Application of the Koopman Operator-Theoretic Framework to Power System Dynamic State Estimation

Marcos Netto,* Venkat Krishnan, Lamine Mili, Yoshihiko Susuki, and Yingchen Zhang

*Postdoctoral Researcher | Sensing and Predictive Analytics Group, Power System Engineering Center, NREL

Abstract: In electric power systems, most state variables are not measured, or even measurable. The large-scale deployment of high-resolution measurement devices, as well as of high-speed, high-bandwidth communications networks, is enabling the development of dynamic state estimators (DSEs), which can provide estimates of the state variables. In this poster, we present a hierarchical decentralized, robust DSE that has been developed by combining model-based and data-driven methods. A two-level hierarchy is proposed, where the lower level consists of model-based DSEs. The state estimates sent from the lower level are received at the upper level, where they are filtered by a robust data-driven DSE that relies on the Koopman operator-theoretic framework.

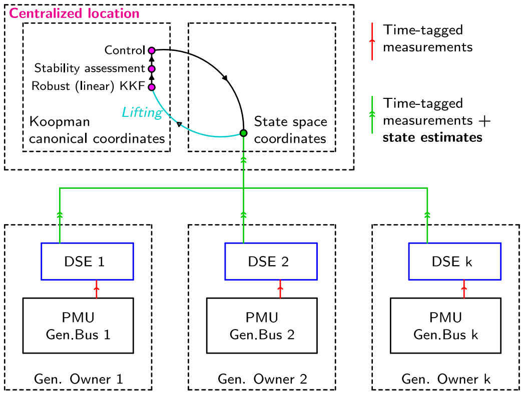

Figure 1: Hybrid framework combining model-based and data-driven methods for power system DSE.

Introduction and Motivation

Consider a discrete, time-invariant, nonlinear dynamical system:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}), \qquad \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k), \tag{1}$$

$\mathbf{x} \in \mathbf{X} \subset \mathbb{R}^n$ is the state vector, $\mathbf{f}: \mathbb{R}^n \to \mathbb{R}^n$ is the discrete map, $\mathbf{y} \in \mathbb{R}^m$ is the observation vector, and $\mathbf{h}: \mathbb{R}^n \to \mathbb{R}^m$ is the observation function.

Assumption: $\mathcal{F}^q = \text{span}\{\phi\}_{i=1}^q$ is a subset of the Koopman eigenfunctions, $\phi(\mathbf{x})$, such that $\mathbf{x}, \mathbf{h}(\mathbf{x}) \in \mathcal{F}^q$. Then, we have that:

$$\mathbf{x}_k = \sum_{i=1}^{q} \phi_i(\mathbf{x}_{k-1})\, \mathbf{v}_i^{(x)} \mu_i, \qquad \mathbf{y}_k = \sum_{i=1}^{q} \phi_i(\mathbf{x}_{k-1})\mathbf{v}_i^{(y)} \mu_i, \tag{2}$$

where $\mathbf{v}$ denotes the Koopman modes and μ denotes the Koopman eigenvalues. Notice that $\{\phi(\mathbf{x}), \mathbf{v}, \mu\}$ are estimated from measured data using the extended dynamic mode decomposition (EDMD) algorithm.

Using (2), Surana and Banaszuk have shown that (1) can be put in the Kalman filter form, as follows:

$$\mathbf{z}_k = \mathbf{A}\mathbf{z}_{k-1} + \mathbf{w}_{k-1}, \qquad \mathbf{y}_k = \mathbf{\Upsilon}^{(y)}\mathbf{z}_k + \mathbf{e}_k, \tag{3}$$

where $\mathbf{A}$ is a block diagonal matrix, $\mathbf{w}_k$ and $\mathbf{e}_k$ are the system and observation error vector, respectively; and $\mathbf{x}_k = \mathbf{\Upsilon}^{(x)}\mathbf{z}_k$.

- In the modeling for power system stability analysis and control, most of the state variables are related to generators. For example, refer to the synchronous generator model given by (4)–(12);
- ω is measured locally, δ is not measured, E'_q and E'_d are not measurable;
- Model-based decentralized DSEs are adopted to get access to estimates of state variables. Consequently, robustness to bad data/data dropouts is of concern because the state estimates are transmitted via communication links (see Fig. 1).

Differential equations

$$\frac{dE'_q}{dt} = \frac{1}{T'_{do}}\left(-E'_q - (X_d - X'_d)\, I_d + E_{fd}\right), \tag{4}$$

$$\frac{dE'_d}{dt} = \frac{1}{T'_{qo}}\left(-E'_d + (X_q - X'_q)\, I_q\right), \tag{5}$$

$$\frac{d\delta}{dt} = \omega - \omega_s, \tag{6}$$

$$\frac{d\omega}{dt} = \frac{\omega_s}{2H}\left(-D(\omega - \omega_s) + T_{mec} - E'_d I_d - E'_q I_q - (X'_q - X'_d)\, I_d I_q\right). \tag{7}$$

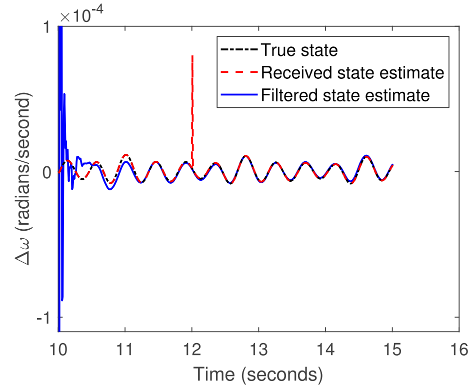

Figure 2: Rotor speed estimation. Case A: Presence of impulsive noise in the state estimate received from the lower level DSE.

Algebraic equations

$$0 = (R_s + R_e)\, I_d - (X'_q + X_{ep})\, I_q - E'_d + V_s \sin(\delta - \theta_{vs}), \tag{8}$$

$$0 = (R_s + R_e)\, I_q + (X'_d + X_{ep})\, I_d - E'_q + V_s \cos(\delta - \theta_{vs}), \tag{9}$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs}), \tag{10}$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs}), \tag{11}$$

$$V_t = \sqrt{V_d^2 + V_q^2}. \tag{12}$$

Variable	Description	Variable	Description
$E'_q, E'_d, \delta, \omega$	State var.	T'_{do}, T'_{qo}	Time constant
$I_d, I_q, V_d,$ V_q, V_t	Gen. algebraic var.	V_s, θ_{vs}	Network algebraic var.
		R_e, X_{ep}	Network parameter
$X'_q, X'_d, X_q, X_d,$ H, D, R_s	Gen. parameter	E_{fd}, T_{mec}	Control input
		ω_s	Constant

Robust Koopman Kalman Filter (GM-KKF)

Define $\zeta_{k|k-1} := \mathbf{z}_k - \mathbf{z}_{k|k-1}$, the error between the true value, $\mathbf{z}_k$, and the predicted value, $\mathbf{z}_{k|k-1}$, which comes from the Kalman filter prediction step. Using the above definition and (3), we write the batch-mode regression:

$$\begin{bmatrix} \mathbf{y}_k \\ \mathbf{z}_{k|k-1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Upsilon}_k^{(y)} \\ \mathbf{I}_q \end{bmatrix} \mathbf{z}_k + \begin{bmatrix} \mathbf{e}_k \\ -\zeta_{k|k-1} \end{bmatrix}, \quad \text{or} \quad \tilde{\mathbf{y}}_k = \tilde{\mathbf{\Upsilon}}_k^{(y)} \mathbf{z}_k + \tilde{\mathbf{e}}_k, \tag{13}$$

$\mathbf{I}_q$ is an Identity matrix. Then, outliers are detected by applying the projection statistics estimator on the point cloud $\mathbf{Y} = [\tilde{\mathbf{y}}_k \;\; \tilde{\mathbf{y}}_{k-1}]$ for every filtering step of the Kalman filter.

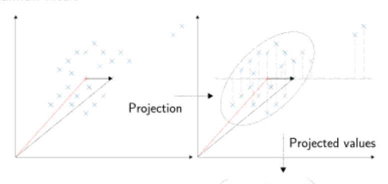

Figure 4: Two-dimensional idealization of the projection statistics estimator. (Left) Compute the median (+) of the point cloud (x). (Right) Project the point cloud to all one-dimensional directions defined by the component-wise median and that pass through the data points. At the end, assign to each point the maximum of the corresponding standardized projections.

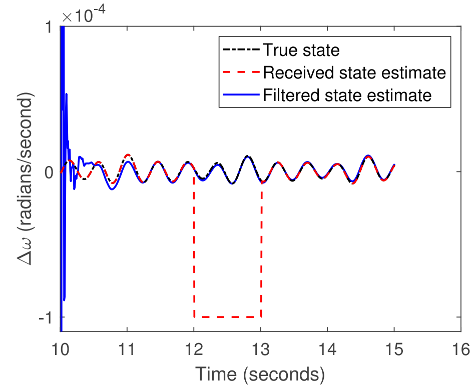

Figure 3: Rotor speed estimation. Case B: Loss of communication with the lower level DSE.

The projection statistics estimator is defined as:

$$PS_i := \max_{\|\mathbf{v}\|=1} \frac{\left|\boldsymbol{\ell}_i^\top \mathbf{v} - \operatorname{median}_j\left(\boldsymbol{\ell}_j^\top \mathbf{v}\right)\right|}{1.4826 \cdot \operatorname{median}_k \left|\boldsymbol{\ell}_k^\top \mathbf{v} - \operatorname{median}_j\left(\boldsymbol{\ell}_j^\top \mathbf{v}\right)\right|}. \tag{14}$$

A PS_i value is computed for each $\{\tilde{y}_{i,k}, \tilde{y}_{i,k-1}\}$ pair and then used to compute the weights, ϖ_i, of a modified Huber estimator, defined as:

$$\rho(r_{s_i}) := \begin{cases} \varpi_i^2\left(\frac{1}{2} r_{s_i}^2\right), & |r_{s_i}| \le \beta, \\ \varpi_i^2\left(\beta|r_{s_i}| - \frac{1}{2}\beta^2\right), & \text{otherwise.} \end{cases} \tag{15}$$

Compared to the non-robust Koopman Kalman filter (KKF) in (3), the GM-KKF shows faster convergence rate due to its batch-mode regression formulation.

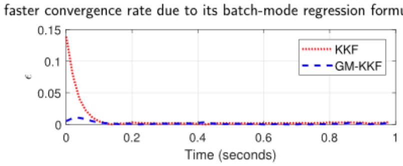

Figure 5: Root mean squared error of the GM-KKF and of (3), which is termed KKF.

Conclusions: Power System Application

The upper level data-driven DSE:

- Is at least 3 times faster than the extended Kalman filter, which is a commonly adopted model-based DSE
- Presents high statistical efficiency when the errors do not follow the Gaussian probability density function
- Is able to suppress the effect of bad data and data dropouts; see Figs. 2 and 3.

This approach was tested on a large-scale synthetic power grid on footprint of Texas. This system has 2,000 nodes and 544 synchronous generators. Numerical results are shown in Figs. 2 and 3 for a specific generator state variable.

References

[1] A. Surana and A. Banaszuk, "Linear observer synthesis for nonlinear systems using Koopman Operator framework," *IFAC-PapersOnLine*, vol. 49, no. 18, pp. 716–723, 2016.

[2] M. Netto and L. Mili, "A Robust Data-Driven Koopman Kalman Filter for Power Systems Dynamic State Estimation," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 7228–7237, Nov. 2018.

[3] M. Netto *et al.*, "A Hybrid Framework Combining Model-Based and Data-Driven Methods for Hierarchical Decentralized Robust Dynamic State Estimation," *IEEE PES General Meeting*, pp. 1–5, Atlanta, GA, 2019.

This work was authored in part by the National Renewable Energy Laboratory, operated by Alliance for Sustainable Energy, LLC, for the U.S. Department of Energy (DOE) under Contract No. DE-AC36-08GO28308. Funding provided by the U.S. Department of Energy Office of Electricity Delivery and Energy Reliability. The views expressed in the article do not necessarily represent the views of the DOE or the U.S. Government. The U.S. Government retains and the publisher, by accepting the article for publication, acknowledges that the U.S. Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this work, or allow others to do so, for U.S. Government purposes.

Operator Theoretic Methods in Dynamic Data Analysis and Control
Institute for Pure and Applied Mathematics (IPAM) – Los Angeles, CA
February 11–15, 2019
NREL/PO-5D00-73236