A Formula for Computing Transmissibility Causing Maximum Possible Drawdown Due to Pumping

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GROUND-WATER HYDRAULICS

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ABSTRACT

By modifying the Theis nonequilibrium formula a relation is found in which the maximum possible drawdown is expressed in terms of a unique value for the aquifer coefficient of transmissibility. The relation is valid for any specified period and rate of pumping, for a given aquifer coefficient of storage, and for any desired radial distance from the center of pumping.

INTRODUCTION

When planning ground-water investigations, it is often desirable to analyze the manner in which the release of ground water from storage affects the time required for changes in head to migrate from a center of pumping to the area of natural discharge. In many instances the effects of release would predominate over such a long period of time that the water from storage would become the controlling feature of the ground-water development in the region. Many ground-water reservoirs are of such large areal extent that pumping throughout any foreseeable economic or practicable pumping period would not cause drawdowns sufficient to recover water from the region of natural discharge. The purpose of this paper is to show the development of a formula describing—for any desired pumping rate, pumping period, and distance from the pumped well—the particular aquifer transmissibility at which the greatest possible drawdown occurs.

It is recognized that in studying any real hydrologic problem, a detailing of the effects of pumping and the release of water from storage requires knowledge of the variation in the coefficients of transmissibility and of storage throughout the region. If these data were known, it would be possible to describe the drawdown that would occur throughout the region because of any given pumping regimen. In the usual absence of such detailed data, however, highly significant perspectives of the hydrologic problem can be obtained through some idealization of the aquifer and use of the equation developed and described in this paper.

EFFECTS OF PUMPING

If a water-table aquifer were infinite in areal extent and transmissibility, pumping from the aquifer would cause no drawdown anywhere. If such an aquifer were infinite in transmissibility but not infinite in areal extent, the effect of pumping would be a uniform drawdown throughout the aquifer and the water table would remain a plane surface. If the aquifer were not infinite in transmissibility there would be a cone of depression that is somewhat steep sided near the well and that flattens out with distance from the well (Theis, 1940); if the aquifer material were relatively impermeable, the drawdown cone near the well would be very steep sided. The relatively steep sides of the drawndown cone would flare out and intersect the horizontal or nearly horizontal water tables representative of the first two sets of conditions postulated in this paragraph.

DEVELOPMENT OF FORMULA

Obviously, for a specified rate and period of pumping, the drawdown near the pumped well increases as successively smaller aquifer

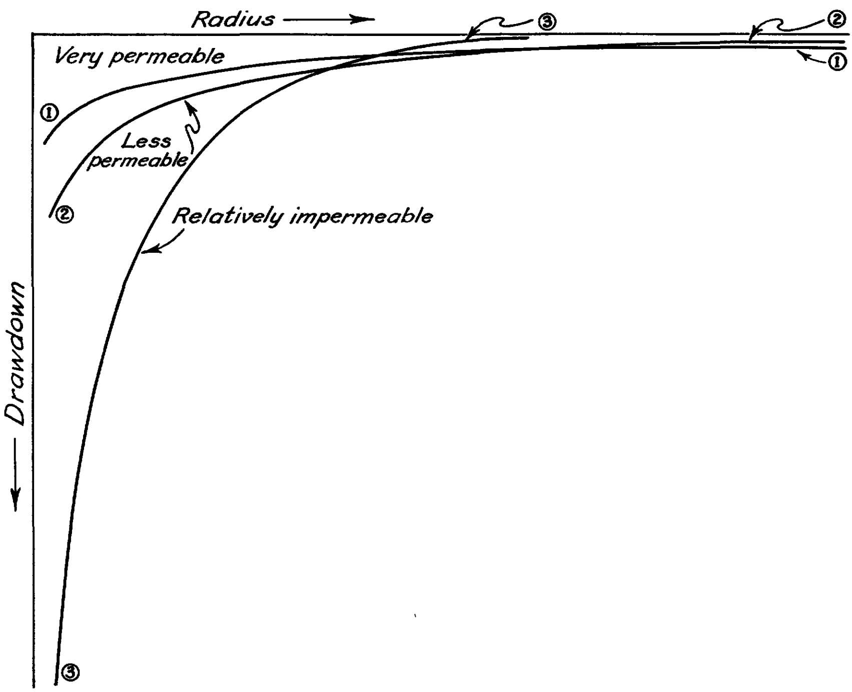

FIGURE 46.—Profiles of drawdown cones near a pumped well.

transmissibilities are considered. Furthermore, as shown in figure 46, the intersection of the drawdown profiles with the plane of the water table occurs at distances that are progressively nearer to the pumped well as successively smaller transmissibilities are considered. The preceding statements suggest that for a specified steady and continuous rate of pumping, from an aquifer having a specified coefficient of storage, there is a unique combination of maximum possible drawdown and aquifer transmissibility for any given elapsed pumping time and distance from the pumped well. The expression for the maximum drawdown, in terms of the coefficient of transmissibility, can be found by differentiating the familiar Theis (1935) formula, which has the nondimensional form (Brown, 1953, p. 851)

$$s=\frac{Q}{4\pi T}W\ (u),\ \text{where}\ u=\frac{Sr^2}{4Tt}.$$

Differentiating

$$\frac{\partial s}{\partial T}=-\frac{Q}{4\pi T^2}\,W(u)+\frac{Q}{4\pi T}\,\frac{\partial W(u)}{\partial u}\cdot\frac{du}{dT}$$

$$\frac{du}{dT}=-\frac{r^2S}{4T^2t}=-\frac{u}{T}$$

$$\frac{\partial s}{\partial T}=-\frac{Q}{4\pi T^2}\left[W(u)+u\,\frac{\partial W(u)}{\partial u}\right].$$

The maximum or minimum may be found by setting $\frac{\partial s}{\partial T}=0$, or

$$W(u)+u\,\frac{\partial W(u)}{\partial u}=0,$$

but

$$\frac{\partial W(u)}{\partial u}=\frac{e^{-u}}{u}$$

Hence

$$W(u)=e^{-u}=-Ei(-u) \qquad \text{(Wenzel, 1942).}$$

The value of u where $e^{-u}=W(u)$ is obtained graphically, as shown in figure 47. The intersection of the two lines, representing plots of $W(u)$ versus u and e^{-u} versus u, occurs at values of $W(u)=$ 0.64738 and $u=0.43482$. From this value of u the transmissibility can be calculated for specified values of storage coefficient, elapsed pumping time, and distance from the pumped well. This value of transmissibility can be used to compute the maximum possible drawdown for a given rate of pumping. Thus the Theis formula can conveniently be rewritten in nondimensional form to give an expression for the maximum drawdown in terms of the aquifer transmissibility:

$$s_{max}=(0.647)\frac{Q}{4\pi T}, \tag{1}$$

where

$$T=\frac{(2.30)r^2S}{4t}.$$

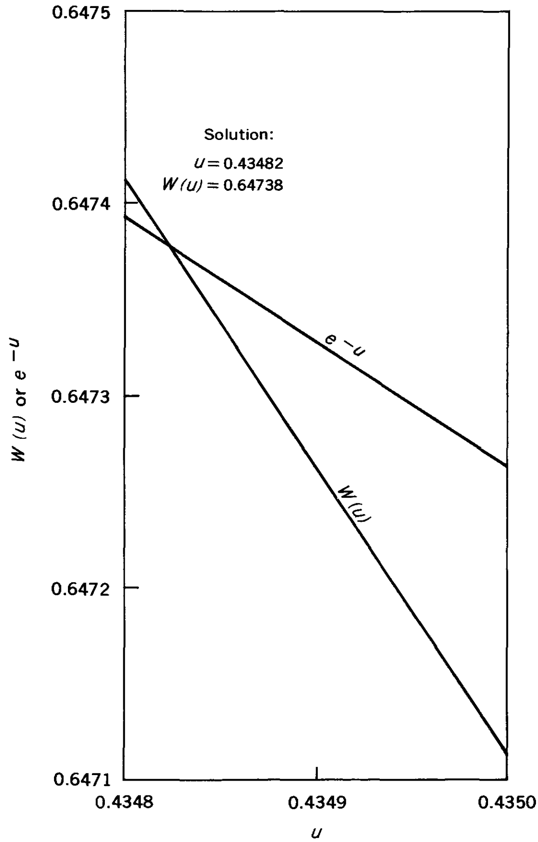

FIGURE 47.—Graph showing point at which $e^{-u}=W(u)$.

ILLUSTRATION OF FORMULA

The implications of equation 1 are illustrated in figure 48, where the maximum possible drawdown is plotted as a function of radius and time, and for the particular value of transmissibility determined for each selected combination of radius and time. The plot is for a pumping rate of 1,000 gpm (gallons per minute) and an aquifer

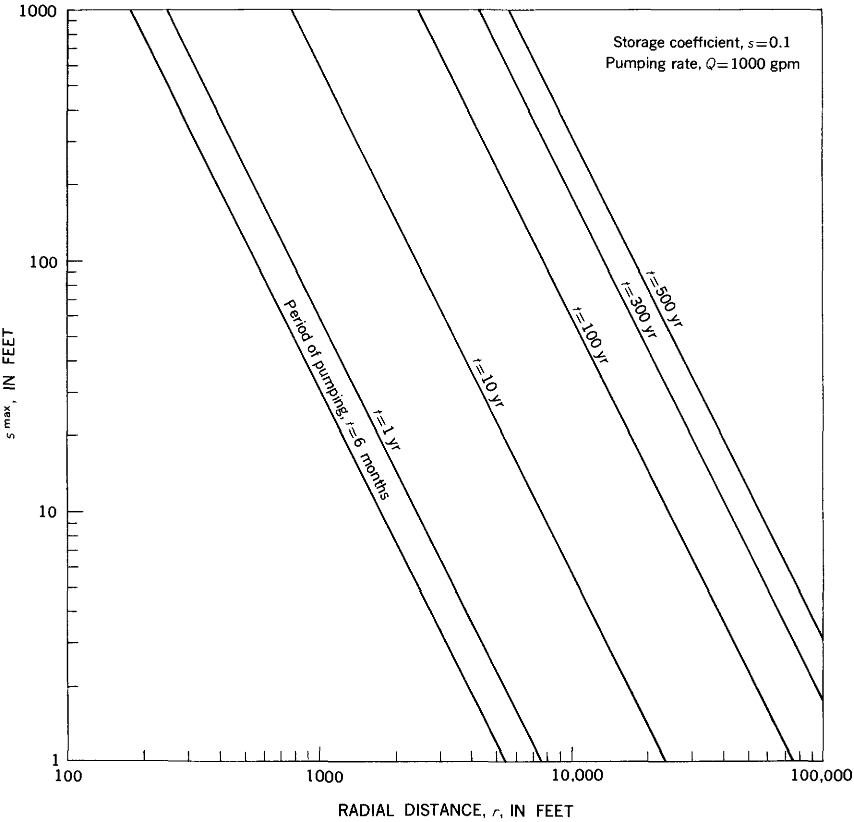

FIGURE 48.—Graph showing maximum possible drawdown for a given radius and time.

storage coefficient of 0.10. Figure 49 also illustrates the significance of equation 1 by showing the transmissibility associated with the maximum drawdown for a given radius and time. This plot is also computed for a pumping rate of 1,000 gpm and a storage coefficient of 0.10. Equation 1 can be used to plot graphs similar to figures 48 and 49 for any other given values of storage coefficient and pumping rate.

SUMMARY

The relation shown as equation 1 can be used to determine quickly the maximum effects of proposed or predicted pumping in a region and to analyze the significance of these effects before proceeding with a hydrologic study. For example, if the maximum possible drawdowns at various points of interest in a very extensive aquifer indicate no drawdown in the region of recharge, it could be concluded

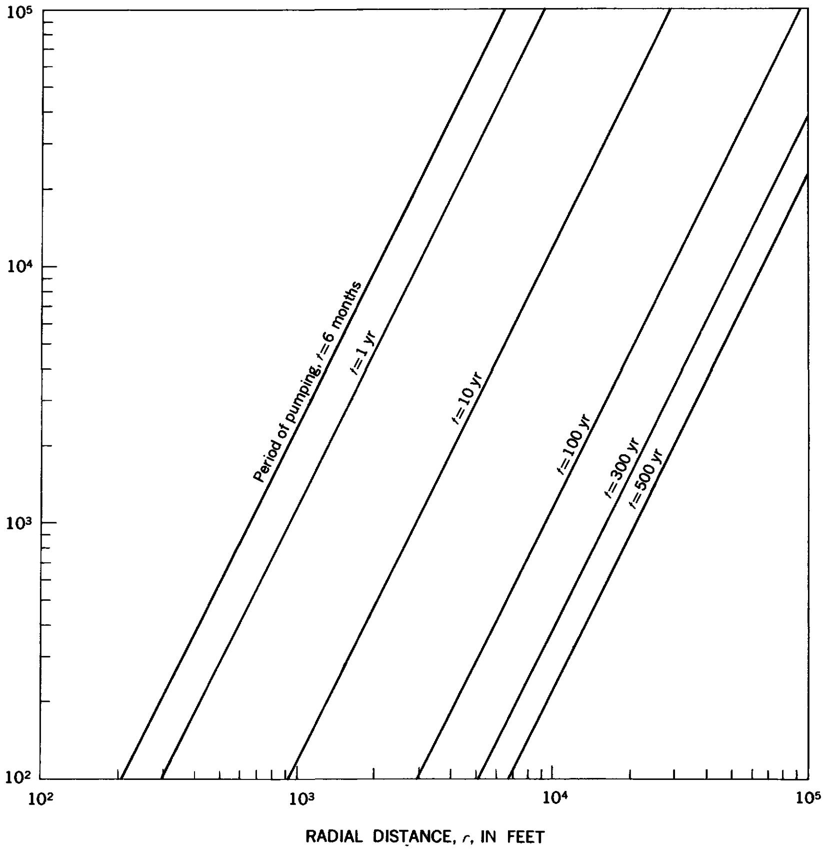

FIGURE 49.—Graph of transmissibility at which drawdown is at a maximum for a given radius and time.

that the problem always would be one of developing ground water from storage and would be independent of any relationships between recharge and the coefficient of transmissibility of the aquifer.

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