TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 91.

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The Energy Losses of the Propeller - I.

By Max M. Munk

April, 1922.

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The Energy Losses of the Propeller - I

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Summary.

The different kinds of energy losses of the propeller and the values of the constants determining them are discussed.

I.

The knowledge of the different kinds of energy losses of the propeller and of the magnitude of the losses in each single case is of great value to the designer. There are three different kinds of energy losses, and the most important has been the least often discussed in the publications of recent years. This is the friction between the air and the blade when whirled through it. Suppose the propeller to be well shaped, so that each blade element is working under a proper angle of attack. Corresponding to the induced drag of an ordinary wing, there are then coming into action the slip stream loss and other similar losses to be discussed afterwards. Besides, there is the friction of the blade surface moved through the air.

The drag coefficient which expresses the relative magnitude of this friction depends, it is true, on the blade section and on its angle of attack or, what amounts to the same thing, on its momentary lift coefficient. But the variability of the drag coefficient for reasonable angles of attack is much smaller than often supposed, the variation for different sections as well as for different angles of attack being small. There is a certain minimum of the drag coefficient existing, which it seems can always be obtained under reasonable conditions by the proper choice of the section, whether the desired lift coefficient be smaller or greater. Hence it is admissible to assume the drag coefficient C_D to be constant for all propellers under those particular conditions for which it is chiefly designed.

The energy loss produced by the drag is the sum of all these losses of each single blade element. Let i be the number of blades, b their breadth at the point considered, v the velocity of the blade element relative to the air, r the distance from axis, dr the length of the blade element, and D the propeller diameter. The entire loss per unit time due to friction is then

$$i \int_0^{D/2} v(v^2 \rho/2)\ b\ C_D\ dr$$

Excepting the velocity v all quantities occurring in this expression are only moderately variable and may be replaced by their mean values for the present purpose. This velocity determines the dynamical pressure $v^2 \rho/2$, and this pressure is the sum of the dynamical pressure of the tangential velocity and of the velocity parallel to the direction of flight, for these two velocities are at right angles to each other and hence the sum of the squares equals the square of the resultant velocity. But the square of the velocity in the direction of flight is so much smaller than the square of the tangential velocity over the greatest part of the propeller blade that it is admissible to neglect it for the following estimation of the magnitude of the drag coefficient and to make a correction for it afterwards. Substitute, therefore, $v = 2\pi r n$ where n denotes the number of revolutions per second. The loss produced by the drag remains then

$$8 \; \rho/2 \; C_D \; b \; \pi^3 n^3 \; i \int_0^{D/2} r^3 \; dr \; .$$

This integral has the value

$$(1) \qquad \frac{1}{8} \; i \; C_D \; \frac{\rho}{2} \; b \; n^3 \; D^4 \pi^3 \; .$$

The thrust calculated in the same way appears

$$(2) \qquad \frac{1}{6} \; i \; C_L \; \frac{\rho}{2} \; b \; n^2 \; D^3 \pi^2 .$$

The lift coefficient C_L can assume very different values as it is not restricted by a lower limit as the drag coefficient is, and its upper limit is rather high. For several reasons, however, the lift coefficient actually used with propellers intended for similar conditions always keeps within narrow limits. A lift coefficient which is too small requires too large a blade area and hence too clumsy a propeller, also C_D/C_L is thus small as a consequence. A very high lift coefficient is not compatible with a small drag coefficient nor with a small ratio of the drag coefficient to the lift coefficient. There is finally the consideration of fairly good efficiency over a wider range of constant revolutions for different conditions of flight. For all these reasons the lift coefficient of propeller blades is far less variable than would appear at first glance, and this holds even more for the ratio

$$\frac{\text{lift coefficient}}{\text{drag coefficient}} .$$

This ratio has a maximum which occurs for moderately high lift coefficients, but the actual lift coefficient will not be very different from the most favorable one, and hence the ratio C_L/C_D can be assumed constant for a rough estimation.

This leads to a convenient approximate formula for the propeller loss due to friction. For the useful work per unit of time is $T V$, and hence the ratio of the loss of friction, as given in equation (1) to the useful work $T V$ where the thrust T is given in equation (2) can be written

$$A \frac{D n \pi}{V} \tag{3}$$

where A is a constant or at least is nearly constant for all good propellers under their best conditions of performance.

The approximation is valid only as long as the ratio of the tip velocity $n D \pi$ to the velocity of flight is great and the efficiency is fairly high. If then the number of revolutions is prescribed, the consideration of the friction alone demands a small diameter. I intend to discuss the question of the best diameter more fully in a later note, but it may be mentioned here that for an unusually small diameter the loss of friction ceases to be the dominant part of the entire loss and the second kind of losses becomes important, calling for a great diameter.

The losses of the second kind are the equivalents of kinetic energy transferred to the air in the form of regularly distributed motion. The chief part is the slip stream loss. It has been discussed so often during the past fifty years that it seems admissible to state the result without repetition of the proof. The ratio of the lost energy to the useful work performed by the propeller is*

$$\frac{1}{2}\left(\sqrt{1 + C_P} - 1\right)$$

where

$$C_P = \frac{T}{V^2 \frac{\rho}{2} D^2 \frac{\pi}{4}}$$

This becomes 1/4 C_P for very small values of C_P but the approximation is not good for greater values of C_P where it gives values that are too great. This expression for the loss is the minimum, occurring for uniform distribution of the thrust over the propeller disc. This condition is not compatible with a finite number of blades, for a blade like any other wing is unable to produce a finite density of lift at its utmost end. Hence

* N.A.C.A. Report No. 114 and Technische Berichte II, p.78.

the finite number of blades involves a small increase of the induced loss.

A third kind of induced losses is a consequence of the rotation around its axis which the slip stream assumes. This kinetic energy however is not entirely lost if the propeller is in front of the fuselage and of the wings. The wings produce a kind of honeycomb effect and straighten out part of the rotation. Besides, the decrease of pressure in front of the fuselage and the radiator diminishes the drag of the airplane.

The additional induced losses can be taken into consideration by introducing an effective diameter D' smaller than the real diameter and using it for the calculation of the energy losses. If, for rough calculation, the approximate formula for the loss, 1/4 C_P, is taken, the result is too great, as said above, and it may be assumed that the additional losses are already contained in it.

The entire efficiency appears now

$$\eta = \frac{1}{1 + A\ D\ n\ \pi/V + \frac{1}{2}\left(\sqrt{1 + C_P} - 1\right)}$$

or approximately

$$1 - \eta \text{ is about } A\ D\ n\ \pi/V + B\ C_P$$

where A and B are constants which do not vary greatly for different propellers under their most favorable conditions. A is expected to be about 3/4 C_D/C_L, B to be in the neighborhood of 1/4 for great velocities of flight.

Another kind of loss is not considerable. This is the loss through the interference of the propeller and the airplane. This loss is not even necessarily positive. The interference in general creates a force between both propeller and airplane, increasing thrust and drag. This alone involves no loss. Furthermore, there is the increase of the drag of the fuselage by the slip stream. This is often added to the original drag of the airplane for matter of convenience, and not considered as a direct loss of the propeller, although it is to be attributed to its existence. The remaining loss of interference is small in general and can probably be neglected.

II.

I have discussed the different kinds of energy losses of the propeller with the intention of determining the most probable value of the respective constants. The data available for this purpose are extremely scarce and unexact, but it seems pertinent and necessary to determine the most probable values and to use them until they can be replaced by more exact ones. The best method for obtaining these at present is the investigation of a stationary propeller of the same dimension and speed as an ordinary propeller, but especially designed for the test. These conditions were fulfilled to a certain extent in the tests of Dr. Schmid.* The scale of his propellers was large enough, but the speed was

* Bendemann: Luftschrauben - Untersuchungen, Muenchen, 1918.

somewhat low. Besides, the sections investigated were comparatively poor, which is not surprising, for the tests were made as early as 1912. With one propeller, the distribution of velocity in front and behind the propeller was determined too.

This test showed that the slip stream loss was exactly 100% of the expected value. The loss due to rotation was confirmed to agree with theory to within 10% error, but here the exactness of the test was far less. This test then justifies the application of the theoretical coefficients for the determination of the induced losses, leaving the determination of the friction losses only to empirical investigation.

The same test of Dr. Schmid gives a lift coefficient $C_L = 0.50$ and a drag coefficient $C_D = 0.023$ calculated by means of the formulas 1 and 2. Thus C_L/C_D appears to be 22. This is in good agreement with the values known from the ordinary wind tunnel tests with wing models, if the induced drag is taken into proper consideration.

The second source of information is free flight tests which lead to the determination of the efficiency of the propeller. These tests consist in gliding tests with stopped engine, thus giving the drag of the airplane. This being known, flights with running engine give the propeller efficiency. With airships, the gliding is replaced by negative accelerated runs with stopped engine. The greatest error of the test comes in owing to the drag of the stopped propeller, which has to be subtracted from the drag obtained from gliding. I have previously published results of both kinds of free flight tests and refer to them, as I am better acquainted with them than with similar tests made by others. The tests with the Brandenburg seaplane gave a maximum efficiency of 71%, so did the tests with the Zeppelin airships. It has to be mentioned that both aircraft had a comparatively low speed and hence their slip stream loss was high. The method of calculation described in the first part of this note gave:

	C_L	C_D
Test of Dr. Schmid	.50	.023
Brandenburg Seaplane	.54	.024
Zeppelin Airships	.54	.025

From these data I conclude that the probable value of the minimum C_D for propeller blades is .024. As explained before, this coefficient refers to too small a dynamical pressure, the dynamical pressure of flight being neglected, and it should be applied when using the formulas 1 and 2 only. Now the mean tangential velocity is about

$$0.7\ \pi\ D\ n = 0.7 \qquad 10 \times 25\pi = 550 \text{ ft/sec.}$$

The velocity of flight is about 140 ft/sec, and the square of this velocity of flight is about 6 1/2% of the square of the mean tangential velocity. The value for C_D appears from this to be 6 1/2 too large. The most probable value of the drag coefficient is then finally

$$C_D = 0.020.$$

The presentation of this important value and of the reasons which lead me to its adoption is the main subject of this note. The value of C_L/C_D would appear to be 22. This value is not changed by the correction of the dynamic speed.

The constants are then expected to be $A = 0.034$ and $B = 0.25$. The maximum efficiency of any propeller is then about

$$1 - 0.034 \frac{D\, n\, \pi}{V} - 0.25 \frac{T}{V^2 \frac{\rho}{2} D^2 \frac{\pi}{4}}$$

For an average value of $C_L = 0.50$ this would give

$$1 - .034 \frac{D\, n\, \pi}{V} - .083 \frac{ib}{D\pi} \left(\frac{D\, n\, \pi}{V}\right)^2$$

An experimental method which also gives information on the air forces of propellers are tests with small propeller models, mostly at low speeds of revolution. These tests of course cannot be used for the determination of the drag coefficient, as this coefficient depends on the Reynolds number. It has been found however that most of these tests give about the same result, so that if others give greatly different values, these tests must be considered as doubtful.