences between concentrations at stations near the west bank and near the east bank have

been as much as 2,400 ppm (parts per million), and at many times the difference has been greater than the mean concentration for the cross section. As the discharge and mean concentration increase, the lateral concentration gradient increases slightly but the percentage change in concentration decreases. The concentration gradient has no apparent relation to the proportion of the flow that is from the Missouri River or from the upper Mississippi River. Only very infrequently have concentrations near the east bank been greater than those near the west bank.

Suspended-sediment discharges at St. Louis from April 1948 to September 1958 (see table 1) ranged from 4,250 to 7,010,000 tons per day and averaged 496,000 tons per day. The annual sediment discharges and concentrations are shown in figure 8. Mean concentrations for water years decreased steadily from 1,690 ppm in 1949 to 403 ppm in 1956, but they increased to 756 ppm in 1958. The monthly trend of suspended-sediment discharges and concentrations (fig. 9) is similar to that of streamflow (fig. 4). The highest suspended-sediment discharges and concentrations are generally in April–July (frequently in June), and the lowest are in November–February.
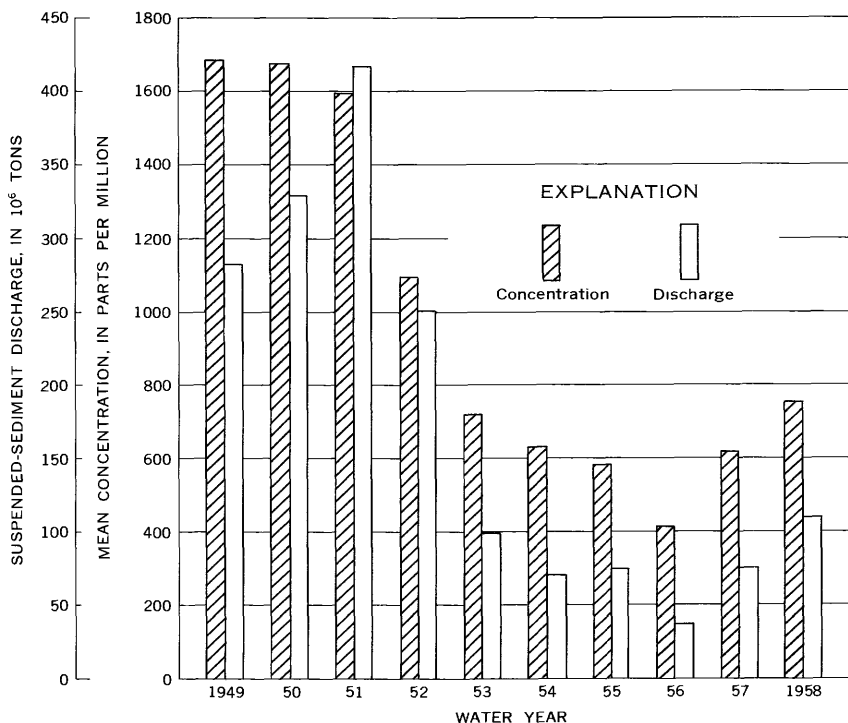
FIGURE 8.—Suspended-sediment concentration and discharge, 1949–58.

TABLE 1.—Monthly and annual summary of streamflow and suspended-sediment discharge, April 1948–September 1958

| Month | Streamflow | | | Suspended sediment | | | | |
| | Cfs-days | Acre-feet | Discharge (tons) | Daily discharge (tons) | | | Concentration (ppm) | |
				Mean	Maximum	Minimum	Weighted mean	Maximum daily
1948								
Apr. 14-30	4,000,000	7,934,000	9,449,000	556,000	1,180,000	284,000	875	[1]1,350
May	6,265,000	12,430,000	19,060,000	615,000	1,240,000	220,000	1,130	[1]1,370
June	5,388,200	10,690,000	34,820,000	1,160,000	3,860,000	170,000	2,390	[1]3,760
July	7,270,000	14,420,000	51,640,000	1,670,000	3,170,000	787,000	2,630	[1]3,380
August	4,422,000	8,771,000	30,040,000	969,000	2,320,000	325,000	2,520	[1]4,090
September	2,371,900	4,705,000	5,631,000	188,000	294,000	82,400	879	[1]1,380
Apr. 14-Sept. 30, 1948	29,717,100	58,950,000	150,600,000	886,000	3,860,000	82,400	1,880	[1]4,090
October	2,240,800	4,445,000	4,064,000	131,000	166,000	89,300	672	[1]896
November	2,641,100	5,239,000	6,549,000	218,000	719,000	114,000	918	[1]2,490
December	1,975,100	3,918,000	1,872,000	60,400	114,000	16,700	351	[1]573
1949								
January	4,590,900	9,106,000	12,770,000	412,000	1,310,000	38,300	1,030	[1]1,370
February	6,492,000	12,880,000	18,330,000	655,000	1,720,000	119,000	1,050	[1]1,710
March	9,933,000	19,700,000	44,480,000	1,430,000	3,480,000	537,000	1,660	[1]2,950
April	9,309,000	18,460,000	52,970,000	1,770,000	3,700,000	399,000	2,110	[1]3,020
May	5,077,000	10,070,000	16,820,000	543,000	1,120,000	252,000	1,230	[1]2,380
June	7,309,000	14,500,000	65,990,000	2,200,000	4,260,000	963,000	3,340	[1]5,110
July	6,024,000	11,950,000	36,540,000	1,180,000	3,120,000	229,000	2,250	[1]4,610
August	3,106,100	6,161,000	5,662,000	183,000	250,000	125,000	675	[1]1,080
September	3,263,500	6,473,000	16,220,000	541,000	2,100,000	98,000	1,840	[1]3,580
Water year 1948-49	61,961,500	122,900,000	282,300,000	773,000	4,260,000	16,700	1,690	[1]5,110
October	3,795,100	7,527,000	12,740,000	411,000	1,690,000	94,200	1,240	[1]2,140
November	2,358,100	4,677,000	3,999,000	133,000	265,000	67,600	628	[1]1,070
December	3,049,600	6,049,000	5,508,000	178,000	885,000	51,300	669	[1]1,390
1950								
January	5,769,000	11,440,000	12,100,000	390,000	1,180,000	146,000	777	[1]1,962
February	4,433,300	3,533,350	7,735,900	276,000	790,000	81,700	643	[1]1,160
March	9,840,000	13,570,000	18,700,000	603,000	2,160,000	149,000	1,010	[1]1,750
April	9,877,000	19,590,000	59,540,000	1,980,000	3,700,000	761,000	2,230	[1]2,930
May	11,422,000	22,660,000	71,730,000	2,310,000	6,420,000	1,070,000	2,330	[1]4,610
June	8,950,000	17,750,000	48,930,000	1,630,000	3,890,000	686,000	2,020	[1]3,010
July	7,088,000	14,060,000	50,740,000	1,640,000	4,890,000	492,000	2,650	[1]4,720
August	5,338,000	10,590,000	29,570,000	954,000	3,030,000	386,000	2,050	[1]4,090
September	3,654,800	7,249,000	8,742,000	291,000	705,000	132,000	886	[1]1,300
Water year 1949-50	72,557,600	144,000,000	330,000,000	904,000	6,420,000	51,300	1,680	[1]4,720

See footnotes at end of table.

TABLE 1.—Monthly and annual summary of streamflow and suspended-sediment discharge, April 1948–September 1958—Continued

| Month | Streamflow | | | Suspended sediment | | | | |
| | | | | Daily discharge (tons) | | | Concentration (ppm) | |
	Cfs-days	Acre-feet	Discharge (tons)	Mean	Maximum	Minimum	Weighted mean	Maximum daily
1950								
October	3,337,400	6,620,000	17,439,000	563,000	2,590,000	125,000	1,940	5,640
November	2,544,900	5,048,000	4,652,000	155,000	238,000	96,000	677	880
December	1,870,400	3,710,000	1,281,300	41,300	87,400	16,200	254	505
1951								
January	2,278,100	4,519,000	1,675,000	54,000	103,000	8,400	272	475
February	4,431,200	8,789,000	15,393,470	550,000	2,560,000	4,340	1,290	2,410
March	7,760,000	15,390,000	25,050,000	808,000	2,910,000	225,000	1,200	2,820
April	13,299,000	26,380,000	69,039,000	2,300,000	4,850,000	999,000	1,920	4,150
May	13,054,000	25,890,000	61,116,000	1,970,000	7,010,000	674,000	1,730	4,820
June	11,171,000	22,160,000	95,871,000	3,200,000	6,810,000	875,000	3,180	6,420
July	20,252,000	40,170,000	56,840,000	1,830,000	2,810,000	1,010,100	1,040	1,720
August	7,449,000	14,770,000	29,642,000	956,000	2,940,000	255,000	1,470	3,170
September	9,186,000	18,220,000	39,249,000	1,310,000	2,060,000	380,000	1,580	2,210
Water year 1950–51	96,633,000	191,700,000	417,247,770	1,140,000	7,010,000	4,340	1,600	6,420
1952								
October	6,199,000	12,300,000	12,327,000	398,000	624,000	222,000	736	1,050
November	7,024,000	13,930,000	15,560,000	519,000	1,390,000	247,000	820	1,570
December	4,355,400	8,639,000	4,513,400	146,000	265,000	46,100	384	581
1952								
January	4,754,000	9,429,000	6,681,900	216,000	308,000	82,900	521	668
February	5,827,000	11,560,000	9,469,000	327,000	644,000	202,000	602	1,020
March	9,764,000	19,370,000	40,173,000	1,300,000	2,710,000	158,000	1,520	2,580
April	14,627,000	29,010,000	69,890,000	2,330,000	3,770,000	576,000	1,770	2,980
May	11,969,000	23,740,000	27,526,000	888,000	1,800,000	480,000	852	2,260
June	6,538,000	12,970,000	26,625,000	888,000	2,920,000	239,000	1,510	3,310
July	6,186,000	12,270,000	25,705,000	829,000	2,010,000	196,000	1,540	3,630
August	5,038,000	9,993,000	7,474,000	241,000	794,000	109,000	549	1,400
September	3,084,800	6,119,000	4,247,600	142,000	426,000	55,400	510	1,210
Water year 1951–52	85,370,000	169,300,000	250,191,900	684,000	3,770,000	46,100	1,090	3,630
October	2,224,000	4,411,000	2,020,500	65,200	89,800	50,000	336	463
November	2,425,200	4,810,000	2,835,300	94,500	176,200	56,200	433	724
December	2,295,400	4,553,000	1,172,800	37,800	91,700	15,100	189	394

1953

January	2,135,800	4,236,000	577,200	18,600	28,700	10,300	100	140
February	3,310,900	6,567,000	3,708,500	132,000	464,000	13,800	415	893
March	5,808,000	11,520,000	11,610,000	375,000	755,000	107,000	740	1,150
April	7,768,000	15,410,000	22,351,000	745,000	2,420,000	236,000	1,070	2,440
May	6,769,000	13,430,000	18,809,000	607,000	1,270,000	212,000	1,030	2,200
June	5,897,000	11,700,000	17,754,000	592,000	1,670,000	197,000	1,120	2,870
July	5,712,000	11,330,000	14,174,700	457,000	1,480,000	93,700	919	2,420
August	4,544,000	9,013,000	3,201,200	103,000	161,000	59,300	261	360
September	2,625,800	5,208,000	1,390,900	46,400	72,900	27,500	196	262
Water year 1952-53	51,515,100	102,200,000	99,605,100	273,000	2,420,000	10,300	716	2,870
October	2,162,000	4,288,000	1,176,500	38,000	47,700	29,100	202	245
November	2,014,800	3,996,000	1,056,000	35,200	43,400	26,700	194	234
December	2,092,500	4,150,000	837,300	27,000	36,900	9,500	147	201

1954

January	1,700,700	3,373,000	351,750	11,300	16,400	6,760	77	105
February	1,674,100	3,321,000	630,980	24,700	95,200	7,210	153	403
March	2,749,000	5,453,000	3,155,100	102,000	512,000	26,500	425	1,290
April	4,360,000	8,648,000	5,371,000	179,000	249,000	113,000	456	688
May	6,538,000	12,970,000	13,929,000	449,000	1,100,000	102,000	789	1,650
June	6,532,000	12,960,000	26,403,000	880,000	1,860,000	138,000	1,500	2,450
July	5,147,000	10,210,000	7,742,200	250,000	1,110,000	47,100	557	1,660
August	3,420,000	6,783,000	6,056,400	195,000	643,000	39,700	656	1,900
September	3,042,300	6,034,000	3,877,500	129,000	653,000	48,300	472	1,520
Water year 1953-54	41,433,300	82,190,000	70,646,730	194,000	1,860,000	6,760	632	2,450
October	5,337,400	10,590,000	9,985,900	322,000	790,000	52,800	693	1,400
November	2,961,900	5,875,000	1,382,100	46,100	128,000	21,000	173	311
December	2,404,100	4,768,000	733,000	23,600	32,400	18,700	113	150

1955

January	3,469,300	6,881,000	3,693,900	119,000	550,000	17,000	394	1,000
February	3,565,500	7,072,000	10,169,600	363,000	2,110,000	13,600	1,060	2,530
March	5,890,000	11,680,000	10,483,000	338,000	550,000	151,000	659	950
April	5,807,000	11,520,000	9,633,000	321,000	910,000	106,000	614	300
May	4,712,700	9,348,000	7,118,200	230,000	610,000	56,000	559	1,100
June	4,847,000	9,614,000	8,744,500	291,000	810,000	98,500	668	1,800
July	3,810,900	7,559,000	9,188,100	296,000	1,010,000	70,100	883	2,400
August	2,537,800	5,034,000	1,616,400	52,100	87,000	31,600	236	340
September	2,213,500	4,390,000	1,592,900	59,400	170,030	32,3?0	295	600
Water year 1954-55	47,557,100	94,300,000	74,530,500	204,000	2,110,000	13,600	580	2,530
October	3,229,400	6,405,000	5,002,400	161,000	850,000	43,700	574	1,400
November	2,003,400	3,974,000	927,800	30,900	50,000	9,500	172	240
December	1,649,000	3,271,000	343,090	11,100	15,000	7,500	77	110

TABLE 1.—*Monthly and annual summary of streamflow and suspended-sediment discharge, April 1948–September 1958—Continued*

Month	Streamflow			Suspended sediment				
				Daily discharge (tons)			Concentration (ppm)	
	Cfs-days	Acre-feet	Discharge (tons)	Mean	Maximum	Minimum	Weighted mean	Maximum daily
1956								
January	1,587,100	3,148,000	258,130	8,330	18,000	4,250	60	120
February	1,716,600	3,405,000	390,670	13,300	31,000	8,600	84	160
March	2,557,100	5,072,000	1,489,300	48,000	114,000	19,900	216	521
April	4,481,000	8,888,000	4,626,500	154,000	224,000	81,500	382	500
May	4,508,000	8,941,000	5,767,000	186,000	550,000	78,000	474	1,050
June	3,400,500	6,745,000	3,997,200	133,000	250,000	73,000	435	645
July	3,700,600	7,340,000	8,430,900	272,000	493,000	70,900	844	1,500
August	3,341,600	6,628,000	8,439,200	143,000	310,000	53,000	492	930
September	2,224,700	4,413,000	1,749,900	58,300	160,000	31,100	291	900
Water year 1955–56	34,399,000	68,230,000	37,422,090	102,000	850,000	4,250	403	1,500
1957								
October	1,822,500	3,615,000	1,251,600	40,400	60,000	30,700	254	360
November	1,750,200	3,471,000	796,900	26,600	41,000	15,000	169	262
December	1,723,100	3,418,000	565,110	18,200	59,000	8,410	121	380
January	1,751,000	3,473,000	596,380	19,200	30,000	6,140	126	190
February	1,931,400	3,831,000	1,072,800	38,300	270,000	15,000	206	847
March	2,830,200	5,614,000	2,446,500	78,900	284,000	21,400	320	720
April	5,652,000	11,210,000	9,712,200	324,000	920,000	62,000	636	1,300
May	7,245,000	14,370,000	13,776,000	444,000	1,020,000	130,000	704	1,230
June	7,095,000	14,070,000	22,378,000	746,000	1,630,000	280,000	1,170	2,000
July	6,856,000	13,600,000	15,456,000	499,000	1,300,000	150,000	835	1,500
August	3,403,700	6,751,000	3,304,000	107,000	280,000	44,000	360	600
September	2,874,800	5,702,000	3,429,200	114,000	200,000	49,000	442	800
Water year 1956–57	44,934,900	89,120,000	74,784,690	205,000	1,630,000	6,140	616	2,000
1958								
October	2,752,800	5,460,000	3,735,000	120,000	397,000	39,000	503	1,040
November	2,702,000	5,359,000	2,350,500	78,400	158,000	48,000	322	600
December	2,903,600	5,759,000	3,154,000	102,000	480,000	23,000	402	1,400
January	2,358,100	4,677,000	1,366,400	44,100	140,000	22,000	215	480
February	2,029,300	4,025,000	1,235,100	44,103	339,000	16,000	225	722
March	5,917,000	11,740,000	16,222,000	523,000	1,420,000	120,000	1,020	1,970
April	4,757,200	9,435,000	6,293,000	210,000	400,000	87,000	490	750
May	4,147,200	8,226,000	8,463,500	273,000	881,000	92,000	756	1,600
June	5,194,200	10,300,000	12,627,000	421,000	1,310,000	130,000	900	1,700
July	9,709,000	19,260,000	30,482,000	983,000	2,600,000	140,000	1,160	2,600
August	6,838,400	13,560,000	13,950,800	450,000	1,520,000	76,000	756	1,300
September	3,647,600	7,235,000	8,246,200	275,000	1,870,000	66,000	837	2,100
Water year 1957–58	52,956,200	105,000,000	108,125,500	296,000	2,600,000	16,000	756	2,600

¹ Maximum measured.
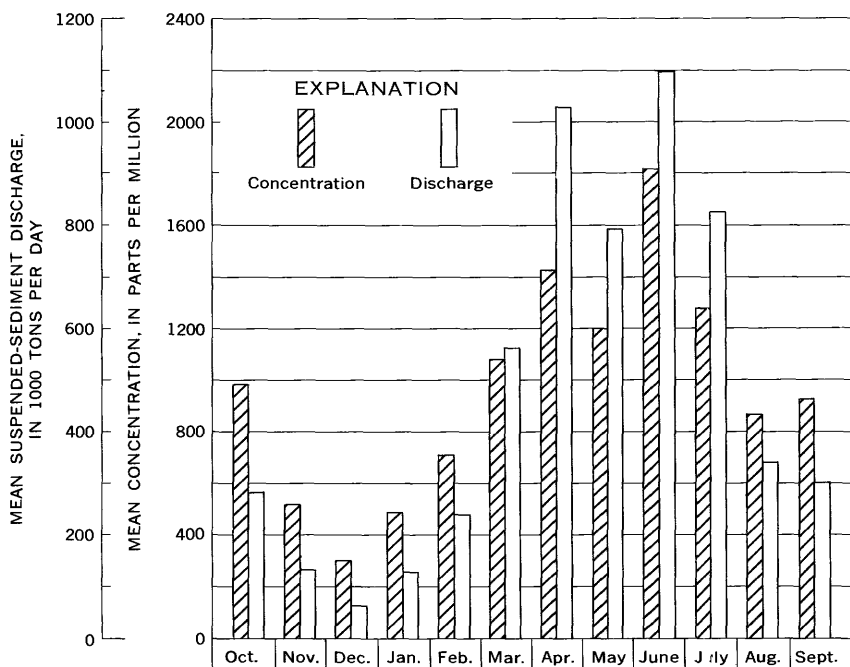
FIGURE 9.—Monthly trend of suspended-sediment concentrations and discharges.

Because of the pronounced decrease in mean concentration from 1949 to 1956, a study was made to determine whether or not the decrease was the result of retention of sediment behind dams that have recently been completed, particularly in the upper Missouri River basin.

The yearly mean concentration is not closely related to the reservoir storage capacity. (See fig. 10.)   The storage capacity was fairly constant from 1948 to the beginning of 1952; however, the mean concentration decreased appreciably.   Also, the storage capacity was much greater in 1954 and 1955 than in previous years, but the mean concentrations were only slightly less.

One reason why the concentration has no apparent close relation to the storage capacity is that any capacity added upstream from existing storage would have little effect on the concentration at St. Louis.   The concentration is expected to be more closely related to the amount of uncontrolled drainage area than to the storage capacity. Figure 10 shows, however, that the concentration is no more closely related to the amount of uncontrolled drainage area than it is to the storage capacity.   The only major change in the uncontrolled drainage area was near the beginning of the 1953 water year, when storage
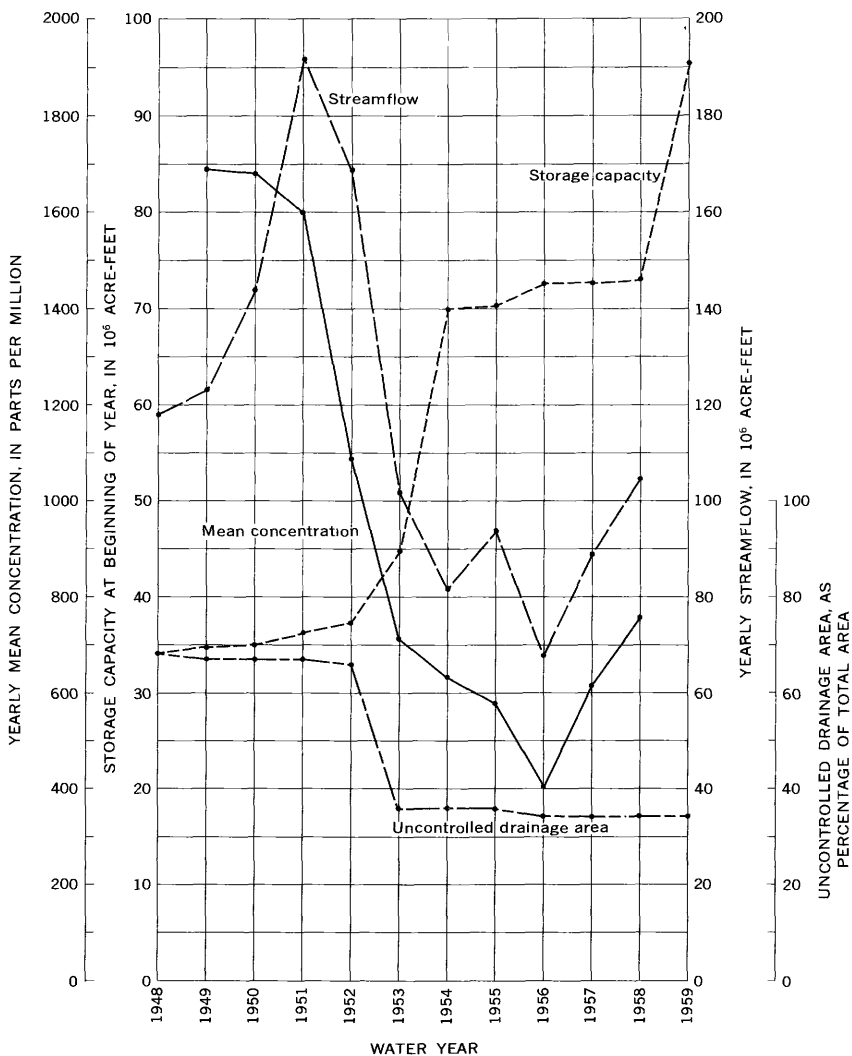
FIGURE 10.—Factors affecting mean concentration.

began at Fort Randall; however, a pronounced decrease in mean concentration took place before 1953.

Except for the first 3 years of sediment record, the mean concentration followed closely the trend of streamflow. (See fig. 10.) Even though the mean concentration is probably affected by the reservoir storage capacity and by the amount of uncontrolled drainage area, the effects are obscured by the close relation of concentration to streamflow. Because the river can pick up material from its bed and banks, the effects of storage on the sediment concentration may not

be apparent as far downstream as St. Louis until many years after the beginning of storage.

A study was made to determine whether or not the relation of suspended-sediment discharge to streamflow had been changed by the upstream storage reservoirs. Measured suspended-sediment discharges were plotted against the corresponding water discharges, and a line was drawn to represent the relation for each year of record. (See fig. 11.) The lines were not intended to be rating curves but were intended to be only lines of general trend.

From 1949 to 1952 the sediment discharge varied with the 2.0–2.4 power of the water discharge. However, from 1953 to 1956 the sediment discharge varied with the 2.9–3.6 power of the water discharge. In 1957 and 1958 the relations were close to those of 1949–52; the sediment discharge varied with the 2.5 and 2.3 powers of the water discharge. The sediment discharges at any given water discharge, however, were generally lower in 1957–58 than in 1949–52. Low water discharges carried less sediment in 1953–56 than in other years, but high water discharges carried more sediment. The results of this study are inconclusive, but they seem to indicate that the trend is toward lower sediment discharges for a given water discharge and, therefore, toward lower yearly mean concentrations.

Suspended sediment from depth-integrated samples representing the entire cross section at St. Louis averaged 47 percent clay, 38 percent silt, and 15 percent sand when analyzed, chemically and mechanically dispersed, in distilled water. The average size distribution of suspended sediment and the smallest and largest percent-finer values that were determined for each size are shown in figure 12.

Logically, particle-size distribution of suspended sed¹ment would be related to discharge, season, temperature, and concentration. The amount of sand would increase as discharge increases because the capacity of the flow to transport sand would increase. The amount of silt and clay would increase when surface runoff laden with silt and clay causes the discharge to increase. Normally, the percentage of silt and clay would be less in the winter than in other seasons because surface runoff would be less. The percentage of sand generally would increase as temperature decreases because the high fluid viscosity at low temperature favors high transport capacities for sand. The percentage of silt and clay generally would increase as concentration increases because the amount of silt and clay is not governed by flow conditions and can increase at a faster rate than the amount of sand. The particle-size distribution must also be governed by the widely different sources of sediment in the large drainage area; however, information for determining the source of the sediment was not available.
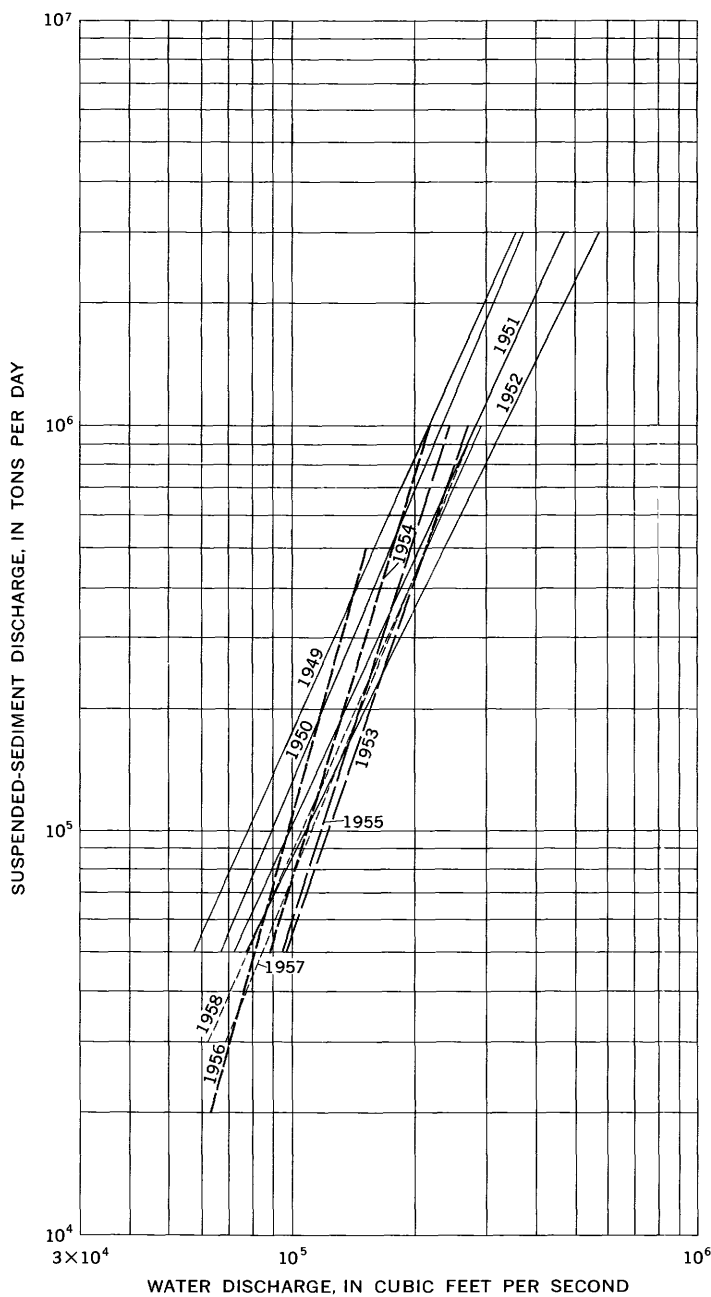
FIGURE 11.—Relations of suspended-sediment discharge to water discharge.
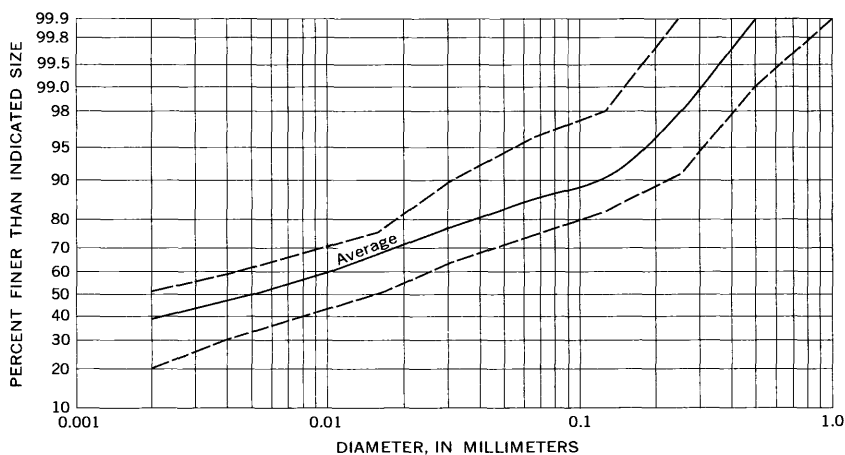
FIGURE 12.—Average and extreme particle-size distributions of suspended sediment.

No consistent relationships were found from graphs in which various particle-size parameters were plotted with discharge, season, temperature, and concentration as independent variables. The expected relationships of particle size to discharge, season, and temperature probably exist but were obscured by the variation of particle size with source area. The expected relationship of particle size to concentration was obscured by the variation of particle size with source area and by the narrow range of variation of concentration.

Because of the lateral change of concentration in the cross section, a lateral change in particle size would be expected. Such a change can be determined only from analyses of samples from individual stations; however, most of the particle-size analyses were for composites rather than for individual samples. From the few analyses of individual samples, no consistent variation in particle size from one side of the river to the other could be detected.

BED MATERIAL

Samples of bed material were obtained with a U.S. BM–48 sampler from May 1951 to March 1958 and with a U.S. BM–54 sampler from April 1958 to July 1959. Both samplers obtain material from the upper 1–1½ inches of the bed. Because of improved design, the BM–54 sampler is less likely than the BM–48 to permit fine material to be washed out as the sample is taken and raised to the bridge. Samples were obtained at 4 points in the cross section from May 1951 to April 1955 and at 10–21 points from May 1955 to July 1959.

Before April 1956, the sieve method was used to analyze all bed-material samples. Since that time, sieve and visual-accumulation-tube methods have been used. Samples were analyzed individually, and the results were averaged to give the particle-size distribution for the entire cross section.

The size distribution of bed material is highly variable with location in the cross section. Near the banks, the material is mostly fine sand and some silt. Near the piers, the material is very coarse; some of the particles are large enough to be retained on a 32-mm sieve. In the main flow, most of the material is between 0.125 and 1.000 mm in diameter.

The size distribution of bed material is also highly variable with time. (See fig. 13.) The data for four sampling points are shown for 1955–59 in figure 13 to indicate the probable reliability of the 1951–55 data. The data for 4 sampling points have more variation than those for 10–21 sampling points and are not completely reliable for individual days; however, they show the trend of bed-material sizes rather well. Figure 13 indicates that the median diameters were
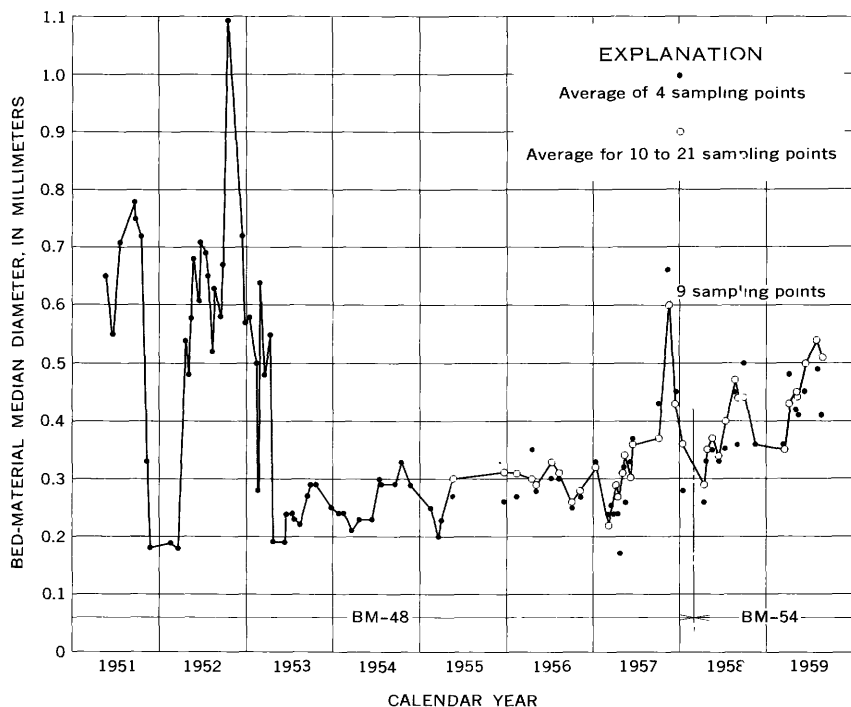
FIGURE 13.—Variation of bed-material size with time.

larger in 1951 and 1952 than in 1953–56 and were generally increasing in 1957–59.

A general relation between median diameter and discharge is suggested by the fact that the discharge was higher in 1951 and 1952 than in 1953–56 and was intermediate in 1957–59. A study was made to determine whether or not the particle size is related to the discharge for short periods. Average median diameters were plotted against mean discharge for 1-day to 2-year periods, and 1 year was the shortest period for which a good relation was found. The relation for 2-year periods is slightly better than the relation for 1-year periods (fig. 14). The absence of short-term relations of median diameter to discharge suggests that the bed-material size is influenced more by the depth of scour than by the selective removal or deposition of fine and medium sands from the upper few millimeters of the bed during short periods of high or low flow. The relations between bed-material size and bed elevation are discussed in the section, "Aggradation and degradation."

For many sets of bed-material samples, a measure of the uniformity of the material was computed. This measure is called the geometric quartile deviation and is computed as $\sqrt{d_{75}/d_{25}}$, where d_{75} and d_{25} are the particle sizes at which either 75 or 25 percent of the bed material by weight is finer. Material that is perfectly uniform in size has a geometric quartile deviation of 1.00, and material that has a wide range of size has a large geometric quartile deviation. Because only the middle 50 percent of the material is considered in computing
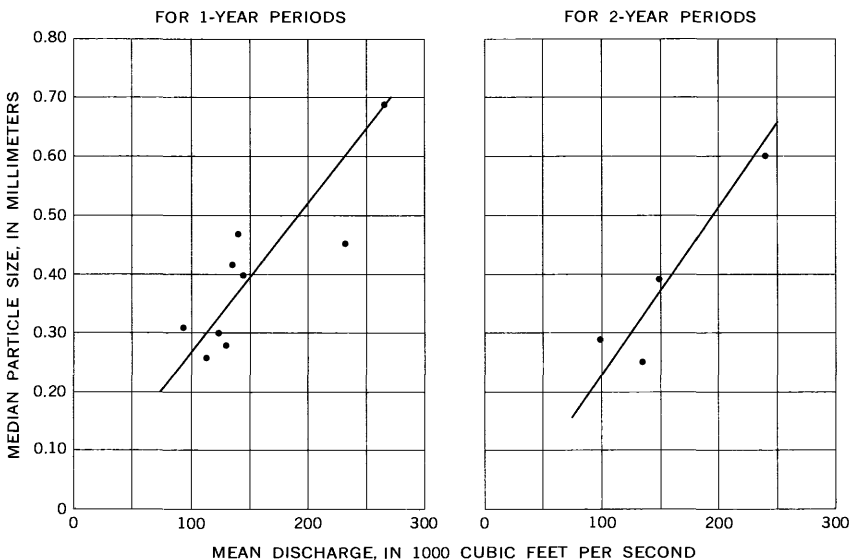
FIGURE 14.—Relation of bed-material size to discharge.

this measure, however, values of 1.00 do not necessarily indicate that all the material is of uniform size. Also, two particle-size distributions that have the same geometric quartile deviation do not necessarily have the same extreme range of sizes. Because this measure is computed from the ratio of two sizes rather than from the difference, the geometric quartile deviations for different samples can be compared without making corrections for different average particle sizes.

The geometric quartile deviations for the bed material in the Mississippi River at St. Louis ranged from 1.1 to 2.5 and averaged 1.5. These values are intermediate in relation to those for some other streams for which bed material has been measured. Bed material of the Colorado River near Blythe, Calif., (median diameter about 0.33 mm) and of the Rio Grande near Bernallilo, N. Mex., (median diameter about 0.30 mm) has geometric quartile deviations of about 1.3 and 1.4, respectively. Bed material of the Kansas River at Wamego, Kans., (median diameter about 0.61 mm) has a geometric quartile deviation of about 2.1.

AGGRADATION AND DEGRADATION

Discharge measurements made about once each week, and sometimes more often, include data on depth and water-surface elevation that provide information on aggradation and degradation at the MacArthur Bridge. A study was made of the factors that affect the streambed elevation. Mean bed elevations were computed for the part of the section between stations 275 and 1,800, so that a constant width could be used and so that the bed elevation would not be affected by the sloping banks. The range of mean bed elevations was from 355.1 to 364.4 feet above mean sea level during the period of sediment record.

The mean bed elevation is not significantly related to the instantaneous discharge, mean velocity, shear velocity, mean depth, nor suspended-sand concentration. The bed elevation and the median diameter of the bed material are fairly closely related (fig. 15). The relation indicates that the particle size is partly dependent on the depth of scour or that the depth of scour and the particle size are mutually dependent on the same causes. If the bed elevation is regarded as the dependent variable, it can be computed from the regression equation

$$h_b = 363.0 - 7.8\, d_{50}$$

where

    h_b is mean bed elevation, in feet above mean sea level
    d_{50} is median diameter of bed material, in millimeters

The standard error of estimate for this equation is 0.91 foot. Multiple

regressions with discharge, mean velocity, shear velocity, water-surface slope, and suspended-sand concentration did not result in significantly better relationships than d_{50} alone.

Failure of the bed elevation to show a significant relation to instantaneous values except for d_{50} suggests that the effect of such variables as discharge, mean velocity, water-surface slope, and suspended-sand concentration is cumulative over periods such as a week or a month. The cumulative effect of discharge on the bed elevation was studied by plotting the mean bed elevations for various periods against the discharges for various other periods that preceded or included the periods for which the mean bed elevations were determined. A relation was found between monthly mean bed elevation and the discharge for a 3-month period (fig. 16), but it changed from year to year.

FLOW RESISTANCE AND SEDIMENT TRANSPORTATION
RESISTANCE TO FLOW

Resistance to flow in sand-bed streams is governed mostly by the size and shape of the irregularities on the bed rather than by the
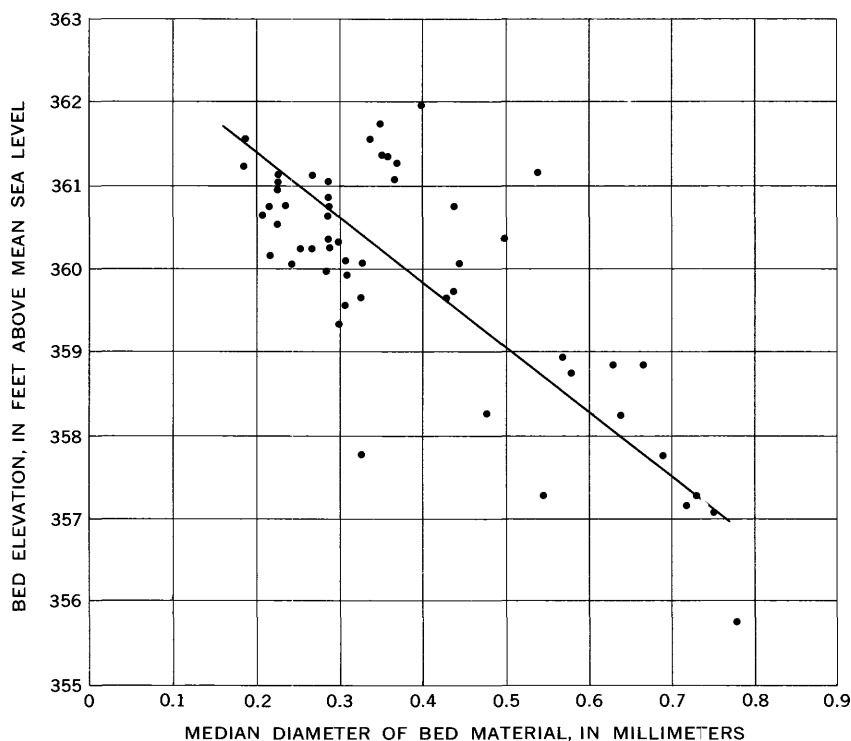
FIGURE 15.—Relation between bed elevation and particle size.
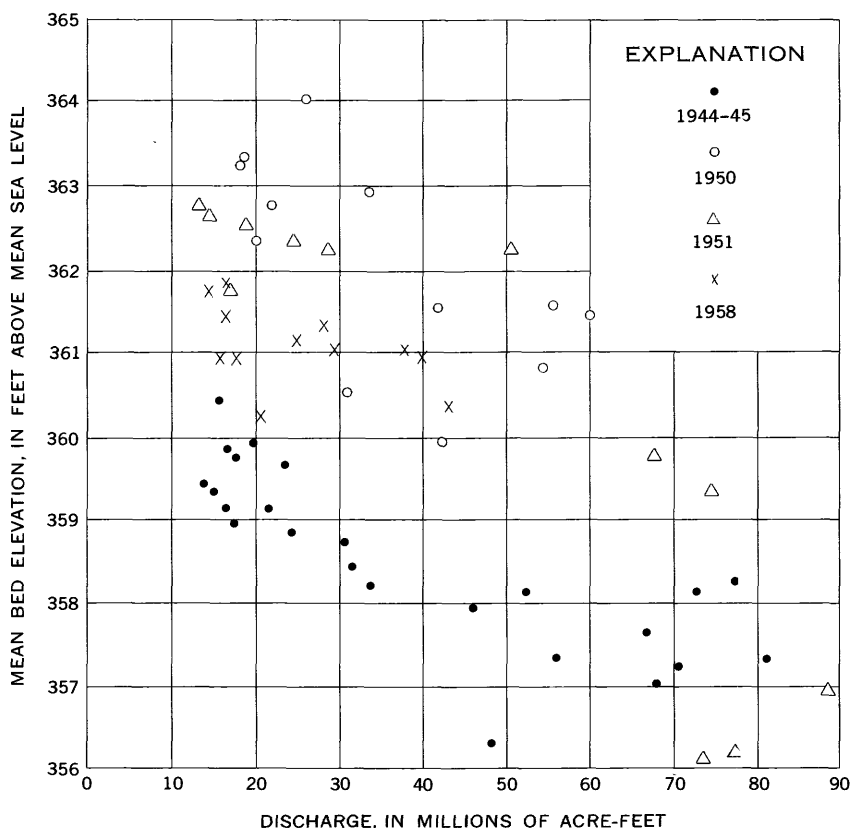
FIGURE 16.—Relation of monthly mean bed elevation to the discharge for 3-month periods. The mean bed elevation for each month has been plotted against the sum of the discharges for that month and the two preceding months.

grain size of the particles in the bed. Small rivers typically have dunes on the bed at low flow, but the dunes are washed out and the bed is fairly flat and regular at high flow. Measured by Manning's n, the resistance when dunes are on the bed is as much as twice the resistance when the bed is flat and regular. In shallow rivers the height of dunes is as much as half the depth of flow. The resistance to flow, however, is governed by the spacing of the dunes as well as by their height. One difficulty in the field study of resistance to flow in nonuniform cross sections is that the bed may be flat and regular in one part of the section but may have dunes in another part; thus, a resistance coefficient for the whole section will represent neither dunes nor a flat bed. Several methods of predicting the resistance to flow in alluvial channels have been proposed. Most of these methods have been based on experiments in laboratory flumes and on a small amount of data from canals and natural streams, most of them small.

Among the few observations of bed configuration in large rivers are those of the lower Mississippi River in Louisiana (Carey and Keller, 1957). Carey and Keller concluded that the large-scale irregularities of the bed vary systematically with changes in discharge and constitute a major element of resistance to flow.

Although the data for the Mississippi River at St. Louis were not obtained primarily for the purpose of research, they can be used to supplement the data for flumes, small rivers, and the lower Mississippi River in the study of resistance to flow. Particularly useful are the data that permit comparison of resistance coefficients and the height and spacing of dunes with data for flumes and small rivers.

Pertinent measured and computed data from nearly concurrent measurements of discharge and sediment are shown in table 2. Hydraulic and sediment characteristics generally remain fairly constant for periods of a few days; therefore, data were considered to be concurrent if the time between the discharge measurement and the bed-material or suspended-sediment measurement was not more than 2 days.

The water-surface slopes in table 2 were determined from gage readings made at 8:00 a.m. each day at gages 4.4 miles upstream and 2.1 miles downstream from MacArthur Bridge. Readings at a gage near MacArthur Bridge were also used in determining the uniformity of slopes in the reaches upstream and downstream from the bridge. Gage readings were furnished by the U.S. Army Corps of Engineers, St. Louis District. During 1950–53, the slopes changed widely from time to time, but the slopes upstream and downstream from the bridge were generally about the same at any given time. The slopes in the upstream reach tended to be slightly steeper than the slopes in the downstream reach. During 1954–59, the slopes were fairly constant, but those in the upstream reach were consistently steeper than those in the downstream reach; in 1956–57, the slopes upstream were about twice as steep as those downstream.

The widths shown in table 2 are the widths of the water surface excluding the two piers, each 27 feet wide. The hydraulic radius was computed by dividing the cross-sectional area, which excluded the area of the piers, by a wetted perimeter computed as the width plus twice the mean depth. The shear velocity was computed as $\sqrt{gRS}$, where g is the acceleration due to gravity, R is the hydraulic radius, and S is the slope of the water surface for the 6.5-mile reach and is assumed to be equal to the energy gradient for most conditions.

In July 1951 the flow was over the banks, and the width increased from about 1,780 to 2,800 feet. Inclusion of 1,000 feet of shallow, slow-moving, overbank flow in the computations would have produced highly nontypical values of mean depth and hydraulic radius.

TABLE 2.—*Summary of hydraulic and*

Date	Hydraulic data							
	Water-surface slope	Discharge (cfs)	Width (ft)	Mean depth (ft)	Hy-draulic radius (ft)	Mean velocity (fps)	Shear velocity (fps)	n
1950								
May 8 _____	0.0000997	330,000	1,681	37.3	35.7	5.26	0.338	0.030
May 22 _____	.0000924	348,000	1,690	37.2	35.6	5.54	.325	.028
June 5 _____	.0000909	327,000	1,692	37.3	35.7	5.18	.323	.030
Aug. 7 _____	.0000428	147,000	1,606	28.0	27.1	3.27	_____	_____
Sept. 5 _____	.0000399	149,000	1,606	28.4	27.4	3.27	_____	_____
Sept. 25 _____	.0000399	128,000	1,594	24.7	23.9	3.26	_____	_____
1951								
Apr. 17 _____	.000106	521,000	1,711	41.3	39.4	7.37	.336	.024
May 21 _____	.0000982	362,000	1,692	40.4	38.6	5.29	.319	.032
June 18 _____	.0000822	322,000	1,683	39.1	37.4	4.89	.315	.031
July 16 _____	.000126	704,000	I 1,770	I 54.8	I 51.6	I 7.26	I .458	I .032
July 22 _____	.000139	763,000	I 1,780	I 56.7	I 53.4	I 7.53	I .439	I .033
July 30 _____	.0000851	468,000	1,746	49.9	47.2	5.37	.330	.033
Sept. 17 _____	.0000632	388,000	1,716	45.0	42.7	5.03	.295	.029
Sept. 24 _____	.0000530	285,000	1,671	39.4	37.6	4.33	_____	_____
Oct. 8 _____	.0000341	208,000	1,647	34.5	33.2	3.66	_____	_____
Oct. 15 _____	.0000355	182,000	1,641	33.0	31.8	3.36	_____	_____
Nov. 13 _____	.0000312	219,000	1,646	33.5	32.1	3.97	_____	_____
Nov. 26 _____	.0000400	233,000	1,651	34.6	33.2	4.08	_____	_____
Dec. 3 _____	.0000370	190,000	1,631	31.0	29.9	3.75	_____	_____
1952								
Feb. 18 _____	.0000544	195,000	1,648	27.9	27.0	4.24	_____	_____
Apr. 28 _____	.000116	670,000	1,834	46.5	44.2	7.86	.496	.026
Apr. 30 _____	.000110	681,000	1,888	46.9	44.7	7.69	.398	.026
June 16 _____	.0000545	198,000	1,648	33.1	31.8	3.63	_____	_____
July 7 _____	.0000502	204,000	1,664	33.5	32.2	3.66	_____	_____
Aug. 19 _____	.0000428	167,000	1,633	30.3	29.2	3.37	_____	_____
Sept. 29 _____	.0000428	74,700	1,558	21.4	20.8	2.24	_____	_____
Oct. 20 _____	.0000355	78,700	1,552	21.6	21.0	2.35	_____	_____
Dec. 8 _____	.0000487	67,500	1,546	20.4	19.9	2.14	_____	_____
Dec. 29 _____	.0000516	67,500	1,549	20.3	19.7	2.15	_____	_____
1953								
Jan. 13 _____	.0000574	62,000	1,544	19.7	19.2	2.04	_____	_____
Feb. 11 _____	.0000530	129,000	1,596	27.1	26.2	2.98	_____	_____
Feb. 16 _____	.0000603	117,000	1,588	25.4	24.6	2.90	.219	.034
Feb. 24 _____	.0000545	204,000	1,646	31.8	30.5	3.90	_____	_____
Mar. 16 _____	.0000487	210,000	1,642	32.3	31.1	3.96	_____	_____
Apr. 6 _____	.0000603	321,000	1,700	40.2	38.4	4.69	.273	.028
Apr. 27 _____	.0000589	241,000	1,654	33.0	31.7	4.42	_____	_____
May 25 _____	.0000603	179,000	1,617	28.1	27.1	3.94	.229	.026
June 8 _____	.0000632	172,000	1,608	27.4	26.5	3.90	.232	.027
July 13 _____	.0000632	197,000	1,635	30.1	29.0	4.00	.243	.028
July 20 _____	.0000603	157,000	1,594	27.9	26.9	3.54	.229	.029
Aug. 10 _____	.0000661	159,000	1,595	27.6	26.6	3.61	.238	.030
Sept. 14 _____	.0000559	80,700	1,546	20.2	19.7	2.59	_____	_____
Sept. 28 _____	.0000516	67,700	1,530	18.4	18.0	2.40	_____	_____
Oct. 19 _____	.0000516	66,600	1,524	18.2	17.8	2.40	_____	_____
Dec. 28 _____	.0000661	61,800	1,523	17.7	17.3	2.30	.192	.035
1954								
Jan. 28 _____	.0000749	45,900	1,487	15.3	15.0	2.01	.190	.039
Feb. 8 _____	.0000807	51,000	1,491	16.1	15.8	2.12	.203	.040
Mar. 22 _____	.0000720	80,500	1,545	19.8	19.3	2.63	.211	.035
Apr. 19 _____	.0000720	138,000	1,581	25.5	24.7	3.42	.239	.031
Apr. 27 _____	.0000720	163,000	1,595	27.5	26.6	3.72	.248	.030
June 7 _____	.0000691	283,000	1,668	34.2	32.9	4.96	.271	.025
July 12 _____	.0000720	173,000	1,597	28.9	27.9	3.74	.254	.031
July 26 _____	.0000603	115,000	1,551	24.1	23.3	3.07	.213	.031
Sept. 21 _____	.0000647	100,000	1,567	22.8	22.1	2.80	.214	.034
Oct. 18 _____	.0000734	214,000	1,622	31.3	30.1	4.22	.267	.029
Dec. 1 _____	.0000720	75,800	1,527	19.7	19.2	2.52	.211	.036

See footnotes at end of table.

sediment data from concurrent measurements

Bed material			Suspended sediment			Water temperature (°F)
Date	d_{50} (mm)	$\sqrt{d_{75}/d_{25}}$	Date	Sand concentration (ppm)	Sand discharge (tons per day)	
1950			*1950*			
------------	------------	------------	May 8 ------------	185	137, 000	58
------------	------------	------------	May 23 ------------	205	186, 000	69
------------	------------	------------	June 7 ------------	234	176, 000	71
------------	------------	------------	Aug. 8 ------------	80	26, 300	76
------------	------------	------------	Sept. 7 ------------	111	37, 000	71
------------	------------	------------	Sept. 26 ------------	89	23, 900	67
1951			*1951*			
------------	------------	------------	Apr. 18 ------------	342	469, 000	47
May 23 ------------	0. 65	1. 58	May 23 ------------	206	199, 000	71
June 19 ------------	. 55	1. 52	------------			74
July 16 ------------	. 71	1. 36	July 16 ------------	175	332, 000	77
------------	------------	------------	July 22 ------------	181	375, 000	81
------------	------------	------------	July 30 ------------	102	133, 000	82
Sept. 19 ------------	. 78	1. 59	Sept. 17 ------------	214	224, 000	70
Sept. 26 ------------	. 75	1. 48	Sept. 26 ------------	78	50, 500	68
------------	------------	------------	Oct. 10 ------------	120	75, 800	[2] 62
Oct. 17 ------------	. 72	1. 86	Oct. 17 ------------	84	38, 600	63
Nov. 13 ------------	. 33	2. 12	Nov. 13 ------------	155	91, 200	47
Nov. 28 ------------	. 18	1. 18	------------			40
------------	------------	------------	Dec. 4 ------------	182	99, 300	43
1952			*1952*			
Feb. 20 ------------	. 19	1. 17	Feb. 20 ------------	221	131, 000	39
Apr. 26 ------------	. 54	1. 82	------------			58
May 2 ------------	. 48	1. 58	May 1 ------------	304	553, 000	65
June 16 ------------	. 61	1. 51	------------			81
July 9 ------------	. 69	1. 85	July 9 ------------	43	24, 300	80
Aug. 19 ------------	. 63	1. 41	Aug. 19 ------------	26	11, 700	79
Sept. 29 ------------	. 67	2. 50	Sept. 29 ------------	6	1, 240	70
------------	------------	------------	Oct. 22 ------------	12	2, 360	52
Dec. 9 ------------	. 68	1. 93	------------			43
Dec. 31 ------------	. 57	2. 09	Dec. 30 ------------	20	3, 600	35
1953			*1953*			
Jan. 14 ------------	. 58	1. 84	Jan. 14 ------------	13	2, 240	41
Feb. 12 ------------	. 50	1. 86	------------			39
Feb. 18 ------------	. 28	1. 84	------------			38
Feb. 26 ------------	. 64	1. 75	Feb. 26 ------------	81	40, 700	39
Mar. 18 ------------	. 48	2. 04	Mar. 18 ------------	100	59, 700	48
Apr. 8 ------------	. 55	2. 16	Apr. 8 ------------	123	92, 300	50
Apr. 27 ------------	. 19	1. 12	------------			54
------------	------------	------------	May 26 ------------	82	39, 400	73
June 10 ------------	. 19	1. 70	June 10 ------------	90	42, 800	80
July 14 ------------	. 24	1. 76	July 14 ------------	64	32, 500	80
July 22 ------------	. 23	1. 43	July 22 ------------	61	26, 400	84
Aug. 10 ------------	. 22	1. 23	Aug. 10 ------------	51	21, 900	79
Sept. 16 ------------	. 27	1. 51	Sept. 15 ------------	13	3, 080	71
Sept. 30 ------------	. 29	1. 78	Sept. 30 ------------	8	1, 530	70
Oct. 20 ------------	. 29	1. 68	Oct. 20 ------------	11	1, 980	65
Dec. 30 ------------	. 25	1. 91	------------			33
1954			*1954*			
Jan. 29 ------------	. 24	1. 60	------------			34
Feb. 8 ------------	. 24	1. 55	Feb. 10 ------------	11	1, 590	38
Mar. 22 ------------	. 21	1. 70	Mar. 22 ------------	32	6, 960	42
Apr. 19 ------------	. 23	1. 30	Apr. 19 ------------	73	27, 000	61
------------	------------	------------	Apr. 27 ------------	101	44, 700	65
June 8 ------------	. 23	1. 57	June 7 ------------	148	115, 000	67
July 12 ------------	. 30	1. 92	------------			81
July 26 ------------	. 29	1. 73	July 26 ------------	17	5, 280	80
Sept. 21 ------------	. 29	1. 81	Sept. 21 ------------	26	6, 870	70
Oct. 19 ------------	. 33	1. 90	Oct. 18 ------------	102	58, 400	57
Nov. 29 ------------	. 29	1. 61	Nov. 29 ------------	26	5, 480	42

TABLE 2.—*Summary of hydraulic and*

Date	Hydraulic data							
	Water-surface slope	Discharge (cfs)	Width (ft)	Mean depth (ft)	Hydraulic radius (ft)	Mean velocity (fps)	Shear velocity (fps)	n
1955								
Feb. 24	.0000720	291,000	1,672	34.0	32.7	5.11	.275	.025
Mar. 30	.0000793	163,000	1,592	26.8	25.9	3.82	.257	.030
May 24	.0000691	98,500	1,540	22.8	22.1	2.81	.222	.035
Dec. 27	.0000822	59,800	1,508	17.0	16.7	2.33	.210	.038
1956								
Feb. 21	.0000822	56,100	1,510	16.9	16.5	2.20	.209	.040
Mar. 27	.0000763	82,400	1,548	19.8	19.3	2.68	.218	.035
Apr. 24	.0000836	172,000	1,630	28.6	27.6	3.69	.272	.034
May 9	.0000836	149,000	1,612	27.7	26.8	3.34	.238	.036
July 9	.0000749	155,000	1,616	27.5	26.6	3.49	.253	.033
Aug. 6	.0000822	105,000	1,566	22.5	21.8	2.98	.240	.035
Oct. 2	.0000938	59,400	1,504	17.2	16.8	2.30	.225	.041
Nov. 5	.0000938	55,200	1,510	15.8	15.5	2.31	.216	.039
1957								
Jan. 21	.0000982	49,200	1,488	15.2	14.9	2.18	.217	.041
Mar. 20	.0000932	69,600	1,520	17.6	17.2	2.61	.227	.037
Mar. 26	.0000874	136,000	1,588	24.5	23.8	3.50	.259	.033
Apr. 9	.0000903	259,000	1,692	32.2	31.0	4.76	.300	.029
Apr. 22	.0000903	193,000	1,638	28.9	27.9	4.08	.285	.032
May 7	.0000845	172,000	1,632	28.1	27.1	3.76	.272	.033
May 20	.0000991	308,000	1,658	36.9	35.3	5.03	.336	.032
June 3	.0000962	219,000	1,678	32.2	31.0	4.05	.310	.036
June 25	.0000932	220,000	1,644	33.0	31.7	4.06	.318	.035
Oct. 7	.0000816	74,100	1,524	18.5	18.1	2.63	.218	.035
1958								
Apr. 14	.0000845	172,000	1,622	27.9	26.9	3.81	.270	.032
Apr. 29	.0000816	135,000	1,596	24.8	24.1	3.41	.252	.033
May 19	.0000787	109,000	1,568	22.1	21.5	3.15	.233	.032
June 17	.0000932	310,000	1,676	35.0	33.6	5.29	.317	.028
July 8	.0000991	235,000	1,638	31.1	29.9	4.62	.309	.031
July 25	.000105	496,000	1,714	44.4	42.2	6.48	.378	.028
Sept. 8	.0000728	106,000	1,562	22.6	22.0	3.00	.227	.033
Sept. 29	.0000583	126,000	1,576	23.4	22.7	3.41		
Nov. 18	.0000787	104,000	1,550	21.1	20.5	3.18	.228	.031
1959								
Mar. 23	.0000991	228,000	1,656	30.6	29.5	4.51	.307	.031
Apr. 6	.000111	301,000	1,710	35.8	34.3	4.92	.350	.034
May 4	.0000874	180,000	1,606	29.5	28.5	3.80	.293	.034
May 13	.0000903	241,000	1,668	32.4	31.2	4.45	.301	.031
June 15	.0000699	141,000	1,576	26.3	25.5	3.40	.240	.032
July 1	.0000612	124,000	1,570	24.5	23.7	3.23	.216	.030

[1] Excluding overbank flow.   [2] Estimated.

Therefore, these values were computed for only the part of the flow that was between the banks.

The variation of water-surface slope with river stage and with time indicates that the water-surface slope sometimes diverged radically from the energy gradient. The divergence between energy gradient and water-surface slope may have been caused by the peculiar conditions of the cross section about 3–4 miles downstream from MacArthur Bridge. During medium discharge, a narrow part of this section

sediment data from concurrent measurements—Continued

Date	Bed material		Date	Suspended sediment		Water tempera-ture (°F)
	d_{50} (mm)	$\sqrt{d_{75}/d_{25}}$		Sand concentration (ppm)	Sand discharge (tons per day)	
1955			*1955*			
Feb. 25	.25	1.57	Feb. 25	220	163,000	35
Apr. 1	.23	1.24	Mar. 31	124	51,900	43
May 24	.30	1.54	May 24	15	4,130	73
Dec. 28	.31	1.46	Dec. 27	22	3,410	36
1956			*1956*			
Feb. 22	.31	1.44	Feb. 21	14	2,120	36
			Mar. 28	79	18,900	49
Apr. 25	.30	1.40	Apr. 24	99	45,400	49
May 10	.29	1.32	May 9	72	30,900	61
July 10	.33	1.35	July 9	64	27,000	77
Aug. 6	.31	1.36	Aug. 6	37	10,500	84
Oct. 2	.26	1.36	Oct. 2	19	3,050	69
Nov. 6	.28	1.46	Nov. 5	18	2,720	58
1957			*1957*			
Jan. 21	.32	1.60				38
Mar. 20	.24	1.42				48
Mar. 26	.23	1.34	Mar. 26	176	64,400	47
Apr. 9	.22	1.37	Apr. 9	307	215,000	49
Apr. 23	.24	1.30				60
May 8	.29	1.40	May 7	63	29,300	66
May 21	.27	1.41	May 21	159	128,000	65
June 5	.31	1.35	June 5	110	63,800	72
June 27	.34	1.40				[2] 78
Oct. 7	.37	1.55	Oct. 7	40	7,950	62
1958			*1958*			
Apr. 14	.29	1.39	Apr. 14	102	47,300	47
Apr. 29	.35	1.41	Apr. 29	96	34,800	56
May 19	.37	1.51	May 19	56	16,400	72
June 17	.34	1.62	June 17	198	166,000	73
July 10	.40	1.49	July 8	116	74,200	78
			July 25	134	180,000	78
Sept. 9	.44	1.57	Sept. 9	54	18,800	78
Oct. 1	.44	1.53	Oct. 1	42	11.800	65
Nov. 20	.36	1.72	Nov. 20	65	21,000	55
1959			*1959*			
Mar. 24	.34	1.43	Mar. 25	212	139,000	47
Apr. 8	.42	1.48	Apr. 8	86	67,000	51
May 6	.46	1.40	May 6	63	24,600	70
May 13	.44	1.48	May 13	94	60,600	67
June 15	.50	1.43	June 15	58	22,100	76
July 2	.54	1.39	July 2	37	13,100	80

near the west bank is fairly deep and the rest of the section is wide and shallow. During very low discharge, the entire flow is near the west bank and a sand bar is exposed in the eastern part of the section. At high discharges, the flow can spread out over the wide section; and, as the discharge increases, the stage does not increase as much as it does at the narrower sections upstream. Therefore, the water-surface slope is greater at high discharges than at medium discharges. The water-surface slope was extremely low at some times of low and

medium discharges, such as in August–November 1951. Normally, the relation of the elevations of the bed at this section and at sections up-stream is such that the low and medium discharges pass witl out any noticeable effect on the water-surface slope. But the upstream sections, such as at MacArthur Bridge, were scoured out during the high flows of 1951 and 1952; and the deep part of the downstream section may have been partly filled in by sediment. After the high flows had passed, the elevation of the bed at the downstream section probably was higher than the elevation of the bed upstream; thus, the water-surface slope was less than normal. Water-surface slopes are normally high when discharges are high, regardless of scour or fill; however, water-surface slopes were extremely low only after up-stream sections had been scoured and sediment probably l ad been deposited in the deep part of the downstream section.

Data for computation of backwater curves had not been obtained when the water-surface slopes were extremely low, so sample computations were made to determine if the low water-surface slopes could reasonably have been caused by unusual nonuniformity of the channel. On November 13, 1951, the average slope in the 6.5-mile reach was 0.0000312, and the mean depth and mean velocity at MacArthur Bridge were 33.5 feet and 3.97 feet per second, respectively. A back-water computation was made for the reach between MacArthur Bridge and the gage 2.1 miles downstream; gradually varied flow was assumed. Solution of the Bernoulli energy equation and the Manning velocity equation by trial showed that a water-surface slope of 0.0000312 could have resulted if the mean velocity 2.1 miles down-stream was 2.9 feet per second and if the average n was 0.035. Inspection of the water-surface slopes indicates that the normal slope is about 0.00009. Because shear velocity and n vary with the square root of the energy gradient, errors in the energy gradient will not cause very large errors in shear velocity nor in n. Therefore, the water-surface slope was assumed to be about equal to the energy gradient when it was not greatly different from 0.00009; but shear velocity and n are not shown in table 2 unless the water-surface slope is greater than 0.00006.

Data obtained with a sonic fathometer by the Corps of Engineers, St. Louis District, show that the normal dune height in the vicinity of MacArthur Bridge is from 2 to 8 feet and that the average dune length is about 250 feet. On April 30, 1952, when the resistance to flow as measured by Manning's n was relatively low (0.026), dunes in the east side of the channel were from 2 to 7 feet high but averaged about 900 feet in length; in the central and western parts of the chan-nel, although the depths were not uniform, the changes were very gradual, and only a few isolated dunes were located.

The range of variation of n from 0.024 to 0.041 indicates that the bed at some times becomes fairly flat and regular. The fact that the minimum values of n exceed the minimum for shallow streams, however, may indicate that the bed never becomes flat across the entire section but that dunes remain in parts of the section even when the discharge and mean velocity are very high.

The relations of n to discharge, mean velocity, and shear velocity are shown in figure 17. The absence of a discontinuity in the relations of n to discharge and mean velocity indicates that the fow resistance of dunes is variable and that, at high flows, part of the section has dunes and part has a relatively flat bed. The facts that n is not closely related to the shear velocity and that the range of shear velocities is not as great as the range of mean velocities are to be expected if the bed sometimes has dunes and sometimes is flat. If dunes are on the bed, the resistance to flow will be great and the velocity will be fairly low; thus, the flow will be fairly deep. If the discharge increases and the bed becomes flat, the resistance will decrease and the velocity will increase. Because of the increased velocity, the increased discharge can be carried at the same depth as before; and if the slopes are about the same, the shear velocities will be about the same. Therefore, the range of shear velocities will be small, and roughness coefficients will vary widely for a given shear velocity.

A method for computing mean velocity from Keulegan's equations was presented by Einstein (1950). In this method the shear velocity is divided into two parts: one part pertains to the shear transmitted to the boundary along the roughness of the grainy sand surface (u_*'), and the other part pertains to the shear transmitted to the boundary in the form of normal pressures at the different sides of the bed irregularities such as ripples and dunes (u_*''). The accuracy of the method depends on the accuracy of the relation of the ratio $\bar{u}/u_*''$ to the shear function Ψ' (Einstein, 1950, fig. 5).

For proper application of Einstein's method, the computations should be made for the average of several cross sections in a reach; however, data were available only for the section at MacArthur Bridge. Because the 6.5-mile reach for which the slopes were measured is rather uniform, the bridge section probably represents the reach about as well as the average of several cross sections would. Therefore, the functions $\bar{u}/u_*''$ and Ψ' were computed from representative data at MacArthur Bridge, and their relation is shown in figure 18. The Mississippi River data confirm Einstein's relation very well except for low values of Ψ'. The lack of agreement at low values of Ψ' can be attributed partly to the fact that at high flows the water-surface slope used in the computations for the Mississippi River is greater than the energy slope and partly to the fact that dunes exist in parts
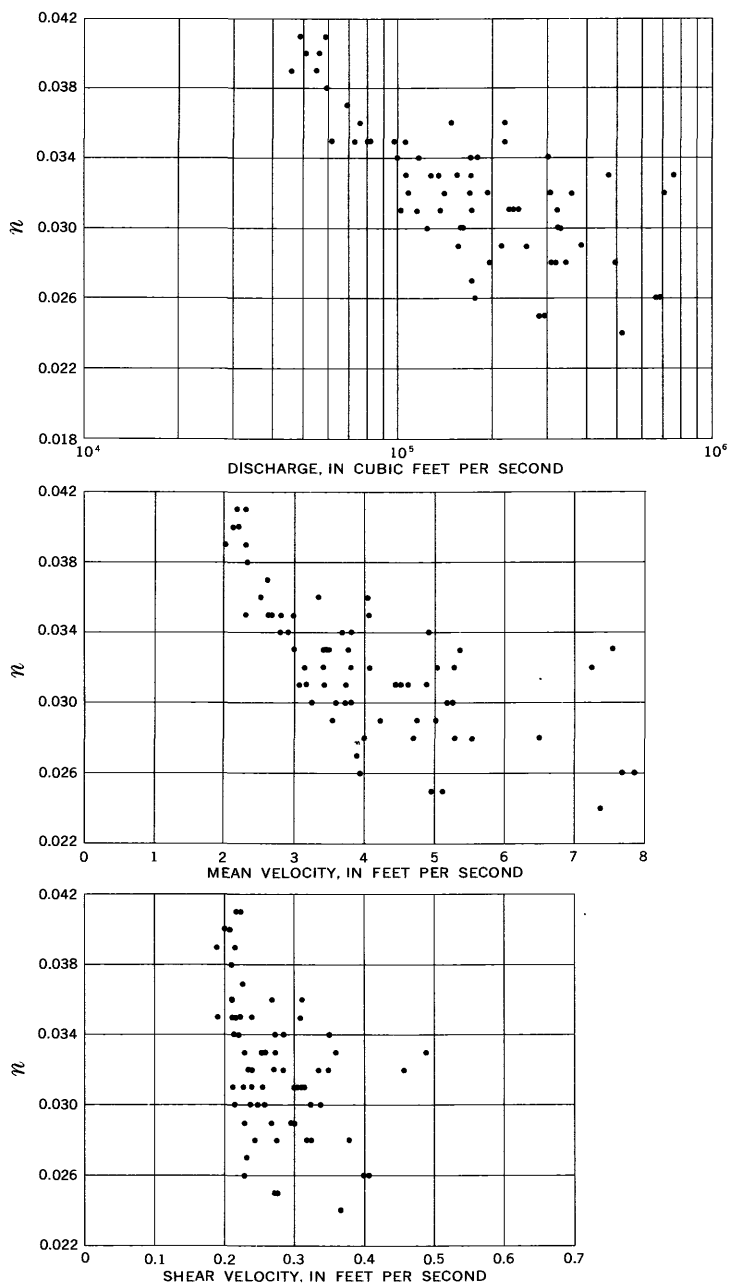
FIGURE 17.—Relation of Manning's n to discharge, mean velocity, and shear velocity.
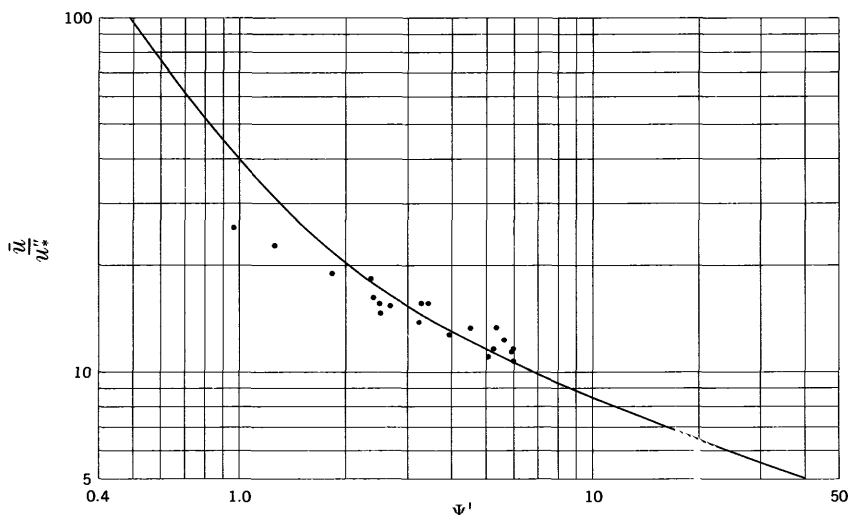
FIGURE 18.—Relation of $\bar{u}/u_*''$ to Ψ'. Curve from Einstein (1950, fig. 5).

of the cross section even though much of the bed is fairly flat and regular.

When gage height is plotted against discharge of more than about 300,000 cfs and consecutive measurements are connected, a loop generally appears in the curve; the gage height for a given discharge is generally lower when the discharge is increasing than when it is decreasing. The largest difference in gage heights for the same discharge for individual rises during the period of sediment record was in April and May 1951; the gage height was 23.8 feet for a discharge of 420,000 cfs when the discharge was increasing and 26.6 feet for the same discharge when the discharge was decreasing (fig. 19). During the same rise, at a gage height of 25.0 feet the increasing discharge was about 450,000 cfs and the decreasing discharge was about 380,000 cfs. Possible conditions that could cause this kind of loop in the relation of gage height to discharge are (1) scour during increasing discharge and fill during decreasing discharge, (2) lower turbulence constant due to higher sediment concentrations during increasing discharge than during decreasing discharge, (3) less channel roughness during increasing discharge than during decreasing discharge, and (4) higher energy gradient during increasing discharge than during decreasing discharge.

Figure 19 shows that, for a given discharge, the mear bed elevation is consistently higher when the discharge is increasing than when it is decreasing. Therefore, scour and fill could not cause the loop in the relation of gage height to discharge.
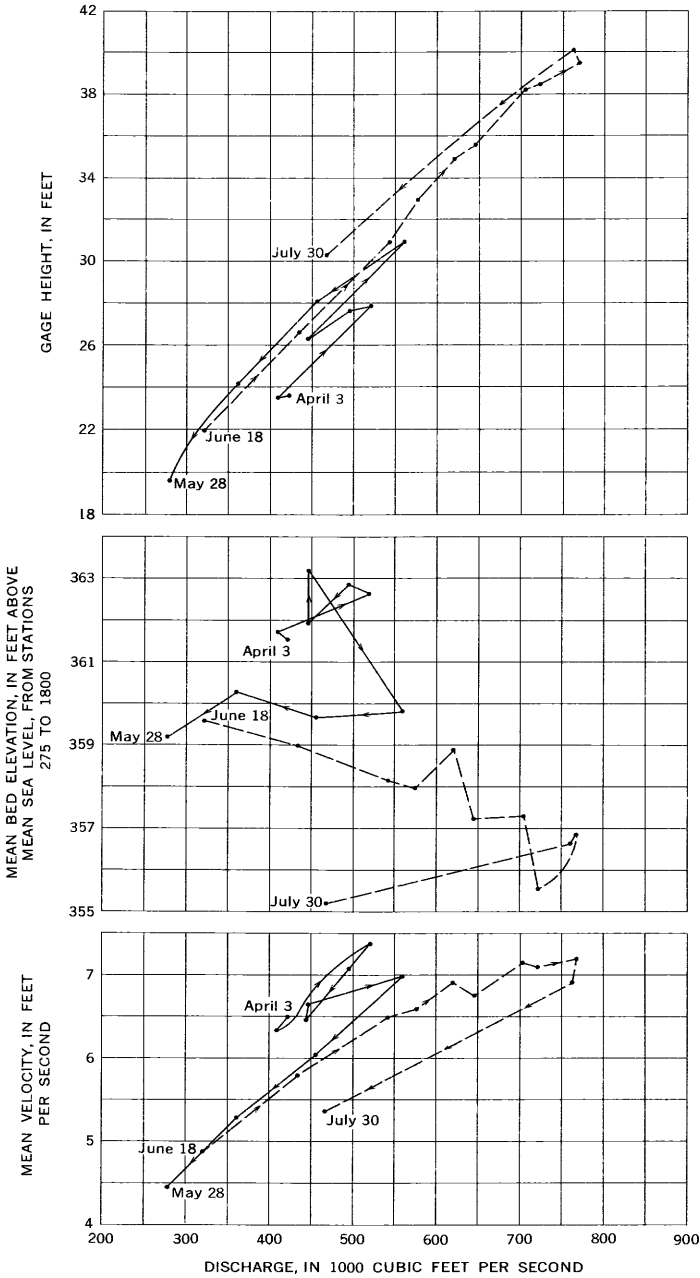
FIGURE 19.—Relation of gage height, bed elevation, and velocity
to discharge during major rises in 1951.

Graphs of mean velocity plotted aga:nst discharge (fig. 19) show that, for a given discharge, the mean velocity is generally much higher for increasing discharge than it is for decreasing discharge. The velocity for decreasing discharge is low enough not only to counterbalance the scour of the bed but to cause a higher gage height. The lower velocity could be caused by higher turbulence constant, by greater channel roughness, by lower energy gradient due to changing discharge, by channel storage between the gage and the measuring section, or by return of overbank flow.

Sample computations were made to determine the change in turbulence constant that would be necessary to cause the observed change in mean velocity. Keulegan's equation for mean velocity in rough channels can be written as

$$\frac{\overline{u}}{u_*} = 6.25 + \frac{2.3}{k} \log \frac{R}{k_s}$$

where

  u_* is shear velocity and equals $\sqrt{gRS_e}$, in which S_e is the energy gradient
  k is the Von Karman coefficient for turbulent exchange
  k_s is a linear measure of the bed roughness

Turbulence constants were computed for increasing and decreasing discharges of 420,000 cfs in April and May 1951. Because the purpose was to determine the change in turbulence constant alone that would cause the observed change in mean velocity, the energy gradient S_e was assumed to be constant at 0.00011, the bed roughness k_s was assumed to be constant at 1.6, and the constant 6.25 for clear water was assumed to be applicable for sediment-laden flow. For increasing discharge of 420,000 cfs in April 1951, the mean velocity was 6.5 feet per second and the hydraulic radius was 37 feet. Solution of Keulegan's equation gave the result $k = 0.27$. For decreasing discharge of 420,000 cfs in May 1951, the mean velocity was 5.8 feet per second and the hydraulic radius was 40 feet. Solution of the equation gave the result $k = 0.35$. Vanoni (1946) reported that k decreased as suspended-sediment concentration increased when the discharge and boundary conditions remained constant. The decrease in k was larger when the suspended sediment was of small particle size than when the suspended sediment was sand. Einstein and Chien (1955) showed a relation of k to a parameter of suspended-sediment concentration and fall velocity. According to this relation, the sediment having high fall velocity (large size) would have a greater effect on k than the sediment having low fall velocity (small size). The suspended-sediment concentration of all particle sizes at an increasing discharge of 420,000 cfs in April 1951 was about twice as great as the concentration at a decreasing discharge of 420,000 cfs in

May 1951. The difference in concentration was mostly in the finer sizes. According to the relation of Einstein and Chien, the observed decrease in concentration would not be enough to increase k from 0.27 to 0.35. Further information on the change in k is given in the section, "Vertical distributions of velocity and sediment concentration."

A sample computation was made to determine what changes in roughness with constant energy gradient would be necessary to cause the observed change in mean velocity. For simplicity, the Manning equation was used instead of the Keulegan equation. For a constant energy gradient, let n_i be the Manning resistance factor during increasing discharge of 420,000 cfs in April 1951, and let n_d be the Manning resistance factor during decreasing discharge of 420,000 cfs in May 1951.
Then

$$6.5 = \frac{1.49}{n_i} 37^{2/3} S_e^{1/2}$$

and

$$5.8 = \frac{1.49}{n_d} 40^{2/3} S_e^{1/2}$$

Solving for n_i,

$$n_i = 0.85 n_d$$

Changes in resistance factor of 15 percent or more are common at St. Louis; therefore, change in roughness alone could account for the loop in the relation of gage height to discharge.

Corbett and others (1943) stated that when the discharge is changing with respect to time, the relation of gage height to discharge may be affected by an increase or a decrease of water-surface slope from that corresponding to steady-flow conditions, conversion of discharge into or out of channel storage, or return of overbank flow. If any of these factors were to account for the loops, the effects would have to be large enough to balance the changes in cross-sectional area due to the changes in bed elevation.

The effect of changes in slope due to changes in discharge was computed from an approximate equation given by Corbett and others (1943, p. 153, eq. 10):

$$\frac{Q_c}{Q_m} = \frac{\sqrt{S_c}}{\sqrt{S_c + (1/U)\,dh/dt}}$$

in which

Q_c is the discharge for a given gage height under conditions of constant discharge

Q_m is the measured changing discharge for the same gage height

S_c is the slope of the water surface under constant discharge conditions

U is the velocity of the flood wave

dh/dt is the rate of change of stage

When the formula is written in terms of discharge Q_c/Q_m, the cross-sectional area must be assumed to be constant for a given gage height. The cross section at MacArthur Bridge is subject to scour and fill; so the formula was rewritten in terms of mean velocity:

$$\frac{\overline{u}_c}{\overline{u}_m} = \frac{\sqrt{S_c}}{\sqrt{S_c + (1/U)dh/dt}}$$

in which

$\overline{u}_c$ is mean velocity under conditions of constant discharge
$\overline{u}_m$ is measured mean velocity under conditions of changing discharge

To compute the maximum effect of changing discharge, the velocity of the flood wave U was computed from the empirical formula (Corbett and others, 1943, p. 155, eq. 14) $U = 1.3\overline{u}$.

During most discharge measurements, the rate of change in gage height was very low.   The effect of change in discharge was computed for two discharge measurements having relatively high rates of change in gage height; for the discharge measurements of June 29 and July 30, 1951, the rates of change in gage height were $+0.036$ and -0.127 foot per hour, respectively.   For these computations, the water-surface slope for constant high discharge was assumed to be 0.00010. The computed ratios $\overline{u}_c/\overline{u}_m$ were 0.994 and 1.029 for June 29 and July 30, respectively.   These results show that the effect of change in discharge was too small to account for the loop in the relation of gage height to discharge.

Because the discharge measurements were made 1.1 miles downstream from the recording gage, channel storage could have some effect on the stage-discharge relationship.   The effect was computed by using the equation (Corbett and others, 1943, p. 156, eq. 16):

$$Q_m - Q_s = bLC \frac{dh}{dt}$$

in which

Q_m is the measured discharge
Q_s is the discharge at the gage
L is the length between the sections at which Q_m and Q_s are determined
C is a constant relating the rate of change of stage dh/dt to its equivalent effect in the reach

Because the reach is nearly uniform, the constant C was taken as 1.00.   The computed differences $Q_m - Q_s$ were $+100$ and -350 cfs for June 29 and July 30, respectively.   The results indicate that channel storage does not affect the stage-discharge relationship significantly.

The return of overbank flow could not account for the loops, because the loops have been observed for stages for which the flow was within the banks at the gage and at least 4 miles upstream and downstream.

The conclusions from the study of the loops in the relation of gage height to discharge are:

1. For a given discharge during an individual rise, the gage height is lower for increasing discharge than for decreasing discharge even though the bed elevation is higher.
2. Change in turbulence constant due to changing sediment concentration could not in itself account for the loops.
3. Change in roughness could in itself account for the loops.
4. Change in energy gradient due to changing discharge, channel storage between the gage and the measuring section, or return of overbank flow could not account for the loops.
5. A combination of changes in roughness due to changing configuration of the bed and changes in turbulence constant due to changing sediment concentration probably account for the loops.

VERTICAL DISTRIBUTIONS OF VELOCITY AND SEDIMENT CONCENTRATION

By B. R. COLBY

The Mississippi River at St. Louis provides ample depths for studies of vertical distributions of velocity and sediment concentration. Because the depth effect is not thoroughly understood for some aspects of sediment transportation, relationships defined for shallow flows should not be arbitrarily assumed to apply to deep flows. Therefore, the available information for the Mississippi River at St. Louis, even though incomplete and perhaps in part inexact (such as for water-surface slopes), may define vertical distributions of velocity and sediment concentration sufficiently well to aid in understanding theoretical and empirical relations for the Mississippi River and other streams.

VELOCITY

A convenient measure of the vertical distribution of velocity at constant shear velocity is the turbulence constant k in equations that were given by Keulegan (1938) and used by Einstein (1950) for point and mean velocities in about the forms

$$u_y = 2.30 \frac{u_*}{k} \log (30.2 \, xy/k_s)$$

$$\bar{u}_s = 2.30 \frac{u_*}{k} \log (12.27 \, xy_0/k_s)$$

In these equations,

u_y and $\overline{u}_s$ are, respectively, the time-averaged velocity at a point and the mean velocity at a station

y is the distance above the streambed

x is a parameter to cover the transition from hydraulically smooth to hydraulically rough boundaries

y_0 is the depth of flow

The equations were derived by Keulegan for clear-water flow and rigid channel boundaries. According to Einstein (1950, p. 8), the d_{65} size of a sediment mixture is the representative size to use as a measure of the roughness of a bed of sediment of mixed particle sizes. If no bed nor bank roughness exists except that due to the stationary grains, Einstein reasoned that d_{65} could be used in place of k_s in the equations and, for no significant sediment movement, that k should about equal 0.40.

These velocity equations may require some modifications for flows over beds of shifting sediment. Vanoni (1946) and Einstein and Chien (1954) reported that k became less than 0.40 for sediment-laden flows over relatively smooth beds. The apparent reason for the reduction of k is a dampening of turbulence because of the sediment in the flows. Another modification is in the roughness k_s. If the streambed has ripples, bars, or dunes, k_s is far greater than d_{65} and presumably depends both on the heights of the dunes, or other major roughness, and on the spacing of the dunes. A large k_s can seldom be evaluated directly from the bed configuration, partly because the bed configuration is usually not known completely and partly because the spacing of the major elements of bed roughness has an uncertain effect on k_s.

Einstein (1950, p. 10) adjusted the shear velocity for flows over beds of shifting sediment and used an equation for mean velocity in about the form

$$\overline{u}_s = 2.30 \, \frac{u_*{}'}{0.40} \, \log \, (12.27 \, x y_0/d_{65})$$

in which $u_*{}'$ is the shear velocity with respect to the grain (the shear velocity that will give the correct mean velocity) and 0.40 is substituted for k. The ratio $u_*{}'/u_*$ is equivalent to the ratio of the actual mean velocity to the mean velocity for a clear-water flow over a stationary bed whose only roughness is due to the unshifting grains and can be represented, at least roughly, by d_{65}. Ratios of $u_*{}'/u_*$ as low as those for the Mississippi River at St. Louis (table 3) indicate, as did the other measure of roughness n, that the bed of the river generally has the major roughness that might be caused by bars, ripples, or dunes.

TABLE 3.—*Ratios of shear velocity with respect to the grain to shea- velocity*

Date	u_*'/u_*	Discharge (cfs)
Apr. 18, 1951	0. 64	510, 000
July 16	. 53	704, 000
June 7, 1954	. 55	283, 000
Mar. 30, 1955	. 45	163, 000
Dec. 27	. 36	59, 800
Feb. 21, 1956	. 34	56, 100
Apr. 24	. 42	172, 000
May 9	. 42	149, 000
July 9	. 43	155, 000
May 13, 1959	. 47	241, 000

Even though k_s cannot be evaluated for the flows over the rough streambed at St. Louis, the vertical distribution should still depend on the ratio of shear velocity to k if the equations of the Keulegan type apply for both point and mean velocities. That is if these equations apply, the difference in point velocities at a distance $10y$ above the streambed and at a distance y above the bed equals 2.30 u_*/k, no matter what k_s may be. For major roughness spaced as widely as bars or dunes may be, the vertical distribution of velocity may be, however, a function of the location of a vertical with respect to a dune or bar as well as of u_*/k. In other words, as Sayre and Albertson (1959, p. 57–62) found for widely spaced baffles and clear-water flow, the k determined from the vertical distribution of velocity may not be the same as the k for the mean-velocity equation.

Some variation of k with change of the observation station can be shown by information at crest and trough of dunes in the Middle Loup River at Dunning, Nebr. Two k's from vertical distributions of velocity at the crest of a dune were 0.32 and 0.62 as compared with k's of 0.11 and 0.19 for stations in a trough downstream from the crest.

Basic data are shown in table 4 for current-meter observations of the vertical distributions of velocity at St. Louis. The k's in table 5 were computed from the data in table 4 and from the velocities of routine discharge measurements.

TABLE 4.—*Vertical distributions of velocity*

[Velocities are averages for about 40–60 sec. unless otherwise noted]

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)
MAR. 2, 1948			**MAY 21, 1948**		
Station 400; total depth, 38.2 ft			**Station 400; total depth, 31.6 ft**		
	1.5	3.08	12:05 p.m.	1.6	2.44
	2.3	3.45		2.6	2.29
	3.8	3.67		3.6	2.61
	5.7	4.19		4.6	2.80
	7.6	4.19		5.6	2.80
	11.5	4.99		6.6	3.06
	15.3	5.10		7.6	3.21
	19.1	5.23		9.6	3.06
	22.9	5.77		11.6	3.21
	26.7	6.09		13.6	3.43
	30.6	6.20		15.6	3.43
	34.4	6.45		17.6	3.50
	37.2	6.45		19.6	3.98
				21.6	3.72
Station 625; total depth, 41.0 ft				23.6	3.80
				25.6	4.16
	1.5	3.01		27.6	4.16
	2.5	3.22	12:23 p.m.	29.6	4.07
	4.1	3.15			
	6.2	3.83	**Station 625; total depth, 33.0 ft**		
	8.2	3.91			
	12.3	4.19			
	16.4	5.10	12:27 p.m.	1.5	2.50
	20.5	5.10		2.0	2.55
	24.6	5.10		3.0	2.74
	28.7	5.58		4.0	3.06
	32.8	5.87		5.0	2.50
	36.9	5.98		7.0	2.74
	40.0	5.87		9.0	2.34
				11.0	2.86
Station 850; total depth, 41.2 ft				13.0	3.65
				15.0	3.65
	1.5	3.45		17.0	3.57
	2.5	3.26		19.0	4.07
	4.1	4.09		21.0	4.16
	6.2	4.19		23.0	4.16
	8.2	4.88		25.0	4.26
	12.4	5.31		27.0	4.75
	16.5	5.67		29.0	4.55
	20.6	5.98	12:45 p.m.	31.0	4.55
	24.7	5.98			
	28.8	6.45	**Station 850; total depth, 35.7 ft**		
	33.0	6.57			
	37.1	6.85	12:55 p.m.	1.5	2.39
	40.2	6.71		2.7	2.92
				3.7	3.06
Station 1,200; total depth, 46.4 ft				4.7	3.21
				5.7	3.13
				7.7	3.37
	1.5	2.51		9.7	3.80
	2.8	3.15		11.7	3.65
	4.6	3.15		13.7	4.16
	7.0	3.67		15.7	3.72
	9.3	4.39		17.7	3.84
	13.9	4.99		19.7	4.07
	18.6	5.10		21.7	4.16
	23.2	5.49		23.7	4.37
	27.8	5.98		25.7	4.16
	32.5	5.87		27.7	4.46
	37.2	6.57		29.7	4.37
	41.8	6.85		31.7	4.55
	45.4	6.85	1:15 p.m.	33.7	4.65

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Distance from bed (ft)	Velocity (fps)	Time	Distance from bed (ft)	Velocity (fps)
MAY 21, 1948—Continued			**MAY 21, 1948—Continued**		
Station 1,050; total depth, 34.3 ft			**Station 1,500; total depth, 38.2 ft**		
1:23 p.m.	0.5	2.05	10:40 a.m.	1.5	2.99
	1.3	2.92		2.2	3.65
	2.3	2.63		3.2	4.16
	3.3	3.21		4.2	3.31
	4.3	3.50		5.2	3.80
	6.3	3.37		6.2	4.07
	8.3	3.65		8.2	4.37
	10.3	3.89		10.2	4.37
	12.3	4.37		12.2	5.08
	14.3	4.26		14.2	4.75
	16.3	4.26		16.2	4.85
	18.3	4.07		18.2	4.65
	20.3	4.16		20.2	4.85
	22.3	4.37		22.2	4.96
	24.3	4.37		24.2	5.32
	26.3	4.37		26.2	5.08
	28.3	4.55		28.2	5.20
	30.3	4.46		30.2	5.20
1:42 p.m.	32.3	4.55		32.2	5.45
				34.2	5.32
Station 1,200; total depth, 38.3 ft			11:05 a.m.	36.2	5.55
1:48 p.m.	1.5	2.55	**Station 1,650; total depth, 40.5 ft**		
	2.3	2.61	10:00 a.m.	1.5	3.72
	3.3	2.58		2.5	3.80
	4.3	2.92		3.5	3.57
	5.3	3.50		4.5	3.98
	6.3	3.21		6.5	4.37
	8.3	3.72		8.5	4.37
	10.3	3.98		10.5	4.46
	12.3	4.20		12.5	4.20
	14.3	4.07		14.5	4.37
	16.3	4.26		16.5	4.46
	18.3	4.07		18.5	4.28
	20.3	4.16		20.5	4.37
	22.3	4.07		22.5	4.55
	24.3	3.98		24.5	4.96
	26.3	4.65		26.5	4.75
	28.3	4.55		28.5	4.75
	30.3	4.16		30.5	4.55
	32.3	4.85		32.5	5.08
	34.3	4.75		34.5	5.45
2:07 p.m.	36.3	4.85		36.5	5.08
			10:24 a.m.	38.5	4.96
Station 1,400; total depth, 42.0 ft			**MAR. 17, 1952**		
2:15 p.m.	1.5	2.55	**Station 500; total depth, 40.1 ft**		
	2.0	3.29	9:20 a.m.	1.5	4.86
	3.0	2.92		4.0	5.71
	4.0	2.86		8.0	6.62
	5.0	2.99		11.9	7.19
	6.0	3.06		15.7	7.35
	8.0	2.86		19.5	7.35
	10.0	2.92		23.3	7.69
	12.0	3.21		27.0	8.07
	14.0	3.21		30.9	8.07
	16.0	4.16	9:35 a.m.	34.8	8.27
	18.0	4.85	**Station 900; total depth, 32.8 ft**		
	20.0	4.96	10.50 a.m.	1.5	6.37
	22.0	4.96		3.5	6.90
	24.0	4.55		6.6	7.35
	26.0	4.37		9.8	7.88
	28.0	4.96		13.0	7.88
	30.0	5.32		16.1	8.32
	32.0	5.32			
	34.0	5.45			
	36.0	5.55			
	38.0	5.45			
2:35 p.m.	40.0	5.45			

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)

MARCH 17, 1952—Continued Station 900; total depth, 32.8—Continued			JUNE 7, 1954—Continued Station 1,300; total depth, 44.6 ft		
---------------------	19.4	8.65	10:11 a.m.------------	1.5	3.40
---------------------	22.1	8.82	---------------------	4.5	4.31
	25.6	9.19	---------------------	8.9	4.41
11:05 a.m.-----------	28.6	9.19	---------------------	17.8	6.01
			---------------------	26.7	6.01
Station 1,275; total depth, 48.1 ft			10:15 a.m.-----------	40.1	6.36

			Station 1,600; total depth, 42.6 ft		
1:30 p.m.------------	1.5	4.31			
---------------------	3.4	5.14	9:13 a.m.------------	1.5	3.68
---------------------	8.3	5.52	---------------------	4.3	4.01
---------------------	13.3	6.37	---------------------	8.4	4.60
---------------------	18.3	6.97	---------------------	17.0	4.70
---------------------	23.1	7.55	---------------------	25.5	5.13
---------------------	28.0	7.60	9:25 a.m.------------	38.3	5.70
---------------------	32.8	7.78			
---------------------	37.7	7.88	**AUG. 10, 1954**		
1:50 p.m.------------	42.3	8.17	**Station 1,000; total depth, 23.1 ft**		

Station 1,600; total depth, 44.1 ft			9:25 a.m.------------	1.5	[1] 2.72
				3.1	[1] 2.45
2:00 p.m.------------	1.5	5.52		5.1	[1] 2.99
---------------------	4.7	6.13		7.1	[1] 3.40
---------------------	9.1	6.75		9.1	[1] 3.05
---------------------	13.4	7.19		11.1	[1] 3.71
---------------------	17.7	7.44		13.1	[1] 3.45
---------------------	21.8	7.69		15.1	[1] 3.48
---------------------	26.1	8.07		17.1	[1] 4.04
---------------------	30.4	7.78		19.1	[1] 3.74
---------------------	34.5	7.78		21.1	[1] 4.12
2:15 p.m.------------	38.7	7.88	9:30 a.m.[2]----------	1.5	[1] 2.48
				3.1	[1] 2.96
APR. 30, 1952				5.1	[1] 2.99
Station 750; total depth, 52.1 ft				7.1	[1] 3.18
				9.1	[1] 3.50
---------------------	5.1	5.42		11.1	[1] 3.65
---------------------	10.3	6.55		13.1	[1] 3.53
---------------------	15.6	7.48		15.1	[1] 3.07
---------------------	20.8	7.80		17.1	[1] 3.90
---------------------	26.0	8.34		19.1	[1] 3.77
---------------------	31.2	8.70		21.1	[1] 3.93
---------------------	36.4	8.73	9:35 a.m.[2]----------	1.5	[1] 2.33
---------------------	41.7	9.12		3.1	[1] 2.46
---------------------	46.9	9.35		5.1	[1] 3.01
				7.1	[1] 3.18
JUNE 7, 1954				9.1	[1] 3.42
Station 475; total depth, 37.5 ft				11.1	[1] 3.25
				13.1	[1] 3.62
11:42 a.m.-----------	1.5	3.16		15.1	[1] 3.50
---------------------	3.7	3.76		17.1	[1] 3.90
---------------------	7.5	3.93		19.1	[1] 3.97
---------------------	15.0	4.41		21.1	[1] 4.04
---------------------	22.5	4.70	9:40 a.m.[2]----------	1.5	[1] 2.81
11:49 a.m.-----------	33.7	5.51		3.1	[1] 2.15
				5.1	[1] 3.09
Station 500; total depth, 34.3 ft				7.1	[1] 3.32
				9.1	[1] 3.65
10:46 a.m.-----------	1.5	4.41		11.1	[1] 3.42
---------------------	3.4	5.13		13.1	[1] 3.59
---------------------	6.8	4.90		15.1	[1] 3.48
---------------------	13.7	6.01		17.1	[1] 3.65
---------------------	20.5	6.24		19.1	[1] 3.81
11:00 a.m.-----------	30.9	6.36	9:45 a.m.------------	21.1	[1] 3.90
			12:35 p.m.-----------	1.5	[1] 2.91
			---------------------	3.1	[1] 3.28
			---------------------	5.1	[1] 3.05
			---------------------	7.1	[1] 3.01

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)

AUG. 10, 1954—Continued

Station 1,000; total depth, 23.1 ft—Continued

Time	Distance from bed (ft)	Velocity (fps)
-------------------	9.1	1 3.50
-------------------	11.1	1 3.30
-------------------	13.1	1 3.42
-------------------	15.1	1 3.87
-------------------	17.1	1 3.74
-------------------	19.1	1 4.12
-------------------	21.1	1 4.12
12:39 p.m.² -----------	1.5	1 2.61
-------------------	3.1	1 2.65
-------------------	5.1	1 3.45
-------------------	7.1	1 2.95
-------------------	9.1	1 3.32
-------------------	11.1	1 3.45
-------------------	13.1	1 3.18
-------------------	15.1	1 3.81
-------------------	17.1	1 3.81
-------------------	19.1	1 3.71
-------------------	21.1	1 4.08
12:43 p.m.² -----------	1.5	1 2.68
-------------------	3.1	1 3.01
-------------------	5.1	1 3.07
-------------------	7.1	1 3.05
-------------------	9.1	1 3.40
-------------------	11.1	1 3.53
-------------------	13.1	1 3.77
-------------------	15.1	1 3.68
-------------------	17.1	1 3.62
-------------------	19.1	1 3.77
-------------------	21.1	1 3.87
12:47 p.m.² -----------	1.5	1 2.50
-------------------	3.1	1 3.05
-------------------	5.1	1 3.12
-------------------	7.1	1 3.20
-------------------	9.1	1 3.39
-------------------	11.1	1 3.05
-------------------	13.1	1 3.71
-------------------	15.1	1 3.59
-------------------	17.1	1 3.81
-------------------	19.1	1 4.04
12:50 p.m. -----------	21.1	1 3.90

AUG. 27, 1954
Station 1,000; total depth, 24.0 ft

Time	Distance from bed (ft)	Velocity (fps)
8:40 a.m. -----------	1.5	3 2.43
-------------------	4.0	3 2.78
-------------------	6.0	3 3.25
-------------------	10.0	3 3.45
-------------------	14.0	3 3.41
-------------------	18.0	3 3.82
-------------------	22.0	3 3.78
9:15 a.m.² -----------	1.5	3 2.54
-------------------	4.0	3 2.89
-------------------	6.0	3 2.98
-------------------	10.0	3 3.23
-------------------	14.0	3 3.58
-------------------	18.0	3 3.54
9:50 a.m. -----------	22.0	3 3.82
11:30 a.m. -----------	1.5	2.52
-------------------	4.0	3.38
-------------------	6.0	3.50
-------------------	10.0	3.83
-------------------	14.0	3.80
-------------------	18.0	3.82
-------------------	22.0	4.30
11:45 a.m.² -----------	1.5	3.07
-------------------	4.0	3.30
-------------------	6.0	3.33
-------------------	10.0	3.57
-------------------	14.0	3.96
-------------------	18.0	3.82
12:00 m. -----------	22.0	4.32

MAR. 31, 1955

Station 500; total depth, 24.7 ft

Time	Distance from bed (ft)	Velocity (fps)
10:45 a.m. -----------	1.5	2.90
-------------------	3.5	3.29
-------------------	4.5	3.42
-------------------	6.5	3.70
-------------------	8.5	3.95
-------------------	10.6	3.95
-------------------	13.6	3.74
-------------------	16.6	4.08
-------------------	19.6	4.33
10:55 a.m. -----------	22.7	4.44
12:30 p.m. -----------	1.5	2.78
-------------------	3.5	3.26
-------------------	4.5	3.49
-------------------	6.5	3.63
-------------------	8.5	3.86
-------------------	10.6	3.63
-------------------	13.6	4.18
-------------------	16.6	4.18
-------------------	19.6	4.33
12:35 p.m. -----------	22.7	4.50

Station 925; total depth, 23.1 ft

Time	Distance from bed (ft)	Velocity (fps)
12:40 p.m. -----------	1.5	3.46
-------------------	5.5	3.86
-------------------	8.5	3.95
-------------------	11.4	4.33
-------------------	14.4	4.28
-------------------	18.3	4.62
12:45 p.m. -----------	21.1	4.58
1:55 p.m. -----------	1.5	3.06
-------------------	5.5	3.38
-------------------	8.5	3.78
-------------------	11.4	3.95
-------------------	14.4	4.13
-------------------	18.3	4.23
2:05 p.m. -----------	21.1	4.62

DEC. 27, 1955

Station 550; total depth, 11.8 ft

Time	Distance from bed (ft)	Velocity (fps)
12:50 p.m. -----------	2.4	2.35
-------------------	4.7	2.40
-------------------	7.1	2.45
12:55 p.m. -----------	10.6	2.76
1:45 p.m. -----------	2.4	2.35
-------------------	4.7	2.45
-------------------	7.1	2.51
1:50 p.m. -----------	10.6	2.69

Station 925; total depth, 15.2 ft

Time	Distance from bed (ft)	Velocity (fps)
2:00 p.m. -----------	1.5	1.51
-------------------	3.0	2.16
-------------------	6.1	2.25
-------------------	9.1	2.63
2:05 p.m. -----------	13.7	2.76
2:50 p.m. -----------	1.5	1.55
-------------------	3.0	2.21
-------------------	6.1	2.45
-------------------	9.1	2.57
2:55 p.m. -----------	13.7	2.76

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)

	DEC. 28, 1955			FEB. 21, 1956—Continued	
	Station 1,275; total depth, 23.7 ft			**Station 1,525; total depth, 18.5 ft**	
9:55 a.m_____	2.4	1.36	3:55 p.m_____	1.9	1.87
_____	4.7	1.39	_____	3.7	2.24
_____	9.5	2.30	_____	7.4	2.59
_____	14.2	2.63	_____	11.1	2.66
10:00 a.m_____	21.3	3.01	_____	16.7	2.72
10:45 a.m_____	2.4	1.42	4:40 p.m_____	1.9	1.95
_____	4.7	1.58	_____	3.7	2.28
_____	9.5	2.01	_____	7.4	2.33
_____	14.2	2.94	_____	11.1	2.54
10:50 a.m_____	21.3	2.94	_____	16.7	2.85

	Station 1,525; total depth, 19.5 ft			APR. 24, 1956	
11:00 a.m_____	1.9	1.88		**Station 475; total depth, 29.0 ft**	
_____	3.9	2.25			
_____	7.8	2.69	12:35 p.m.[2]_____	2.9	3.04
_____	11.7	2.82	_____	5.8	3.34
11:05 a.m_____	18.3	3.01	_____	11.6	3.56
			_____	17.4	4.04
	FEB. 21, 1956		_____	26.1	4.13

	Station 550; total depth, 12.0 ft			**Station 900; total depth, 26.8 ft**	
11:30 a.m_____	1.5	1.53	1:30 p.m.[2]_____	2.7	3.04
_____	2.4	2.24	_____	5.4	3.26
_____	4.8	2.03	_____	10.7	4.23
_____	7.2	2.33	_____	16.1	4.33
_____	10.8	2.59	_____	24.1	4.72
12:55 p.m_____	1.5	1.68	2:15 p.m.[2]_____	2.7	3.18
_____	2.4	1.91	_____	5.4	3.26
_____	4.8	2.24	_____	10.7	3.87
_____	7.2	2.43	_____	16.1	3.95
_____	10.8	2.54	_____	24.1	4.62

	Station 925; total depth, 14.4 ft			**Station 1,300; total depth, 43.0 ft**	
1:10 p.m_____	1.5	1.79	2:30 p.m.[2]_____	4.3	2.48
_____	2.9	1.99	_____	8.6	2.09
_____	5.8	2.08	_____	17.2	3.78
_____	8.6	2.33	_____	25.8	4.33
_____	13.0	2.85	_____	39.0	4.44
2:25 p.m_____	1.5	1.76	3:15 p.m.[2]_____	4.3	1.76
_____	2.9	1.95	_____	8.6	2.97
_____	5.8	2.48	_____	17.2	3.26
_____	8.6	2.54	_____	25.8	4.33
_____	13.0	2.59	_____	39.0	4.53

	Station 1,275; total depth, 20.5 ft			**Station 1,575; total depth, 32.0 ft**	
2:40 p.m_____	2.1	1.79	3:25 p.m.[2]_____	3.2	2.91
_____	4.1	1.76	_____	6.4	3.71
_____	8.2	2.38	_____	12.8	3.87
_____	12.3	2.85	_____	19.2	4.23
_____	18.5	2.97	_____	28.8	4.72
3:35 p.m_____	2.1	1.79	4:15 p.m.[2]_____	3.2	3.26
_____	4.1	1.87	_____	6.4	3.63
_____	8.2	2.48	_____	12.8	4.35
_____	12.3	2.85	_____	19.2	4.35
_____	18.5	3.11	_____	28.8	5.04

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)
MAY 9, 1956			**JULY 9, 1956—Continued**		
Station 475; total depth, 27.6 ft			Station 925; total depth, 25.7 ft		
10:45 a.m.² _____	2.8	2.65			
_____	5.5	3.03	12:05 p.m_____	2.6	2.79
_____	10.0	3.18	_____	5.1	3.63
_____	16.6	3.86	_____	10.3	3.95
_____	24.8	4.23	_____	15.4	3.95
11:25 a.m.² _____	2.8	2.78	_____	23.1	4.44
_____	5.5	3.10	12:35 p.m_____	2.6	3.04
_____	10.0	3.34	_____	5.1	2.97
_____	16.6	3.95	_____	10.3	3.95
_____	24.8	4.13	_____	15.4	4.23
			_____	23.1	4.33
Station 900; total depth, 25.9 ft					
			Station 1,275; total depth, 38.8 ft		
11:40 a.m.² _____	2.1	2.72			
_____	5.2	3.18	12:40 p.m_____	3.9	1.99
_____	10.4	3.95	_____	7.8	2.20
_____	15.5	4.13	_____	15.5	3.11
_____	23.3	4.53	_____	23.3	4.13
12:25 p.m.² _____	2.1	2.23	_____	34.9	4.82
_____	5.2	2.84	1:10 p.m_____	3.9	2.48
_____	10.4	3.49	_____	7.8	2.04
_____	15.5	4.04	_____	15.5	3.26
_____	23.3	4.44	_____	23.3	4.23
			_____	34.9	4.72
Station 1,300; total depth, 41.5 ft					
			Station 1,575; total depth, 27.0 ft		
1:00 p.m.² _____	4.1	2.03			
_____	6.3	2.59	1:15 p.m_____	3.0	2.43
_____	16.6	3.78	_____	6.0	3.49
_____	24.9	3.95	_____	12.0	3.56
_____	37.3	4.62	_____	18.0	4.23
1:35 p.m.² _____	4.1	1.98	_____	27.0	4.62
_____	6.3	2.18	1:45 p.m_____	3.0	2.66
_____	16.6	2.65	_____	6.0	3.26
_____	24.9	3.86	_____	12.0	3.71
_____	37.3	4.44	_____	18.0	4.23
			_____	27.0	4.62
Station 1,577; total depth, 31.5 ft					
			NOV. 5, 1956		
1:45 p.m.² _____	3.1	3.34	Station 600; total depth, 10.5 ft		
_____	6.3	3.95			
_____	12.6	4.13	_____	1.5	1.80
_____	18.7	4.33	_____	2.1	1.91
_____	28.3	4.44	_____	4.2	2.38
2:25 p.m.² _____	3.1	3.18	_____	6.3	2.33
_____	6.3	4.04	_____	9.5	2.33
_____	12.6	4.23	_____	1.5	1.69
_____	18.7	4.33	_____	2.1	1.95
_____	28.3	4.72	_____	4.2	2.24
			_____	6.3	2.28
JULY 9, 1956			_____	9.5	2.43
Station 500; total depth, 25.8 ft			Station 1,000; total depth, 15.5 ft		
			_____	1.5	1.69
11:30 a.m_____	2.6	2.85	_____	3.1	1.80
_____	5.2	3.34	_____	6.2	2.43
_____	10.3	3.42	_____	9.3	2.54
_____	15.5	3.71	_____	13.9	2.48
_____	23.2	4.13	_____	1.5	1.69
12:00 m_____	2.6	2.79	_____	3.1	1.91
_____	5.2	2.91	_____	6.2	2.19
_____	10.3	3.42	_____	9.2	2.66
_____	15.5	3.63	_____	13.9	2.43
_____	23.2	4.23			

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)

NOV. 5, 1956—Continued

Station 1,375; total depth, 39.3 ft

Time	Distance from bed (ft)	Velocity (fps)
	1.5	1.61
	3.9	1.47
	7.9	1.69
	15.7	2.43
	23.6	2.72
	35.4	3.34
	1.5	1.21
	3.9	1.15
	7.9	1.65
	15.7	2.24
	23.6	2.91
	35.4	3.26

Station 1,600; total depth, 18.0 ft

Time	Distance from bed (ft)	Velocity (fps)
	1.5	1.54
	1.8	2.24
	3.6	2.24
	7.2	2.43
	10.8	2.97
	16.2	2.91
	1.5	1.67
	1.8	1.87
	3.6	2.33
	7.2	2.60
	10.8	2.97
	16.2	2.97

APR. 17, 1958

Station 500; total depth, 25.1 ft

Time	Distance from bed (ft)	Velocity (fps)
11:45 a.m	1.5	2.26
	2.6	2.36
	5.0	3.08
	10.0	4.00
	15.1	3.91
11:51 a.m	22.6	4.39
12:10 p.m	1.5	1.71
	2.6	2.22
	5.0	3.31
	10.0	3.52
	15.1	3.82
12:16 p.m	22.6	4.09

Station 950; total depth, 28.0 ft

Time	Distance from bed (ft)	Velocity (fps)
11:13 a.m	1.5	2.31
	2.8	2.82
	5.6	3.15
	11.2	3.52
	16.8	3.74
11:18 a.m	25.2	4.18
11:32 a.m	1.5	2.11
	2.8	3.01
	5.6	2.94
	11.2	3.45
	16.8	4.28
11:40 a.m	25.2	4.57

APR. 17, 1958—Continued

Station 1,250; total depth, 32.0 ft

Time	Distance from bed (ft)	Velocity (fps)
10:35 a.m	1.5	3.08
	2.2	3.21
	6.4	3.23
	12.8	4.48
	19.2	4.57
10:40 a.m	28.8	4.87
11:00 a.m	1.5	3.08
	2.2	3.31
	6.4	3.59
	12.8	4.57
	19.2	4.67
11:07 a.m	28.8	5.10

Station 1,550; total depth, 27.3 ft

Time	Distance from bed (ft)	Velocity (fps)
	1.5	3.45
10:00 a.m	2.7	4.08
	5.5	4.00
	10.9	4.18
	16.4	4.87
10:05 a.m	24.6	5.22
10:25 a.m	1.5	3.59
	2.7	3.39
	5.5	4.28
	10.9	4.39
	16.4	4.98
10:20 a.m	24.6	5.10

MAY 13, 1959

Station 375; total depth, 28.2 ft

Time	Distance from bed (ft)	Velocity (fps)
9:20 a.m.[2]	2.8	2.69
	5.6	3.59
	11.3	4.57
	16.9	4.41
	22.6	4.38
9:10 a.m.[2]	25.4	4.29

Station 575; total depth, 30.0 ft

Time	Distance from bed (ft)	Velocity (fps)
9:40 a.m.[2]	3.0	3.30
	12.0	4.47
	18.0	4.66
	24.0	4.47
9:30 a.m.[2]	27.0	5.09

Station 774; total depth, 32.3 ft

Time	Distance from bed (ft)	Velocity (fps)
9:55 a.m.[2]	3.2	3.32
	12.9	4.29
	19.4	5.09
	25.8	5.21
9:45 a.m.[2]	29.1	5.21

Station 924; total depth, 32.8 ft

Time	Distance from bed (ft)	Velocity (fps)
10:05 a.m.[2]	3.3	4.66
	13.1	5.09
	19.7	4.98
	26.2	5.47
10:00 a.m	29.5	5.34

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)

MAY 13, 1959—Continued

Station 1,074; total depth, 35.0 ft

Time	Distance	Velocity
10:20 a.m.² _____	3. 5	3. 99
_____	14. 0	5. 34
_____	21. 0	5. 66
_____	28. 0	5. 66
10:10 a.m.² _____	31. 5	5. 96

Station 1,200; total depth, 35.5 ft

Time	Distance	Velocity
10:40 a.m.² _____	3. 6	3. 52
_____	14. 2	5. 34
_____	21. 3	5. 34
_____	28. 4	5. 76
10:30 a.m.² _____	31. 9	6. 07

Station 1,300; total depth, 44.3 ft

Time	Distance	Velocity
11:00 a.m.² _____	4. 5	3. 59
_____	17. 7	5. 34
_____	26. 6	5. 66
_____	35. 4	5. 86
10:50 a.m.² _____	39. 9	6. 19

Station 1,425; total depth, 43.2 ft

Time	Distance	Velocity
11:20 a.m.² _____	4. 3	3. 52
_____	17. 3	5. 09
___;_____	25. 9	5. 76
_____	34. 6	5. 86
11:10 a.m.² _____	38. 9	6. 19

Station 1,551; total depth, 36.2 ft

Time	Distance	Velocity
11:35 a.m.² _____	3. 6	3. 82
_____	14. 5	5. 09
_____	21. 7	5. 66
_____	29. 0	6. 43
11:25 a.m.² _____	32. 6	6. 43

Station 1,675; total depth, 39.4 ft

Time	Distance	Velocity
11:50 a.m.² _____	3. 9	3. 26
_____	15. 8	4. 18
_____	23. 6	4. 76
_____	31. 5	4. 98
11:40 a.m.² _____	35. 5	4. 57

JULY 10, 1959

Station 710; total depth, 32.7 ft

Time	Distance	Velocity
9:20 a.m. _____	1. 5	1. 82
_____	3. 3	1. 63
_____	6. 5	1. 93
_____	9. 8	2. 89
_____	13. 1	2. 70
_____	16. 3	3. 23
_____	19. 6	3. 40
_____	22. 9	3. 76
_____	26. 2	3. 61
9:33 a.m. _____	29. 4	3. 84

JULY 10, 1959—Continued

Station 750; total depth, 26.3 ft

Time	Distance	Velocity
9:05 a.m. _____	1. 5	2. 41
_____	2. 6	2. 64
_____	5. 3	3. 02
_____	7. 9	3. 34
_____	10. 5	3. 47
_____	13. 1	3. 47
_____	15. 8	3. 76
_____	18. 4	4. 30
_____	21. 0	4. 10
9:15 a.m. _____	23. 7	3. 92

Station 800; total depth, 25.3 ft

Time	Distance	Velocity
8:40 a.m. _____	1. 5	2. 70
_____	2. 5	2. 64
_____	5. 1	3. 16
_____	7. 6	3. 16
_____	10. 1	2. 95
_____	12. 7	3. 53
_____	15. 2	4. 10
_____	17. 7	3. 84
_____	20. 2	4. 20
8:50 a.m. _____	22. 8	3. 84

Station 1,000; total depth, 29 5 ft

Time	Distance	Velocity
8:20 a.m. _____	1. 5	2. 77
_____	2. 9	2. 64
_____	5. 9	3. 09
_____	8. 9	3. 92
_____	11. 8	3. 47
_____	14. 7	4. 10
_____	17. 7	4. 20
_____	20. 7	4. 01
_____	23. 6	4. 20
8:30 a.m. _____	26. 5	4. 41

APR. 14, 1960

Station 351; total depth, 42.7 ft

Time	Distance	Velocity
9:00 a.m.² _____	4. 4	6. 47
_____	8. 5	6. 35
_____	17. 1	7. 02
_____	25. 7	7. 50
_____	34. 2	7. 85
9:05 a.m.² _____	38. 4	7. 85

Station 525; total depth, 46.6 ft

Time	Distance	Velocity
9:15 a.m.² _____	4. 7	4. 69
_____	9. 3	5. 51
_____	18. 7	7. 50
_____	28. 0	7. 50
_____	37. 3	7. 50
9:20 a.m.² _____	41. 9	7. 67

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

Time	Measuring point		Time	Measuring point	
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)
APR. 14, 1960—Continued			APR. 14, 1960—Continued		
Station 750; total depth, 48.4 ft			Station 1,701; total depth, 52.6 ft		
9:45 a.m.[2]	4.8	5.37	11:55 a.m.[2]	5.3	6.11
	9.7	6.60		10.5	6.35
	19.4	7.17		21.1	6.60
	29.0	7.50		31.6	6.73
	38.7	8.24		42.1	7.50
9:50 a.m.[2]	43.6	8.04	12:00 m	47.3	7.02
Station 924; total depth, 51.2 ft			JUNE 21, 1960		
10:05 a.m.[2]	5.1	5.37	Station 375; total depth, 23.3 ft		
	10.2	5.37			
	20.5	7.17			
	30.7	7.85	9:25 a.m.[2]	2.3	3.07
	41.0	8.24		4.7	3.07
10:10 a.m.[2]	46.1	8.04		9.3	3.21
				14.0	3.99
Station 1,074; total depth, 50.0 ft				18.6	4.18
			9:15 a.m.[2]	21.0	4.76
10:25 a.m.[2]	5.0	6.11			
	10.0	6.23	Station 575; total depth, 22.7 ft		
	20.0	7.17			
	30.0	7.85			
	40.0	7.85	9:50 a.m.[2]	2.3	3.38
10:30 a.m.[2]	45.0	8.97		4.5	3.99
				9.1	3.99
Station 1,200; total depth, 54.7 ft				13.6	4.47
				18.2	4.87
10:35 a.m	5.4	6.11	9:40 a.m.[2]	20.4	4.76
	10.9	6.11			
	22.1	7.85	Station 750; total depth, 31.0 ft		
	32.8	8.04			
	43.8	9.35			
10:40 a.m.[2]	49.2	9.55	10:15 a.m.[2]	3.1	3.58
				6.2	3.42
Station 1,320; total depth, 58.2 ft				12.4	3.74
				18.6	4.28
10:50 a.m.[2]	5.8	7.17		24.8	4.57
	11.6	8.24	10:05 a.m.[2]	27.9	4.38
	23.3	8.79			
	34.9	9.55	Station 950; total depth, 31.7 ft		
	46.6	9.98			
10:55 a.m.[2]	52.4	9.98	10:30 a.m.[2]	3.2	3.66
				6.3	4.08
Station 1,451; total depth, 61.6 ft				12.7	3.99
				19.0	4.98
11:15 a.m.[2]	6.1	7.33		25.4	4.87
	12.3	8.62	10:20 a.m.[2]	28.5	5.22
	24.7	8.79			
	37.0	9.55	Station 1,100; total depth, 36.8 ft		
	49.3	10.71			
11:20 a.m.[2]	55.4	10.46	10:45 a.m.[2]	3.7	2.87
Station 1,551; total depth, 61.1 ft				7.4	3.82
				14.7	4.18
11:30 a.m.[2]	6.1	8.14		22.1	4.66
	12.2	8.45		29.4	4.87
	24.4	9.55	10:35 a.m.[2]	33.1	5.22
	36.7	10.71			
	48.9	9.35			
11:35 a.m.[2]	55.0	9.76			

See footnotes at end of table, p. 54.

TABLE 4.—*Vertical distributions of velocity*—Continued

| Time | Measuring point | | Time | Measuring point | |
	Distance from bed (ft)	Velocity (fps)		Distance from bed (ft)	Velocity (fps)
JUNE 21, 1960—Continued **Station 1,200; total depth, 38.0 ft**			**JUNE 21, 1960—Continued** **Station 1,601; total depth, 40.5 ft**		
10:55 a.m.[2]_____	3. 8	3. 07	11:30 a.m.[2]_____	4. 0	3. 82
_____	7. 6	4. 08	_____	8. 1	3. 90
_____	15. 2	5. 09	_____	16. 2	4. 57
_____	22. 8	4. 57	_____	24. 3	4. 66
_____	30. 4	5. 48	_____	32. 4	5. 09
10:45 a.m.[2]_____	34. 2	5. 22	11:20 a.m.[2]_____	36. 5	5. 34
Station 1,320; total depth, 51.4 ft			**Station 1,701; total depth, 44.0 ft**		
11:10 a.m.[2]_____	5. 2	2. 50	11:45 a.m.[2]_____	4. 4	2. 62
_____	10. 3	4. 38	_____	8. 8	2. 56
_____	20. 6	4. 57	_____	17. 6	3. 38
_____	30. 8	5. 48	_____	26. 4	3. 58
_____	41. 1	5. 48	_____	35. 2	3. 74
11:00 a.m._____	46. 3	5. 48	11:35 a.m.[2]_____	39. 6	3. 58

[1] Average for about 11–15 seconds.   [2] Estimated.   [3] Average for about 115–130 seconds.

Velocity distributions at each of four or more stations in the cross section defined the average k's that are shown in columns 4 and 5 of table 5. The k's in column 4 were obtained by averaging the computed k's for the individual stations; the k's in column 5 were computed from the averages of the differences in velocity for a 10 to 1 change in distance above the streambed for the individual stations. T e difference between the two types of average k's is a rough measure of the uniformity of the k's for the individual stations and is small unless, as on May 13, 1959, k for one of the stations is large. The averages of the k's in columns 5 and 6 are 0.35 and 0.33, respectively.

As relatively few vertical distributions of velocity were defined by current-meter observations, k was also computed from the velocities of routine discharge measurements. Differences between the velocities at 0.2 and at 0.8 of the depth for all stations except those near banks and piers were added, and the total was divided by the number of stations to obtain the average velocity difference per station. The Keulegan type of equation for point velocity indicates that the velocity difference equals 2.30 (u_*/k) log 4 or 1.39 u_*/k for a relative change of 4 to 1 (0.8 to 0.2) in the distance above the streambed. Hence, k for the cross section was computed by dividing 1.39 u_* by the average velocity difference per station.

TABLE 5.—*Turbulence constants based on vertical distributions of velocity*

Date	Daily mean discharge (1,000 cfs)	Water-surface slope	Average k for cross section from—		
			Average of k's for stations	Average velocity difference at stations	Discharge measurements
(1)	(2)	(3)	(4)	(5)	(6)
1948					
Mar. 2	327	0.0000551	0.21	0.21	0.21
May 21	214	.0000589	.36	.36	--------------
1950					
May 8	330	.0000997	--------------	--------------	.33
May 22	347	.0000924	--------------	--------------	.32
June 5	326	.0000909	--------------	--------------	.31
1951					
Apr. 17	527	.000106	--------------	--------------	.37
Apr. 19	496	.000107	--------------	--------------	.31
July 16	704	.000126	--------------	--------------	.38
July 22	765	.000139	--------------	--------------	.34
July 30	472	.0000851	--------------	--------------	.27
Sept. 17	387	.0000632	--------------	--------------	.22
1952					
Mar. 17	411	.0000661	.30	.28	--------------
Apr. 30	682	.000110	[1] .24	--------------	.35
1954					
June 7	281	.0000691	.40	.40	.34
Aug. 9	115	.0000647	--------------	--------------	.33
Aug. 10	111	.0000633	[1] .35	--------------	--------------
Aug. 27	118	.0000720	[1] .44	--------------	--------------
1955					
Mar. 30, 31	163	.0000793	.41	.40	.37
Dec. 27, 28	57.4	.0000822	.37	.36	.39
1956					
Feb. 21	56.1	.0000822	.40	.40	.58
Apr. 24	169	.0000836	.39	.37	.32
May 9	159	.0000836	.37	.35	.32
July 9	154	.0000749	.34	.32	.32
1958					
Apr. 17	164	.0000962	.40	.39	--------------
1959					
May 13	240	.0000903	.40	.34	.25
July 10	150	.0000670	.32	.32	--------------
1960					
Apr. 14	[2] 581	.000131	.40	.35	.37
June 21	[2] 211	.0000728	.34	.34	.33

[1] One station only.
[2] Measured discharge.

The 11 k's computed from discharge measurements averaged almost the same—0.345 as compared with 0.349 (table 5, cols. 6, 5), as the 11 k's determined on the same days from several velocity observations at each of a few stations. Theoretically, the k's in column 5, which are based on measured velocities at only a few stations, should represent the average vertical distribution of velocity more closely for the measuring stations and less closely for the whole cross section

than the k's in column 6, which are based on velocities at about 20–30 stations. Actually, the differences in the two types of k's may be due mainly to experimental errors.

The method for computing k from discharge measurements is simple and rapid and usually requires no additional fieldwork except determining a reasonably accurate energy gradient. Although the velocity differences may be much less consistent from one station to another in shallow flows over dune beds than in the deep flows of the Mississippi River, the vertical distributions defined by many observations at only a few stations are also likely to be less consistent. For such shallow flows, the velocity differences for the large number of stations that are included in a discharge measurement, especially if the stations are spaced randomly with respect to the dunes, may provide a more representative vertical distribution of velocity for the cross section than the vertical distribution that can be obtained from detailed information for only a few stations.

The turbulence constants, mostly computed from discharge measurements, are fairly consistent (fig. 20) with those reported by Einstein and Chien (1955, fig. 15). Individual k's scatter considerably from the curve perhaps partly because of the rough streambed of the Mississippi River at St. Louis. Also, the data for flume studies and for the Missouri River, collected primarily for research studies, probably were more accurate than some of the routinely determined water-surface slopes at St. Louis and some velocity differences computed from discharge measurements. The k's based on 22 discharge measurements for the Mississippi River at St. Louis (table 5) average 0.33, which is considerably less than 0.40 but somewhat greater than would be indicated by the curve (fig. 20) that was defined by Einstein and Chien.

SEDIMENT CONCENTRATION

The usual equation for the vertical distribution of suspended sediment of a particular fall velocity is

$$\frac{c_y}{c_a} = \left(\frac{y_0 - y}{y} \frac{a}{y_0 - a} \right)^z$$

in which

c_y and c_a are concentrations of sediment of a particular fall velocity at distances y and a, respectively, above the streambed

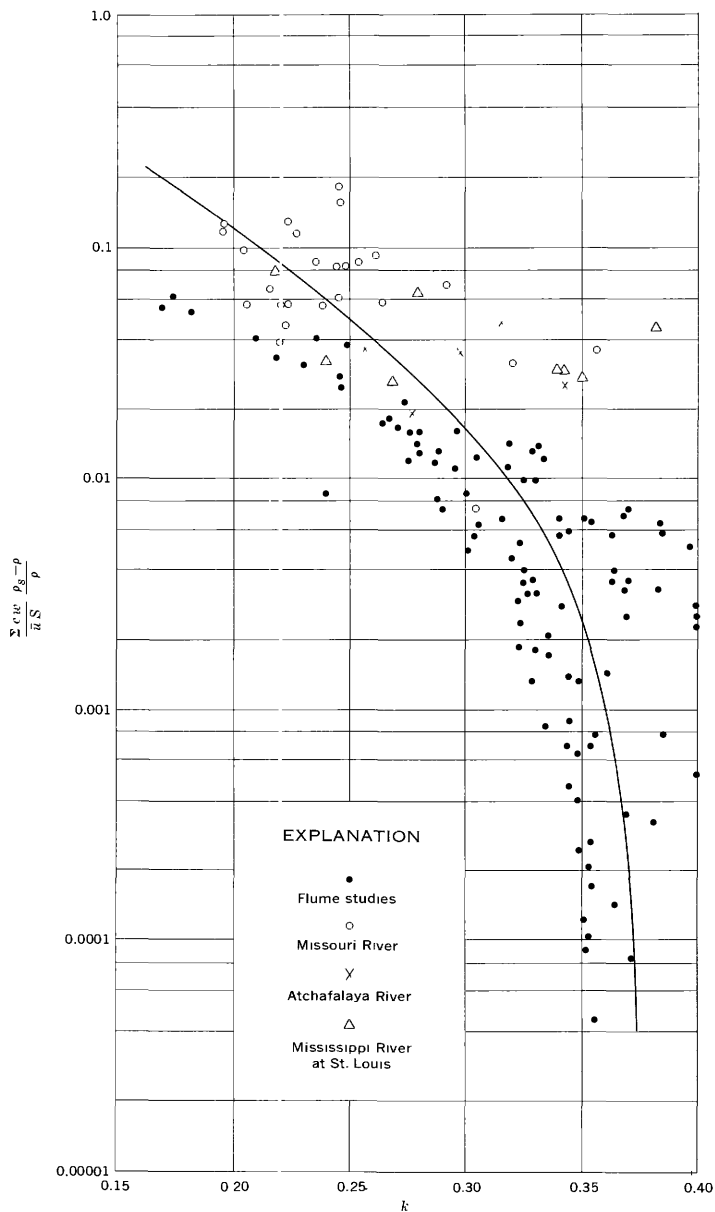z theoretically equals the fall velocity w divided by ku_*

FIGURE 20.—Effect of sediment on the turbulence constant computed from the vertical distribution of velocity. Data for the Mississippi River added to figure 15 of Einstein and Chien (1955).

TABLE 6.— *Values of z_1 from point samples*

Date	Sampling station	z_1 for indicated particle-size range			Date	Sampling station	z_1 for indicated particle-size range		
		<0.062 mm	0.062–0.125 mm	0.125–0.250 mm			<0.062 mm	0.062–0.125 mm	0.125–0.250 mm
1950					*1952*—Con.				
May 8	400	0.01	0.12	0.38	May 1—Con.	1275	0.03	0.21	0.29
	1200	.02	------	------		1575	.04	------	------
	1500	.06	------	.28	*1953*				
May 23	400	.02	------	------	Apr. 24	500	.03	.26	.56
	625	.02	------	------		900	.06	.27	.83
	900	.01	.02	------		1275	.05	.35	1.32
	1100	.04	------	------		1575	.01	.26	.77
	1250	.01	------	.14	Sept. 15	525	.01	.30	------
	1400	.01	------	.41		925	.01	.30	.73
	1500	0	------	.24		1275	0	.30	.59
	1650	.02	------	.21		1550	.03	.40	.99
June 7	400	.02	------	.21	*1954*				
	1200	.02	------	.12	June 7	475	.01	.22	------
	1500	.02	------	------		900	.02	.25	.75
Aug. 8	400	.05	------	------		1275	.10	.26	1.02
	1200	.05	.08	.07		1600	.04	.27	.72
	1500	.02	.12	------	*1955*				
Sept. 7	400	.06	.38	.43	Mar. 31	500	.01	.13	.49
	1200	.04	------	------		925	.02	.27	.42
	1500	.02	.07	------		1275	------	.21	.41
Sept. 26	400	.03	.47	.32		1575	------	.14	.32
	1200	.01	.49	.35	Dec. 27	550	.03	.26	.82
	1500	.15	.37	.47		925	0	.23	.61
1951						1275	.08	.10	.74
Apr. 18	450	.02	------	------	*1956*				
	925	------	.05	------	Feb. 21	550	0	.41	.43
	1250	.06	.27	------		925	.01	.38	.58
	1600	.01	.14	.36		1275	.02	.05	.67
July 16	900	.02	.11	------		1575	------	.22	.50
	1275	------	.09	------	Apr. 24	475	.02	.29	.49
	1600	------	------	.23		900	------	.39	.48
July 22	450	.01	------	------		1300	------	.18	.48
	875	------	------	.32		1575	0	.23	.48
	1250	------	------	.27	May 9	475	.02	.26	.76
	1600	------	.21	------		900	.04	.41	.70
July 30	450	.02	------	.20		1300	.01	.28	.34
	875	.01	------	------		1575	.05	------	.95
	1250	.02	.10	.25	July 9	500	.02	.33	.75
Sept. 17	475	.02	.22	------		925	------	------	.81
	900	.02	------	------		1275	.07	.31	.67
	1275	------	.16	------	*1959*				
	1600	------	.14	.38	May 13	375	.01	.04	.62
Sept. 26	475	.01	.18	.13		575	.02	.03	.17
	900	------	.20	------		775	0	.13	.35
	1275	.03	.21	------		925	.02	.25	.49
	1600	.02	.23	------		1075	.03	.33	.57
Oct. 17	475	.03	.03	.48		1200	.03	.27	.42
	900	.02	.37	.50		1300	.04	.23	.59
	1275	.07	.27	------		1425	.02	.23	.38
	1600	.05	------	.68		1550	.02	.22	.33
Nov. 13	475	.02	.30	.51		1675	.03	.16	.53
	900	------	.24	------	July 10	710	.02	.21	.17
	1275	------	.48	------		750	.02	.13	.96
	1600	------	.14	------		800	.02	.05	.35
1952						1000	.01	.15	.55
May 1	450	.01	.24	.28					
	850	.01	.09	------					

This equation is based on the assumption that the water surface and streambed are plane and parallel. In general, the form of the equation has been found to be correct except that the actual exponent, called z_1, generally differs materially from $w/(ku_*)$ (Anderson, 1942; Vanoni, 1946; Einstein and Chien, 1954).

The z_i's for the Mississippi River at St. Louis were computed for most times when point-integrated sediment samples from each of several points at a station were analyzed for sediment concentration and particle size. Each z_1 for a station and a size range is listed in table 6 and was determined by measuring the slope of the straight line through the plotted points of the logarithm of the concentration for a size range against the logarithm of $(y_0 - y)/y$. Some plotted points were too irregularly and widely scattered to define a line, and the z_1's based on the slopes of other lines were obviously inconsistent from one size range to the other. In general, the z_1's were more consistent and seemed to be more dependable after March 31, 1955, when the size analyses of the sands were made with the visual-accumulation tube (see table 7), than the z_1's on or before March 31, 1955, when the size analyses of sands were made with the bottom-withdrawal tube.

TABLE 7.—*Particle-size analyses of suspended sediment, point-integrated samples*

[Method of analysis: C, chemically dispersed; M, mechanically dispersed; P, pipet; V, visual accumulation tube; W, in distilled water]

Date	Time	Discharge (cfs)	Water temperature (°F)	Sampling station	Total depth (ft)	Sampling point		Suspended sediment								Method of analysis
						Depth (ft)	Concentration (ppm)	Percent finer than indicated size, in millimeters								
								0.002	0.004	0.016	0.062	0.125	0.250	0.500	1.000	
1955																
Dec. 27	12:52 p.m.	58,000	36	550	11.6	10.6	218	---	---	---	78	84	100	---	---	V
	1:02	58,000	---	550	11.6	10.4	293	---	---	---	65	76	100	---	---	V
	1:12	58,000	---	550	11.6	9.3	188	---	---	---	86	91	100	---	---	V
	1:22	58,000	---	550	11.6	7.0	202	---	---	---	90	96	100	---	---	V
	1:32	58,000	---	550	11.6	4.6	163	---	---	---	86	98	100	---	---	V
	1:40	58,000	36	550	11.6	1.2	166	---	---	---	97	99	100	---	---	V
	2:02	58,000	36	925	15.2	14.2	190	---	---	---	71	80	100	---	---	V
	2:12	58,000	---	925	15.2	13.7	164	---	---	---	79	85	100	---	---	V
	2:22	58,000	---	925	15.2	12.2	164	---	---	---	82	87	100	---	---	V
	2:32	58,000	---	925	15.2	9.1	162	---	---	---	82	91	100	---	---	V
	2:40	58,000	---	925	15.2	6.1	153	---	---	---	90	95	100	---	---	V
	2:45	58,000	36	925	15.2	1.5	125	---	---	---	95	99	100	---	---	V
	9:56 a.m.	58,000	37	1,275	23.7	22.7	107	---	---	---	53	58	100	---	---	V
	10:08	58,000	---	1,275	23.7	21.3	100	---	---	---	55	65	100	---	---	V
	10:18	58,000	---	1,275	23.7	19.0	121	---	---	---	50	61	100	---	---	V
	10:28	58,000	---	1,275	23.7	14.2	84	---	---	---	69	79	100	---	---	V
	10:32	58,000	---	1,275	23.7	9.5	64	---	---	---	77	89	100	---	---	V
	10:38 a.m.	58,000	36	1,275	23.7	2.4	50	---	---	---	83	96	100	---	---	V
1956																
Feb. 21	11:36 a.m.	56,100	36	550	11.9	10.9	138	---	---	---	76	81	97	100	---	V
	11:50	56,100	---	550	11.9	10.7	115	---	---	---	75	85	98	100	---	V
	12:03 p.m.	56,100	---	550	11.9	9.5	121	---	---	---	75	84	100	---	---	V
	12:14	56,100	---	550	11.9	7.1	103	---	---	---	82	92	100	---	---	V
	12:28	56,100	---	550	11.9	4.8	102	---	---	---	85	92	100	---	---	V
	12:42	55,125	---	550	11.9	1.2	102	---	---	---	98	100	100	---	---	V
	1:22	56,100	36	925	14.3	13.3	112	---	---	---	68	77	100	---	---	V
	1:30	56,100	---	925	14.3	11.4	98	---	---	---	70	83	100	---	---	V
	1:44	56,100	---	925	14.3	8.6	80	---	---	---	83	93	100	---	---	V
	1:58	56,100	---	925	14.3	5.7	88	---	---	---	87	91	100	---	---	V
	2:08	56,100	---	925	14.3	1.4	78	---	---	---	97	100	100	---	---	V

Apr. 24	2:37	56,100	35	1,275	22.3	21.3	67					54	64	95	100		V
	2:52	56,100		1,275	22.3	20.1	94					53	61	100	100		V
	3:02	56,100		1,275	22.3	17.8	80					58	64	92			V
	3:12	56,100		1,275	22.3	18.4	60					70	82	100			V
	3:22	56,100		1,275	22.3	8.9	49					86	97	100			V
	3:30	56,100		1,275	22.3	9.2	46					86	100	100			V
	3:52	56,100		1,525	18.5	17.5	90					36	52	90	100		V
	4:02	56,100		1,525	18.5	16.7	95					36	47	96	100		V
	4:11	56,100		1,525	18.5	14.8	69					52	65	96			V
	4:20	56,100		1,525	18.5	11.1	60					64	77	100			V
	4:28	56,100		1,525	18.5	7.4	46					66	82	100			V
	4:37 p.m.	56,100	36	1,525	18.5	1.9	61					67	93	100			V
	12:45 p.m.	170,000		475	27.8	26.8	696	20		40		53	61	86	100		VPWM
	12:50	170,000		475	27.8	25.0	508	23		47		63	75	88	100		VPWM
	12:55	170,000	51	475	27.8	22.2	421	25		58		78	86	98			VPWM
	1:00	170,000		475	27.8	16.7	415	25		60		78	86	100			VPWM
	1:05	170,000		475	27.8	11.1	354	28		66		88	94	100			VPWM
	1:15	170,000		475	27.8	2.8	359	31		63		89	94	100			VPWM
	1:40	170,000		900	27.0	26.0	878	9		20		22	34	88	98		VPWM
	1:45	170,000		900	27.0	24.3	558	16		30		39	52	94	100		VPWM
	1:50	170,000		900	27.0	21.6	420	16		40		51	63	98	100		VPWM
	1:55	170,000		900	27.0	16.2	434	22		43		50	63	96	100		VPWM
	2:00	170,000		900	27.0	10.8	332	29		48		69	77	96	100		VPWM
	2:10	170,000	49	900	27.0	2.7	272	30		64		81	85	100			VPWM
	2:40	170,000		1,300	43.9	42.9	587	12		31		34	39	88	100		VPWM
	2:45	170,000		1,300	43.9	39.5	519	12		33		37	40	88	97		VPWM
	2:50	170,000		1,300	43.9	35.1	377	23		51		57	60	95	100		VPWM
	3:00	170,000		1,300	43.9	26.3	317	28		58		67	70	100			VPWM
	3:05	170,000		1,300	43.9	17.6	245	34		71		83	89	100			VPWM
	3:10	170,000	49	1,300	43.9	4.4	251	43		68		85	90	99			VPWM
	3:35	170,000		1,575	30.6	29.6	455	19		39		46	49	86	100		VPWM
	3:40	170,000		1,575	30.6	27.5	364	22		51		56	62	88	100		VPWM
	3:50	170,000		1,575	30.6	24.5	343	26		57		66	68	94	100		VPWM
	4:00	170,000		1,575	30.6	18.4	309	32		65		72	77	98	99		VPWM
	4:05	170,000		1,575	30.6	12.2	262	32		66		78	85	97	100		VPWM
	4:10 p.m.	170,000		1,575	30.6	3.1	224	38		83		93	95	100			VPWM
May 9	10:55 a.m.	159,000		475	27.8	26.8	1,260	11		30		71	76	88	97		VPWM
	11:00	159,000		475	27.8	25.0	1,030	16		36		78	91	88	100		VPWM
	11:05	159,000		475	27.8	22.2	987	17		37		89	93	99	100		VPWM
	11:10	159,000		475	27.8	16.7	886	18		40		96	98	100			VPWM
	11:15	159,000		475	27.8	11.1	887	21		42		97	98	100			VPWM
	11:20	159,000	61	475	27.8	2.8	838	24		42		97	99	100		100	VPWM

TABLE 7.—*Particle-size analyses of suspended sediment, point-integrated samples*—Continued

Date	Time	Discharge (cfs)	Water temperature (°F)	Sampling station	Total depth (ft)	Sampling point Depth (ft)	Sampling point Concentration (ppm)	0.002	0.004	0.016	0.062	0.125	0.250	0.500	1.000	Method of analysis
1966—Con.																
May 9—Con.	11:50 a.m.	159,000	------	900	27.0	26.0	1,210	12	----	22	52	60	98	100	----	VPWM
	11:55	159,000	------	900	27.0	24.3	898	14	----	28	65	72	83	99	100	VPWM
	12:00 m.	159,000	------	900	27.0	21.6	817	17	----	33	76	84	89	100	----	VPWM
	12:05 p.m.	159,000	------	900	27.0	16.2	663	17	----	36	85	89	100	----	----	VPWM
	12:15	159,000	60	900	27.0	10.8	598	19	----	39	90	93	100	----	----	VPWM
	12:20	159,000	------	900	27.0	2.7	526	22	----	43	96	99	100	----	----	VPWM
	1:10	159,000	------	1,300	43.8	42.8	501	12	----	20	40	49	85	100	----	VPWM
	1:05	159,000	------	1,300	43.8	39.4	314	24	----	35	66	74	98	100	----	VPWM
	1:00	159,000	------	1,300	43.8	35.0	340	18	----	27	56	63	97	100	----	VPWM
	1:20	159,000	------	1,300	43.8	26.3	236	16	----	26	70	76	100	----	----	VPWM
	1:25	159,000	------	1,300	43.8	17.5	320	20	----	34	80	69	98	----	----	VPWM
	1:30	159,000	60	1,300	43.8	4.4	205	32	----	44	90	94	100	100	----	VPWM
	1:55	160,000	------	1,575	32.8	31.8	821	4	----	11	24	25	61	100	----	VPWM
	2:00	160,000	------	1,575	32.8	29.5	377	8	----	23	44	45	77	100	----	VPWM
	2:05	160,000	------	1,575	32.8	26.2	293	15	----	29	60	61	93	100	----	VPWM
	2:10	160,000	------	1,575	32.8	19.7	190	4	----	38	79	86	99	100	----	VPWM
	2:13	160,000	------	1,575	32.8	13.1	165	14	----	51	86	95	100	----	----	VPWM
	2:20 p.m.	160,000	62	1,575	32.8	3.3	159	16	----	37	100	100	100	----	----	VPWM
July 9	11:32 a.m.	154,000	------	500	25.8	24.8	2,150	53	58	72	89	92	100	----	----	VPWCM
	11:32	154,000	------	500	25.8	23.2	1,930	52	59	72	94	96	100	----	----	VPWCM
	11:32	154,000	------	500	25.8	20.6	1,910	52	58	77	95	97	100	----	----	VPWCM
	11:32	154,000	------	500	25.8	15.5	1,850	53	61	80	98	99	100	----	----	VPWCM
	11:32	154,000	------	500	25.8	10.3	1,720	59	67	82	99	100	----	----	----	VPWCM
	11:32	154,000	------	500	25.8	2.6	1,770	56	64	82	99	100	----	----	----	VPWCM
	12:10 p.m.	153,000	------	925	25.7	24.7	1,590	51	58	73	93	95	100	----	----	VPWCM
	12:10	153,000	------	925	25.7	23.1	1,550	61	65	80	94	97	100	----	----	VPWCM
	12:10	153,000	------	925	25.7	20.6	1,460	57	63	80	97	99	100	----	----	VPWCM
	12:10	153,000	------	925	25.7	15.4	915	58	69	74	98	99	100	----	----	PWCM
	12:10	153,000	------	925	25.7	10.3	1,480	60	67	84	¹99	99	----	----	----	PWCM
	12:10	153,000	------	925	25.7	2.6	1,440	63	64	80	¹100	100	----	----	----	PWCM

VPWCM	---		100	99	83	78	64	53	50	1,010	37.8	38.8	1,275		153,000	12:45
VPWCM	---		100	99	83	78	57	52	47	1,120	34.9	38.8	1,275		153,000	12:45
VPWCM	---		100	100	89	85	58	53	44	777	31.0	38.8	1,275		153,000	12:45
VPWCM	---		100	100	94	90	76	56	47	709	23.3	38.8	1,275		153,000	12:45
VPWCM	---		100	100	97	94	72	64	53	638	15.5	38.8	1,275		153,000	12:45
PWCM	---		100	100	99[1]	99	77	71	53	467	3.9	38.8	1,275		153,000	12:45
VPWCM	---		100	98	67	59	44	32		312	29.0	30.0	1,575		153,000	1:18
VPWCM	---		100	98	85	76	60	54		231	27.0	30.0	1,575		153,000	1:18
VPWCM	---			100	87	81	58	51		245	24.0	30.0	1,575		153,000	1:18
PWCM	---						72	72		254	18.0	30.0	1,575		153,000	1:18
PWCM	---						79	64		193	12.0	30.0	1,575		153,000	1:18
PWCM	---						82	82		165	3.0	30.0	1,575		153,000	1:18 p.m.

1969

May 13

V	---	100	96	90	83				887	37.5	41.7	1,675	68	239,000	6:30 a.m.
V	---		100	97	90				809	33.4	41.7	1,675		239,000	6:36
V	---		98	91	83				870	25.0	41.7	1,675		239,000	6:39
V	---			100	95				732	16.7	41.7	1,675		239,000	6:42
V	---	100		99	95				713	4.2	41.7	1,675		239,000	6:45
V	---	100	89	82	77				1,110	35.5	39.4	1,550	68	239,000	6:50
V	---		96	89	84				936	31.6	39.4	1,550		239,000	6:54
V	---		96	92	90				1,190	23.6	39.4	1,550		239,000	6:57
V	---		99	98	94				810	15.8	39.4	1,550		240,000	7:00
V	---		100	97	94				826	3.9	39.4	1,550		240,000	7:03
V	---	100	97	90	85				1,110	40.0	44.4	1,425	68	240,000	7:07
V	---		99	93	89				1,080	35.5	44.4	1,425		240,000	7:10
V	---		98	92	89				1,040	26.6	44.4	1,425		240,000	7:13
V	---		100	95	92				1,010	17.8	44.4	1,425		240,000	7:16
V	---		100	98	95				942	4.4	44.4	1,425		240,000	7:19
V	---	100	98	92	89				1,280	40.5	45.0	1,300	68	240,000	7:24
V	---		94	87	83				1,370	36.0	45.0	1,300		240,000	7:27
V	---		99	97	94				1,150	27.0	45.0	1,300		240,000	7:30
V	---		100	98	96				1,130	18.0	45.0	1,300		240,000	7:33
V	---		100	99	97				1,060	4.5	45.0	1,300		240,000	7:36
V	---	100	98	91	89				1,510	33.1	36.8	1,200	68	240,000	7:40
V	---		98	92	90				1,500	29.4	36.8	1,200		240,000	7:44
V	---		99	94	92				1,430	22.1	36.8	1,200		240,000	7:47
V	---		100	95	96				1,350	14.7	35.3	1,200		240,300	7:50
V	---		100	98	97				1,270	3.7	36.8	1,200		240,000	7:53
V	---	100	85	80	78				2,000	34.4	38.2	1,075	67	240,000	7:59
V	---		99	96	94				1,660	30.6	38.2	1,075		240,000	8:02
V	---		100	98	97				1,620	22.9	38.2	1,075		240,000	8:05
V	---		100	97	96				1,690	15.3	38.2	1,075		240,000	8:08
V	---		100	100	99				1,430	3.8	38.2	1,075		240,000	8:00

See footnotes at end of table.

TABLE 7.—Particle-size analyses of suspended sediment, point-integrated samples—Continued

Date	Time	Discharge (cfs)	Water temperature (°F)	Sampling station	Total depth (ft)	Sampling point Depth (ft)	Sampling point Concentration (ppm)	Suspended sediment — Percent finer than indicated size, in millimeters 0.002	0.004	0.016	0.062	0.125	0.250	0.500	1.000	Method of analys
1950—Con.																
May 13—Con.	8:15 a.m.	240,000	67	925	34.2	30.8	2,050				90	92	98	100		▷
	8:18	240,000		925	34.2	27.4	2,020				93	95	100			▷
	8:21	240,000		925	34.2	20.5	2,010				94	96	100			▷
	8:24	240,000		925	34.2	13.7	1,890				96	98	100			▷
	8:27	240,000		925	34.2	3.4	1,750				98	99	100			▷
	8:32	240,000	68	775	32.6	29.3	2,210				94	95	99	100		▷
	8:35	240,000		775	32.6	26.1	2,150				94	96	100			▷
	8:38	240,000		775	32.6	19.6	2,100				97	98	100			▷
	8:41	240,000		775	32.6	13.0	2,160				97	98	100			▷
	8:44	240,000		775	32.6	3.3	2,080				98	99	100			▷
	8:48	240,000	67	575	30.0	27.0	2,460				95	96	100			▷
	8:51	240,000		575	30.0	24.0	2,380				96	97	99	100		▷
	8:54	240,000		575	30.0	18.0	2,350				97	98	100			▷
	8:57	240,000		575	30.0	12.0	2,290				97	98	100			▷
	9:00	240,000		575	30.0	3.0	2,250				98	99	100			▷
	9:05	240,000	67	375	28.4	25.6	2,590				95	96	100			▷
	9:08	240,000		375	28.4	22.7	2,490				97	98	100			▷
	9:11	240,000		375	28.4	17.0	2,410				98	99	100			▷
	9:14	240,000		375	28.4	11.4	2,430				98	99	100			▷
	9:17 a.m.	240,000		375	28.4	2.8	2,420				99	100	100			▷
July 10 [2]	5:26 a.m.	154,000		1,000	31.7	30.2	2,340				96.5	97.8	99.7	100.0		▷
	5:34	154,000		1,000	31.7	28.5	2,290				96.6	98.0	99.7	100.0		▷
	5:40	154,000		1,000	31.7	25.4	1,970				97.7	98.8	100.0			▷
	5:45	154,000		1,000	31.7	22.2	2,270				98.5	99.5	100.0			▷
	5:50	153,000		1,000	31.7	19.0	2,240				98.5	99.5	100.0			▷
	5:56	153,000		1,000	31.7	15.8	2,180				98.9	99.8	100.0			▷
	6:01	153,000		1,000	31.7	12.7	2,140				99.0	99.8	100.0			▷
	6:06	153,000		1,000	31.7	9.5	2,200				98.9	99.6	100.0			▷
	6:11	153,000		1,000	31.7	6.3	2,180				99.2	100.0	100.0			▷
	6:16	153,000		1,000	31.7	3.2	2,140				99.2	99.8	100.0			▷

Time														
6:56	152,000	—	800	26.0	24.5	2,560	—	—	97.8	98.7	99.7	100.0	—	>
7:02	152,000	—	800	26.0	23.4	2,540	—	—	98.5	99.4	100.0	—	—	>
7:08	152,000	—	800	26.0	20.8	2,540	—	—	98.9	99.6	100.0	—	—	>
7:13	152,000	—	800	26.0	18.2	2,520	—	—	98.7	99.7	100.0	—	—	>
7:20	152,000	—	800	26.0	16.6	2,540	—	—	99.0	99.7	100.0	—	—	>
7:24	152,000	—	800	26.0	13.0	2,490	—	—	98.5	99.6	100.0	—	—	>
7:28	152,000	—	800	26.0	10.4	2,500	—	—	98.8	99.6	100.0	—	—	>
7:34	152,000	—	800	26.0	7.8	2,500	—	—	99.2	100.0	—	100.0	—	>
7:40	152,000	—	800	26.0	6.2	2,500	—	—	99.2	99.2	99.9	—	—	>
7:44	151,000	—	800	26.0	2.6	2,420	—	—	99.6	100.0	—	—	—	>
8:16	151,000	—	750	26.2	24.7	2,570	—	—	98.5	99.2	100.0	—	—	>
8:22	151,000	—	750	26.2	23.6	2,590	—	—	98.3	99.2	100.0	—	—	>
8:27	151,000	—	750	26.2	21.0	2,550	—	—	98.9	99.6	100.0	—	—	>
8:32	151,000	—	750	26.2	18.3	2,580	—	—	98.6	99.6	100.0	—	—	>
8:38	151,000	—	750	26.2	15.7	2,590	—	—	99.0	99.7	100.0	—	—	>
8:43	151,000	—	750	26.2	13.1	2,600	—	—	98.4	99.2	99.7	100.0	—	>
8:48	151,000	—	750	26.2	10.5	2,490	—	—	99.2	99.7	100.0	—	—	>
8:52	151,000	—	750	26.2	7.9	2,500	—	—	99.2	99.8	100.0	—	—	>
8:57	150,000	—	750	26.2	6.2	2,440	—	—	99.9	99.9	100.0	—	—	>
9:01	150,000	—	750	26.2	2.6	2,500	—	—	99.6	100.0	—	—	—	>
9:06	150,000	—	710	35.2	33.7	2,540	—	—	99.0	99.6	100.0	—	—	>
9:12	150,000	—	710	35.2	31.7	2,670	—	—	98.3	99.5	100.0	—	—	>
9:18	150,000	—	710	35.2	28.2	2,630	—	—	98.4	99.6	100.0	—	—	>
9:24	150,000	—	710	35.2	24.6	2,560	—	—	99.0	99.7	100.0	—	—	>
9:30	150,000	—	710	35.2	21.1	2,620	—	—	98.0	98.9	99.8	100.0	—	>
9:36	150,000	—	710	35.2	17.6	2,670	—	—	99.0	99.7	100.0	—	—	>
9:41	150,000	—	710	35.2	14.1	2,580	—	—	99.1	100.0	—	100.0	—	>
9:46	150,000	—	710	35.2	10.6	2,540	—	—	98.9	99.9	—	—	—	>
9:51	150,000	—	710	35.2	7.0	2,640	—	—	99.5	100.0	—	—	—	>
9:56 a.m.	150,000	—	710	35.2	3.5	2,630	—	—	99.5	100.0	—	—	—	>

[1] Estimated.

[2] Percent-finer values accurate to ±0.2 because large amounts of material were analyzed.

The average z_1 for a cross section and for a range of sand size was plotted against $w/(ku_*)$ to define the relationship (fig. 21) between
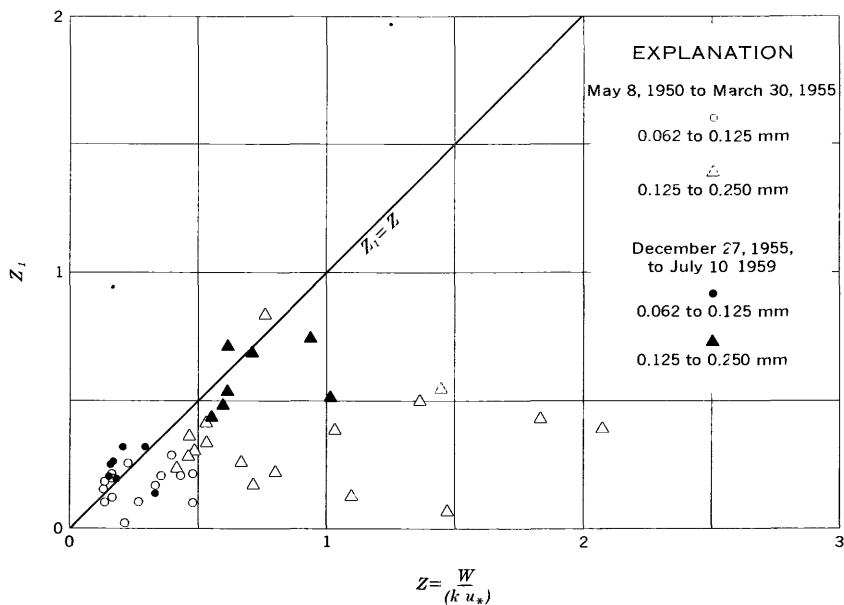
FIGURE 21.—z_1 as a function of $w/(ku_*)$.

z_1 and z. (A size range including both silt and clay is too wide to have a single representative fall velocity; so, the z_1's for the range of smallest sizes were not plotted.) The z_1's and z's through March 31, 1955, were based, respectively, on size analyses of sands that were made with the bottom-withdrawal tube and on k's that were computed from the discharge measurements; they are less consistent than the z_1's and z's for later times. Although the points scatter widely in figure 21, especially for the earlier determinations, the average of all the z_1's in table 6 for the size range from 0.062 to 0.125 mm is 0.20, which compares reasonably well with the average z of 0.27. If based only on the seven determinations after March 31, 1955, the average z_1 and z are 0.24 and 0.21, respectively. Thus, for this size range the average z_1 is roughly the same as the average z. The ratio of the average z_1 for the size range from 0.125 to 0.250 mm to the average z_1 for the size range from 0.062 to 0.125 mm is 0.41/0.20 (equal 2.0:1.0) for all the determinations or 0.59/0.24 (equals 2.5:1.0) for the last seven determinations. The ratio of fall velocities of representative

particles for the two size ranges varies with temperature but averages about $3.3:1.0$. These fall velocities were based on equations given by Rubey (1933). Hence, the average z_1's for all the determinations indicate a variation of z_1 with about the 0.58 power of the fall velocity, and the average z_1's for the last seven determinations indicate a variation of z_1 with about the 0.77 power of the fall velocity. For some shallow streams in Nebraska, z_1 for sands varied with about the 0.7 power of fall velocities computed from Rubey's equations (Colby and Hembree, 1955, p. 70; Hubbell and Matejka, 1959, p. 72). Except for the difference between the theoretical variation and the actual variation of z_1 with changing fall velocity, the theoretical equation for the vertical distribution of sediment concentration seems to apply reasonably well for the Mississippi River at St. Louis.

The relationship (fig. 21) of z_1 to $w/(ku_*)$ for the deep flows over rough beds at St. Louis seems to be generally similar and about as good as the relationship that was reported by Einstein and Chien (1954, p. 3 and fig. 1) for flows over a plane bed in the Missouri River at Omaha. That is, the z_1's and z's are approximately equal for low z's, but the z_1's do not increase nearly so fast as the z's. In general, z_1 seems to be the same function of $w/(ku_*)$ for deep flows over dune beds as for flows over plane beds. However, this conclusion is inconsistent with the suggestion by Einstein (1950, p. 28, 59) that z be computed from the equation $z = w/(0.40\ u_*')$. The difference between $w/(0.40u_*')$ and $w/(ku_*)$ is large at St. Louis. As k averaged about 0.34 and the ratio of u_*' to u_* for the 10 determinations in table 3 averaged 0.46, the z's computed according to Einstein's equation might be about $0.34/(0.40)(0.46)$ or roughly 1.8 times as large as those that were computed from $w/(ku_*)$. Vanoni and Brooks (1957, p. 73–75) noted that the observed vertical distributions of sediment over dune beds in certain laboratory experiments were inconsistent with any decrease in u_* such as Einstein suggested. A recent reexamination of Einstein's derivation of the equation for z indicates that the classical equation $z = w/(ku_*)$ probably should have been obtained for flow over dune beds as well as for flow over plane beds. Although the basic assumptions on which the derivation is based do not apply accurately for flow over dune beds, the only obvious change in the equation for z for flow over dune beds as compared with flow over plane beds is that the k for widely spaced major roughness elements, such as dunes, may not be a true turbulence constant.

BEDLOAD DISCHARGE AND BED-MATERIAL DISCHARGE

Samples obtained with samplers currently available for practical field use can be used to determine the sediment discharge in the zone

traversed by the sampler or the size distribution of the material in the bed, but they cannot be used to determine the discharge of material on the bed or in close proximity to the bed. No samplers are as yet available that will measure the bedload discharge or the total sediment discharge in a natural stream. In various rivers the measured suspended-sediment discharge probably ranges from about 40 percent to nearly 100 percent of the total sediment discharge. The bed-material discharge affects the stage-discharge relationship and builds bars that may obstruct the channel for navigation; so it is a significant part of the sediment discharge even though it may be a fairly small percentage of the total. Therefore, the relation of bed-material discharge to velocity and temperature was studied, and several different equations were used to compute bedload discharge and bed-material discharge.

Because the bed-material discharge consists of particles of the same size as those present in significant quantities in the bed, the size distributions of samples from the bed were examined to determine the sizes that are part of the bed-material discharge. On the average, the bed-material samples had very small amounts of material finer than 0.125 mm. Therefore, the bed-material discharge is considered in this report to be the rate of transport of all particles larger than 0.125 mm, regardless of the mode of transport.

The measured bed-material discharge (see "Definitions") is computed from the measured concentration of particles larger than 0.125 mm in the sampled zone and the streamflow for the whole cross section. Therefore, it includes all the bed-material discharge in the sampled zone and part of the bed-material discharge in the unsampled zone. The measured bed-material discharge was closely related to the mean velocity and to the water temperature (fig. 22). The measured bed-material discharge increased with about the fourth power of the mean velocity; and, for a given mean velocity, the high discharges were associated with low water temperatures.

Several different equations were used to compute bedload discharges and bed-material discharges, and the results are shown in table 8. For most of the computations, the hydraulic data and sediment data were obtained from measurements made within a 3-day period and are considered to be concurrent. For the other computations, hydraulic data were obtained from rating curves of discharge, depth, and velocity plotted against gage height. The results were studied to determine the equations that are best suited for application to this river and to similar rivers.

Because an accurate measurement of bedload discharge or total bed-material discharge is not possible, an absolute determination of
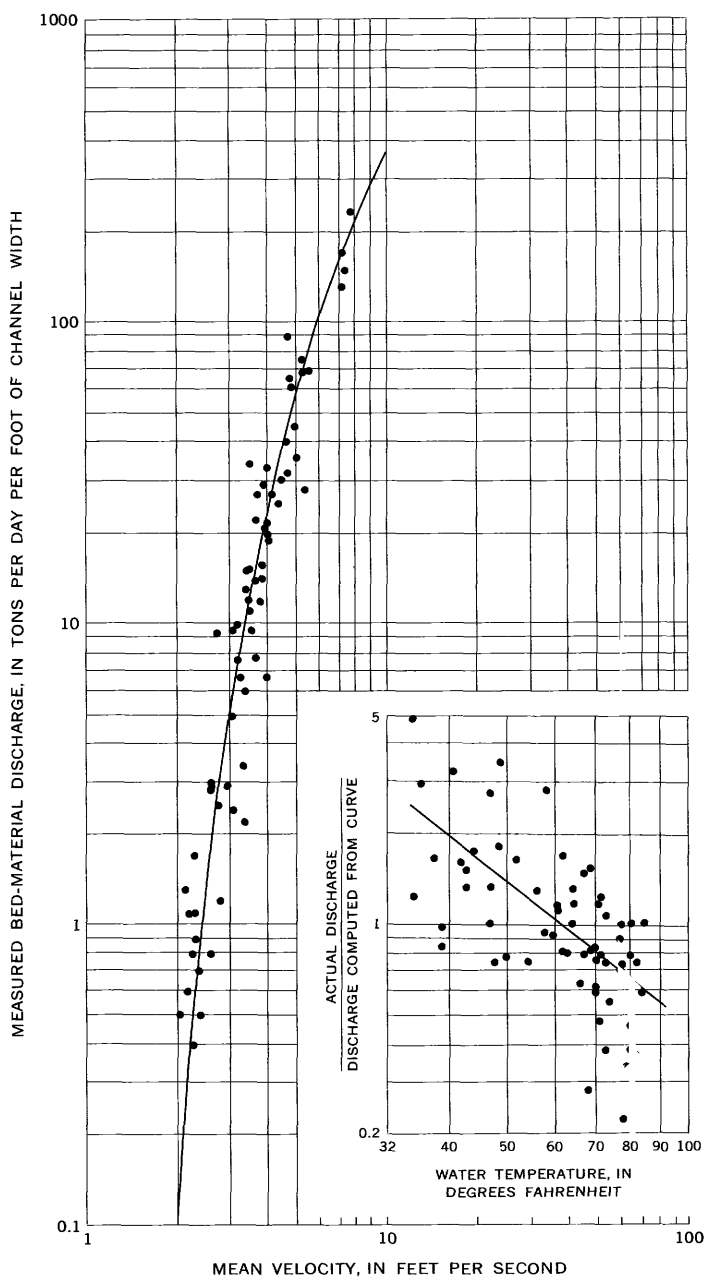
FIGURE 22.—Relation of measured bed-material discharge to mean velocity and water temperature.

TABLE 8.—*Computed bedload and bed-material discharges*

Date	Streamflow (cfs)	Mean velocity (fps)	Measured bed-material discharge[1] (tons per day)	Sediment discharge computed by indicated method (tons per day)								
				Bedload discharge					Bed-material discharge			
				Schok-litsch	Meyer-Peter and Müller	Kalinske	Modified Einstein	Bagnold	Straub	Modified Einstein	Colby	Bagnold
1951												
May 23	359,000	5.26	94,000	10,000			8,400	38,000		190,000	160,000	210,000
July 16	704,000	[2]7.26	250,000	33,000			22,000	90,000	540,000	470,000	370,000	530,000
July 22	763,000	[2]7.53	300,000	26,000			24,000		780,000	560,000		
July 30	468,000	5.37	46,000				8,600	24,000		120,000	120,000	120,000
Sept. 17	387,000	5.03	61,000				6,100			190,000		
Sept. 26	238,000	3.93	11,000	1,900			1,800	14,000		52,000	35,000	74,000
Oct. 10	234,000	3.91	30,000				2,300			43,000		
Oct. 17	170,000	3.31	11,000				800	6,100		18,000	28,000	20,000
Nov. 13	225,000	3.97	55,000	540			3,800	3,200		68,000	89,000	13,000
Dec. 4	202,000	3.88	52,000	630			2,000			75,000		
1952												
Feb. 20	222,000	6.36	96,000	3,300			6,900	3,500		380,000	400,000	330,000
Mar. 17	411,000		250,000	9,400			30,000			500,000	620,000	
May 1	674,000	7.96	440,000	20,000				59,000				
June 19	198,000	3.62	5,600	1,600			1,500	10,000		28,000	20,000	51,000
July 9	210,000	3.70	13,000	2,000			1,400			48,000	29,000	
July 24	178,000	3.37	4,500	1,300			770	7,000		18,000	18,000	
Aug. 8	161,000	3.13	5,200	680			680			10,000	14,000	
Aug. 19	167,000	3.37	5,600	970			990			21,000	15,000	29,000
Sept. 11	107,000	2.68	2,600	330			240	4,000		3,400	2,300	
Sept. 29	74,700	2.15	540	240			57	4,000		1,000	5,000	8,800
Dec. 29	67,500		2,000	190	150	2,500	57	4,800	17,000	2,900	5,000	16,000
1953												
Jan. 13	62,000	2.04	750	200	140	2,000	41	5,300		940	2,800	20,000
May 25	179,000	3.94	26,000	2,800	2,200	3,000	2,000		68,000	60,000	48,000	
July 10	197,000	4.00	25,000	2,800	2,700	3,800	2,900	6,500	79,000	63,000	46,000	30,000
Aug. 10	159,000	3.51	15,000	2,700	1,200	3,400	1,300	5,273	72,000	34,000	30,000	23,000
1954												
Mar. 22	80,500	2.63	4,400	1,200	710	2,800	360	3,200		9,100	9,800	14,000
Apr. 19	138,000	3.42	21,000	2,600	1,400	3,500	1,400	5,500	69,000	37,000	36,000	25,000
July 26	115,000	3.07	3,700	1,100	980	3,200	1,100	4,900	33,000	11,000	9,400	22,000
Oct. 18	214,000	4.22	44,000	3,500	3,300	4,900	4,300	11,000	88,000	93,000	73,000	53,000

Date												
1955												
Feb. 24	291,000	5.11	110,000	6,000	4,000	4,000	5,000	9,700	130,000	240,000	170,000	47,000
Mar. 30	163,000	3.82	---	3,800	2,100	4,000	2,100	6,900	96,000	---	---	32,000
Dec. 27	59,800	2.33	5,300	580	380	3,000	270	4,700	26,000	6,100	8,600	20,000
1956												
Nov. 5	55,200	2.31	2,500	790	430	3,000	270	4,800	32,000	4,800	5,300	20,000
1957												
Apr. 9	259,000	4.76	150,000	5,900	4,400	4,900	4,600	10,000	180,000	210,000	220,000	57,000
May 7	172,000	3.76	20,000	---	---	---	2,200	11,000	---	34,000	---	---
May 21	308,000	4.96	100,000	---	---	---	5,000	18,000	---	130,000	---	---
June 5	219,000	4.01	64,000	---	---	---	2,900	17,000	---	---	---	---
June 28	220,000	4.06	21,000	---	---	---	2,900	---	---	28,000	45,000	---
Oct. 7	74,100	2.63	4,500	---	---	---	400	6,300	---	6,900	10,000	27,000
Nov. 15	81,200	2.89	7,200	---	---	---	560	---	---	12,000	15,000	---
1958												
Apr. 14	172,000	3.81	33,000	---	---	---	2,300	10,000	---	42,000	57,000	50,000
Apr. 29	135,000	3.41	24,000	---	---	---	1,400	9,000	---	29,000	41,000	47,000
May 19	109,000	3.15	7,700	---	---	---	1,400	8,000	---	12,000	19,000	36,000
June 17	310,000	5.29	130,000	---	---	---	7,400	20,000	---	180,000	190,000	100,000
July 8	235,000	4.62	53,000	---	---	---	5,000	21,000	---	93,000	89,000	110,000
Aug. 27	102,000	2.89	1,800	---	---	---	580	---	---	5,100	8,900	---
Sept. 9	128,000	3.32	3,600	---	---	---	1,300	8,400	---	9,400	16,000	38,000
1959												
Mar. 25	243,000	4.78	110,000	---	---	---	5,900	18,000	---	160,000	---	---
Apr. 8	287,000	4.80	45,000	---	---	---	14,000	33,000	---	91,000	---	---
May 13	241,000	4.45	40,000	---	---	---	6,500	21,000	120,000	90,000	---	---

[1] The part of the measured suspended-sediment discharge that is coarser than 0.125 mm.
[2] Excluding overbank flow.

the accuracy of an equation cannot be made. Indirect methods of evaluating the equations must be used. The measurable quantity that is probably most closely related to the bedload discharge and to the total bed-material discharge is the measured part of the bed-material discharge. This quantity was used as the primary means of evaluating the results of the different equations.

Measurements in rivers that have natural contractions or artificial turbulence flumes where the total sediment discharge is in suspension have shown that the bed-material discharge is closely related to the mean velocity in a normal section of the river. The bed-material discharge varies with about the third power of the mean velocity. Therefore, the mean velocity was used as a secondary means of evaluating the results of the equations.

Bedload discharges were computed with the Schoklitsch, the Meyer-Peter and Müller, the Kalinske, the modified Einstein, and the Bagnold methods.

Schoklitsch's equation (Shulits, 1935) expresses the bedload discharge as a function of the particle diameter, the slope, the streamflow, and a critical discharge that depends on the particle diameter:

$$q_b = \frac{86.7}{\sqrt{12d}} \, S_e^{3/2} \left(q - 0.00532 \, \frac{12d}{S_e^{4/3}} \right)$$

where

$\quad q_b$ is bedload discharge, in pounds per second per foot of width
$\quad d$ is particle diameter, in feet
$\quad S_e$ is "hydraulic slope," presumably the slope of the energy gradient
$\quad q$ is streamflow, in cubic feet per second per foot of width

The equation can be used to compute the bedload discharges by ranges of particle size. Bedload discharges computed for the whole width with Schoklitsch's equation and converted to tons per day are shown plotted against the measured bed-material discharge in figure 23A. The dashed lines in figure 23 are lines of equality and are shown, not to imply that the computed bedload discharge should be equal to the measured discharge of particles larger than 0.125 mm, but to indicate the relative magnitudes of the two discharges. The bedload discharges increased with about the 0.8 power of the measured bed-material discharge and with about the fourth power of the mean velocity.

Meyer-Peter and Müller's equation (1948) for bedload discharge in wide channels with negligible influence from the banks can be
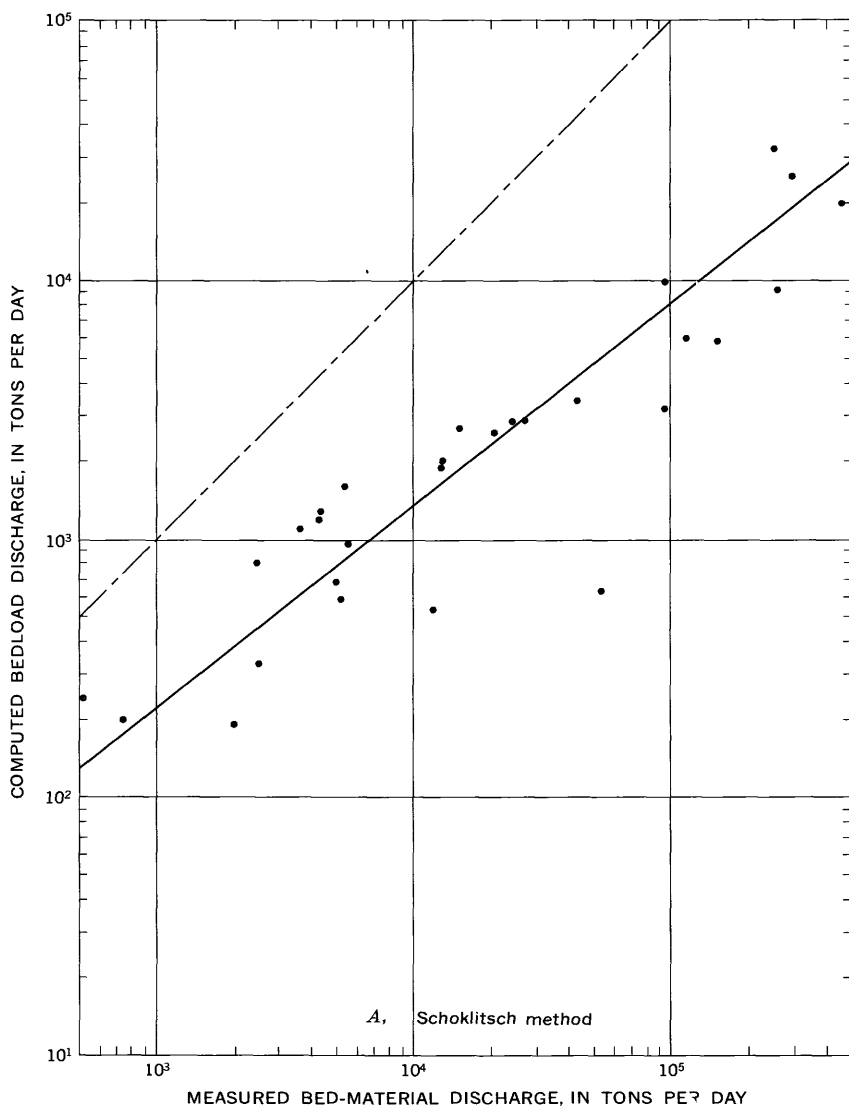
FIGURE 23.—Bedload discharges computed by various methods.   A, Schoklitsch method; B, Meyer-Peter and Müller method; C, Kalinske method; D, modified Einstein method; E, Bagnold method.   (B–E on following pages.)

written for units of meters, metric tons, and seconds as

$$\gamma \left(\frac{k_t}{k_r}\right)^{3/2} \overline{y}_0 S_e = 0.047(\gamma_s - \gamma) \ d_e + 0.25 \left(\frac{\gamma}{g}\right)^{1/3} \left(\frac{\gamma_s - \gamma}{\gamma_s}\right)^{2/3} q_b{}^{2/3}$$

where

γ is specific weight of water, in metric tons per cubic meter

k_t is equal to $\overline{u}/y_0{}^{2/3} \ S_e{}^{1/2}$, a discharge coefficient

10^5

10^4

10^3

10^2

10^1

COMPUTED BEDLOAD DISCHARGE, IN TONS PER DAY

$B,$ Meyer-Peter and Müller method

10^3 10^4 10^5

MEASURED BED-MATERIAL DISCHARGE, IN TONS PER DAY
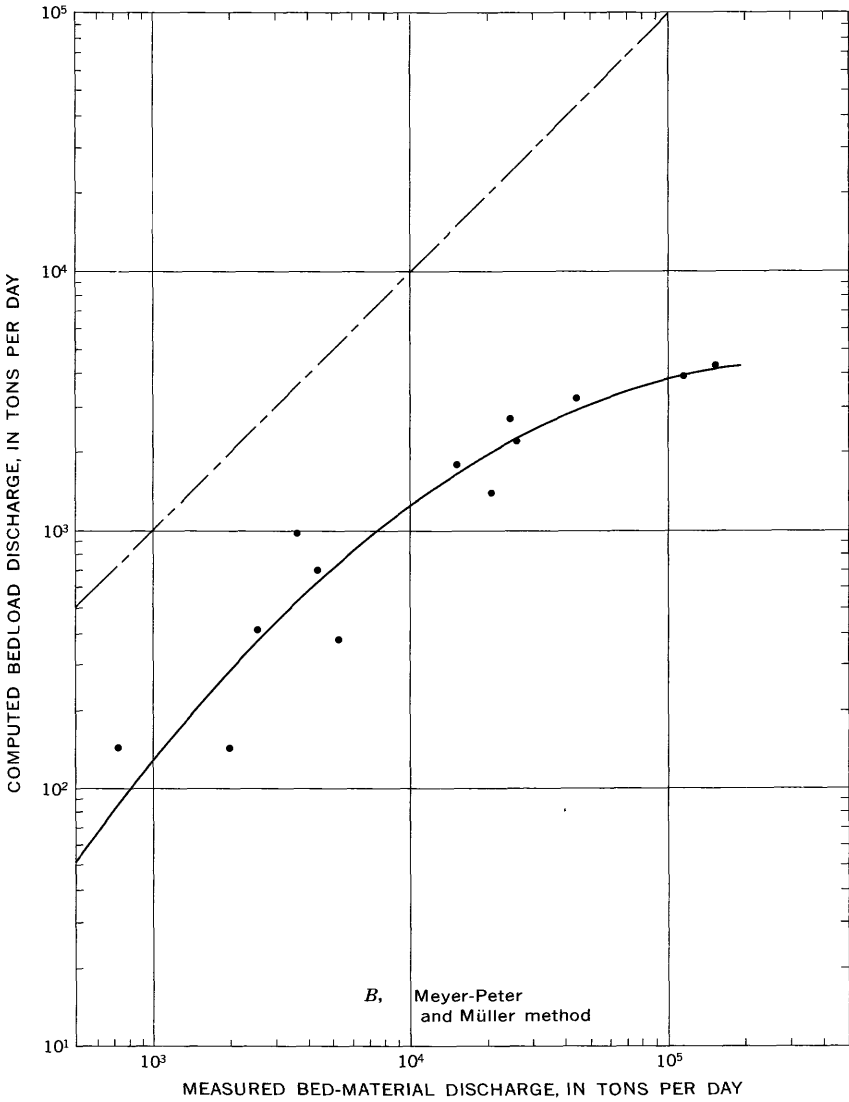
k_r is equal to $26/(d_{90})^{1/6}$, an approximation of the maximum discharge coefficient that would occur with bed material of a given size and a plane bed with no bed-material movement

d_{90} is the particle size, in meters, at which 90 percent of the bed material by weight is finer

γ_s is specific weight of the sediment, in metric tons per cubic meter

d_e is effective diameter of bed material, in meters

q_b is bedload discharge, in metric tons per second per meter of width

The ratio k_t/k_r depends on the form of the bed; so the equation is one of the few that account for the effects of bed forms on the bedload

C, Kalinske method

MEASURED BED-MATERIAL DISCHARGE, IN TONS PER DAY

(vertical axis) COMPUTED BEDLOAD DISCHARGE, IN TONS PER DAY
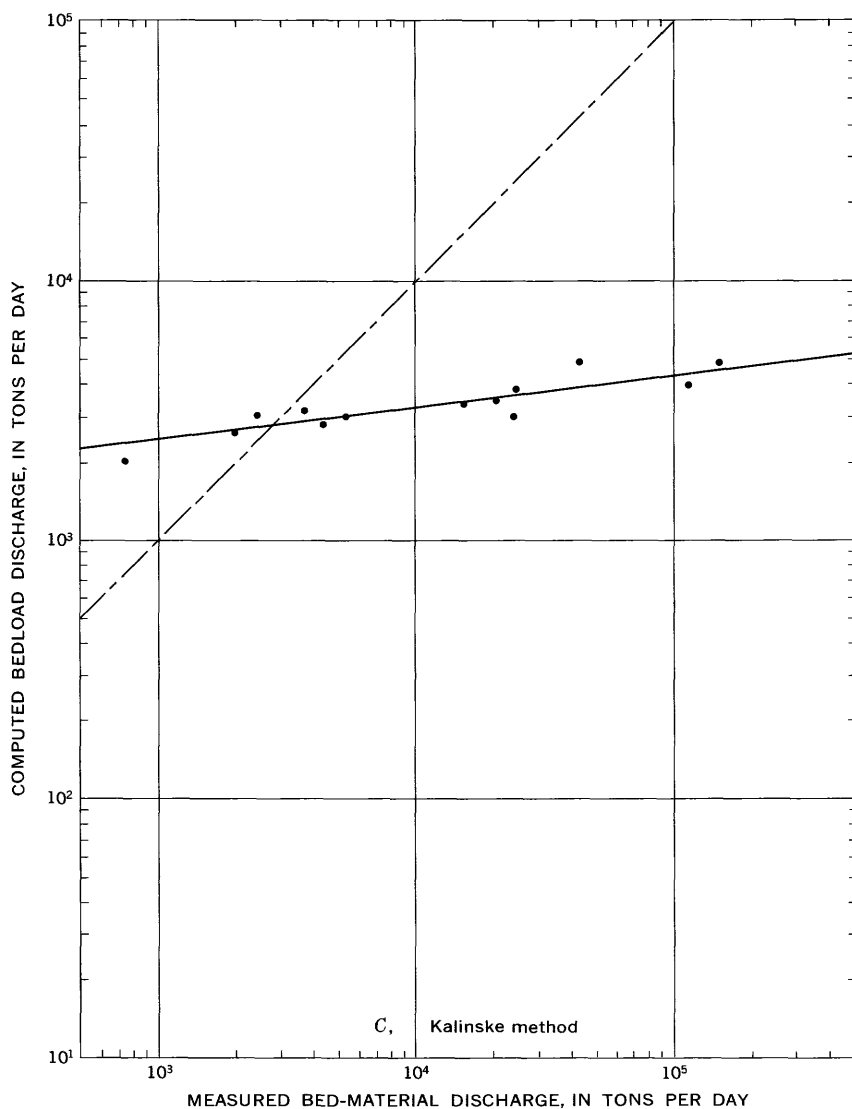
discharge. The equation can be used to compute bedload discharge by ranges of particle size. Bedload discharges computed for the whole width from this equation and converted to tons per day are shown plotted against measured bed-material discharge in figure 23B. Bedload discharges increased with about the first power of the measured bed-material discharge for low and medium discharges and with about the one-third power for high discharges. Bedload discharges increased with about the fourth power of the mean velocity.

D,   Modified Einstein method

MEASURED BED-MATERIAL DISCHARGE, IN TONS PER DAY

COMPUTED BEDLOAD DISCHARGE, IN TONS PER DAY
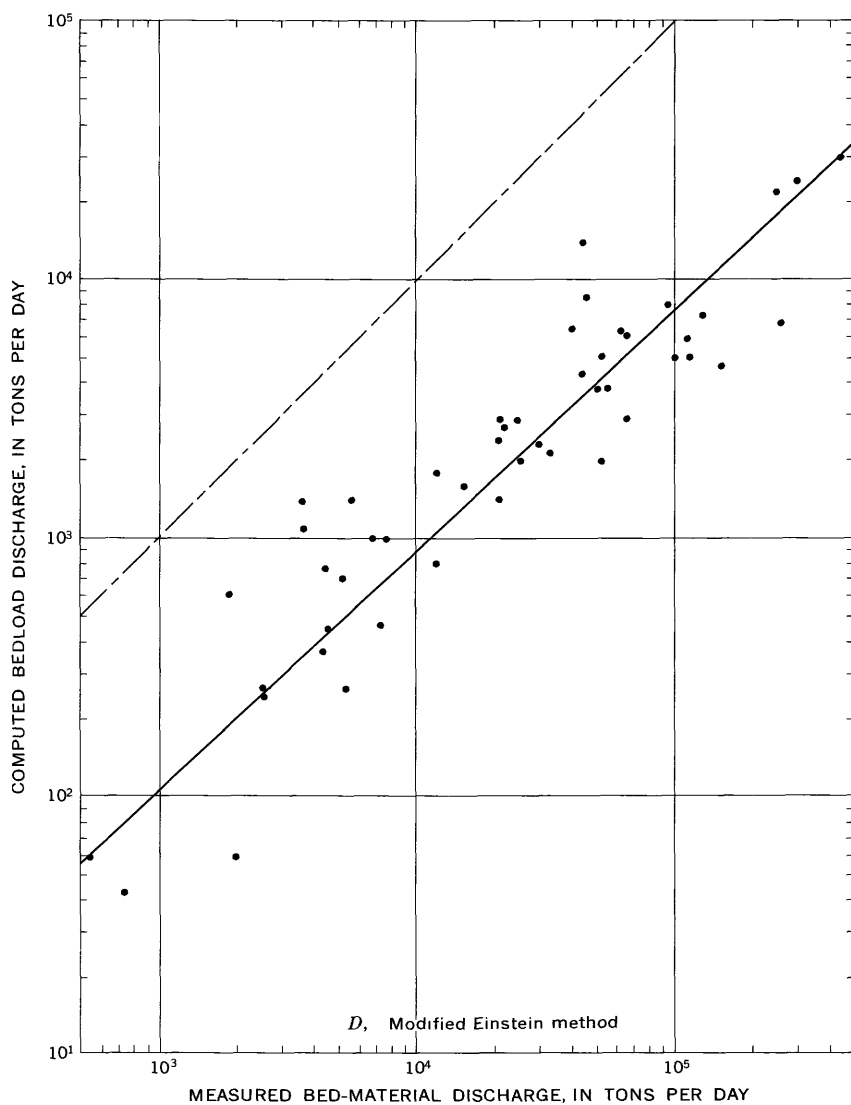
Kalinske's equation (1947) is used for computing bedload discharges by ranges of particle size.   The equation can be written

$$q_b = u_* \gamma_s d_i p_i 7.3 \frac{\overline{U}_s}{\overline{U}}$$

where

q_b is bedload discharge of a size range, in pounds per second per foot of width

u_* is shear velocity, in feet per second

MEASURED BED-MATERIAL DISCHARGE, IN TONS PER DAY
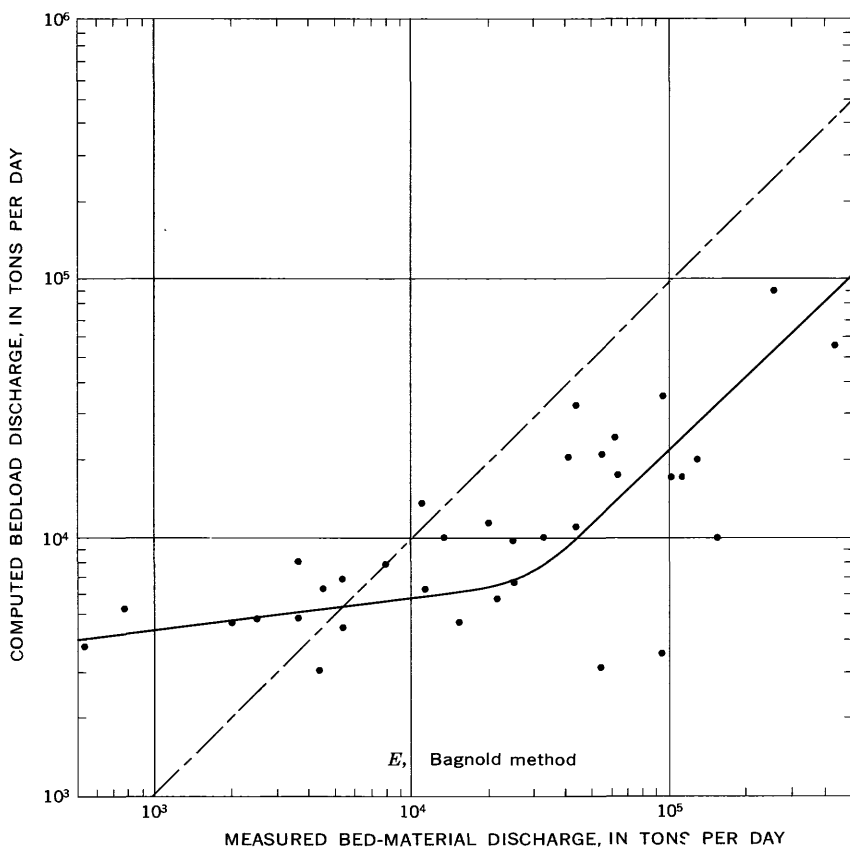
γ_s is specific weight of sediment, in pounds per cubic foot
d_i is geometric mean diameter of a size fraction of bed material, in feet
p_i is the fraction of bed area occupied by the particles in a size range
$\overline{U_g}/\overline{U}$ is the ratio of mean speed of movement of the grains to mean fluid velocity at the grain level

The factor $\overline{U_g}/\overline{U}$ depends on the relative intensity of turbulence and on the relative bed-shear stress. Bedload discharges were computed for the whole width and converted to tons per day. In figure 23C, bedload discharges increased with only the 0.13 power of the measured bed-material discharge. The bedload discharges increased with only the first power of the mean velocity.

The modified Einstein procedure (Colby and Hembree, 1955) can be used to compute bedload discharge, bed-material discharge, and total sediment discharge. In the procedure is used a maximum amount of measured data, which include suspended-sediment concentration, particle-size distribution, mean velocity, and temperature. Bedload discharges increased with about the 0.95 power of the

measured bed-material discharge (fig. 23D) and with about the fourth power of the mean velocity.

Bagnold (1956, p. 250) defined the material composing "bed load" as "that part of the load whose normal immersed weight component is in normal equilibrium with the grain stress * * * This stress is transmitted downwards via the dispersed grains to the stationary grains of the bed upon which it therefore ultimately rests." Thus the bedload discharges computed by Bagnold's equations may include some material that would be in suspension and that might be sampled by suspended-sediment samplers. For the particular conditions of the Mississippi River at St. Louis, Bagnold's equations can be simplified to

$$g_b = 5,100 B(y_0 S)^{3/2}$$

where

g_b is "bed load" discharge, in pounds per second per foot of width
B is a mathematical abbreviation for a group of parameters that depend on the particle size
y_0 is depth of flow, in feet
S is slope of the bed surface, assumed to be the same as the slope of the water surface for a 6.5-mile reach

Bedload discharges were computed for the whole width and converted to tons per day. They were nearly constant at low and medium discharges and increased with about the first power of the measured bed-material discharge at high discharges (fig. 23E). They increased with about the third power of the mean velocity.

Bed-material discharges have been computed using Straub's equation, the modified Einstein procedure, and Bagnold's method, and by adding the unmeasured sediment discharge computed with Colby's method to the measured bed-material discharge.

Straub's equation (U.S. Congress, 1935) is intended for computing "bed load." However, because τ_c and Θ for the equation were determined from data for bed-material discharge and because sediment discharges computed with the equation are about the same in magnitude as those computed with other methods for bed-material discharge, the results are regarded as bed-material discharges rather than bedload discharges. The equation can be written as

$$q_{BM} = \Theta y_0 S_e \left(y_0 S_e - \frac{\tau_c}{\gamma} \right)$$

where

q_{BM} is bed-material discharge, in pounds per second per foot of width
Θ is the sediment characteristic
y_0 is the depth of flow, in feet
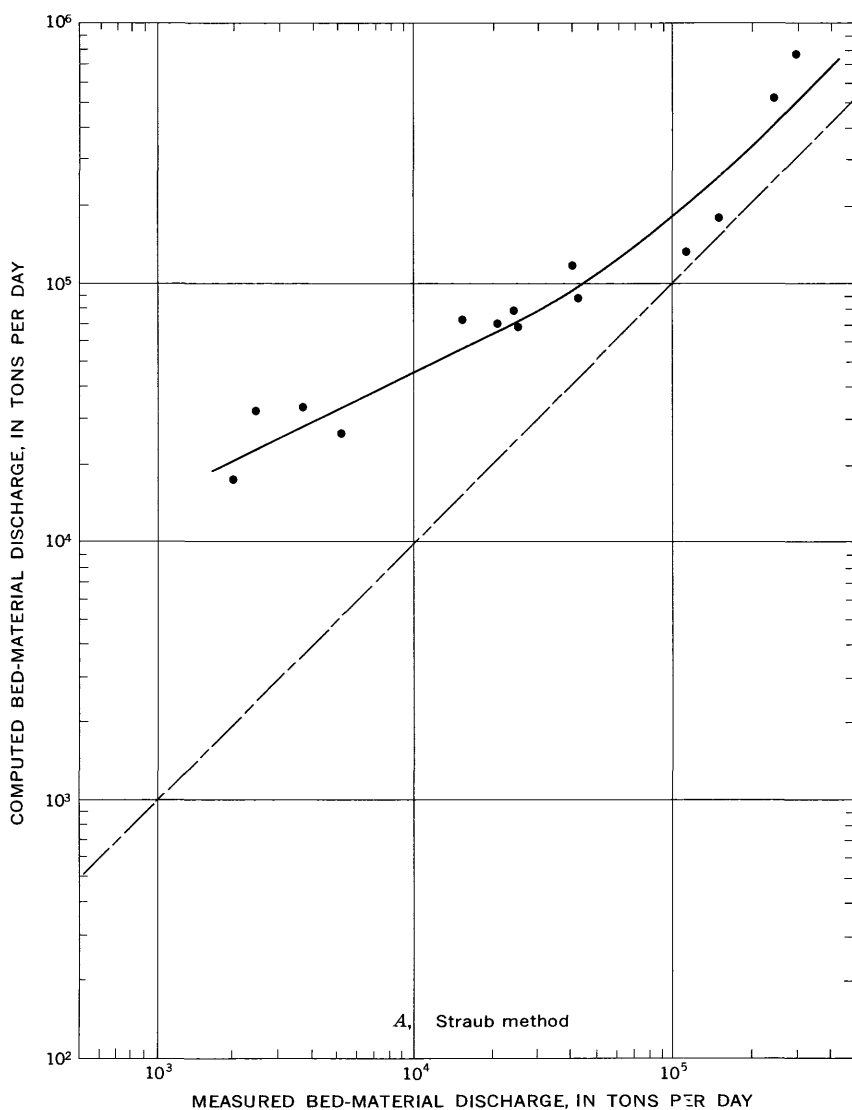
FIGURE 24.—Bed-material discharges computed by various methods. A, Straub method; B, modified Einstein method; C, Colby method; D, Bagnold method. (B–D on following pages.)

τ_c is the transporting force required to start movement of the bed material, in pounds per square foot

γ is the specific weight of water, in pounds per cubic foot

Bed-material discharges computed for the whole width with Straub's equation and converted to tons per day are shown plotted against measured bed-material discharge in figure 24A. The dashed lines in figure 24, as in figure 23, are lines of equality. Computed bed-material

B,   Modified Einstein method

MEASURED BED-MATERIAL DISCHARGE, IN TONS PER DAY
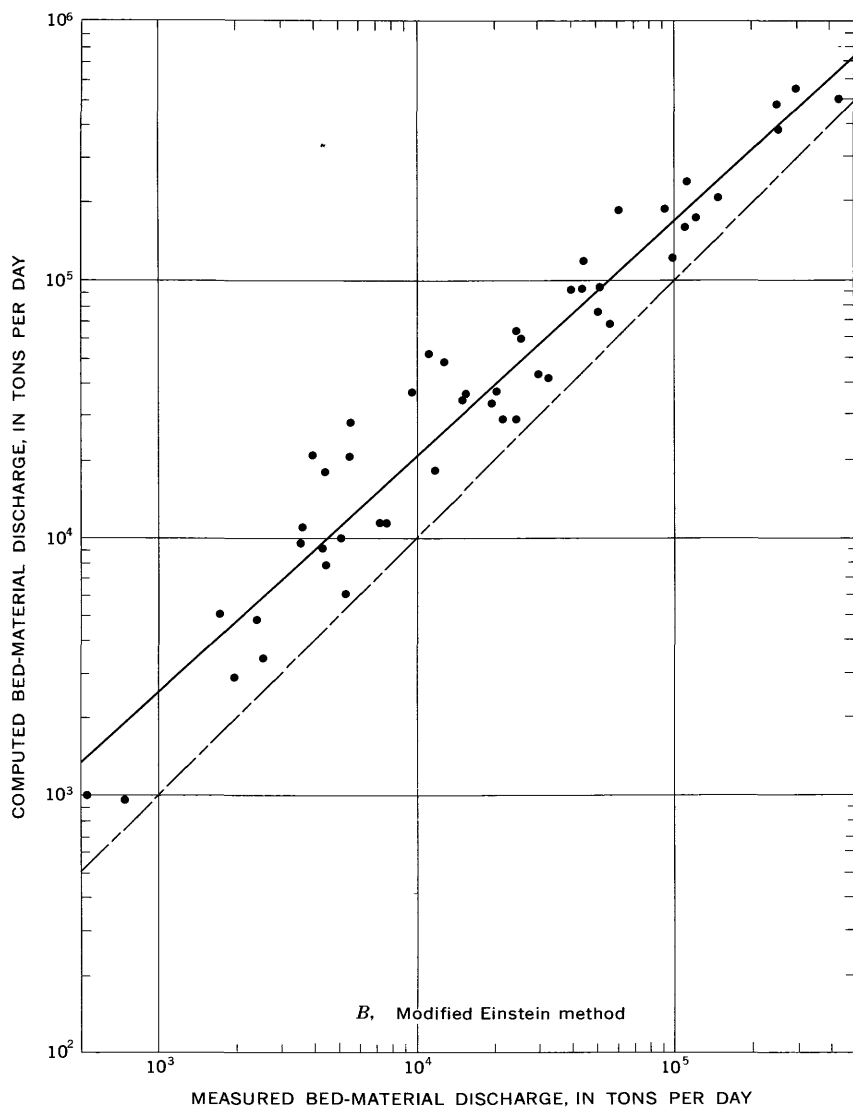
discharge will not be less than the measured bed-material discharge unless one or both of the discharges are in error. The discharges from Straub's equation increased with only the one-half power of measured bed-material discharge at low discharges and with about the first power at high discharges. They increased with about the third power of the mean velocity.

Bed-material discharges computed with the modified Einstein procedure (Colby and Hembree, 1955) increased with the 0.9 power of the measured bed-material discharge (fig. 24B). The scatter of
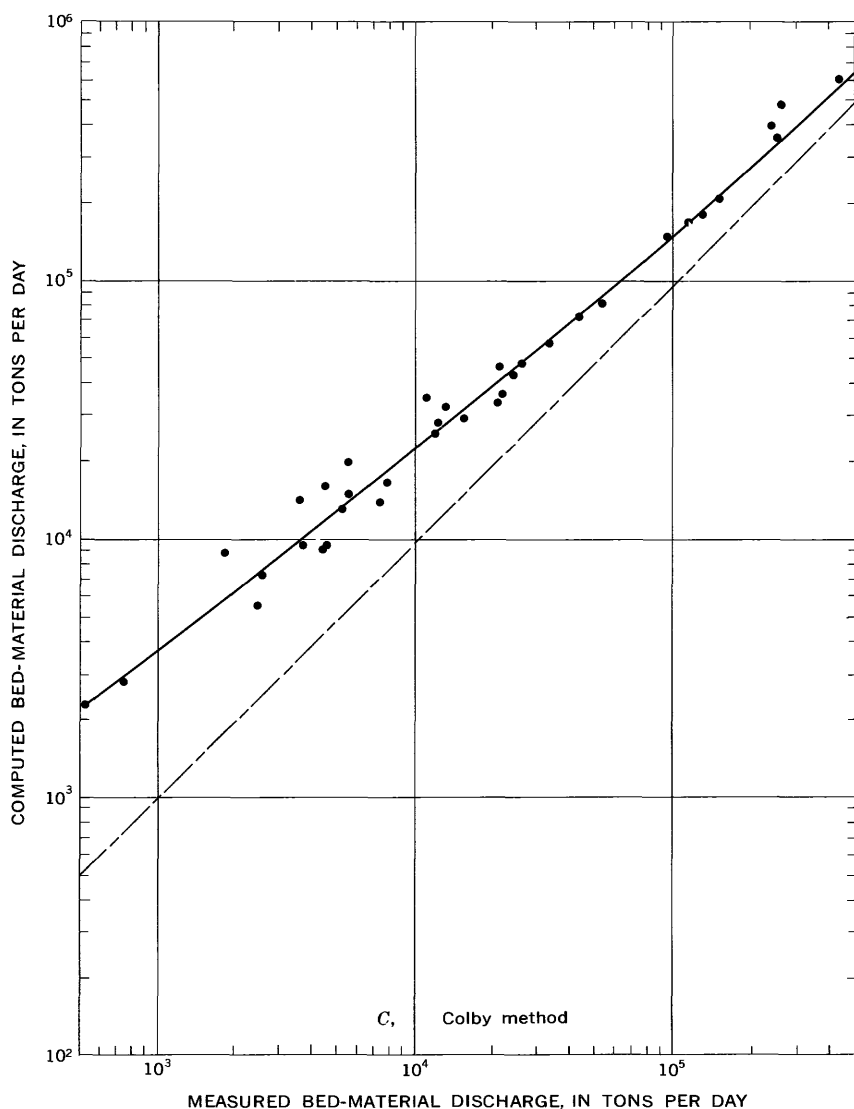
points is not excessive.   The bed-material discharges increased with about the fifth power of the mean velocity.

Colby (1957) presented a method for computing unmeasured sediment discharge from the relation to mean velocity, depth, and concentration of suspended sands.   Total bed-material discharges can be computed by adding the unmeasured sediment discharge to the measured bed-material discharge.   The bed-material discharges increased with about the three-fourths power of the measured bed-

material discharge (fig. 24*C*) and with about the fourth power of the mean velocity.
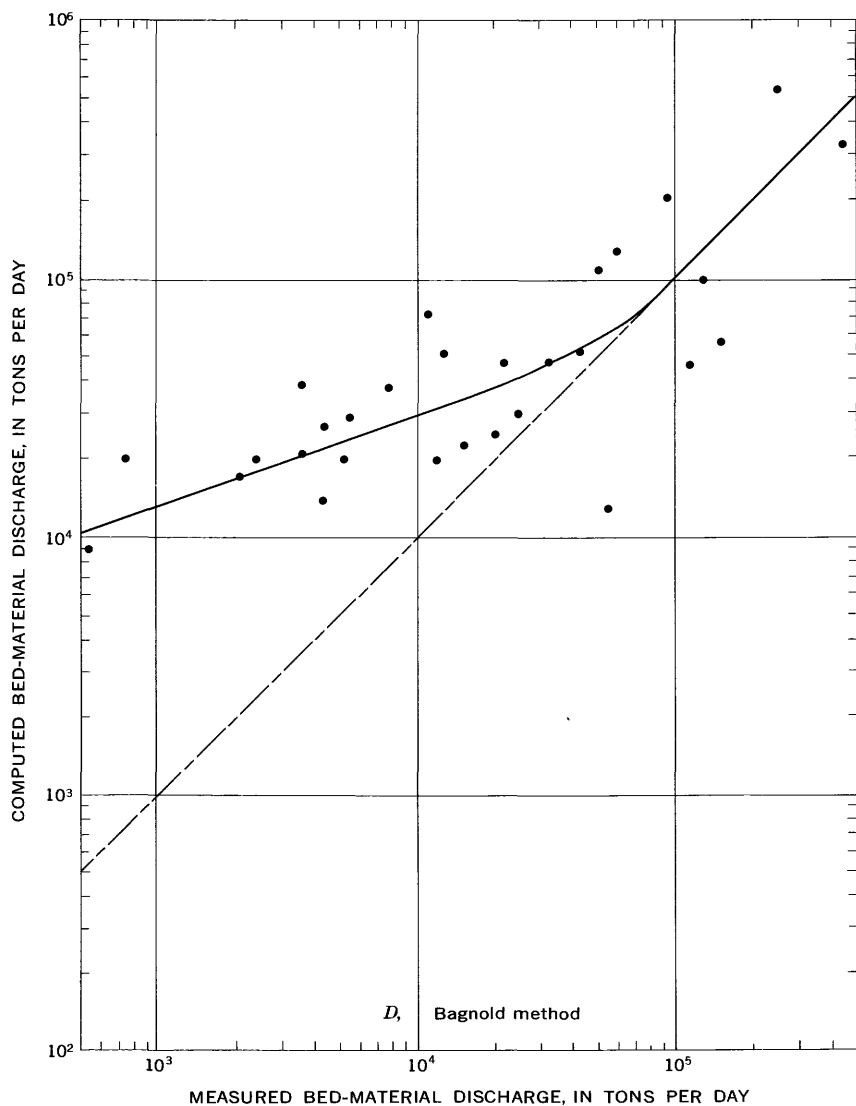
Bagnold (1956) gave functions for "total transport rate." This term was interpreted to be the total transport rate of particle sizes that are in the bed, and it should be comparable to the bed-material discharge that was computed by other methods. Bed-material discharges from Bagnold's functions increased with about the one-half power of the measured bed-material discharge for low discharges and with the first power for high discharges (fig. 24*D*). For high dis-

charges, the bed-material discharges from Bagnold's functions were about equal to the measured bed-material discharges, whereas in reality the total bed-material discharge must be greater than the measured part of the bed-material discharge.

For comparison of the results obtained with the different equations, the average curves from figures 23 and 24 were plotted together in figures 25 and 26, respectively. The equality lines are also shown.

Figure 25 shows that the Schoklitsch equation and the modified Einstein procedure gave about the same results for bedload discharge. The Meyer-Peter and Müller equation also gave similar results except at high discharge, where it gave lower bedload discharges than the other two equations. The Kalinske equation gave bedload discharges that are not in agreement with the results of any other equation. Kalinske bedload discharges increased very slowly with measured bed-material discharge; therefore, this equation is probably not suited for application to rivers similar to the Mississippi. Because Bagnold's definition of "bed load" is different from that for the other equations, the results are not in agreement with those from any of the other equations.

This study indicates that the modified Einstein, the Schoklitsch, and the Meyer-Peter and Müller bedload methods are the most suitable for the Mississippi River and similar rivers.

Figure 26 shows that the bed-material discharges computed by the modified Einstein method and those determined by adding the unmeasured sediment discharge from Colby's procedure to the measured bed-material discharge are in very good agreement. The figure indicates that the results are logical and reasonable; the computed discharges are greater than the measured discharges, and they increase at a slightly slower rate than the measured discharges. The Straub equation gives results that are higher than those for the modified Einstein and the Colby methods in the low range and are about the same in the high range. Bed-material discharges from Bagnold's functions are similar to those computed from Straub's equation, but on the average they are about equal to the measured bed-material discharge in the high range.

This study indicates that, when the required data—such as measured velocities, concentrations, and particle-size distributions—are available, the most suitable methods of computing bed-material discharge for the Mississippi River and similar rivers are the modified Einstein method and the addition of the unmeasured sediment discharge from Colby's procedure to the measured bed-material discharge. When such data are not available, the Straub equation is the best method to use; but it probably will give results that are too high in the low range.
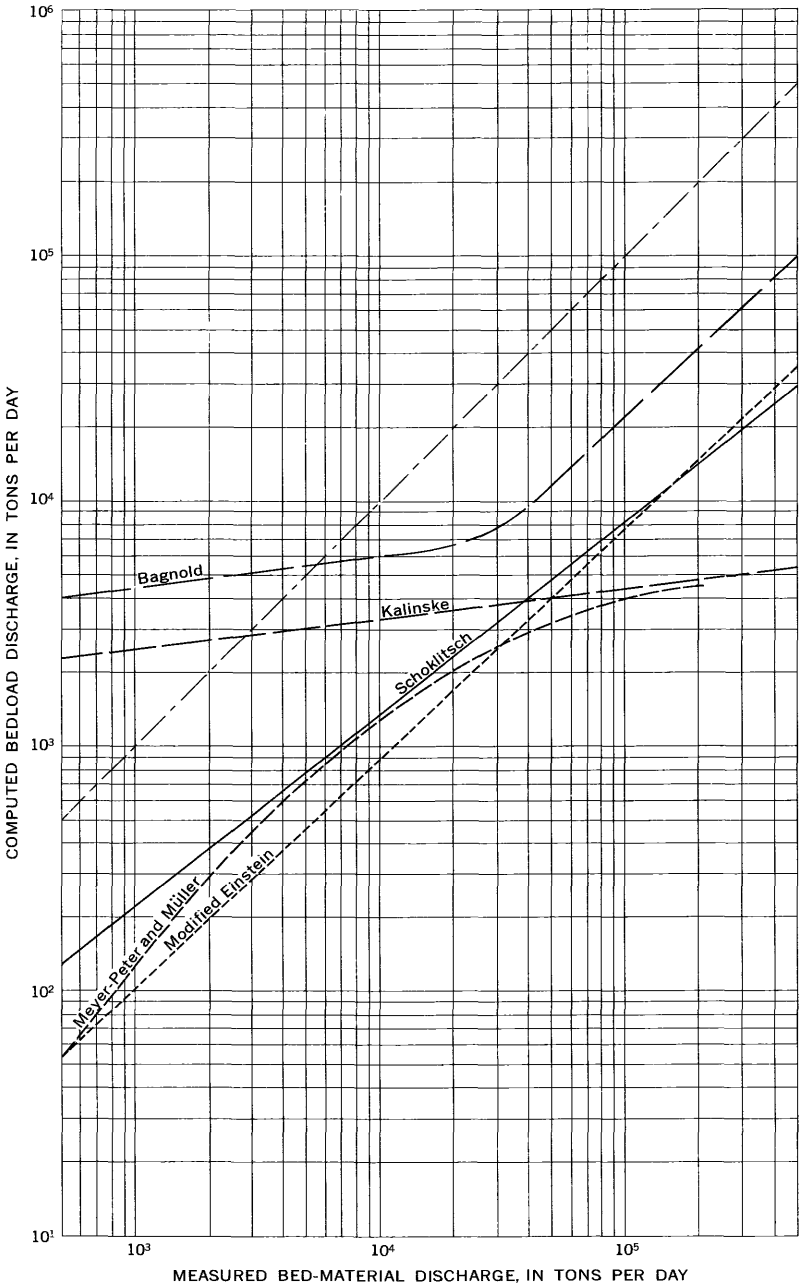
FIGURE 25.—Comparison of results from different methods of computing bedload discharge.
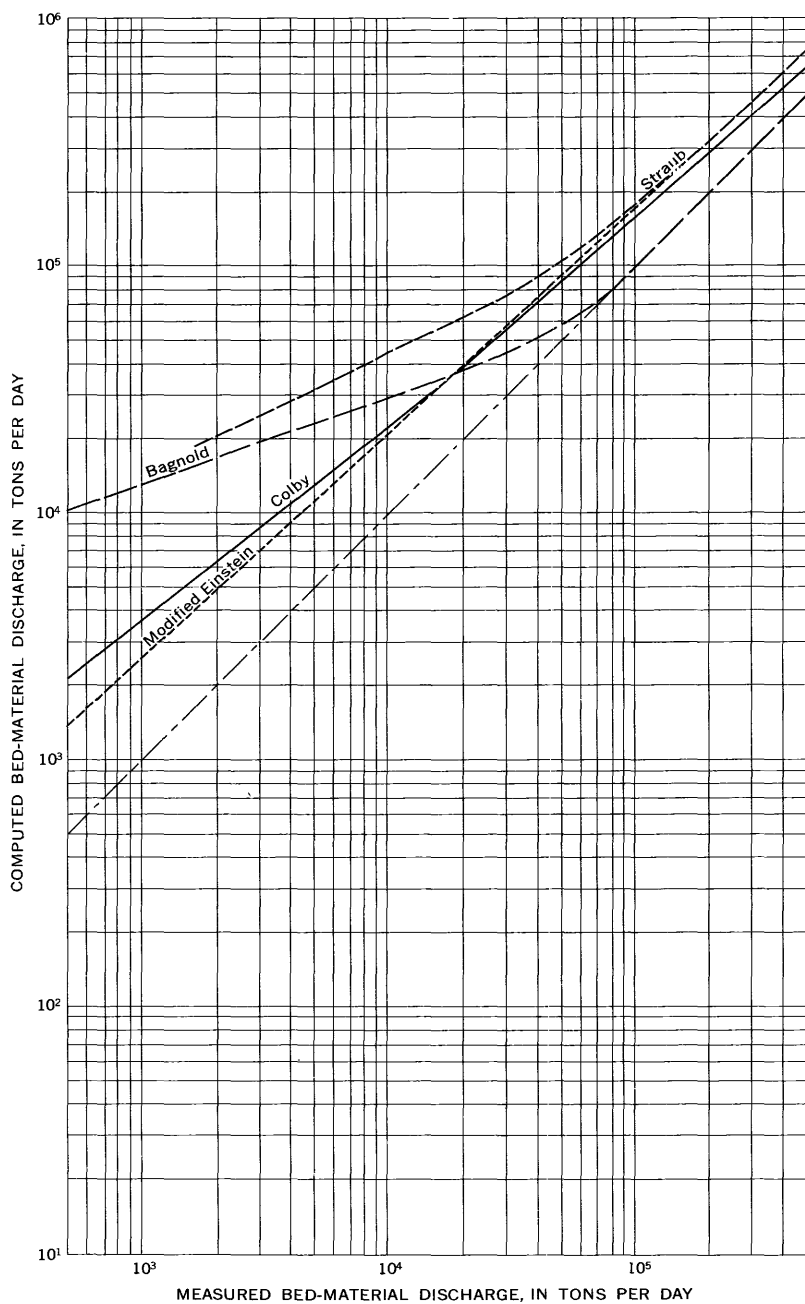
FIGURE 26.—Comparison of results from different methods of computing bed-material discharge.

As an example of the relative magnitudes of measured su⸜pended-sediment discharges and total sediment discharges that could be expected over fairly long periods of time, bed-material discharges and total sediment discharges were computed for the 1958 water year. The year 1958 was chosen both because conditions during that year represent those after most of the dams on the Missouri River main stem had been closed and because the mean flow of 145,000 cfs was fairly close to the long-term mean of 175,000 cfs. Rating curves of mean depth, mean velocity, and width were prepared from the weekly discharge measurements. For each day on which no discharge measurement was made, the depth, velocity, and width were determined from these curves. Water temperatures were measured or estimated for each day. These depths, velocities, widths, and water temperatures were used with figure 22 to compute the bed-material discharge in the sampled zone. These bed-material discharges were expressed as concentrations by dividing by the streamflow and by the units-conversion constant 0.0027. The unmeasured sediment discharge was then computed by Colby's method (1957).

These computations resulted in an unmeasured sediment discharge of 5,800,000 tons and a total sediment discharge of 114,000,000 tons for the year, compared to a measured suspended-sediment discharge of 108,125,500 tons. The suspended-sediment discharge was about 95 percent of the total sediment discharge, and the bed-material discharge, 15,800,000 tons, was about 14 percent of the total sediment discharge. These percentages probably are typical for any years except those having extreme conditions.

SUMMARY

An average of about 45 percent of the flow at St. Louis is from the Missouri River, which was partly controlled by reservoirs having a total capacity of 90,300,000 acre-feet in 1956. The Mississippi River at Alton, Ill., upstream from the Missouri River, was partly controlled by lakes and reservoirs having a total capacity of 4,890,000 acre-feet in 1956.

The relative rates of increase of width, depth, and velocity with discharge at the MacArthur Bridge can be expressed by the simple power functions

$$b = 680 Q^{0.07}$$
$$\bar{y}_0 = 0.16 \; Q^{0.43}$$
$$\bar{u} = 0.0092 \; Q^{0.50}$$

The average recurrence interval of the peak discharge in July 1951 of 782,000 cfs is 9 years, and the average recurrence interval of a discharge of 990,000 cfs or more is 100 years. Peak discharges

and minimum discharges will be less extreme in the future than in the past because of the increased storage capacity in the drainage basin.

The flows of the Missouri and upper Mississippi Rivers have not become mixed at St. Louis; so the river has a lateral gradient of suspended-sediment concentration. The concentration near the west bank has been as much as 2,400 ppm greater than the concentration near the east bank.

Suspended-sediment discharges from April 1948 to September 1958 ranged from 4,250 to 7,010,000 tons per day and averaged 496,000 tons per day. Mean concentrations for water years decreased steadily from 1,690 ppm in 1949 to 403 ppm in 1956, but they increased to 756 ppm in 1958. Effects of the new reservoirs in the Missouri River basin on the concentration were obscured by the close relation of concentration to streamflow.

The measured suspended-sediment discharge averaged 47 percent clay, 38 percent silt, and 15 percent sand. Variations of particle size were due mainly to differences in the source areas of the sediment.

Most of the bed material in the main flow is between 0.125 and 1.00 mm in diameter. The average of median diameters is related to the discharge for periods of 1 year and longer. Geometric quartile deviations of the bed material ranged from 1.1 to 2.5 and averaged 1.5.

The mean bed elevation from stations 275 to 1,800 ranged from 355.1 to 364.4 feet above mean sea level and was related to the median diameter of bed material by the regression equation $h_b = 363.0 - 7.8\,d_{50}$, for which the standard error of estimate was 0.91 foot.

The resistance to flow as measured by Manning's n ranged from 0.024 to 0.041 and was related to the discharge and mean velocity but not to the shear velocity. Normal dune height is 2–8 feet, and average length is about 250 feet. When the resistance to flow was low, much of bed was fairly flat and the few dunes were much longer than the average. Data obtained at St. Louis confirm Einstein's relation of $\bar{u}/u_*{}'$ to Ψ' very well except for low values of Ψ'. Loops in the relation of gage height to fairly high discharge are caused by a combination of changes in roughness and turbulence constant.

Turbulence constants (Von Karman's k) computed from velocity measurements at 5–10 points in the vertical and from routine velocity measurements at 2 points in the vertical averaged 0.35 and 0.33, respectively. Turbulence constants were fairly consistent with those reported by Einstein and Chien (1955).

The exponent z_1 of the vertical distribution of concentration for different size ranges varied with about the 0.77 power of the fall velocity. Except for the difference between the theoretical variation and the actual variation of z_1 with changing fall velocity, the theoreti-

cal equation for the vertical distribution of sediment concentration seems to apply reasonably well for the Mississippi River at St. Louis. The relations of computed bedload discharge to the measured bed-material discharge and to the mean velocity indicated that the most suitable methods for computing bedload discharge of the Mississippi River and similar rivers are the modified Einstein, Schoklitsch, and Meyer-Peter and Müller methods. The relations of computed bed-material discharge to the measured bed-material discharge and to the mean velocity indicated that the most suitable methods for computing bed-material discharge are the modified Einstein procedure or the addition of the unmeasured sediment discharge from Colby's procedure (1957) to the measured bed-material discharge when data on velocity, concentration, and particle-size distribution are available; the Straub equation is the most suitable method when such data are not available. Measured suspended-sediment discharge averaged about 95 percent of the total sediment discharge, and the bed-material discharge averaged about 14 percent of the total sediment discharge.

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