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STATE COORDINATES AND POLYCONIC MAPS

SOME NOTES ON THE STATE PLANE-COORDINATE SYSTEMS AND THEIR RELATION TO POLYCONIC QUADRANGLE MAPS

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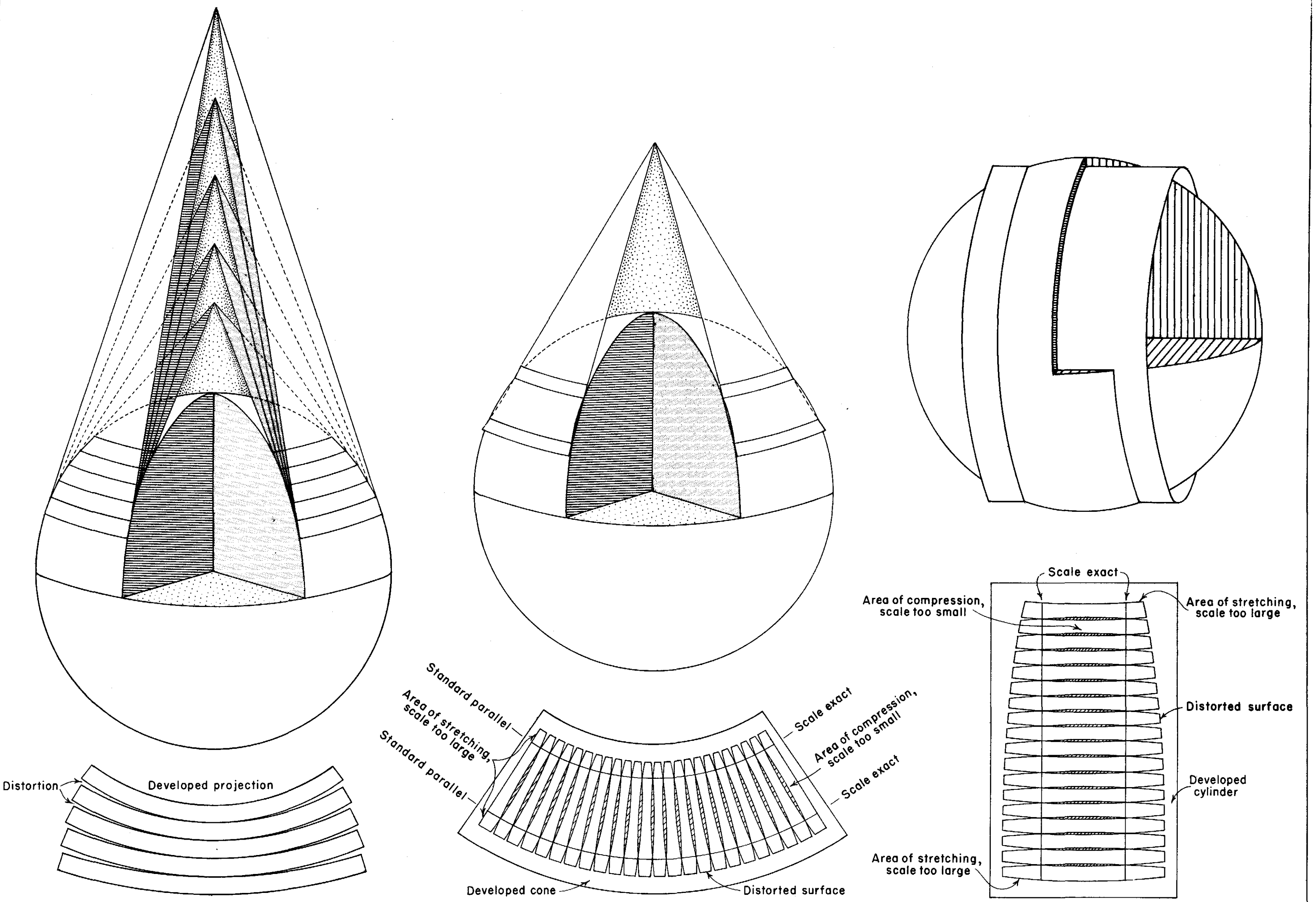
Since the introduction of the State plane-coordinate systems on the Lambert and transverse Mercator projections in 1935, numerous questions have arisen among prospective users of these systems concerning their applicability in connection with Geological Survey maps on the polyconic projection. Some of the questions deal with the basic properties of the several projections; some question the legitimacy of combining, in a single map, data based on more than one projection; some concern the ease of matching adjoining sheets to form a map of a larger area. An attempt will be made in this discussion to clarify the relationship between the projections and to answer some of the questions that have been propounded and some that are expected to recur from time to time.

As it is impossible to represent a portion of a spherical or spheroidal surface on a plane sheet without some distortion, many types of projection have been devised for mapping the earth's surface and at the same time keeping the resulting distortion within permissible limits. Each projection has its own advantages and disadvantages. No one system is superior to all others in every respect, and the choice of projection for a particular map must depend on the size, shape, and location of the area to be mapped, the scale of the proposed map, and the use to which it is to be put. For a small area, many projections are available in which the distortion can be kept within such small limits that it is impossible to distinguish between them. As the area to be mapped increases in size, the distortions inherent in the various projections become increasingly significant, and the map maker is confronted with the necessity of choosing a projection that will best suit his own particular needs. When the area is too large to be represented conveniently on a single sheet at the map scale, it must be divided into smaller areas. It is desirable, then, that the projection be such as to permit the joining of adjacent sheets so that corresponding edge details will match properly.



POLYCONIC PROJECTION
Infinite number of cones tangent to spheroid
(Each strip of infinitesimal width)

LAMBERT PROJECTION
Single cone cutting spheroid

TRANSVERSE MERCATOR PROJECTION
Cylinder cutting spheroid

FIGURE I. DIAGRAMS SHOWING DEVELOPMENT OF RESPECTIVE PROJECTIONS AND TYPE AND LOCATION OF PRINCIPAL DISTORTIONS. (ALL VERY HIGHLY EXAGGERATED)

Only three projections will be considered in this discussion: the polyconic projection, used for all quadrangle sheets of the Geological Survey; the Lambert conformal conic projection, used for many of the State plane-coordinate systems; and the transverse Mercator projection, used for the remainder of the State plane-coordinate systems.

POLYCONIC PROJECTION

The polyconic projection has been generally accepted as the best for a small, regular-shaped area such as the standard quadrangle. In this projection, the earth's surface is projected onto an infinite number of cones, each tangent to the ellipsoid at its own parallel of latitude. The developed parallels are therefore true circles, each of a different radius, and each centered at a different point on the extension of the central meridian of the projection. The central meridian is a straight line; all other meridians are slightly concave toward it. Scale is true along the central meridian and along each parallel of latitude; at all other points, and in other directions, the scale is slightly exaggerated. Although the polyconic projection does not comply strictly with most of the desirable specifications for a map projection--it is not conformal, scale is not uniform, shapes and areas are not retained exactly--it comes closer to compliance with more of these specifications than any other available projection. The distortions are kept to a minimum in all parts of the map sheet, and, for small areas, the earth's surface may be represented with a greater fidelity on this projection than on any of the others. On a standard quadrangle sheet, the distortions will be so small that it is impossible to plot or measure them at map scale, so that, for practical purposes, a polyconic projection of a single quadrangle may be considered a true portrayal of that portion of the earth's surface. The principal weaknesses of the polyconic projection are two: it cannot be used on a large area without noticeable distortion; and, although adjacent sheets can be matched continuously in one direction or strip, two or more strips cannot be matched for any great distance without developing gaps along the abutting edges. The situation is somewhat analogous to that of an orange peel. If the peel is cut into a large number of tiny quadrangular pieces, each may be placed on a flat surface without much stretching; but it is impossible to reassemble a large number of pieces on a flat surface, except in a single strip. Similarly, a single large piece cannot be laid flat without stretching or tearing it.

LAMBERT PROJECTION

The Lambert projection portrays a portion of the earth's surface on the developed surface of secant cone. Along the two parallels of latitude at which the cone cuts the spheroid, the scale is exact. In the area between these parallels, the map scale will be too small, and in the area outside these parallels, it will be too great. The mapped area may be extended without limit in the east-west direction, but is restricted within narrow limits in the north-south direction. Scale distortions increase rapidly as the distance increases beyond the two standard parallels. Therefore the use of this projection is limited to relatively narrow east-west bands and to States of that shape or States that

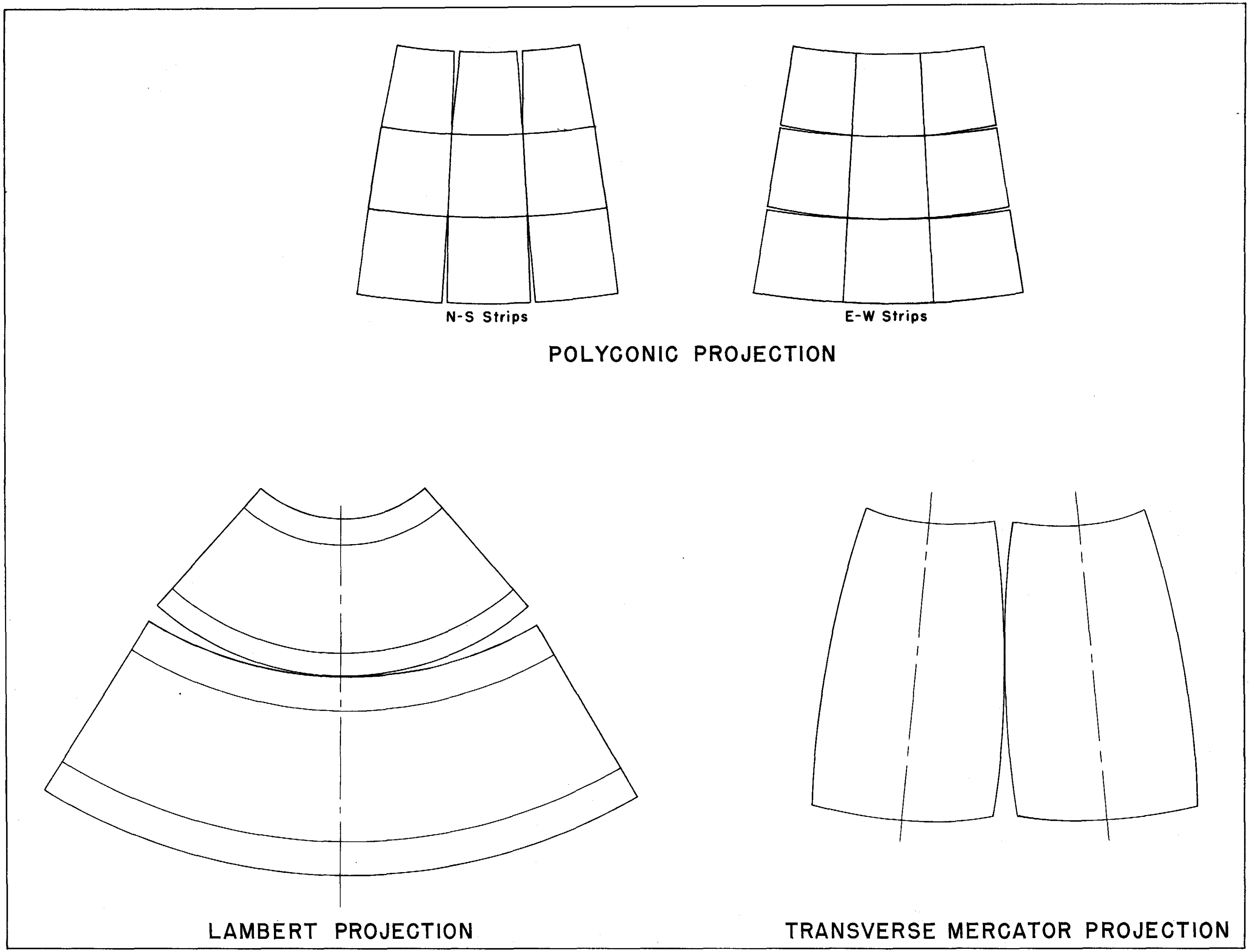


FIGURE 2. DIAGRAMS SHOWING MATCHING OF QUADRANGLE SHEETS AND ZONES ON THE RESPECTIVE PROJECTIONS

can be cut into zones of that shape. The principal advantages of this projection are that the scale distortions are systematic and can be predetermined and that a map sheet of any portion of a zone can be matched perfectly with an adjoining sheet in the same zone. However, a map of one zone cannot be matched with a map of an adjoining zone.

TRANSVERSE MERCATOR PROJECTION

For the transverse Mercator projection, picture a portion of the earth's surface projected onto the developed surface of a tangent cylinder with its axis in the plane of the equator, so that the line of tangency is the central meridian of the area. Then, reduce the scale of the projection slightly so that it will be too small in the region of the central meridian. The scale will then be exact along two lines parallel to the central meridian and spaced symmetrically on both sides of it. In the entire region between these two parallel lines, the scale will be too small; in the regions beyond them, it will be too great. The mapped area may be extended without limit in the north-south direction, but is restricted to narrow limits in the east-west direction. Scale distortions increase rapidly as the distance increases beyond these parallel lines. This projection is limited to relatively narrow north-south bands, and to States and zones of that shape. Its advantages and disadvantages are essentially the same as those of the Lambert projection.

COMPARISON OF PROJECTIONS

The Lambert and transverse Mercator projections were selected as the bases of the State plane-coordinate systems because of their mathematical simplicity, because the scale factor at any point can be readily determined, and because the limiting scale factor can be controlled by proper choice of zone width. On either of these systems, there is a unique relationship between grid coordinates and geographic coordinates, and the interchange between coordinates is relatively easy. Both projections are conformal. As already explained, a polyconic projection for a single quadrangle map of 1 degree extent, or less, contains so little distortion that it may be considered a true representation of the earth's surface. Therefore, a polyconic projection of a quadrangle may be fitted into the appropriate portion of any State coordinate grid by a mere adjustment of the scale.

For the mapping of a large area, such as an entire State or State zone, two methods are available when the scale is too large to permit representation of the entire area on a single sheet. Each section, quadrangle, or county may be mapped on the polyconic projection, with the State grid shown, without perceptible error. Then, when several sections are assembled to compile the larger map, gaps may be expected to develop along the abutting sheet edges. The other available method is to adopt the projection of the State grid and to map each smaller portion of the area as a part of the larger projection. With this method, all the sheets will fit together perfectly, but the internal distortion will be greater because of the variation in map scale. A map compiled by this method would not match with a similar map of an adjoining State or zone.

For a picture of the magnitude of the distortions involved, consider some of the State coordinate systems actually in use. These systems (with one exception) have been designed so that the greatest scale distortion factor at any point in the widest zones will not exceed 1 part in 10,000. For a map distance of 20 inches, this represents a map error of only 0.002 inch. The maximum cumulative error in a map distance of 100 inches (8 ft. 4 in.) is only 0.01 inch. It may be stressed that these extreme errors can exist only in the widest of the zones adopted, and only at critical portions of these zones. In general, the distortions at other points in the zones will be considerably smaller.

It follows, therefore, that map detail may be transferred readily between a polyconic quadrangle map and either of the State coordinate systems, in any part of a State zone, without perceptible error. Furthermore, the State coordinate grid may be shown on a polyconic quadrangle map without any qualms about the inconsistency of mixing projections. While differences will exist theoretically, they will be of infinitesimal magnitude and may be considered nonexistent.

DISTORTIONS

At this point, mention should be made of the distortions that may be developed in a map through the physical instability of the medium on which it is drawn or printed. It is well known that paper will shrink or expand with changes in the surrounding atmospheric conditions, as well as from other causes. The change in dimensions of any sheet of paper will not necessarily be the same in all directions, and changes of shape, as well as of size, may result. These changes can be so great as to obliterate completely the theoretical distortions and scale changes in the map projection. It is not unusual for a dimensional change as great as 1/8 inch to occur in a single quadrangle map sheet on ordinary paper. Under such conditions, it is manifestly impossible to assemble perfectly a number of adjoining sheets, each with its own unpredictable paper distortion, regardless of the projection on which the maps were prepared. Many media are available in which distortions can be reduced or controlled, and such improved materials should always be used, when feasible, in the preparation of maps. Unless materials of exceptional stability are employed, it may be assumed that the theoretical differences between the projections referred to will always be less than the probable distortions of the paper, or other materials on which the map may be produced.

PROCEDURE

In conclusion, the following procedure is offered as a simple, yet accurate, method of transferring topographic detail from a polyconic quadrangle map to a State coordinate grid:

1. Construct a rectangular grid to the desired scale, to represent the State grid.
2. Compute the plane coordinates, on the State system, of the projection corners of the quadrangle, and plot these corners on the grid.

3. Connect the contiguous plotted corners, if desired, to form a graticule of meridians and parallels.

4. By the usual drafting methods, adjust the scale of the quadrangle map to that of the graticule and transfer the detail as desired.

Conversely, to show the State grid on a quadrangle map, compute the geographic coordinates of the appropriate grid intersections, plot the computed points on the quadrangle map, and connect the appropriate plotted points by straight lines.

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