TECHNICAL NOTES.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

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SYLPHON DIAPHRAGMS

A Method for Predicting their Performance for Purposes of Instrument Design.

By H. N. Eaton and G. H. Keulegan,
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May, 1922.

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Introduction.

This technical note was prepared for the National Advisory Committee for Aeronautics as a part of the report on the "Investigation of Diaphragms for Aeronautic Instruments," and the purpose of this paper is to show that the characteristic performance of a sylphon diaphragm can be predicted from a knowledge of its stiffness and of its dimensions. The proof is based on a mathematical analysis of this type of diaphragm, together with enough experimental data to prove the validity of the assumptions and the sufficiency of the analysis. Equations are developed for the performance of sylphons under various conditions of loading, both for concentrated loads and for hydrostatic pressure.

The results of the investigation will be useful in the design of instruments or devices containing sylphons, since, by measuring certain dimensions of the diaphragm and the deflection produced by a known concentrated load (to determine the stiffness), the designer will be able to predict the action of the sylphon under the above-mentioned types of loading within the limits defined below.

The load-deflection curve of a sylphon is linear over a considerable range, and over this range the errors due to imperfect elasticity (i.e. drift, hysteresis, and after-effect)* have been found to be less than one per cent of the maximum deflection and so can be neglected, as far as the object of this paper is concerned. The discussion which follows is limited, therefore, to the range of loading for which the load-deflection curve of the sylphon is a straight line and does not include a consideration of the effect of drift, hysteresis and after-effect.

Assumptions.

The following assumptions will be made:

(a) The load-deflection curve of the sylphon for concentrated loads is linear. (This curve will be called the characteristic curve of the sylphon - see Fig. 2.)

(b) The errors due to imperfect elasticity (drift, hysteresis and after-effect) can be neglected.

(c) The external space between two successive corrugations is equal in volume to the internal space between two successive corrugations for deflections within the linear range.

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* These terms will be defined as follows:
Drift is the change of displacement under constant load.
Hysteresis is the excess of displacement with loads decreasing, over the displacement at the same load, with loads increasing.
After-effect is the residual displacement at any time after removal of the load.

(d) The cross-sectional area of the sylphon remains constant.

It has already been stated that assumptions (a) and (b) are warranted from experimental evidence. The range over which they hold good will be discussed later for a particular sylphon.

Assumption (c) is justified by the excellent agreement between results obtained experimentally and those obtained analytically by making use of this assumption. Assumption (d) is sufficiently accurate for the present purpose.

Notation

The following notation will be used:

L = concentrated central load.

P = difference of pressure between inside and outside of sylphon.

d_1 = internal diameter of sylphon) See Fig. 1.

d_2 = external " " ")

y = deflection of upper face under load, measured from neutral position of upper face (i.e. under no load).

A = maximum cross-section perpendicular to axis

$$\left(\text{i.e. } A = \frac{\pi d_2^2}{4}\right).$$

A_q = equivalent area.

l = length of sylphon.

h = depth of one corrugation.

V = volume of the sylphon.

v = average volume of sylphon per unit length $= \frac{V}{l}$.

W = work done on or by the sylphon.

n = number of corrugations.

S_s = spring stiffness = $\frac{L}{y}$

S_c = stiffness of sylphon for concentrated loads = $\frac{L}{y}$

S_d = " " " " distributed loads = $\frac{P}{y}$

S = stiffness of combined sylphon and spring when hydrostatic pressure is supplied to the sylphon.

g = width of gap between spring and sylphon before being coupled together.

THEORETICAL DISCUSSION.

Laws of Deflection.

Consider a sylphon diaphragm with its axis vertical, its lower surface fixed, and with its upper surface consisting of a rigid plate (see Fig. 1). When concentrated loads are considered, it will be assumed that they are applied vertically at the center of the rigid plate. The discussion which follows applies only to the range of deflections for which the load-deflection curve is linear.

From assumption (a) and the definition of stiffness there follows:

(1) $L = S_c y$

It will now be proved that a relation

(2) $P = S_d y$

similar to that expressed by equation (1) exists for loads produced by hydrostatic pressure.

Consider the system consisting of a sylphon with an applied concentrated load L. The interior of the sylphon is open to the outside air. While the sylphon is yielding to the influence of this load, the effective force, i.e., the force tending to deflect the sylphon, is $(L - S_c y)$. Hence during the infinitesimal distance dy the work of deformation is $(L - S_c y)\,dy$. The total work of deformation or the increase in potential energy of the system when the deflection has reached a value y_1 is

$$(3)\quad W = \int_{y=o}^{y=y_1} (L - S_c y)\,dy = L y_1 - \tfrac{1}{2} S_c y_1^2 = \tfrac{1}{2} S_c y_1^2$$

Now suppose a hydrostatic pressure P applied to the sylphon tending to nullify the deflection produced by L. Then if y is the deflection of the sylphon measured from its original position under no load, we can express P as a continuous and single-valued function of $y_1 - y$ as follows. Denote $y_1 - y$ by D.

$$(4)\quad P = A_1 D + A_2 D^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ A_n D^n$$

where A_1, A_2, ... A_n are constants.

Suppose that P is taken sufficiently great exactly to nullify the deflection produced by L. During the cycle just completed, no unconservative forces have been introduced, if we neglect the hysteresis and internal friction in accordance with assumption (b) and ignore the small effect due to viscosity of the air. Consequently, when the deflection has been reduced to zero by the application of the pressure P, the work thus done can be equated to $\frac{1}{2} S_c y_1^2$, the potential energy which was added to the system when it deflected under the influence of the load L.

Therefore

$$(5)\quad W = \int_{D=o}^{D=y_1} KPdD = \int_{P=o}^{P=P_1} PdV = \frac{1}{2} S_c y_1^2$$

Substituting from (4) the value of P

$$W = \int_{D=o}^{D=y_1} K[A_1 D + A_2 D^2 + \ldots A_n D^n]\, dD = \frac{1}{2} S_c y_1^2$$

or

$$(6)\quad W = \frac{KA_1 y_1^2}{2} + \frac{KA_2 y_1^3}{3} + \ldots \frac{KA_{n-1}\, y_1^n}{n} = \frac{1}{2} S_c y_1^2$$

This equation must hold for any value of y_1 : hence the coefficients of y_1^2 must vanish identically; i.e.

$$(6a)\quad \begin{cases} KA_1 = S_c \\ \text{and } A_2 = A_3 = \ldots\ldots A_n = 0 \end{cases}$$

Equation (4) now becomes

$$(4a)\quad \begin{cases} P = A_1 D \\ \text{or } P = A_1 (y_1 - y) \end{cases}$$

Since we are considering only the linear portion of the deflection curve of the sylphon, it makes no difference where the initial position is taken: consequently (4a) may be written

$$(4b)\quad P = A_1 y$$

where y is now measured from the position of the upper surface of the sylphon before the pressure P was applied. If we replace the constant A_1 by S_d, (4b) becomes identical with (2) and thus proves the validity of the latter. Experiment also verifies equation (2) (see Fig. 3).

Equivalent Area.

Dividing equation (1) by equation (2) there results

$$\begin{cases} \dfrac{L}{P} = \dfrac{S_c}{S_d} \\ L = P\dfrac{S_c}{S_d} \text{ (y constant)} \end{cases}$$

Now the ratio S_c/S_d has the same dimensions as has area. It is also clear that this ratio for a given sylphon is dependent only on the geometrical and physical characteristics of the sylphon. Consequently it may be considered as a certain proportion of the maximum cross-sectional area A of the sylphon and will be called the <u>equivalent area</u> of the sylphon. The physical significance of the equivalent area may be seen most clearly perhaps by considering that a given hydrostatic pressure P produces the same deflection of the sylphon as does a certain concentrated central load L, and that the equivalent area is defined as the ratio of this L to the given P. Its usefulness consists in the facts that it is a constant for a given sylphon and that it enables one to predict the performance of the sylphon under any specified conditions, once the deflection for one concentrated load is known. The value of the ratio $\frac{S_c}{S_d}$ or A_q will now be derived in terms of known constants and dimensions of the sylphon.

Returning to equation (5) we have

(5) $$\int_{P=o}^{P=P_1} PdV = \frac{1}{2} S_c y_1^2$$

$$dV = d(lv) = vdl^* = vdy$$

and $$P = S_d y \text{ from } (2)$$

$$\therefore \int_{y=o}^{y=y_1} S_d vydy = \frac{1}{2} S_c y_1^2$$

or

$$\frac{1}{2} S_d v y_1^2 = \frac{1}{2} S_c y_1^2$$

whence

(8) $$\frac{S_c}{S_d} = A_q = v$$

Now v may be computed from the assumption that the sylphon is made up of successive cylinders each of height h and alternately of diameter d_1 and d_2.

If there are 2n corrugations then

$$v = \frac{V}{l} = \frac{\pi nh\ (d_1^2 + d_2^2)}{2\ nh} = \frac{\pi}{8} (d_1^2 + d_2^2)$$

or

(9) $$A_q = \frac{\pi}{8} (d_1^2 + d_2^2)$$

Experimental verification of this result will be given later.

For purposes of comparison it will often be convenient to express the equivalent area as a percentage of the maximum area. Thus,

(10) $$\frac{A_q}{A} = \frac{d_1^2 + d_2^2}{2d_2^2}$$

(10a) or $$100 \frac{A_q}{A} = 100 \frac{d_1^2 + d_2^2}{2d_2^2} \text{ per cent.}$$

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* See assumption (c).

Performance of Sylphon Under any Combination of Loads.

From equations (1), (2), (7), and (8).

(11) $L \pm A_q P = S_c (y_1 \pm y_2)$

where y_1 is the deflection produced by L

and y_2 is the deflection produced by P

Equation (11) contains two constants, A_q and S_c, which depend only upon the characteristics of the sylphon. A_q can be computed from purely geometrical considerations, but S_c must be determined by experiment. From a single deflection with a concentrated load it is possible to obtain S_c, provided care is taken that the range of loading for which the load-deflection curve is linear is not exceeded. It is also possible to determine A_q from a second experiment with distributed load, but it is usually preferable to compute the value of this constant from equation (9).

Performance of Sylphon and Spring in Combination.

Suppose that a sylphon is distended by a spring either internal or external (see Fig. 4). The performance of the spring may be expressed by

(12) $L = S_s y$

Assume that, with the spring and sylphon mounted but not coupled together, there exists a gap g between the couplings. If the two are now coupled together, g is reduced to zero and the top of the sylphon is deflected an amount y_1. y_1 can be computed from the known values of g, S_c, and S_s and, consequently, the reaction of the spring and the sylphon upon each other L_1 can be computed, as will be shown.

$$L_1 = S_c y_1 = S_s (g - y_1)$$

$$(13) \quad \begin{cases} y_1 = \dfrac{S_s g}{S_c + S_s} \\[2ex] g - y_1 = \dfrac{S_c g}{S_c + S_s} \end{cases}$$

and

$$(14) \quad L_1 = S_c y_1 = \frac{S_c S_s}{S_c + S_s} g$$

Now let a pressure P be applied externally (or suction internally). Equating the force exerted by the spring to the elastic resistance of the sylphon and the load, there results

$$L_1 + S_s y = L_1 - S_c y + A_q P$$

where y is measured from the neutral position of the couplings when the sylphon and spring are coupled together.

Then (15) $A_q P = (S_c + S_s)\ y$

or

$$(15a) \quad P = (S_d + \frac{S_s}{A_q})\ y$$

The quantity in parentheses is the stiffness of the combination of spring and sylphon and (15a) may be written

$$(15b) \quad P = Sy$$

Experimental Verification of Formula for Equivalent Area.

Four sylphons were tested in order that experimental confirmation of the preceding analysis might be obtained. The construction characteristics of these sylphons are given in the following table:

Table 1.

		No. 1	No. 2	No. 3	No. 4
Internal Diameter	cm.	4.1	9.5	4.1	9.5
External Diameter	cm.	6.0	11.9	6.13	11.6
Thickness of Material	cm.	0.011	0.025	0.025	0.025
Number of Corrugations		11	10	12	21
Material (Brass)					

Fig. 2 shows the characteristic curves for these sylphons. The value of S_c can be obtained from these curves, since from

$$(1) \qquad S_c = \frac{L}{y}$$

Fig. 3 shows performance curves for Sylphon No. 2 determined experimentally. Extension and loads tending to produce extension of the sylphon are considered positive. The curves are linear over the range shown in Fig. 3. It was found that, when a load producing a deflection of -0.35 cm. was applied, the curve was no longer linear. The points in which the lines L = const. cut the axis of deflections is obtained from the characteristic curve of the sylphon. The points in which these lines cut the axis of pressure may be computed from equation (11) by putting

$y_1 - y_2 = 0$, when

$$P = \frac{L}{A_q}$$

or, if preferred, the lines may be drawn through the proper points on the axis of deflections with the required slope. This slope is easily proven to be $+S_d$ from equation (11). The slope appears to be negative in Fig. 3, but it should be remembered that the pressures shown here are negative.

Fig. 3 shows that Sylphon No. 2 will give linear load-deflection curves from a deflection of about -0.2 cm. to at least +0.4 cm. The positive limit of linear deflections may be much higher than +0.4 cm., but the tests were not carried to the limit.

From a consideration of the way in which the family of curves in Fig. 3 was constructed, it is clear that each intersection of a line L = const. with the axis of pressures can be used to determine the value of A_q. Considering such a point we have the relation

$$A_q = \frac{L_1}{P_1}$$

This will be utilized to provide a check on the values of A_q as determined analytically.

Table 2.

Equivalent Areas of Sylphon.

Sylphon No. 1.

P gms/cm²	L gms.	A cm²	A_q cm² Experimental	$\frac{A_q}{A}$ % Computed	$\frac{A_q}{A}$ % Exper'l
4.32	91.0	28.3	21.1		–
2.14	45.0		21.0		
1.07	23.0		21.5		
0.42	9.0		21.4		
Av.			21.25	73.4	75.2

The difference between experimental and computed values is 2.4%.

Sylphon No. 2

P gms/cm²	L gms.	A cm²	A_q cm² Experimental	$\frac{A_q}{A}$ % Computed	$\frac{A_q}{A}$ % Exper'l
15.75	1361.0	111.3	86.4		
10.40	908.0		87.3		
5.00	453.0		90.6		
2.65	227.0		85.7		
1.55	136.0		87.8		
1.00	91.0		91.0		
0.45	45.0		100.0		
Av.			89.83	81.7	80.7

The difference is 1.3%.

Sylphon No. 3.

P gms/cm²	L gms.	A cm²	A_q cm² Experimental	$\frac{A_q}{A}$ % Computed	$\frac{A_q}{A}$ % Exper'l
64.5	1361.0	29.55	21.1		
44.0	908.0		20.6		
21.5	453.0		21.1		
10.7	227.0		21.2		
Av.			21.00	72.4	71.2

The difference is 1.7%.

Sylphon No. 4.

P gms/cm²	L gms.	A cm²	A_q cm² Experimental	$\frac{A_q}{A}$ % Computed	$\frac{A_q}{A}$ % Exper'l
26.0	2270.0	105.7	87.3		
20.8	1814.0		87.2		
15.3	1361.0		89.0		
10.2	907.0		88.8		
5.0	453.0		90.6		
Av.			88.58	83.4	83.8

The difference is 0.5%

The agreement between the experimental and calculated values of A_q indicates that the equivalent area for any sylphon is independent of the elastic properties of the sylphon and may be computed from the expression

$$A_q = \frac{\pi}{8}\,(d_1^2 + d_2^2) \tag{9}$$

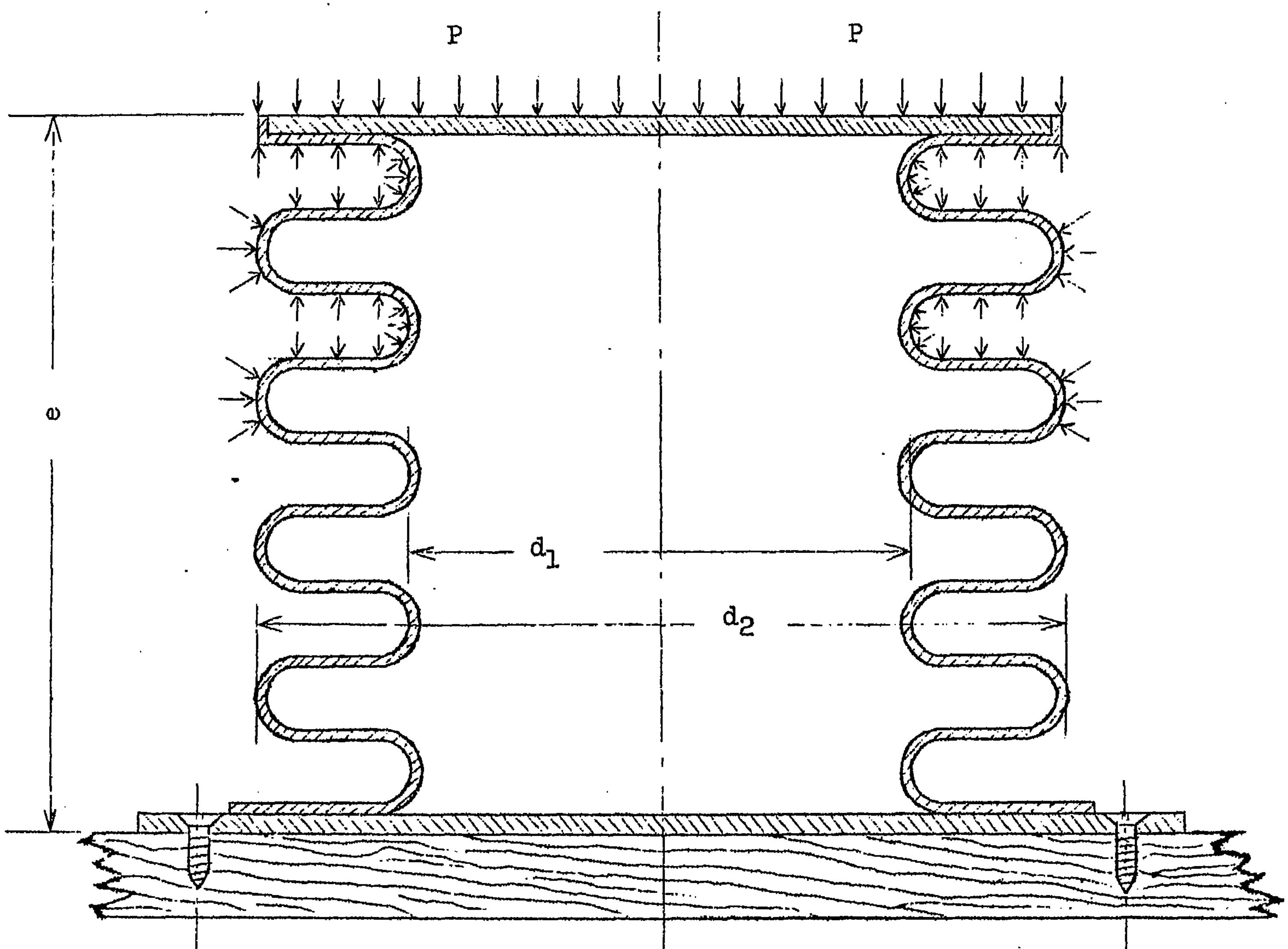

Fig. 1.

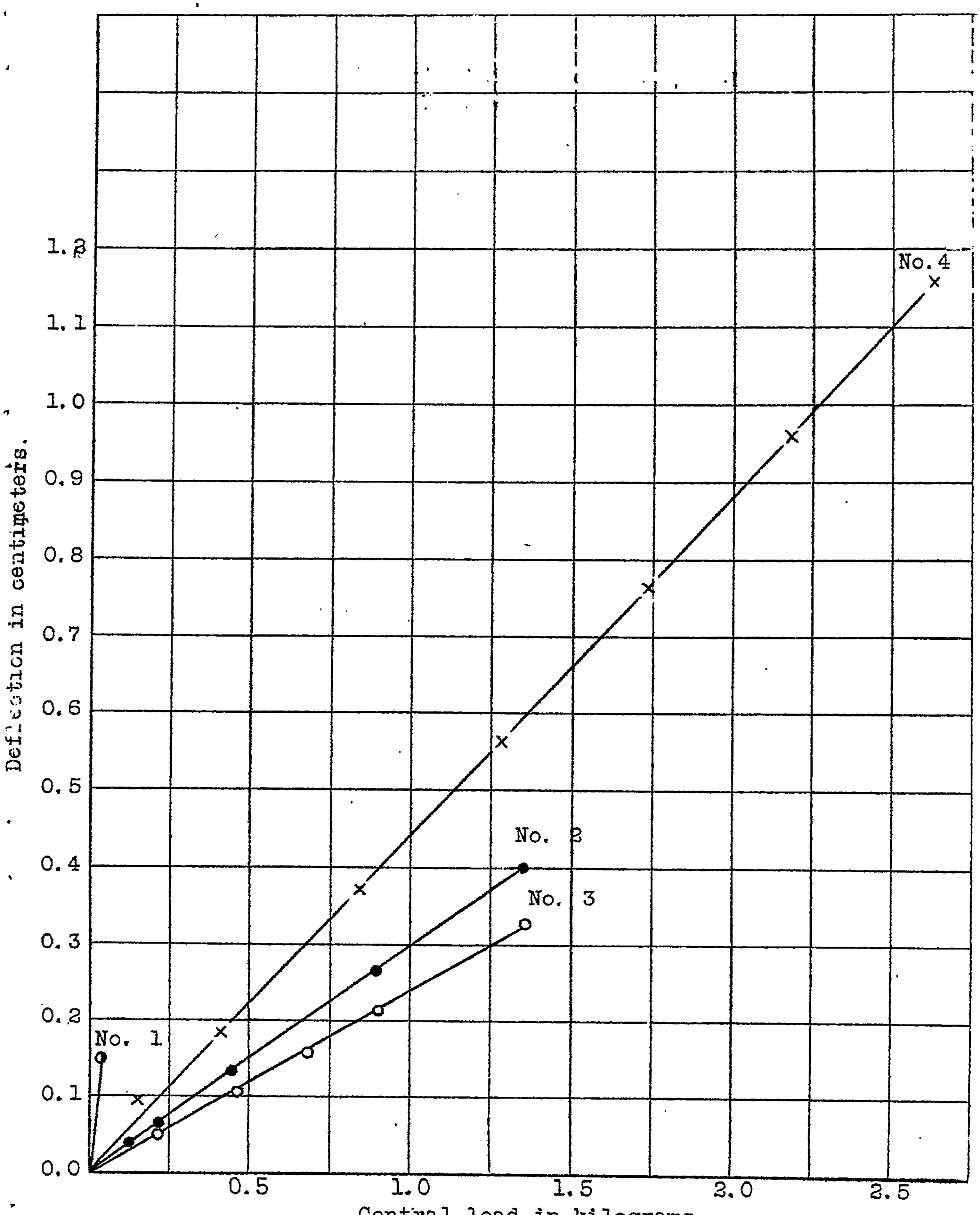

Fig. 2. Characteristic curves for sylphons. Nos. 1 to 4.

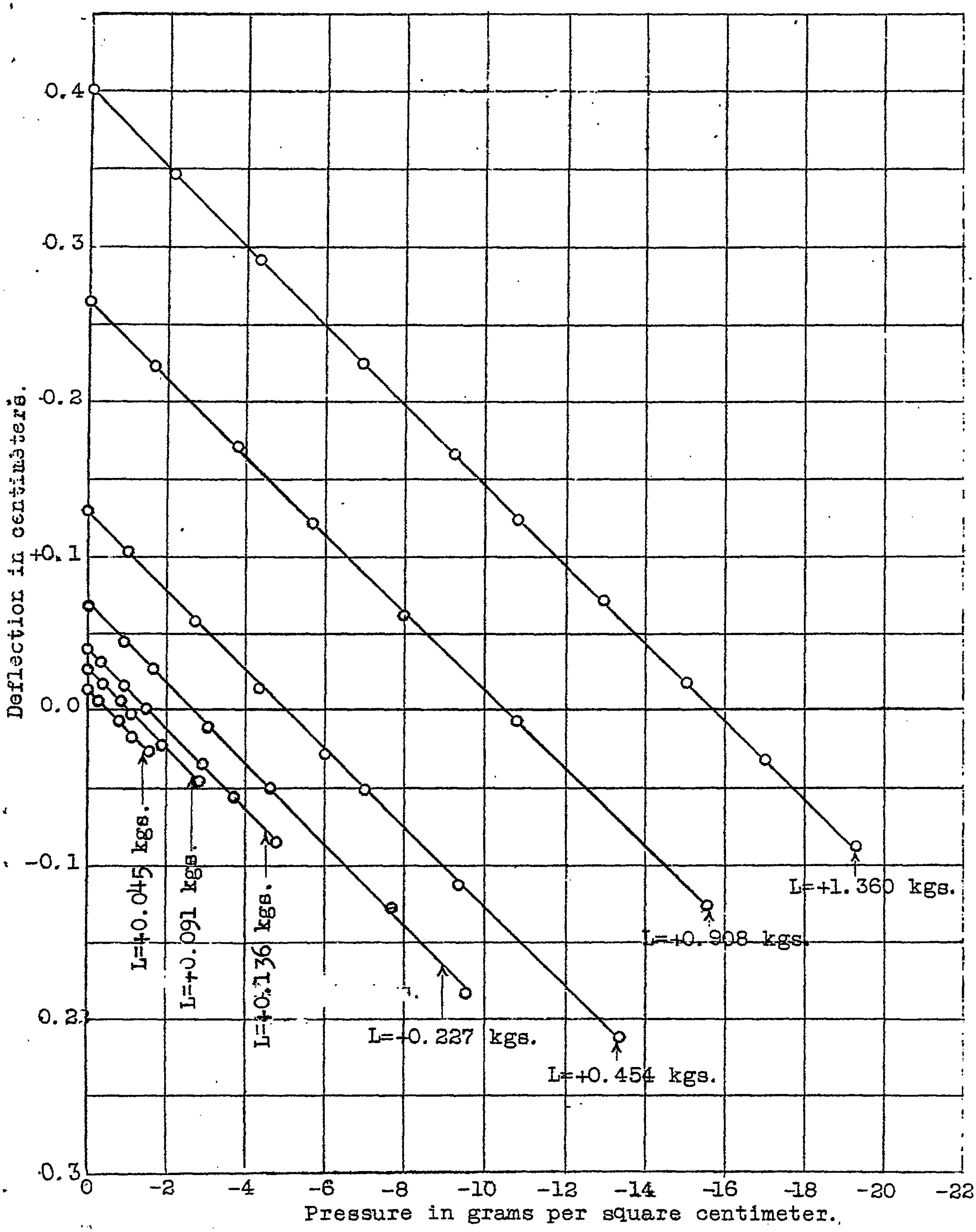

Fig. 3. Deflection of sylphon No. 2 with variable pressure under constant central load.

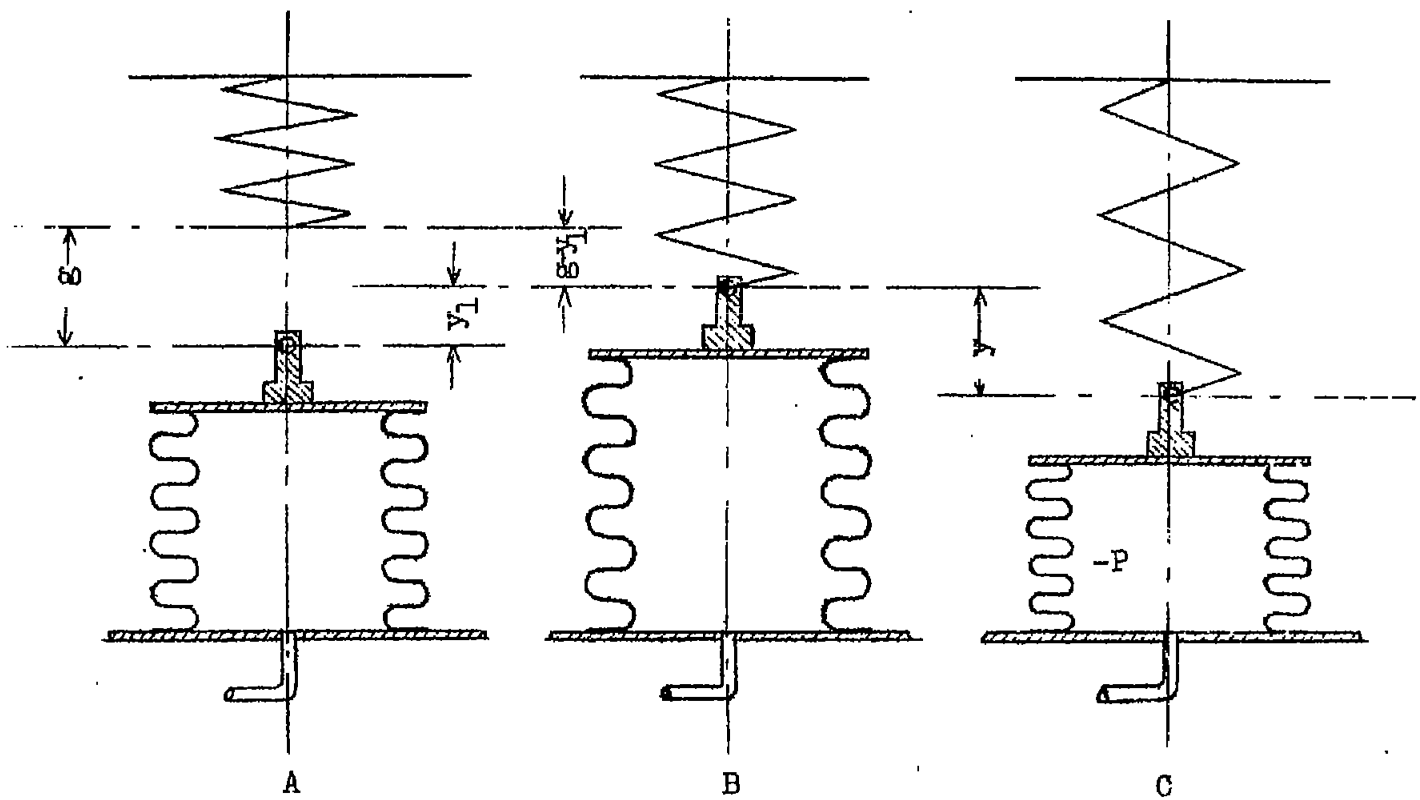

Fig. 4. Sylphon and spring in combination.