AUTO-EXHAUST PROPORTIONAL SAMPLER

STABILITY ANALYSIS

by

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Discussion

This report presents the results of a stability analysis performed on the auto-exhaust proportional sampler control system. The system functions to control the flow rate of a sample of auto-exhaust gas proportional to the flow rate of air into the engine carburetor. The exhaust gas sample is collected in a bag for later analysis in the laboratory.

To control the sample flow rate a pressure transducer senses the pressure across a laminar flow element attached to the inlet of the carburetor and produces a d.c. voltage directly proportional to the pressure drop. A second pressure transducer is used to produce a d.c. voltage proportional to the pressure across a laminar flow element in the sample line. These two voltages are compared in the input circuit of an electronic amplifier to produce an error voltage. The error voltage is amplified and used to drive a motor which positions a throttling valve in series with the laminar flow element in the sample line. The throttling valve is positioned in a direction to reduce the error to zero, thereby, maintaining the pressure across the laminar flow element in the sample line equal to that across the laminar flow element at the inlet to the carburetor. The flow rate in the sample line is thus directly proportional to the flow rate into the carburetor, the proportion depending on the relative size of the two laminar flow elements.

The performance characteristics of prime importance for the system include; steady state accuracy, speed of response to transients, and steady state stability. For a clearer understanding of the factors which determine these characteristics it is convenient to represent the system in terms of a functional block diagram as shown on figure 1.

From this diagram it can be seen that the feedback loop contains the amplifier, motor and gear train, throttling valve, and the sample line laminar element and pressure transducer. The carburetor laminar element and pressure transducer are external to the loop and serve to form the reference input.

In general the steady state accuracy of the system depends on the relative linearity of the two laminar flow elements and their pressure transducers and on the magnitude of error required to overcome friction in the motor and valve. To minimize error due to friction the amplifier gain should be high and the motor should have high stall torque characteristics. For this system errors due to friction are negligible since friction is low, amplifier gain is extremely high and the stall torque characteristics of the motor are adequate. In addition the friction is the same for both directions so that any small error would tend to average out.

The speed of response depends on the gain and dynamic characteristics of all the components and on the maximum rate at which the flow area of the throttling valve can be changed. The rate at which the flow area can be changed depends on the maximum speed of the motor, the gear ratio, and the relation between flow area and angular rotation of the valve. For fast response the characteristic delays and lags of the components should be small and their gains high, especially the gain from motor speed to rate of change of valve area. Thus for both fast response and good accuracy the gain of the components should be high, however, this can lead to instability.

In general instability in a system is caused by the transmission of a signal around a loop in such a manner and magnitude so as to reinforce the original signal and cause it to build in magnitude. Although as shown on the diagram, the feedback is negative and acts to reduce the original signal, the characteristics of the components within the loop cause the signal to be shifted in time. For a sine wave or oscillation this results in a phase shift. A phase shift of 180° is equivalent to a reversal of sign so that for this condition the feedback becomes positive and acts to reinforce the original signal. If the loop gain is equal to or greater than one the feedback signal will be equal to or greater than the original signal and oscillations will build up and be sustained. This leads to the Simplified Nyquist stability criteria, which states in effect, a negative feedback system will be unstable and will oscillate at a frequency where the phase shift around the loop is 180° if the loop gain at that frequency is equal to or greater than one. Conversely a system will be stable if the loop gain is less than one at the frequency where the phase shift around the loop is 180°. To provide adequate stability margin, a rule of thumb that is frequently used is to design for a loop gain of one or slightly less at a frequency where the phase shift is 135°. This corresponds to a stability phase margin of 45°.

From the foregoing it is evident that a high loop gain is desirable for fast response and good accuracy however the maximum valve that can be used is limited by the requirements for stability. The analysis of the proportional sampler thus resolves into four general steps:

1. Determination of the magnitude and phase characteristics of the components within the feedback loop as a function of frequency.
2. Determination of the variation in loop gain as a function of environmental conditions.
3. Definition of maximum loop gain that will satisfy stability characteristics.
4. Distribution of loop gain among components to provide minimum error and maximum speed of response.

The analysis, procedures used, and results obtained are presented in the appendix.

Conclusions and Recommendations

1. The proportional sampler can be stabilized and still provide adequate accuracy and speed of response without the addition of stabilization circuitry.
2. The amplifier gain is somewhat high, however, its range of adjustment is adequate.
3. The 10 to 1 gear ratio should be used.
4. The relief valve on the tank whould be set to maintain tank pressure at 3 in. of Hg below atmospheric. The pump should have adequate capacity to hold the relief valve open at maximum sample flow conditions.
5. The flow area of the throttling valve as a function of angular displacement should be such that, when tested with the sample line laminar flow element and with tank pressure set at -3 in. of Hg, the slope of the curve of pressure across the laminar flow element plotted verses angular displacement of the valve is .006 $\pm$10% in. of H_2O per degree at a pressure 1 in. of H_2O across the laminar flow element.
6. Transient testing should be performed on the final system to demonstrate adequate recovery time following a throttle chop.
7. Work still needs to be done to minimize the loading effect of the integrator or counter circuits on the input circuit to the amplifier.

Appendix

General equations representing the dynamic characteristics of the components within the feedback loop can be written as follows.

<u>Amplifier Motor and Gear Train</u>

The speed of the output shaft of the gear train is proportional to the voltage at the input to the amplifier, however, the speed will not change instantaneously with a step change in voltage but will build up at a rate determined by the viscous friction and electrical damping. This can be represented by:

$$N = K_1 e - T_m \frac{dN}{dt} \qquad (1)$$

where N = output shaft speed, rev/sec

K_1 = amplifier and motor steady state gain, rev/sec/volt.

e = amplifier input volgage, volts.

T_m = motor time constant, sec.

t = time, sec.

Output shaft angular position can be represented by:

$$\theta = 360 \int N dt \qquad (2)$$

where θ = output shaft angular position, degrees.

<u>Throttling Valve and Laminar Element in Sample Line</u>

The flow through the sample line and thus the pressure across the laminar element is proportional to the position of the throttling valve. Because of the compressibility of the gas and the volume in the system the pressure across the laminar element will not change instantaneously with a change in valve position.

Thus:

$$P_s = K_2\theta - T_v \frac{dP_s}{dt} \qquad (3)$$

where P_s = pressure across the laminar flow element in the sample line, in. H_2O

K_2 = throttling valve and laminar element steady state gain, in. H_2O/ Degree.

T_v = gas-volume time constant, sec.

Sample Line Pressure Transducer

The output voltage of the pressure transducer is proportional to the pressure across the laminar flow element. Thus:

$$V_s = K_3 \; P_s \qquad (4)$$

where V_s = sample line laminar element pressure transducer voltage, volts

K_3 = pressure transducer steady state gain, volts /in. H_2O

Amplifier Input Circuit

The input voltage to the amplifier is the difference between the voltages of the two pressure transducers. Thus:

$$e = V_R - V_s \qquad (5)$$

where V_R = Carburetor laminar element pressure transducer voltage, volts.

Equations 1 through 5 can be linearized and represented in operational form to yield:

$$\Delta N = \frac{K_1}{T_m p+1} \; \Delta e \qquad (6)$$

$$\Delta\theta = \frac{360}{p} \quad \Delta N \qquad (7)$$

$$\Delta P_s = \frac{K_2}{T_v p + 1} \Delta \theta \qquad (8)$$

$$\Delta V_s = K_3 \, \Delta P_s \qquad (9)$$

$$\Delta e = \Delta V_R - \Delta V_s \qquad (10)$$

where Δ represents a small change in a variable

$$p = \frac{d}{dt}$$

Equations 6 through 10 can be represented in block diagram form as shown on figure 2.

The value of K_1 was measured by disconnecting the valve from the gear train output shaft and recording shaft speed for various values of voltage $(V_R - V_s)$. These results are plotted on figure 3 in terms of motor shaft speed versus error voltage. K_1 is the slope of this line divided by the gear ratio. From figure 3 it can be seen that the value of K_1 will depend on the magnitude of the input signal. For stability purposes the effective gain can be approximated by the slope of the dashed line.

K_2 was determined by taking steady state data of valve position, pressure across the valve, and pressure across the laminar flow element. The valve was set at various angles, at each angle the flow in the sample line was restricted various amounts by clamping the tube. At each angle, readings of pressure across the valve and pressure across the laminar element were taken for the various amounts of restriction. The results are plotted on figure 4. Lines of constant pressure across the sample line are superimposed on this graph.

Figure 5 is a cross plot of figure 4. The value of K_2 can be determined from the slope of these lines at the operating point. It can be seen that the value of K_2 will depend on the pressure in the tank as well as the pressure across the laminar flow element. Figure 6 shows the variation in K_2 as a function of tank pressure at a flow corresponding to a pressure across the laminar element of 1 inch of water. This figure shows that the gain of the valve increases as tank pressure is reduced. Thus the tank relief valve setting will have an effect on the stability of the system tending to decrease stability for lower tank pressure settings.

The value of K_3 was determined by taking readings of pressure across the laminar flow element and corresponding readings of pressure transducer voltages. These results are plotted on figure 7.

To determine the value of T_v a frequency response of V_s versus valve position was run. This was accomplished by oscillating the valve through an angle of about $\pm$ 30 degrees by means of an external drive attached to the valve. The valve was oscillated at various frequencies. At each frequency a photograph of valve position and transducer voltage, V_s, displayed on a dual beam oscilloscope was taken. These photographs are shown on figure 8 along with the reduction of the data. The value of T_v can be determined from this data in the following manner. For steady state response to a sinusoidal signal p can be replaced by $j\omega$ where $\omega = 2\pi f$ and $j = \sqrt{-1}$. Thus equation 6 combined with equation 7 becomes

$$\Delta V_s = K_2 K_3 \frac{1}{T_v j\omega + 1} \Delta\theta \qquad (9)$$

Only the last term in equation 9 need be considered to determine T_v. The test data was plotted on figure 9. Using straight line approximations to curve fit the data, T_v was found to be approximately .025 sec.

The product of equations 6 through 10 yields the open loop transfer function and describes the characteristics of the components within the loop.

$$\Delta V_c = \frac{360\ K_1\ K_2\ K_3}{p(T_v p + 1)(T_m p + 1)} \Delta e \qquad (11)$$

The time constant of the motor T_m is the only value that was not determined by test, however, representative values for a motor of this size and loading, range from about .01 to .02 seconds.

Figure 10 shows a plot of the open loop transfer function, (equation 11) with p replaced by $j\omega$, for assumptions of $T_m = 0$, .01, and .02 seconds. This plot is for the 10:1 gear train, a tank pressure of -3 in. of Hg, and low gain throttling valve. This curve indicates that the phase margin is from about 47° to 57° depending on the time constant of the motor.

It should be noted that the value of K_1 is a function of the magnitude of the input signal. The value near null is about 1/4 the effective gain for large signals. This has the effect of modifying the rule of thumb for 45° of phase margin to a value of 15° to 20° provided the plot is based on the maximum gain condition as is the case on figure 10. As a consequence the gain of the present system can be increased by a factor of 2 to nearly 3 without causing instability.

Figure 11 shows the response of the system to throttle bursts and chops. These traces show that the recovery from a chop is very poor. This can be explained as follows: When the throttle is chopped V_R decreases about 1.6 volts in .1 second. The error needs to be only about .18 volts to be sufficiently large to cause maximum motor speed. This takes slightly less than .015 seconds. From figure 3 it can be seen that the maximum speed the motor can attain is 27.3 rev. per second even for error signals much larger than .18 volts.

The maximum rate that V_s can be reduced is

$$(27.3)(1/10)(360)(.00205)(1.6) = 3.22 \text{ volts /sec.}$$

This is in close agreement with the slope of the trace of V_s following the initial dip caused by the decrease in exhaust pressure immediately following the chop.

This can be improved considerably by increasing the gain of the valve so that a given motor speed will cause a higher rate of change of flow area. If the gain of the valve is increased by about 3, the recovery time will be reduced to nearly 1/3 that indicated by the trace. According to figure 10 this much increase in gain might cause instability if the motor time constant is as high as .02 seconds. However, if the system is unstable or marginal, the gain of the amplifier can be reduced the appropriate amount by adjustment without degrading the speed of response.

PROPORTIONAL SAMPLER FUNCTIONAL BLOCK DIAGRAM

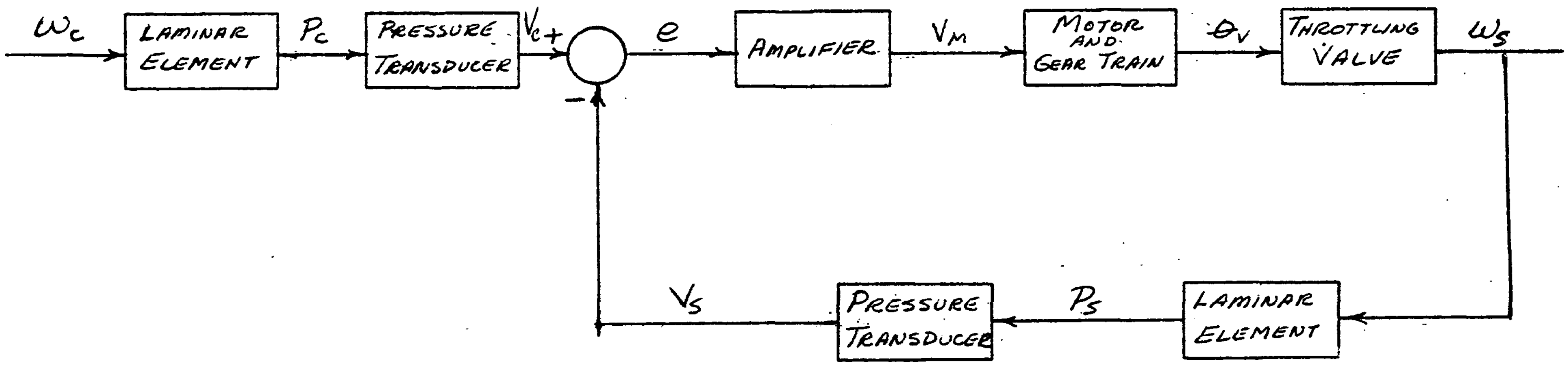


FIGURE 1

Proportional Sampler Block Diagram

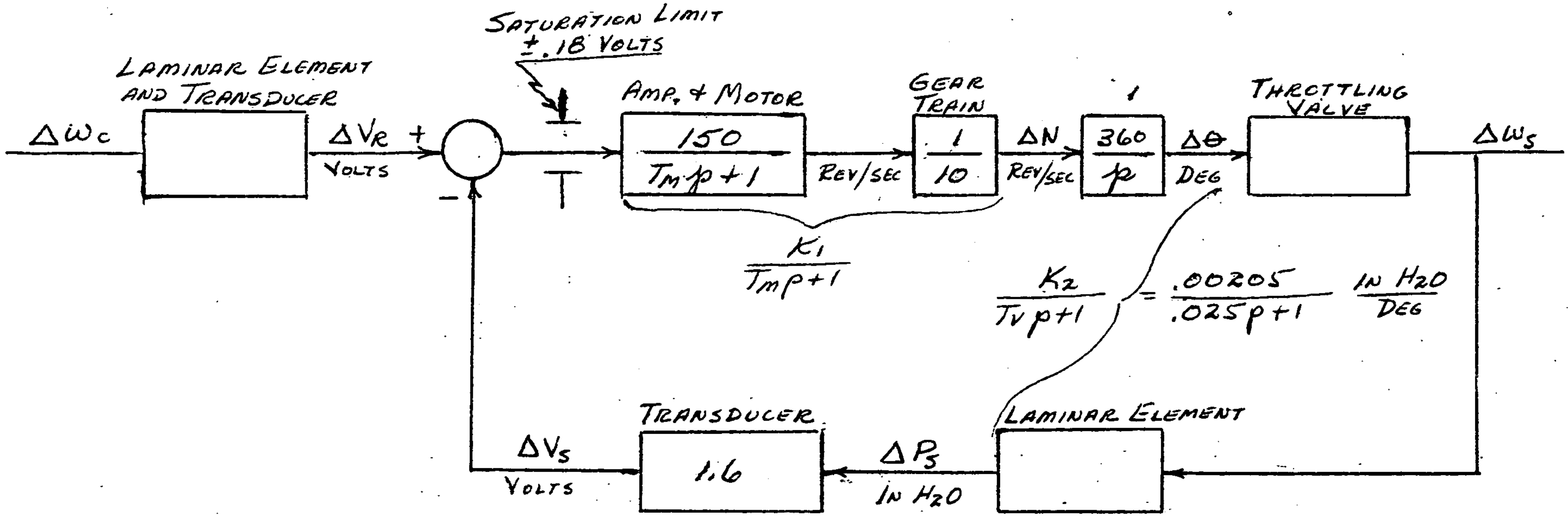


Figure 2

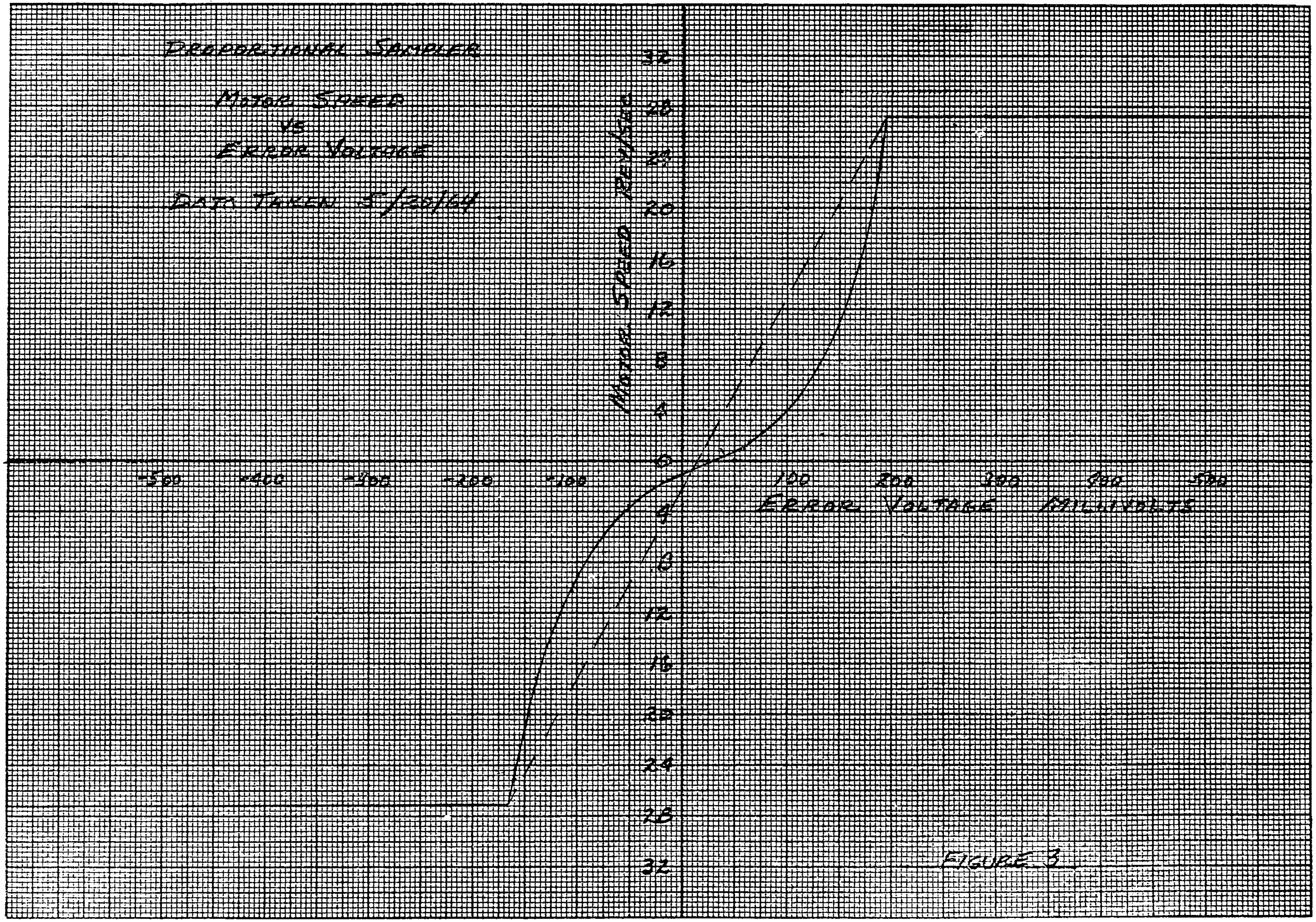

PROPORTIONAL SAMPLER
MOTOR SPEED
VS
ERROR VOLTAGE
DATA TAKEN 5/20/64
MOTOR SPEED REV/SEC
32
28
24
20
16
12
8
4
0
-500
-400
-300
-200
-100
100
200
300
400
500
ERROR VOLTAGE MILLIVOLTS

FIGURE 3

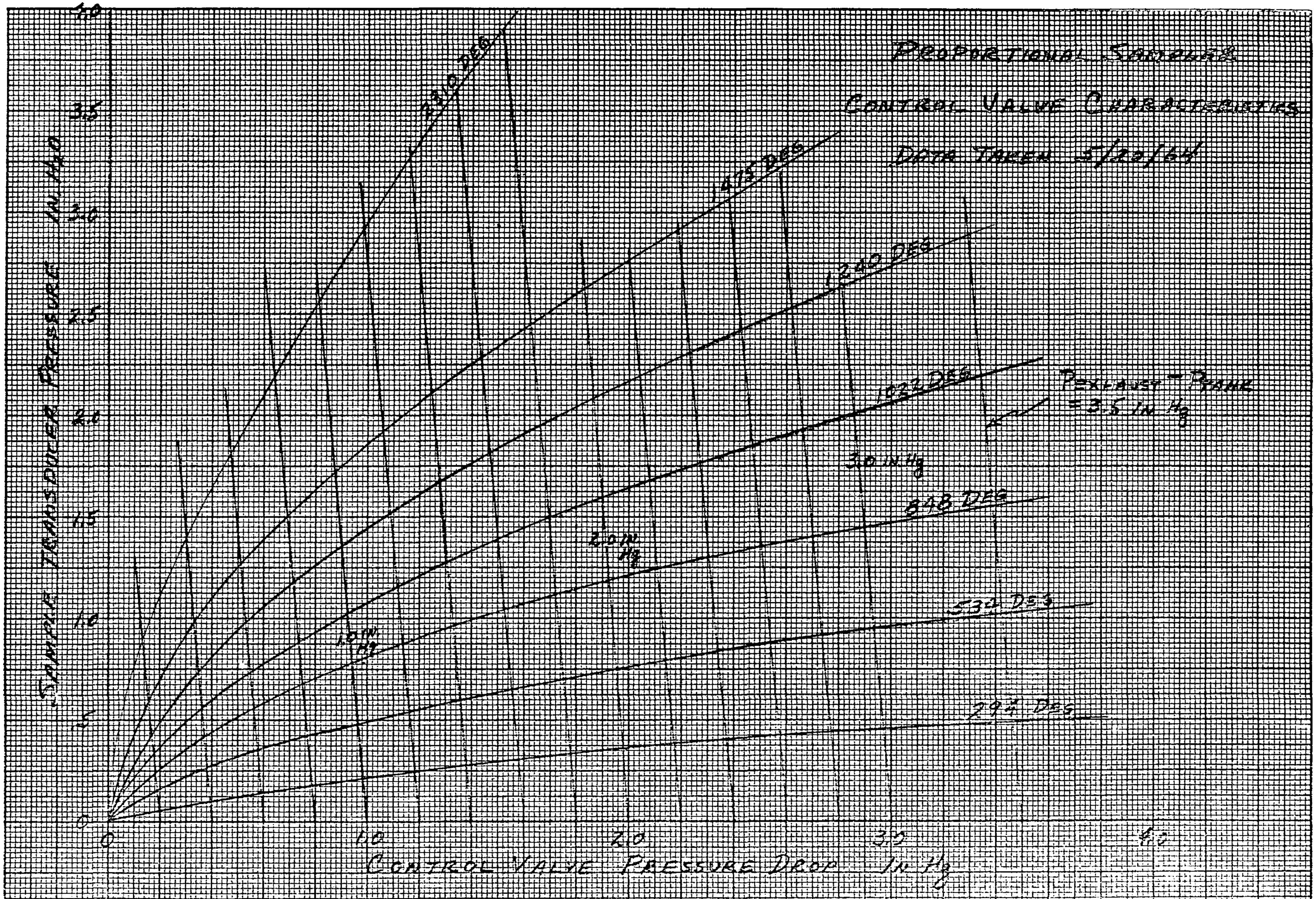

PROPORTIONAL SAMPLER
CONTROL VALVE CHARACTERISTICS
DATA TAKEN 5/22/64
SAMPLE TRANSDUCER PRESSURE IN H_2O
CONTROL VALVE PRESSURE DROP IN Hg
2310 DEG
1475 DEG
1240 DEG
1022 DEG
848 DEG
534 DEG
294 DEG
1.0 IN Hg
2.0 IN Hg
3.0 IN Hg
P EXHAUST - P TANK = 3.5 IN Hg
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
1.0
2.0
3.0
4.0

FIGURE 4

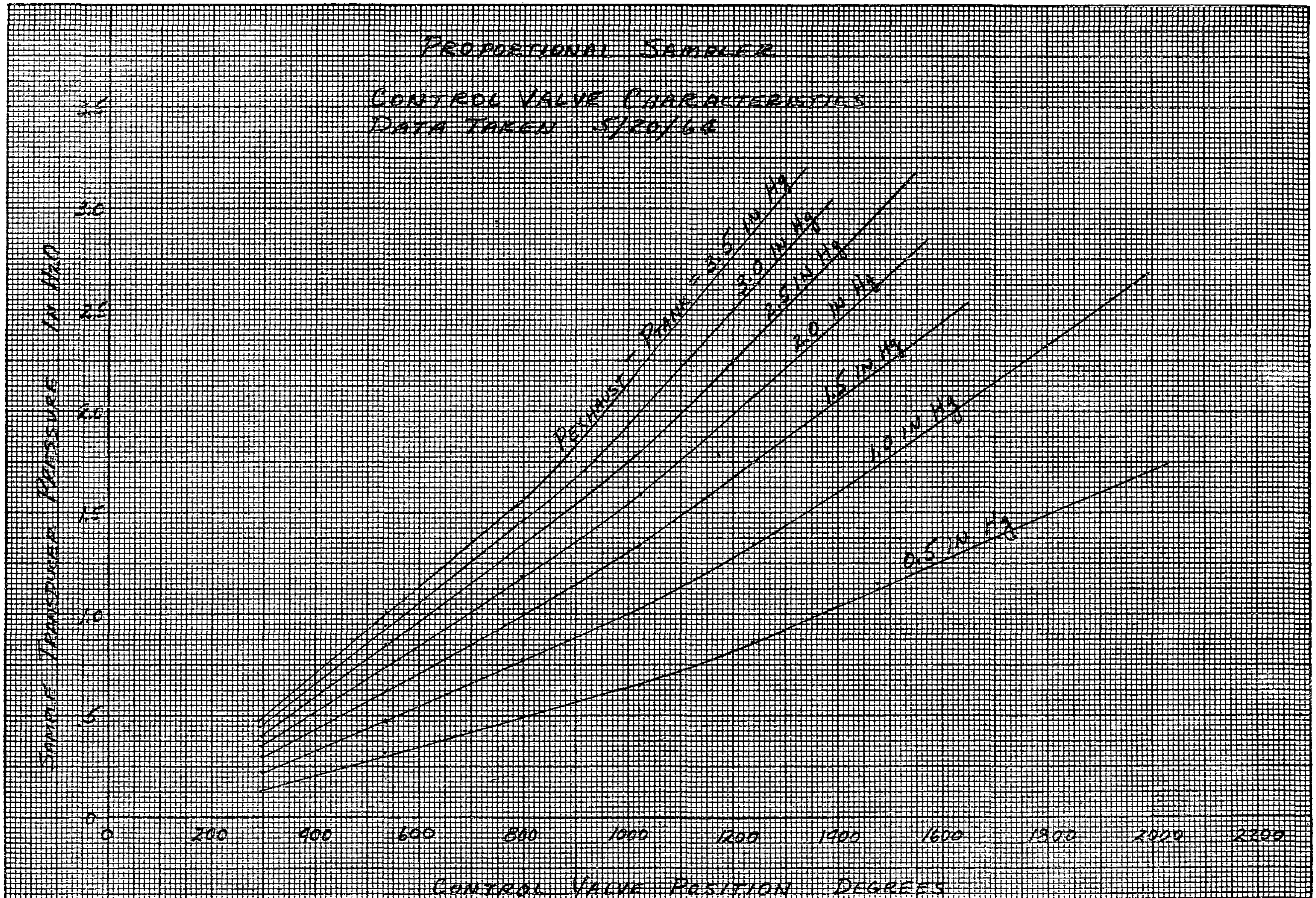

PROPORTIONAL SAMPLER
CONTROL VALVE CHARACTERISTICS
DATA TAKEN 5/20/64
PEXHAUST − PTANK = 3.5 IN HG
3.0 IN HG
2.5 IN HG
2.0 IN HG
1.5 IN HG
1.0 IN HG
0.5 IN HG
SAMPLE TRANSDUCER PRESSURE IN H2O
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
CONTROL VALVE POSITION DEGREES

FIGURE 5

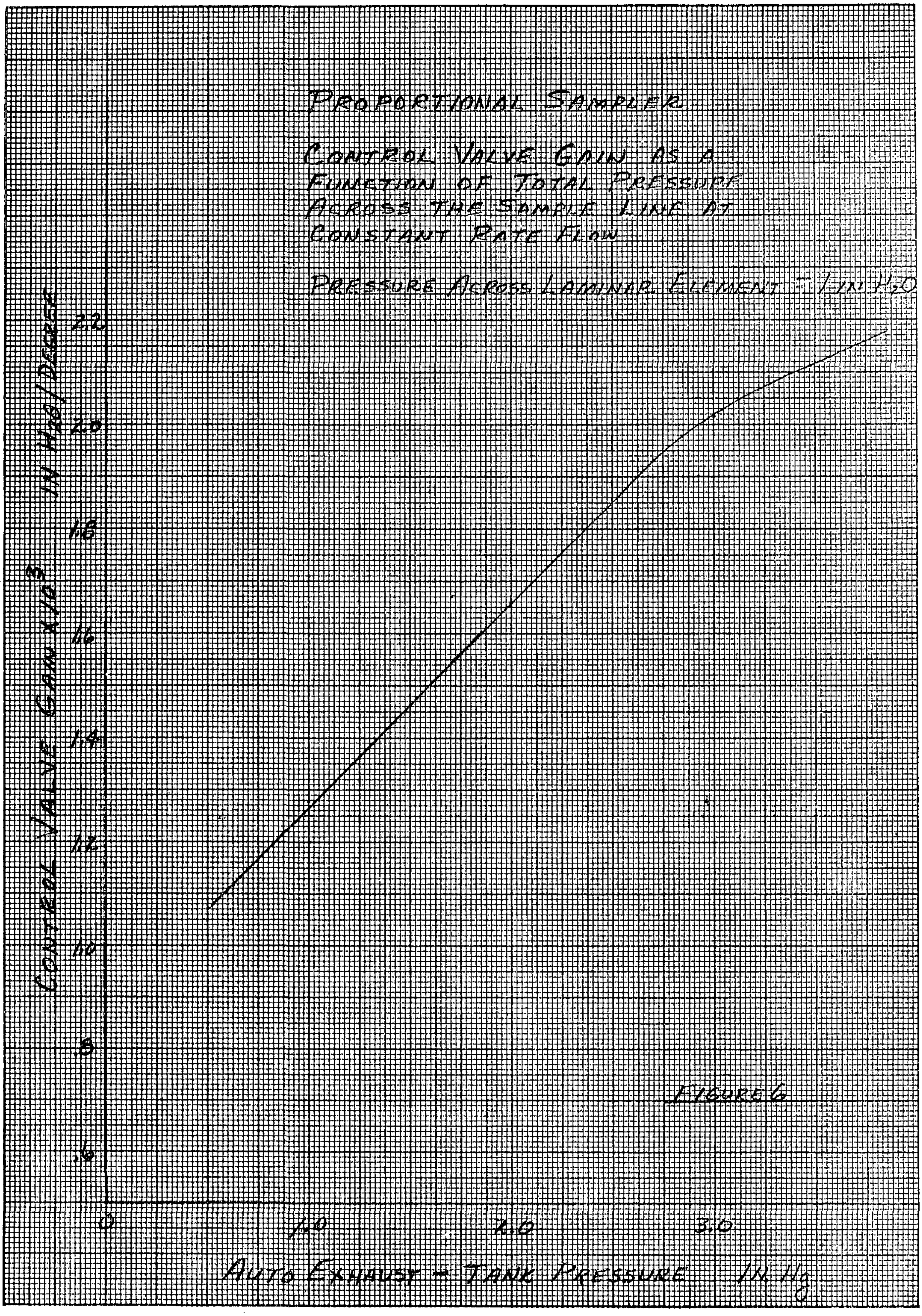

PROPORTIONAL SAMPLER
CONTROL VALVE GAIN AS A FUNCTION OF TOTAL PRESSURE ACROSS THE SAMPLE LINE AT CONSTANT RATE FLOW
PRESSURE ACROSS LAMINAR ELEMENT = 1 IN H_2O
CONTROL VALVE GAIN X 10^3 IN H_2O/DEGREE
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
0
1.0
2.0
3.0
AUTO EXHAUST – TANK PRESSURE IN Hg

FIGURE 6

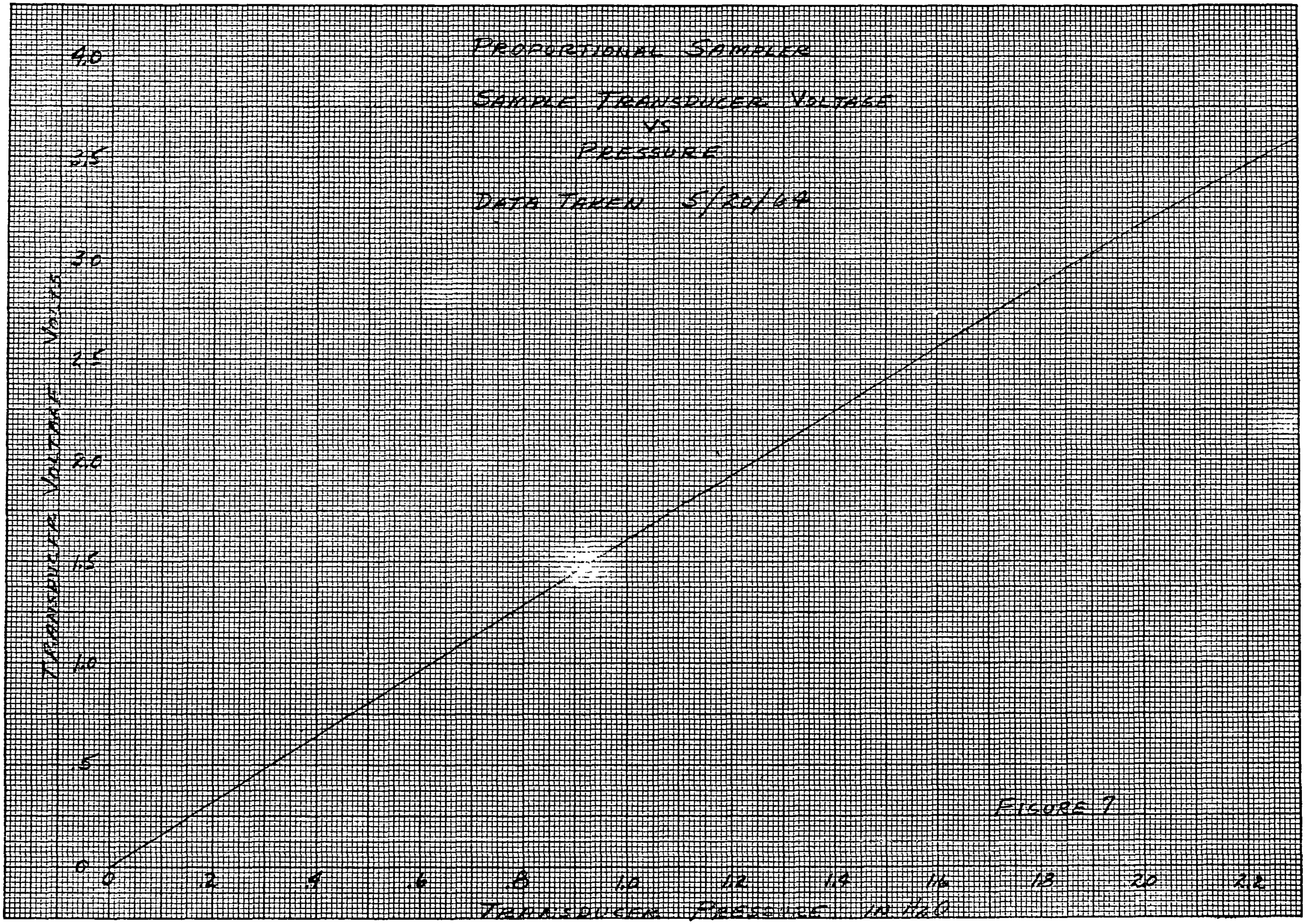
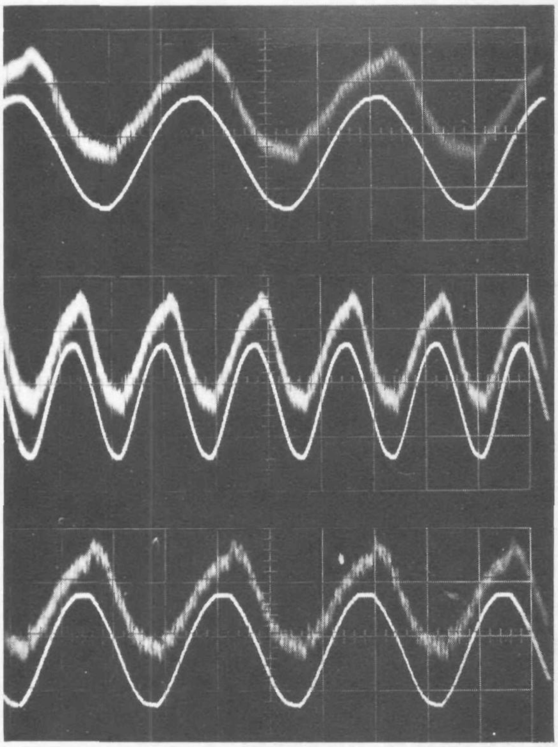

PROPORTIONAL SAMPLER
SAMPLE TRANSDUCER VOLTAGE
VS
PRESSURE
DATA TAKEN 5/20/64
TRANSDUCER VOLTAGE VOLTS
4.0
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
TRANSDUCER PRESSURE IN H2O

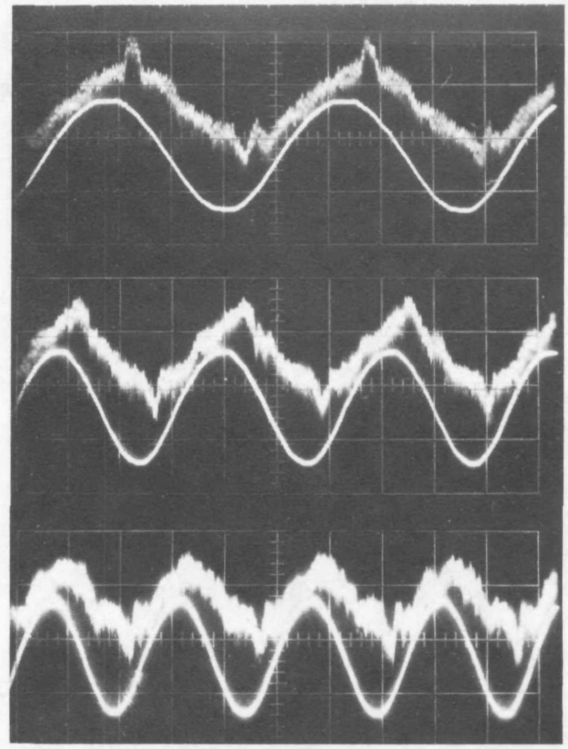
FIGURE 7

FREQUENCY RESPONSE FROM VALVE POSITION TO SAMPLE TRANSDUCER V_s 5/19/64

1.
2.
3.



4.
5.
6.



RUN	PERIOD	f	ω	RELATIVE MAGNITUDE	PHASE
1	1.74 SEC	.575 CPS	3.6 RAD/SEC	1.0	
2	.87	1.15	7.2	1.0	
3	.552	1.81	11.3	1.0	19°
4	.232	4.31	27	.835	34°
5	.16	6.27	39.2	.71	41°
6	.127	7.88	49.5	.55	45°

FIGURE 8

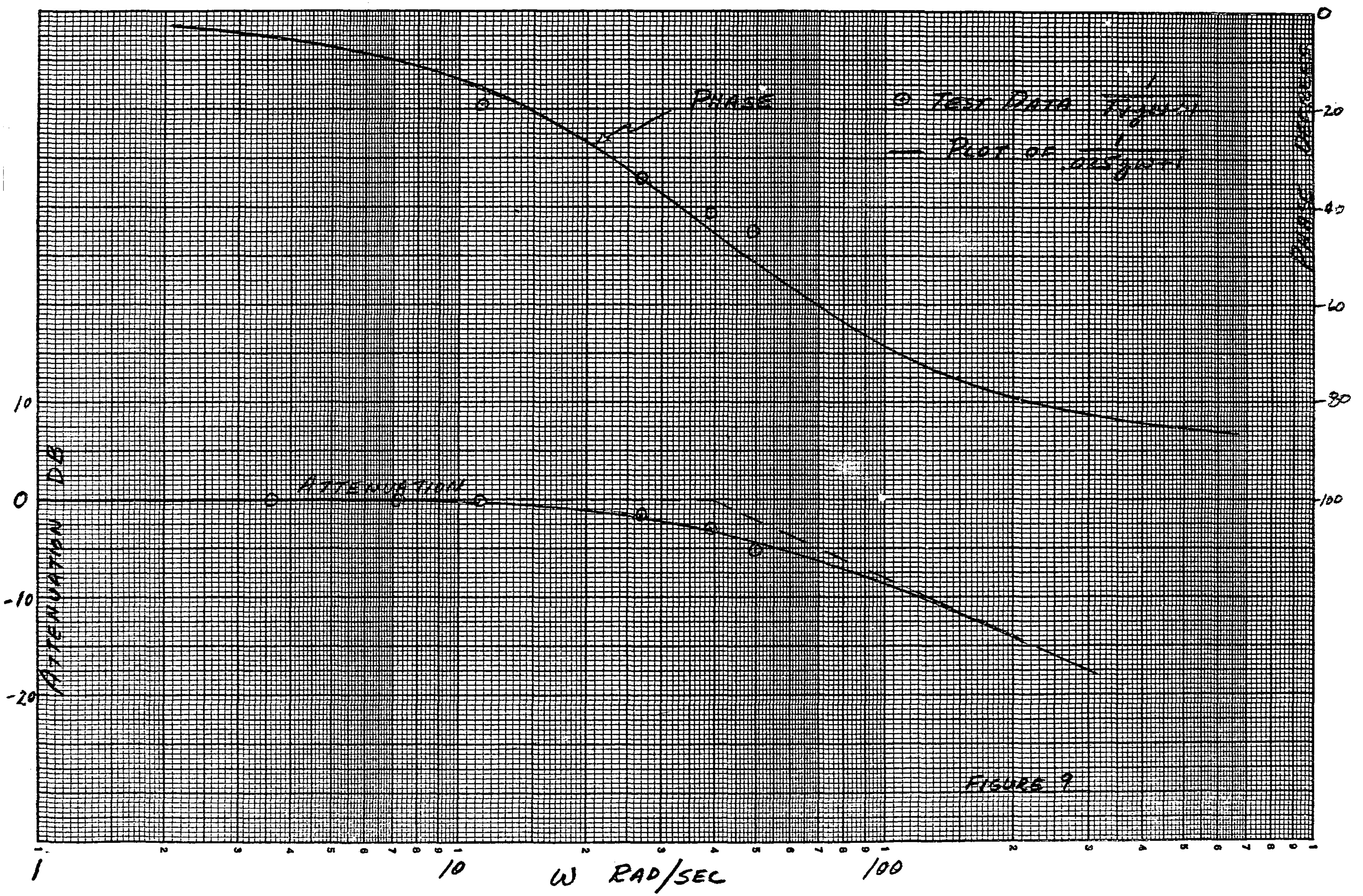

PHASE
⊙ TEST DATA
— PLOT OF $\frac{1}{.025j\omega+1}$
PHASE DEGREES
0
-20
-40
-60
-80
-100
ATTENUATION
ATTENUATION DB
10
0
-10
-20
1
10
100
W RAD/SEC

FIGURE 9

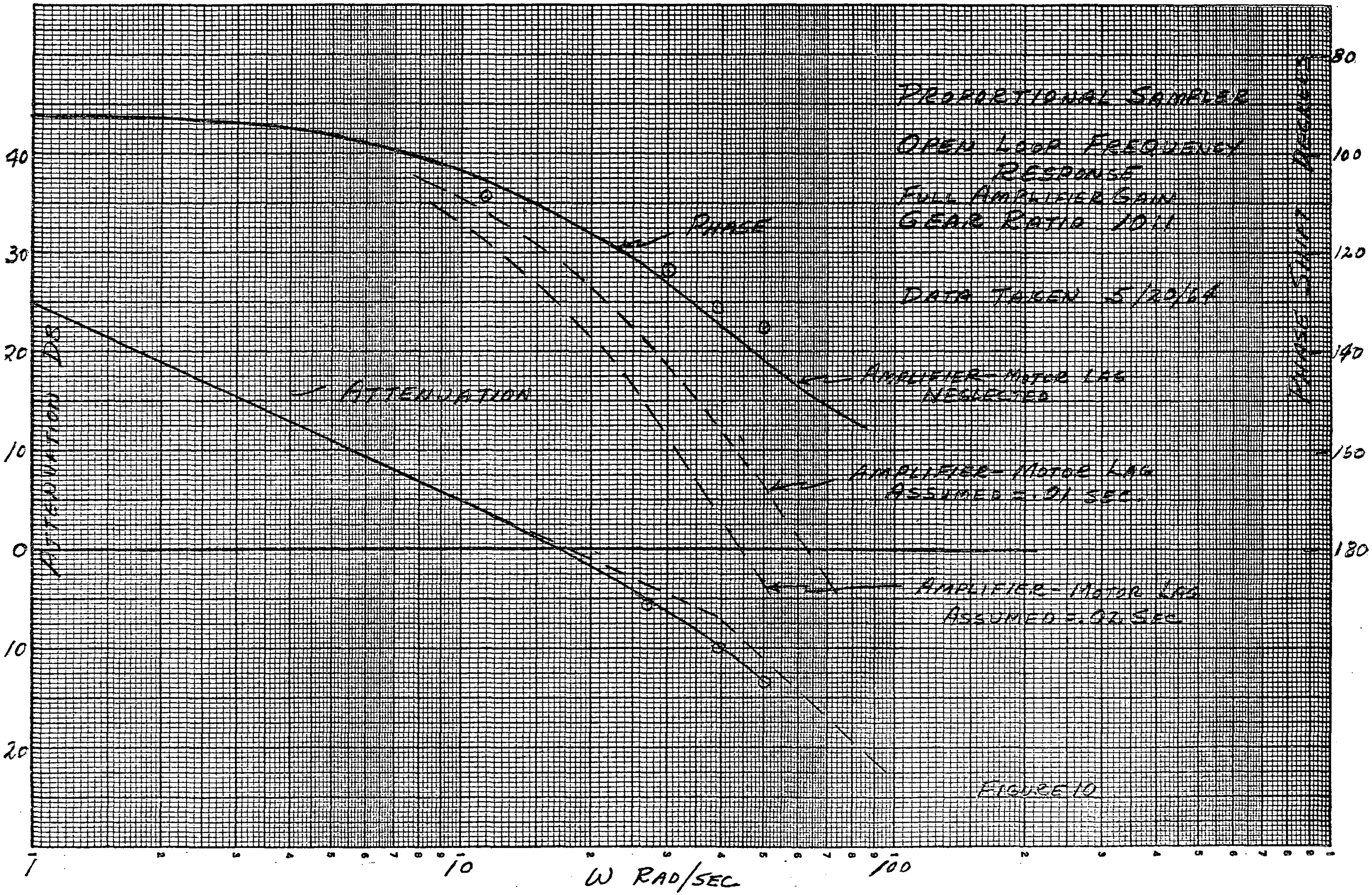
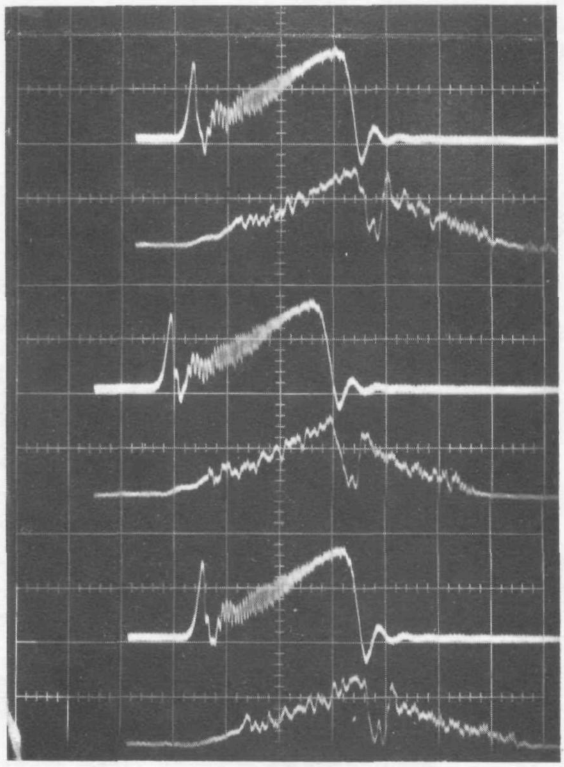

PROPORTIONAL SAMPLER
OPEN LOOP FREQUENCY RESPONSE
FULL AMPLIFIER GAIN
GEAR RATIO 10:1
DATA TAKEN 5/20/64
PHASE
ATTENUATION
AMPLIFIER-MOTOR LAG NEGLECTED
AMPLIFIER-MOTOR LAG ASSUMED = .01 SEC.
AMPLIFIER-MOTOR LAG ASSUMED = .02 SEC
ATTENUATION DB
PHASE SHIFT DEGREES
W RAD/SEC
40
30
20
10
0
10
20
80
100
120
140
160
180
1
10
100

FIGURE 10



PROPORTIONAL SAMPLER TRANSIENT RESPONSE TO THROTTLE BURST AND CHOP

FIGURE 11