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CHARLES D. WALCOTT, DIRECTOR

EXPERIMENTS WITH WINDMILLS

BY

THOMAS O. PERRY

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ILLUSTRATIONS.

LETTER OF TRANSMITTAL.

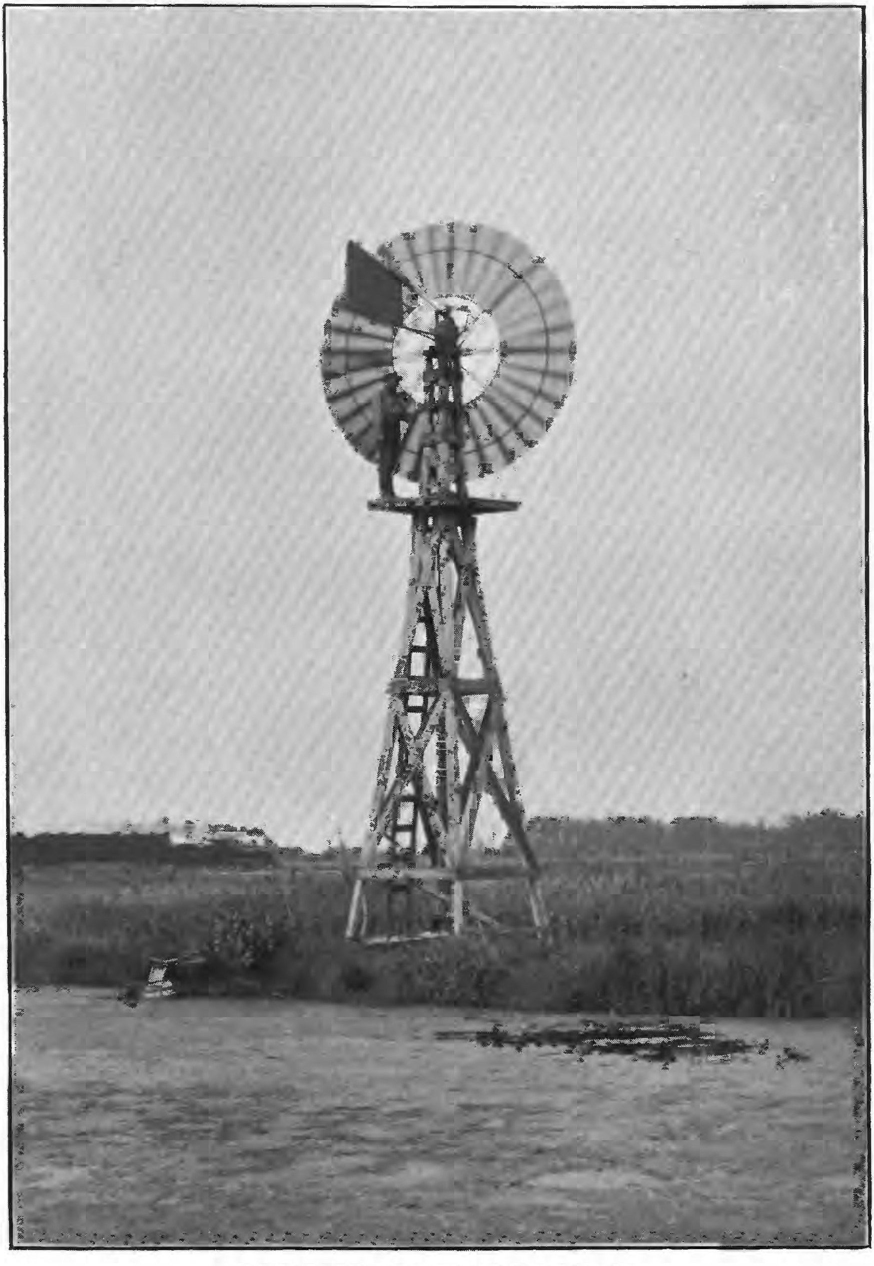
DEPARTMENT OF THE INTERIOR,
UNITED STATES GEOLOGICAL SURVEY,
DIVISION OF HYDROGRAPHY,
Washington, July 15, 1898.

SIR: I have the honor to transmit herewith a manuscript entitled Experiments with Windmills, prepared by Mr. Thomas O. Perry. These experiments, as described by Mr. Perry, were carried on during the years 1882 and 1883 for the United States Wind Engine and Pump Company, of Batavia, Illinois. As a result of these experiments radical changes and improvements were made in the windmills. As a matter of business policy the company did not desire that the results of these tests should be made known for some years. After the expiration of a certain time, however, the data have been placed at the disposal of the public through the kindness of the officers of the company and the efforts of Mr. Perry. Although as the result of this work great changes have been made in windmills, many of the suggestions made have not yet been put into practice and may serve as a foundation for further work along this line. The importance of the windmill as a means of utilizing the water resources of a part of the country is so great that all available information on the subject should be diffused and brought to the attention of persons who can make use of the facts. I respectfully request, therefore, that this manuscript, with a brief introduction, be printed as one of the series of pamphlets on water supply and irrigation.

Very respectfully,

F. H. NEWELL,
Hydrographer in Charge.

Hon. CHARLES D. WALCOTT,
Director United States Geological Survey.



WINDMILL PUMPING WATER FOR IRRIGATION.

INTRODUCTION.

By F. H. NEWELL.

During the progress of investigation of the extent to which the arid lands can be reclaimed by irrigation and of the related question of the occurrence of water underground attention has been continually drawn to the practical methods of bringing the underground water to the surface. Throughout a great part of the arid and semiarid region there are localities where water can be obtained at a short distance from the surface. The amount, although not large in the aggregate when compared with the quantity in some notable stream or lake, is yet inexhaustible by the ordinary methods of pumping. If, therefore, this water which exists from 10 to 50 feet beneath the surface could be cheaply raised, it would be practicable to utilize for agriculture tracts which otherwise have little or no value.

The irrigation of 20 acres in the midst of a section or township of land is, figuratively speaking, a mere drop in the bucket; but the reclamation of this small area generally means the utilization of adjoining lands. If, for example, 20 acres of some forage crop like alfalfa is made possible, this will result in obtaining a considerable amount of winter feed used in the sustenance of a herd which can be pastured upon the surrounding dry land. The successful cultivation of this 20 acres may thus directly or indirectly support a family, and, with increased experience and adaptation to the surrounding conditions, the family may in turn give place to a rural community. Given the existence of sufficient water underground to irrigate the 20 acres, the first question is that of ways and means of bringing the water to the surface.

The force which is ever present, making itself persistently felt throughout the Great Plains region, is the wind, which blows almost continuously. It carries the dust before it, cuts out the traveled roads, carries away the fine earth of the tilled fields, and builds up a fine loess, almost everywhere to be found. The wind, which has so long been considered as an annoyance and mischief-maker, has sufficient strength to perform the work of bringing water to the surface, if only suitable means of directing its energy can be discovered.

The windmill is the best-known method of converting wind energy into work. In one form or another it has been used from times antedating the dark ages. In the twelfth century windmills, built either by individuals or by communities, were common. Some of these mills were of enormous size. In the German type the whole building on which the mill was placed was constructed in such a manner as to turn on a post in order to bring the sails into the wind. In the Dutch form the building was fixed, but the head of the mill could be turned into the wind. The most notable use of these early mills was in Holland, where the land was drained by pumping water from behind the dikes into the sea. In 1391 the bishop of Utrecht, holding that the wind of the whole province belonged exclusively to him, gave to the convent at Windsheim express permission to build a windmill wherever it was thought proper. In so doing he overruled a neighboring lord who declared that the wind in the district belonged to him. Three years later the city of Haarlem obtained leave from Albert, Count Palatine of the Rhine, to build a windmill, using the wind of the country.[1]

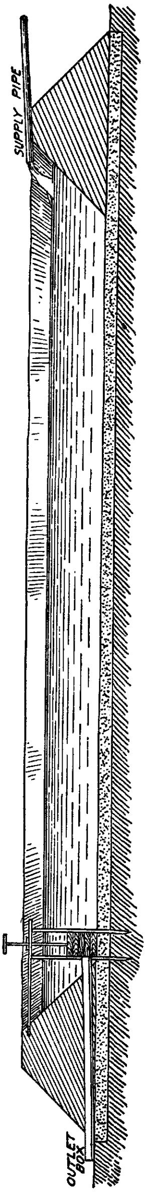
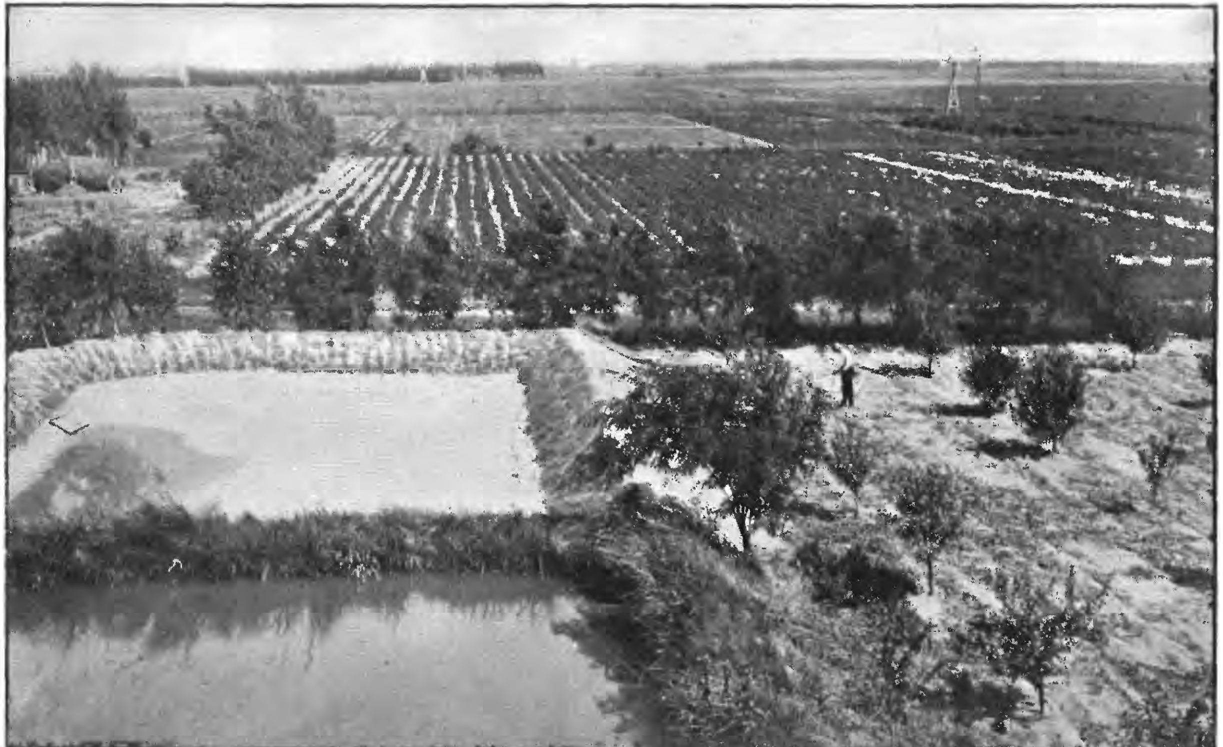


FIG. 1.—Vertical section through reservoir and outlet.

The huge, clumsy windmills of European make, one of which was erected at Lawrence, Kansas, within the memory of the present inhabitants, have within a few decades given place in this country to the light, rapidly running forms. Thousands of these have been made by various firms throughout the country. At first wood was used almost exclusively, but this is being rapidly displaced by metal, especially by thin steel plates and forgings. Although millions of dollars have been invested in the manufacture and purchase of mills and much attention has been given to the mechanical details and the saving in weight and cost, yet comparatively little study has been bestowed upon the actual efficiency of the various forms and upon their development toward theoretical ideals.

A view of gardens cultivated by water pumped by windmills is shown in the accompanying plate (Pl. II). This picture has been taken from a windmill platform. In the foreground is a small reservoir, divided by a bank in the middle, so that one part may be used independently of the other. The part nearer the observer is the older; the second part is a recent addition, rendered necessary by the increase of the area cultivated. Without windmills the cultivation of the tract of country shown in this picture would be impossible. It is doubtful if a single cow could find subsistence on the area which now supports a family.

[1] The Windmill as a Prime Mover, by Alfred R. Wolff, p. 51.



RESERVOIR AND GARDEN.

Another small reservoir and windmill is shown on Pl. III. This reservoir is in the corner of a small suburban truck farm, to which it furnishes water both for vegetables and trees. The owner has stocked a small pond with fish and from it obtains an ample supply for his own use. On the right-hand side is shown the outlet gate, through which water is discharged into a ditch, and that in turn empties into furrows running near the fruit trees and traversing the cultivated ground.

In fig. 1 is given a section through one of these small reservoirs, showing at the bottom the puddled earth or clay that prevents the water from seeping into the adjacent ground. On this puddled earth the banks are built at a height of from 4 to 10 feet. These are usually built by plowing and scraping up the earth from the outside, the tramp-

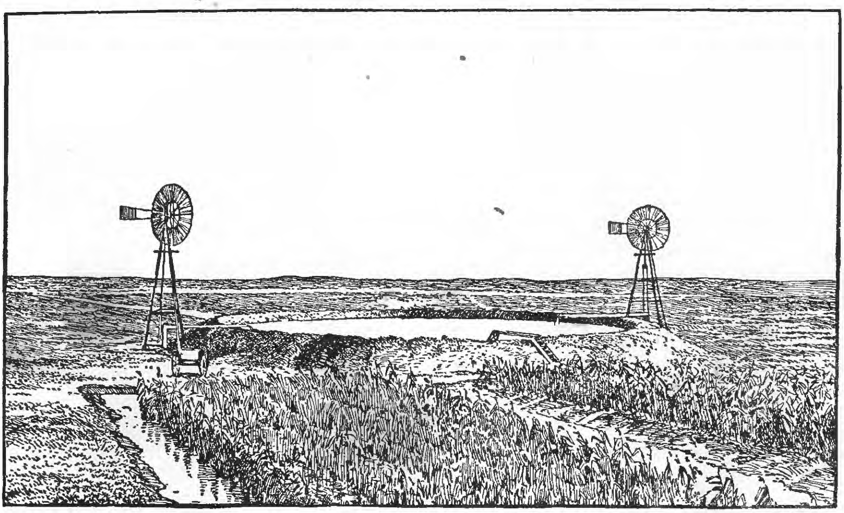


FIG. 2.—Windmills and circular reservoir.

ing of the horses and the men serving to consolidate it. When the bank has been built to the proper height it is smoothed and sodded. On the right-hand side of the figure is the pipe or wooden flume from the windmill, and on the left-hand side is shown the outlet box, which is usually built of 2-inch plank. This is closed by some simple form of wooden gate or valve, either lifted by means of a screw or hinged so as to open outward, and is held in place by the pressure of the water against it.

The square reservoir is the form usually adopted. The mills, as in the other cases, are placed on each side, pumping through short wooden flumes over the bank. These reservoirs are not only used for holding water for irrigation, but, as before stated, with a little care serve as ponds for raising fish.

A view of one of these small square reservoirs is shown on the accompanying plate (Pl. IV). The banks are obscured by the luxuriant growth of weeds. The surface of the water is a little higher than the general level of the land, so that a supply can be drawn by gravity directly to the adjacent fields.

Fig. 2 gives a nearer view of one of these earth reservoirs, which in this case has been built nearly circular in form. The two windmills which supply the water are placed upon opposite sides, in order that the pumps may be as far apart as possible. In many instances three or even four mills, each of moderate size, are placed around a reservoir of considerable size. The banks, made of earth, are covered with sod to protect them from washing by the rain and by the waves during times of high winds.

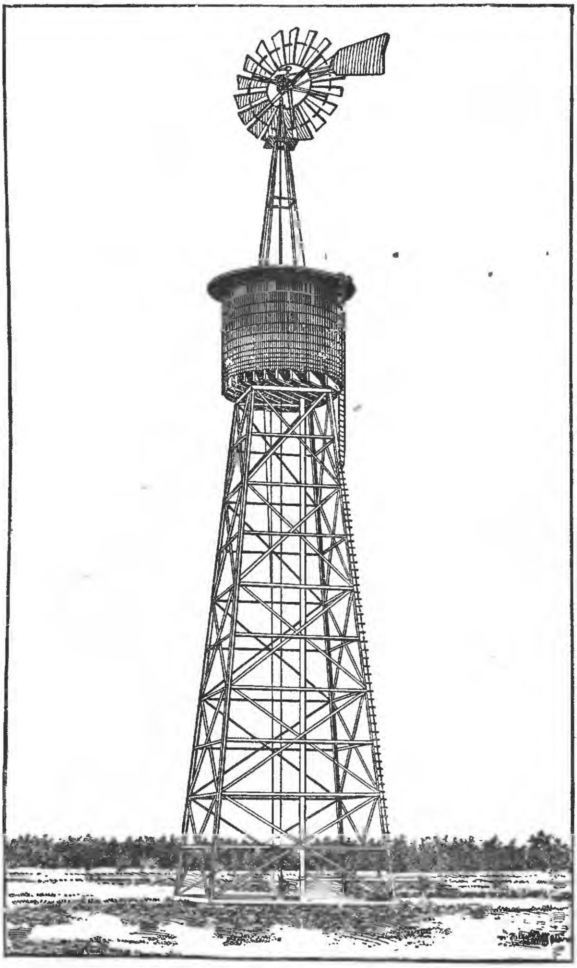
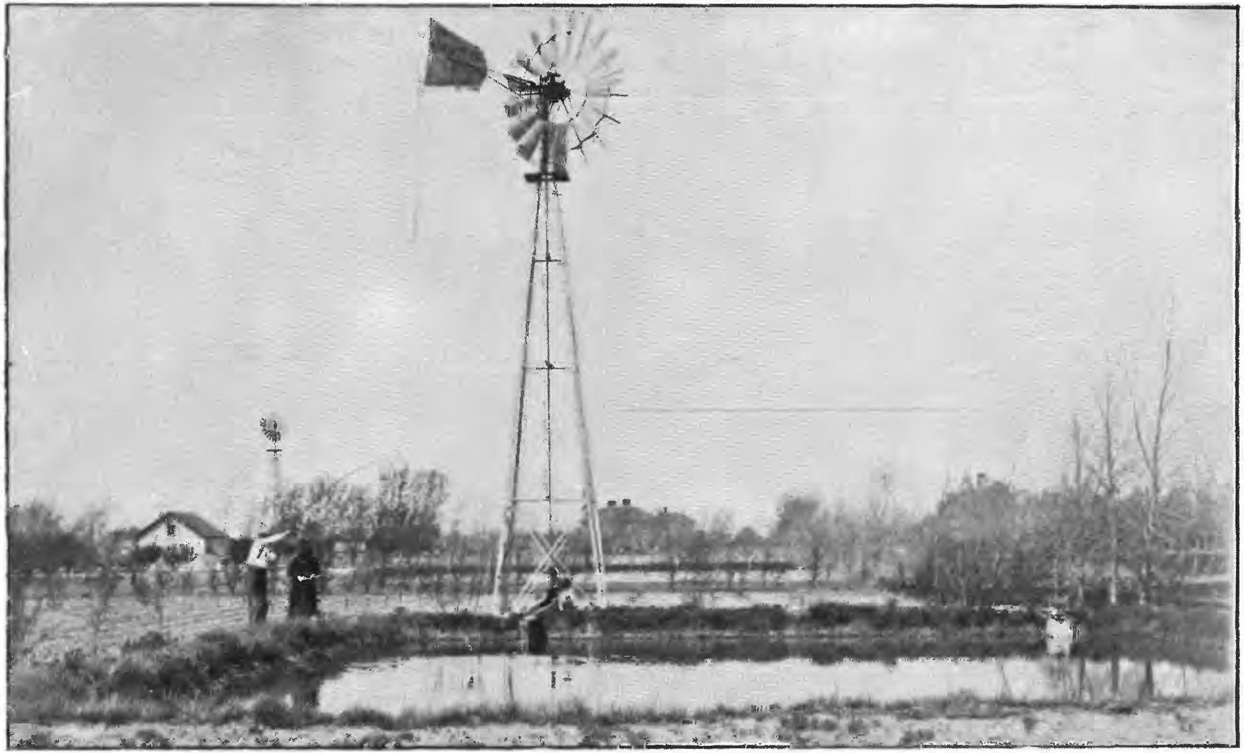


Fig. 3.—Steel windmill and tower carrying tank.

Fig. 3 represents an adaptation of a windmill for use in domestic water supply or for furnishing water to a village or small town under considerable pressure. The wind engine is erected on the top of a high steel tower, which also supports a wooden tank with suitable cover to protect the water from loss by evaporation. This device is generally employed by railroad companies at stations on the Great Plains, where the wind may be depended upon to force a sufficient supply into the tank for use by locomotives or for the railroad shops and offices. Many towns also depend for their water supply upon a windmill pumping water into an elevated tank, particularly where the general surface is so nearly level



WINDMILL AND SMALL RESERVOIR.

SMALL IRRIGATION RESERVOIR.

that it is impossible to construct a small earth reservoir within reasonable distance of the principal buildings.

The device shown in Pl. V, *A*, which has been employed to a considerable extent in the Great Plains region, is usually constructed by the owners. The merit of the device is its cheapness. It may be built mainly of old lumber and other material that can often be found about the farm, such as axles or other gear from old farm machinery, bale wire for staying the sails, and pieces of wood or metal which may be classed as old junk. The machine can not be recommended on the ground of efficiency or economy. If a farmer has sufficient capital to purchase and erect a good windmill he will undoubtedly succeed better than by spending his time in making the cheaper device. On the other hand, in situations where, as is often the case in a dry region, the farmer has lost crops year after year, has exhausted his resources, and is on the verge of bankruptcy, a contrivance of this kind may serve to save a small crop and give him a new start. In such instances there usually will be found pieces of broken-down machinery about the farm. Time and labor are commonly of little value where the ordinary farming operations have been unsuccessful, so that by the exercise of a little ingenuity the material and energy that otherwise would be wasted may be turned to advantage.

The mill or engine consists of a shaft of wood or iron placed horizontally and supported at each end. Upon this sails are fastened by arms extending out at right angles. On each end of the shaft is attached a crank, and each of these cranks in turn drives some simple form of homemade pump. The lower half of the mill is boxed in, and thus forms a small building without roof, above which project the arms carrying the sails.

As illustrating another homemade device, Pl. V, *B*, has been introduced. This mill and water elevator, invented by the owner, has been successfully used to furnish water for irrigation, and, although not by any means an economical device, nor one that can be recommended, it has served its purpose. In other words, while as a rule it is economical to purchase the best, there are circumstances and times when for special reasons the best mill can not be had, but it is still practicable to construct a machine which will accomplish the desired end, that of getting water from the ground upon the land.

These examples might be almost indefinitely multiplied, but are sufficient to demonstrate the principle that with energy and ingenuity a start toward irrigation can be made. When, however, some experience has been had in irrigation and newer mills are being procured, it is highly essential for continued success that something better than the ordinary form of mill be obtained. Many of these have been designed for some other purpose than that of raising large quantities of water through a short distance for irrigation. Some, for example, have been built with the idea of pumping a small quantity from great depth for watering stock. Such mills, as a rule, do not fill the requirements of the irrigator. Thousands of windmills are in use and thousands more will be purchased, involving expenditures on the part of farmers aggregating millions of dollars. A saving of even a small percentage in cost and economy is therefore a matter of considerable importance in the continued development of the water resources of the country.

Fig. 4 shows two windmills so arranged as to pump into a reservoir built of earth, placed upon the highest part of a farm. Running night and day the pumps supply such a quantity of water to the reservoir that when needed a considerable volume can be drawn at once, filling

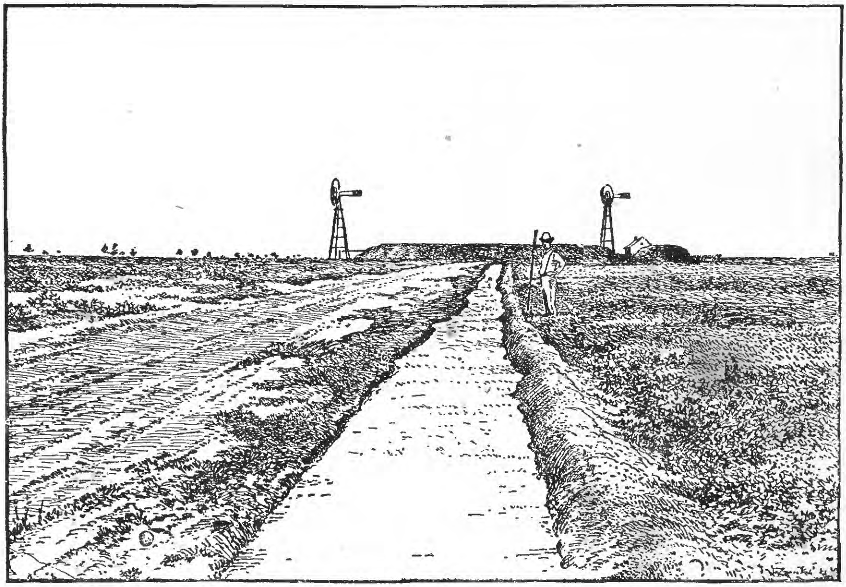
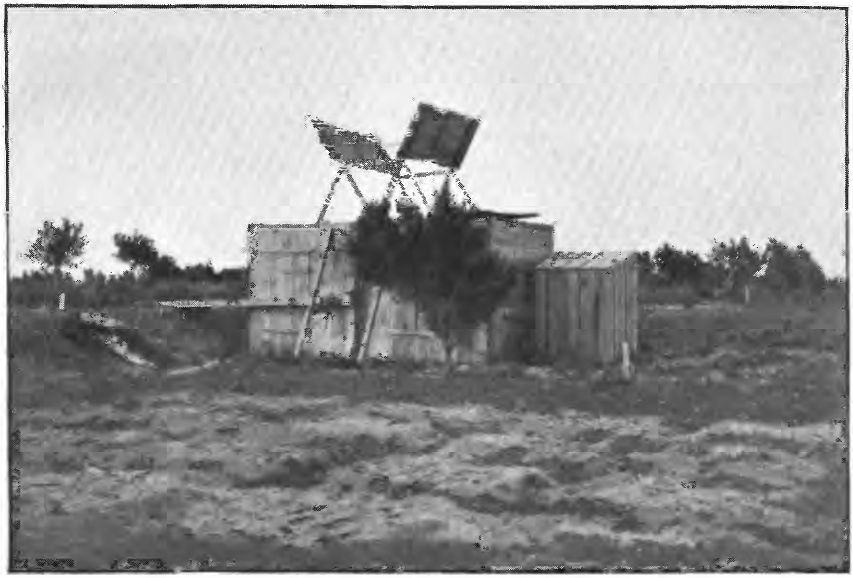


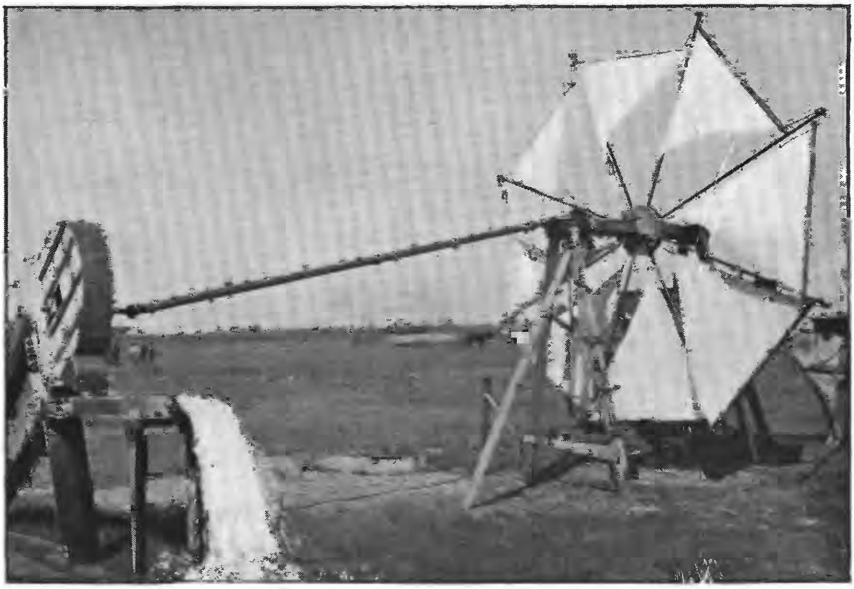
FIG. 4.—Irrigating ditch leading from earth reservoir filled by windmills.

a ditch of as large size as is ordinarily built for irrigating a field of from 10 to 20 acres.

Fig. 5 illustrates one of the methods used to distribute small quantities of water, such as may be had from the tank or reservoir of a windmill. In the drawing the banks are represented as made of cement or of wooden planks, such as are used in portions of California, where water has especial value. The usual practice, however, within the Great Plains region is to make the ditches of earth and to place the little gates in wooden frames. Sometimes instead of using gates the irrigator controls the water by shoveling earth into the ditches and furrows, backing up the water until it overflows the bank or comes out through on opening made by means of the shovel. As shown in the figure, the water flowing in the main stream from right to left is checked by small gates and forced to flow laterally through furrows,



A. HOMEMADE WIND ENGINE, AS USED ON THE GREAT PLAINS.



B DEFENDER WINDMILL AND WATER ELEVATOR

and is again checked in these and forced out upon the beds lying on each side of the furrow. When a certain portion is wet the small gates are adjusted so as to force the water over other portions.

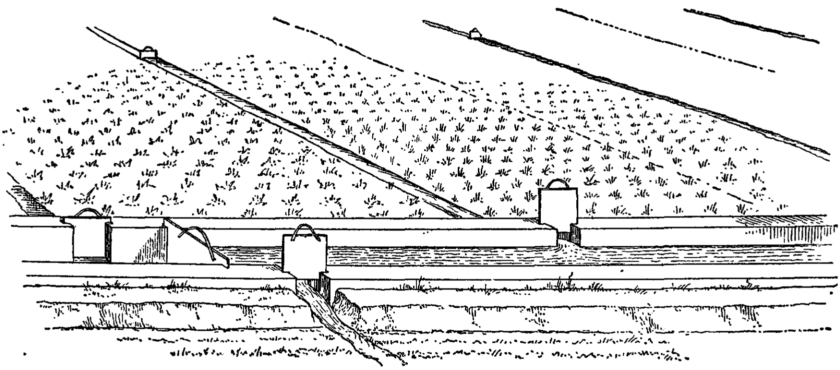


FIG. 5.—Distribution of water to small beds.

Where land has a decided fall or is rolling and it is too expensive to terrace, a system of distributing the water must be devised suited to the contour of the ground. In the case shown by the accompanying figure (fig. 6) the water enters the distributing ditches at the upper left-hand

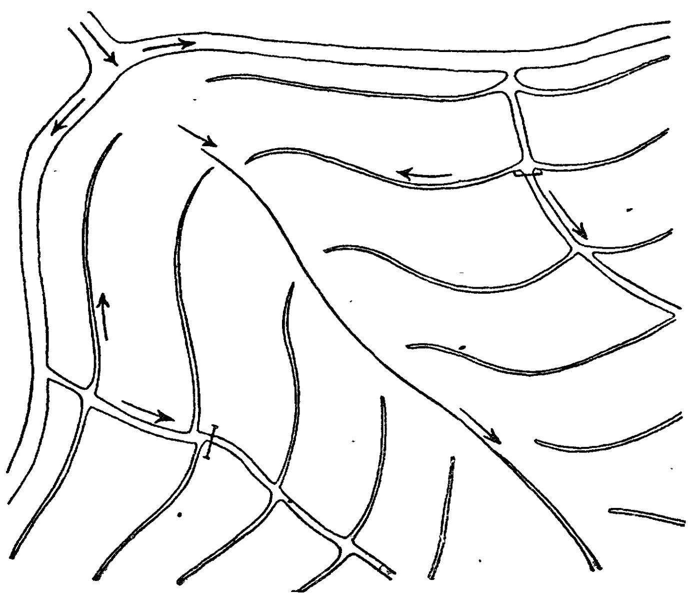


FIG. 6.—Distribution system adapted to irregular land.

corner and, dividing, flows through these and again into still smaller ditches, from which it is turned laterally into furrows. The flow is checked from point to point by little dams or temporary obstructions of earth. After the water passes out upon a field any surplus is caught by a small trench shown in the figure as trending diagonally toward the right-hand lower corner, and from this in turn the water can be caused to flow out upon the lower fields, so that the excess or seepage from the higher portions is not wasted but is employed in the irrigation of the lower parts of the farm.

There are many other devices by which water pumped by windmills may be stored and distributed, but those briefly described above are among the most common. It is sufficient at this time to make mention of these in order to bring out the fact that experiment has demonstrated that it is practicable to irrigate land by water pumped by windmills. Many farmers in various parts of the United States are dependent for their living on this system of agriculture, and others are seeking to gain knowledge by which they may better their condition by providing a water supply for use in times of drought. This is true not only of the West, but of the East and the South, especially in regions where the soil, though fertile, is light and becomes dry if the rains do not occur with great regularity. Even on the Atlantic seaboard the early truck farmers are experimenting with devices of this character, copying methods which have already proved successful in Kansas. Modifications in mills and other machinery are doubtless necessary to suit local conditions. For a given purpose one mill may be far better than another, and of two wind engines similar in cost one may be actually worth two or three times as much as the other.

The dynamometric experiments carried on by Mr. Perry, as described in the following pages, have brought out many important points upon which depend the relative efficiency of various mills and the advantages of one type over another. Some of these points have already been considered and made use of in mills now on the market, while others of equal or greater importance are awaiting practical application and ingenuity. It is hoped that by the publication of these facts interest will be aroused and inventors will be stimulated to continue work along various lines, leading to further improvement and to a consequent cheapening of the cost of raising water by wind power.

EXPERIMENTS WITH WINDMILLS.

By THOMAS O. PERRY.

GENERAL STATEMENT.

The experiments with wind wheels described in the following pages were commenced June 1, 1882, and concluded September 15, 1883. At that time wind wheels in this country were nearly all made with narrow wooden slats for sails, set at various angles with the plane of the wheel, ranging from 35 to 45 degrees.

The slats were usually placed as close together as possible without having their projections on the plane of the wheel overlap. The proportions of sail surface and their angles of weather were apparently arrived at without any well-defined purpose. The only experiments made in the United States, so far as could be learned, related to starting forces only. They did not include the measurement of work in foot-pounds. The only well-defined experiments with wind wheels, the records of which were available, were those made in England by John Smeaton, F. R. S., about one hundred and twenty-five years before. Smeaton's paper, On the Construction and Effects of Windmill Sails, was read before the Royal Society May 31 and June 14, 1759, and to this day has remained the only definite available source of information on the subject treated. The paper was republished in Tredgold's Tracts on Hydraulics, a copy of which was received before these experiments were commenced.

It had been especially noted that Smeaton's angles of weather were much less than those in common use in this country, and note had also been taken of his statement to the effect that the work of a wind wheel is not increased, but diminished, by crowding it with sails so that their total surface exceeds about seven-eighths of the circular area containing them. The universal practice here was to crowd the wheel with slats until the total sail surface exceeded the total area of the annular zone containing them by more than one-fifth of the whole zone; according to Smeaton, this was adding to the greatest possible effective sail area more than 39 per cent additional area of worse than useless surface. If a large part of the material could be saved, and at the same time a considerable increase in power be effected, no slight gain would be secured. Smeaton's experiments were made so long ago that they were disregarded, if they were ever noticed, by the builders of modern wind wheels. Our experiments fully confirmed Smeaton's results in regard to the greatest effective amount of sail area, although we do not necessarily accept Smeaton's judgment that it is not profitable to increase the total area of sails beyond about 37 per cent of the annular zone containing them.

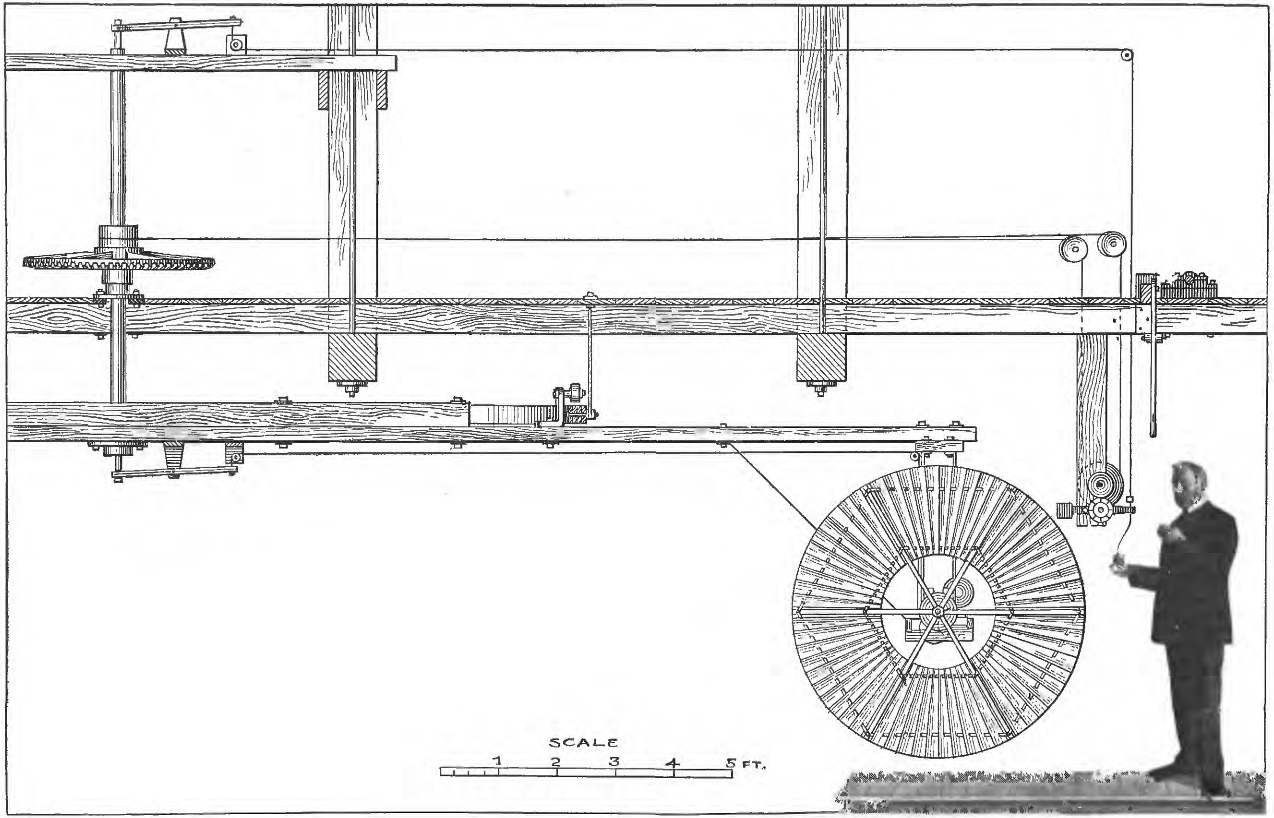
In wheel No. 3 we followed closely Smeaton's angles of weather, but did not obtain so good results as with greater angles. In fact, we were not able to obtain the best results with weather angles so small as Smeaton's in any of our wind wheels. Nor did our sail speeds, as compared with wind velocity, nearly approach the speeds obtained by Smeaton. Even our unloaded wheels did not show the sail speed attained by the best of Smeaton's wheels when loaded for maximum work.

This difference in the ratio of sail speed to wind velocity constituted the greatest disagreement between his results and ours. Our loads at the maximum of work were smaller as compared with greatest loads, and the speeds of revolution at maximum work as compared with speeds of unloaded wheels were smaller for our wheels than for Smeaton's. The general laws established by Smeaton, as enunciated in his eight "maxims," were substantially confirmed by our experiments. The law of cubes pertaining to maximum products and the law of squares pertaining to greatest loads, or starting forces, were more exactly fulfilled in our experiments than in Smeaton's. The differences in results as regards angles of weather, speeds, and best loads might have been due to differences in the construction of the wheels.

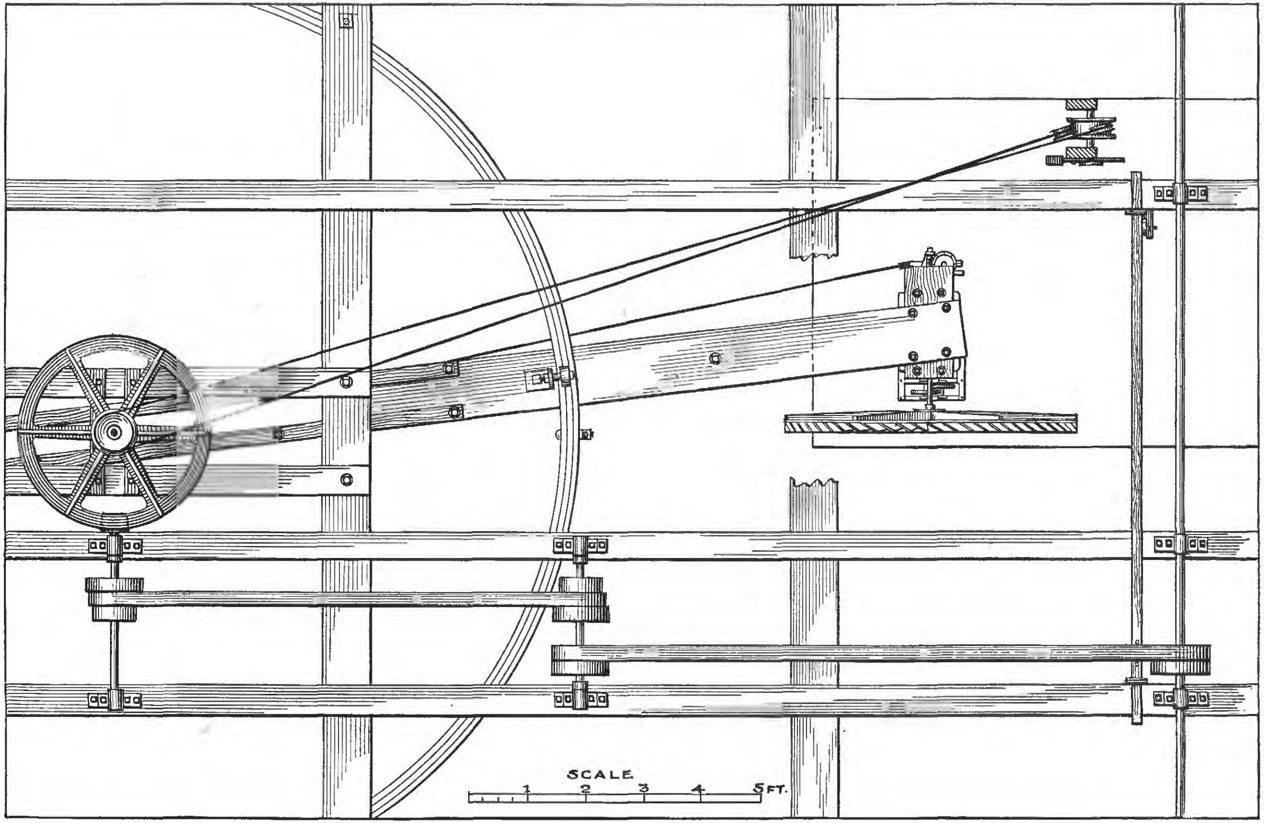
Smeaton constructed his wheels after the manner of windmill construction in his time, and we constructed our wheels mostly after the methods adopted by American manufacturers at the time we experimented. Our methods of experiment were similar, though Smeaton's experiments were conducted on a much smaller scale with smaller wheels, and his appliances generally were crude as compared with our own.

Smeaton allowed about one-eighth of his total applied load as the equivalent additional load due to friction. This friction, as Smeaton determined it, was in reality friction at starting only, and it must have been much in excess of the friction under the rapid revolutions of his wheels, and helped considerably to swell his products.

Still another allowance must be made in comparing Smeaton's efficiencies with ours, on account of the relative shortness of the sweep, which carried his wheels in a circle against still air. The velocity of the centers of his wheels was taken as the equivalent velocity of wind. As a matter of fact, some point beyond the center of the wind wheel, farther away from the axis of the sweep, should have been taken to represent wind velocity. The distance from the axis of Smeaton's



ELEVATION OF APPARATUS USED IN WHEEL TESTS.



PLAN OF APPARATUS USED IN WHEEL TESTS.

sweep to the extremity of his sail as it revolved was, when greatest, nearly double its least distance, and the energy of the wind acting on the tip of his sail was more than seven times as great at the maximum as at the minimum distance. To compute accurately the total wind energy intercepted by the wheel, after taking into account the various velocities of impingement, would be an interesting problem in integral calculus.

The differential expression for work is easily found, but the integration is not so simple. An approximate calculation shows that the wind energy intercepted by Smeaton's wheel was more than 7 per cent in excess of what it would have been had all points of the wheel met the air with the same velocity as at the center. The difference is not so large as one would expect from the great difference in energy at extreme positions. We doubt whether anything should be deducted from Smeaton's products on this account, for only a very limited portion of the sails could have a proper relative velocity with respect to the wind. Our wheels were, of course, similarly affected, although in considerably less degree.

Weisbach's theoretical formulæ for wind action, if correct, would cast a cloud over Smeaton's results; for he concludes his rather elaborate mathematical discussion with these words: "According to these experiments [Smeaton's] the action of the wind in general upon sail wheels is greater than is given either by the theory or by Coulomb's experiments." Coulomb's very incomplete experiments seem to have been a great source of consolation to those mathematicians whose theories would not permit the realization of efficiencies so great as Smeaton obtained.

Our own experience with wind wheels in actual use, in widely different lines of work, and the testimony of many persons using them, convince us that the efficiencies we obtained and which Smeaton obtained in artificial wind are not so great as those commonly obtained now in natural wind. We consider our experiments valuable principally on account of the comparisons made and the insight they afford into the causes of waste. Through the courtesy of Mr. H. N. Wade, manager of the United States Wind Engine and Pump Company, of Batavia, Illinois, we are enabled to reproduce the original records of our experiments on wind wheels as they were made in 1883.

APPARATUS.

Our experiments were conducted in a room about 36 feet wide, 48 feet long, and 19 feet high from floor to roof trusses.

Pls. VI and VII represent, in elevation and plan, as a whole, the apparatus that was used and show portions of the roof trusses underneath which the sweep was suspended. The rear end of the sweep, not shown, carried a counterweight, which caused the whole apparatus to balance on the central shaft. Two rollers carried by the sweep on opposite sides of the suspending shaft ran lightly over a large circular track and prevented vertical oscillations. Pls. VIII and IX present enlarged views, showing the details of the dynamometer and its method of application.

These four plates and the illustrations of wind wheels Nos. 3, 6, 19, 24, 29, 39, 40, 44, 60, and 61 were recently prepared from data in our possession. The illustrations of wheels Nos. 2, 35, and 48 are reproductions of the drawings made for the original records, as are also all the other illustrative figures which are shown in connection with the original records. In computing the figures denoting efficiencies, shown in connection with the illustrations of wheels Nos. 3 and 6, axle friction was called 0.15 pound instead of the 0.3 pound given in the tables, as explained under the heading "Corrections for axle friction."

In these two cases, and also in computing the efficiency of wheel No. 2, allowance was made for temperature as recorded and for atmospheric density, according to the records of the United States Signal Service station at Chicago for 2 p. m. on the days of experiments.

The efficiencies shown in connection with all the other illustrations of wind wheels were obtained by simply multiplying the efficiency for No. 2 by the ratios of products given in the various tables where direct comparison with No. 2 is shown.

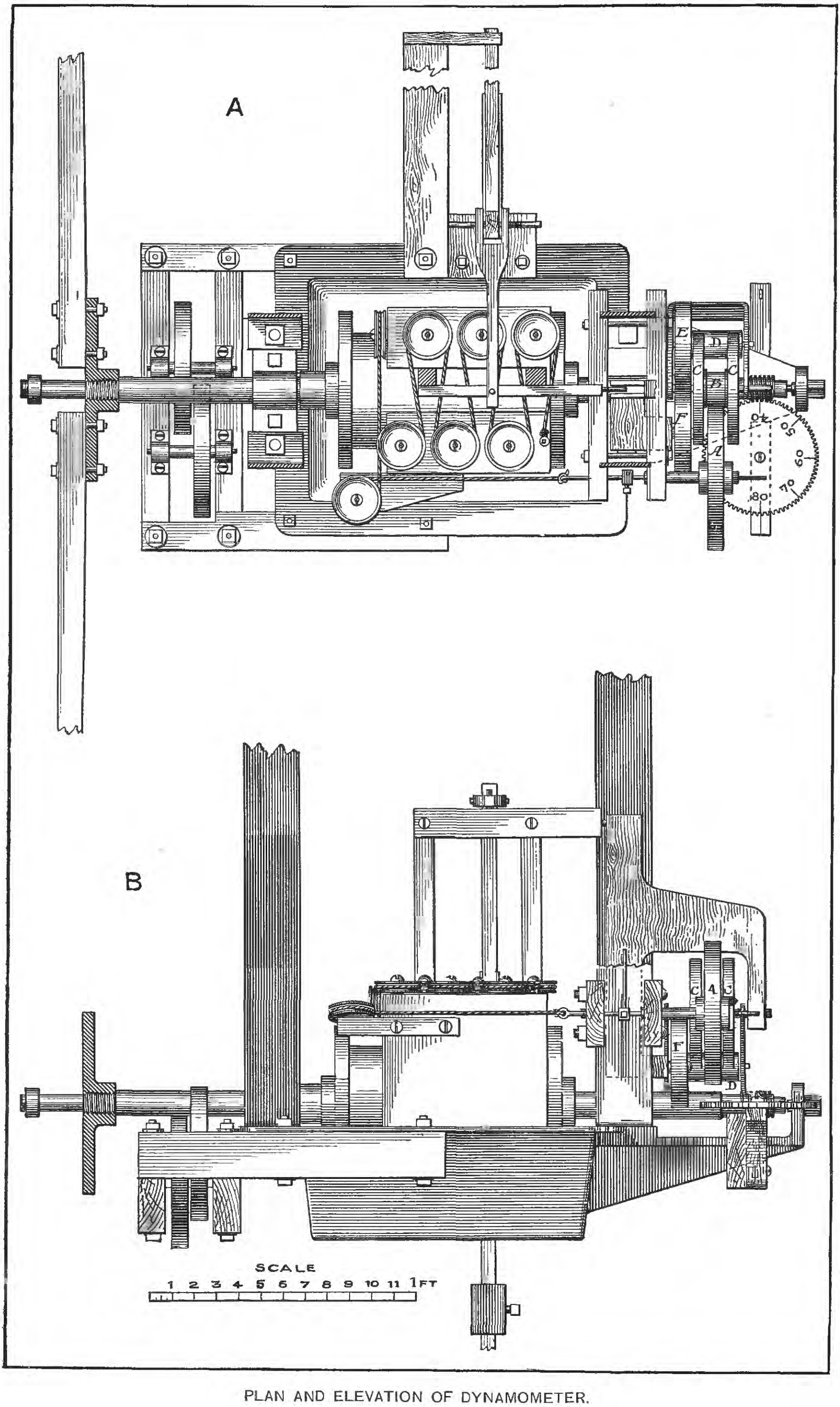
The original records of 1883, immediately following, end on page 72. The comments following page 72, including the tables on pages 74 and 77, have recently been prepared.

ANGLE OF WEATHER.

The term "angle of weather," used in connection with the following tables, means the angle made by the face of the sail with the plane of the wheel.

ARTIFICIAL WIND.

In conducting these experiments with windmills, it was necessary that results to be compared should be obtained in wind of uniform velocity. We therefore made use of artificial wind, obtained by carrying the wind wheels in a circle against still air on the end of a long sweep suspended beneath the roof trusses of a large room used for setting up tanks. The sweep was made to revolve horizontally around its vertical axis by means of gearing, pulleys, and belts connected with a line shaft driven by an 80-horsepower Reynolds-Corliss steam engine, which furnished the motive power for the works of the United States Wind Engine and Pump Company. The distance from axis of sweep to center of wind wheel was 14 feet, so that the velocity of wind against the wind wheel in miles per hour would be indicated very closely by the number of revolutions per minute made by the sweep. To attain absolute accuracy the length of sweep should have been 14.006 feet. Hence it was only necessary to note the number of turns per minute



PLAN AND ELEVATION OF DYNAMOMETER.

made by the sweep in order to know the velocity of wind in miles per hour with sufficient accuracy.

For counting the number of turns of the sweep a toothed wheel, containing 80 teeth, was used, which was made to revolve at the same rate as the sweep. Thus the fractions of a turn could be easily determined to within 0.0125, and we made a practice of setting down the fractional turns to within 0.00125, thus securing relative accuracy in determining the velocity of the wind to within 0.00125 of a mile per hour.

The face of the wind wheel was set at right angles to the direction of the wind. The frame supporting the shaft of the wind wheel was suspended about 3 feet below the sweep by four iron straps, presenting their thin edges to the wind, so that the action of the wind upon the wheel should be obstructed as little as possible by the sweep or otherwise. The motion of the sweep undoubtedly carried the air of the room with it to some extent, but not so as to produce a noticeable current.

DYNAMOMETER.

The ordinary Prony friction brake was used, with such modifications and additions as circumstances demanded.

The brake consisted of two pine blocks, clamped vertically upon a brass cylinder 5¼ inches in diameter, attached to the shaft of the wind wheel. But instead of using two bolts, with thumb nuts for adjusting the brake, one bolt was used as a hinge beneath the cylinder, and the adjustment was made by means of a cord passing across from one block to the other around small iron sheaves, which turned on common wood screws fastened into the tops of the two blocks, so that the pressure of the blocks against the cylinder corresponded to about sixteen times the tension of the cord.

The adjusting cord was carried back from the brake some distance in a direction parallel to the axis of the cylinder to a sheave fixed to the frame supporting the shaft of the wind wheel. From that point the cord was carried by means of sheaves, levers, etc., up through the hollow shaft supporting the sweep and finally fastened to one end of a lever. The other end of the lever was hinged and provided with a sliding weight, by means of which the tension of the cord could be adjusted from the station of observation while the sweep was in motion. The brake was provided with a horizontal graduated arm carrying a weight, which could be moved in or out to correspond to any desired load. Suitable stops were provided, allowing a limited motion of the brake and graduated arm, which could be conveniently watched from the station of observation, just beyond the reach of the sweep. The brake and graduated arm were balanced, as regards the action both of gravity and of centrifugal force, before applying the loads, which were balanced by the friction of the brake. The thrust of the rear end of the shaft of the wind wheel was sustained by a steel point.

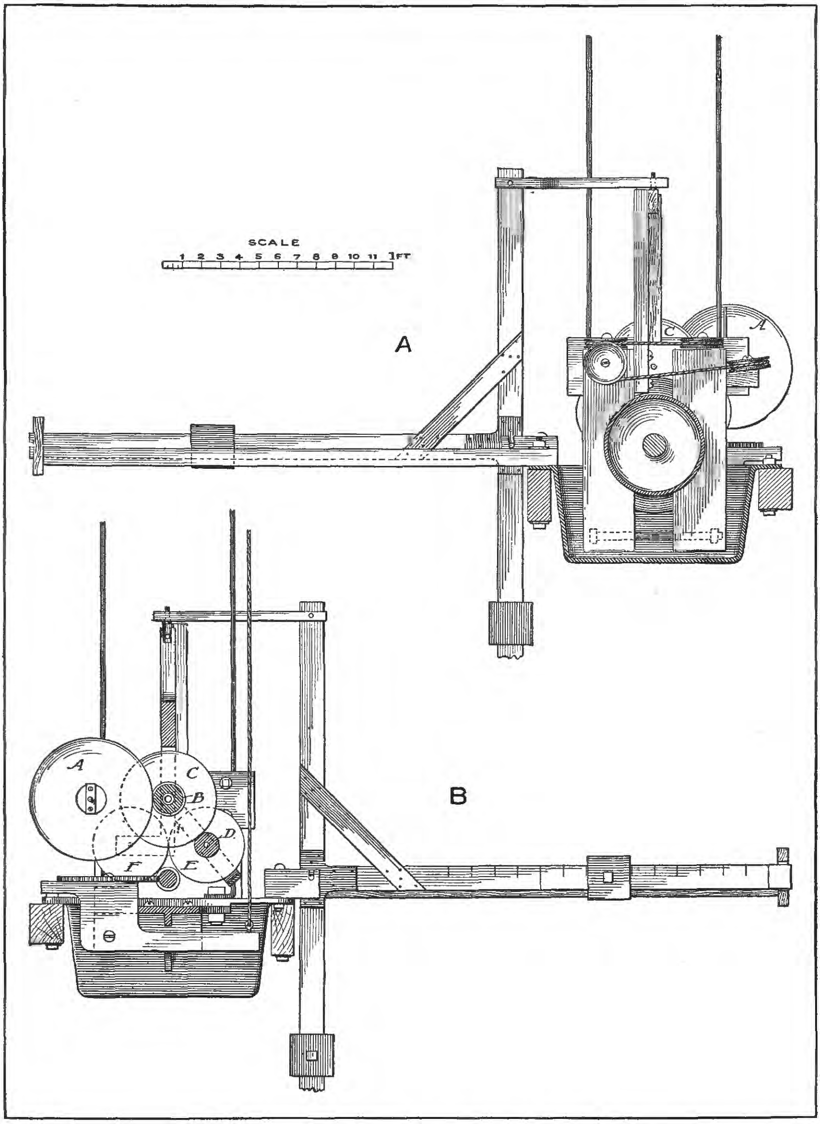
METHOD OF EXPERIMENT.

For noting the number of turns of the wind wheel, the rear end of the shaft was provided with a worm, with which a toothed wheel could be thrown in or out of contact at the pleasure of the operator from the station of observation. Each turn of the wind wheel would cause the toothed wheel to make 0.0125 of a revolution, as there were 80 teeth in the wheel, and the fractional turns could easily be estimated to within 0.1 of a revolution of the wind wheel. The weight on the graduated arm was always expressed in pounds and decimal parts at 1 foot from the center of shaft of the wind wheel. During use the brake was freely lubricated with machine oil.

For a trial load the weight on the graduated arm was set at some number, say 1 pound. Then the counter which was to record the speed of the wind wheel was set at 0, as was also the counter which was to record the velocity of the wind. The sweep having been set in motion, the speed of the wind wheel was checked by the brake, which was adjusted from the station of the observer by regulating the tension of the adjusting cord until the weight on the graduated arm was just lifted by the friction, and no more. As soon as the friction and weight were well balanced the counters which were to record the speed of the wheel and velocity of wind were thrown in by a single movement of a lever, and at the expiration of one minute they were both instantaneously thrown out. Then the sweep was stopped at the station of observation, the number of revolutions of the wind wheel recorded, and also the velocity of the wind.

To the load applied was added the friction of the journals, also expressed in pounds at 1 foot from the center of the shaft. This sum made the total load, which was multiplied by the number of turns of the wind wheel.

Then a second product was obtained in precisely the same manner with a greater load applied; and if greater than the first, a still greater load was applied and the corresponding product obtained. A succession of products were thus obtained corresponding to a succession of loads gradually increased until the products began to decrease on account of the diminishing speed of the wheel. The greatest product corresponded to the best load for the wheel, and the actual work performed by the wheel in each case could be obtained by multiplying the product by 6.2832 feet, equal to the circumference of the circle whose radius is 1 foot. In obtaining these products measurements were taken several times for each load and the total load was multiplied by the average turns of the wheel per minute. The average wind was also taken for record. The velocity of wind was supposed to be nearly uniform, but varied slightly with the speed of the engine, and the variation was sufficient to affect results considerably in some cases; hence the necessity of recording the velocity of the wind in each case. In comparing



END VIEWS OF DYNAMOMETER.

results some allowance also had to be made for differences in velocities of wind.

In testing the speed of the wheel unloaded the brake was removed, making the load nothing but the journal friction.

STARTING FORCES.

The determination of starting forces was attended with some difficulty. In the first place, it became necessary to define what should be considered the starting force of a wind wheel—whether it should be considered the greatest load under which the wheel would turn, or the greatest load which the wheel would balance without turning. It was also necessary to decide what motion of the wheel should be defined as turning. The wheel with a certain load would sometimes perform one revolution in ten or twenty minutes, if that could be called turning; between that and proper continuous turning there were many gradations. It was impracticable to decide when the wheel reached the lowest limit of speed which could properly be called turning. Therefore we have considered the starting force as the greatest load the wheel would balance without turning. Our method of determining starting forces was to clamp the brake so tightly on the cylinder that the wheel could not turn, but could lift the load applied such small distance as was allowed by the stops which limited the motion of the brake. The sweep was then set in motion and timed for one minute, during which time the load on the graduated arm was watched, its alternating upward and downward movements were noted, and judgment was exercised as to the balance between the load applied and the starting force of the wind wheel. The average velocity of wind during these tests was always recorded after March 13, 1883, but not always before that date.

In lifting the load the starting friction of the journals was also overcome by the wind wheel, but when the weight descended the load overcame the starting friction of journals in addition to the starting force of wind, and as the load almost constantly moved up and down when well balanced, nothing should be added to loads applied for friction in case of starting forces. Yet the friction has been added in the tables, the same as to other loads applied, in order to accommodate the statements at the head of columns for loads. On account of the starting friction being great and uncertain, and for other reasons, the starting forces recorded prior to March 13, 1883, are not so much to be relied upon as those recorded after that date; nor were the later tests for starting forces nearly so accurate as those for products.

JOURNAL FRICTION.

For journal friction previous to March 13, 1883, see description in connection with wind wheel No. 11, page 38. After the date above given the journal friction was greatly reduced by the use of large antifriction wheels in place of the front box, just behind the wind wheel and in front of cylinder, where the weight of the wind wheel and cylinder was mostly supported. In place of the rear box a small steel pin was inserted in the end of the shaft and turned in a brass box. The end thrust of the shaft, due to wind pressure and tension of the adjusting cord, was sustained by a steel point, as before.

As thus improved, one-half an ounce hung to a thread wound around the outside of wheel No. 37 overcame friction at very slow motion, and was almost if not quite equal to starting friction. The brake was removed during this test.

Let $x =$ weight 12 inches from axis required to overcome friction.
30 inches = distance from axis to thread.

$\frac{1}{2}$ ounce $= \frac{1}{32}$ pound.

$$12\,x = \tfrac{30}{32}$$

$$x = 0.0785 = \text{friction at very slow motion or at starting.}$$

With brake and wind wheel removed 26.50 ounces hung by a thread wound around the forward end of shaft 0.7969 inch in diameter overcame friction at very slow motion and was almost equal to starting friction, and 24.25 ounces hung in the same way overcame friction at 50 turns per minute, and would not overcome friction when speed was much less. 0.8125 inch = diameter outside of thread, 0.4024 inch = radius to center of thread.

Friction at 50 turns per minute is to starting friction as 24.25 is to 26.50. Hence, $0.0785 \times \frac{24.25}{26.50} = 0.0714$ pound = journal friction at 50 turns per minute, with wind wheel on and brake removed. The weight of the brake would, of course, add a little to the friction, which we have uniformly called 0.1 in the tables, as explained on page 28, under the table.

The 0.3-pound friction added to loads applied prior to March 13, 1883, was friction at very slow motion and was undoubtedly much in excess of friction at speed of wheels corresponding to maximum products, as is evinced by the fact that notwithstanding the great reduction of starting friction the tables do not show for the same applied loads, or even unloaded, very much increase of speed of wheels after March 13, 1883, over speeds indicated previous to that date. If 0.1 pound had been added to applied loads previous to March 13, 1883, the products at maximum and for higher speeds of wheels would probably have been more nearly correct. It was this uncertainty and ignorance regarding friction at different speeds that led us to reduce as much as possible this source of error. However, the error thus arising does not greatly affect comparative values of products recorded prior to March 13, 1883, although products affected by allowance of 0.3 pound for friction should not, unless corrected, be compared with products recorded after above date.

COMPARATIVE STANDARD WHEEL.

In the earlier stages of these experiments we acted on the supposition that only such variations in the state of the atmosphere as were indicated by the barometer and thermometer should affect the work performed by a wheel tested at different times in wind of the same velocity. But later we were convinced that the readings of the barometer and thermometer would not always account for variations in products given by the same wheel on different days. We were also convinced of the uselessness of speculating as to, or trying to understand and forestall, all the causes that might combine to affect results. Therefore we adopted the practice of comparing the maximum product of each wheel tested with the maximum product of wheel No. 2, obtained in close proximity as to time and under similar conditions in all respects. This could be done with a good deal of certainty, and no further trouble was experienced on account of contradictory results. After the best loads had been ascertained by trial, comparative tests were made by trying first one wheel and then the other in succession until a considerable number of measurements had been taken. Then as many measurements as possible, giving the same average velocity of wind for both wheels, were selected from each set of experiments.

In this way the product obtained was usually the average of 8 to 10 measurements of each wheel made under the same conditions of time, wind, weather, and whatever else might affect the results. The product of No. 2 was thus used as a standard of measure, and the ratio of products indicated the relative value of the wheel compared.

For further particulars consult tables of experiments of later date than March 13, 1883.

Results of experiments performed in March and May, 1883, with wheel No. 2.

Velocity of wind per hour.	Load applied + 0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Thermometer.	Date.
Miles.	*Pounds.*			°*F.*	1883.
6.437	0.1	43.78	....	62	May 14
6.371	1.1	23.90	26.290	54	May 14
6.381	2.4	0	....	62	May 14
8.417	.1	59.05	....	53	Mar. 14
8.403	1.9	32.875	62.463	50	Mar. 14
8.405	4.2	0	....	53	Mar. 14
10.898	.1	77.85	....	46	May 21
11.041	3.3	41.06	135.498	60	May 17
10.976	6.9	0	....	46	May 21

Previous to making the above experiments, March 13, 1883, the dynamometer was improved by the addition of an automatic brake adjuster and antifriction wheels, which reduced starting friction from about 0.3 to about 0.08 pound, and made journal friction at 50 turns per minute about 0.071 pound. In the table friction was called 0.1 pound for convenience, as the absolute friction in each case could not easily be obtained, and the error does not materially affect comparative results. (See pages 25 and 26 for "Journal friction.")

Average products of wheel No. 2, which was used as a standard for comparison, were obtained by selecting from numerous tests several numbers—generally 8 to 10—which corresponded to wind of the same average velocity as wind for wheels compared in each case; the difference of averages in no case exceeded 0.001 mile per hour in the table on page 47.

Measurements for maximum products of two wheels compared were always taken as far as possible under the same conditions of wind, temperature, barometer, weather, time, etc.

RESULTS OF ORIGINAL TESTS.

TABLES AND DESCRIPTIONS OF WIND WHEELS.

WHEEL NO. 1.—*Results of experiments.*

Velocity of wind per hour.	Load applied +0.3 for friction.	Turns of wheel per minute.	Product.
Miles.	*Pounds.*		
8.427	0.3	59.08	17.724
8.431	1.2	42.60	51.120
8.385	1.4	39.90	55.860
8.412	1.6	35.69	57.104
8.452	1.8	31.69	57.042
8.432	2.0	28.02	56.040
....	2.85	0	0

Experiments made June 1 and 2, 1882.
Load expressed in pounds applied 1 foot from center of wheel.
Circumference of circle whose radius is 1 foot, 6.2832 feet.
Work per minute at the maximum, $57.104 \times 6.283 = 358.784$ foot-pounds
Diameter of wheel, 5 feet.
Number of slats or sails, 30.
Length of slats, 18 inches.
Width of slat at outer end, 3.56 inches.
Width of slat at inner end, 1.344 inches.
Total area of slats or sails, 9.19 square feet.
Angle of weather, 35 degrees.
Portion of available annular space filled by sails, 0.5.

The spaces between are equal to the spaces occupied by the slats. This wheel is the same as No. 2 with one-half the slats omitted.

Slats, plain surfaces, made of white pine three-sixteenths of an inch thick, with forward edges trimmed to an edge.

WHEEL NO. 2.—*Results of experiments.*

Velocity of wind per hour.	Load applied+0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			° *F.*	1882.
8.411	0.3	57.95	17.385	69	....
8.381	0.9	49.00	44.100	....	....
8.451	1.05	47.00	49.350	....	....
8.435	1.3	43.08	56.004	77	June 7
8.445	1.5	40.80	61.200	73	June 7
8.412	1.7	37.30	63.410	70	June 7
8.460	1.8	35.85	64.530	78	....
8.436	1.9	34.89	66.291	72	June 8
8.429	2.0	33.43	66.860	78	....
8.443	2.1	31.33	65.793	69	June 8
8.421	2.3	27.60	63.480	73	June 8
8.446	2.5	24.48	61.200	75	June 8
8.416	2.7	21.70	58.590	76	June 8
8.398	2.9	20.00	58.000	....	....
8.338	3.1	15.80	48.980	....	....
....	3.81	0	0	....	June 9

Total area of slat sails 18.380 square feet.

Angle of weather, 35 degrees.

The same as No. 1, with double the number of slats—that is, the projections of the slats upon the plane of the wheel equaled the entire available annular area. An area as shown in fig. 8 at each of the six arms constituted the space not available. Slats held in place by 12 straight notched pieces extending from arm to arm the same as in No. 1.

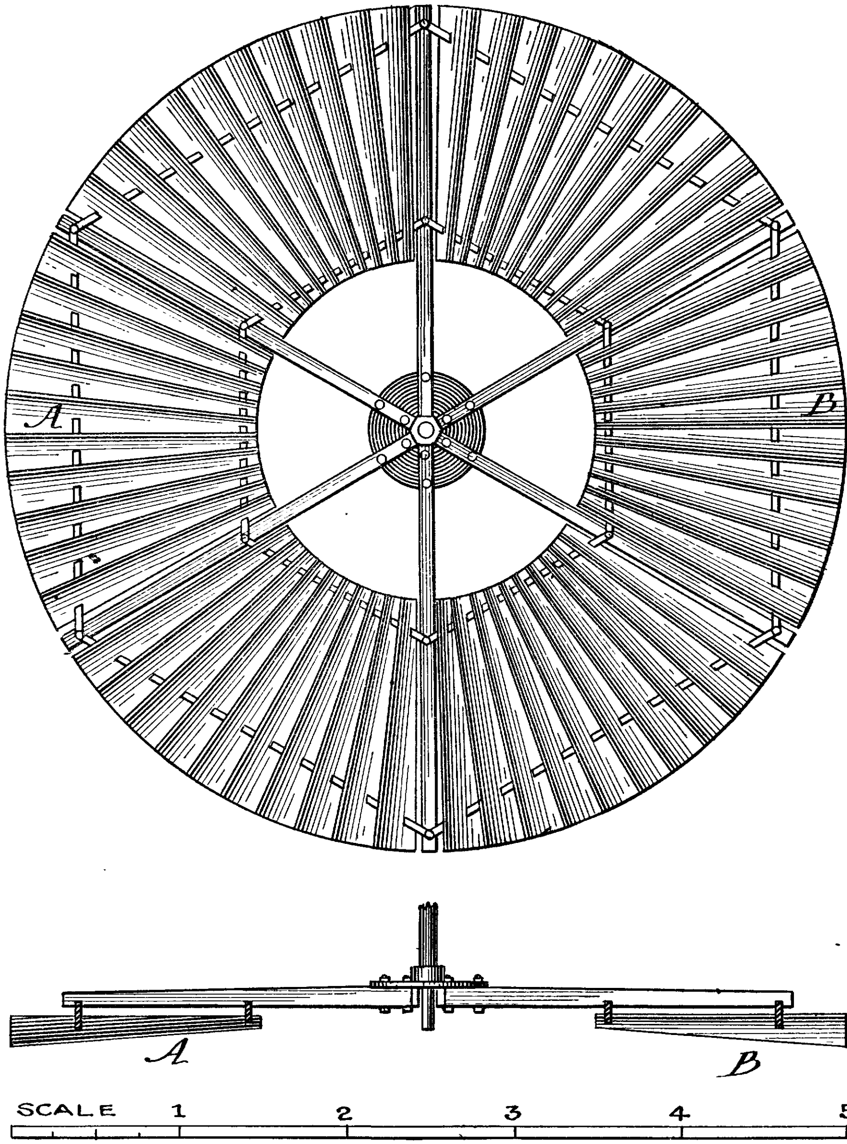


FIG. 7.—Elevation and section of wheel No. 2. (Sail area, 18.380 square feet; efficiency, 0.154.)

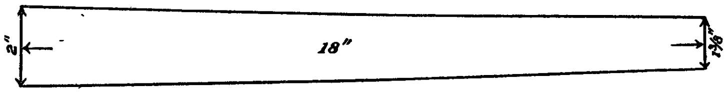


FIG. 8.—Outline of area not available.

WHEEL NO. 3.—*Results of experiments.*

Velocity of wind per hour.	Loads applied+0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			° *F.*	1882.
8.476	0.3	90.48	27.144	68	June 22
8.435	.95	69.40	65.930	68	June 22
8.441	1.05	64.90	68.145	68	June 21
8.428	1.1	62.15	68.365	88	June 22
8.447	1.2	59.65	71.580	69	June 19
8.436	1.3	55.36	71.968	83	June 22
8.402	1.4	51.10	71.540	69	June 21
8.423	1.5	47.48	71.220	74	June 19 June 22
8.441	1.7	41.00	69.700	71	June 22
8.404	1.9	32.00	60.800	71	June 22
....	2.4	0	0	....	June 22

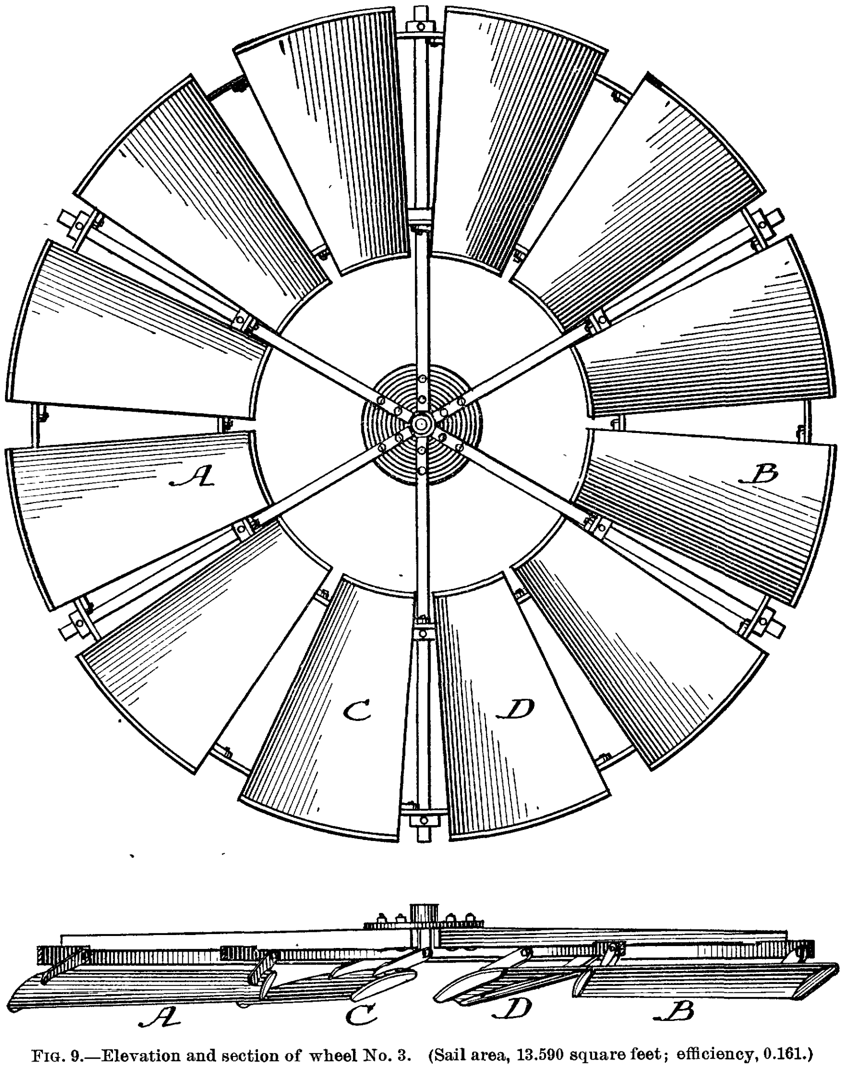


FIG. 9.—Elevation and section of wheel No. 3. (Sail area, 13.590 square feet; efficiency, 0.161.)

Diameter of wheel, 5 feet.

Number of sails, 12.

Sails of lineal dimensions as shown in figs. 10 and 11, made of white-pine boards three-sixteenths of an inch thick and fastened by hinges near each end to 12 pine bands bent to arcs of circle and secured to the 6 arms.

Total sail surface, 13.59 square feet.

Sails concave toward the wind.

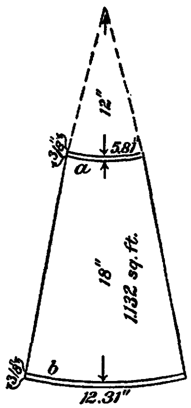


FIG. 10.—Dimensions of sails of wheel No. 3. 18 inches is the extreme length of sail.

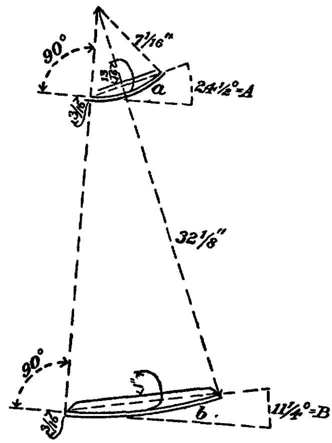


FIG. 11.—Angles of weather of sails of wheel No. 3. *A*, Angle of weather at inner ends of sails; *B*, Angle of weather at outer ends of sails.

WHEEL NO. 4.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			° *F.*	1882.
8.354	0.3	89.43	26.829	76	June 29
8.372	0.9	74.54	67.086	76	June 29
8.375	1.1	70.03	77.033	76	June 29
8.324	1.3	62.75	81.575	68	June 29
8.461	1.5	60.72	91.080	73	June 28
8.467	1.7	55.93	95.081	73	June 28
8.465	1.9	48.75	92.625	73	June 28
8.366	2.1	42.78	89.838	68	June 29
8.386	2.3	38.50	88.550	68	June 29
8.369	2.5	29.13	72.825	68	June 29
....	3.3	0	0	....	June 29

Wheel the same as No. 3, except that the inclination of the sails was changed, making the angle of weather 20 degrees at outer ends of sails and 30 degrees at inner ends of sails.

WHEEL NO. 5.—*Results of experiments.* (*a*)

Velocity of wind per hour.	Loads applied +0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			°*F.*	1882.
8.444	0.3	86.78	26.034	74	July 12
8.499	0.9	73.56	66.204	74	July 12
8.479	1.1	68.72	75.592	74	July 12
8.493	1.3	64.18	83.434	71	July 5
8.419	1.5	57.98	86.970	71	July 5
8.422	1.7	53.86	91.562	71	July 5
8.468	1.9	50.83	96.577	71	July 5
8.465	2.1	46.10	96.810	74	July 5
8.456	2.3	41.60	95.680	74	July 5
8.414	2.5	33.86	84.650	63	July 6
8.424	2.7	27.70	74.790	63	July 12
8.440	2.9	19.62	56.898	63	July 12
....	3.7	0	0	....	July 5
8.491	1.9	51.88	98.572	71	....

a See wheel No. 44, p. 57.

Wheel the same as No. 3, except that the inclination of the sails was changed, making the angle of weather 22.5 degrees at outer ends of sails and 32.5 degrees at inner ends of sails.

WHEEL NO. 6.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			°*F.*	1882.
8.373	0.3	69.03	20.709	32	Dec. 9
8.355	1.5	47.30	70.950	34	Dec. 9
8.348	1.7	44.00	74.800	34	Dec. 9
8.405	1.8	42.10	75.780	35	Dec. 9
8.317	1.9	38.28	72.732	35	Dec. 9
a 8.368	1.3	50.83	66.079	38	Dec. 9
....	3.4	0	0	33	Dec. 8

a When reversed, so as to present its back side to the wind.

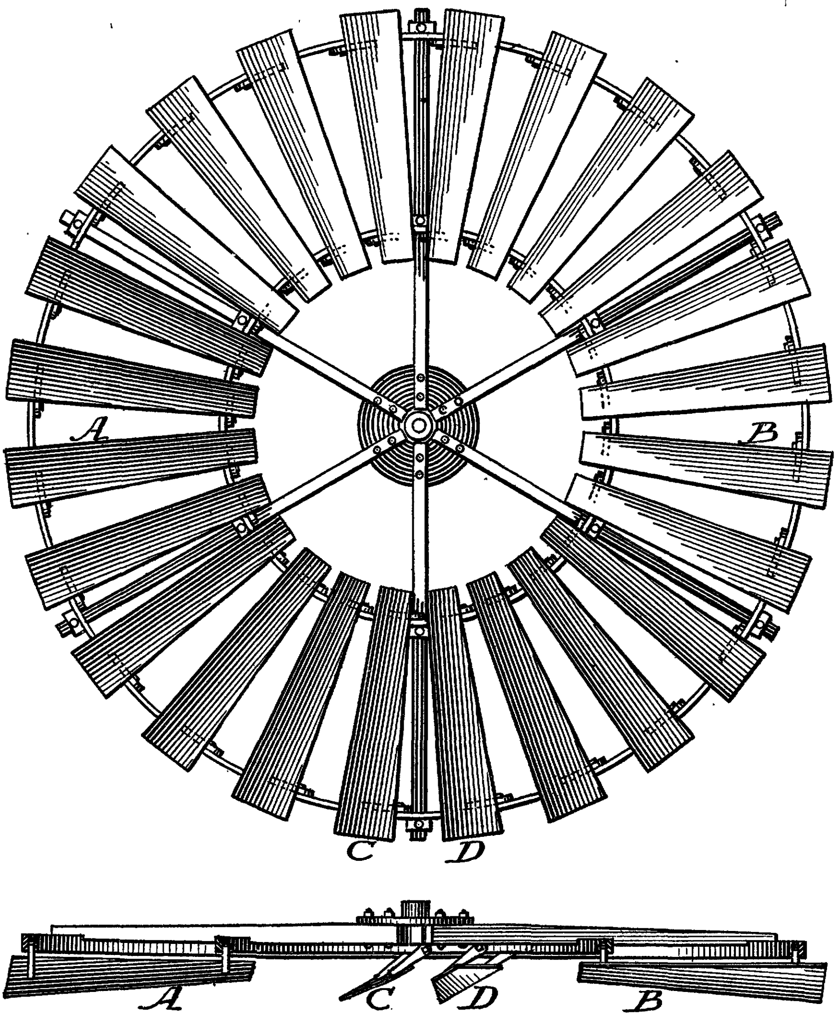


FIG. 12.—Elevation and section of wheel No. 6. (Sail area, 10.688 square feet; efficiency, 0.163.)

Diameter of wheel, 5 feet.

Arms and bands the same as in wheel No. 3.

Number of sails, 24. Sails of lineal dimensions as in fig. 13; made of white-pine boards three-sixteenths of an inch thick, and hinged to the bands as in No. 3.

Sails plain, with forward edges trimmed to an edge.

Total sail surface, 10.688 square feet.

Angle of weather, 30 degrees.

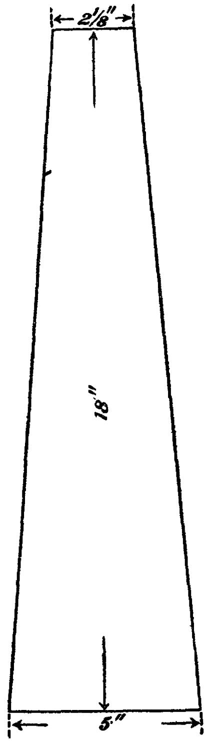


FIG. 13.—Dimensions of sails of wheel No. 6.

WHEEL NO. 7.—*Results of experiments.*

Velocity of wind per hour.	Loads applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			°*F.*	1882.
8.361	0.3	73.60	22.080	42	Dec. 12
8.380	1.3	52.50	68.250	39	Dec. 12
8.416	1.5	47.40	71.100	40	Dec. 11
8.359	1.7	40.74	69.258	40	Dec. 12
....	2.8	0	0	42	Dec. 12

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 25 degrees.

WHEEL NO. 8.—*Results of experiments.*

Velocity of wind per hour.	Loads applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			°*F.*
8.333	0.3	62.67	18.801	....
8.344	1.3	46.83	60.879	48
8.403	1.5	44.23	66.345	48
8.375	1.7	40.23	68.391	39
8.366	1.9	36.84	69.996	41
8.368	2.1	32.80	68.880	47
8.354	2.3	27.70	63.710	47
....	3.55	0	0	....

Experiments made December 13, 1882.

Wheel the same as No 6, except that the inclination of the sails was changed, making the angle of weather 35 degrees.

WHEEL NO. 9—*Results of experiments.*

Velocity of wind per hour.	Loads applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			°*F.*
8.374	0.3	66.40	19.920	43
8.361	1.3	49.73	64.649	38
8.375	1.5	47.15	70.725	39
8.350	1.7	42.30	71.910	40
8.364	1.9	38.20	72.580	40
8.344	2.1	34.58	72.618	42
8.323	2.3	30.70	70.610	42
....	3.4	0	0	....

Experiments made December 14, 1882.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 32.5 degrees.

WHEEL NO. 10.—*Results of experiments.*

Velocity of wind per hour.	Loads applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			°*F.*	1882.
8.347	0.3	72.23	21.690	34	Dec. 15
8.336	1.1	54.77	60.247	45	Dec. 14
8.329	1.3	53.30	69.290	45	Dec. 14
8.345	1.5	48.70	73.050	33	Dec. 15
8.358	1.7	43.78	74.426	34	....
8.338	1.9	39.33	74.727	35	....
8.351	2.1	35.06	73.626	39	....
....	3.25	0	0	....	....

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 27.5 degrees.

WHEEL NO. 11.—*Results of experiments.*

Velocity of wind per hour.	Loads applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			°*F.*
8.415	0.3	77.60	23.280	41
8.353	1.3	51.64	67.132	39
8.391	1.5	45.46	68.190	40
8.401	1.7	39.52	67.184	40
8.410	1.9	*a* 33.14	62.966	41
....	2.1	(*b*)	....	....
....	2.3	0	0	....

a Started slowly and with difficulty, though well balanced. Once needed help to start.
b Ran slowly two or three minutes and stopped.

Experiments made December 18, 1882.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 20 degrees.

A journal friction test made with wheel No. 11 by hanging a 1¾-ounce weight by a thread wound around the outside of the wheel showed that this weight did not overcome starting friction, but was sufficient to accelerate motion after the wheel was started at moderate speed.

Let $x =$ journal friction, at 1 foot (12 inches) from axis.

30 inches = distance of 1¾-ounce weight from axis.

$1.75 \times 30 = 12\,x.$

$x = 4.375$ ounces $= 0.2734$ pound $=$ friction of journals at moderate speed.

The journal friction diminishes somewhat with the increase of speed, although authorities give the same coefficient of friction at all speeds. Some variation in journal friction is also due to difference in weights of wheels and in lubrication; and as the above test was made with the brake removed, we have considered it approximately correct to call journal friction = 0.3 pound in all cases. See, however, later conclusions.

WHEEL No. 12.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.405	0.3	75.60	22.680	44
8.385	.95	56.18	53.371	40
8.376	1.1	50.78	55.858	40
8.341	1.3	*a* 43.88	57.044	41
8.376	1.5	*b* 35.42	53.130	42
....	1.6	(*c*)	....	....
....	1.7	0	0	....

a Started slowly.
b Needed assistance to start in every instance, though without assistance it would barely move, at the rate, perhaps, of one turn in ten minutes.
c Ran three minutes and stopped, after having been started at moderate speed.

Experiments made December 19, 1882.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 15 degrees.

WHEEL No. 13.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.	Date.
Miles.	*Pounds.*			° *F.*	1882.
8.369	0.3	56.33	16.899	45	Dec. 20
8.377	1.7	37.13	63.121	45	Dec. 19
8.353	1.9	34.10	64.790	43	Dec. 20
8.352	2.1	31.30	65.730	41	Dec. 20
8.407	2.3	28.70	66.010	42	Dec. 20
8.375	2.5	25.38	63.450	42	Dec. 20
8.328	2.7	20.63	55.701	44	Dec. 20
8.351	2.9	17.25	50.025	46	Dec. 20
....	3.1	(*a*)	....	....	Dec. 20
....	3.2	(*b*)	....	....	Dec. 20
....	3.7	0	0	....	Dec. 20

a Ran very slowly after starting, and stopped several times.
b Stopped and started occasionally.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 40 degrees.

WHEEL NO. 14.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.327	0.3	48.60	14.580	47
8.335	1.5	33.60	50.400	46
8.375	1.7	32.68	55.556	46
8.393	1.9	29.88	56.772	47
8.336	2.1	26.95	56.595	46
8.364	2.3	24.72	56.856	46
8.427	2.5	21.82	54.550	46
8.381	2.7	17.98	48.546	46
8.409	2.9	15.10	43.790	46
8.349	3.1	12.47	38.657	46
....	3.3	(*a*)	....	....
....	3.8	0	0	....

a Ran a minute or two, stopped, and afterwards started slightly.

Experiments made December 21, 1882.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 45 degrees.

WHEEL NO. 15.—*Results of experiments.*

Velocity of wind per hour.	Load applied +0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.444	0.3	46.84	14.052	30
8.402	1.7	29.36	49.912	28
8.429	1.9	27.40	52.600	30
8.479	2.1	25.30	53.130	31
8.457	2.3	22.92	52.716	33
8.443	2.5	20.17	50.425	32
8.440	2.7	17.71	47.817	33
8.438	3.1	10.20	31.620	35
....	3.3	(*a*)	....	30
....	3.5	(*b*)	....	30
8.406	3.95	0	0	35

a Turned very slowly and irregularly. *b* Barely moved.

Experiments made January 5 and 6, 1883.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 47.5 degrees.

WHEEL NO. 16.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			*° F.*
8.443	0.3	42.40	12.720	32
8.444	1.7	27.75	47.175	32
8.440	1.9	24.92	47.348	30
8.385	2.1	23.27	48.867	30.5
8.447	2.3	20.67	47.514	38
8.419	2.5	18.45	46.125	37
8.505	2.7	16.23	43.821	37
8.426	2.9	*a* 12.33	35.757	37
8.484	3.1	(*b*)	....	36
....	3.3	(*c*)	....	36
8.257	3.55	0	0	35
8.426	3.9	0	0	36

a Almost needed help to start, and once stopped within one minute.
b Stopped after running one minute.
c Stopped after running very slowly a short time.

Experiments made January 6 and 8, 1883.

Wheel the same as No. 6, except that the inclination of the sails was changed, making the angle of weather 50 degrees.

Comparative results with plain sails.

A	B	C	D	E	F	G	H	I
No. of wheel.	Angle of weather.	Velocity of wind per hour at maximum.	Greatest load.	Load at maximum.	Turns of wheel per minute at maximum.	Turns of wheel per minute unloaded.	Products at maximum work, E × F.	Total sail surface.
	°	*Miles.*	*Pounds.*	*Pounds.*				*Sq. ft.*
1	35	8.412	2.85	1.6	35.69	59.08	57.104	9.190
2	35	8.429	3.81	2.0	33.43	57.95	66.860	18.380
12	15	8.341	1.7	1.3	43.88	75.60	57.044	10.688
11	20	8.391	2.3	1.5	45.46	77.60	68.190	10.688
7	25	8.416	2.8	1.5	47.40	73.60	71.100	10.688
10	27.5	8.358	3.25	1.7	43.78	72.23	74.426	10.688
6	30	8.348	3.4	1.7	44.00	69.03	74.800	10.688
9	32.5	8.364	3.4	1.9	38.20	66.40	72.580	10.688
8	35	8.366	3.55	1.9	36.84	62.67	69.996	10.688
13	40	8.352	3.7	2.1	31.30	56.33	65.730	10.688
14	45	8.336	3.8	2.1	26.95	48.60	56.595	10.688
15	47.5	8.479	3.95	2.1	25.30	46.84	53.130	10.688
16	50	8.385	3.9	2.1	23.27	42.40	48.867	10.688

No. 2 modeled after Halliday 10-foot mill, and contained 60 sails 18 inches long and 3.56 inches wide at outer end.

No. 1 contained 30 sails 18 inches long and 3.56 inches wide at outer end.

Nos. 6–16 contained 24 sails 18 inches long and 5 inches wide at outer end.

If the separate tables are consulted, it will be found that in making out the comparative table, we have not taken the highest products for wheels Nos. 6, 9, 10, 13, and 14, in column H. As the angle of weather increases from 15 degrees to 47.5 degrees, the loads at maximum should obviously increase or at least should not decrease in any case within those limits. This law would apparently have been violated if we had taken the highest products in the above-mentioned cases. So, in Nos. 6, 9, 10, 13, and 14, we have taken the loads and products immediately preceding those corresponding to maximum. By so doing we made column E appear consistent with the law of increasing loads without materially changing the values of products in column H. For it will be noted that in no case does the product we have set down in column H fall short of the highest we might have taken by so much as unity.

The majority of the slight discrepancies in column H are clearly accounted for by the variation in wind as shown in column C. But in Nos. 9 and 10 there was evidently a slight fault in adjusting the friction of the brake to the load applied. Slight errors of this nature are unavoidable, although they may be reduced to a minimum by the exercise of care and repeated trials. (See page 27.)

In the foregoing experiments seldom less than six measurements were made for the determination of each product and sometimes the unsteadiness of the wind required many more trials for the determination of a product. The variations in wind, though not great, sometimes made a difference of one or two turns of the wheel per minute. The indicator which registered the number of turns of the wheel was not originally intended to indicate fractions of a turn, although the fractional turns were actually set down by estimation to within one-tenth of a turn. But the indicator could not be counted on as absolutely correct to within less than half a turn, so that an error of half a turn too much or too little might sometimes account for a variation of one turn of the wheel. Since, however, the indicator was just as liable to make the fractional error one way as the other, the errors in average results would be probably diminished in proportion to the number of tests from which the averages were deduced.

The indicator consisted of a toothed wheel which was thrown in contact with a worm on the shaft of the wheel, and the teeth were liable to strike the worm so as to accelerate or retard the indicator to the extent of half the pitch of the teeth or less.

An examination of the several tables shows that there are generally three or four consecutive products corresponding to different loads, none of which vary much from the greatest product, and when the two highest products are nearly identical, as often happens, it requires but a very slight error in any one of two or three chances for variation to make the highest product correspond to either of two consecutive loads.

WHEEL NO. 17.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F*
8.392	0.3	73.90	22.170	32
8.379	1.3	52.62	68.406	30
8.363	1.5	46.55	69.825	30
8.390	1.7	42.80	72.760	31
8.408	1.9	38.70	73.530	31
8.373	2.1	33.30	69.930	31
8.400	3.05	0	0	32
a 8.391	.3	79.83	23.949	32
a 8.380	1.3	54.15	70.395	32
a 8.374	1.5	47.60	71.400	31
a 8.361	1.7	40.70	69.190	32

a Reversed.

Experiments made January 10 and 11, 1883.

Wheel the same as No. 6, except that the sails were twisted, making the angle of weather 25 degrees at outer extremities of sails and 35 degrees at inner extremities of sails.

The twisting made the sails somewhat convex on the front side and concave on the reversed side.

WHEEL NO. 18.—*Results of experiments.* (*a*)

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.430	0.3	75.58	22.674	37
8.370	1.3	53.48	69.524	37
8.362	1.5	50.64	75.960	38
8.386	1.7	48.08	81.736	38
8.397	1.9	43.31	82.289	39
8.393	2.1	39.42	82.782	40
8.357	2.3	33.65	77.395	39
8.368	2.5	*b* 28.31	70.775	40
8.419	2.7	*c* 21.83	58.941	37
8.440	3.4	0	0	38

a See table on p. 47.
b Started slowly.
c Started very slowly—almost needed help.

Experiments made January 13 and 17, 1883.

Wheel contained 24 sails of lineal dimensions as shown in fig. 14, made of white-pine boards three-sixteenths inch thick, beveled on forward edges, and hinged to the bands the same as in wheel No. 6.

Total area of sail surface, 12.937 square feet.

Angle of weather, 25 degrees.

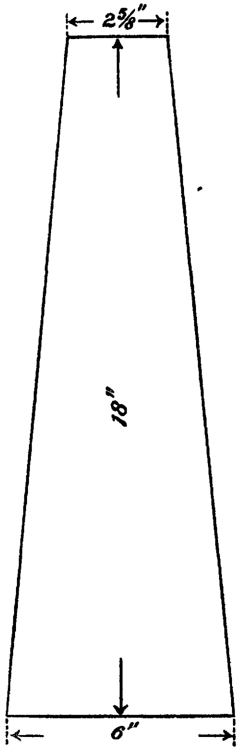


FIG. 14.—Dimensions of sails of wheel No. 18.

WHEEL NO. 19.—*Results of experiments.* (*a*)

Velocity of wind per hour.	Load applied +0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.375	0.3	72.26	21.678	33
8.375	1.3	53.78	69.914	34
8.466	1.5	50.32	75.480	28
8.475	1.7	47.17	80.189	28
8.379	1.9	44.48	84.512	29
8.422	2.1	42.10	88.410	30
8.439	2.3	38.20	87.860	31
8.400	2.5	32.95	82.375	31
8.397	2.7	26.98	72.846	31
8.392	2.9	22.03	63.887	32
8.370	3.1	*b* 16.03	49.693	32
8.329	3.6	0	0	32

a See table on page 47. *b* Needed assistance to start.

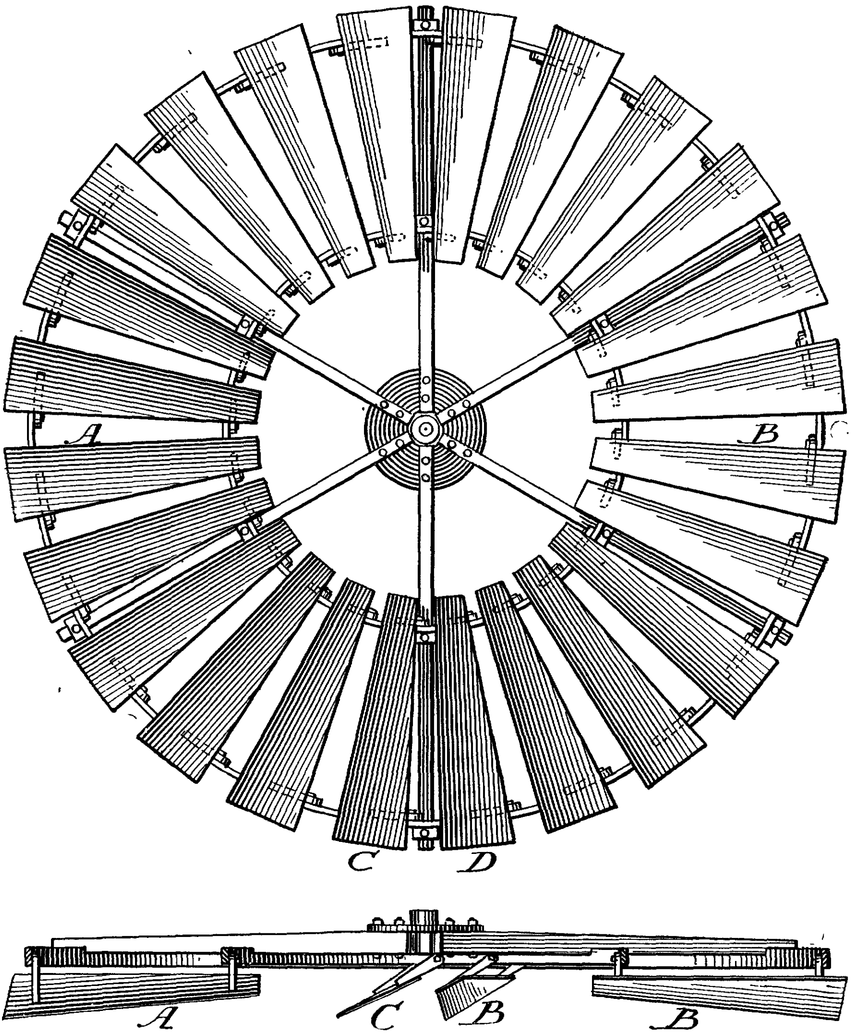


FIG. 15.—Elevation and section of wheel No. 19. (Sail area, 12.937 square feet; efficiency, 0.185).

Experiments made January 18, 1883.

Wheel the same as No. 18, except that the inclination of the sails was changed, making the angle of weather 27.5 degrees.

WHEEL NO. 23.—*Results of experiments.*

Velocity of wind per hour.	Load applied + 0.3 for friction.	Turns of wheel per minute.	Product.	Thermometer.
Miles.	*Pounds.*			° *F.*
8.422	0.3	73.46	22.038	34
8.420	1.3	55.60	72.280	30
8.428	1.5	50.83	76.245	30
8.486	1.7	47.64	80.988	30
8.444	1.9	43.21	82.099	30
8.400	2.1	38.14	80.094	30
8.402	2.3	34.30	78.890	31
8.382	2.5	30.12	75.300	31
8.434	*a* 2.7	24.23	65.421	32
8.399	3.55	0	0	32

a Needed assistance to start.

Experiments made January 25, 1883.

Wheel the same as No. 18, except that the angle of weather was 25 degrees at outer ends of sails and 30 degrees at inner ends.

COMPARISON OF SEVERAL WHEELS.

WHEELS NOS. 2 AND 50.—*Results of experiments.*

No. of wheel.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Date.
	Miles.	*Pounds.*		1883.
2	11.022	0.1	77.75	July 2
2	11.054	3.3	38.48	July 2
2	10.926	6.8—	0	July 2
50	11.093	0.1	77.45	July 2
50	11.054	3.3	36.36	July 2
50	10.981	6.6	0	July 2

Wheel No. 50 was No. 2 with the center of wheel filled in by a cone whose base was 24 inches in diameter and whose slant height was 18 inches. Base of cone rested against the front of arms of wheel. Cone made of stiff paper, supported by light wooden frame.

WHEELS NOS. 18 TO 22.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
18	25	8.516	0.1	77.13	....	....	....	Apr. 10
18	25	8.485	1.5	46.20	69.300	58.083—	1.193	Apr. 10
18	25	8.444	3	0	....	....	....	Apr. 10
19	27.5	8.471	0.1	74.94	....	....	....	Apr. 12
19	27.5	8.500	1.9	39.57	75.185	62.719+	1.204	Apr. 11
19	27.5	8.455	3.3	0	....	....	....	Apr. 11
20	30	8.464	0.1	70.78	....	....	....	Apr. 14
20	30	8.486	1.9	39.87	75.753	62.966—	1.203	Apr. 12
20	30	8.475	3.55	0	....	....	....	Apr. 14
21	32.5	8.478	0.1	67.97	....	....	....	Apr. 16
21	32.5	8.463	1.9	38.72	73.568	*a*62.225=	1.182	Apr. 16
21	32.5	8.400	3.7	0	....	....	....	Apr. 16
22	35	8.498	0.1	64.20	....	....	....	Apr. 17
22	35	8.461	2.1	31.96	67.166	58.957—	1.138	Apr. 17
22	35	8.504	3.9	0	....	....	....	Apr. 17

a The sign = indicates that the number affected needs no correction.

These wheels differed from each other only in angles of weather.

WHEELS NOS. 24 TO 28.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
26	25	8.455	0.1	75.38	....	....	....	Apr. 9
26	25	8.434	1.7	43.74	74.358	*a* 62.966+	1.181	Apr. 9
26	25	8.448	3.25	0	....	....	....	Apr, 9
24	27.5	8.428	0.1	72.60	....	....	....	Apr. 7
24	27.5	8.426	1.9	38.87	73.853	*a* 60.762—	1.212	Apr. 6
24	27.5	8.369	3.5	0	....	....	....	Apr. 7
25	30	8.377	0.1	69.00	....	....	....	Apr. 5
25	30	8.450	1.9	38.95	74.005	*b* 61.902—	1.196	Apr. 6
25	30	8.425	3.7	0	....	....	....	Apr. 5
28	32.5	8.437	0.1	67.07	....	....	....	Apr. 4
28	32.5	8.391	2.1	34.49	72.429	*c* 61.142—	1.184	Apr. 4
28	32.5	8.379	3.9	0	....	....	....	Apr. 4
27	35	8.473	0.1	63.65	....	....	....	Apr. 3
27	35	8.401	2.1	33.74	70.854	*d* 61.370+	1.154	Apr. 3
27	35	8.454	4.1	0	....	....	....	Apr. 2

a Wind, 8.432 miles per hour.
b Wind, 8.455 miles per hour.
c Wind, 8.398 miles per hour.
d Wind, 8.395 miles per hour.

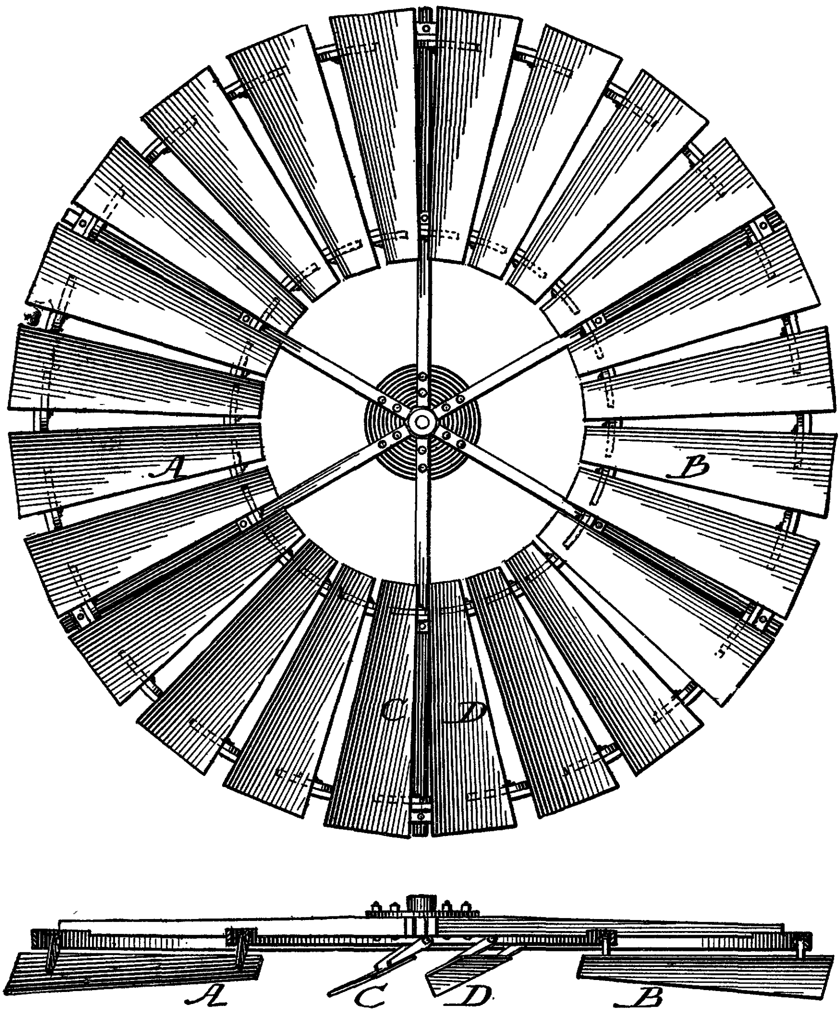


FIG. 16.—Elevation and section of wheel No. 24. (Sail area, 15.000 square feet; efficiency, 0.187).

Wheels Nos. 24 to 28 differed from each other only in angles of weather, each having 24 plain sails 18 inches long, 7 inches wide at outer ends, and 3 inches wide at inner ends. Wheels otherwise the same as No. 6.

Total area of sail surface, 15.000 square feet.

WHEELS NOS. 29 TO 33.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
33	25	8.434	0.1	75.46	....	....	....	Mar. 14
33	25	8.401	1.9	37.80	71.820	*a* 62.463—	1.150	Mar. 14
33	25	8.341	3.5	0	....	....	....	Mar. 14
29	27.5	8.421	0.1	73.10	....	....	....	Mar. 27
29	27.5	8.406	1.9	39.10	74.290	*b* 61.845—	1.201	Mar. 27
29	27.5	8.454	3.7	0	....	....	....	Mar. 27
30	30	8.348	0.1	69.18	....	....	....	Mar. 29
30	30	8.429	2.1	35.15	73.815	*c* 62.478+	1.181	Mar. 28
30	30	8.399	4.0	0	....	....	....	Mar. 28
31	32.5	8.407	0.1	66.73	....	....	....	Mar. 30
31	32.5	8.418	2.1	34.94	73.374	*d* 61.864—	1.186	Mar. 30
31	32.5	8.422	4.1	0	....	....	....	Mar. 30
32	35	8.410	0.1	62.90	....	....	....	Mar. 31
32	35	8.454	2.1	33.43	70.203	*e* 61.408+	1.143	Mar. 31
32	35	8.444	4.3	0	....	....	....	Mar. 31

a Wind, 8.403 miles per hour.
b Wind, 8.410 miles per hour.
c Wind, 8.427 miles per hour.
d Wind, 8.419 miles per hour.
e Wind, 8.446 miles per hour.

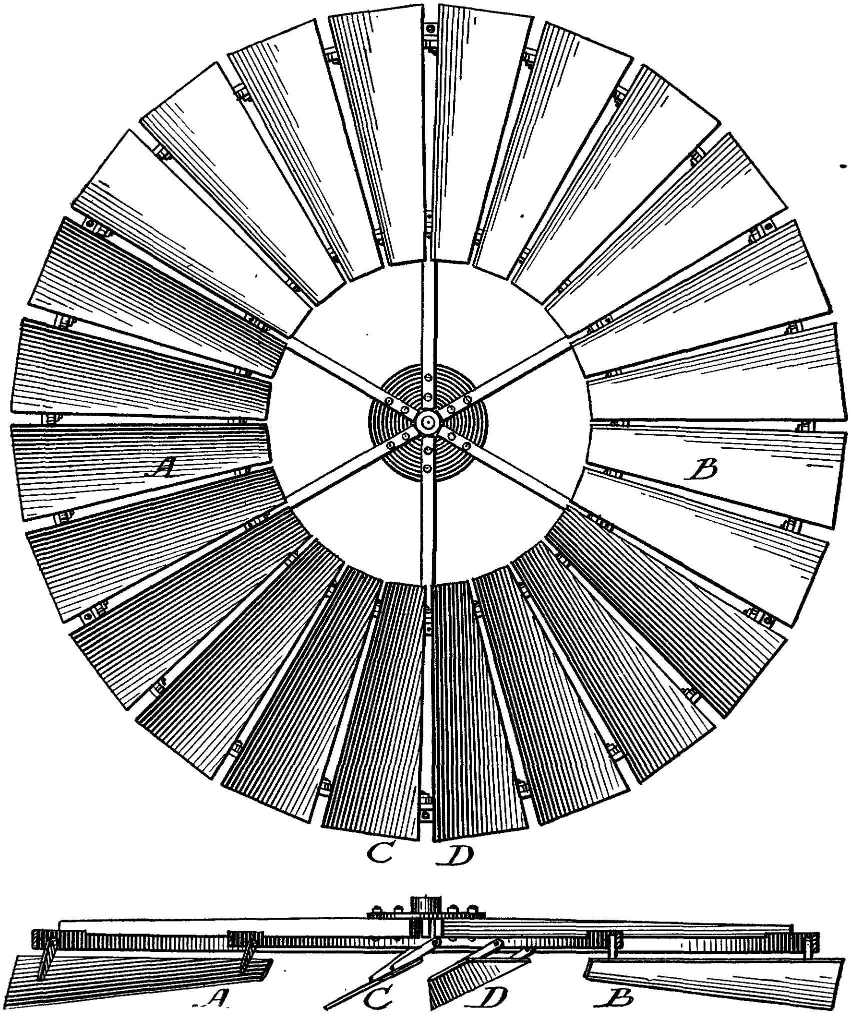


FIG. 17.—Elevation and section of wheel No. 29. (Sail area, 17.016 square feet; efficiency, 0.185.)

Wheels Nos. 29 to 33 differed from each other only in angles of weather, each having 24 plain sails 18 inches long, $8\frac{1}{16}$ inches wide at outer ends, and $3\frac{9}{32}$ inches wide at inner ends.

Total area of sail surface, 17.016 square feet.

Wheels otherwise the same as No. 6.

Dynamometer improved by addition of antifriction wheels and automatic brake adjuster, first used March 13, 1883, with wheel No. 33.

WHEELS NOS. 19, 20, AND 34.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
19	27.5	8.471	0.1	74.94	....	....	....	Apr. 12
19	27.5	8.500	1.9	39.57	75.183	*a* 62.719+	1.204	Apr. 11
19	27.5	8.455	3.3	0	....	....	....	Apr. 11
20	30	8.464	0.1	70.78	....	....	....	Apr. 14
20	30	8.486	1.9	39.87	75.753	*b* 62.966—	1.203	Apr. 13
20	30	8.475	3.55	0	....	....	....	Apr. 14
34	25, 35	8.498	0.1	76.47	....	....	....	Apr. 19
34	25, 35	8.502	1.7	43.77	74.409	*c* 61.655+	1.207	Apr. 18
34	25, 35	8.487	3.5	0	....	....	....	Apr. 19

a Wind, 8.499 miles per hour. *b* Wind, 8.487 miles per hour. *c* Wind, 8.501 miles per hour.

Wheels Nos. 19, 20, and 34 differed from No. 18 only in angles of weather.

The sails of No. 34 were twisted, making angles of weather 25 degrees at outer ends of sails and 35 degrees at inner ends.

WHEELS NOS. 35 TO 38.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
38	22.5	8.524	0.1	84.65	....	....	....	Apr. 28
38	22.5	8.488	1.5	52.51	78.765	*a* 63.802=	1.235	Apr. 28
38	22.5	8.521	2.9	0	....	....	....	Apr. 28
36	25	8.525	0.1	80.67	....	....	....	Apr. 26
36	25	8.499	1.7	47.05	79.985	*b* 62.852—	1.273	Apr. 26
36	25	8.413	3.1	0	....	....	....	Apr. 26
35	27.5	8.413	0.1	76.78	....	....	....	Apr. 24
35	27.5	8.424	1.7	46.64	79.288	*c* 63.726—	1.244	Apr. 25
35	27.5	8.445	3.4	0	....	....	....	Apr. 25
37	30	8.459	0.1	73.12	....	....	....	Apr. 27
37	30	8.477	1.9	40.41	76.779	*d* 62.928	1.220	Apr. 27
37	30	8.473	3.6	0	....	....	....	Apr. 27

a Wind, 8.488 miles per hour. *c* Wind, 8.425 miles per hour.
b Wind, 8.500 miles per hour. *d* Wind, 8.478 miles per hour.

Wheels Nos. 35, 36, 37, and 38 contained 12 plain sails of lineal dimensions, as shown in fig. 19. Each sail made by fastening together 2 sails of No. 18. Wheels otherwise the same as No. 18, and differed from each other only in angles of weather.

Total area of sail surface, 12.937 square feet.

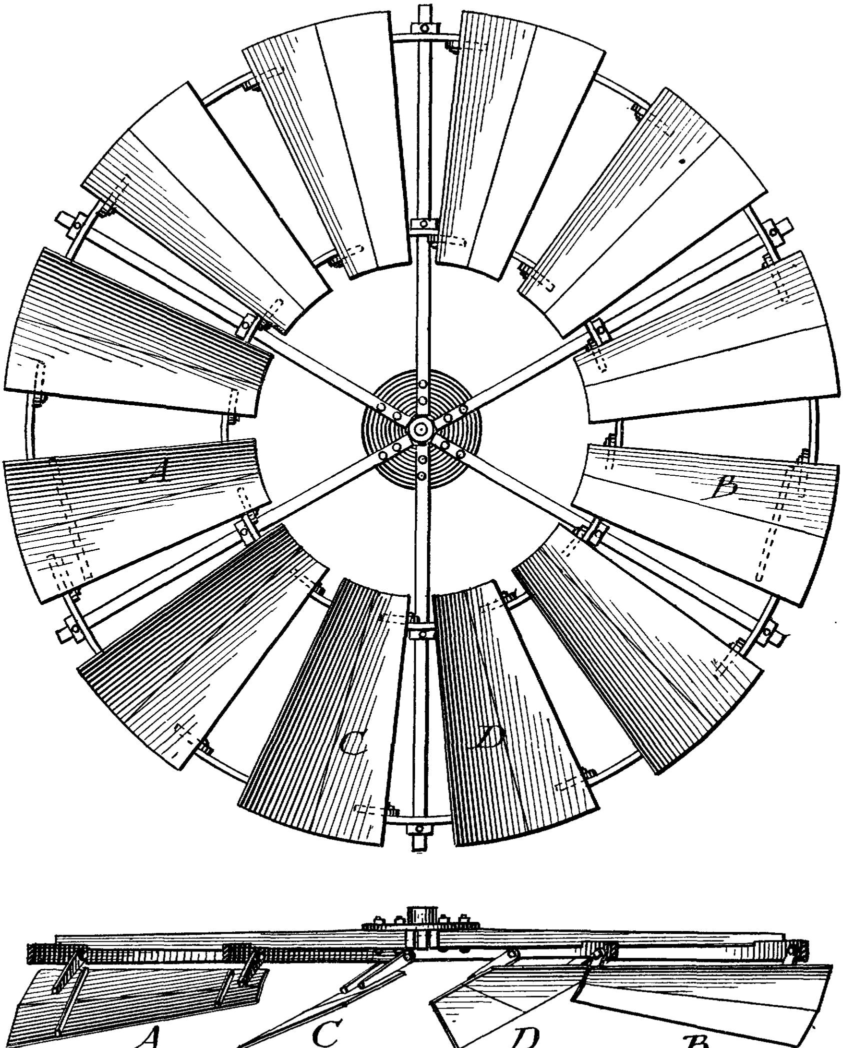


FIG. 18.—Elevation and section of wheel No. 35. (Sail area, 12.937 square feet; efficiency, 0.192.)

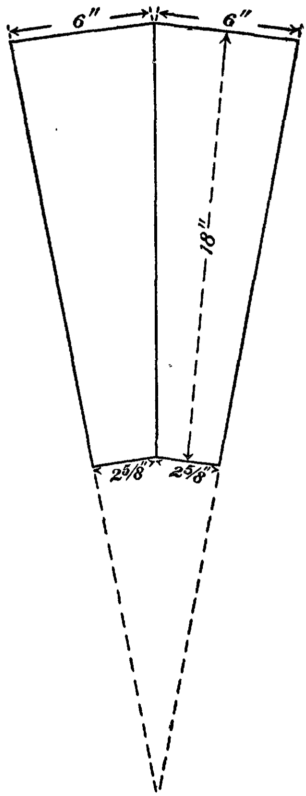


FIG. 19.—Dimensions of sails of wheels. Nos. 35, 36, 37, and 38.

WHEELS NOS. 40 TO 43.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
41	25	8.461	0.1	84.70	....	....	....	May 5
41	25	8.496	1.9	46.81	88.939	65.474=	1.358	May 5
41	25	8.496	3.45	0	....	....	....	May 5
40	27.5	8.498	0.1	82.13	....	....	....	May 3
40	27.5	8.459	1.9	45.74	86.906	62.928=	1.381	May 3
40	27.5	8.519	3.75	0	....	....	....	May 3
42	30	8.491	0.1	77.95	....	....	....	May 7
42	30	8.464	1.9	43.15	81.985	61.180=	1.339	May 7
42	30	8.475	3.9	0	....	....	....	May 7
43	25, 30	8.528	0.1	84.92	....	....	....	May 8
43	25, 30	8.476	1.9	45.92	87.248	62.396=	1.398	May 8
43	25, 30	8.495	3.6	0	....	....	....	May 8

Wind for No. 2 the same as for wheel compared in each case. Wheels contained 12 angular concave sails of lineal dimensions as shown in fig. 21. Each sail made by fastening together two sails of No. 18 at an angle of 165 degrees so as to present concave surface to the wind. Wheels differed only in angles of weather. The sails of No. 43 were twisted, making the angles of weather 25 degrees at outer ends of sails and 30 degrees at inner ends. Total area of sail surface, 12.937 square feet.

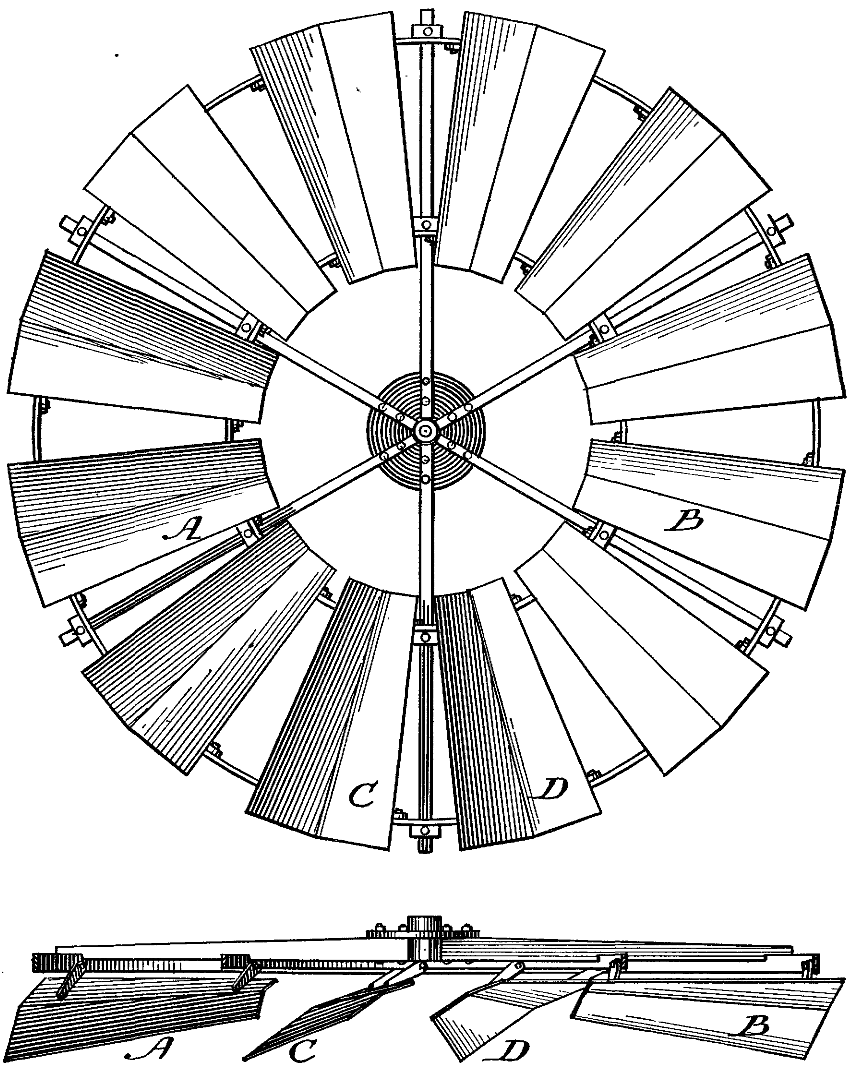


FIG. 20.—Elevation and section of wheel No. 40. (Sail area, 12.937 square feet; efficiency, 0.213.)

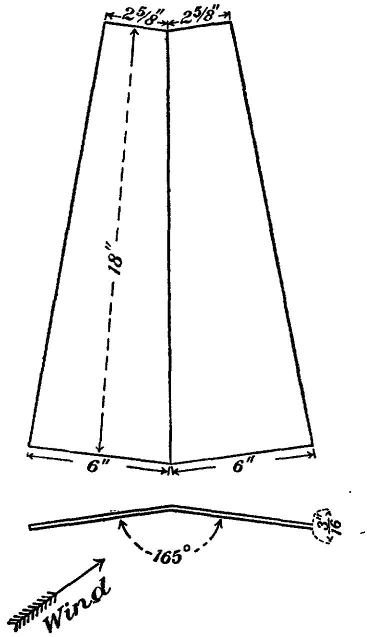


FIG. 21.—Dimensions of concave sails of wheels Nos. 40, 41, 42, and 43.

WHEELS NOS. 40 TO 42.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
41	25	6.444	0.1	65.33	....	....	....	May 24
41	25	10.963	0.1	111.10	....	....	....	May 29
41	25	6.459	1.1	35.07	38.577	28.270=	1.365	May 24
41	25	11.022	3.3	55.45	182.985	a137.346+	1.332	May 29
41	25	6.404	2.0	0	....	....	....	May 24
41	25	11.111	5.6	0	....	....	....	May 29
40	27.5	6.372	0.1	60.27	....	....	....	May 14
40	27.5	10.890	0.1	106.73	....	....	....	May 21
40	27.5	6.371	1.1	32.90	36.190	26.290=	1.377	May 14
40	27.5	11.041	3.3	56.32	185.856	135.498=	1.372	May 17
40	27.5	6.403	2.1	0	....	....	....	May 14
40	27.5	10.944	6.3	0	....	....	....	May 21
42	30	6.454	0.1	59.13	....	....	....	May 23
42	30	10.974	0.1	103.50	....	....	....	May 23
42	30	6.429	1.2	29.65	35.580	26.037=	1.367	May 23
42	30	10.997	3.7	50.62	187.294	138.897=	1.348	May 22
42	30	6.411	2.3	0	....	....	....	May 23
42	30	11.063	6.9	0	....	....	....	May 22

a Wind, 11.021 miles per hour.

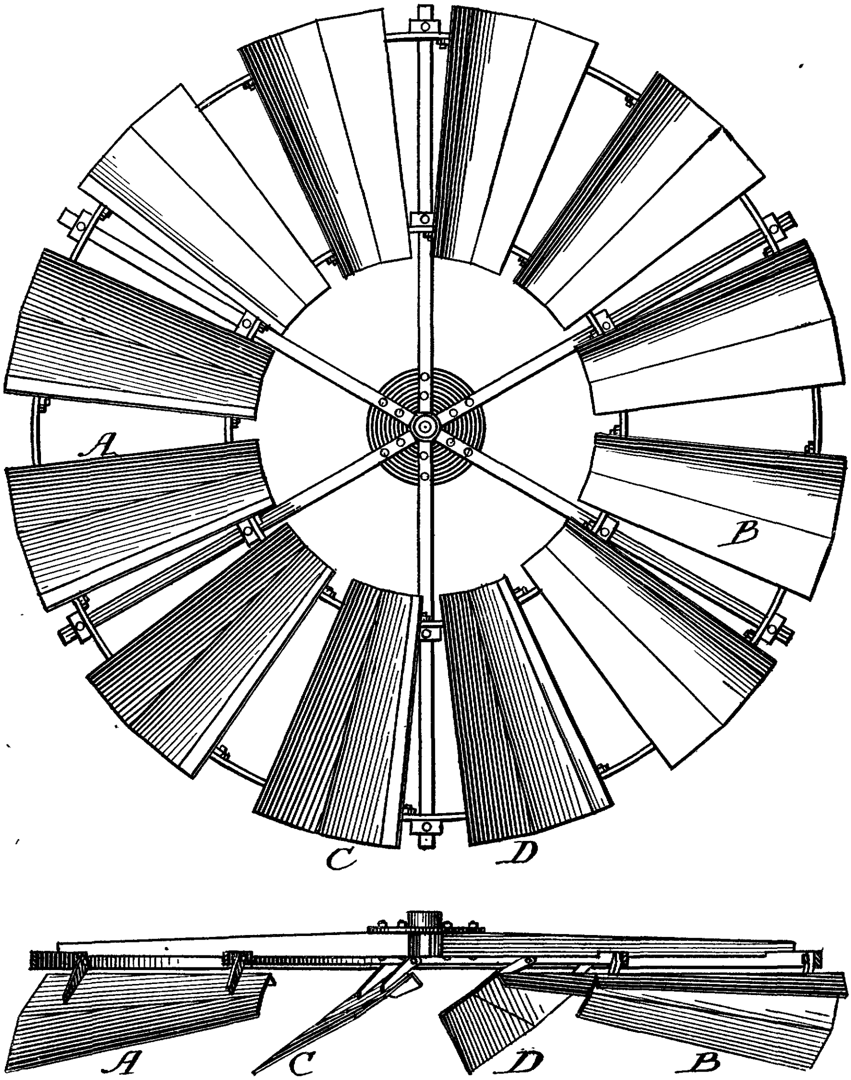


FIG. 22.—Elevation and section of wheel No. 39. (Sail area, 13.072 square feet; efficiency, 0.164.)

WHEELS NOS. 35, 39, 40, AND 44.—*Results of experiments.*

No. of wheel.	Angle of weather.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
35	27.5	8.413	0.1	76.78	....	....	....	Apr. 24
35	27.5	8.424	1.7	46.64	79.288	*a*63.726—	1.244	Apr. 25
35	27.5	8.445	3.4	0	....	....	....	Apr. 25
39	27.5	8.472	0.1	64.52	....	....	....	May 1
39	27.5	8.512	1.9	35.57	67.583	63.650=	1.062	May 1
39	27.5	8.431	3.7	0	....	....	....	May 1
40	27.5	8.498	0.1	82.13	....	....	....	May 3
40	27.5	8.459	1.9	45.74	86.906	62.928=	1.381	May 3
40	27.5	8.519	3.75	0	....	....	....	May 3
44	25, 30	8.482	0.1	84.88	....	....	....	May 9
44	25, 30	8.477	1.9	44.35	84.265	60.952=	1.382	May 9
44	25, 30	8.475	3.6	0	....	....	....	May 9

a Wind, 8.425 miles per hour.

Wheel No. 39 contained 12 sails of lineal dimensions as shown in fig. 23, made by adding to the sails of No. 35, along their rear edges, strips as illustrated, so as to present concave surface to the wind. But the line *ab* was considered the face of the sail as regards angle of weather. For No. 40 see previous description. Sails angular concave.

Wheel No. 44 was the same as No. 3 except that the angles of weather were 25 degrees at outer ends of sails and 30 degrees at inner ends. Sails circular concave.

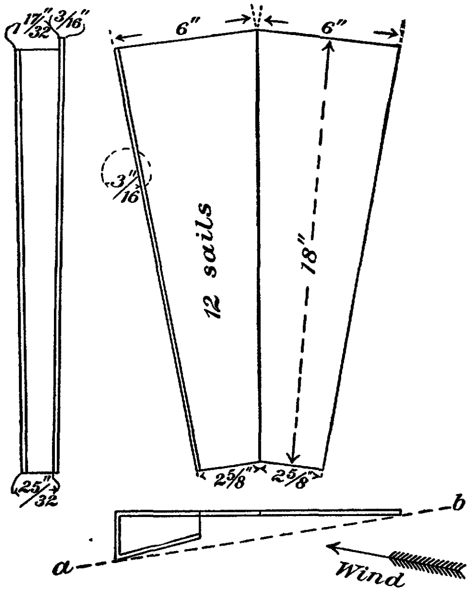


FIG. 23.—Dimensions of sails of wheel No. 39.

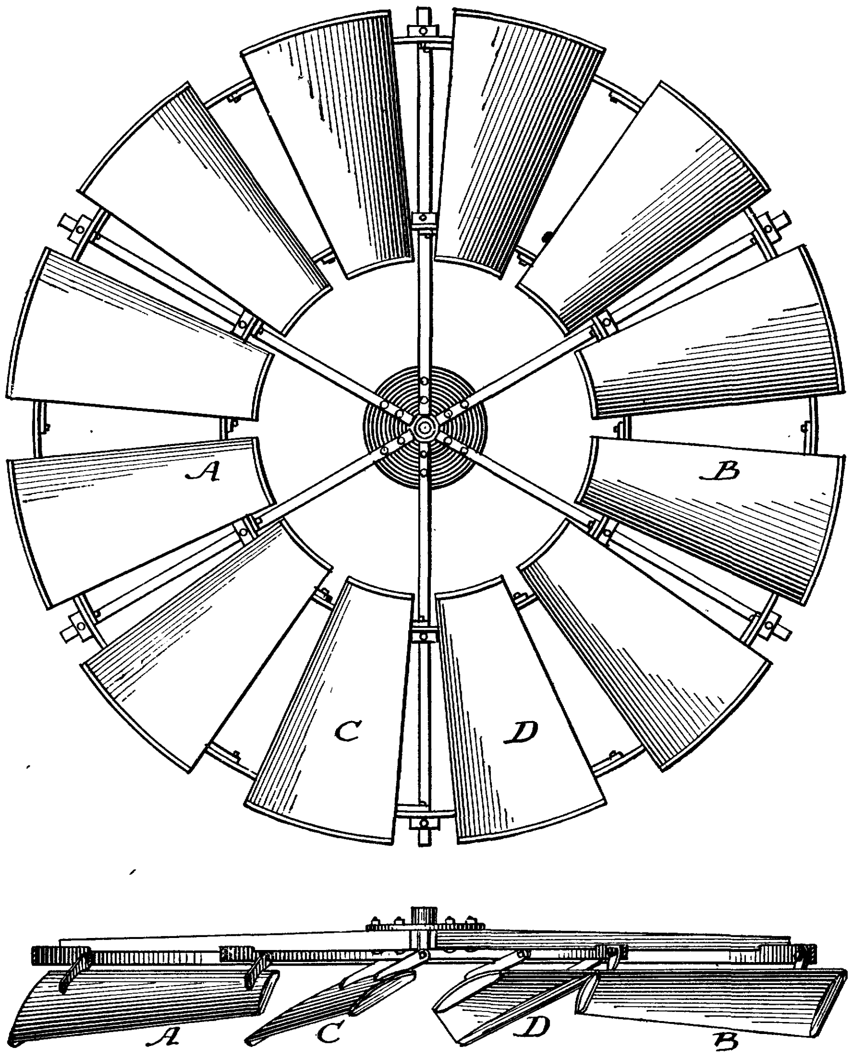


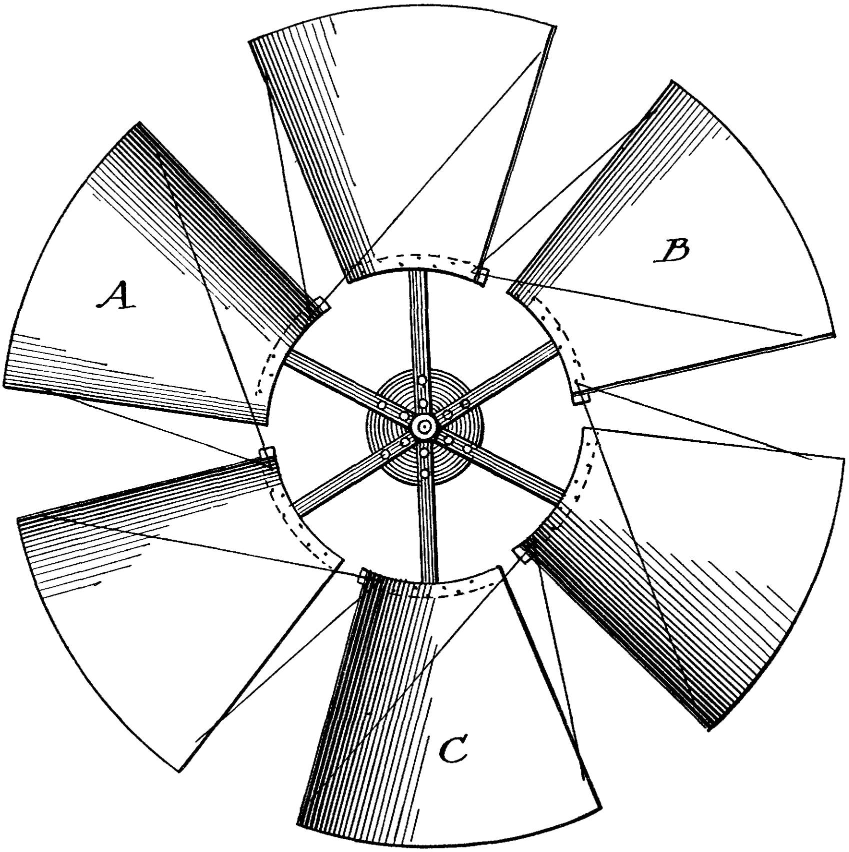
FIG. 24.—Elevation and section of wheel No. 44. (Sail area, 13.590 square feet; efficiency, 0.213.)

WHEELS NOS. 45 TO 49.—*Results of experiments.*

No. of wheel.	Angle of weather. *a*	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	°	*Miles.*	*Pounds.*					1883.
46	17.5 at *ab*; 20 at *cd*; 25 at *ef*; 27.5 at *gh*	11.025	0.1	160.90	....	....	......	June 25
46		10.979	2.7	85.19	230.013	*b*135.201—	1.701	June 25
46		10.905	4.8	0	....	....	......	June 25
45	20 at *ab*; 22.5 at *cd*; 27.5 at *ef*; 30 at *gh*	10.918	0.1	156.63	....	....	......	June 21
45		11.001	2.9	79.99	231.971	*c*129.195+	1.796	June 20
45		11.021	5.2	0	....	....	......	June 21
47	22.5 at *ab*; 25 at *cd*; 30 at *ef*; 32.5 at *gh*	11.026	0.1	148.68	....	....	......	June 27
47		10.980	3.3	73.31	241.923	132.264=	1.829	June 26
47		10.944	6.0	0	....	....	......	June 27
48	25 at *ab*; 27.5 at *cd*; 32.5 at *ef*; 35 at *gh*	11.125	0.1	141.97	....	....	......	June 27
48		10.935	3.7	66.79	247.123	132.033=	1.872	June 28
48		11.014	6.5	0	....	....	......	June 28
49	27.5 at *ab*; 30 at *cd*; 35 at *ef*; 37.5 at *gh*	11.187	0.1	136.30	....	....	......	June 29
49		10.994	3.9	58.82	229.398	127.116=	1.805	June 29
49		10.993	6.6	0	....	....	......	July 2

a Letters in this column refer to fig. 26, p. 61.
b Wind, 10.980 miles per hour.
c Wind, 11.000 miles per hour.

$1.09 = \frac{u}{v} = x =$ ratio of velocity of wind to velocity of outer extremities of sails of No. 48 for maximum work. See page 71.



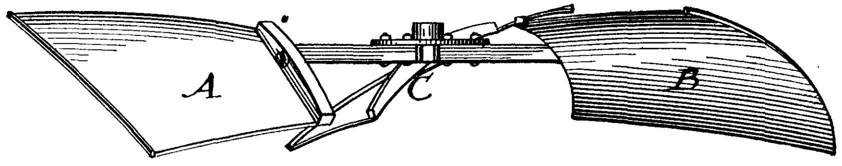


FIG. 25.—Elevation and section of wheel No. 48. (Sail area, 14.159 square feet; efficiency, 0.283.)

Wheels Nos. 45 to 49 differed from each other only in angles of weather, and contained six sails of lineal dimensions as shown in fig. 26. Sails were made by gluing together several layers of paper which formed pasteboard about one-sixteenth of an inch thick. The backs of front and rear edges of each sail were bordered by thin pine strips. Backs of outer edges of sails were likewise bordered and held in shape by stiff curved pine pieces. The inner end of each sail was fastened securely to a thick curved pine piece, which was bolted against the outer end of a short ash arm, so that the sail conld be turned on the bolt and set at any desired angle of weather. Thus the arms did not extend beyond the inner ends of the sails. The outer corners of each sail were held in position by two fine brass wires extending to the front inner corner of the succeeding sail. The front and rear edges also curved somewhat, so that the sails presented double concave surfaces to the wind. In constructing these wheels we aimed at avoiding all unnecessary obstruction to the passage of wind between the sails.

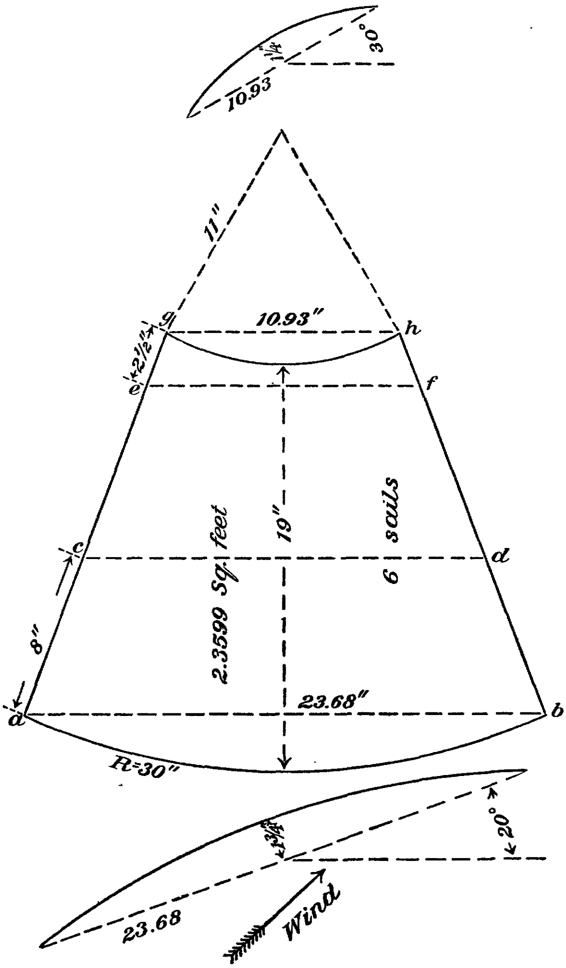


FIG. 26.—Dimensions of sails of wheels Nos. 45 to 49.

WHEELS NOS. 2, 60, AND 61.—*Results of experiments.*

No. of wheel.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Date.
	Miles.	*Pounds.*					1883.
60	8.451	1.9	56.50	107.350	58.273—	1.842	Sept. 15
61	8.452	1.9	38.13	*a* 72.447	58.273=	1.243	Sept. 15
2	8.452	1.9	30.67	58.273	58.273=	1.000	Sept. 15

a This product may not have been a maximum, as only the one load, 1.9 pounds, was tried with No. 61.

Wheel No. 60 was the same as No. 48, except that the sails were considerably warped out of original shape from long standing and drying.

Wheel No. 61 was the same as No. 60, except that along the back of each sail a rectangular piece of pine 1 by 1¼ by 17 inches was fastened to make an obstruction representing an extension of the arm, as illustrated in figs. 28 and 29.

The relative efficiency of No. 60 as compared with No. 61 is indicated by the equation

$$\frac{107.350}{72.447}=1.482$$

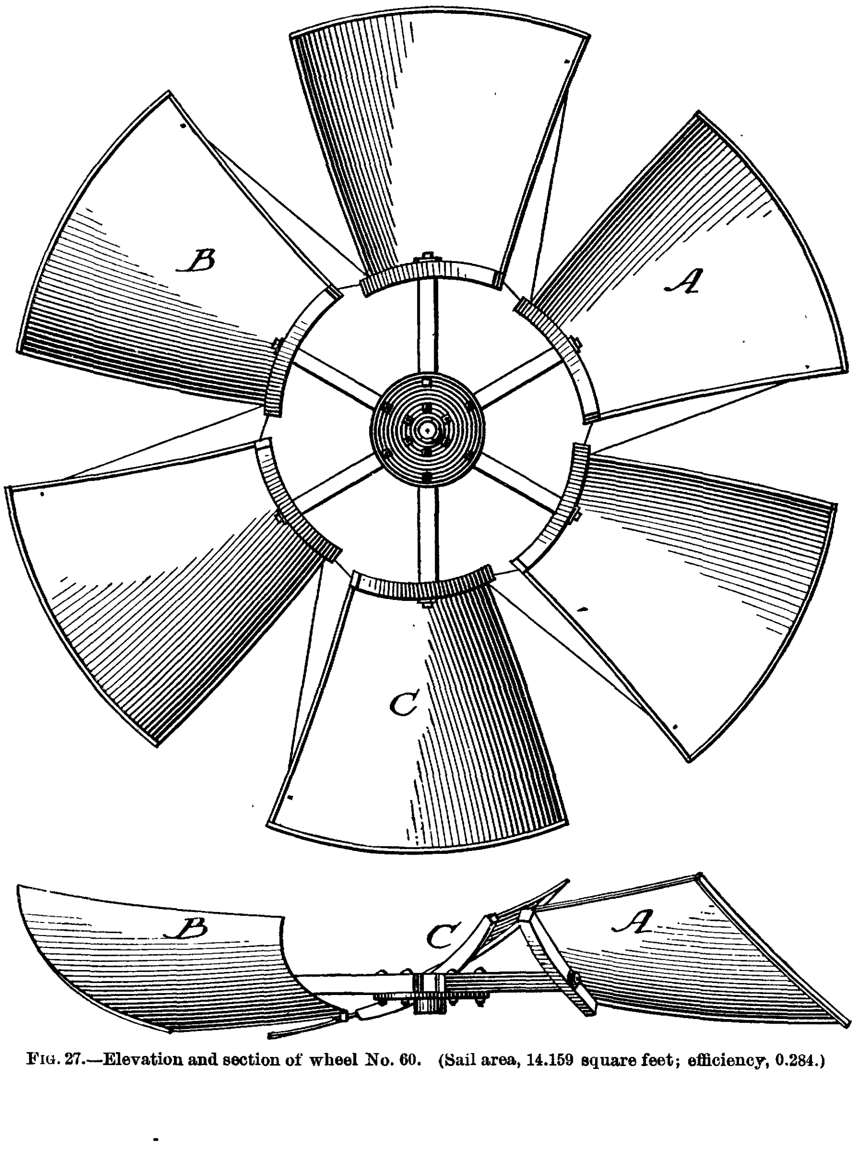


FIG. 27.—Elevation and section of wheel No. 60. (Sail area, 14.159 square feet; efficiency, 0.284.)

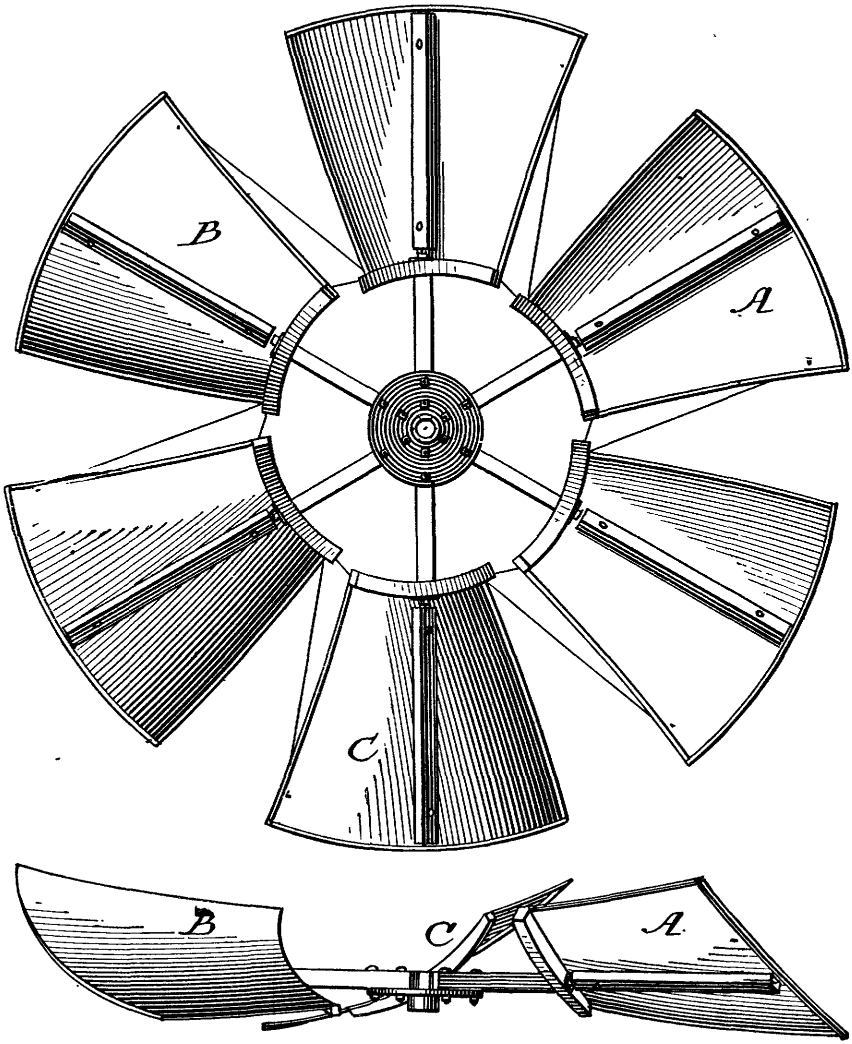


FIG. 28.—Elevation and section of wheel No. 61. (Sail area, 14.159 square feet; efficiency, 0.191.)

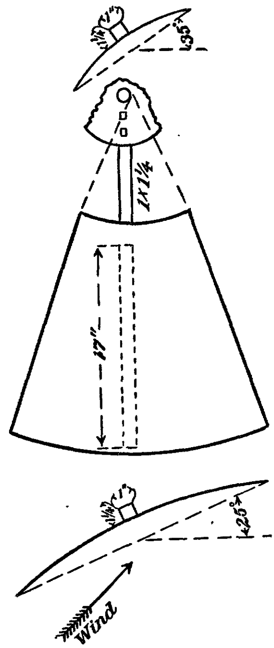


FIG. 29.—Dimensions of sails of wheel No. 61 with obstruction on back.

WHEELS NOS. 2, 49, AND 53 TO 57.—*Results of experiments.*

No of wheel.	Angle of deflector.	Velocity of wind per hour.	Loads applied +0.1 for friction.	Turns of wheel per minute.	Date.
	°	*Miles.*	*Pounds.*		1883.
54	30	10.989	0.1	79.40	July 3
53	45	11.156	0.1	81.80	July 3
55	60	10.985	0.1	80.52	July 3
2	....	11.007	0.1	77.57	July 3
2	....	10.975	3.3	37.83	July 5
56	50	10.968	3.3	40.63	July 5
57	50	11.054	3.5	62.63	July 5
49	....	11.032	3.5	64.22	July 5

Wheels Nos. 53, 54, 55, and 56 were the No. 2 wheel, with stationary rectangular board, $\frac{1}{4}$ by $9\frac{13}{16}$ by 22 inches, placed in front of sails and with its face set at various angles with plane of wheel, as indicated in table, to act as deflector. Board supported by arm outside of wheel on side farthest from axis of sweep; its lower edge at height of axis of wheel, horizontal, and parallel to plane of wheel. Inner end of board 9 inches from axis of wheel prolonged.

Wheel No. 57 was wheel No. 49, with the same board, described above, similarly used as deflector, except that its lower edge was placed so as to clear the forward edges of sails by about 2 inches at each end.

In the case of Nos. 53 to 56 the center of board deflector was about 7 inches from center of sails.

WHEELS NOS. 2, 49, 51, 52, 58, AND 59.—*Results of experiments.*

No. of wheel.	Position of obstruction.	Velocity of wind per hour.	Load applied +0.1 for friction.	Turns of wheel per minute.	Date.
		Miles.	*Pounds.*		1883.
2	....	11.141	0.1	78.58	July 3
51	Before ..	11.130	0.1	75.73	July 3
52	Behind..	11.045	0.1	76.16	July 3
49	....	10.966	3.5	64.53	July 5
58	Before ..	10.972	3.5	55.54	July 5
59	Behind..	10.972	3.5	60.06	July 5

Wheel No. 51 was No. 2, with rectangular board ¼ by 9$\frac{13}{16}$ by 22 inches placed horizontally in front of the side of wheel farthest from axis of sweep, its face parallel to plane of wheel so as to act as obstruction to wind. Board held at its outer end by means of an arm outside of wheel.

Distance from board to centers of sails, 8 inches.

Distance from inner end of board to axis of wheel prolonged, 9 inches.

Wheel No. 52 was the same as No. 51, except that the board was placed 8 inches behind centers of sails.

Wheel No. 58 was No. 49, with obstrution board above described similarly placed, 8¼ inches in front of centers of sails.

Wheel No. 59 was the same as No. 58, except that the board was placed 8¼ inches behind centers of sails.

SAILS AT BEST ANGLES OF WEATHER.

Efficiency of various wind wheels at best angles of weather.

No. of wheel.	Area of sail surface.	Angles of weather.	Wind per hour.	Turns of wheel unloaded.	Starting forces.	Load at maximum.	Turns of wheel at maximum.	Product at maximum.	Product of No. 2 at maximum.	Ratio of products.	Value per square foot of sail surface.	Number and indicated character of sails.
	Sq. ft.	°	*Miles.*		*Lbs.*	*Lbs.*						
1	9.190	35	8.452	60.0	.....	1.6	31.69	50.704	....	....	....	30 a
2	18.380	35	8.403	59.0	4.1	1.9	32.875	62.463	62.463	1.000	0.0544	60 a
10	10.688	27.5	8.338	72.0	.....	1.7	39.33	66.861	....	....	....	24 b
19	12.937	27.5	8.500	75.5	3.2	1.9	39.57	75.183	62.719	1.204	.0931	24 c
34	12.937	25, 35	8.502	76.5	3.4	1.7	43.77	74.409	61.655	1.207	.0938	24 c
24	15.000	27.5	8.426	72.7	3.4	1.9	38.87	73.853	60.762	1.212	.0808	24 d
29	17.016	27.5	8.406	73.0	3.6	1.9	39.10	74.290	61.845	1.201	.0706	24 e
35	12.937	27.5	8.424	77.0	3.3	1.7	46.64	79.288	63.726	1.244	.0962	12 c c
36	12.937	25	8.499	80.0	3.0	1.7	47.05	79.988	62.852	1.273	.0984	12 c c
44	13.590	25, 30	8.477	85.0	3.5	1.9	44.35	84.265	60.953	1.382	.1017	12 f
40	12.937	27.5	8.459	81.5	3.6	1.9	45.74	86.906	62.928	1.381	.1067	12 c c
43	12.937	25, 30	8.476	84.5	3.5	1.9	45.92	87.248	62.396	1.398	.1081	12 c c
48	14.159	25, 27.5, 32.5, 35	10.935	141.0	6.1	3.7	66.79	247.123	132.033	1.872	.1322	6 g

The velocities of wind given in this table are strictly correct only for products at maximum, though they are very nearly correct also for products of No. 2 at maximum. The velocities of wind are only approximately correct for turns of wheels unloaded and for starting forces. Nothing was added to starting forces for friction, and the 0.1 pound was added to starting forces in preceding tables only for convenience.[1] For further particulars consult preceding tables.

RELATIONS OF THREE DIFFERENT VELOCITIES OF WIND.

Relation of different velocities of wind to maximum products.

Angle of weather.	No. of wheel.	Wind per hour.		Product at maximum.		$\frac{b}{a}$	$\left(\frac{b}{a}\right)^3$	$\frac{B}{A}$	$\frac{B}{A}-\left(\frac{b}{a}\right)^3$	$A\times\left(\frac{b}{a}\right)^3$
		a	b	A	B					
°		*Miles.*	*Miles.*							
35	2	6.371	8.459	26.290	62.928	1.328—	2.342	2.394	+0.052	61.571
	2	8.459	11.041	62.928	135.498	1.305+	2.223	2.153	—.070	139.889
	2	6.371	11.041	26.290	135.498	1.733	5.199	5.154	—.045	136.682
27.5	40	6.371	8.459	36.190	86.906	1.328—	2.342	2.401	+.059	84.757
	40	8.459	11.041	86.906	185.856	1.305+	2.223	2.139	—.084	193.192
	40	6.371	11.041	36.190	185.856	1.733	5.199	5.135	—.064	188.152
25	41	6.459	8.496	38.577	88.939	1.315+	2.274+	2.306	+.032	87.724
	41	8.496	11.022	88.939	182.985	1.297+	2.182	2.057+	—.125	194.065
	41	6.459	11.022	38.577	182.985	1.706+	4.965+	4.743	—.222	191.535
30	42	6.429	8.464	35.580	81.985	1.317+	2.284+	2.304+	+.020	81.265
	42	8.464	10.997	81.985	187.294	1.299+	2.192	2.284+	+.092	179.711
	42	6.429	10.997	35.580	187.294	1.707+	4.974	5.264	+.290	176.975

Conclusion.—Maximum products vary as the cubes of the velocities of wind. Some allowance must be made for the differences in time and condition of air under which the various products were obtained; considering these differences, there is no unaccountable variation from the law of cubes.

Relation of different velocities of wind to starting forces.

Angle of weather.	No. of wheel.	Wind per hour.		Starting forces.		$\frac{b}{a}$	$\left(\frac{b}{a}\right)^2$	$\frac{B}{A}$
		a	b	A	B			
°		*Miles.*	*Miles.*	*Pounds.*	*Pounds.*			
35	2	6.380	8.405	2.3	4.1	1.319—	1.740	1.783
	2	8.405	10.989	4.1	7.0	1.307	1.708	1.704
	2	6.380	10.989	2.3	7.0	1.722	2.965	3.043
27.5	40	6.403	8.519	2.0	3.65	1.330	1.769—	1.825
	40	8.519	10.944	3.65	6.2	1.285	1.651	1.699
	40	6.403	10.944	2.0	6.2	1.709	2.921—	3.100
25	41	6.404	8.496	1.9	3.35	1.327—	1.761	1.763
	41	8.496	11.111	3.35	5.5	1.308—	1.711—	1.642
	41	6.404	11.111	1.9	5.5	1.735	3.010	2.895
30	42	6.411	8.475	2.2	3.8	1.322	1.748	1.727
	42	8.475	11.063	3.8	6.8	1.305—	1.703	1.789
	42	6.411	11.063	2.2	6.8	1.725	2.976—	3.091

Conclusion.—Starting forces vary as the squares of the velocities of wind.

[1] See starting forces, "S," p. 25.

Relation of different velocities of wind to speed of unloaded wheels.

Angle of weather.	Nos. of wheels.	Wind per hour.		Turns unloaded.		$\frac{b}{a}$	$\frac{B}{A}$
		a	b	A	B		
°		*Miles.*	*Miles.*				
35	2	6.437	8.511	43.78	59.62	1.322	1.362
	2	8.511	10.996	59.62	78.68	1.292	1.320
	2	6.437	10.996	43.78	78.68	1.708	1.797
27½	40	6.372	8.498	60.27	82.13	1.334	1.363
	40	8.498	10.890	82.13	106.73	1.281	1.300
	40	6.372	10.890	60.27	106.73	1.709	1.771
25	41	6.444	8.461	[1] 65.33	84.70	1.313	1.296
	41	8.461	10.963	84.70	111.10	1.296	1.312
	41	6.444	10.963	[1] 65.33	111.10	1.701	1.701
30	42	6.454	8.491	59.13	77.95	1.316	1.318
	42	8.491	10.974	77.95	103.50	1.292	1.328
	42	6.454	10.974	59.13	103.50	1.700	1.750

[1] Too great because of outside wind and perhaps other causes.

Conclusion.—Speeds of unloaded wheels increase in somewhat greater ratio than the velocities of wind.

All wind wheels tested were 5 feet in external diameter, and all sails were 18 inches long except in the case of Nos. 46 to 49 and Nos. 60 and 61, in which wheels the sails were 19 inches long, 1 inch being added for support at inner ends.

AUTOMATIC BRAKE ADJUSTER.

On March 13, 1883, the dynamometer was improved by the addition of an automatic brake adjuster. Previous to this date the brake was adjusted as described on page 23 through regulating the tension of the adjusting cord by sliding a weight in and out on a lever attached to the cord.

This method required time and patience in order to get an exact balance between the load applied and the friction of brake, and it was also necessary to exercise judgment in determining when the balance was exactly even. Furthermore, an exact balance was not always retained during a test of one minute, as required for accuracy, and frequent readjustments were constantly necessary. The labor involved in adjusting and maintaining a proper balance of friction and load became very tiresome when kept up for from six to eight hours continuously, as was frequently the case; and weariness was detrimental to the perceptions as well as to the judgment. Therefore, an automatic adjuster became very desirable, both for relief to the mind and for the increase of accuracy.

The automatic brake adjuster, after various attempts and failures, was finally constructed as follows: The brake and adjusting cord were left substantially as previously described on page 23; but one end of a short rod three-sixteenths inch in diameter was attached to the cord where it was brought back from the brake in a direction parallel to the axis of the shaft. The other end of this rod had a full thread, 32 to the inch, and a nut which was fastened to a wooden wheel *A*, $6\frac{1}{2}$ inches in diameter. This wheel was supported on a hollow tube through which the rod passed, and against the outer end of which the nut rested. The nut communicated tension to the adjusting cord, which was tightened by revolving the wheel *A* in one direction and loosened by turning the wheel in the opposite direction. This wheel *A* was driven by a small wheel *B*, $1\frac{1}{2}$ inches in diameter, fastened to a large wheel *C*, 5 inches in diameter. This wheel *C* was driven by another small wheel *D*, $1\frac{5}{16}$ inches in diameter, fastened to a large wheel *E*, 4 inches in diameter, the periphery of which was constantly in contact with another wheel *F* of the same dimensions. The wheels *E* and *F* were supported by a tilting frame which oscillated on a supporting rod passing through the centers of the wheels *B* and *C*, but not touching them. The wheels *E* and *F* hung in close proximity to the rear end of wind wheel shaft, $1\frac{3}{16}$ inches in diameter, so that a slight movement of the tilting frame would bring either of the wheels *E* or *F* in contact with the shaft; and if wheel *E* came in contact with the shaft while revolving, the adjusting cord would be drawn tighter, and if wheel *F* came in contact with the shaft, the tension of the cord would be relaxed. The tilting frame was connected with the brake and partook of its oscillating motion, so that if the friction of the brake was too small to lift the load applied, wheel *E* would be in contact with the shaft and would cause an increase in the friction of the brake until the load was lifted and contact between wheel *E* and the shaft broken. If the friction of the brake was too great, the load applied was raised until wheel *F* came in contact with the shaft so as to cause a diminution of friction until contact of wheel *F* with the shaft was broken. It was very easy to tell at any time if friction of brake was too little or too great, by simply watching the direction of movement of wheels *E* and *F*. This did not require the exercise of judgment, as errors of balance were greatly magnified to the eye and made self-correcting. All contacts between the various wheels, *A*, *B*, *C*, *D*, *E*, *F*, and shaft were frictional. Wheels *B*, *C*, were entirely supported by contact of peripheries with wheels *A* and *D*, and wheel *E* was so hung to a separately hinged support that its own weight produced a uniform pressure against wheel *F*. The entire automatic system was supported independently of the brake; the connection with the brake served only to give the necessary oscillatory movement to the tilting frame, which was also balanced both as regards the action of gravity and centrifugal force when the sweep was in motion.

Some power of the wheel was necessarily consumed by the automatic adjuster, but it may be ascertained, from the description and dimensions given, that a point in the wind wheel 1 foot from the center of shaft would move in tightening the adjusting cord about 67,019 times as far as the threaded rod attached to the cord; from this it follows that the pressure of the brake against the cylinder was $67{,}019 \times 16 = 1{,}072{,}304$ times as great as the force required at 1 foot from the center of shaft to produce the pressure, without considering the friction of the automatic system. And if we make the very liberal allowance of one-third additional force required to overcome the friction of the automatic system and call the friction of brake 0.05 of pressure, remembering that the diameter of cylinder was $5\frac{1}{4}$ inches, we may readily find that the force required to produce friction of brake equal to any load would be only about $\frac{1}{13964}$ part of that load at the same distance from center of shaft. Hence, it will be seen that the power consumed by the automatic brake adjuster was too minute materially to affect products and called for no correction.

THEORY OF THE ACTION OF THE WIND UPON THE SAILS OF WINDMILLS.

Where a fluid moves along in a current, the direct force of the current is expressed in pounds by the formula Mv, in which v represents the velocity of flow in feet per second and M is the mass of fluid which flows per second past a fixed point. $M=\frac{W}{g}$, W representing weight in pounds of fluid passing the fixed point per second, and $g=32.2$, the constant for gravity. [1]

Where a current acts against a surface and its direction is simply changed by the surface without impeding its velocity, a direct pressure $= Mv$ is exerted by the impinging current in the direction of its motion, and an equal pressure is exerted by the current as it escapes from the surface, $= Mv$, in a direction opposite to the direction of escape.

If the current impinges against a surface which is itself in motion, the relative velocities of the impinging and escaping currents must be considered in determining the forces which act upon the moving surface. Of the two equal forces due to impingement and escape of current the sum of those components acting in the direction of motion of the surface constitutes the useful effort designated by P, and if the velocity per second of motion of surface is called u, the useful work yielded per second by the current will be represented by Pu. The actual energy of the current is represented [2] by $\frac{Mv^2}{2}=\frac{Wv^2}{2g}$. The ratio of useful work to actual energy is termed efficiency, and is represented [3] by the symbol $1-k=\frac{2Pug}{Wv^2}$.

In deducing formulæ to represent the action of wind upon the sails of windmills we have proceeded upon the assumption that the sail

[1] See Rankine's Steam Engine, articles 14 and 144, Case III.
[2] See Rankine's Steam Engine, article 31.
[3] See Rankine's Steam Engine, article 92.

should be of such form as most effectually to change the air current into a direction as nearly as possible opposite to the direction of the sail's motion. We have also assumed, in accordance with principles stated by Rankine concerning the action of water on vanes, that the receiving edge of the sail should be tangent to the relative direction of the impinging current. These conditions are represented in fig. 30, in which $DE = v =$ direction and velocity of wind, $AE = u =$ direction and velocity of sail, and $\alpha =$ angle DEA, which the direction of wind makes with direction of motion of sail.

Then, $DA =$ relative direction and velocity of current with reference to sail AB.

Let $BG = AD =$ relative direction and velocity of current as it escapes from the sail.

$\beta =$ angle BGF which relative direction of escaping current makes with direction of sail's motion.

Then

$$DC = v \sin \alpha.$$
$$AC = u - v \cos \alpha.$$
$$AD = \sqrt{v^2 \sin^2 \alpha + (u - v \cos \alpha)^2}.$$
$$GF = GB \cos \beta = AD \cos \beta.$$
$$GF = \cos \beta \sqrt{v^2 \sin^2 \alpha + u^2 - 2\,uv \cos \alpha + v^2 \cos^2 \alpha}.$$
$$GF = \cos \beta \sqrt{v^2 + u^2 - 2uv \cos \alpha}.$$

If $Q =$ volume in cubic foot of air acting on sail in 1 second, and $d =$ weight in pounds of 1 cubic foot of air, we have for useful effort

$$P = \frac{dQ}{g}(GF - AC).$$
$$P = \frac{dQ}{g}[\cos \beta \sqrt{u^2 + v^2 - 2\,uv \cos \alpha} - u + v \cos \alpha].$$

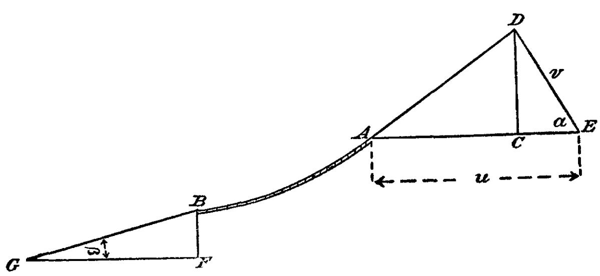


FIG. 30.—Action of wind on a sail.

The expression for useful work becomes

$$Pu = \frac{dQ}{g}[u \cos \beta \sqrt{u^2 + v^2 - 2\,uv \cos \alpha} - u^2 + uv \cos \alpha]. \qquad \text{(A)}$$

Dividing by $\frac{Mv^2}{2} = \frac{dQv^2}{2g}$ we have for efficiency

$$1 - k = 2\frac{u}{v} \cos \beta \sqrt{1 + \frac{u^2}{v^2} - 2\frac{u}{v} \cos \alpha} - 2\frac{u^2}{v^2} + 2\frac{u}{v} \cos \alpha.$$

Or letting $\frac{u}{v}=x$,

$$1-k=2x\cos\beta\sqrt{1+x^2-2x\cos\alpha}-2x^2+2x\cos\alpha. \tag{B}$$

By substituting $-\cos\gamma$ for $\cos\beta$ in formula (A) the expression for Pu becomes the same as formula (6) on page 165 of Rankine's Steam Engine.

To find an expression for x corresponding to maximum efficiency, for general values of α and β would, as Rankine remarks, be more trouble than it is worth. But in the case of the windmill we know that $\alpha=90°$, which makes $\cos\alpha=0$. Hence, formula (B) becomes

$$1-k=2\cos\beta\sqrt{x^2+x^4}-2x^2. \tag{C}$$

The relation between x and β corresponding to maximum and minimum efficiency will be expressed by the formula

$$x^2=-\tfrac{1}{2}\pm\tfrac{1}{2}\sqrt{\frac{1}{1-\cos^2\beta}}=-\tfrac{1}{2}\pm\frac{1}{2\sin\beta}. \tag{D}$$

From this it appears that if $\beta=0$, as it should for maximum efficiency, x becomes infinite. That is, the theoretical efficiency of the windmill becomes greater as the speed of the sails increases indefinitely.

If $\beta=0$ formula (C) shows that

For $x=1$, $1-k=.828$; for $x=2$, $1-k=.944$;
For $x=3$, $1-k=.973$; for $x=10$, $1-k=.9974$;

or almost perfect efficiency when the speed of the sails is 10 times the velocity of the wind. Perfect efficiency, however, could not be reached unless $x=\infty$.

Of course the formula does not take into account the retarding effect of friction, resistance of medium, and other causes which make it impossible to realize in practice anything like the efficiency indicated by the formula; and in practice β can never equal 0 as it should for perfect efficiency.

Letting $x=1$ and assigning more probable values to β, formula (C) gives

For $\beta=10°$, $1-k=.785$.
For $\beta=20°$, $1-k=.657$.
For $\beta=30°$, $1-k=.449$.

Theoretically the analysis of the action of wind on the sails of a windmill corresponds to the analysis of the action of water on a reaction wheel or turbine without guide blades.[1] In the windmill, as commonly made, angle α in our formula (B) is necessarily 90°; but the formula suggests guide blades or deflectors corresponding to those of parallel-

[1] See Rankine's Steam Engine, pp. 197 and 206.

flow turbine water wheels. For if we give to x and β any probable finite values, we may find the values of angle α corresponding to maximum efficiency from the formula,

$$\cos \alpha = \tfrac{1}{2}\left(x + \frac{1}{x} - x \cos^2 \beta\right) \tag{E}$$

in which—

For $x = 3$ and $\beta = 10^\circ$, $\alpha = 77^\circ\ 46'$;
For $x = 2$ and $\beta = 10^\circ$, $\alpha = 73^\circ\ 44'$;
For $x = 3$ and $\beta = 20^\circ$, $\alpha = 70^\circ\ 00'$;
For $x = 2$ and $\beta = 20^\circ$, $\alpha = 68^\circ\ 28'$;
For $x = 1$ and $\beta = 20^\circ$, $\alpha = 56^\circ\ 03'$.

These assumptions for x and β are plausible, as x varies greatly between the outer and inner extremities of a sail, and β should be assumed as the average angle of escape of the whole cylinder of air intercepted by the wheel, and not as the angle merely of that portion of air which comes immediately in contact with the sails, or which the sails directly intercept.

It is to be noticed that formula (B) does not take into account as such the angle of weather, which seems to be incidental to practice but not to theory.

As we have already pointed out, our formula (A) is substantially the same as Rankine's general formula for the action of water on vanes. But Rankine, instead of applying his general formula to windmills, breaks entirely away from it, and uses another formula specially deduced for the "case of a flat vane oblique to the jet," and which he calls the "easiest method of solution."[1] But that his "easiest method" is erroneous even in the case specified, is evidenced by the fact that a very different result is obtained by the application of his general formula.

We pointed out the nature of Rankine's error in a paper dated December, 1881, which was submitted to Prof. R. H. Thurston, of Stevens Institute of Technology. Other writers on windmills have made the same mistake. The theory and formula which we have here given are abstracted from a paper also prepared for Professor Thurston, dated March 6, 1882.

Theoretical considerations were kept in view during our dynamometric experiments. It will be seen from the preceding tables of experiments how closely the results obtained in practice correspond to theory.

DISCUSSION OF RESULTS.

THERMOMETRIC AND BAROMETRIC INFLUENCES.

The preceding pages are substantially an unchanged reproduction of the records made by the writer for the United States Wind Engine and Pump Company of Batavia, Illinois, during and immediately after the experiments in 1883. The only alteration we have presumed to

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[1] See Rankine's Steam Engine, pp. 169 and 215.

make is in omiting the records of barometric readings which were made during the earlier experiments. No barometric readings were taken or recorded after December 21, 1882.

CONSTRUCTION OF TABLES.

In tables on pages 28 to 46 results are recorded for a greater or less number of loads increasing usually by increments of two-tenths pound. Although the results are always given in the order of increasing loads, they were not always obtained in that order, as is indicated by the marginal dates at the right-hand side of some of the tables. Enough tables were prepared in this manner to show how in general the speeds of revolution decreased as the loads increased, and how the products increased with the increasing loads up to a certain point, which marks the maximum, and then decreased continually with further increments of load until the vanishing point was reached. It was not necessary to try many different loads in order to find the best load corresponding to the maximum product. This could be ascertained by a few trials, and as our main object was to get at the relative merits of the wheels tested, we subsequently abandoned the practice of trying so many different loads, and confined our attention more strictly to the determination of maximum products, starting forces, and unloaded speeds; so that the remaining tables on pages 47 to 65 show in general only these three results for each wheel and comparisons of results with different wheels. Special explanations relative to special tables are given elsewhere.

EXTREME RESULTS.

In addition to the original tables already presented, we have collected together in the following table the maximum products given by wind wheel No. 2 at the various dates it was tested in connection with other wheels. It may be observed that the lowest maximum product was 58.083, recorded on April 10, 1883, and that the highest product for wind of nearly the same velocity was 65.474, recorded on May 5, 1883. The products for the same wheel thus vary nearly 13 per cent on different days. Taking the temperature and barometric pressure of Chicago on the two days above mentioned as approximately the same as at Batavia, 35 miles distant due west, we are able to account for but little more than 2 per cent of the variation. Notice also that in wind of about 11 miles per hour, the lowest maximum product was 124.839, recorded on July 5, 1883, and that the highest product was 138.897, recorded on May 22, 1883. Here a variation of over 11 per cent appears for the same wheel on different days. In this case the difference in temperature and barometric pressure at Chicago might account for a little more than 8 per cent of the variation.

Maximum product of wheel No. 2 at different dates.

Velocity of wind per hour.	Product at maximum.	Date.	Velocity of wind per hour.	Product at maximum.	Date.
Miles.		1883.	*Miles.*		1883.
8.485	58.083—	Apr. 10	8.478	62.928—	Apr. 27
8.499	62.719+	Apr. 11	8.496	65.474=	May 5
8.487	62.966—	Apr. 12	8.459	62.928=	May 3
8.463	62.225	Apr. 16	8.464	61.180=	May 7
8.461	58.957—	Apr. 17	8.476	62.396=	May 8
8.432	62.966+	Apr. 9	8.512	63.650=	May 1
8.432	60.762	Apr. 6	8.477	60.952	May 9
8.455	61.902—	Apr. 6	8.452	58.273	Sept. 15
8.398	61.142	Apr. 4	11.041	135.498	May 17
8.395	61.370+	Apr. 3	11.054	126.984	July 2
8.403	62.463—	Mar. 14	11.021	137.346	May 29
8.410	61.845—	Mar. 27	10.997	138.897	May 22
8.427	62.478+	Mar. 28	10.980	135.201	June 25
8.419	61.864—	Mar. 30	11.000	129.195	June 20
8.446	61.408+	Mar. 31	10.980	132.264	June 26
8.501	61.655+	Apr. 18	10.935	132.033	June 28
8.488	63.802	Apr. 28	10.994	127.116	June 29
8.500	62.852—	Apr. 26	10.975	124.839	July 5
8.425	63.726—	Apr. 25			

INFLUENCE OF OUTSIDE WIND.

Experiments were conducted in a closed room to insure the exclusion of all natural wind. The exclusion of wind from the inclosed space, however, was not perfect. The wind circling around the outside of the building undoubtedly at times caused a slight circulation of air inside, which may have been more potent in causing variations in results than either thermometric or barometric influences. The windows were always kept carefully closed during experiments, and the doors also were guarded as much as possible. The doors, however, were necessarily used, and we distinctly remember being obliged to discard many measurements on account of doors being opened during tests on windy days. But unless a door was opened during a test we considered that errors from influence of outside wind were sufficiently guarded against by our practice of comparing all results with those obtained with wheel No. 2, under like conditions, as explained in the original records.

STANDARD OF COMPARISON.

As stated in the original record, wheel No. 2, which we always used as a standard for comparison, was modeled after the Halliday 10-foot wind wheel; that is to say, the slats, arms, etc., were the same in number and of the same relative dimensions on a scale of one-half. There was, however, this difference, that the Halliday was what is called a section wheel, having pivoted sections for the purpose of regulation. Wheel No. 2 had all its parts rigidly fixed with reference to each other, and therefore was more like the solid wheels in common use at the time. The solid wheels were really copies of the Halliday, so far as the sails or slats and the arms were concerned. The only essential difference between section and solid wind wheels related to the method of stopping and governing their motion. Different makers of both styles of wheels had their own notions about angles of weather and proportions of sail surface. The Halliday 10-foot wheel had its slats set at an angle of 35° with the plane of motion. The slats were as thin as was consistent with safety, considering that their material was wood, and they were trimmed to a sharp edge, so as to cut the air as much as possible.

Nearly all wind wheels in use at the time of our experiments in 1882–83 were made with narrow wooden slats, similar in general appearance to our wheel No. 2. Some makers adopted greater angles of weather than 35 degrees, and even let the sails overlap so as to boast of greater sail area for their wheels as a basis for claiming greater power. A 45-degree angle of weather was not uncommon, and the slats were often made thicker than those of the Halliday wheel; besides, they were not always trimmed to an edge in front.

In adopting as our standard of comparison a wheel modeled after the 10-foot Halliday, we believe that we made use of the best model available to represent wind wheels in general as then constructed.

CORRECTIONS FOR AXLE FRICTION.

In the earlier tables 0.3 pound was added to all applied loads. This 0.3 pound was arrived at as explained after the table on page 38, and was in reality an estimate of frictional resistance to starting, or to slow motion. As authorities stated that friction was the same at all velocities, we adopted 0.3 pound as approximately representing the frictional resistance of the journals without regard to speed. We afterwards learned that the axial friction diminished greatly with increase of speed of revolution, as is made evident by comparing the speeds of the same wheels after starting friction was greatly reduced with the speeds before reduction of friction under the same applied loads and also unloaded. (See original records after table on page 38.)

Wheels Nos. 2, 18, and 19, especially, may be compared. Subtract 0.3 from loads in the tables giving average results for wheels Nos. 2, 18, and 19, on pages 29, 43, and 44, and subtract 0.1 in tables on pages 27 and 47 to obtain applied loads. Evidently there is very little difference between the axial frictions in the two cases for unloaded wheels.

We can not now, after a lapse of more than fifteen years, undertake to assert what the axial friction previous to March 13, 1883, should have been called for different speeds. But if in the table on page 29 we call friction 0.15 pound, we shall find the maximum product to be 61.846 instead of 66.860, as there recorded. And as this correction would make this table agree substantially with the general results of the table on page 74, it seems probable that 0.15 pound is about the right amount to allow for friction in the first twenty-one tables to obtain maximum products.

If in the table on page 43 for wheel No. 18 we call friction 0.15 pound, the maximum product will become 76.869, obtained January 13, 1883. On April 10, 1883, the same wheel, No. 18, gave as maximum product 69.300, showing a variation of more than 12 per cent. But the United States Signal Service records in Chicago show for these two dates a difference of more than 1 inch in the barometric column, and about 39 degrees difference in temperature, which will account for a variation of about 12 per cent. Again, if in the table on page 44 for wheel No. 19 we call friction 0.15 pound, the maximum product will become 82.130, obtained January 18, 1883. On April 11, 1883, the same wheel, No. 19, gave as maximum product 75.185, showing a variation of over 9 per cent. The differences in temperature and barometric pressure as given for these two dates by the Signal Service records at Chicago would account for a variation in atmospheric density of just about 9 per cent. These three wheels, Nos. 2, 18, and 19, are the only ones tested both before and after the reduction of friction on March 13, 1883. While the products as recorded in the tables in which 0.3 pound was added to loads are all too large, their comparative values are not greatly distorted by the excessive allowance for axial friction.

The smaller allowance for axial friction will sometimes make the maximum product correspond to the next greater applied load, as in the case of wheel No. 19, page 44. But in the tables on pages 29 and 43, after allowing 0.15 pound instead of 0.3 pound for friction, the maximum products still correspond to the same applied loads and the same speeds of revolution as before.

AERIAL FRICTION.

So far we have not attempted to estimate the friction and resistances of the air itself, although the tables on pages 57, 59, and 61 are very suggestive on this point, and theoretical considerations also indicate the great importance of this subject, as will hereafter appear.

RELATION BETWEEN SPEED AND LOAD.

The following table shows the relation between speed and load for various wind wheels:

Table showing the relation between speed and load.

No. of wheel.	Angles of weather.	A Turns per minute at maximum.	B Turns per minute unloaded.	$\frac{A}{B}$	C Load at maximum.	D Starting force.	$\frac{C}{D}$	Sail surface.
	°							*Square feet.*
1	35	31.69	59.08	.54	1.65	2.55	.65	9.19
2	35	33.43	57.95	.58	1.85	3.55	.52	18.38
3	24.5, 11.25	47.48	90.48	.52	1.35	2.10	.64	13.59
4	20, 30	55.93	89.43	.63	1.55	3.00	.52	13.59
5	22.5, 32.5	46.10	86.78	.53	1.95	3.4	.57	13.59
6	30	42.10	69.03	.61	1.65	3.1	.53	10.688
7	25	47.40	73.60	.64	1.35	2.5	.54	10.688
8	35	36.84	62.67	.59	1.75	3.25	.54	10.688
9	32.5	34.58	66.40	.52	1.95	3.1	.63	10.688
10	27.5	39.33	72.23	.54	1.75	2.95	.59	10.688
11	20	45.46	77.60	.59	1.35	2.00	.68	10.688
12	15	43.88	75.60	.58	1.15	1.4	.82	10.688
13	40	28.70	56.33	.51	2.15	3.4	.63	10.688
14	45	24.72	48.60	.51	2.15	3.5	.61	10.688
15	47.5	25.30	46.84	.54	1.95	3.65	.53	10.688
16	50	20.67	42.40	.49	2.15	3.60	.60	10.688
17	25, 35	38.70	73.90	.52	1.75	2.75	.64	10.688
18	25	39.42	75.58	.52	1.95	3.1	.63	12.937
19	27.5	38.20	72.26	.529	2.15	3.3	.65	12.937
20	30	39.87	70.78	.56	1.9	3.45	.55	12.937
21	32.5	38.72	67.97	.57	1.9	3.6	.53	12.937
22	35	31.96	64.20	.50	2.1	3.8	.55	12.937
23	25, 30	43.21	73.46	.59	1.75	3.25	.54	12.937
24	27.5	38.87	72.60	.54	1.8	3.4	.53	15.000
25	30	38.95	69.00	.56	1.8	3.6	.50	15.000
26	25	43.74	75.38	.58	1.7	3.15	.54	15.000
27	35	33.74	63.65	.53	2.1	4.0	.53	15.000
28	32.5	34.49	67.07	.51	2.1	3.8	.55	15.000
29	27.5	39.10	73.10	.53	1.9	3.6	.53	17.016
30	30	35.15	69.18	.57	2.1	3.9	.54	17.016
31	32.5	34.94	66.73	.52	2.1	4.0	.52	17.016
32	35	33.43	62.90	.53	2.1	4.2	.50	17.016
33	25	37.80	75.46	.50	1.9	3.4	.56	17.016
34	25, 35	43.77	76.47	.57	1.7	3.4	.50	12.937
35	27.5	46.64	76.78	.61	1.7	3.3	.52	12.937
36	25	47.05	80.67	.58	1.7	3.0	.57	12.937
37	30	40.41	73.12	.55	1.9	3.5	.54	12.937
38	22.5	52.51	84.65	.62	1.5	2.8	.54	12.937
39	27.5	35.57	64.52	.55	1.9	3.6	.53	13.072

Table showing the relation between speed and load—Continued.

No. of wheel.	Angles of weather.	A Turns per minute at maximum.	B Turns per minute unloaded.	$\frac{A}{B}$	C Load at maximum.	D Starting force.	$\frac{C}{D}$	Sail surface.
	°							*Square feet.*
40	27.5	45.74	82.13	.56	1.9	3.65	.52	12.937
41	25	46.81	84.70	.55	1.9	3.35	.53	12.937
42	30	43.15	77.95	.55	1.9	3.8	.50	12.937
43	25, 30	45.92	84.92	.54	1.9	3.5	.54	12.937
44	25, 30	44.35	84.88	.52	1.9	3.5	.54	13.590
45	20, 30	79.99	156.63	.51	2.9	5.1	.57	14.159
46	17.5, 27.5	85.19	160.90	.52	2.7	4.7	.57	14.159
47	22.5, 32.5	73.31	148.68	.49	3.3	5.9	.56	14.159
48	25, 35	66.79	141.97	.47	3.7	6.4	.58	14.159
49	27.5, 37.5	58.82	136.30	.43	3.9	6.5	.60	14.159

BEST SPEEDS FOR WIND WHEELS.

In the fifth column of the preceding table the decimal fractions $\frac{A}{B}$ indicate for the various wind wheels the ratio of speeds at maximum work to the speed of wheels unloaded.

By consulting the original tables on pages 28 to 46, it may be noticed that there are generally four or five products nearly as great as the maximum product for each wheel. Where the two greatest products are nearly the same, a very slight difference in velocity of wind, or other cause, may make the maximum result correspond to either of two different loads or two different speeds. This may account for certain apparently rather wide variations in the columns $\frac{A}{B}$ and $\frac{C}{D}$.

In general the best speed for most of these wind wheels is about 0.55 of the unloaded speed, and a variation of speed between 0.45 and 0.70 of the unloaded speed will not make a very great difference in the amount of work performed, provided the load is varied to suit the difference in speed.

BEST LOADS FOR WIND WHEELS.

In the eighth column of the preceding table the decimal fractions $\frac{C}{D}$ indicate the ratios of loads at the maximum of work to the greatest loads that the wheels can start without continuous turning.

In collecting from the original tables the numbers D, which represent starting forces, the original figures were corrected so as to exclude what was originally added for axial friction. (See "Starting forces" on page 25.) Where 0.3 pound was added to applied loads prior to March 13, 1883, the figures were corrected for C in the sixth column

WINDMILL AND RESERVOIR.

to correspond to an axial friction of 0.15 pound, as explained under the heading "Corrections for axial friction," page 75.

In general, it may be said that the ratio of best loads to starting forces is about 0.55, and that there may be a variation in the load between 0.50 and 0.65 of the starting force without serious impairment of efficiency.

CONICAL DEFLECTOR IN CENTER OF WHEELS.

Wheel No. 50 was tested because our attention was called to a windmill patent in which a conical deflector was shown. Besides saving the wind that otherwise would pass through the center of wheel unoccupied by sails, an advantage was also claimed because the air was made to act with greater leverage on account of encountering the sails farther from the axis. The experiment showed there could be no possible advantage from such an arrangement.

AIR-CUSHIONED SAIL SURFACES.

When wheel No. 39 was constructed, the narrow strips of board along the edges of the sails were intended not to obstruct the spaces between sails, but to afford air cushions on the faces of the sails against which the wind might act to better advantage than against plain surfaces. It had become apparent that plain sails were decidedly inferior to sails with concave faces, whether circular or angular. (Note experiments with wheels Nos. 40, 43, and 44.)

It was contended by certain windmill experts that the narrow strips added to plain sails, as used in wheel No. 39, would answer the same purpose as the sail forms of wheels Nos. 40, 43, and 44, and that wooden wheels could be more easily constructed in that way. One of the experts had constructed wind wheels similar to wheel 39. It was immediately due to the suggestion of another prominent windmill official that wheel No. 39 was made and tested. For ourselves, we had much more decided opinions after the test than before, notwithstanding the fact that our mathematical solutions as presented in this paper were written out and made public before any of our experiments were commenced. Whatever value there may have been in the air-cushioned sail faces of wheel No. 39, the advantage was more than nullified by the obstruction of spaces between sails and the bad contours of their backs.

OBSTRUCTIONS ON THE BACK OF SAILS.

Our experience with wheel No. 39 suggested that the arms and bands also might be detrimental to the proper smooth and peaceful flow of air between the sails. So we next constructed and tested wheels Nos. 45 to 49, which have no bands and no arms along the backs of the sails. The results, as shown in the table on page 59, speak for themselves.

It may be noticed that wheel No. 47 carried 3.3 pounds as best load, the same as the best load of wheel No. 2, over which it shows a gain in power of about 83 per cent. The gain, therefore, in this case was entirely in speed. Notice also the ratio between the speeds of the two wheels running unloaded, $\frac{148.68}{77.85}=1.91$. The denominator, 77.85, is taken from the table on page 27.

WASTE OF POWER DUE TO THE ARMS OF WIND WHEELS.

Two or three months after making the many experiments the summary of which is recorded on page 59, it occurred to us that it would be interesting to know and easy to ascertain something definite as to the gain in power effected by the removal of those portions of the arms which usually are made to extend along the backs of sails or between the sails of wind wheels. To represent the portions of arms previously removed, we prepared six pieces of pine wood 1 by $1\frac{1}{4}$ by 17 inches. These could be quickly attached to the backs of the pasteboard sails by two little screws in each, and as quickly removed, as often as necessary.

We found, on retesting wheel No. 48 in September, that it showed a gain over wheel No. 2 of 84 per cent instead of the 87 per cent gained when tested in June. We attributed this small loss to the drying and warping of the sails, and therefore designated the retested wheel as No. 60. The same wheel, with the six pine strips above mentioned fastened to the backs of the six sails, was designated as wheel No. 61. We found by repeated trials, as usual, that wheel No. 60 required, for the wind velocity taken to produce a maximum product, a load of 1.9 pounds.

This load was the same as the best load for wheel No. 2. The coincidence of best loads made the comparison of wheels No. 60 and No. 2 merely a matter of recording speeds. No time had to be consumed in changing loads and readjustments of the brake, and the two wheels were replaced by each other alternately in rapid succession.

The same method was pursued in comparing wheel No. 60 with wheel No. 61, except that in this case, instead of removing from the axle and replacing the entire wheels, we merely placed and removed in succession the six strips of pine. The table on page 61 gives the averages of results.

It is possible that 1.9 pounds may not have been exactly the best load for wheel No. 61. To facilitate the speed comparisons and render mistakes of brake adjustment impossible, the load was not changed and the adjustment of the brake was left undisturbed during these alternate tests of the two wheels. We consider it altogether probable that an exact determination and application of the best load for wheel No. 61 would not have materially changed the product as recorded. At any rate, simply removing the six strips of pine from the backs of the sails caused an increase in the speed of the wheel with the same load, from 38.13 to 56.50 turns per minute, or 48 per cent.

TWISTED SAILS.

That the angle of weather of a sail should be greatest at the end nearest the axle and least at its outer end where its speed is comparatively great, seems self-evident, and the advantages to be gained by warping or twisting the sails so as to make the weather angles increase as they approach the center have always been recognized, at least in theory, by windmill manufacturers, although comparatively few American wind wheels are made with such sails. Our best results, as shown in the table on page 59, were obtained with warped sails, but it does not appear from previous experiments that the warping was by any means the most important feature of our best wheels.

The tables on pages 51 and 53 afford, in the comparisons of wheels Nos. 34 and 43, having twisted sails, with wheels as nearly like them as possible without twisted sails, an apparent test of the advantages gained by making the weather angles greater at the inner ends of the sails. Judged by these two wheels the advantage is very slight.

We are now satisfied that merely twisting the sails can not do much good for the reason that what is gained in this way by better disposition of the forward or receiving edges of the sails is largely lost by the consequent less advantageous disposition of the rear edges from which the wind escapes.

As we have pointed out in the discussion of the theory of the windmill, the rear edge of a sail should be nearly parallel to the plane of motion without regard to radial distance, while the forward edge should be parallel to the relative direction of the impinging current of air, which depends on radial distance.

This theoretical disposition would make the angles of weather greatest where the sail speed is least, but is a very different thing from the twisting of the sails, as in wheels Nos. 34 and 43.

Fig. 26, on page 61, shows a sail form more nearly in accord with our theory, which makes not only the angle of weather, but also the curvature of the sail, greater with nearer approach to the center of the wheel.

DEFLECTORS IN FRONT OF SAILS.

In our discussion of the theory of the windmill on pages 71 and 72 we have called attention to the fact that our formula (E) indicates that the efficiency of a wind wheel might be increased if the air currents, just before meeting the sails, should be deflected so as to meet the plane of motion at a less angle than 90 degrees. In order to test somewhat this mathematical suggestion, the experiments recorded in the table on page 64 were made. Only a single deflector was used, and that a plane surface. It may be noticed that when placed in front of wheel No. 2 at various angles the deflector caused some increase of speed in every instance, while the load remained the same. But when placed in front of wheel No. 49 its effect as an obstruction overbalanced its value as a deflector.

It did not appear that deflectors would be advantageous in front of the most efficient wheel, nor of sufficient value in connection with the wheel of relatively low efficiency to justify the expense of such cumbersome appliances. We did not, therefore, consider it worth while to experiment further with deflectors.

OBSTRUCTIONS IN FRONT AND IN THE REAR OF WIND WHEELS.

The experiments recorded on page 65 we were prompted to make on account of the contention by certain windmill experts, that an obstruction behind a wind wheel caused about the same loss of power as if placed in front.

Many so-called vaneless windmills had their supporting masts, or tower tops, in front of the wheels, while the ordinary mills with vanes had their supporting masts behind the wheels.

The obstructing boards described under the table on page 65 were intended to represent such obstructions as would be representative of supporting masts placed as indicated. In the case of wheel No. 2, unloaded, the obstruction when placed in front caused a loss of speed amounting to a little more than 3.5 per cent, and when placed behind the loss amounted to a little over 3 per cent.

In the case of wheel No. 49, loaded, the loss of speed from placing the obstruction in front amounted to nearly 14 per cent, and the loss of speed from placing the obstruction behind was nearly 7 per cent.

It may be observed that 3.5 pounds is less than the best load for wheel No. 49 (3.9 pounds), but it should be borne in mind that with all wheels a considerable variation in the load made but little difference with the product when near the maximum.

It might have been profitable if we had extended these experiments. It is evident that obstructions even when relegated to the rear are not obliterated.

MAXIMUM PRODUCTS IN DIFFERENT VELOCITIES OF WIND.

The table on page 66 indicates that maximum products vary as the cubes of the velocities of wind almost as exactly in fact as in theory. If there had been an exact correspondence between the results of experiment and the law of cubes, the figures under $\left(\frac{b}{a}\right)^3$ would exactly equal those opposite under $\frac{B}{A}$. The figures under $\frac{B}{A}$ are in some instances a little greater and sometimes a little less than those opposite under $\left(\frac{b}{a}\right)^3$, as indicated by the plus and minus signs before the differences given under $\frac{B}{A}-\left(\frac{b}{a}\right)^3$. The plus and minus signs happen to be equal in number, showing a remarkable general agreement with the law of cubes. Again, the law of cubes would require that the numbers under A multiplied by $\left(\frac{b}{a}\right)^3$ should equal the corresponding numbers under B. How nearly the law was actually fulfilled may be seen by comparing the last computed column, $A \times \left(\frac{b}{a}\right)^3$, with the real products of column B.

STARTING FORCES AND LOADS AT THE MAXIMUM IN DIFFERENT VELOCITIES OF WIND.

Under the second table on page 66 we find the conclusion that "starting forces vary as the squares of the velocities of wind." If the law of squares had been strictly confirmed, the computed ratios under $\left(\frac{b}{a}\right)^2$ would have just equaled the actual ratios of experiment as given under $\frac{B}{A}$. The agreement is quite close, but the numbers in column $\frac{B}{A}$ are more often greater than the corresponding numbers in column $\left(\frac{b}{a}\right)^2$. So it appears that starting forces really increase in at least as great a ratio as would conform to the law of squares. But, as we have explained before, it should be remembered that what we have called starting forces could neither be defined nor determined with the same accuracy as was attainable in the case of maximum products. Nor could the loads corresponding to maximum products be determined so accurately as the maximum products themselves, since two or three different loads would generally give about the same product when the product was near the maximum. This fact made it all the more easy to ascertain the maximum products correctly, since a little variation from the best load was compensated for by the consequent variation in the speed of the wheel.

The table on page 77 shows, on the whole, a fairly constant ratio between loads at the maximum and starting forces, from which we may draw the conclusion that loads at the maximum vary as the squares of the velocities of wind.

SPEEDS OF UNLOADED WHEELS IN DIFFERENT VELOCITIES OF WIND.

In the table on page 67 an inspection of columns $\frac{b}{a}$ and $\frac{B}{A}$ will show that in every case but two the counted turns of the unloaded wheel in the higher wind exceeds somewhat the number of turns computed on the supposition that the speed of an unloaded wheel should vary directly as the velocity of wind. The two exceptional cases may be accounted for. We attribute this variation from the natural law in part to the fact, previously discussed, that the axial friction diminishes as the speed increases. Strictly speaking, the wheels, on account of unavoidable friction at the axle, could not be entirely unloaded.

ACTION OF AIR ON THE SAILS OF A WIND WHEEL.

In fig. 31, AB, AB are supposed to represent cross sections of two sails, one following the other as in a wind wheel. DE represents the direction and velocity of the wind with respect to the earth. AE represents the direction and velocity of the sail sections, at right angles to DE. Then DA represents the relative direction of wind with respect to the sails in motion, and the front faces AA of the sails are set parallel to DA. The rear faces of the sails B are set parallel to the plane of the wheel's motion, according to correct theory.

As one sail necessarily follows another in a wind wheel, the air can

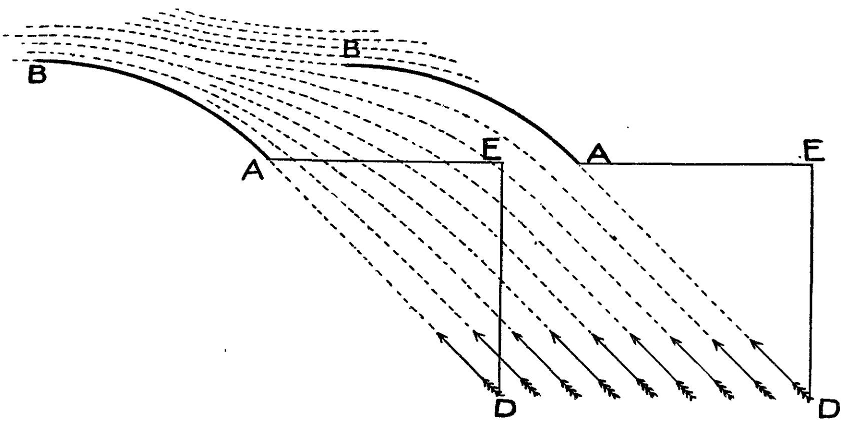

FIG 31.—Action of air on the sails of a wind wheel.

not escape in a direction exactly opposite to the sail's motion, and as the space between sails unavoidably contracts toward the rear edges these edges may in practice conform in some measure to the best possible direction of escape instead of being strictly parallel to the plane of the wheel. The placing of arms on the backs of or between the sails would render a necessary evil worse on account of further contraction of space. Besides, it should be noted that the backs of the sails, as well as their faces, play an important part in deflecting the wind. For it is a well-known fact that air currents cling to and follow the surfaces along which they flow. Therefore, the backs of the sails should be smooth and free from obstructions. It is evident that all the air passing between the sails is deflected and not merely that portion which comes immediately in contact with the faces of the sails. The arrows and dotted lines in the figure indicate the relative direction and deflection of air as it flows through the wheel.

THEORETICAL EFFICIENCY OF THE WIND WHEEL.

Our general formulæ and deductions previously given under theory of the windmill are presented just as they were originally recorded near the beginning of the year 1882. We now see no reason for altering them in any respect, but there has been so much confusion among mathematical writers on wind action, that we wish to make the matter perfectly clear in its essential points.

It is not necessary to resort to the calculus, nor even to produce the general formulæ (A) and (B) in order to arrive at the theoretical efficiency of a wind wheel. A very simple, graphical solution may be shown as follows: In figs. 32 to 35, AB represents the direction and velocity of the wind with respect to the earth; AC represents the direction and velocity of the sail, a section of which is represented by the curved line CD; the velocity of wind AB is the same for each of the four cases; and the several sail sections are supposed to be taken at the extremity of a sail in fig. 32, at a distance from the center of the wheel equal to two-thirds the radius in fig. 33, at one-third the radial distance in fig. 34, and at one-sixth in fig. 35. We have assumed that

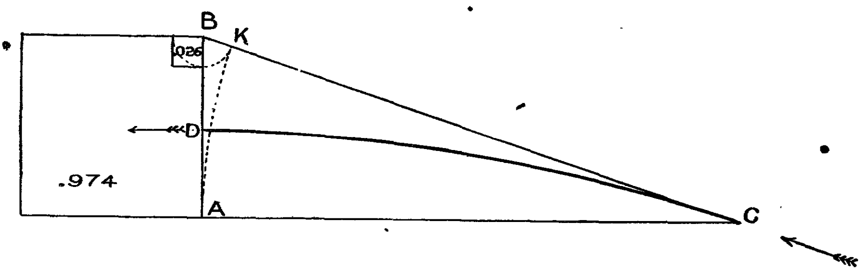

Fig. 32.

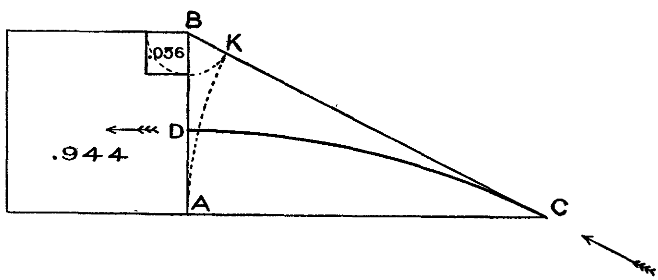

Fig 33.

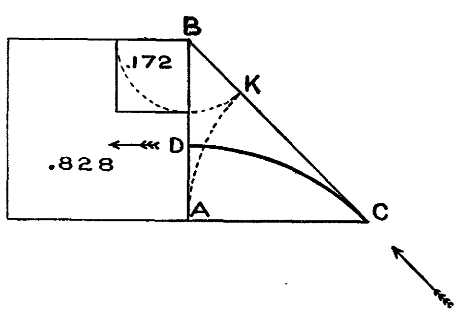

Fig. 34.

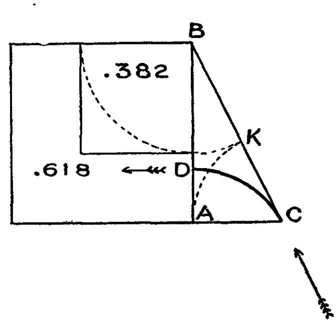

Fig. 35.

FIGS. 32–35.—Theoretical efficiency of the windmill.

the outer extremity of the sail travels three times as fast as the wind. That is to say,

$$AC = 3 \times AB \text{ in fig. 32},$$

$$AC = 2 \times AB \text{ in fig. 33},$$

$$AC = 1 \times AB \text{ in fig. 34},$$

$$AC = \tfrac{1}{2} \times AB \text{ in fig. 35}.$$

CB represents the relative direction and velocity of the wind with respect to the sail in each case. Neglecting friction, the relative velocity of the wind is not changed by contact with the sail. The air escapes from the sail at D with the same relative velocity it had at C; but its course is changed so that it escapes in a direction parallel and opposite to that of the sail's motion. Now, since the sail is traveling with an actual velocity AC in one direction while the air is escaping with a relative velocity CB in the opposite direction, the difference between CB and AC = BK will represent the actual velocity of the air after it has escaped from the sail. The remnant of velocity BK is all there is left of the wind's original motion. The sail has absorbed the rest and converted into work a corresponding amount of energy. A certain quantity of wind started with a velocity AB and gave up all its energy except that corresponding to a velocity BK.

Now, the potential energy of the same quantity of air in motion varies as the square of its velocity, as in the case of any other substance. Hence a square constructed on AB and another square constructed on BK will represent relatively the original potential energy of the wind and the energy it still retains after encountering the sail. These squares are shown in the figures. The small squares represent the waste of energy or that not consumed by the sail. The relative areas of these squares may be readily computed. Taking the large squares as unity the small square equals 0.026 in fig. 32, 0.056 in fig. 33, 0.172 in fig. 34, and 0.382 in fig. 35. The efficiency is represented by the difference between the squares; thus, the efficiency in fig. 32 is 0.974; in fig. 33, 0.944; in fig. 34, 0.828, and in fig. 35, 0.618.

The small squares correspond to the letter k in the formula (B) on page 71. Compare these efficiencies with the values of $1-k$ as originally obtained by a very different process. There would have been no difference if the decimals had been equally carried out in both cases.

The potential energy of the wind, as well as the useful work resulting from its action on a sail, varies as the cube of its velocity, because the quantity of air coming in contact with the sail increases or diminishes with the change of velocity in the same proportion. It is only when we consider the result of a fixed quantity of air in motion, that its energy varies as the square of velocity and may be represented by square areas as illustrated above.

The work of the same quantity of air varies as the square of velocity, because its resultant reaction against the sail varies directly as the velocity and the sail also travels with the same proportional velocity. If, for instance, 1 pound of air, acting on a sail, has its velocity doubled, the sail travels with double velocity under doubled pressure, so that the work is increased fourfold; and, as in reality the doubling of velocity would cause 2 pounds of natural wind to meet the sail in the same time, the result is doubled again, making it eightfold in theory as well as in practice.

THEORETICAL USEFUL EFFORT OF WIND.

In the figs. 36 to 39, let AB represent the direction and velocity of wind with respect to the earth. In fig. 36 let AC = 3 AB represent the direction and velocity of the outer extremity of a wind wheel; in fig. 37 at two-thirds of the radius from center, AC = 2 AB; in fig. 38 at one-third the radius from center, AC = AB, and in fig. 39 at one-sixth the radius, AC = $\frac{1}{2}$ AB. The circular arc CD in each figure represents correctly what the cross section of the sail at each point should be according to theory. To make the calculation simple and specific, we will consider the action of 1 pound of air only, and that moving at an original velocity of 32.2 feet per second.

We take this particular velocity because it requires the constant action of a 1-pound force to give to 1 pound of air, or to 1 pound of any substance, a velocity of 32.2 feet per second.

To give other velocities to the 1 pound of air, requires the impelling action of a force proportional to the velocity imparted in a second of time. That is to say, the force required to impart to 1 pound of air any velocity, v, is represented in pounds by $\frac{v}{32.2}$.

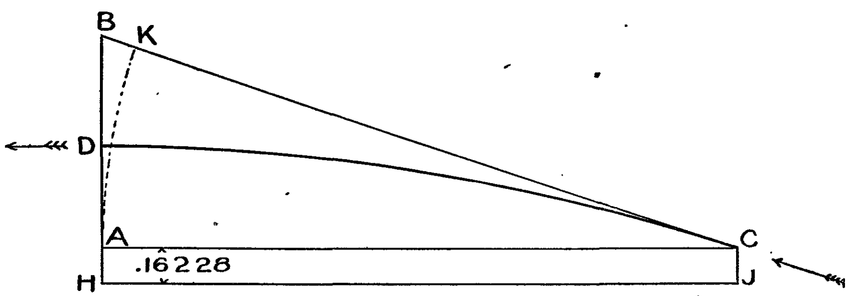

Fig. 36.

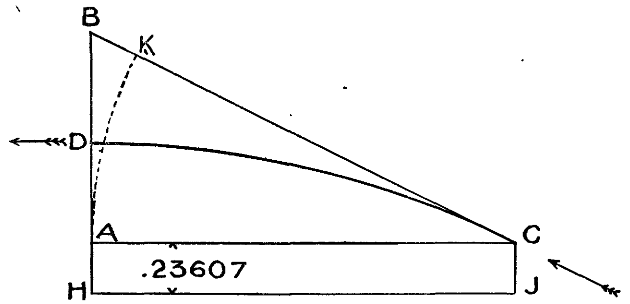

Fig. 37.

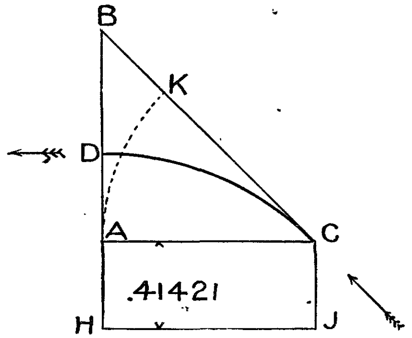

Fig. 38.

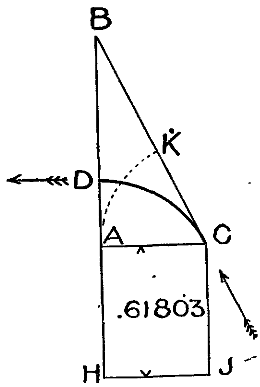

Fig. 39.

FIGS. 36-39.—Theoretical useful effort of wind.

The sail of a wind wheel moves at right angles to the direction of wind or in the direction AC, but the wind meets the sail in motion in the relative direction and with the relative velocity CB, and has with reference to the sail, due to the sail's motion, an opposite motion equal to CA in the direction C to A in addition to its original motion. The length of the line CB represents the velocity with which the air flows along the surface of the sail with undiminished relative velocity, neglecting friction, and the difference between CB and CA, or BK, represents the relative velocity acquired in a second by the air in a direction parallel to CA. That is to say, by action against the sail during one second, the 1 pound of air has acquired a relative velocity BK in a direction parallel to CA, in addition to the relative velocity CA which it had before meeting the sail. This newly acquired relative velocity BK required for its generation a force reacting between the sail and the air proportional to the acceleration of velocity. If BK were equal to 32.2 feet, the reaction would be 1 pound. But the reaction is less than 1 pound in the same ratio that BK is less than AB, which we assumed to be 32.2 feet.

AB:BK:: 1 pound: Useful effort of wind.

So $\frac{BK}{AB}$ represents the ratio between original wind pressure in the

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direction of its original motion and the useful effort which pushes the sail in the direction of its motion. If we lay off on BA prolonged AH=BK and complete the rectangles ACJH, the areas of the four rectangles in figs. 36 to 39, will represent relatively the work theoretically performed in each case. The effort AH may be readily computed. For the action of 1 pound of air, at velocity of 32.2 feet per second, we find that the useful effort is 0.162 pound when the sail moves 3 times as fast as the wind, 0.236 pound when the sail moves twice as fast as the wind, 0.414 pound when the sail moves as fast as the wind, and 0.618 pound when the sail moves one-half as fast as the wind.

AERIAL RESISTANCE TO MOTION.

It should be noted that by far the most striking and important results of our experiments with wind wheels were produced by cutting out from between the sails obstructions which produced aerial resistance to motion; and it should be profitable to study the causes which make these aerial obstructions apparently a matter of greater moment than all the other features of our investigation.

In the two preceding articles we have shown that the greater the sail velocities, theoretically the higher the efficiencies, and also that the useful efforts become smaller as the sail velocities increase.

When the sail travels three times as fast as the wind, the theoretical efficiency is 0.974, very close to perfection; but for a wind velocity of 32.2 feet per second, each pound of air gives a useful effort of only 0.162 pound. It is evident at once that while high sail velocity seems desirable, it does not require great aerial resistance to counteract a large percentage of the useful effort. When the sail travels three times as fast as the the wind, a flat resisting surface of about 1 square inch set perpendicular to the direction of motion would counteract entirely the useful effort due to the action of 1 pound of air. This is computed as follows:

At a velocity of 32.2 feet per second, 1 pound of air per second under ordinary conditions represents a stream of about 59 square inches area in cross section; from which it follows that 59 square inches of flat surface carried normally against still air at the rate of 32.2 feet per second meets with a resistance equal to 1 pound, and at a velocity three times as great meets with a resistance of 9 pounds. But the total useful effort in this case is only 0.162 pound.

Therefore we obtain $F=\frac{59\times 0.162}{9}=1.062$ square inches = flat resisting surface required completely to neutralize all useful effort of wind whose normal pressure against 59 square inches of flat surface equals 1 pound. In like manner we find that F equals 3.418 square inches where the sail velocity is twice the wind velocity, 24.426 square inches where the sail velocity equals the wind velocity, and 145.848 square inches where the sail velocity is one-half the wind velocity.

Or if we let f be the percentage of wind area represented by F, we find that $f=0.018$ when sail velocity is three times wind velocity, 0.059 when sail velocity is twice the wind velocity, 0.414 when sail velocity is equal to wind velocity, and 2.472 when sail velocity is one-half wind velocity.

These percentages hold good for all velocities of wind and emphasize the necessity of reducing to the utmost aerial resistances to motion, if we are to attain high efficiency in a wind wheel.

BEST NUMBER OF SAILS.

An inspection of the table on page 65 shows that wheel No. 35 was more than 3 per cent better than wheel No. 19, although the amounts of sail surface and the angles of weather were the same in both. Wheel No. 35 had exactly the same sails as No. 19, except that each sail of No. 35 contained two of No. 19 placed edge to edge, making 12 broad sails instead of 24 narrow ones. By simply changing the angle of weather from 27.5 degrees to 25 degrees, we made No. 35 into No. 36 and thus gained nearly 6 per cent over No. 19. The wide sails did better with less angle of weather. Wheel No. 6, with the same number of sails as No. 19 but narrower, required a greater angle of weather, 30 degrees.

It is apparent that for a given total of sail area it is better to divide the surface between fewer sails. It may be noted that wheel No. 48, with six sails, gave, in proportion to total sail area, nearly two and a half times the efficiency of No. 2 with 60 sails. Reducing the number of sails reduces the aerial resistance to motion due to the number of edges, and leaves relatively freer interstices for the flow of air between the sails.

BEST RELATIVE AREAS OF SAIL SURFACE.

Tables on pages 47 and 49 give the following results for wheels Nos. 19, 24, and 29:

Effect of area of sail surface on efficiency.

	No. 19.	No. 24.	No. 29.
Square feet of sail surface	12.937	15.000	17.016
Ratio of maximum products (a)	1.204	1.212	1.201

a Compared with wheel No. 2.

These three wheels were nearly of the same efficiency. As compared with the area of the zone containing the sails, No. 19 was about three-fourths full, No. 24 was about seven-eighths full, and No. 29 was what we call full, as the total area of the sails about equaled the area of the zone containing them. These wheels all contained the same number of sails set at the same angle, and differed only in width of the sails. It is clear that nothing was gained by making the total sail area more than seven-eighths of the zone, and that very little was gained by filling beyond three fourths of the zone—surely not enough to pay for the extra material.

From the table on page 65 we obtain the relative values of these three wheels per square foot of sail surface, viz, for No. 19, 0.0931; for No. 24, 0.0808; and for No. 29, 0.0706. Notice also the relative values per square foot of sail surface as given for other wheels in this table.

ACTUAL EFFICIENCY OF THE WIND WHEEL.

To obtain the actual efficiency of wind wheel No. 2, taking the result recorded in the table on page 27, we first obtain the maximum work performed by multiplying the maximum product by the circumference of a circle whose diameter is 2 feet. (See note under the table on page 28.)

62.463 = maximum product in the table.

6.283 feet = circumference of circle 2 feet in diameter.

$62.463 \times 6.283 = 392.455$ foot-pounds per minute = work of wind wheel No. 2 at maximum.

The potential energy of the wind is expressed by the well-known formula $\frac{W v^2}{2 g} = \frac{W}{32.2} \times \frac{v^2}{2}$ in which we make W = weight of air intercepted per second by the total area of the wind wheel. v = velocity of wind in feet per second, and $g = 32.2$, the velocity acquired each second by a body whose motion is not resisted when impelled by a constant force equal to its own weight. The temperature as recorded was 50° F., and on the same day at 2 p. m., in Chicago, the barometer stood at 28.983. Hence 0.075 pound may be taken as the weight of 1 cubic foot of air, 19.635 square feet as the total area of the wheel which was 5 feet in diameter, and 12.286 feet per second, or 8.403 miles per hour, as the velocity of wind. Thus, $W = 19.635 \times 12.286 \times 0.075 = 18.093$ pounds per second, $v^2 = 150.946$, and $\frac{W v^2}{2 g} = \frac{18.093 \times 150.946}{2 \times 32.2} = 42.4078$ foot-pounds per second, or 2544.468 foot-pounds per minute. The portion of actual energy of wind utilized by wind wheel No. 2, or its efficiency, is accordingly $\frac{392.455}{2544.468} = 0.154 +$.

To obtain the efficiency of any other wind wheel whose maximum product is directly compared with maximum product of wheel No. 2 as shown in the tables of later date than March 13, 1883, it is only necessary to multiply 0.154, the efficiency of wheel No. 2, by the ratio of products obtainable from the various tables.

For example, in the table on page 59 we find 1.872 given as the ratio of maximum products for wheels No. 48 and No. 2. Hence, $0.154 \times 1.872 = 0.288$ is the efficiency of wheel No. 48. This is the highest efficiency attained by any of the wheels whose products are recorded in the preceding tables.

WORK OF VARIABLE WIND WITH VARIABLE LOAD.

In most localities the velocity of the wind is exceedingly variable. The average of velocity recorded for an hour by an anemometer gives no idea of the total actual energy of the wind.

It might in conception be actually of uniform velocity, but where the average velocity is 10 miles per hour it is quite as likely to blow one-half the time at the rate of 20 miles and during the other half not at all. The potential energy of 20-mile wind for half an hour is four times as great as that of a uniform 10-mile wind for a whole hour, although the average velocity per hour in each case is the same.

This is taking an extreme case, but it shows how the wind as it blows may and does develop much more energy than could be derived from uniform wind of equal velocity as usually recorded. It would be desirable, if possible, to have the load applied to a wind wheel vary as the square of the velocity of wind, in order that the work might be the greatest possible. Any fixed load which allows the wheel to run freely most of the time is generally altogether too small during much of the time as well as too great at other times. For most uses to which wind wheels are especially adapted an automatic regulation of the load to meet the varying wind would make the best form of wind wheel regulation.

REGULATION OF WIND WHEELS.

Ordinarily wind wheels in this country are made to regulate themselves automatically, so that they can not attain a very rapid rate of rotation. This practice of not allowing the wheel to run at more than a very moderate speed is due to the fact that our wind wheels have been developed mostly in connection with the operation of common reciprocating pumps attached directly to a crank on the shaft of the wheel. Such direct connection necessitates a slow motion of the wheel, in order that the piston speed of the pump may not exceed its economical limit. The consequence is that many wind wheels are allowed to do only a small part of the work which they are capable of developing. Too much attention has been paid to restraining automatically the speed of wind wheels by means which involve a waste of power. So far as the safety of the wheel itself is concerned there should be no need of restraining its speed in any wind under 40 miles per hour; and it should be borne in mind that strains on the wheel, due to any given amount of work, may be diminished in the same ratio that the speed of the wheel is increased.

The gearing of a wind wheel, whether it is geared up or down, should be made to suit the work and the nature of the device through which the power of the wheel is applied to use. When the machine driven requires a high speed of revolution, much gearing up may be avoided if the wind wheel is allowed to revolve rapidly. Back gearing for slow running machines should be resorted to whenever a direct connection with the machine to be driven will not allow the driven mechanism and the wind wheel both to have their best speeds with reference to work and wind.

For any given velocity of wind the speed of the wind wheel should not change, no matter what the nature of the work or of the machine through which work is performed; but the gearing should be such as to suit the load to the wind.

LIMITATION TO SIZE OF WIND WHEELS ON ACCOUNT OF WEIGHT.

Probably in no other form of motor does a gain in efficiency effect so great a saving in weight as in the wind wheel. The strength of other motors needs to be proportional to their own power. A 4-horsepower steam engine for instance does not need to weigh more than four times as much as a 1-horsepower engine, as the strains to which it is subjected are only four times as great. But a 4-horsepower wind wheel needs to be eight times as heavy as a 1-horsepower wheel in order to resist equally well the violence of exceptional storms. It is the strains of storms, and not its own working strains, that determine the required strength of a wind wheel.

Consider a single sail A supported on the end of an arm fastened to a central spider. Let $L =$ length of the arm from center of wind pressure on sail to point of attachment to spider. If the wind wheel is doubled in diameter, keeping the same proportions, L will be doubled and the area of A will be four times as great. Hence the arm needs to resist four times the wind pressure acting at double the former distance out from its point of attachment, and needs to have eight times its former strength. This eightfold strength is secured by doubling all lineal dimensions, as evidently should be done, when the diameter of a wind wheel is doubled.

But the doubling of all lineal dimensions makes the wheel weigh eight times as much, and as its area is only four times as great, it follows that, in proportion to its power, the weight of the wheel is doubled.

As actually made, all the dimensions of large wind mills do not conform strictly to the proportions of small mills, and often the large wheels are built after different models from small ones. But it is well-known that the large wheels as built do not resist storms so well as small ones, and costly experience has taught manufacturers to make the weight of their large wheels at least approximately what calculation requires.

On account of the disproportionate weight of large wind wheels, the towers which support them also have to be made considerably heavier than would be the case if the large wheels required no more material in proportion to power than do small wheels. If the tower for a large wind wheel were of correspondingly greater height, the weights of towers would follow the same law that should control the weights of wheels.

It is evident that small wind wheels are more efficient in proportion to weight than large ones, and that the cost of construction, as the wheels are made larger, is increased in much greater proportion than the gain in power.

EFFICIENCY OF WIND WHEELS AS AFFECTED BY DIAMETER.

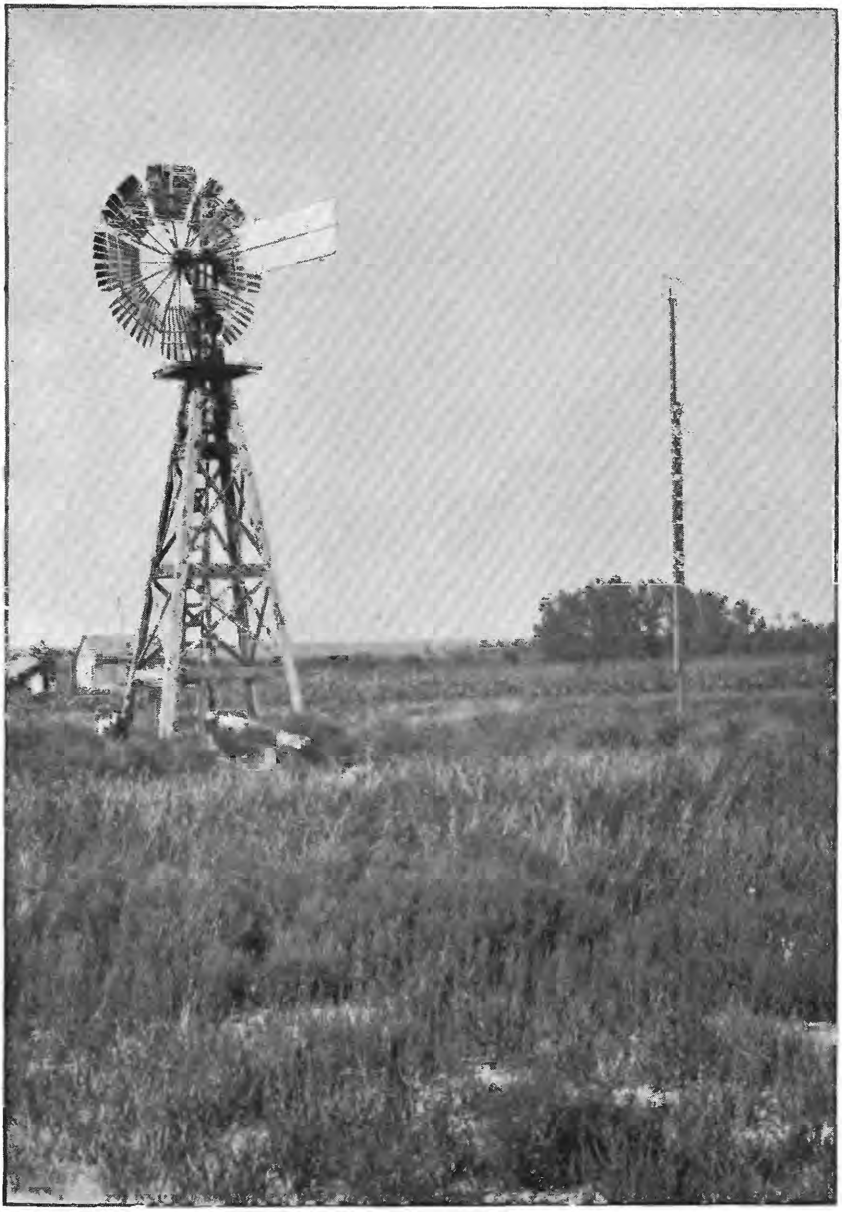
Another reason why small wind wheels are more efficient than large ones is that the wind meets all parts of the area of a small wheel with greater uniformity of velocity. It needs no special acuteness of observation to discover that the wind strikes different portions of large wheels with great unevenness of velocity.

Before commencing the experiments here recorded we attempted to measure the power of a wind wheel 22 feet in diameter in natural wind. By way of preparation we made a special anemometer, which indicated at sight the velocity of wind. We placed the anemometer as near to the wind wheel as possible, expecting to be able to note where its pointer stood and know the wind velocity for a minute of time, during which the revolutions of the wheel carrying a known load might be counted. The first thing we learned was that the anemometer never pointed steadily to any uniform velocity of wind even for one-fourth of a minute, and we next observed that the anemometer would sometimes almost stop running when the wind wheel showed an extra spurt of speed; also the wind wheel would slow down while the speed of the anemometer was accelerated. We could often hear the wind whistling through the sails on the opposite side of the wind wheel, while very little wind was felt on the near side close to the anemometer. We were unable to obtain a single measurement which we considered worth preservation. We could not determine even the best load for any wind, to say nothing about comparative results. We do not doubt that localities might be found where the wind would blow with greater uniformity, but our experience in trying to measure the power of wind wheels in natural wind led us to provide for artificial wind before proceeding further. The slight variations in the best artificial wind we could command caused a great abundance of vexations, which made the obtaining of accurate average results a matter of tedious labor.

HEIGHT OF TOWERS.

Nothing like steady wind can be obtained near the ground in an inhabited country. Buildings, trees, and other obstructions set the wind to whirling and cause it to flow in sinuous streams of very uneven velocity, concentrated in one place at the expense of another.

There is no effective remedy except in the elevation of the wind wheels several feet at least above all obstructions, even if the obstructions are isolated and a thousand feet away. In order that a large wind wheel may be as efficient in proportion to its area as a small wheel, its height above the ground must be greater. So it is probably not far out of the way to say that the weights of the towers as well as



WINDMILL ON LOW TOWER ON THE GREAT PLAINS.

the weights of the wind wheels, for equal safety and efficiency, should be nearly proportional to the cubes of the diameters of the wheels.

MULTIPLICATION OF WIND POWER.

We have shown that for equal safety in storms the weights of wind wheels of different sizes and like forms should be proportioned to the cubes of their diameters. It would require four 12-foot wheels to equal the area and power of one 24-foot wheel, if the larger wheel is proportionately elevated. But the weight of the one 24-foot wheel would be twice as great as the combined weight of the four 12-foot wheels, and the weight of the one higher tower would probably be twice that of the four shorter towers combined. Hence it would seem that in proportion to the power obtained in each case, the one 24-foot wheel would cost twice as much in material. The thought naturally presents itself that the four 12-foot wheels ought in some way to be combined so as to act in unison for concentrating a great amount of power where it is desirable to use the power at only one point, as in driving one machine of large dimensions. If the four wheels were coupled together rigidly the trouble from uneven reception of wind which is experienced in large wheels would be augmented. The problem has not been worked out, but we may imagine a number of wind wheels, each compressing air according to its own ability and delivering it at any distance into a common reservoir. Natural elevations would be selected as locations for windmills, and such a plant could not be rendered useless for the time by an accident to one or two of the wind wheels. There would necessarily be considerable loss in compressing air, but a low-pressure system might be devised that would greatly reduce the waste. Some waste of power attends every mode of transmission. In seeking to make a gain in power of 100 per cent in proportion to cost of plant, the loss of an extra 25 per cent in transmission might well be tolerated.

There may, however, be other and better methods for accomplishing the object in view than by the means we have ventured to suggest.

POWER OF TWELVE-FOOT WIND WHEEL.

The maximum product of our best experimental wheel, No. 48, is given as 247.123 in the table on page 59, and the velocity of wind as 10.935 miles per hour.

The constant 6.283 multiplied by products gives the foot-pounds of work per minute. Hence $247.123 \times 6.283 = 1552.674$ foot-pounds = work per minute of No. 48 in wind of 10.935 miles per hour.

The work of 5-mile wind compared with the work of 10.935-mile wind is $\left(\frac{5}{10.935}\right)^3$ or .0956. Hence, the work of No. 48 is, in 5-mile wind, $1552.674 \times .0956$ or 148.345 foot-pounds. The work of a 12-foot wheel in 5-mile wind is $148.345 \times \left(\frac{12}{5}\right)^2$ or 854.467 foot-pounds, which is equal to 0.0259 horsepower.

It is, therefore, approximately correct to call the power of a 12-foot wind wheel in 5-mile wind equal to one-fortieth of a horsepower. Computing the power of other wind velocities according to the law of cubes, we obtain for the power of a 12-foot wheel, one-fortieth or 0.025 horsepower in 5-mile wind, 0.2 horsepower in 10-mile wind, 0.675 horsepower in 15-mile wind, 1.6 horsepower in 20-mile wind, 3.125 horsepower in 25-mile wind, 5.4 horsepower in 30-mile wind, 8.575 horsepower in 35-mile wind, and 12.8 horsepower in 40-mile wind.

Most 12-foot wind wheels, as now made, are strong enough to stand the strain of furnishing 13-horsepower, and this we believe is not beyond the actual achievement of some 12-foot wind wheels in 40-mile wind.

VALUE OF MATHEMATICAL FORMULÆ IN EXPERIMENTAL INVESTIGATIONS.

The formulæ which Professor Rankine and other mathematicians have applied to wind wheels give for maximum theoretical efficiency only 50 per cent of the potential energy of the wind actually intercepted by the sails. Our experimental wind wheel No. 48 realized an efficiency of more than 44 per cent, if we consider only the wind which would be intercepted by a surface equal to the projections of the sails on the plane of the wheel. So, if we should accept previous mathematical calculations as correct, we have already made a wind wheel which utilizes 88 per cent of the greatest possible theoretical result, and the remaining 12 per cent margin would not furnish very great incentive to attempts at improvement. Mathematicians hitherto have treated the sails of wind wheels as flat surfaces. Even for flat surfaces their theoretical results have been entirely too small; but it is not our purpose to discuss previous mathematical errors. Wind-wheel sails of proper form are not flat, but curved surfaces, and in analyzing the action of wind on curved sails we have not contradicted Rankine's formulæ, but have confined ourselves to making such suppositions as may be actually true in practice. Rankine's formulæ for the action of fluids on curved vanes give as a theoretical result a maximum efficiency of 100 per cent. This is correct. It shows that we have not made a very efficient wind wheel yet, and affords hope for the attainment of still better results in the future. We know that there is a great waste of energy somewhere, even in the best wind wheels we have been able to produce, but we are encouraged to think that much of the waste can be avoided. We do not claim, therefore, that our experiments have revealed the whole truth. In fact we believe that only a beginning has been made, and fully expect to see our best results greatly surpassed. In our view, the results of our experiments, as here recorded, are far richer in the suggestions offered than in the efficiencies realized.

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