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Technical Report

AUTOMOBILE EXHAUST EMISSION MODAL ANALYSIS MODEL

Paul Kunselman and H.T. McAdams

Calspan Report No. NA-5194-D-3

Calspan Corporation
Buffalo, New York 14221

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ENVIRONMENTAL PROTECTION AGENCY
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ABSTRACT

This report on modal analysis of automobile emissions was prepared for the United States Environmental Protection Agency's Division of Certification and Surveillance, Ann Arbor, Michigan under EPA Contract No. 68-01-0435.

The mathematical model and allied computer programs which are described in this report enable an analyst to calculate the amounts of hydrocarbons, carbon monoxide and oxides of nitrogen emitted by individual vehicles or vehicle groups over any specified driving sequence. The model requires as input the amounts of the three pollutants given off by individual vehicles over short duration driving sequences (modes) in which speed is a monotonic function of time.

The validity of the model is investigated by using it to predict emissions for individual vehicles over the Surveillance Driving Sequence (SDS) and the first 505 seconds of the hot Federal Test Procedure (FTP) driving sequence. The ability of the model to predict actual vehicle emissions is compared with the reproducibility of the actual vehicle emissions measured in replicated tests. Using this analysis, the model performs extremely well. The model has maximum effectiveness as a predictor of group emission characteristics of warmed-up vehicles. It is understood that the mathematical model should be used only within the region of speed and acceleration space which is spanned by the input modal data.

It should be noted that because of the specifics of the study design and the data collection process, the same vehicles were tested over the SDS and FTP. Therefore, the test of model performance may have been more favorable than would have been the case if the data had been obtained from two different vehicle fleets.

TEXT

1. INTRODUCTION

In many geographic regions, the major portion of hydrocarbons (HC), carbon monoxide (CO), and nitrogen oxides (NO_x) present in the environment is due to motor vehicle emissions. The impact of motor vehicles on the environment in a given location is a function of many factors. Among these factors are the emission characteristics of individual vehicles, the mix of vehicles in a particular traffic way, the numerical concentration of vehicles per mile or per unit of area, and the driving pattern in which the vehicles are employed. This driving pattern is influenced by the functional use of the traffic artery (i.e., whether inter or intra city, whether it serves industrial or recreational purposes, etc.) as well as a number of other items. Other items include the design of the highway (e.g., whether it is designed for high or low speed) and the extent to which it passes through and is limited by population density.

It has been well established that emission rates for a particular automobile depend upon the manner in which the vehicle is operated -- that is, emission rates are different for different accelerations or decelerations. In a particular trip taken by an automobile in traveling from Point A to Point B, that automobile will exhibit a particular time profile of acceleration and velocity. The trip may entail a number of starts and stops, as well as a range of speeds determined by traffic conditions, local speed controls, and other factors. As the vehicle travels from A to B, this time profile or "driving sequence", together with the emission characteristics of the vehicle, determines the pollution contribution to the atmosphere. Due to the fact that emissions, expressed in grams per miles, vary along the route, the distribution of the vehicle emissions, as well as the total contribution of pollutants to the atmosphere, can be determined. Indeed, the traffic way can be considered as a line source of pollution, the strength of which depends on vehicle density, vehicle mix, driving sequence, and the emission characteristics of individual automobiles. Finally, the local concentration of pollution along this route is determined by this line source distribution mechanism acting in concert with meteorological transport processes such as wind and diffusion.

To assess the impact of vehicular emissions on a particular section of highway requires, therefore, a number of data inputs. These include characterization of both traffic and emission parameters. Traffic parameters include numerical traffic density, traffic composition (makes and models of vehicles), and traffic flow characteristics (speed, starts and stops, rates of acceleration and deceleration). Emission parameters include emission rates for various categories of vehicles where these rates are expressed as functions of driving variables such as speed and acceleration.

The required traffic parameters can be readily obtained by monitoring traffic along the route in question. The number of vehicles passing various points along the way per unit of time can be counted, and the total number of vehicles can be broken down into homogeneous groups according to make, model, age or other factors influencing emissions. Moreover, speeds and accelerations prevailing along the route can be measured by observing a "tagged" automobile or by instrumenting a vehicle and injecting this vehicle as a "probe" into the traffic stream.

In contrast to the relatively straightforward approach to traffic parameter assessment, the evaluation of applicable vehicle emission functions proves to be quite difficult unless a means can be found for modeling the infinite multiplicity of driving sequences which can arise. It is to be noted, for example, that the EPA Surveillance Driving Sequence is only one of the infinitude of possible sequences. Standard emission tests based on a prescribed driving sequence serve the purpose of comparing vehicles according to a standard set of operating conditions and make it possible to implement emission control standards and to check compliance with these standards. However, they are not structured in such a way as to readily provide the ability to predict vehicle emissions over an arbitrary driving sequence. A generalized prediction capability can be accomplished by breaking the standard sequence, or any other available sequence, into segments having specified speeds and accelerations. Then, these segments can be appropriately recombined to form other driving sequences. In this way, one hopes to be able to approximate any desired driving sequence by appropriately weighting the various segments according to their time duration in the sequence to be modeled. The segments are referred to as operating "modes" and any analysis based on the use of these modes as a "modal analysis".

1.1 MODAL ANALYSIS OF VEHICLE EMISSIONS

In 1971, the Surveillance Driving Sequence (SDS) was developed by EPA to measure vehicle emissions over a variety of steady state and transient driving conditions. The acceleration and deceleration modes represented in the SDS consist of all possible combinations of the following five speeds: 0 mph, 15 mph, 30 mph, 45 mph, 60 mph. The average acceleration or deceleration rate observed for each mode in the Los Angeles basin is used during operation of 20 of the transient modes. In addition, 6 of the transient modes are repeated using accel/decel rates higher or lower than the average rate in order to determine the affect of accel/decel rate on emissions. These acceleration and decelerations were chosen to represent the full range of accelerations and decelerations observed in the CAPE-10 project.*

The concept of modal analysis examined in this report employs as input data the emissions measured for the 37 distinct modes of the SDS. These modes can be characterized by an average speed and an average acceleration. Of the 37 modes, 5 are regarded as "steady state" -- that is, the acceleration is zero. The five modes represent average speeds of 0, 15, 30, 45 and 60 miles per hour. The other 32 modes represent either periods of acceleration or deceleration and are characterized by an average acceleration, which is constant, and an average speed. The importance of the speed constraint can be appreciated by noting that if a vehicle accelerates from 0 to 15 miles per hour in t seconds, its emissions response is not the same as when it accelerates from 15 to 30 miles per hour or 45 to 60 miles per hour in the same time of t seconds.

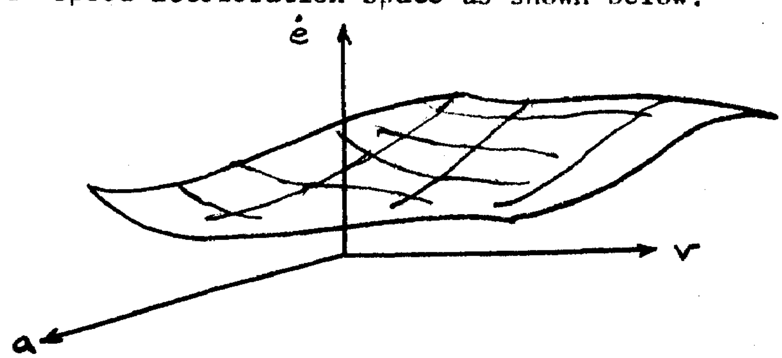
The mathematical model which has been developed to predict vehicle emissions over any specified driving sequence is derived from vehicle data on emissions from the 37 modes of the SDS. One difficulty presented by the use of these discrete modes as inputs to a continuous driving sequence model is that during much of the sequence, the vehicle may be operating at velocities and accelerations not included in the set of five steady state and 32 accel/decel modes. For example, a vehicle traveling at 23 mph is neither in the 15 mph or 30 mph steady state mode. To arrive at a continuous predictive model, one must be able to interpolate or otherwise estimate the appropriate emission rates for

* "Construction of Chassis Dynamometer Test Cycles", Scott Research Laboratories, Inc., November 18, 1971.

all combinations of speed and acceleration encountered in the driving sequence.

The primary contribution of this report is the development of a scheme whereby emissions from the 37 discrete modes can be expanded into a continuous function of time. Any driving sequence can be reduced to a speed time profile. Since acceleration is a function of speed change and time, both speed and acceleration can be expressed as continuous functions of time. The emission rate of a vehicle at a given point in time is dependent upon its speed and acceleration. Using these functional relationships, it is possible to integrate the emission rate function according to the time history of speed and acceleration associated with the driving sequence in question.

An essential feature of the model presented in this report is a regression function which can, for purposes of visualization, be represented as a "surface" in speed-acceleration space as shown below.



Emission Response Surface

For any point (v , a) in the speed-acceleration plane there corresponds an instantaneous emission rate $\dot{e}$ (v,a). The surface can be represented by a mathematical equation of the form: $\dot{e} = f(v,a)$ in which the function f contains a number of adjustable constants. These constants can be selected to represent the emission characteristics of a particular automobile or can be selected to represent the mean emission characteristics of a collection of automobiles. This collection need not be homogeneous with regard to make, model, age or other identifying characteristics of automobiles. However, if a comparison of homogeneous sets of vehicles is desired, a characteristic emission function for each set can be derived. In short, the model presented in this report can be applied to individual vehicles or to composite groups of vehicles selected in whatever way is meaningful to pollution assessment. The flexibility of the model in this connection rests on the fact that the emission rate function is developed as a linear function of adjustable constants which can be particularized to an individual vehicle. Since the pooling of emissions for a composite group of vehicles is itself a linear summing operation, the composite emission function can be derived in a straightforward manner as a weighted linear sum of the emission functions for individual vehicles. Determination of the best process for pooling emissions from a collection of vehicles is beyond the scope of this report, but, to provide perspective for the use of the modal analysis, it is essential that certain aspects of composite emissions modeling be considered since they affect the modal analysis model.

1.2 COMPOSITE ANALYSIS OF VEHICLE EMISSIONS

Computation of the emissions emanating from a particular traffic way in a given period of time is a composite of the emissions produced by all the vehicles which traversed that traffic way during that time. Quite clearly, it is not possible to assess the instantaneous emission rate functions for each automobile. However, if the composition of the vehicle mix is known or can be determined or postulated, one can define what might be called a "pilot mix" or analog of the actual traffic composition. For example, suppose that a fraction p_1 of the vehicles belong to Category 1, a fraction p_2 of the vehicles belong to Category 2, and so on, and that the number of vehicles traversing the traffic way per unit of time is N. An analog of this mix is n vehicles, where $n \ll N$, in which there are $p_1 n$ vehicles of Category 1, $p_2 n$ vehicles of Category 2, etc. in the sample. If the constants for the emission-rate function are determined for each of the n vehicles in the sample, these constants can be averaged over all vehicles comprising the sample to produce an emission rate function which is "typical" for the mix. This composite emission rate function can then be used to compute a typical or average emission for the specified driving sequence, where by "typical" is meant "representative of the vehicle mix in question." By multiplying this average emission by N, the total number of vehicles traversing the traffic way per unit of time, an estimate of the total emission contribution in the time can be obtained. Note that, by virtue of the additive nature of the model, the result will be the same as if separate contributions to the composite were computed for each vehicle in the sample by means of its own specific emission-rate function, and then these individual emission outputs were added and scaled up by multiplying by the factor N/n.

The approach taken above can be referred to as the method of proportional sampling -- that is, the number of vehicles in each category in the sample is proportional to the corresponding number of vehicles in each type in the population. An alternative approach is one in which no attempt is made to produce an analog of the mix in the population but rather an emission-rate function for each category is established independently of all other categories. For example, n_1 vehicles of Category 1 would be subjected to modal analysis and an average emission rate function determined for Category 1 vehicles. Similarly, n_2 vehicles of Category 2 would be analyzed to determine an average emission rate function for Category 2 vehicles, and so on. It is presumed that the number of vehicles tested in each category (that is, n_1, n_2....) would be such that the desired precision is realized; among other things, these numbers would depend on the intra-class variability. Then, given that each category of vehicle has been characterized by an emission-rate function, a composite emission-rate function for any mix of vehicles could be computed as a weighted average of the emission-rate functions for the several categories.

In view of the flexibility of the modal-analysis model, the definition of homogeneous categories of vehicles is not necessary. Indeed, categories of vehicles can be constructed arbitrarily, so long as these arbitrary categories are useful in the particular problem under study and can be weighted appropriately in the composite result. Nevertheless, it was considered of interest to examine available modal data to determine if any significant groupings were evident and whether these groupings might influence the application of the emission model. In this connection it was found, by discriminant-function analysis, that vehicles in Denver exhibit somewhat different emission-rate functions from comparable vehicles in other cities (see Appendix 5). The only way in which this observation affects the use of the model, however, is in the choice of input data. In short, to model traffic ways or to compare alternatives in Denver or in high-altitude locations one must use input modal data appropriate to these locations. In all other respects, the application of the emission model would be unaffected.

The model is particularly valuable if the analyst wishes to examine alternatives, such as alternative routes or highway designs, before an actual highway is built. By postulating the anticipated mix of vehicles and the anticipated driving sequences, the relative desirability of alternatives can be ranked according to their pollution impact.

1.3 REPORT SCOPE AND PREVIEW

In the ensuing sections of this report a methodology will be presented that will enable an analyst to predict the amount of hydrocarbons (HC), carbon monoxide (CO) and oxides of nitrogen (NO_x) given off by individual or specified distributions of light duty vehicles as these vehicles move from point A to point B by some defined speed-time profile.

The methodology will be presented in a somewhat classical modeling approach. First, a description of the problem and the proposed model objectives will be given in terms of the data that are available (the iconic model). Secondly, a mathematical model will be developed that parallels the iconic model in its objectives. This model will be amenable to computer implementation. The model's performance will then be analyzed to see if it is able to meet the objectives set for it.

The proposed methodology is intended to be flexible enough to accomodate changes in emission parameters which are expected to result from improvements in emission control systems. This flexibility will also allow modification or extensions of the model's objectives as the need for such modifications arise.

2.0 PROBLEM DEFINITION

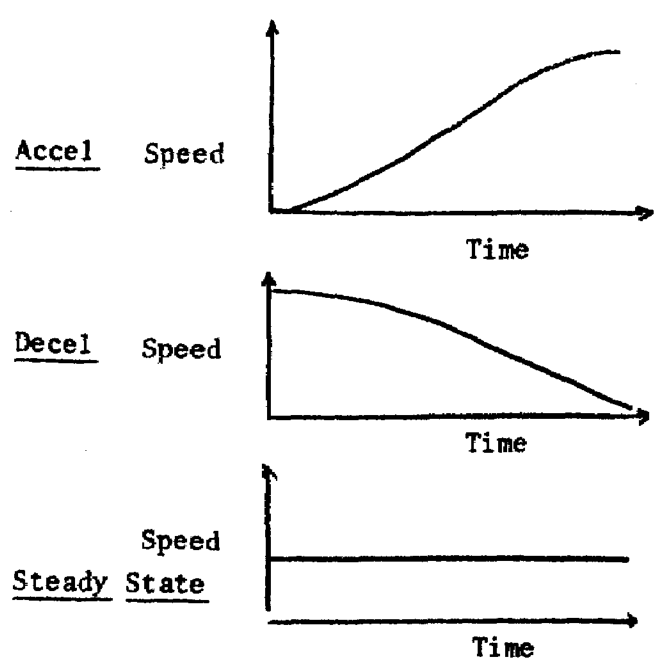
Problem definition can best be understood in terms of the inputs and outputs of the model. Input data consist of vehicle modal emission measurements and speed versus time profiles for specified driving sequences. Output data consist of estimates of emissions for any given driving sequence.

2.1 INPUT

Vehicle emission data are given for 1020 individual light duty vehicles that represent variations in model year, manufacturer, geographic location, engine and drive train equipment, accumulated mileage, state of maintenance and attached pollution abatement devices. The individual vehicle characteristics and emission data were obtained by Automotive Environmental Systems, Inc. under EPA Contract No. 68-04-0042.*

For each of the 1020 vehicles in the data base, the following emission data are given for the three pollutants (HC, CO and NO_x) under consideration:

(A) Modal Emission Data: The amount of each pollutant emitted in each of 37 defined speed-time profiles. There are three cases: speed is monotonically increasing and acceleration is constant and positive over time (accel); speed is monotonically decreasing and acceleration is constant and negative over time (decel); speed is constant over time and acceleration is zero (steady-state), as shown below.



* APTD-1497 "A Study of Emissions from Light Duty Vehicles in Six Cities", March, 1973.

The 37 speed-time curves are referred to as modes. There are 32 accel/decel modes and 5 steady state modes. (See Appendix I for modal specifications.)

(B) Driving Sequence Emission Data

(1) The total amount of each pollutant emitted during the Surveillance Driving Sequence: The Surveillance Driving Sequence represents a speed-time curve of duration 1054 seconds; it is made up of the 32 accel/decel speed-time curves joined together by the 5 steady state modes. The Surveillance Driving Sequence was performed after the vehicle had performed the Federal Test Procedure Driving Sequence. Therefore, emissions measured over the Surveillance Driving Sequence represent emissions from a warmed-up vehicle. (See Appendix II for the speed-time values in the Surveillance Driving Sequence.)

(2) Emissions measured for each vehicle twice using the FTP, once from a cold start and once from a hot start: Emissions were collected from the "transient" and "stabilized" portions of these tests and the data are reported as "cold transient", "cold stabilized", "hot transient" and "hot stabilized" values. Emissions from the Federal Short Cycle were also measured. In the study of model effectiveness, the values used were the amount of each pollutant given off during the FTP "hot transient" driving sequence (see Appendix II for the first 505 seconds of the Federal Test Procedure Driving Sequence). Hot transient data were used since the model has maximum effectiveness as a predictor of emissions for warmed-up vehicles. The use of data from the Federal Short Cycle to measure the effectiveness of the model has been left for future work.

NOTE: The total amount of a pollutant emitted by a vehicle as it executes a driving sequence is often referred to as the "bag value".

2.2 OUTPUT DATA

Given the modal emission data on an individual vehicle, the basic objective of the model is to develop a method which can predict the emission response of this vehicle over any specified driving sequence. The predictive ability of the model is restricted to accelerations and speeds in the sequence which do not exceed the range of accelerations and speeds spanned by the input modal data. In addition, it is desired to extend these individual vehicle responses so that the emission responses of specified homogeneous groups of vehicles can be predicted.

3.0 OVERVIEW OF THE MATHEMATICAL MODEL

In this section a general description of the mathematical model development is given. A more detailed version of the model appears in Appendix III.

3.1 THE EMISSION RATE FUNCTION

The mathematical model used to describe the emission response of a vehicle or group of vehicles is built around the concept of an instantaneous emission rate. (The instantaneous emission rate is defined as the rate at which a pollutant is given off at a specific point in time.)

If the amount of a pollutant emitted by a vehicle from time = 0 to any time = t is denoted by e(t), then the instantaneous emission rate function $\dot{e}(t)$ is defined as the time rate of change of e(t).

$$(1) \qquad \dot{e}(t) = \frac{d[e(t)]}{dt}$$

The instantaneous emission rate at a specified time T is the value of the emission rate function evaluated at this time:

$$(2) \qquad \dot{e}(T) = \left(\frac{de(t)}{dt}\right)_{t = T}$$

In the development of this model, it has been assumed that the instantaneous emission rate of a vehicle is a function of its speed, v and acceleration, a. Since speed and acceleration are considered to be time dependent, the emission rate function can be expressed as

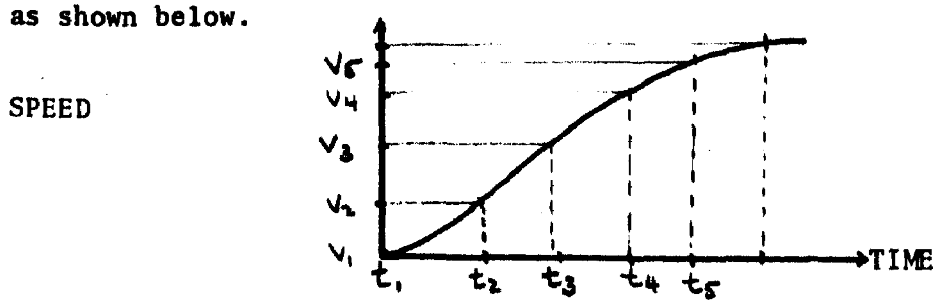
$$(3) \qquad \dot{e}(t) = \dot{e}(v(t),\ a(t)) = \dot{e}\ (v,a)$$

Inherent in the definition of a driving sequence is a speed-time (and therefore acceleration time) profile. The amount of a pollutant given off by the vehicle over a driving sequence lasting T seconds is then given by integrating the emission rate function over the speed-time curve for the driving sequence of interest:

$$e(T) = \int_0^T \dot{e}(v(t), a(t))dt \quad (4)$$

where v(t) and a(t) are the values of speed and acceleration at time = t specified by the driving sequence.

In practice the driving sequences are specified by a series of speed-time points along the speed-time curve that are equidistant in time, as shown below.



where $t_2 - t_1 = t_3 - t_2 = \cdots\cdots\cdots = t_n - t_{n-1} = \Delta t$

The integration in equation (4) is then approximated by the following summation:

$$e(T) = \sum_{i=1}^{N-1} \dot{e}(\hat{v}_i, \hat{a}_i) \; \Delta t \quad (5)$$

where

$$\hat{v}_i = \frac{v_{i+1} + v_i}{2}$$

$$\hat{a}_i = \frac{v_{i+1} - v_i}{\Delta t}$$

$$N \Delta t = T$$

At this point, it is necessary to determine a suitable functional form of the instantaneous emission rate function in terms of speed and acceleration.

3.2 STEADY STATE AND ACCEL/DECEL* EMISSION RATE FUNCTIONS

It is necessary to determine a functional form for the emission rate function for the steady state case and the case of accel/decel. In the steady state case (acceleration equals zero, constant speed) the emission rate function is a function of speed only. This case is presented first.

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* If a(t) = acceleration at time t, then $a(t) < 0 \Rightarrow$ decel. $a(t) > 0 \Rightarrow$ accel.

For each of three pollutants, steady state emission rates averaged over the 1020 vehicles in the data base were plotted against speed. Inspection of these plots (Figures 1, 2 and 3) suggested that the steady state emission rate function $\dot{e}_s$ could be expressed as a quadratic function of speed:

$$\dot{e}_s(v) = s_1 + s_2 v + s_3 v^2 \tag{6}$$

where s_1, s_2 and s_3 are constants.

In the case of non-zero acceleration (accel/decel), the assumption is made that the acceleration occurring at a given speed is a perturbation to the steady state emission rate at this speed. This perturbation can be accounted for by letting the coefficients s_1, s_2 and s_3 become functions of acceleration. If it is assumed that quadratic functions of acceleration represent good approximations to these coefficients, the coefficients can be expressed as follows:

$$(7)\begin{cases} s_1 = s_1(a) = q_{11} + q_{12} a + q_{13} a^2 \\ s_2 = s_2(a) = q_{21} + q_{22} a + q_{23} a^2 \\ s_3 = s_3(a) = q_{31} + q_{32} a + q_{33} a^2 \end{cases}$$

where the q's are constants. The emission rate function used during times of non-zero acceleration $\dot{e}_A$ can then be written in the form:

$$\dot{e}_A(v,a) = b_1 + b_2 v + b_3 a + b_4 av + b_5 v^2 + b_6 a^2 + b_7 v^2 a + b_8 a^2 v + b_9 a^2 v^2 \tag{8}$$

where the b's are constants and can be expressed in terms of the q's. It is noted that if a = 0 equation (8) reduces to:

$$\dot{e}_A(v, a = 0) = b_1 + b_2 v + b_5 v^2 \tag{9}$$

which has the identical form as the equation for $\dot{e}_s$. Thus, $\dot{e}_A$ could be used to determine emissions for both steady state and non-zero acceleration periods. At this point in the discussion, however, separate functions for steady state and accel/decel emission rates will be retained; the reason for doing so will be given later in this report.

The instantaneous emission rate function $\dot{e}$ for a given vehicle and pollutant is a composite function given by:

$$\text{(10)} \qquad \dot{e}(v,a) = h(a)\dot{e}_S(v) + (1 - h(a))\,\dot{e}_A(v,a)$$

where h(a) is a weighting function which is bounded by the values 0 and 1 and which is dependent on acceleration. Note that h(a) allows for a smooth, continuous transition from steady state to accel/decel emission rate functions or vice versa.

The next step is to evaluate the twelve coefficients (b_i, i = 1,9; s_i, i = 1,3) for each vehicle and pollutant. These coefficients will completely specify the instantaneous emission rate function describing this vehicle's response with respect to the given pollutant.

3.3 DETERMINATION OF THE COEFFICIENTS (b_i, s_i)

The coefficients that specify the instantaneous emission rate function could be determined by a straightforward application of the least squares regression method if values of the instantaneous emission rates were available. However, the data base on vehicle emissions does not contain any instantaneous emission rate observations for accel/decel modes; instead, the observations reported are the total amounts of the pollutants collected over each mode or the average emission rate for the mode (which covers many speeds). In this light, the following method allows the determination of the coefficients that specify the accel/decel instantaneous emission rate function.

(A) Specification of the Accel/Decel Emission Rate Function

It can be shown that if the proposed form of the instantaneous accel/decel emission rate function is used to evaluate the functional form of the average emission rate function, the same coefficients that specify the emission rate function also appear in the average emission rate function in a linear fashion (see Appendix III). Now, the values for the average emission rate can be determined for each mode by dividing the amount of pollutant given off in the mode by the time in mode. A standard least squares regression analysis can then be performed on the average emission rate function which will determine the values of the coefficients that specify the instantaneous emission rate function. For example, suppose the instantaneous emission rate function is given as:

$$\dot{e}(v,a) = b_1 + b_2 v + b_3 va \quad ^*$$

Then the average emission rate function $\langle \dot{e}(v,a) \rangle_T$ over T seconds is defined as:

$$\langle \dot{e}(v,a) \rangle_T \equiv \frac{1}{T} \int_0^T \dot{e}(v,a)\, dt = \frac{1}{T} e(T)$$

Substituting the functional form of the instantaneous emission rate function into the integral gives:

$$\langle \dot{e}(v,a) \rangle_T = \frac{1}{T} \int_0^T (b_1 + b_2 v + b_3 va)\, dt = e(T)/T$$

$$\langle \dot{e}(v,a) \rangle_T = \frac{1}{T} \int_0^T b_1\, dt + \frac{1}{T} \int_0^T b_2 v\, dt + \frac{1}{T} \int_0^T b_3 va\, dt$$

Let $\bar{v} = \frac{1}{T} \int_0^T v\, dt$, $\overline{av} = \frac{1}{T} \int_0^T av\, dt$, and since $\frac{1}{T} \int_0^T dt = 1$, have

$$e(T)/T = b_1 + b_2 \bar{v} + b_3 \overline{av}$$

The total emission e(T) given off in each mode and the time in each mode T are known. $\bar{v}$, and $\overline{av}$ can be determined for each mode. The coefficients (b_i) can therefore be obtained through least squares regression analysis applied to the average emission rate function.

For the general model, there are 9 coefficients to determine and 32 accel/decel modes. A least squares regression analysis can be performed on an individual vehicle or on the mean of a group of vehicles. This approach forms a logical bridge from the experimental observations to the specification of the accel/decel emission rate function.

(B) Specification of the Steady State Emission Rate Function

In the case of steady state conditions, the speed does not change with time. Thus, the average emission rate is equal to the instantaneous emission rate. Values of the steady state emission rate function are then available from the experimental observations, and the coefficients are evaluated directly using least squares regression techniques.

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* This expression in only an example, deliberately simplified for illustrative purposes.

There is, however, one problem that crops up with the above straightforward least squares approach. The values of the emission rate vary greatly between speed zero (idle) and a speed of 60mph. As a result, the least squares approach sometimes produces a steady state emission rate function which predicts negative emission rates for certain speeds. In this event, the function is adjusted by means of a constraint on its minimum value. The two lowest emission rates measured experimentally are determined and averaged. Similarly, the speeds corresponding to these two rates are also averaged. The average rate and average speed determined in this way are then taken as the coordinates of the minimum point of the emission rate function. Two of the three coefficients that specify the steady-state emission rate function are thus determined; the third is computed by the least squares method subject to this constraint (see Appendix II for details).

3.4 THE COMPOSITE EMISSION RATE FUNCTION

As stated earlier, two separate emission rate functions were desired in order to describe accel/decel and steady state conditions. The two functions are then joined by means of the weighting function as defined by equation (10). The reason for retaining a separate function for steady state conditions when the accel/decel function appears flexible enough to handle the steady state case is that the accel/decel rate function also produces negative emission rates in some cases for steady state speeds; any efforts to modify the coefficients to constrain the function to yield only positive steady state emission rates would produce serious errors when the function is used to evaluate accel/decel emission rates. The composite emission rate function allows the freedom to adjust the coefficients of the steady state emission rate function without disturbing the accuracy of the accel-decel emission rate function.

3.5 VEHICLE AND VEHICLE GROUP CHARACTERIZATION

Once the emission rate function for a vehicle and pollutant is specified, it can be used to obtain this vehicle's response, over any given driving sequence, by integrating the rate function over the speed-time curve defined by the driving sequence. Each vehicle is characterized by 36 parameters or coefficients; 12 parameters for the specification of each emission rate function describing the HC, CO, and NO_x response.

The characterization of a group of vehicles can be achieved by defining the emission rate function for the average vehicle within the group:

Let b_{ijk} = k'th coefficient in the emission rate function for the j'th vehicle within the group and i'th kind of pollutant.

N_g = number vehicles in the group.

$\bar{b}_{ik}$ = k'th coefficient in the emission rate function describing the average vehicle's i'th kind of pollutant response.

Then,

$$\bar{b}_{ik} = \frac{1}{N_g} \sum_{j=1}^{N_g} b_{ijk} \qquad (11)$$

Thus, the group emission rate functions are determined by averaging the coefficients which make up the emission rate functions of each vehicle in the group. The emission response of the group over any driving sequence is then determined by multiplying the average vehicle's response by the number of vehicles in the group. The average vehicle's response is obtained by integrating its rate function over the speed-time curve specified by the driving sequence.

4.0 COMPUTER IMPLEMENTATION

The computer version of the mathematical model to calculate emissions given off from individual vehicles and vehicle groups over any specified driving sequence is made up of two main programs:

Main Program I

A main program to compute emissions from individual vehicles over any specified driving sequence.

Main Program II

A main program to compute emissions from a specified group of vehicles over any specified driving sequence.

The main programs are used to read in the speed-time values of the driving sequences, to perform any filtering operations needed to define vehicle groups, to write out calculated emission values, and to call in proper sequence the following set of routines which perform the majority of calculations:

Subroutine SETUP

Output: Subroutine SETUP determines the basis function factor arrays for the accel/decel and steady state emission rate functions. These arrays are labeled AA and AS, respectively.

Input: Speed-time values for the Surveillance Driving Sequence.

Utilization: Called once in each main program.

Other Subroutines Used: Subroutine INVERS (to calculate the inverse of a matrix).

Subroutine EDOT

Output: Subroutine EDOT calculates the 36 coefficients specifying the three emission rate functions for an individual vehicle.

Input: (i) The amount of each pollutant given off by an individual vehicle in each of 37 modes.

(ii) The basis function factor arrays AA and AS determined by subroutine SETUP.

Utilization: EDOT is called once for each individual vehicle considered in main programs I and II.

Other Subroutines Used: Subroutine PAD.

Subroutine PAD

Output: Subroutine PAD calculates a set of 3 coefficients that specify the steady state emission rate function for an individual vehicle such that the emission rate function does not produce any negative emission rates.

Input: The amount of pollutant given off by an individual vehicle in the five steady state modes.

Utilization Called once from subroutine EDOT for each individual vehicle that originally had a steady state emission rate function that produced negative emission rates.

Other Subroutines Used: None.

Subroutine EDGRP

Output: A set of 36 coefficients that specify the emission rate functions for the average vehicle within a group of vehicles.

Input: (i) The 36 coefficients specifying the emission rate functions of individual vehicles determined by subroutine EDOT.

(ii) Sequence indicator INT.

Utilization: Called by Main Program II only; once for each vehicle in the group (INT = 1 for first vehicle in the group, INT = 2 for all the following vehicles in the group), and once after all vehicles in the group have been considered (INT = 3).

Other Subroutines Used: None.

Subroutine ESUM

Output: The amounts of the three pollutants (HC, CO, NO_x) given off by an individual vehicle or a group average vehicle over any specified driving sequence.

Input: (i) The 36 coefficients that specify the emission rate functions determined by subroutine EDOT (Program I) or subroutine EDGRP (Program II).

(ii) The velocity-time values for the driving sequence under consideration.

Utilization: Called once for each vehicle in Main Program I, called once for the group under consideration in Main Program II.

Other Subroutines Used: None.

Subroutine INVERS

Output: The inverse of any two-dimensional square matrix of dimension less than 20.

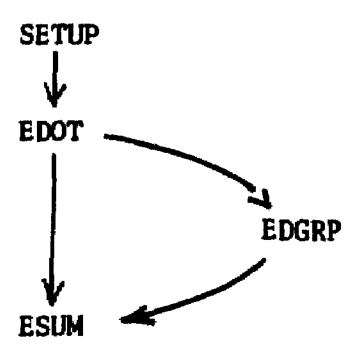
Input: (i) The matrix whose inverse is desired.

(ii) Dimension of the matrix.

Utilization: Called once by subroutine SETUP.

Other Subroutines Used: None.

Flow charts for the main programs (see Figures 4 and 5) show the calling order for the four main subroutines: SETUP, EDOT, EDGRP and ESUM. In any main program the following calling sequence of the main subroutines must be strictly observed:



Listings of the main programs and subroutines are given in Appendix IV.

5.0 MODEL PERFORMANCE

Evaluation of the performance of the model can be approached only by comparing computed and measured quantities. For this purpose, it was found convenient to use the Surveillance Driving Sequence and the first 505 seconds of the Hot Federal Test Procedure driving cycles as measurable quantities and to compare these quantities with the corresponding outputs predicted by the model. Also, it must be appreciated that the degree of agreement between computed and observed results will vary from vehicle-to-vehicle and that ultimate evaluation of the validity of the model must take into account this statistical variability. Toward this end, several statistical quantities were employed, as discussed below.

5.1 STATISTICAL INDICATORS OF PERFORMANCE

<u>Notation</u>

O_{ij} = observed amount of i^{th} kind of pollutant given off by j^{th} vehicle over a specified driving sequence (observed bag value)

C_{ij} = calculated amount of i^{th} kind of pollutant given off by j^{th} vehicle over a specified driving sequence (calculated bag value)

N_c = number of vehicles in sample (1020)

R_{ij} = bag value error = $O_{ij} - C_{ij}$

To analyze the performance of the emission rate model in predicting bag values, the following statistics are evaluated:

(a) <u>The Mean Bag Error or Bias</u> ($\overline{R}_i$) for each type of pollutant:

$$\overline{R}_i = 1/N_c \sum_{j=1}^{N_c} (O_{ij} - C_{ij})$$

or

$$\overline{R}_i = 1/N_c \sum_{j=1}^{N_c} R_{ij}$$

(b) The Standard Deviation of the Bag Error (σ_{R_i}) for each pollutant:

$$\sigma_{R_i} = \sqrt{\left(\frac{1}{N_c - 1}\right) \sum_{j=1}^{N_c} (R_{ij} - \bar{R}_i)^2}$$

(c) Root Mean Square Deviation of the Bag Error (RMS_i) for each pollutant:

$$RMS_i = \sqrt{\bar{R}_i^2 + \sigma_{R_i}^2}$$

(d), (e), (f) The Mean, Standard Deviation and Root Mean Square Deviation of the Bag Error expressed in terms of percent of the observed mean bag value for each pollutant ($\bar{O}_i$).

$$\bar{O}_i = \sum_{j=1}^{N_c} O_{ij} / N_c$$

$$(\bar{R}_i / \bar{O}_i) \cdot 100\,\%$$

$$(\sigma_{R_i} / \bar{O}_i) \cdot 100\,\%$$

$$\left(\sqrt{\bar{R}_i^2 + \sigma_{R_i}^2} \,/\, \bar{O}_i\right) \cdot 100\,\%$$

The mean, standard deviation, and root mean square deviation of the bag error together provide insight into how the bag errors are distributed. Expressing these statistics in terms of percent of the observed mean gives an indication of how serious the bag error distribution is.

5.2 PERFORMANCE RESULTS

The values of the statistics for bag values obtained in the Surveillance Driving Sequence and first 505 sec. of the Hot Federal Test Procedure's driving sequence (hot transient) are given in Tables 1 and 2.

A visual inspection of the distribution of bag value errors is offered by Tables 3 through 8 and corresponding histograms on Figures 6 through 11.

5.3 DISCUSSION AND EVALUATION

To focus attention on the adequacy of the model, it is helpful to condense Table 1 and Table 2 to a somewhat more concise form, as shown below.

PERCENT RMS ERROR
BETWEEN CALCULATED AND OBSERVED BAG VALUES
FOR 1020 VEHICLES

	Surveillance Driving Sequence	First 505 Seconds Federal Test Procedure
HC	26.1	32.0
CO	23.9	29.1
NO_x	27.1	28.0

The percent RMS error is defined as: $\left\{ \sqrt{\sigma_R^2 + \bar{R}^2} / \bar{O} \right\} 100\%$

and represents the combined systematic and random errors. It is a particularly meaningful quantity if one assumes that the mean or expected difference between the calculated and observed values should be zero. As will be noted in Tables 1 and 2, the RMS values are largely dominated by the random error component, as represented by σR^2. Moreover, these tables, together with the histograms showing the error distributions, suggest that the difference between computed and observed results cluster rather closely around the average error $\bar{R}$ and that this average value deviates from zero by only a few percent of the average measured bag values.

A logical question arises, however, as to the interpretation which should be put on such terms as "cluster rather closely around the average" or on such quantitative measures of performance as "25% RMS error". Against what criterion are these measures of performance to be judged and is the model to be judged satisfactory or unsatisfactory?

To answer this question, one must consider the quality of the input data and the manner in which errors in the input propagate into errors in the output. In particular, one must inquire as to the repeatability of emission measurements performed on the same vehicle and ostensibly under identical test conditions. If the results of the Surveillance Driving Sequence or any other specified driving sequence fail to repeat on replicate tests, this failure can not be traced to the inadequacy of a computational model, because no such model is involved. The accruement of instantaneous emissions over the driving sequence is a physical, not a mathematical, process of integration, and the vehicle and the measuring instrumentation constitute the only "computer" in the system.

Of the 1020 vehicles in the input data set, 61 had been tested twice each. Thus there were available 61 "replicate" measurements from which can be obtained a measure of repeatability of measurements.

This measure of repeatability can be easily obtained as follows. Consider a particular vehicle, and compute the mean $\bar{X}_k$ of the two replicate measurements. Then compute the quantity

$$\hat{\sigma}_k^2 = \frac{(X_{1k} - \bar{X}_k)^2 + (X_{2k} - \bar{X}_k)^2}{N-1}$$

where X_{1k} and X_{2k} are the two replicate measurements for the k^{th} vehicle. Since $N = 2$ in this case, the formula for $\hat{\sigma}_k^2$ reduces to the simple form

$$\hat{\sigma}_k^2 = 1/2 \cdot (X_{1k} - X_{2k})^2$$

Now let us assume that the quantity $\hat{\sigma}_k^2$ is one estimate of the variance of replicate determinations and that each of the other 60 pairs of values provide an additional estimate. These 61 estimates can be pooled or averaged to obtain:

$$\hat{\sigma}^2 = 1/61 \sum_{k=1}^{61} \hat{\sigma}_k^2$$

as a best estimate of the variance of replicate values. Similarly, the quantities $\overline{X}_k$ (k = 1, 2,........, 61) can be pooled to obtain an estimate of the mean $\overline{X}$ for the total collection of vehicles, and the quantity $\hat{\sigma}_k/\overline{X}_k$ can be taken as a relative or percent standard deviation characterizing the repeatability of measurements.

The values $\hat{\sigma}_k/\overline{X}_k$ are shown below for the Surveillance Driving Sequence and for the first 505 seconds of the Federal Test Procedure.

PERCENT STANDARD DEVIATION BETWEEN
REPLICATE BAG VALUES FOR 61 VEHICLES

	Surveillance Driving Sequence	First 505 Seconds Federal Test Procedure
HC	68.6	70.6
CO	14.4	26.9
NO_x	15.5	15.8

Comparison of these values with those obtained by comparing calculated and measured results suggests that the errors are comparable in the two cases. Consequently, it is concluded that the model is performing quite acceptably and that, indeed, its performance is substantially limited by the variability inherent in the test measurements themselves.

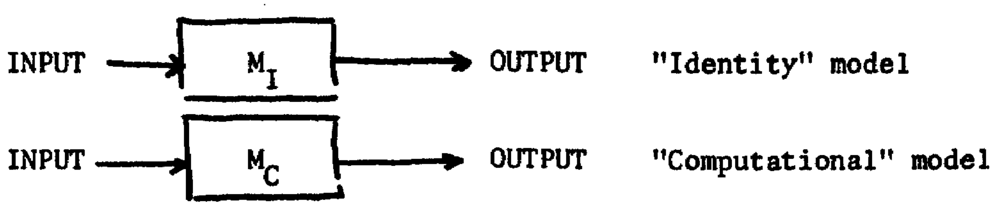
Further support for this point of view is found in Tables 9, 10 and 11. Based on the 61 replicates, the quantities $\overline{X}$, $\hat{\sigma}$ and $(\hat{\sigma}/\overline{X})100\%$ are presented for each of the 37 modes as well as the Surveillance Driving Sequence and the FTP. The percent standard deviations for individual modes range from 30% to nearly 100% for HC, from about 20% to 85% for CO, and from about 20% to nearly 138% of NO_x. These errors are reflected as errors in the determination of the regression coefficients, and these errors in turn determine the error of estimating the instantaneous emission rate at any point in the (a,v) - space. Procedures are available to trace the error propagation through this rather involved process and to produce "variance maps" in (a,v) - space,* but this type of analysis is beyond the scope of this report. Moreover, even if such a variance surface were available, one must further translate this surface into its effect on the integrated emissions for a particular driving cycle. In view of the relatively large errors in modal input data, however, the 25% to 35% RMS errors obtained for model performance do not appear unreasonable.

Further insight into this matter can be had by an elementary and straightforward application of analysis of variance as follows. Denote by X_{ijk} (i = 1, 2; j = 1, 2; k = 1, 2,......, 61) the j^{th} replicate of the k^{th} vehicle, where i = 1 denotes measured values and i = 2 denotes values computed from the model. For each vehicle, therefore, there are four values of total emission for each of the pollutants HC, CO and NO_x. These are:

X_{11k} = Bag value measured for first replicate
X_{12k} = Bag value measured for second replicate
X_{21k} = Bag value computed for first replicate
X_{22k} = Bag value computed for second replicate

These four values can each be decomposed into components representing the effects of the model, the effects of replication, and the interaction between replications and models.

Let us visualize the effect of the model as shown in the sketch below



In the "identity model", the measured emissions are subjected to no computation, the bag values being those obtained directly from the measurement process itself. In the "computational model", the modal measurements are used as the basis for generation of an emission-rate surface, and bag values are computed by integra-

* H. T. McAdams, "A Computer Method for Hypsometric Analysis of Abrasive Surfaces," Advances in Machine Tool Design and Research, 1968, Pergamon Press, Oxford, 1969, pp. 1149-1171.

tion over the appropriate driving sequence. Thus if the data from replicate tests are fed to these two "models" as inputs, the outputs will differ because of the difference in the "transfer functions," M_I and M_C, of the two models. The difference between the outputs of the models can be called the "model effect" and is a measure of the extent to which the computationally integrated results fail to agree with the physically integrated results. In previous discussion we have referred to this difference as a measure of the "validity" of the computational model. In the present analysis, however, we wish to examine this difference in relation to the repeatability of the input measurements.

An appropriate statistical model for the analysis is

$$X_{ijk} = \mu_k + \alpha_{ik} + \beta_{jk} + (\alpha\beta)_{ijk}$$

$i = 1, 2;\ j = 1,2;\ k = 1,2,\ldots, 61.$

where X_{ijk} is the output of the i^{th} model for the j^{th} replicate on the k^{th} vehicle. The convention, of course, is that $i = 1$ denotes the identity or physical model and $i = 2$ denotes the computational model. The quantity μ_k is the mean of the four output values for each vehicle and can be thought of as the common or reference value against which model and replication effects can be compared. The quantity α_{ik} is a departure from this mean occasioned by the effect of the particular model; since $\alpha_{1k} + \alpha_{2k} = 0$, one of the models will be represented as a negative departure, the other as a positive departure from the mean. The quantity β_{jk} is a departure from the mean occasioned by replication. As far as β_{jk} is concerned, it is assumed that the difference between replicates, in the statistical sense, is the same for the identity model and for that computational model; hence, β_{jk} represents a pooled estimate incorporating both expressions of the replication effect. Since $\beta_{1k} - \beta_{2k} = 0$, one of the replicates will be represented as a negative departure from the mean, the other as a positive departure from the mean.

In reality, there is a distinct possibility that replication will be influenced by the model--that is, it might be anticipated that replicate results emerging from the computational model might be different from replicate results emerging from the identity model. If such is the case, then there is <u>inter-</u>

action between replication and models. This interaction is measured by the term $(\alpha\beta)_{ijk}$, which represents a "correction", in a sense, to the assumption that repeatability of the output does not depend on whether one is considering the identity or the computational model.

Decomposition of X_{ijk} into components is easily accomplished by the matrix transformation

$$1/4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} X_{11k} \\ X_{12k} \\ X_{21k} \\ X_{22k} \end{bmatrix} = \begin{bmatrix} \mu_k \\ \alpha_{ik} \\ \beta_{jk} \\ (\alpha\beta)_{ijk} \end{bmatrix}$$

Moreover, α_{ik}^2, β_{jk}^2 and $(\alpha\beta)^2_{ijk}$ represent, respectively, the mean squares for models, replication and interaction in a two-way analysis of variance. Each of these mean squares has one degree of freedom.

Our analysis is completed by computing these mean squares for each of the 61 vehicles and averaging these values. These results are presented in Table 12 under the heading "mean squares". Viewed directly, however, these mean squares are somewhat misleading, because the statistical expectation of the mean squares for--say, the models effect--is not the variance σ_α^2 associated with models but rather:

$$2\sigma_\alpha^2 + \sigma_{\alpha\beta}^2$$

where $\sigma_{\alpha\beta}^2$ is the interaction variance. Similarly, the expected mean squares for replications is not the variance σ_β^2 associated with replications but rather

$$2\sigma_\beta^2 + \sigma_{\alpha\beta}^2$$

The expected value of the mean squares for interaction is $\sigma_{\alpha\beta}^2$, however. By solving the system of equations:

$$2\sigma_\alpha^2 + \sigma_{\alpha\beta}^2 = \text{models mean squares}$$

$$2\sigma_{\beta}^{2} + \sigma_{\alpha\beta}^{2} = \text{replicates mean squares}$$

$$\sigma_{\alpha\beta}^{2} = \text{interaction mean squares}$$

one can extract the variance components σ_{α}^{2}, σ_{β}^{2} and $\sigma_{\alpha\beta}^{2}$. These are displayed in Table 12 under the heading "Variance Components".

Though the analysis for HC, CO, and NO_x as well as the analysis for the two driving sequences give different results, the general impression is that the models and replications effects are of comparable magnitude (note, in particular, the results for CO and NO_x for the Surveillance Driving Sequence). In the case of HC, it appears that the replications effect is much larger than the models effect for both the first 505 seconds of the Federal Test Procedure and for the Surveillance Driving Sequence. However, examination of the data for individual vehicles revealed that there was one vehicle for which the bag values replicated so poorly that the case might be considered an outlier. This single vehicle is largely responsible for the large replications mean square for HC.

Special attention must be given to the interaction components. Though there are several examples in which this component is of appreciable magnitude, it can not be concluded that the computational model has poorer repeatability than the identity model, because all of the variance components denote magnitude only, not direction. Indeed, examination of the data for individual vehicles reveals that the interaction effect is about as likely to be negative as positive. Often the interaction is occasioned by the fact that the ranking of the two replicates is reversed when one goes from the identity model to the computational model. For example, in the identity model the first replication might yield a higher bag value than the second, but in the computational model the reverse might be true, yet the magnitude of the difference between the two replicates might be the same in both cases.

In conclusion, the computational model performs remarkably well in view of the relatively large errors in the modal emission rates which serve as inputs. Since the model reproduces the measured bag values about as well as a replicate does, it is postulated that the model can predict emissions form a non-standard driving sequence about as well as might be expected from an actual test performed on that driving sequence. Further experience with the model is needed, however, to be more assertive on this point. However, the model has maximum effectiveness as a predictor of vehicle group emission characteristics, since the input variability of a homogeneous group of vehicles is in general less than the input variability of any individual vehicle.

6.0 SUMMARY AND CONCLUSIONS

In the preceeding discussion, a method was presented to calculate the amounts of HC, CO and NO_x emitted by individual vehicles and vehicle groups over any specified driving sequence. The method uses, as inputs, the modal emission data on individual warmed-up vehicles. It is to be understood, of course, that the mathematical model should be used only within the region of the speed and acceleration space which is spanned by the input modal data. The model has maximum effectiveness as a predictor of group emissions for warmed-up vehicles.

The method, given in terms of a vehicle emissions model, is characterized by the concept of an instantaneous emission rate. From this concept, the emissions response of individual vehicles and vehicle groups are given in terms of instantaneous emission rate functions. The development of the instantaneous emission rate functions for a vehicle contains two important computational features: the assessment of the coefficients that specify the instantaneous emission rate function and a method to bound the steady state emission rate function so that the function is non-negative (does not produce negative emission rates) on the speed interval (0,60) mph.

In a least squares fitting procedure, it is possible to obtain negative predicted values for some points on the speed and acceleration/deceleration surface. Such a possibility is most likely in extrapolated areas of the surface or areas with very few actual data points. Due to the complexity of the prediction procedure over the range of accel/decel space, the current model does not check for negative emissions for each possible point in the prediction surface of each vehicle. This type of problem did not occur for the set of vehicles considered when appropriate weighting functions were used. However, a test for negative emissions over accel/decel space and an appropriate mathematical correction is planned as a future refinement to the model.

The instantaneous emission rate function can be used to characterize an individual vehicle's emission response over any driving sequence as well as to describe a vehicle group's emission response. The latter is accomplished by the determination of the group's average vehicle emission rate function. Further, a means of investigating the homogeneity of hypothesized vehicle groups using linear discriminant function analysis is presented. (Appendix V)

The content of the discussion and structure of the model allows for the immediate use of the emissions model by an analyst to predict vehicle emission and serves as a base for further research in predicting vehicle emissions and their effect on the environment.

TABLES

TABLE 1: BAG VALUE STATISTICS FOR THE SURVEILLANCE DRIVING SEQUENCE

POLLUTANT	$\bar{O}$	$\bar{R}$	$\sigma_{\bar{R}}^2$	$\sigma_{\bar{R}}$	$\sqrt{\bar{R}^2 + \sigma_{\bar{R}}^2}$	$\frac{\bar{R}}{\bar{O}} .100\%$	$\frac{\sigma_R}{\bar{O}} .100\%$	$\frac{\sqrt{\bar{R}^2 + \sigma_{\bar{R}}^2}}{\bar{O}} .100\%$
HC	53.5	7.2	143.3	12.0	14.0	13.5	22.4	26.1
CO	625.0	43.1	20420.8	143.0	149.3	6.9	22.9	23.9
NO_x	48.2	-2.7	163.0	12.8	13.0	-5.6	26.5	27.1

TABLE 2: BAG VALUE STATISTICS FOR THE FIRST 505 SEC. OF THE FEDERAL TEST PROCEDURE DRIVING SEQUENCE

POLLUTANT	$\bar{O}$	$\bar{R}$	σ_R^2	σ_R	$\sqrt{\bar{R}^2 + \sigma_R^2}$	$\frac{\bar{R}}{\bar{O}} \cdot 100\%$	$\frac{\sigma_R}{\bar{O}} \cdot 100\%$	$\frac{\sqrt{\bar{R}^2 + \sigma_R^2}}{\bar{O}} \cdot 100\%$
HC	21.0	2.8	37.3	6.1	6.7	13.4	29.0	32.0
CO	223.7	9.2	4158.0	64.5	65.1	4.1	28.8	29.1
NO_x	17.2	0.5	22.9	4.8	4.8	2.9	27.8	28.0

TABLE 3: DISTRIBUTION OF HC BAG VALUE ERROR (OBSERVED - CALCULATED) FROM THE SURVEILLANCE DRIVING SEQUENCE

ERROR (GMS)	NUMBER OF VEHICLES
-70 to -65	1
-65 to -60	1
-60 to -55	0
-55 to -50	0
-50 to -45	0
-45 to -40	0
-40 to -35	1
-35 to -30	0
-30 to -25	2
-25 to -20	4
-20 to -15	5
-15 to -10	9
-10 to - 5	24
- 5 to - 0	149
0 to 5	309
5 to 10	246
10 to 15	97
15 to 20	80
20 to 25	37
25 to 30	16
30 to 35	18
35 to 40	3
40 to 45	4
45 to 50	5
50 to 55	1
55 to 60	0
60 to 65	2
65 to 70	2
70 to 75	0
75 to 80	2
80 to 85	1
85 to 90	0
90 to 95	0
95 to 100	0
100 to 105	0
105 to 110	0
110 to 115	0
115 to 120	1
TOTAL	1020

TABLE 4: DISTRIBUTION OF CO BAG VALUE ERROR (OBSERVED - CALCULATED) FROM THE SURVEILLANCE DRIVING SEQUENCE

ERROR (GMS)	NUMBER OF VEHICLES
-750 to 700	1
-750 to -650	0
-650 to -600	0
-600 to 550	0
-550 to -500	1
-500 to -450	1
-450 to -400	0
-400 to -350	1
-350 to -300	4
-300 to -250	3
-250 to -200	10
-200 to -150	22
-150 to -100	57
-100 to - 50	94
- 50 to 0	170
0 to 50	272
50 to 100	143
100 to 150	92
150 to 200	53
200 to 250	32
250 to 300	27
300 to 350	6
350 to 400	7
400 to 450	7
450 to 500	2
500 to 550	5
550 to 600	2
600 to 650	1
650 to 700	2
700 to 750	1
750 to 800	0
800 to 850	0
850 to 900	0
900 to 950	4
TOTAL	1020

TABLE 5: DISTRIBUTION OF NO_x BAG VALUE ERROR (OBSERVED - CALCULATED) FROM THE SURVEILLANCE DRIVING SEQUENCE

ERROR (GMS)	NUMBER OF VEHICLES
-65 to -60	1
-60 to -55	0
-55 to -50	0
-50 to -45	2
-45 to -40	1
-40 to -35	3
-35 to -30	8
-30 to -25	13
-25 to -20	26
-20 to -15	81
-15 to -10	98
-10 to -5	203
-5 to 0	298
0 to 5	145
5 to 10	60
10 to 15	44
15 to 20	22
20 to 25	17
25 to 30	14
30 to 35	5
35 to 40	6
40 to 45	1
45 to 50	0
50 to 55	1
55 to 170	1
TOTAL	1020

TABLE 6: DISTRIBUTION OF HC BAG VALUE ERROR (OBSERVED - CALCULATED) FROM FIRST 505 SEC. OF THE FEDERAL TEST PROCEDURE DRIVING SEQUENCE (HOT TRANSIENT PORTION)

ERROR (GMS)	NUMBER OF VEHICLES
-45 to -40	1
-40 to -35	0
-35 to -30	1
-30 to -25	2
-25 to -20	3
-20 to -15	5
-15 to -10	3
-10 to -5	21
-5 to 0	158
0 to 5	592
5 to 10	172
10 to 15	35
15 to 20	11
20 to 25	9
25 to 30	1
30 to 35	3
35 to 40	1
40 to 45	1
45 to 50	0
50 to 55	0
55 to 60	0
60 to 65	1

TABLE 7: DISTRIBUTION OF CO BAG VALUE ERROR (OBSERVED - CALCULATED) FROM FIRST 505 SEC. OF FEDERAL TEST PROCEDURE DRIVING SEQUENCE (HOT TRANSIENT PORTION)

ERROR (GMS)	NUMBER OF VEHICLES
-350 to -300	2
-300 to -250	1
-250 to -200	1
-200 to -150	8
-150 to -100	24
-100 to - 50	72
- 50 to 0	279
0 to 50	483
50 to 100	103
100 to 150	30
150 to 200	3
200 to 250	5
250 to 300	0
300 to 350	1
350 to 400	1
400 to 1000	1
TOTAL	1020

TABLE 8: DISTRIBUTION OF NO_x BAG VALUE ERROR (OBSERVED - CALCULATED) FROM FIRST 505 SEC. OF FEDERAL TEST PROCEDURE DRIVING SEQUENCE (HOT TRANSIENT PORTION)

ERROR (GMS)	NUMBER OF VEHICLES
-25 to -20	2
-20 to -15	1
-15 to -10	13
-10 to - 5	60
- 5 to 0	468
0 to 5	351
5 to 10	88
10 to 15	24
15 to 20	10
20 to 25	3

TABLE 9

REPLICATE MODAL ANALYSES OF HC FOR 61 VEHICLES

MODE	$\bar{X}$ (gms/min.)	$\hat{\sigma}$(gms/min)	$\frac{\hat{\sigma}}{\bar{X}}$ .100%
1	3.8570	2.2342	57.93
2	1.6284	1.1856	72.81
3	2.4522	1.2589	51.34
4	2.8327	2.7032	95.43
5	3.5073	3.2759	93.40
6	1.9178	1.0114	52.74
7	5.2240	3.2673	62.54
8	2.7310	2.1971	80.45
9	4.4227	3.7429	84.63
10	2.9622	1.3290	44.86
11	4.9806	3.6829	73.95
12	3.0453	1.3054	42.87
13	4.9745	4.1268	82.96
14	3.2702	1.8522	56.64
15	1.8480	0.8133	44.01
16	1.4517	0.5117	35.25
17	4.2265	3.4144	80.79
18	2.2881	1.5467	67.60
19	3.7102	3.0242	81.51
20	2.3403	1.3852	59.19
21	5.7121	4.1975	73.48
22	3.1666	1.2412	39.20
23	3.5338	2.8711	81.25
24	4.2144	3.7651	89.34
25	2.9225	1.6542	56.60
26	1.8284	1.0534	57.61
27	4.2448	3.3618	79.20
28	3.0080	0.9896	32.90
29	3.1500	2.6789	85.04
30	4.5983	3.8230	83.14
31	2.6819	1.4452	53.89
32	1.9091	1.1396	59.69
33	1.2829	0.3898	30.39
34	1.2182	0.9104	74.73
35	1.6903	1.0114	59.84
36	2.5522	2.3242	91.06
37	3.2911	2.6043	79.13
FTP (gms.)	21.3255	15.0555	70.60
SDS (gms.)	54.4599	37.3647	68.61

TABLE 10

REPLICATE MODAL ANALYSES OF CO FOR 61 VEHICLES

MODE	$\overline{X}$ (gms/min)	$\hat{\sigma}$ (gms/min)	$\frac{\hat{\sigma}}{\overline{X}} \cdot 100\%$
1	49.4840	14.9649	30.24
2	15.3812	13.0886	85.09
3	32.4560	9.8602	30.38
4	36.1708	11.1998	30.96
5	40.5807	11.3075	27.86
6	17.4203	8.4930	48.75
7	96.5027	23.5758	24.43
8	24.6048	10.4125	42.32
9	65.0038	31.8901	49.06
10	22.3535	9.3043	41.62
11	90.2337	50.5725	56.05
12	21.4964	9.9618	46.34
13	80.1931	50.5111	62.99
14	25.9958	5.0928	19.59
15	16.1499	5.1828	32.09
16	19.9149	5.4151	27.19
17	63.0919	16.0394	25.42
18	20.5962	7.7882	37.81
19	59.6194	25.6127	42.96
20	21.1044	8.6146	40.82
21	106.7290	51.5901	48.34
22	24.1967	9.0618	37.45
23	50.1886	11.6789	23.27
24	71.2064	50.5271	70.96
25	25.8109	10.7750	41.75
26	17.0626	9.7663	57.24
27	65.9579	17.3853	26.36
28	25.9421	12.5521	48.38
29	44.9232	13.5433	30.15
30	85.6044	49.4810	57.80
31	26.3899	10.8379	41.07
32	18.1339	8.8996	49.09
33	16.9898	4.7618	28.03
34	17.3020	5.1765	29.92
35	16.7257	4.7752	28.55
36	28.9744	7.7927	26.90
37	45.2389	8.8608	19.59
FTP (gms)	239.8225	64.6112	26.94
SDS (gms)	677.9143	97.8159	14.43

TABLE 11

REPLICATE MODAL ANALYSIS OF NO_x FOR 61 VEHICLES

Mode	$\overline{X}$ (gm/min)	$\hat{\sigma}$ (gm/min)	$\hat{\sigma}/\overline{X}$ .100%
1	3.5689	1.0305	028.87
2	0.6435	0.2893	044.96
3	0.9107	0.4624	050.77
4	2.8518	0.9324	032.70
5	5.2695	1.1874	022.53
6	1.5707	0.6046	038.50
7	6.9947	2.2683	032.43
8	2.9842	1.2236	041.00
9	7.9071	2.5482	032.23
10	1.8646	0.8339	044.72
11	7.2462	1.6195	022.35
12	1.7380	0.7112	040.92
13	7.0501	1.4354	020.36
14	1.9580	0.8544	043.64
15	0.6813	0.3585	052.62
16	0.3179	0.3019	094.94
17	4.9960	1.1497	023.01
18	1.1132	0.6661	059.83
19	4.8965	1.3169	026.89
20	0.9471	0.3277	034.59
21	6.6968	1.6761	025.03
22	1.3989	0.5392	038.54
23	2.9016	0.9026	031.11
24	6.9010	1.3887	020.12
25	2.0334	0.7985	039.27
26	0.6391	0.2832	044.31
27	6.5950	1.2575	019.07
28	1.3037	0.4668	035.80
29	2.5598	0.6831	026.69
30	6.6468	1.7541	026.39
31	2.1487	0.8939	041.60
32	0.6203	0.2489	040.12
33	0.1174	0.1618	137.83
34	0.1826	0.2170	118.81
35	1.0192	0.2395	023.50
36	3.1829	0.6877	021.61
37	6.3341	1.2201	019.26
FTP (gm)	18.2670	2.8891	15.82
SDS (gm)	50.8930	7.8712	15.47

TABLE 12

VARIANCE COMPONENT ANALYSIS FOR 61 REPLICATE TESTS

First 505 Seconds - Federal Test Procedure

Source	Mean Squares			Variance Component		
	HC	CO	NO_x	HC	CO	NO_x
Models	5.74	832.57	3.39	1.54	79.59	1.01
Replications	94.00	784.67	2.64	45.67	55.64	0.63
Interaction	2.65	673.39	2.37	2.65	673.39	1.37

Surveillance Driving Sequence

Source	Mean Squares			Variance Component		
	HC	CO	NO_x	HC	CO	NO_x
Models	24.95	5560.34	29.76	7.75	1394.90	12.31
Replications	600.84	4759.36	26.37	295.65	994.41	10.61
Interaction	9.54	2770.54	5.13	9.54	2770.54	5.13

FIGURES

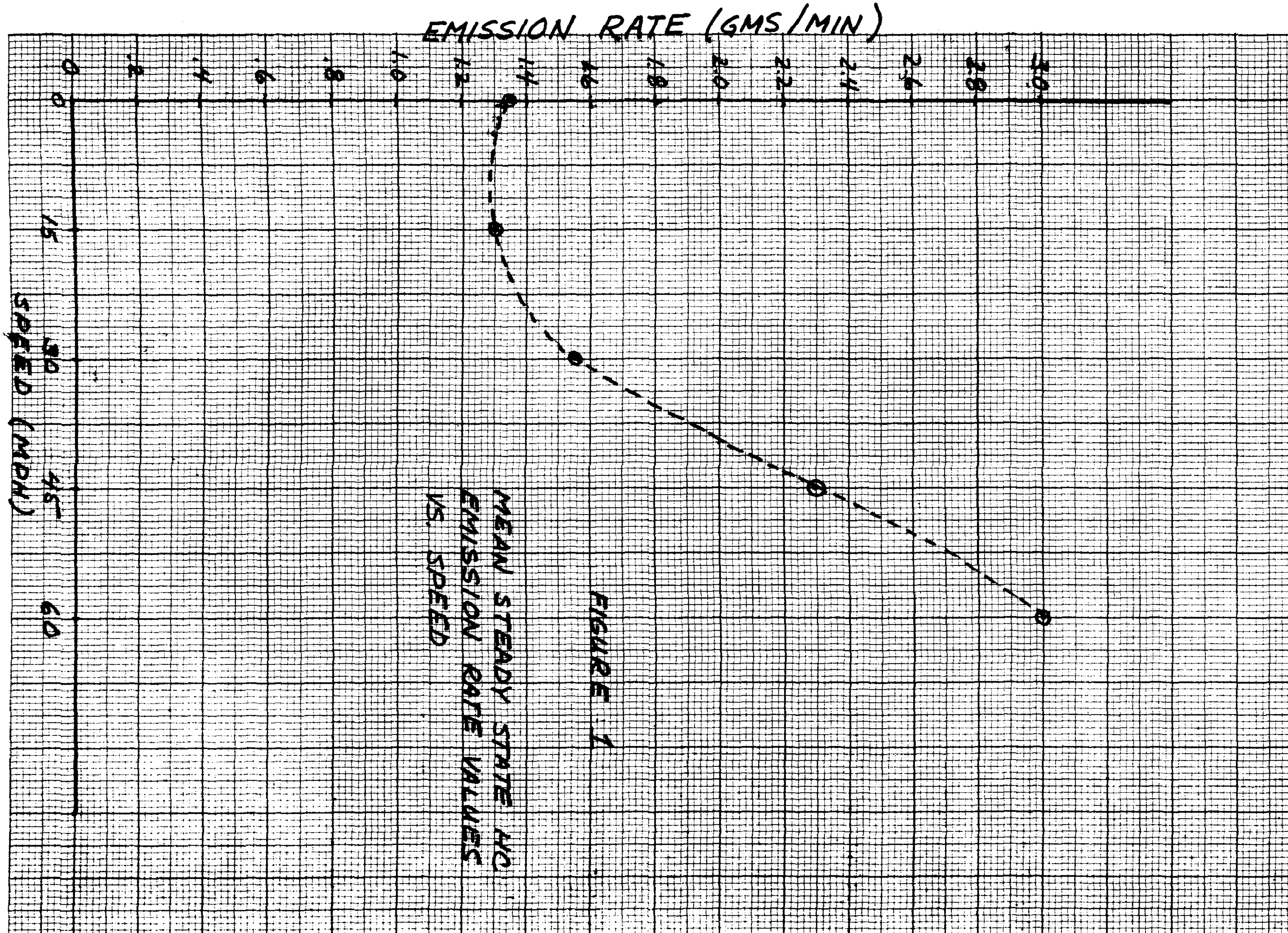


FIGURE 1
MEAN STEADY STATE HC EMISSION RATE VALUES VS. SPEED

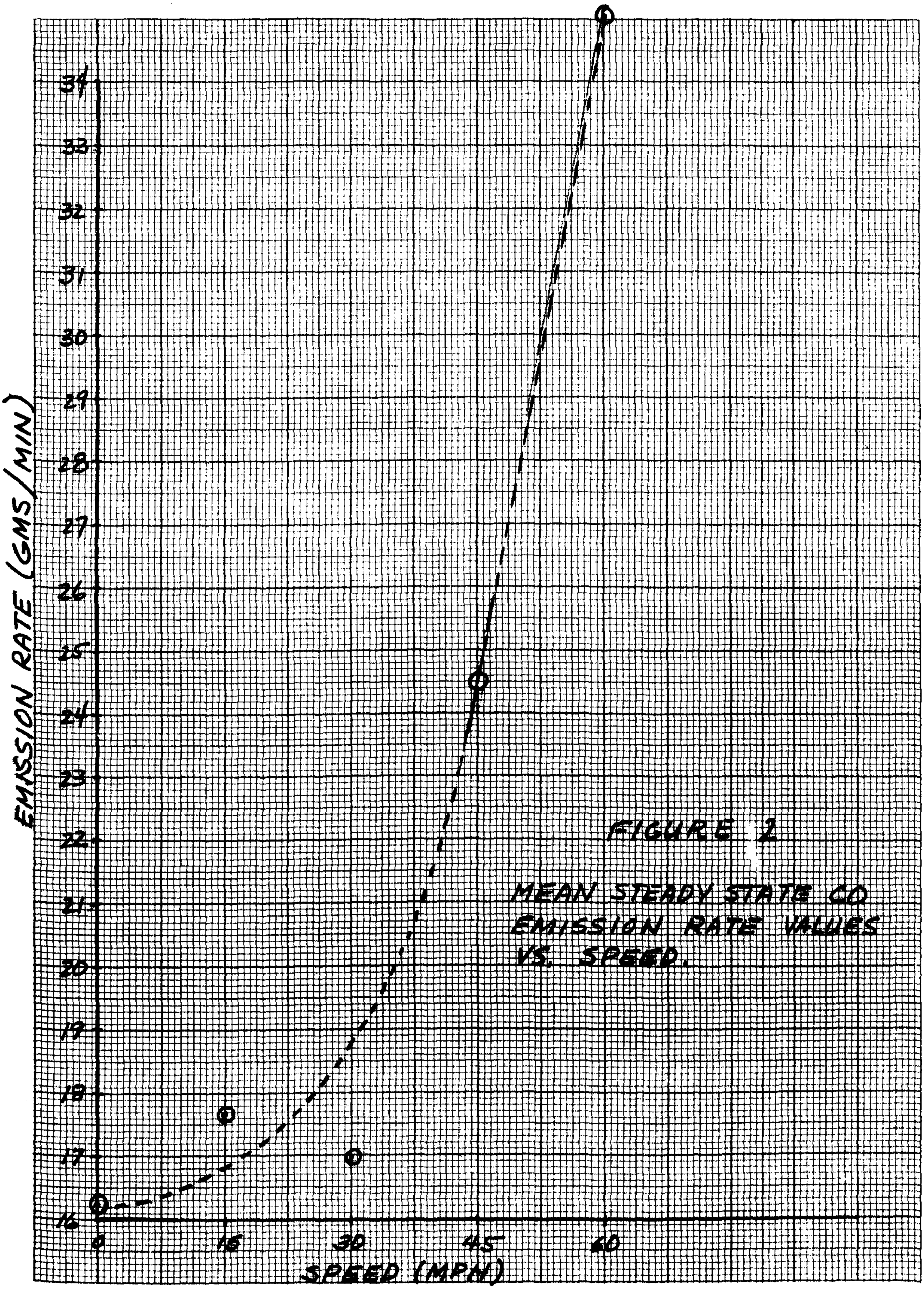
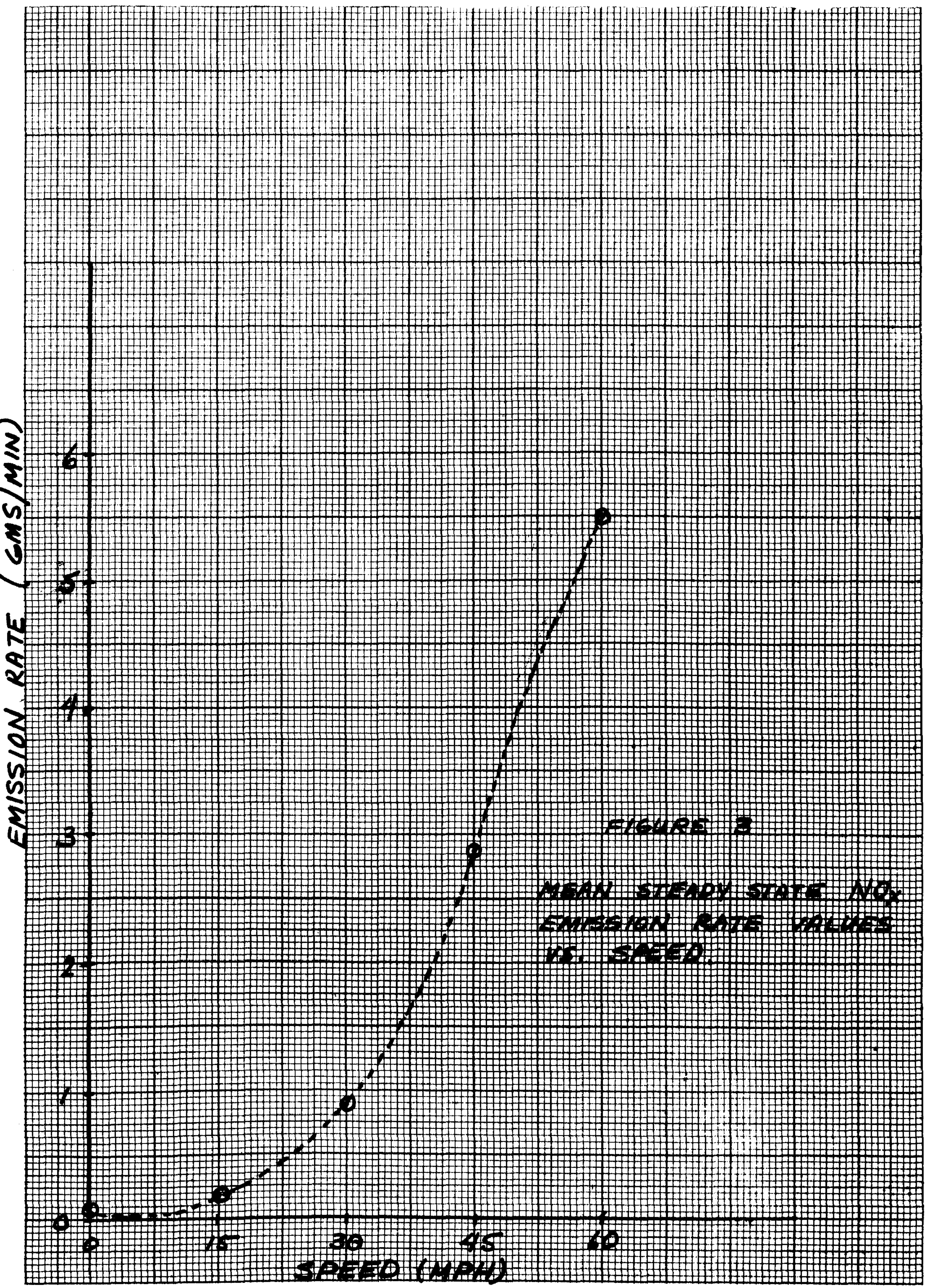


FIGURE 2

MEAN STEADY STATE CO EMISSION RATE VALUES VS. SPEED.



BEE 20x20 TO INCH

FIG 4: MAIN PROGRAM I

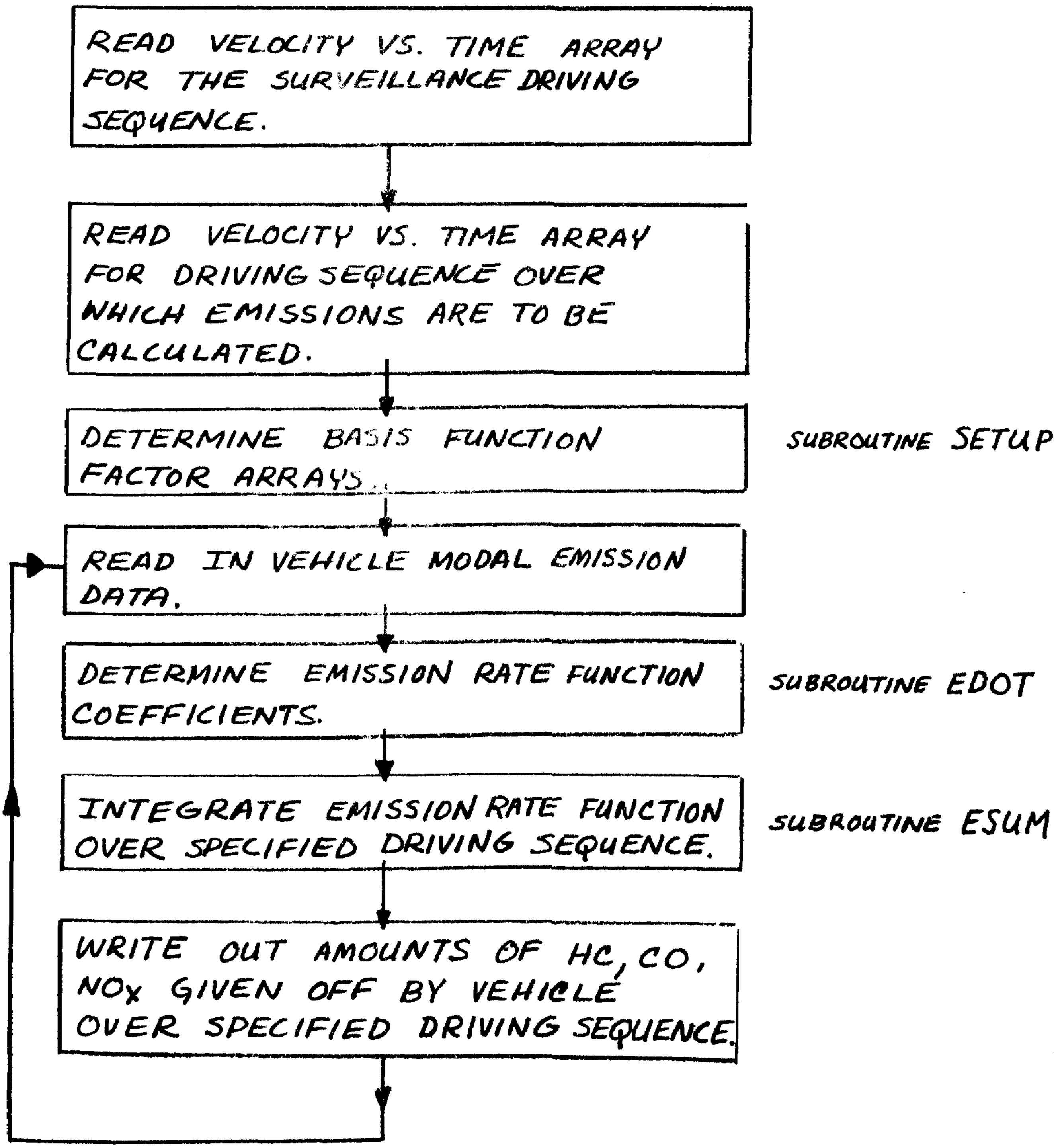
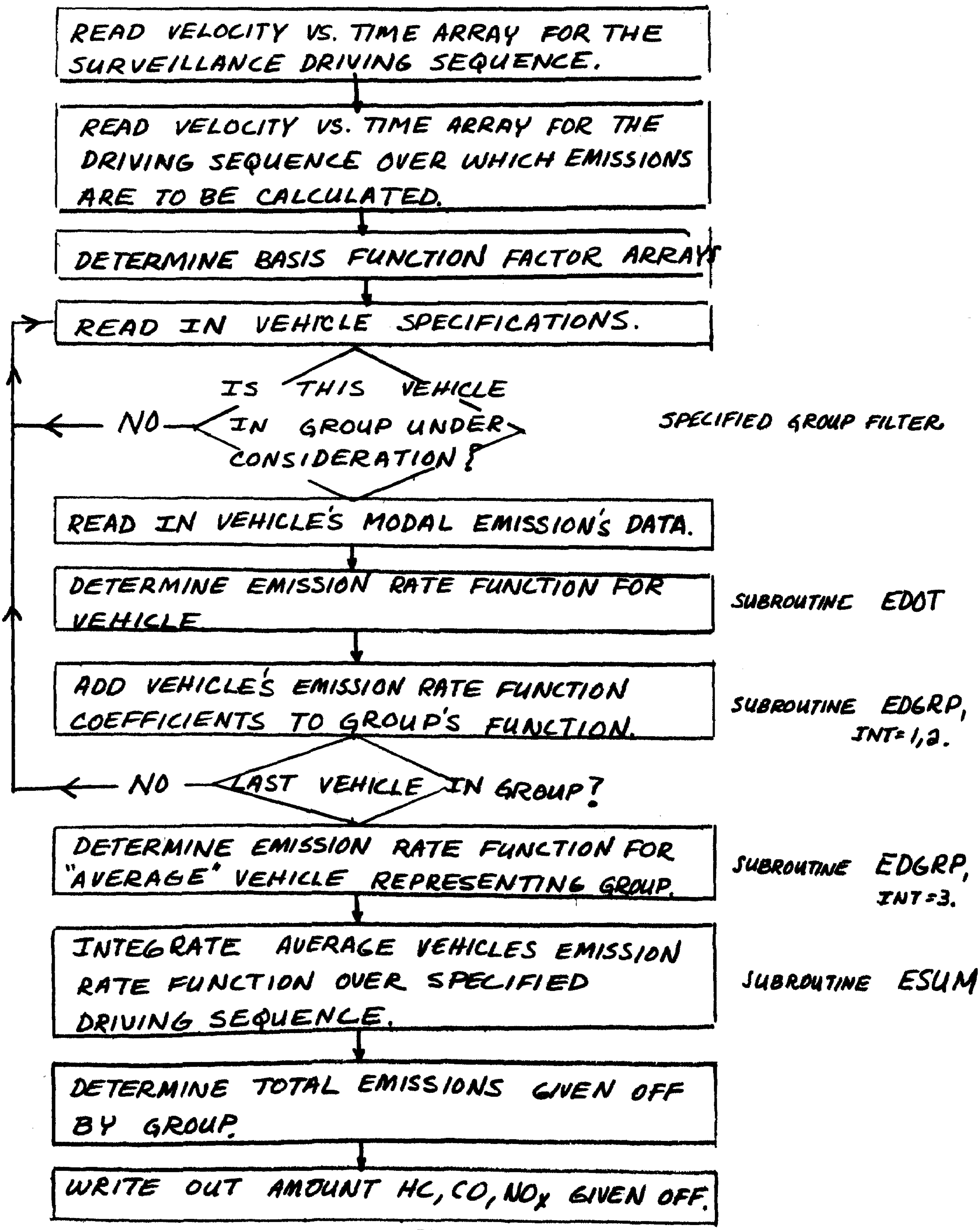


FIG 5: MAIN PROGRAM II



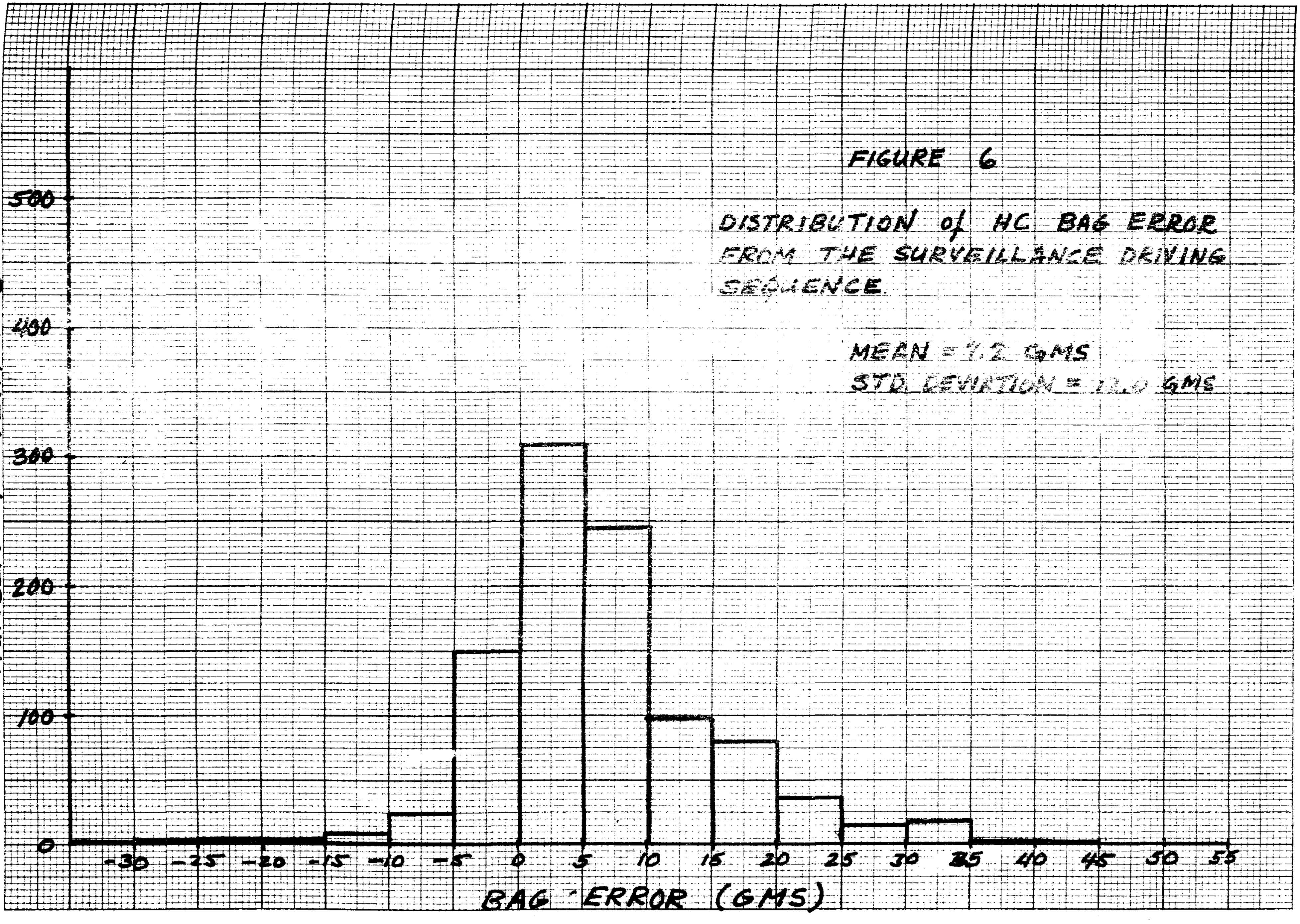


FIGURE 6

DISTRIBUTION of HC BAG ERROR FROM THE SURVEILLANCE DRIVING SEQUENCE.

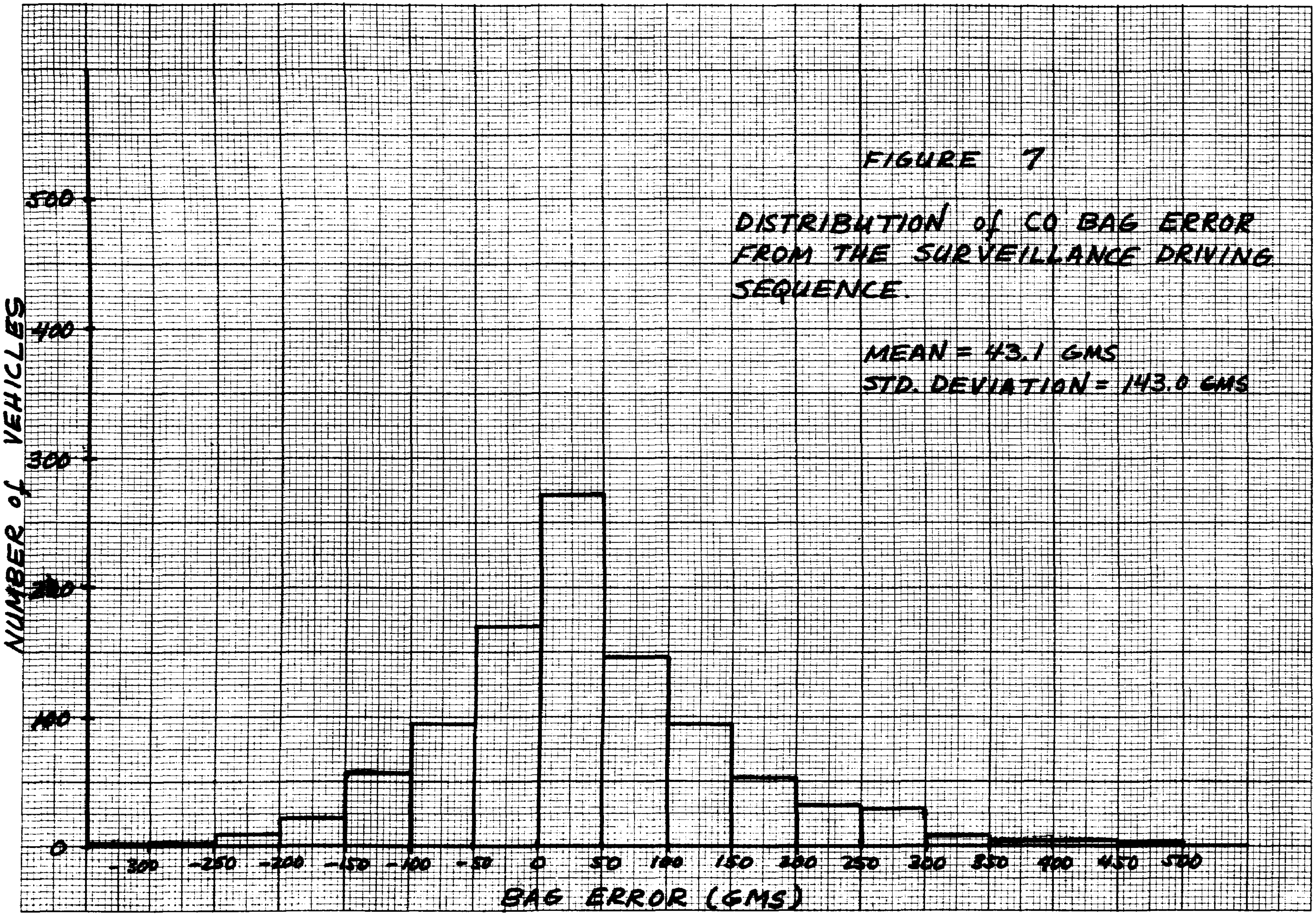


FIGURE 7
DISTRIBUTION of CO BAG ERROR FROM THE SURVEILLANCE DRIVING SEQUENCE.
MEAN = 43.1 GMS
STD. DEVIATION = 143.0 GMS
NUMBER of VEHICLES
500
400
300
0
-300
-250
-200
-150
-100
-50
0
50
100
150
200
250
300
350
400
450
500
BAG ERROR (GMS)

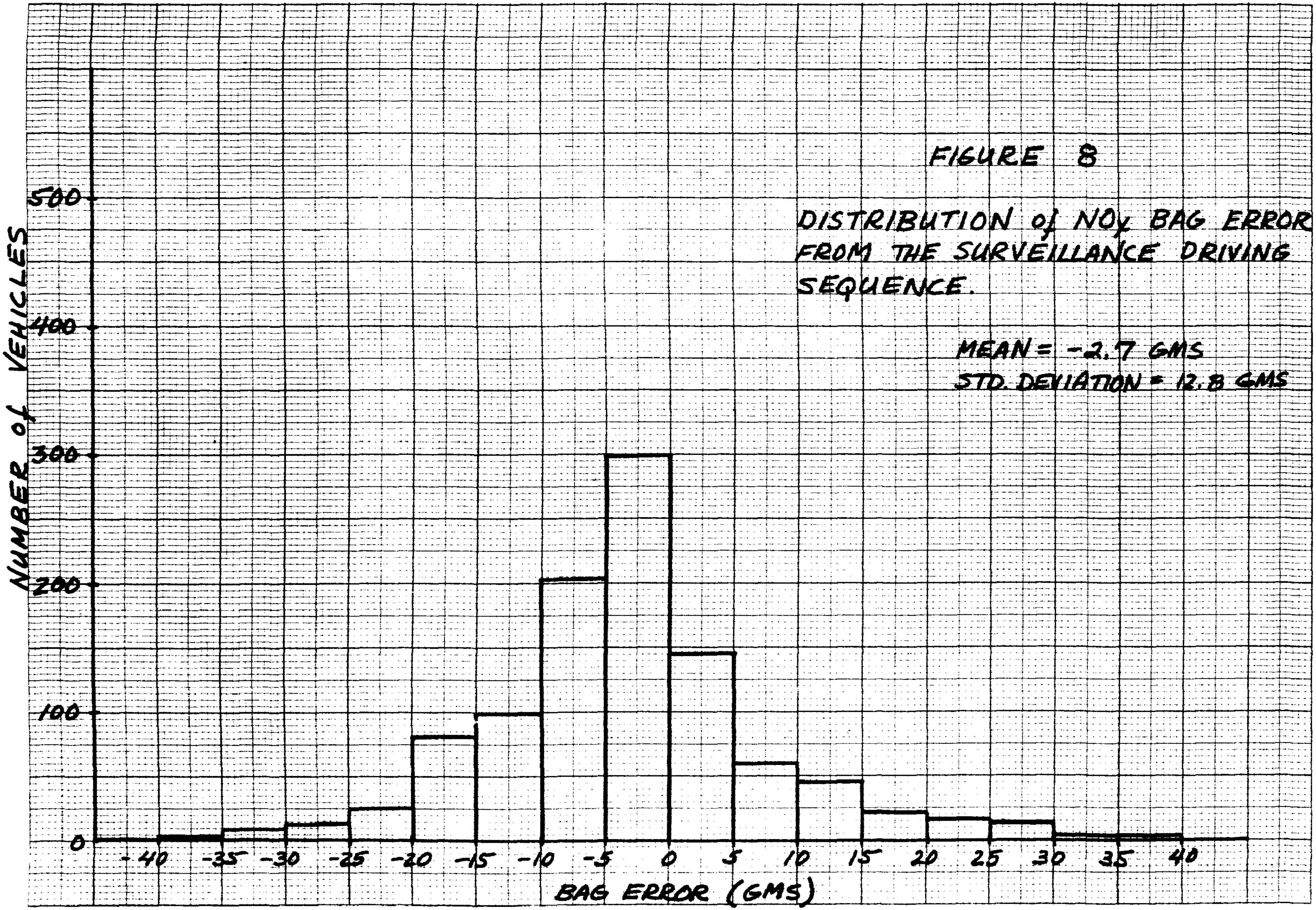


FIGURE 8
DISTRIBUTION of NOx BAG ERROR FROM THE SURVEILLANCE DRIVING SEQUENCE.
MEAN = -2.7 GMS
STD. DEVIATION = 12.8 GMS
NUMBER of VEHICLES
500
400
300
200
100
0
-40 -35 -30 -25 -20 -15 -10 -5 0 5 10 15 20 25 30 35 40
BAG ERROR (GMS)

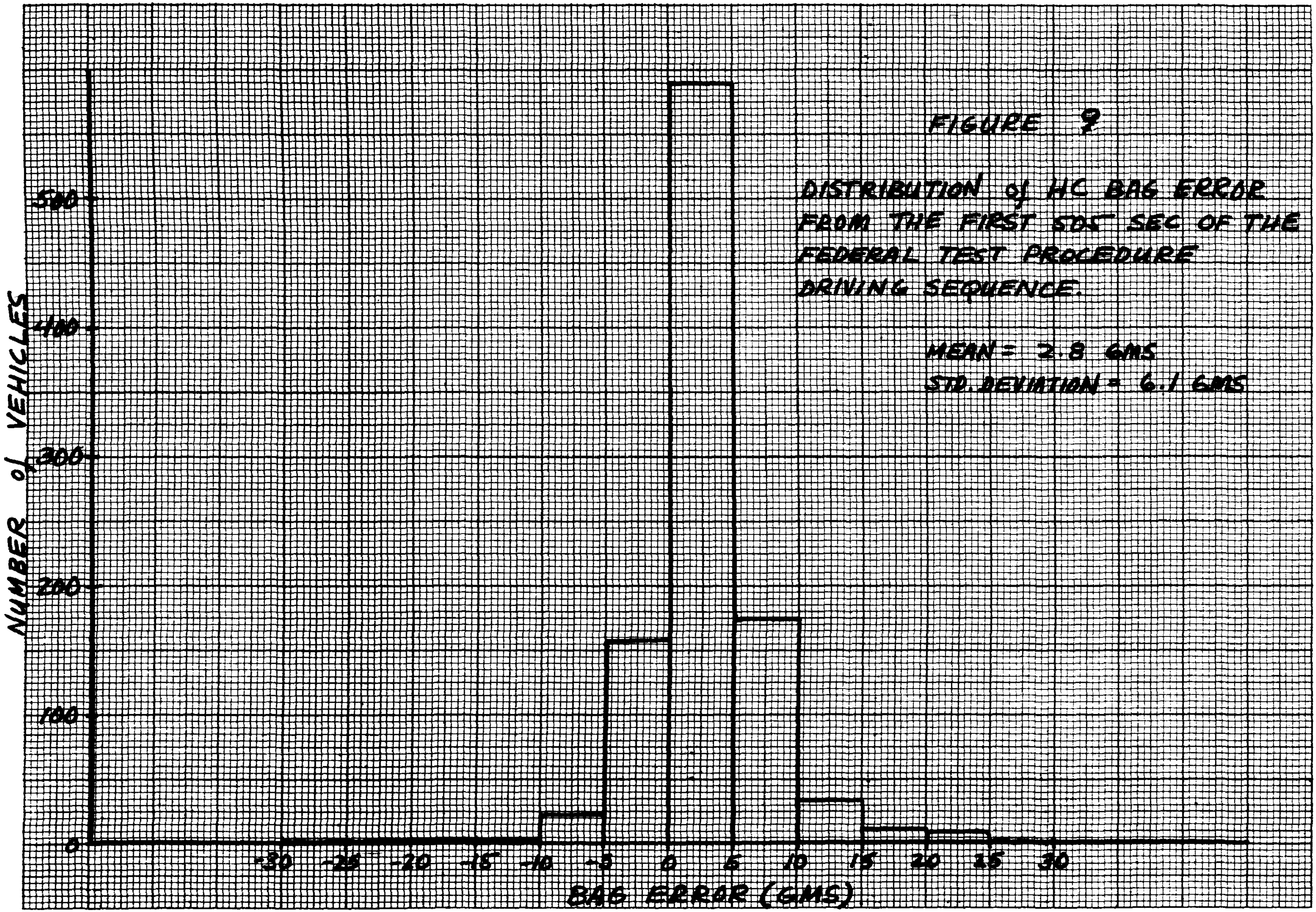


FIGURE 9
DISTRIBUTION of HC BAG ERROR FROM THE FIRST 505 SEC OF THE FEDERAL TEST PROCEDURE DRIVING SEQUENCE.
MEAN = 2.8 GMS
STD. DEVIATION = 6.1 GMS
NUMBER of VEHICLES
0
100
200
300
400
500
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
BAG ERROR (GMS)

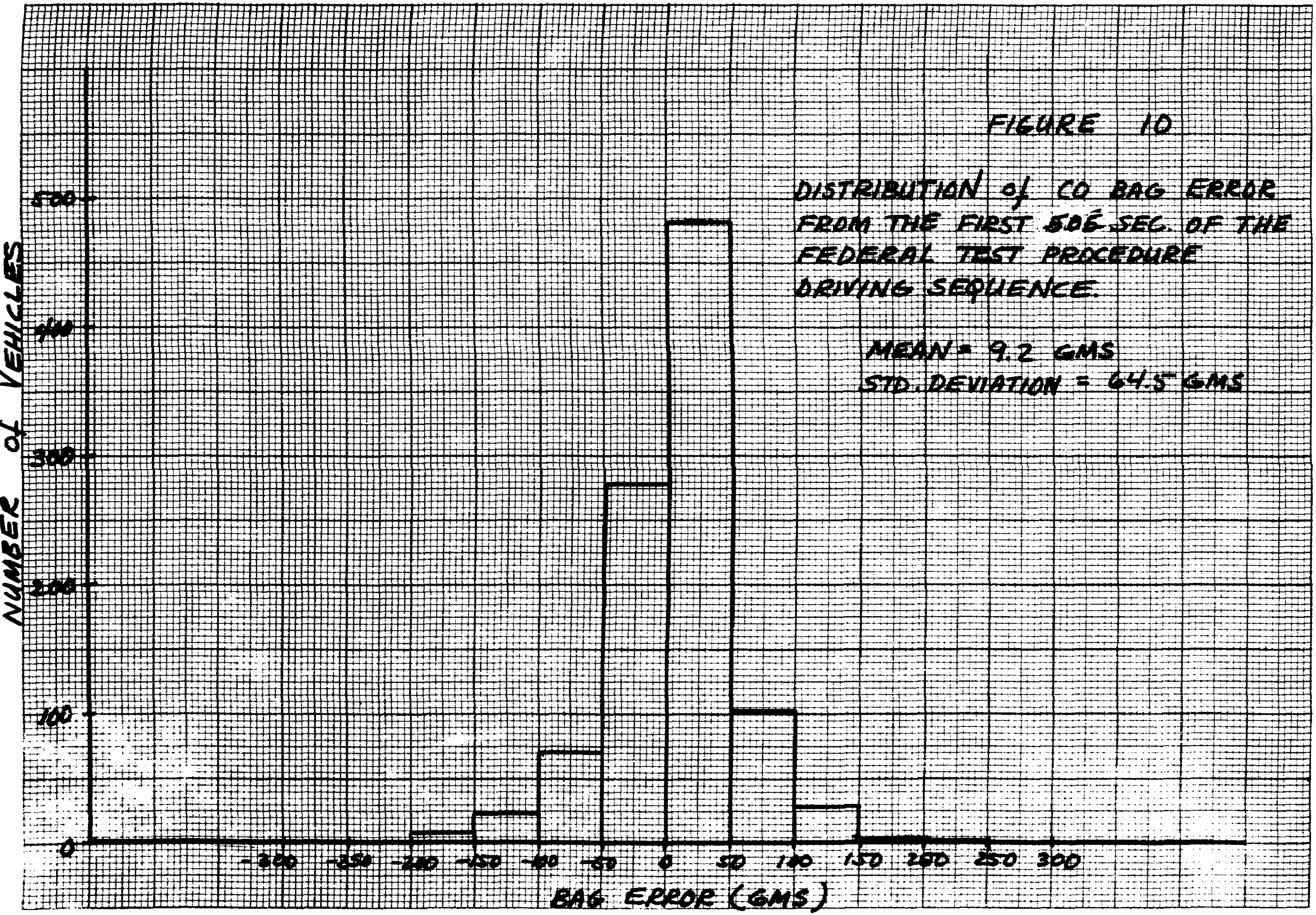


FIGURE 10
DISTRIBUTION of CO BAG ERROR FROM THE FIRST 505 SEC. OF THE FEDERAL TEST PROCEDURE DRIVING SEQUENCE.
MEAN = 9.2 GMS
STD. DEVIATION = 64.5 GMS
NUMBER of VEHICLES
500
400
300
200
100
0
-300
-250
-200
-150
-100
-50
0
50
100
150
200
250
300
BAG ERROR (GMS)

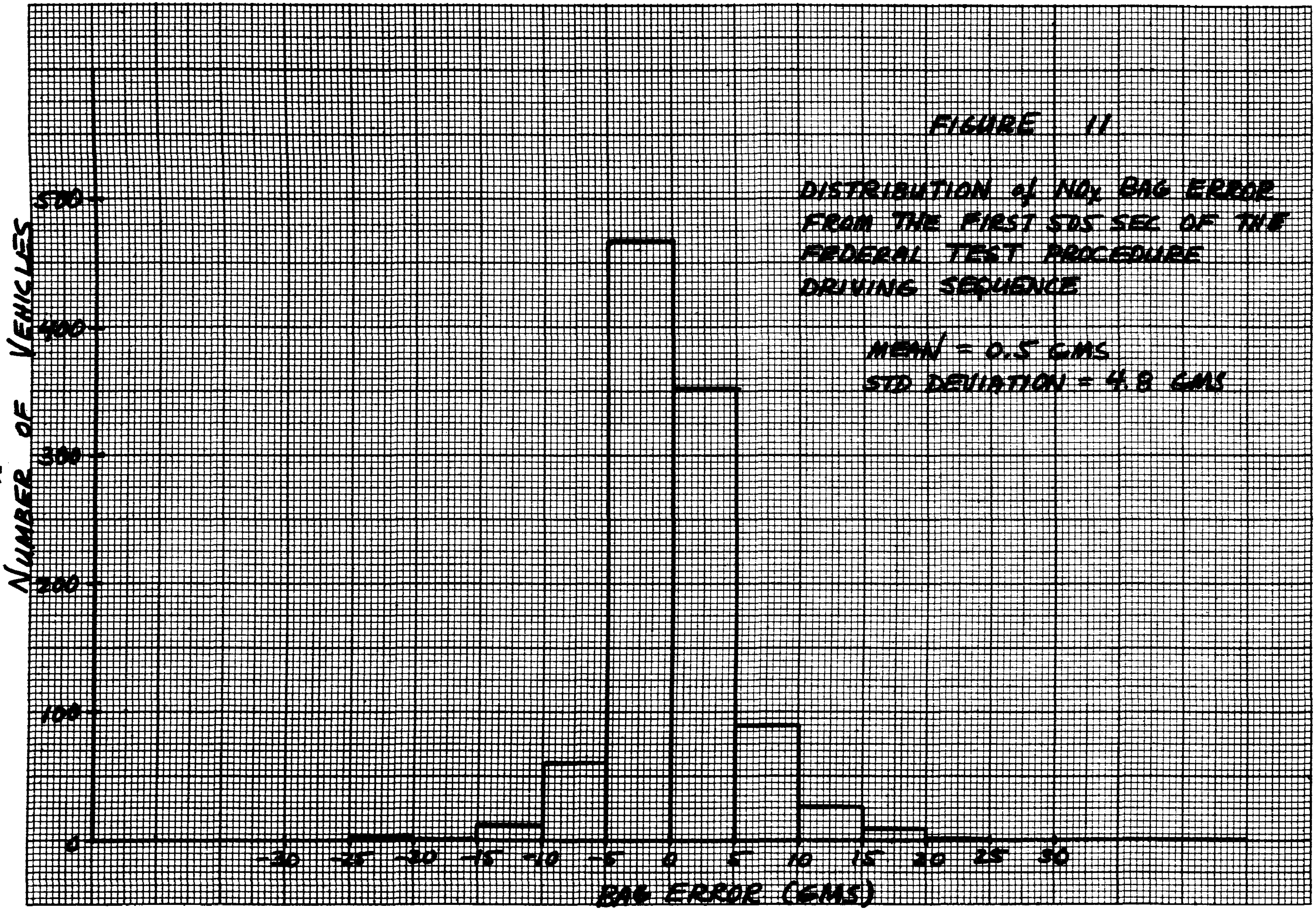


FIGURE 11
DISTRIBUTION of NOx BAG ERROR FROM THE FIRST 505 SEC OF THE FEDERAL TEST PROCEDURE DRIVING SEQUENCE
MEAN = 0.5 GMS
STD DEVIATION = 4.8 GMS
NUMBER OF VEHICLES
500
400
300
200
100
0
-30
-25
-20
-15
-10
-5
0
5
10
15
20
25
30
BAG ERROR (GMS)

APPENDICES

APPENDIX I

APPENDIX I

Specification of the 37 Modes and Evaluation of the Average Values of the Basis Functions

(I) Modal Specifications: Table 1-1

(II) Evaluation of the Average Values of The Basis Functions

The nine functions of speed (v) and acceleration (a) which form the basis functions of the instantaneous emission rate function must be averaged over each mode in order to determine the coefficients which specify the emission rate functions (See Appendix III).

To demonstrate how these average values are evaluated for the basis functions: 1.0, v, a, va, v^2, va^2, v^2a^2, consider the case of determining the average value of the 3rd basis function va over the i'th mode.

Let: T_i = duration of i'th mode

$V_i(t)$ = speed at time = t in the i'th mode, $t \leq T_i$

$a_i(t) = \frac{dV_i(t)}{dt}$ acceleration at time = t in the i'th mode, $t \leq T_i$

then by definition:

$$\overline{va}\,\Big|_i \equiv \frac{1}{T_i}\int_0^{T_i} V_i(t)\,a_i(t)\,dt$$

This integration is evaluated by the following approximation:

$$\overline{va}\,\Big|_i = \frac{1}{T_i}\sum_{j=1}^{N_i+1} \frac{(V_{ij} + V_{i,j+1})}{2}\;\frac{(V_{i,j+1} - V_{i,j})}{\Delta t}\;\Delta t$$

where: V_{i1} = initial speed of mode:

$\frac{V_{ij} + V_{i,j+1}}{2}$ = average speed over the j'th time interval of mode i

$$\frac{V_{i,j+1} - V_{i,j}}{\Delta t}$$ = average acceleration over the j th time interval of mode i

and, $$N_i \Delta t = T_i$$

The averages of the other 9 basis functions are similarily determined. Values for the averages of the basis functions over each mode are given in Table 1-2.

Table 1-1

MODAL SPECIFICATIONS

MODE	DURATION(SEC)	DISTANCE(MI)	SPEED (MPH) AT ONE SEC. INTERVALS										
1	12.0	0.06020	0.0	1.8	5.1	9.1	13.2	17.1	20.6	23.4	25.7	27.3	28.6	29.7
			30.0											
2	16.0	0.07410	30.0	29.6	28.9	28.1	26.9	25.4	23.5	21.2	18.5	15.6	12.5	9.4
			6.4	3.7	1.6	0.3	0.0							
3	8.0	0.02010	0.0	1.7	4.6	7.6	10.3	12.3	13.7	14.5	15.0			
4	11.0	0.07050	15.0	15.9	17.3	18.9	20.6	22.4	24.2	25.9	27.5	28.8	29.8	30.0
5	13.0	0.13600	30.0	30.7	31.7	32.9	34.2	35.6	37.0	38.5	39.9	41.2	42.4	43.5
			44.4	45.0										
6	12.0	0.12680	45.0	44.7	44.1	43.4	42.2	40.7	38.9	36.8	34.6	32.6	31.0	30.1
			30.0											
7	17.0	0.21630	30.0	30.0	31.0	32.5	34.3	36.2	38.3	40.5	42.7	45.0	47.3	49.5
			51.6	53.6	55.5	57.2	58.6	59.8						
8	12.0	0.17160	60.0	60.0	59.7	59.1	58.3	57.2	55.7	53.8	51.7	49.6	47.5	45.9
			45.1											
9	14.0	0.20430	45.0	45.6	46.5	47.6	48.7	50.0	51.3	52.6	53.9	55.2	56.4	57.5
			58.5	59.3	60.0									
10	30.0	0.33670	60.0	59.7	59.3	58.8	58.3	57.6	56.9	56.0	54.9	53.8	52.4	50.9
			49.2	47.4	45.5	43.4	41.1	38.8	36.4	34.0	31.5	29.0	26.6	24.3
			22.1	20.2	18.4	17.0	15.9	15.2	15.0					
11	26.0	0.31360	15.0	16.7	20.0	23.1	26.1	28.9	31.6	34.2	36.6	38.9	41.1	43.1
			45.0	46.8	48.5	50.1	51.5	52.8	54.0	55.1	56.1	56.9	57.7	58.3
			58.9	59.3	60.0									
12	21.0	0.19730	60.0	59.4	58.5	57.4	56.1	54.4	52.4	49.9	46.9	43.6	39.8	35.7
			31.2	26.6	21.9	17.3	12.8	8.7	5.1	2.3	0.5	0.0		
13	32.0	0.33130	0.0	1.5	5.2	8.6	12.0	15.3	18.4	21.4	24.2	27.0	29.6	32.1
			34.5	36.8	39.0	41.0	43.0	44.8	46.5	48.1	49.6	51.0	52.3	53.5
			54.5	55.5	56.4	57.2	57.9	58.5	58.9	59.3	60.0			
14	23.0	0.29940	60.0	59.7	59.3	58.9	58.3	57.6	56.7	55.7	54.5	53.1	51.5	49.7
			47.8	45.8	43.7	41.6	39.4	37.3	35.4	33.6	32.1	30.9	30.2	30.0
15	9.0	0.05790	30.0	29.5	28.7	27.3	25.3	22.7	19.9	17.3	15.4	15.0		
16	8.0	0.01730	15.0	14.4	13.3	11.3	8.5	5.2	2.1	0.2	0.0			
17	22.0	0.17590	0.0	2.4	6.3	10.0	13.5	16.8	19.9	22.8	25.5	28.0	30.3	32.4
			34.4	36.2	37.8	39.2	40.5	41.6	42.6	43.4	44.0	44.5	45.0	
18	16.0	0.13920	45.0	44.5	43.9	43.0	41.8	40.2	38.2	35.8	33.1	30.1	26.9	23.7
			20.7	18.1	16.1	15.1	15.0							
19	18.0	0.15280	15.0	15.9	17.3	18.9	20.7	22.6	24.5	26.5	28.7	30.8	32.9	34.9
			36.9	38.8	40.5	42.0	43.4	44.5	45.0					
20	19.0	0.13040	45.0	44.5	43.7	42.7	41.4	39.8	37.8	35.3	32.5	29.2	25.6	21.8
			17.8	13.8	9.9	6.4	3.5	1.3	0.1	0.0				
21	25.0	0.26540	0.0	2.6	7.1	11.5	15.7	19.6	23.4	26.9	30.3	33.4	36.4	39.2
			41.8	44.2	46.4	48.5	50.3	52.0	53.5	54.9	56.1	57.1	58.0	58.7
			59.3	60.0										
22	28.0	0.26340	60.0	59.6	59.0	58.3	57.4	56.5	55.3	54.0	52.4	50.6	48.5	46.2
			43.6	40.8	37.8	34.6	31.3	27.9	24.3	20.8	17.3	14.0	10.7	7.8
			5.2	3.0	1.3	0.3	0.0							
23	15.0	0.07370	0.0	1.2	3.5	6.4	9.6	12.8	15.9	18.8	21.3	23.5	25.2	26.7
			27.8	28.7	29.6	30.0								
24	25.0	0.31340	30.0	30.5	31.4	32.4	33.6	34.8	36.0	37.4	38.7	40.2	41.6	43.1
			44.6	46.0	47.5	48.9	50.4	51.7	53.0	54.3	55.5	56.6	57.6	58.5
			59.3	60.0										
25	18.0	0.23620	60.0	59.6	59.1	58.4	57.6	56.5	55.1	53.4	51.5	49.3	46.8	44.3
			41.6	38.9	36.3	34.0	32.1	30.7	30.0					
26	10.0	0.04440	30.0	29.1	27.7	25.4	22.0	17.6	12.5	7.4	2.9	0.3	0.0	
27	38.0	0.40090	0.0	0.9	4.0	6.9	9.8	12.6	15.3	17.9	20.4	22.8	25.2	27.4
			29.6	31.7	33.7	35.7	37.5	39.3	41.0	42.5	44.2	45.6	47.0	48.4
			49.6	50.8	51.9	52.9	53.8	54.7	55.5	56.3	56.9	57.5	58.1	58.5
			58.9	59.3	60.0									
28	35.0	0.32930	60.0	59.7	59.2	58.7	58.1	57.4	56.7	55.8	54.8	53.7	52.4	50.9
			49.3	47.6	45.7	43.6	41.4	39.0	36.6	34.0	31.3	28.6	25.8	22.9
			20.1	17.3	14.6	12.0	9.5	7.2	5.2	3.4	1.9	0.8	0.2	0.0
29	18.0	0.08860	0.0	0.8	2.7	4.9	7.5	10.1	12.8	15.4	17.8	20.1	22.1	23.8
			25.2	26.5	27.4	28.3	29.0	29.8	30.0					
30	21.0	0.25990	30.0	30.7	31.8	33.1	34.5	36.0	37.6	39.3	41.0	42.8	44.6	46.3
			48.1	49.8	51.5	53.0	54.6	56.0	57.2	58.4	59.3	60.0		
31	14.0	0.18130	60.0	59.5	58.7	57.6	56.1	54.0	51.5	48.5	45.1	41.6	38.0	34.7
			32.1	30.4	30.0									
32	13.0	0.05920	30.0	29.4	28.5	27.2	25.4	22.9	19.9	16.4	12.5	8.6	5.0	2.1
			0.3	0.0										
33	60.0	0.0	SPEED = 0.0FOR 60 SEC											
34	60.0	0.25000	SPEED = 15.0FOR 60 SEC											
35	60.0	0.50000	SPEED = 30.0FOR 60 SEC											
36	60.0	0.75000	SPEED = 45.0FOR 60 SEC											
37	60.0	1.00000	SPEED = 60.0FOR 60 SEC											

Table 1-2

VALUES OF THE AVERAGES OF THE BASIS FUNCTIONS OVER EACH MODE

MODE	1	$\overline{v}$	$\overline{a}$	$\overline{va}$	$\overline{v^2}$	$\overline{a^2}$	$\overline{v^2a}$	$\overline{va^2}$	$\overline{v^2a^2}$
1	1.0000	18.0500	2.5000	37.5000	423.5988	7.7567	747.7868	104.8537	1840.1320
2	1.0000	16.6625	-1.8750	-28.1250	386.9459	4.4550	-561.5264	61.9900	1103.4938
3	1.0000	9.0375	1.8750	14.0625	105.4887	4.3150	139.7132	29.2004	255.4498
4	1.0000	23.0727	1.3636	30.6818	556.5805	2.0800	715.6334	46.5847	1075.2698
5	1.0000	37.6538	1.1538	43.2692	1440.5222	1.4046	1644.0825	52.5375	1986.8048
6	1.0000	38.0500	-1.2500	-46.8750	1476.2113	2.0417	-1780.9423	75.4888	2818.3750
7	1.0000	45.8000	1.7647	79.4118	2192.9562	3.4094	3705.3125	152.8414	7064.6548
8	1.0000	53.0083	-1.2500	-65.6250	2838.4180	2.0500	-3468.4389	106.5081	5561.0050
9	1.0000	52.5428	1.0714	56.2500	2783.1422	1.2014	2973.0977	62.9974	3322.1119
10	1.0000	40.4033	-1.5000	-56.2500	1873.3556	2.8120	-2362.0152	100.7045	3951.2093
11	1.0000	43.4154	1.7308	64.9038	2067.4378	3.7085	2725.2108	123.4465	4606.5663
12	1.0000	33.8333	-2.8571	-85.7143	1572.5804	10.2076	-3425.2217	282.9109	10063.8078
13	1.0000	38.2375	1.8750	56.2500	1789.5073	4.4462	2248.9990	107.7057	3670.1761
14	1.0000	46.8609	-1.3043	-58.6956	2303.4566	2.1313	-2738.8101	93.5297	4220.4915
15	1.0000	23.1778	-1.6667	-37.5000	564.6190	3.5533	-874.3014	78.0627	1761.9989
16	1.0000	7.8125	-1.8750	-14.0625	89.8487	4.6950	-139.5406	32.8369	290.9055
17	1.0000	28.8454	2.0455	46.0227	1016.1811	5.2827	1379.3948	96.7243	2467.4704
18	1.0000	31.3250	-1.8750	-56.2500	1093.6045	4.5450	-1827.1115	131.2360	4035.1918
19	1.0000	30.5500	1.6667	50.0000	1026.4346	2.9778	1624.5391	88.6821	2826.1665
20	1.0000	24.7158	-2.3684	-53.2895	861.2722	7.1937	-1596.6755	149.6970	3992.4466
21	1.0000	38.2760	2.4000	72.0000	1791.6623	7.2248	2877.9475	176.2425	6001.1068
22	1.0000	33.8750	-2.1429	-64.2857	1575.9649	5.7421	-2570.0153	159.2388	5668.8549
23	1.0000	17.7333	2.0000	30.0000	411.6827	4.8573	598.9142	65.4714	1148.7460
24	1.0000	45.1440	1.2000	54.0000	2128.2651	1.5128	2519.8353	67.7954	3133.7277
25	1.0000	47.2333	-1.6667	-75.0000	2334.0895	3.4022	-3499.3613	149.2304	6730.2482
26	1.0000	15.9900	-3.0000	-45.0000	369.3002	11.8100	-895.7151	164.2715	2912.0867
27	1.0000	38.0053	1.5789	47.3684	1770.6486	3.1421	1894.1427	75.7415	2585.3045
28	1.0000	33.8686	-1.7143	-51.4286	1575.8225	3.6766	-2056.4181	101.8697	3624.8628
29	1.0000	17.7333	1.6667	25.0000	412.2008	3.3911	499.3674	45.6602	803.7473
30	1.0000	45.2667	1.4286	64.2857	2140.4352	2.1562	2999.7180	96.6297	4465.1001
31	1.0000	46.6286	-2.1429	-96.4286	2283.4472	5.8200	-4498.5436	255.5805	11536.7248
32	1.0000	16.4000	-2.3077	-34.6154	379.8860	6.8246	-690.4485	94.9561	1686.8623
33	1.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	1.0000	15.0000	0.0	0.0	225.0000	0.0	0.0	0.0	0.0
35	1.0000	30.0000	0.0	0.0	900.0000	0.0	0.0	0.0	0.0
36	1.0000	45.0000	0.0	0.0	2025.0000	0.0	0.0	0.0	0.0
37	1.0000	60.0000	0.0	0.0	3600.0000	0.0	0.0	0.0	0.0

APPENDIX II

Speed vs. time curves
for the Surveillance Driving Sequence
and first 505 seconds of the Federal
Test Procedure Driving Sequence.

SURVEILLANCE ACCELERATION-DECELERATION DRIVING SEQUENCE

TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)
1.	0.0	58.	0.0	115.	32.9	172.	38.3	229.	47.6	286.	15.0	343.	59.4	400.	57.2	457.	30.0
2.	0.0	59.	0.0	116.	34.2	173.	40.5	230.	48.7	287.	15.0	344.	58.5	401.	57.9	458.	30.0
3.	0.0	60.	0.0	117.	35.6	174.	42.7	231.	50.0	288.	15.0	345.	57.4	402.	59.5	459.	29.5
4.	0.0	61.	0.0	118.	37.0	175.	45.0	232.	51.3	289.	15.0	346.	56.1	403.	58.9	460.	28.7
5.	0.0	62.	0.0	119.	38.5	176.	47.3	233.	52.6	290.	15.0	347.	54.4	404.	59.3	461.	27.3
6.	0.0	63.	0.0	120.	39.9	177.	49.5	234.	53.9	291.	15.0	348.	52.4	405.	60.0	462.	25.3
7.	0.0	64.	1.7	121.	41.2	178.	51.6	235.	55.2	292.	15.0	349.	49.9	406.	60.0	463.	22.7
8.	0.0	65.	4.6	122.	42.4	179.	53.6	236.	56.4	293.	15.0	350.	46.9	407.	60.0	464.	19.7
9.	0.0	66.	7.6	123.	43.5	180.	55.5	237.	57.5	294.	15.0	351.	43.6	408.	60.0	465.	17.3
10.	0.0	67.	10.3	124.	44.4	181.	57.2	238.	58.5	295.	15.0	352.	39.8	409.	60.0	466.	15.4
11.	1.8	68.	12.3	125.	45.0	182.	58.6	239.	59.3	296.	15.0	353.	35.7	410.	60.0	467.	15.0
12.	5.1	69.	13.7	126.	45.0	183.	59.8	240.	60.0	297.	15.0	354.	31.2	411.	60.0	468.	15.0
13.	9.1	70.	14.6	127.	45.0	184.	60.0	241.	60.0	298.	15.0	355.	26.6	412.	60.0	469.	15.0
14.	13.2	71.	15.0	128.	45.0	185.	60.0	242.	60.0	299.	15.0	356.	21.9	413.	60.0	470.	15.0
15.	17.1	72.	15.0	129.	45.0	186.	60.0	243.	60.0	300.	15.0	357.	17.3	414.	60.0	471.	15.0
16.	20.6	73.	15.0	130.	45.0	187.	60.0	244.	60.0	301.	15.0	358.	12.8	415.	60.0	472.	15.0
17.	23.4	74.	15.0	131.	45.0	188.	60.0	245.	60.0	302.	16.7	359.	8.7	416.	60.0	473.	15.0
18.	25.7	75.	15.0	132.	45.0	189.	60.0	246.	60.0	303.	20.0	360.	5.1	417.	60.0	474.	15.0
19.	27.3	76.	15.0	133.	45.0	190.	60.0	247.	60.0	304.	23.1	361.	2.3	418.	60.0	475.	15.0
20.	28.6	77.	15.0	134.	45.0	191.	60.0	248.	60.0	305.	26.1	362.	0.5	419.	60.0	476.	15.0
21.	29.7	78.	15.0	135.	45.0	192.	60.0	249.	60.0	306.	28.9	363.	0.0	420.	60.0	477.	15.0
22.	30.0	79.	15.0	136.	45.0	193.	60.0	250.	60.0	307.	31.6	364.	0.0	421.	59.7	478.	15.0
23.	30.0	80.	15.0	137.	45.0	194.	60.0	251.	60.0	308.	34.2	365.	0.0	422.	59.3	479.	15.0
24.	30.0	81.	15.0	138.	45.0	195.	60.0	252.	60.0	309.	36.6	366.	0.0	423.	58.9	480.	15.0
25.	30.0	82.	15.0	139.	45.0	196.	60.0	253.	60.0	310.	38.9	367.	0.0	424.	58.3	481.	15.0
26.	30.0	83.	15.0	140.	45.0	197.	60.0	254.	60.0	311.	41.1	368.	0.0	425.	57.6	482.	15.0
27.	30.0	84.	15.0	141.	44.7	198.	60.0	255.	60.0	312.	43.1	369.	0.0	426.	56.7	483.	14.4
28.	30.0	85.	15.0	142.	44.1	199.	60.0	256.	60.0	313.	45.0	370.	0.0	427.	55.7	484.	13.3
29.	30.0	86.	15.0	143.	43.4	200.	59.7	257.	59.7	314.	46.8	371.	0.0	428.	54.5	485.	11.3
30.	30.0	87.	15.9	144.	42.2	201.	59.1	258.	59.3	315.	48.5	372.	0.0	429.	53.1	486.	8.5
31.	30.0	88.	17.3	145.	40.7	202.	58.3	259.	58.8	316.	50.1	373.	0.0	430.	51.5	487.	5.2
32.	30.0	89.	18.9	146.	38.9	203.	57.2	260.	58.3	317.	51.5	374.	1.5	431.	49.7	488.	2.1
33.	30.0	90.	20.6	147.	36.8	204.	55.7	261.	57.6	318.	52.8	375.	5.2	432.	47.8	489.	0.2
34.	30.0	91.	22.4	148.	34.6	205.	53.8	262.	56.9	319.	54.0	376.	8.6	433.	45.8	490.	0.0
35.	30.0	92.	24.2	149.	32.6	206.	51.7	263.	56.0	320.	55.1	377.	12.0	434.	43.7	491.	0.0
36.	30.0	93.	25.9	150.	31.0	207.	49.6	264.	54.9	321.	56.1	378.	15.3	435.	41.6	492.	0.0
37.	30.0	94.	27.5	151.	30.1	208.	47.5	265.	53.8	322.	56.9	379.	18.4	436.	39.4	493.	0.0
38.	29.6	95.	28.8	152.	30.0	209.	45.9	266.	52.4	323.	57.7	380.	21.4	437.	37.3	494.	0.0
39.	28.9	96.	29.8	153.	30.0	210.	45.1	267.	50.9	324.	58.3	381.	24.2	438.	35.4	495.	0.0
40.	28.1	97.	30.0	154.	30.0	211.	45.0	268.	49.2	325.	58.9	382.	27.0	439.	33.6	496.	0.0
41.	26.9	98.	30.0	155.	30.0	212.	45.0	269.	47.4	326.	59.3	383.	29.6	440.	32.1	497.	0.0
42.	25.4	99.	30.0	156.	30.0	213.	45.0	270.	45.5	327.	60.0	384.	32.1	441.	30.9	498.	0.0
43.	23.5	100.	30.0	157.	30.0	214.	45.0	271.	43.4	328.	60.0	385.	34.5	442.	30.2	499.	0.0
44.	21.2	101.	30.0	158.	30.0	215.	45.0	272.	41.1	329.	60.0	386.	36.8	443.	30.0	500.	0.0
45.	18.5	102.	30.0	159.	30.0	216.	45.0	273.	38.8	330.	60.0	387.	39.0	444.	30.0	501.	2.4
46.	15.6	103.	30.0	160.	30.0	217.	45.0	274.	36.4	331.	60.0	388.	41.0	445.	30.0	502.	6.3
47.	12.5	104.	30.0	161.	30.0	218.	45.0	275.	34.0	332.	60.0	389.	43.0	446.	30.0	503.	10.0
48.	9.4	105.	30.0	162.	30.0	219.	45.0	276.	31.5	333.	60.0	390.	44.8	447.	30.0	504.	13.5
49.	6.4	106.	30.0	163.	30.0	220.	45.0	277.	29.0	334.	60.0	391.	46.5	448.	30.0	505.	16.8
50.	3.7	107.	30.0	164.	30.0	221.	45.0	278.	26.6	335.	60.0	392.	48.1	449.	30.0	506.	19.9
51.	1.6	108.	30.0	165.	30.0	222.	45.0	279.	24.3	336.	60.0	393.	49.6	450.	30.0	507.	22.8
52.	0.3	109.	30.0	166.	30.0	223.	45.0	280.	22.1	337.	60.0	394.	51.0	451.	30.0	508.	25.5
53.	0.0	110.	30.0	167.	30.0	224.	45.0	281.	20.2	338.	60.0	395.	52.3	452.	30.0	509.	28.0
54.	0.0	111.	30.0	168.	31.0	225.	45.0	282.	18.4	339.	60.0	396.	53.5	453.	30.0	510.	30.3
55.	0.0	112.	30.0	169.	32.5	226.	45.0	283.	17.0	340.	60.0	397.	54.5	454.	30.0	511.	32.4
56.	0.0	113.	30.7	170.	34.3	227.	45.6	284.	15.9	341.	60.0	398.	55.5	455.	30.0	512.	34.4
57.	0.0	114.	31.7	171.	36.2	228.	46.5	285.	15.2	342.	60.0	399.	56.4	456.	30.0	513.	36.2

TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)	TIME (SEC)	SPEED (MPH)
514.	37.8	574.	24.5	634.	15.7	694.	5.2	754.	50.4	814.	29.1	874.	60.0	934.	4.9	994.	60.0
515.	39.2	575.	26.6	635.	19.6	695.	3.0	755.	51.7	815.	27.7	875.	60.0	935.	7.5	995.	60.0
516.	40.5	576.	28.7	636.	23.4	696.	1.3	756.	53.0	816.	25.4	876.	60.0	936.	10.1	996.	60.0
517.	41.6	577.	30.8	637.	26.9	697.	0.3	757.	54.3	817.	22.0	877.	60.0	937.	12.8	997.	60.0
518.	42.6	578.	32.9	638.	30.3	698.	0.0	758.	55.5	818.	17.6	878.	60.0	938.	15.4	998.	60.0
519.	43.4	579.	34.9	639.	33.4	699.	0.0	759.	56.6	819.	12.5	879.	60.0	939.	17.8	999.	60.0
520.	44.0	580.	36.9	640.	36.4	700.	0.0	760.	57.6	820.	7.4	880.	60.0	940.	20.1	1000.	60.0
521.	44.5	581.	38.8	641.	39.2	701.	0.0	761.	58.5	821.	2.9	881.	60.0	941.	22.1	1001.	59.5
522.	45.0	582.	40.5	642.	41.8	702.	0.0	762.	59.3	822.	0.3	882.	60.0	942.	23.8	1002.	58.7
523.	45.0	583.	42.0	643.	44.2	703.	0.0	763.	60.0	823.	0.0	883.	60.0	943.	25.2	1003.	57.6
524.	45.0	584.	43.4	644.	46.4	704.	0.0	764.	60.0	824.	0.0	884.	60.0	944.	26.5	1004.	56.1
525.	45.0	585.	44.5	645.	48.5	705.	0.0	765.	60.0	825.	0.0	885.	60.0	945.	27.4	1005.	54.0
526.	45.0	586.	45.0	646.	50.3	706.	0.0	766.	60.0	826.	0.0	886.	60.0	946.	28.3	1006.	51.5
527.	45.0	587.	45.0	647.	52.0	707.	0.0	767.	60.0	827.	0.0	887.	59.7	947.	29.0	1007.	48.5
528.	45.0	588.	45.0	648.	53.5	708.	0.0	768.	60.0	828.	0.0	888.	59.2	948.	29.8	1008.	45.1
529.	45.0	589.	45.0	649.	54.9	709.	1.2	769.	60.0	829.	0.0	889.	58.7	949.	30.0	1009.	41.6
530.	45.0	590.	45.0	650.	56.1	710.	3.5	770.	60.0	830.	0.0	890.	58.1	950.	30.0	1010.	38.0
531.	45.0	591.	45.0	651.	57.1	711.	6.4	771.	60.0	831.	0.0	891.	57.4	951.	30.0	1011.	34.7
532.	45.0	592.	45.0	652.	58.0	712.	9.6	772.	60.0	832.	0.0	892.	56.7	952.	30.0	1012.	32.1
533.	45.0	593.	45.0	653.	58.7	713.	12.8	773.	60.0	833.	0.0	893.	55.8	953.	30.0	1013.	30.4
534.	45.0	594.	45.0	654.	59.3	714.	15.9	774.	60.0	834.	0.9	894.	54.8	954.	30.0	1014.	30.0
535.	45.0	595.	45.0	655.	60.0	715.	18.8	775.	60.0	835.	4.0	895.	53.7	955.	30.0	1015.	30.0
536.	45.0	596.	45.0	656.	60.0	716.	21.3	776.	60.0	836.	6.9	896.	52.4	956.	30.0	1016.	30.0
537.	45.0	597.	45.0	657.	60.0	717.	23.5	777.	60.0	837.	9.8	897.	50.9	957.	30.0	1017.	30.0
538.	44.5	598.	45.0	658.	60.0	718.	25.2	778.	60.0	838.	12.6	898.	49.3	958.	30.0	1018.	30.0
539.	43.9	599.	45.0	659.	60.0	719.	26.7	779.	60.0	839.	15.3	899.	47.6	959.	30.0	1019.	30.0
540.	43.0	600.	45.0	660.	60.0	720.	27.8	780.	59.6	840.	17.9	900.	45.7	960.	30.0	1020.	30.0
541.	41.8	601.	45.0	661.	60.0	721.	28.7	781.	59.1	841.	20.4	901.	43.6	961.	30.0	1021.	30.0
542.	40.2	602.	44.5	662.	60.0	722.	29.6	782.	58.4	842.	22.8	902.	41.4	962.	30.0	1022.	30.0
543.	38.2	603.	43.7	663.	60.0	723.	30.0	783.	57.6	843.	25.2	903.	39.0	963.	30.0	1023.	30.0
544.	35.8	604.	42.7	664.	60.0	724.	30.0	784.	56.5	844.	27.4	904.	36.6	964.	30.0	1024.	30.0
545.	33.1	605.	41.4	665.	60.0	725.	30.0	785.	55.1	845.	29.6	905.	34.0	965.	30.7	1025.	30.0
546.	30.1	606.	39.8	666.	60.0	726.	30.0	786.	53.4	846.	31.7	906.	31.3	966.	31.8	1076.	30.0
547.	26.9	607.	37.8	667.	60.0	727.	30.0	787.	51.5	847.	33.7	907.	28.6	967.	33.1	1027.	30.0
548.	23.7	608.	35.3	668.	60.0	728.	30.0	788.	49.3	848.	35.7	908.	25.8	968.	34.5	1028.	30.0
549.	20.7	609.	32.5	669.	60.0	729.	30.0	789.	46.8	849.	37.5	909.	22.9	969.	36.0	1029.	30.0
550.	18.1	610.	29.2	670.	60.0	730.	30.0	790.	44.3	850.	39.3	910.	20.1	970.	37.6	1030.	29.4
551.	16.1	611.	25.6	671.	59.6	731.	30.0	791.	41.6	851.	41.0	911.	17.3	971.	39.3	1031.	28.5
552.	15.1	612.	21.8	672.	59.0	732.	30.0	792.	38.9	852.	42.6	912.	14.6	972.	41.0	1032.	27.2
553.	15.0	613.	17.8	673.	58.3	733.	30.0	793.	36.3	853.	44.2	913.	12.0	973.	42.8	1033.	25.4
554.	15.0	614.	13.8	674.	57.4	734.	30.0	794.	34.0	854.	45.6	914.	9.5	974.	44.6	1034.	22.9
555.	15.0	615.	9.9	675.	56.5	735.	30.0	795.	32.1	855.	47.0	915.	7.2	975.	46.3	1035.	19.9
556.	15.0	616.	6.4	676.	55.3	736.	30.0	796.	30.7	856.	48.4	916.	5.2	976.	48.1	1036.	16.4
557.	15.0	617.	3.5	677.	54.0	737.	30.0	797.	30.0	857.	49.6	917.	3.4	977.	49.8	1037.	12.5
558.	15.0	618.	1.3	678.	52.4	738.	30.0	798.	30.0	858.	50.8	918.	1.9	978.	51.5	1038.	8.6
559.	15.0	619.	0.1	679.	50.6	739.	30.5	799.	30.0	859.	51.9	919.	0.8	979.	53.0	1039.	5.0
560.	15.0	620.	0.0	680.	48.5	740.	31.4	800.	30.0	860.	52.9	920.	0.2	980.	54.6	1040.	2.1
561.	15.0	621.	0.0	681.	46.2	741.	32.4	801.	30.0	861.	53.8	921.	0.0	981.	56.0	1041.	0.3
562.	15.0	622.	0.0	682.	43.6	742.	33.6	802.	30.0	862.	54.7	922.	0.0	982.	57.2	1042.	0.0
563.	15.0	623.	0.0	683.	40.8	743.	34.8	803.	30.0	863.	55.5	923.	0.0	983.	58.4	1043.	0.0
564.	15.0	624.	0.0	684.	37.8	744.	36.0	804.	30.0	864.	56.3	924.	0.0	984.	59.3	1044.	0.0
565.	15.0	625.	0.0	685.	34.6	745.	37.4	805.	30.0	865.	56.9	925.	0.0	985.	60.0	1045.	0.0
566.	15.0	626.	0.0	686.	31.3	746.	38.7	806.	30.0	866.	57.5	926.	0.0	986.	60.0	1046.	0.0
567.	15.0	627.	0.0	687.	27.9	747.	40.2	807.	30.0	867.	58.1	927.	0.0	987.	60.0	1047.	0.0
568.	15.0	628.	0.0	688.	24.3	748.	41.6	808.	30.0	868.	58.5	928.	0.0	988.	60.0	1048.	0.0
569.	15.9	629.	0.0	689.	20.8	749.	43.1	809.	30.0	869.	58.9	929.	0.0	989.	60.0	1049.	0.0
570.	17.3	630.	0.0	690.	17.3	750.	44.6	810.	30.0	870.	59.3	930.	0.0	990.	60.0	1050.	0.0
571.	18.9	631.	2.6	691.	14.0	751.	46.0	811.	30.0	871.	60.0	931.	0.0	991.	60.0	1051.	0.0
572.	20.7	632.	7.1	692.	10.7	752.	47.5	812.	30.0	872.	60.0	932.	0.8	992.	60.0	1052.	0.0
573.	22.6	633.	11.5	693.	7.8	753.	48.9	813.	30.0	873.	60.0	933.	2.7	993.	60.0	1053.	0.0
																1054.	0.0

FEDERAL TEST PROCEDURE

TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)
0	0.0	55	15.8	110	31.2	165	6.6	220	50.0
1	0.0	56	17.7	111	31.8	166	9.9	221	50.6
2	0.0	57	19.8	112	32.2	167	13.2	222	51.0
3	0.0	58	21.6	113	32.4	168	16.5	223	51.5
4	0.0	59	23.2	114	32.2	169	19.8	224	52.2
5	0.0	60	24.2	115	31.7	170	22.2	225	53.2
6	0.0	61	24.6	116	28.6	171	24.3	226	54.1
7	0.0	62	24.9	117	25.3	172	25.8	227	54.6
8	0.0	63	25.0	118	22.0	173	26.4	228	54.9
9	0.0	64	24.6	119	18.7	174	25.7	229	55.0
10	0.0	65	24.5	120	15.4	175	25.1	230	54.9
11	0.0	66	24.7	121	12.1	176	24.7	231	54.6
12	0.0	67	24.8	122	8.8	177	25.0	232	54.6
13	0.0	68	24.7	123	5.5	178	25.2	233	54.8
14	0.0	69	24.6	124	2.2	179	25.4	234	55.1
15	0.0	70	24.6	125	0.0	180	25.8	235	55.5
16	0.0	71	25.1	126	0.0	181	27.2	236	55.7
17	0.0	72	25.6	127	0.0	182	26.5	237	56.1
18	0.0	73	25.7	128	0.0	183	24.0	238	56.3
19	0.0	74	25.4	129	0.0	184	22.7	239	56.6
20	0.0	75	24.9	130	0.0	185	19.4	240	56.7
21	3.0	76	25.0	131	0.0	186	17.7	241	56.7
22	5.9	77	25.4	132	0.0	187	17.2	242	56.5
23	8.6	78	26.0	133	0.0	188	18.1	243	56.5
24	11.5	79	26.0	134	0.0	189	18.6	244	56.5
25	14.3	80	25.7	135	0.0	190	20.0	245	56.5
26	16.9	81	26.1	136	0.0	191	22.2	246	56.5
27	17.3	82	26.7	137	0.0	192	24.5	247	56.5
28	18.1	83	27.5	138	0.0	193	27.3	248	56.4
29	20.7	84	28.6	139	0.0	194	30.5	249	56.1
30	21.7	85	29.3	140	0.0	195	33.5	250	55.8
31	22.4	86	29.8	141	0.0	196	36.2	251	55.1
32	22.5	87	30.1	142	0.0	197	37.3	252	54.6
33	22.1	88	30.4	143	0.0	198	39.3	253	54.2
34	21.5	89	30.7	144	0.0	199	40.5	254	54.0
35	20.9	90	30.7	145	0.0	200	42.1	255	53.7
36	20.4	91	30.5	146	0.0	201	43.5	256	53.6
37	19.8	92	30.4	147	0.0	202	45.1	257	53.9
38	17.0	93	30.3	148	0.0	203	46.0	258	54.0
39	14.9	94	30.4	149	0.0	204	46.8	259	54.1
40	14.9	95	30.8	150	0.0	205	47.5	260	54.1
41	15.2	96	30.4	151	0.0	206	47.5	261	53.8
42	15.5	97	29.9	152	0.0	207	47.3	262	53.4
43	16.0	98	29.5	153	0.0	208	47.2	263	53.0
44	17.1	99	29.8	154	0.0	209	47.0	264	52.6
45	19.1	100	30.3	155	0.0	210	47.0	265	52.1
46	21.1	101	30.7	156	0.0	211	47.0	266	52.4
47	22.7	102	30.9	157	0.0	212	47.0	267	52.0
48	22.9	103	31.0	158	0.0	213	47.0	268	51.9
49	22.7	104	30.9	159	0.0	214	47.2	269	51.7
50	22.6	105	30.4	160	0.0	215	47.4	270	51.5
51	21.3	106	29.8	161	0.0	216	47.9	271	51.6
52	19.0	107	29.9	162	0.0	217	48.5	272	51.8
53	17.1	108	30.2	163	0.0	218	49.1	273	52.1
54	15.8	109	30.7	164	3.3	219	49.5	274	52.5

TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)	TIME(SEC)	SPEED(MPH)
275	53.0	330	8.0	385	33.5	440	0.0	495	28.0
276	53.5	331	4.7	386	31.4	441	0.0	496	25.5
277	54.0	332	1.4	387	29.0	442	0.0	497	22.5
278	54.9	333	0.0	388	25.7	443	0.0	498	19.8
279	55.4	334	0.0	389	23.0	444	0.0	499	16.5
280	55.6	335	0.0	390	20.3	445	0.0	500	13.2
281	56.0	336	0.0	391	17.5	446	0.0	501	10.3
282	56.0	337	0.0	392	14.5	447	0.0	502	7.2
283	55.8	338	0.0	393	12.0	448	3.3	503	4.0
284	55.2	339	0.0	394	8.7	449	6.6	504	1.0
285	54.5	340	0.0	395	5.4	450	9.9		
286	53.6	341	0.0	396	2.1	451	13.2		
287	52.5	342	0.0	397	0.0	452	16.5		
288	51.5	343	0.0	398	0.0	453	19.8		
289	51.5	344	0.0	399	0.0	454	23.1		
290	51.5	345	0.0	400	0.0	455	26.4		
291	51.1	346	0.0	401	0.0	456	27.8		
292	50.1	347	1.0	402	0.0	457	29.1		
293	50.0	348	4.3	403	2.6	458	31.5		
294	50.1	349	7.6	404	5.9	459	33.0		
295	50.0	350	10.9	405	9.2	460	33.6		
296	49.6	351	14.2	406	12.5	461	34.8		
297	49.5	352	17.3	407	15.8	462	35.1		
298	49.5	353	20.0	408	19.1	463	35.6		
299	49.5	354	22.5	409	22.4	464	36.1		
300	49.1	355	23.7	410	25.0	465	36.6		
301	48.6	356	25.2	411	25.6	466	36.1		
302	48.1	357	26.6	412	27.5	467	36.2		
303	47.2	358	28.1	413	29.0	468	36.0		
304	46.1	359	30.0	414	30.0	469	35.7		
305	45.0	360	30.8	415	30.1	470	36.6		
306	43.8	361	31.6	416	30.0	471	36.0		
307	42.6	362	32.1	417	29.7	472	35.0		
308	41.5	363	32.8	418	29.3	473	35.5		
309	40.3	364	33.6	419	28.8	474	35.4		
310	38.5	365	34.5	420	28.0	475	35.2		
311	37.0	366	34.6	421	25.0	476	35.2		
312	35.2	367	34.9	422	21.7	477	35.2		
313	33.8	368	34.8	423	18.4	478	35.2		
314	32.5	369	34.5	424	15.1	479	35.2		
315	31.5	370	34.7	425	11.8	480	35.2		
316	30.6	371	35.5	426	8.5	481	35.0		
317	30.5	372	36.0	427	5.2	482	35.1		
318	30.0	373	36.0	428	1.9	483	35.2		
319	29.0	374	36.0	429	0.0	484	35.5		
320	27.5	375	36.0	430	0.0	485	35.2		
321	24.8	376	36.0	431	0.0	486	35.0		
322	21.5	377	36.0	432	0.0	487	35.0		
323	20.1	378	36.1	433	0.0	488	35.0		
324	19.1	379	36.4	434	0.0	489	34.8		
325	18.5	380	36.5	435	0.0	490	34.6		
326	17.0	381	36.4	436	0.0	491	34.5		
327	15.5	382	36.0	437	0.0	492	33.5		
328	12.5	383	35.1	438	0.0	493	32.0		
329	10.8	384	34.1	439	0.0	494	30.1		

APPENDIX III

THE MATHEMATICAL MODEL

The mathematical model presents a method to calculate the amount of a particular pollutant given off by a vehicle or vehicle group over any specified driving sequence given the modal emissions data on the individual vehicles.

<u>Notation</u>

$e(t)$ = amount of a pollutant given off by a vehicle from time = 0 to time = t.

$e(t) = \frac{de(t)}{dt}$ = instantaneous emission rate function.

$e_A(t)$ = accel/decel instantaneous emission rate function.

$e_s(t)$ = steady state instantaneous emission rate function.

E_j = amount of a pollutant given off by a vehicle in the j'th mode.

T_j = duration of jth mode. (in seconds)

$V_j(t)$ = speed at time = t in the j'th mode

$v(t)$ = speed at time = t for any driving sequence

$a_j(t) = \frac{dv_j(t)}{dt}$ = acceleration at time = t in the j'th mode

$a(t) = \frac{dv(t)}{dt}$ = acceleration at time = t in any driving sequence.

$\langle \dot{e}(v,a) \rangle_{T_j} = \frac{1}{T_j} \int_0^{T_j} \dot{e}\, dt$ = average value of the emission rate function for the j'th mode.

(A) Functional Form of the Steady State and Accel/Decel Emission Rate Functions

Based on Figures 1, 2, 3 (Appendix VI) where average steady state emission rates over the 1020 vehicles in the data base are plotted versus speed, the assumption is made that the steady state emission rate function $\dot{e}_S$ can be represented as a quadratic function of speed:

(1) $$\dot{e}_S(V) = S_1 + S_2 V + S_3 V^2$$

where S_1, S_2, and S_3 are constants.

The functional form of an emission rate function that will be applicable during periods of acceleration/deceleration is obtained by assuming the constant coefficients (S_i) become functions of the acceleration and that this functional dependence can be sufficiently represented by quadratic functions of acceleration. Therefore:

(2) $$\begin{aligned} S_1 &= S_1(a) = q_{11} + q_{12}\,a + q_{13}\,a^2 \\ S_2 &= S_2(a) = q_{21} + q_{22}\,a + q_{23}\,a^2 \\ S_3 &= S_3(a) = q_{31} + q_{32}\,a + q_{33}\,a^2 \end{aligned}$$

where the q's are constants
and

(3) $$\dot{e}_A(V,a) = S_1(a) + S_2(a)\,V + S_3(a)\,V^2$$

(4) $$\dot{e}_A(V,a) = (q_{11} + q_{12}a + q_{13}a^2) + (q_{21} + q_{22}a + q_{23}a^2)\cdot V + (q_{31} + q_{32}a + q_{33}a^2)\cdot V^2$$

Upon multiplying and redefining the constant coefficients we have the following expression for the accel/decel emission rate function

(5) $$\dot{e}_A(V,a) = b_1 + b_2 V + b_3 a + b_4 Va + b_5 V^2 + b_6 a^2 + b_7 V^2 a + b_8 a^2 V + b_9 V^2 a^2$$

(B) The Instantaneous Emission Rate Function

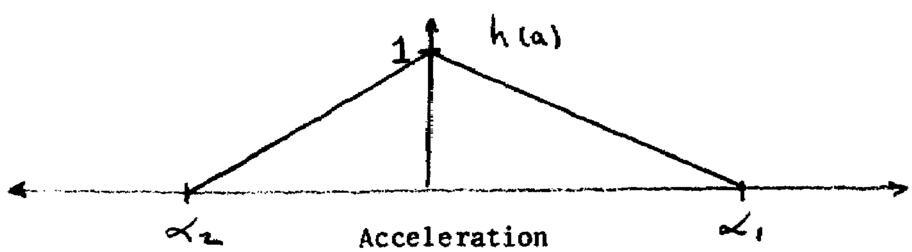
Although the accel/decel emission rate function is functionally identical to the steady-state rate function when the acceleration is zero, both functions will be retained to allow independent adjustment of the coefficients (b_i and S_i) specifying the accel/decel and steady state emission rate functions.

The two functions can be joined by an acceleration dependent weighting function h(a) to form the instantaneous emission rate function:

(6) $$\dot{e}(v,a) = h(a)\,\dot{e}_S(v) + (1-h(a))\,\dot{e}_A(v,a)$$

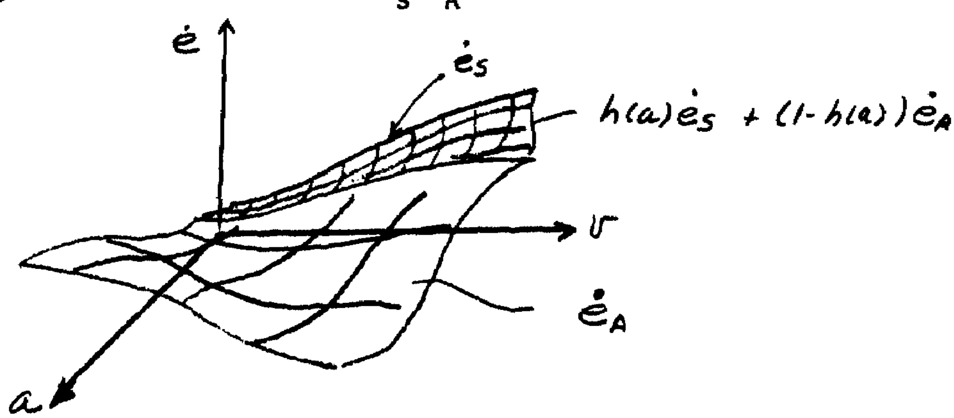
where h(a) =
$$\begin{cases} -\frac{1}{\alpha_1}a + 1, & \alpha_1 > a > 0 \\ -\frac{1}{\alpha_2}a + 1, & \alpha_2 < a < 0 \\ 0, & a \geq \alpha_1 \\ 0, & a \leq \alpha_2 \end{cases}$$

or



By specifying the constants $\alpha 1$ and $\alpha 2$ the weightings of the two rate functions will vary between 0 and 1 in a continuous manner when the transition is made between accel/decel and steady state periods of driving. If the accel/decel and steady state emission rate functions are thought of as a response surface S in (v,a)-space then $h(a)\dot{e}_S + (1 - h(a))\dot{e}_A$: $h(a) \neq 0$ can be visualized as a ramp function that joins the two responses $\dot{e}_S, \dot{e}_A$.

Ex:



In practice α_1 and α_2 have been arbitrarily set to -1.2 sec and 1.0 mph/sec respectively.

(C) Specification of The Emission Rate Function

Given the functional form of the emission rate function, the coefficients (b_i, S_i) must be determined in order to specify the emission rate function which characterizes a vehicle's emission response.

(1) Determination of The Accel/Decel Emission Rate Function Coefficients (b_i)

The only modal emission observations available are the total amount of each pollutant given off in each mode of the SDS; there are no measures of the instantaneous emission rate given for accel/decel modes. Therefore, the following procedure is used to determine the coefficients (b_i) that specify the instaneous emission rate function for a given vehicle and pollutant:

Let:

$$f_1 = 1.0 \,, \quad f_2 = v \,, \quad f_3 = a$$

$$f_4 = av \,, \quad f_5 = v^2 \,, \quad f_6 = a^2$$

$$f_7 = v^2 a \,, \quad f_8 = a^2 v \,, \quad f_9 = v^2 a^2$$

where the $f_i \,, \; i = 1, 9 \Rightarrow$ basis functions of the accel/decel emission rate function.

Then using equation (5) we have:

(6) $$\dot{e}_A(v,a) = \sum_{i=1}^{9} b_i f_i$$

For an accel/decel mode the weighting function h(a) $\simeq$ 0, then:

(7) $$\dot{e}_A(v,a) \cong \dot{e}(v,a)$$

Now consider the average emission rate over the k' th mode:

(8) $$\langle \dot{e}(v,a) \rangle_{T_k} = \frac{1}{T_k} \int_0^{T_k} \dot{e}(v,a)\, dt$$

by equation (7)

$$(9) \quad \langle \dot{e}(v,a) \rangle_{T_k} = \langle \dot{e}_A(v,a) \rangle_{T_k} = \frac{1}{T_k} \int_0^{T_k} \dot{e}_A(v,a)\, dt$$

However, the average emission rate over the k th mode is also equal to the total emission amount (E_k) divided by the time in mode T_k ---- both observable quantities. Therefore:

$$(10) \quad \langle \dot{e}_A(v,a) \rangle_{T_k} = E_k / T_k$$

and

$$(11) \quad \frac{E_k}{T_k} = \frac{1}{T_k} \int_0^{T_k} \dot{e}_A(v,a)\, dt = \frac{1}{T_k} \int_0^{T_k} \left(\sum_i b_i f_i \right) dt$$

Since the b_i are constants:

$$(12) \quad E_k / T_k = 1/T_k \sum b_i \left(\int_0^{T_k} f_i\, dt \right)$$

Now, $1/T_k \int_0^{T_k} f_i\, dt$ is just the average value of the i'th basis function over the K'th mode, which can be easly evaluated knowing the speed as function of time for the k'th mode $v_k(t)$, and acceleration as a function of time, $a_k(t)$.

Let, $$\bar{f}_{ik} = \frac{1}{T_k} \int_0^{T_k} f_i\, dt$$

(see Appendix I for values of $\bar{f}_{ik}$)

Thus,

$$(13) \quad \frac{E_k}{T_k} = \sum_{i=1}^{9} b_i \bar{f}_{ik}$$

and since E_K/T_K is known for all modes and the $\bar{f}_{ik}$ can be evaluated then the b_i can be determined using standard least squares regression techniques.

The result of the least squares regression method can best be given by defining the following matrices and elements:

$Y, Y(k) = \frac{E_k}{T_k}$ (response) k =1, 32

(32 accel/decel modes)

$X, X(i,k) = \bar{f}_{ik}$ $\quad$ i = 1,9

$B, B(i) = b_i$

and using the convention

$Y' =$ Transpose of Y

$Y^{-1} =$ Inverse of Y

we have by equation (13):

(14) $Y = BX$

The method of least squares then gives:

(15) $B = (X' X)^{-1} X' Y$

Let: $A_A \equiv (X' X)^{-1} X'$ $\quad$ (A_A basis function factor array)

Then, (16) $B = A_A Y$

and the i'th coefficient is given by (b_i):

(16) $b_i = \sum_{i=1}^{32} A_A (i,k)\ Y(k)$ $\quad$, i = 1, 9

In summary, the average emission rate, which can be evaluated by using experimental observables, is used to determine the coefficients that specify the instantaneous accel/decel emission rate function.

(2) Determination of the Steady State Emission Rate Function Coefficients (S_i)

In the case of steady state emissions the average emission rate is equal to the instantaneous emission rate because the speed is constant in time.

Let: $f_{1k} = 1.0$, $f_{2k} = V_k$, $f_{3k} = V_k^2$

then, by equation (1):

(17) $$\dot{e}_S(V_k) = \sum_{i=1}^{3} S_i f_{ik}$$

Consider the k'th steady state mode , $h(a) \simeq 1.0$

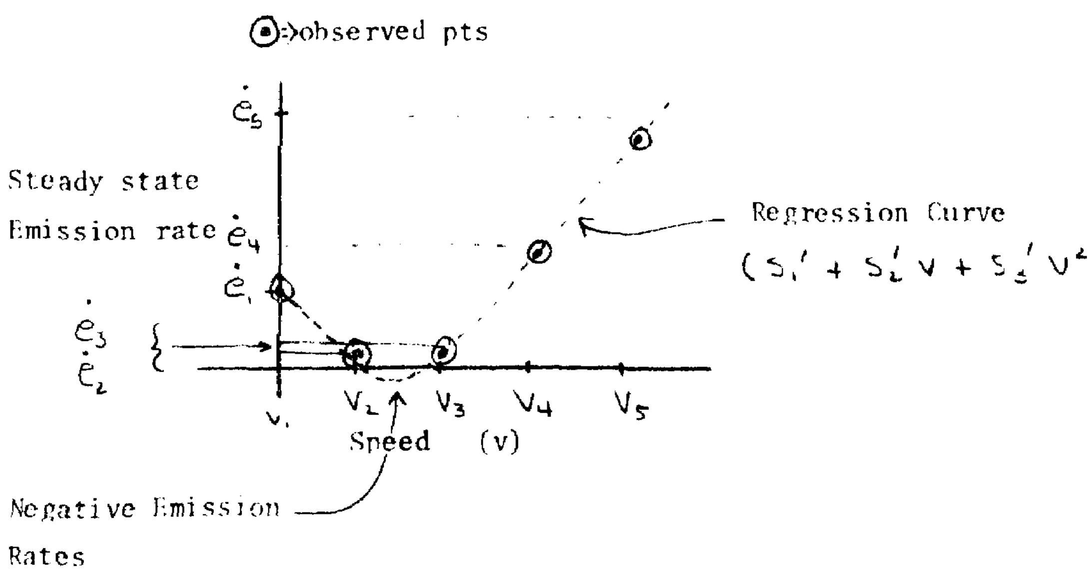
(18)
$$\langle \dot{e}(v,a) \rangle_{T_k} = \langle \dot{e}_S(v) \rangle_{T_k} = \frac{E_k}{T_k} = \frac{1}{T_k} \int_0^{T_k} \left(\sum S_i f_{ik} \right) dt$$

since both S_i and f_{ik} are now constants

(19) $$\frac{E_k}{T_k} = \sum_i S_i f_{ik} \int_0^{T_k} dt \Big/ T_k = \sum_i S_i f_{ik}$$

The coefficients can now be determined using standard least squares regression techniques.

Consider the situation in which the S_i have been determined by the above procedure and the specified emission rate function produces negative emission rates as shown below.



When this happens (each steady state emission rate function is tested for this possibility) a new set of coefficients (S_i) is calculated in a manner that will guarantee the steady state emission rate function will be positive for all values of speed between 0 and 60 mph. The procedure to determine the new set of coefficients is described below.

The minimum of the regression curve representing the steady state emission rate function is forced throught the point given by the average of the 2 lowest emission rates and the average of their speeds. In our example above this point $(\bar{e}, \bar{v})$ is given by

$$\bar{e} = \dot{e}_2 + \dot{e}_3 / 2 \quad , \quad \bar{v} = v_2 + v_3 / 2$$

Requiring the minimum of the regression curve (steady state emission rate function) to go through $(\bar{e}, \bar{v})$ specifys two of the three coefficients. Since $\dot{e}_s$ is a quadratic function of speed, the coordinates of the minimum are given by:

$$\left(\frac{4 S_1 S_3 - S_2^2}{4 S_3} \, , \; - \frac{S_2}{2 S_3} \right)$$

Therefore

$$\bar{v} = - S_2 / 2 S_3$$

$$\bar{e} = 4 S_1 S_3 - S_2^2 / 4 S_3$$

Solving for S_1 and S_2 in terms of S_3, v and e we obtain:

$$S_2 = - 2 \bar{v} S_3$$

$$S_1 = \bar{e} + \bar{v}^2$$

substituting these values into equation (1)

(20) $$\dot{e}_s (v) = \bar{e} + \bar{v}^2 S_3 + (-2 \bar{v} S_3) v + S_3 v^2$$

Regrouping:

(21) $$\dot{e}_s (v) = \bar{e} + S_3 (\bar{v}^2 - 2 \bar{v} v + v^2)$$

The only coefficient left to determine is now S_3 which is obtained by the standard least squares method.

This method will remove any negative emission rates over the 0 to 60 mph speed interval and still retain the general trend of the observed data.

(D) Characterization of Individual Vehicle and Vehicle Group Emission Responses

Assuming input modal data is available, each vehicle's emission response can be characterized by the 36 coefficients that specify the three emission rate functions for HC, CO and NO_x (12 coefficients per function).

A vehicle group can be similarly characterized by determining the coefficients for the average vehicle in the group.

Let: b_{ijk} = k'th coefficient of the emission rate function for the j th vehicle in group sample and i th kind of pollutant

N_g = number of vehicle in sample representing group

$\bar{b}_{ik}$ = k'th coefficient of the emission rate function for the average vehicle for i th kind of pollutant

The coefficients describing the average vehicle's emission rate function is thus given by:

(22)
$$\bar{b}_{ik} = \frac{1}{N_g} \sum_{j=1}^{N_g} b_{ijk}$$

(e) Determination of Individual Vehicle and Vehicle Group Emissions Over a Specified Driving Sequence

The amount of a pollutant e(T) given off by a vehicle in undergoing a driving sequence from time = 0 to time = T is obtained by integrating the instantaneous emission rate function describing this vehicle's response with respect to pollutant under consideration over the sequence:

(23)
$$\dot{e}(T) = \int_0^T \dot{e}(v,a)\,dt = \int_0^T \left(h(a)\,\dot{e}_S(v) + (1-h(a))\,\dot{e}_A(v,a)\right) dt$$
$$= \int_0^T \Big\{ h(a)(S_1 + S_2 v + S_3 v^2) + (1-h(a))(b_1 + b_2 v + b_3 a + b_4 a v + b_5 v^2 + b_6 a^2 + b_7 v^2 a + b_8 a^2 v + b_9 a^2 v^2) \Big\}\, dt$$

The integral is then approximated by the following summation:

$$(24) \quad e(T) = \sum_{i=1}^{N} \Big\{ h(a_i)(s_1 + s_2 v_i + s_3 v_i^2) + (1 - h(a_i))(b_1 + b_2 v_i + b_3 a_i + b_4 a_i v_i + b_5 v_i^2 + b_6 a_i^2 + b_7 v_i^2 a_i + b_8 a_i^2 v_i + b_9 v_i^2 a_i^2) \Big\}$$

where

$$N \Delta t = T$$

$$(25) \quad v_i = v(t_i) + v(t_{i+1}) / 2$$

$$a_i = v(t_{i+1}) - v(t_i) / \Delta t$$

The total amount of a pollutant given off by a vehicle group is obtained by integrating the average vehicle's emission rate function over the sequence and then multiplying the average vehicle's emission response by the number of vehicles in the group.

APPENDIX IV

Program Listings

(1) Main Program I

(2) Main Program II

(3) Subroutine SETUP

(4) Subroutine EDOT

(5) Subroutine PAD

(6) Subroutine ESUM

(7) Subroutine EDGRP

(8) Subroutine INVERS

```
          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                              SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                   C      ******************  MAIN PROGRAM I   **************************
                   C      MAIN PROGRAM I DETERMINES THE AMOUNT OF EMISSIONS GIVEN OFF BY
                   C      INDIVIDUAL VEHICLES OVER A DRIVING SEQUENCE SPECIFIED BY ARR'VVT'.
                   C
                   C      VTM(I)=>VELOCITY VS. TIME(IN ONE SECOND INTERVALS) OF THE SURVEIL-
                   C      -LANCE DRIVING SEQUENCE.VTM(I)=VELOCITY(MPH) AT TIME (I-1)SEC
                   C      (REAL*4)
                   C
                   C      VVT(I)=>VELOCITY VS. TIME(IN ONE SECOND INTERVALS) OF ANY DRIVING
                   C      SEQUENCE OVER WHICH  EMISSIONS ARE TO BE CALCULATED.VVT(I)=VELOC-
                   C      -ITY AT TIME (I-1) SEC. (REAL*4)
                   C
                   C      AMTC(I,J)=> AMOUNT OF I'TH EMMITTANT GIVEN OFF IN J'TH MODE.
                   C
                   C      DS(I)=DISTANCE(MILES)TRAVELED IN I'TH MODE.NOTE,STEADY STATE MODES
                   C      ARE 60 SEC IN DURATION.
                   C
                   C      ***************************************************************
  ISN 0003                DIMENSION ITAB(20,2),IDAT(4,26),RDAT(121,26),DS(37)
  ISN 0004                DIMENSION VTM(1055),VVT(2000),AMTC(3,37),C(3)
  ISN 0005                REAL*8 AA(9,32),AS(3,5),BAD(3,12)
  ISN 0006                DATA DS          /.0602,.0741,.0201,.0705,.1360,.1268,.2163,.1716
                         C,.2043,.3367,.3136,.1973,.3313,.2994,.0579,.0173,.1759,.1392,.1528
                         C,.1304,.2654,.2634,.0737,.3134,.2362,.0444,.4009,.3293,.0886,.2599
                         C,.1813,.0592,.0000,.2500,.5000,.7500,1.000/
  ISN 0007                DEFINE FILE 99(75,3256,U,N1)
                   C
                   C       READ IN SURVEILLANCE DRIVING SEQUENCE
                   C
  ISN 0008                DO 3000 I=1,100
  ISN 0009                NX1=((I-1)*16)+1
  ISN 0010                NX2=NX1+15
  ISN 0011                READ(5,100)(VTM(K),K=NX1,NX2)
  ISN 0012            100 FORMAT(16F5.0)
  ISN 0013                IF(VTM(NX1).GT.99.0)GOTO3111
  ISN 0015           3000 CONTINUE
  ISN 0016           3111 CONTINUE
                   C
                   C      READ IN DRIVING SEQUENCE OVER WHICH EMMISSIONS ARE TO BE CALCULATED
                   C
                   C      IN THIS EXAMPLE VVT=> FIRST 505 SEC. OF FTP
                   C
  ISN 0017                DO 1500 I=1,100
  ISN 0018                NX1=((I-1)*16)+1
  ISN 0019                NX2=NX1+15
  ISN 0020                READ(5,100)(VVT(K),K=NX1,NX2)
  ISN 0021                IF(VVT(NX1).GT.99.0)GOTO1555
  ISN 0023           1500 CONTINUE
  ISN 0024           1555 CONTINUE
                   C
                   C      SET UP BASIS FUNCTION FACTOR ARRAYS AA,AS.
                   C
  ISN 0025                CALL SETUP(VTM,AA,AS)
                   C
  ISN 0026                WRITE(6,500)
```

```
ISN 0027        500 FORMAT(1H1)
            C       READ IN INDIVIDUAL VEHICLE MODAL EMISSIONS DATA.
            C       PUT MODAL EMISSIONS DATA INTO ARRAY AMTC
            C
            C        IN THIS EXAMPLE THE MODAL EMSSIONS DATA IS READ OFF A DISK FILE
            C
ISN 0028            NCART=0
ISN 0029            READ(99'75)ITAB
ISN 0030            DO 2000 IY=57,71
ISN 0031            IREC=IY-56
ISN 0032            JSTART=ITAB(IREC,1)
ISN 0033            JEND=ITAB(IREC,2)
ISN 0034            DO 2001 J=JSTART,JEND
ISN 0035            READ(99'J)((IDAT(L,K),L=1,4),(RDAT(L,K),L=1,121),K=1,26)
ISN 0036            DO 2002 K=1,26
ISN 0037            IF(IDAT(1,K).EQ.-9)GOTO2001
ISN 0039            NCART=NCART+1
ISN 0040            DO 1000 IR=1,37
ISN 0041            DD=1.0
ISN 0042            IF(IR.LE.32)DD=DS(IR)
ISN 0044            DO 1001 IC=1,3
ISN 0045            IW=((IR-1)*3)+10+IC
ISN 0046            AMTC(IC,IR)=RDAT(IW,K)*DD
ISN 0047       1001 CONTINUE
ISN 0048       1000 CONTINUE
            C
            C       DETERMINE INDIVIDUAL VEHICLE EMISSION RATE FUNCTION COEFFICIENTS
            C
ISN 0049            CALL EDOT(AMTC,AA,AS,BAD)
            C
            C
            C       INTEGRATE THIS VEHICLES EMISSION RATE FUNCTION OVER THE
            C       DRIVING SEQUENCE.
            C
            C
ISN 0050            CALL ESUM(VVT,506,BAD,C(1),C(2),C(3),DIST)
            C
            C
            C       WRITE OUT EMISSION RESULTS....
            C
ISN 0051            WRITE(6,501)NCART,(C(L),L=1,3)
ISN 0052        501 FORMAT(1H ,'CAR=',I10,2X,'HC=',F10.2,2X,'CO=',F10.2,2X,'NOX=',F10.
                  C2,)
ISN 0053       2002 CONTINUE
ISN 0054       2001 CONTINUE
ISN 0055       2000 CONTINUE
ISN 0056       1234 STOP
ISN 0057            END
```

```
        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                          SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
                C      **********************  MAIN PROGRAM II   ************************
                C
                C
                C      MAIN PROGRAM II DETERMINES THE AMOUNT OF EMISSIONS GIVEN OFF BY
                C      A GROUP OF VEHICLES  SPECIFIED BY THE FILTER OVER THE DRIVING SEQ.
                C      SPECIFIED BY ARRAY ' VVT'.
                C
                C      VTM(I)=>VELOCITY VS. TIME(IN ONE SECOND INTERVALS) OF THE SURVEIL-
                C      -LANCE DRIVING SEQUENCE.VTM(I)=VELOCITY(MPH) AT TIME (I-1)SEC
                C      (REAL*4)
                C
                C      VVT(I)=>VELOCITY VS. TIME(IN ONE SECOND INTERVALS) OF ANY DRIVING
                C      SEQUENCE OVER WHICH EMMISSIONS ARE TO BE CALCULATED.VVT(I)=VELOC-
                C      -ITY AT TIME (I-1) SEC. (REAL*4)
                C
                C      AMTC(I,J)=> AMOUNT OF I'TH EMMITTANT GIVEN OFF IN J'TH MODE.
                C
                C      DS(I)=DISTANCE(MILES)TRAVELED IN I'TH MODE.NOTE,STEADY STATE MODES
                C      ARE 60 SEC IN DURATION.
                C
                C      ******************************************************************
  ISN 0002             DIMENSION ITAB(20,2),IDAT(4,26),RDAT(121,26),DS(37)
  ISN 0003             DIMENSION VTM(1055),VVT(2000),AMTC(3,37),C(3)
  ISN 0004             REAL*8 AA(9,32),AS(3,5),BAD(3,12)
  ISN 0005             DATA DS             /.0602,.0741,.0201,.0705,.1360,.1268,.2163,.1716
                     C,.2043,.3367,.3136,.1973,.3313,.2994,.0579,.0173,.1759,.1392,.1528
                     C,.1304,.2654,.2634,.0737,.3134,.2362,.0444,.4009,.3293,.0886,.2599
                     C,.1813,.0592,.0000,.2500,.5000,.7500,1.000/
  ISN 0006             DEFINE FILE 99(75,3256,U,N1)
                C
                C       READ IN SURVEILLANCE DRIVING SEQUENCE
                C
  ISN 0007             DO 3000 I=1,100
  ISN 0008             NX1=((I-1)*16)+1
  ISN 0009             NX2=NX1+15
  ISN 0010             READ(5,100)(VTM(K),K=NX1,NX2)
  ISN 0011         100 FORMAT(16F5.0)
  ISN 0012             IF(VTM(NX1).GT.99.0)GOTO3111
  ISN 0014        3000 CONTINUE
  ISN 0015        3111 CONTINUE
                C
                C      READ IN DRIVING SEQUENCE OVER WHICH EMMISSIONS ARE TO BE CALCULATED
                C
                C      IN THIS EXAMPLE VVT=> FIRST 505 SEC. OF FTP
                C
  ISN 0016             DO 1500 I=1,100
  ISN 0017             NX1=((I-1)*16)+1
  ISN 0018             NX2=NX1+15
  ISN 0019             READ(5,100)(VVT(K),K=NX1,NX2)
  ISN 0020             IF(VVT(NX1).GT.99.0)GOTO1555
  ISN 0022        1500 CONTINUE
  ISN 0023        1555 CONTINUE
                C
                C      SET UP BASIS FUNCTION FACTOR ARRAYS AA,AS.
                C
```

```
ISN 0024          CALL SETUP(VTM,AA,AS)
            C
            C      READ IN INDIVIDUAL VEHICLE MODAL EMISSION DATA
            C     FILTER VEHICILES FOR GROUP REPRESENTATIVES
            C
            C      IN  THIS EXAMPLE THE DATA IS BEING READ OFF A DISC FILE AND THE
            C     INIVIDUAL VEHICLE MODAL EMISSION DATA IS PUT INTO ARRAY AMTC
            C     IN THIS EXAMPLE THE GROUP IS= DENVER,PRE EMISSION CONTROL.
            C
ISN 0025          NVIG=0
ISN 0026          READ(99'75)ITAB
ISN 0027          DO 2000 IY=57,71
ISN 0028          IREC=IY-56
ISN 0029          JSTART=ITAB(IREC,1)
ISN 0030          JEND=ITAB(IREC,2)
ISN 0031          DO 2001 J=JSTART,JEND
ISN 0032          READ(99'J)((IDAT(L,K),L=1,4),(RDAT(L,K),L=1,121),K=1,26)
ISN 0033          DO 2002 K=1,26
ISN 0034          IF(IDAT(1,K).EQ.-9)GOTO2001
            C
            C      FILTER VEHICLES=> PASS ONLY DENVER,PRE EMISSION CONTROL=>PRE67
            C
ISN 0036          NYR=IDAT(1,K)/1000000
ISN 0037          NLOC=IDAT(3,K)/100000
ISN 0038          IF(NYR.LE.67.AND.NLOC.EQ.5)GOTO1313
ISN 0040          GOTO2002
ISN 0041     1313 NVIG=NVIG+1
ISN 0042          DO 1000 IR=1,37
ISN 0043          DD=1.0
ISN 0044          IF(IR.LE.32)DD=DS(IR)
ISN 0046          DO 1001 IC=1,3
ISN 0047          IW=((IR-1)*3)+10+IC
ISN 0048          AMTC(IC,IR)=RDAT(IW,K)*DD
ISN 0049     1001 CONTINUE
ISN 0050     1000 CONTINUE
            C
            C     DETERMINE INDIVIDUAL VEHICLE EMISSION RATE FUNCTION COEFFICIENTS
            C
ISN 0051          CALL EDOT(AMTC,AA,AS,BAD)
            C
            C     ENTER THIS VEHICLES'S COEFFICIENTS INTO THE GROUP'S EMISSION RATE
            C     FUNCTION
            C
ISN 0052          IF(NVIG.EQ.1)INT=1
ISN 0054          IF(NVIG.GT.1)INT=2
ISN 0056          CALL EDGRP(BAD,INT)
            C
            C
ISN 0057     2002 CONTINUE
ISN 0058     2001 CONTINUE
ISN 0059     2000 CONTINUE
            C
            C     NOW THAT ALL VEHICLES IN GROUP HAVE BEEN CONSIDERED,INT=3
            C     DETERMINE THIS GROUP'S EMISSION RATE FUNCTION COEFFICIENTS.
            C
ISN 0060          CALL EDGRP(BAD,3)
            C
            C
```

```
            C       INTEGRATE THIS GROUP'S EMISSION RATE FUNCTION OVER THE DRIVING
            C       SEQUENCE.
            C
ISN 0061            CALL ESUM(VVT,506,BAD,C(1),C(2),C(3),DIST)
            C
            C       DETERMINE TOATAL EMISSION=>MULTIPLY EACH EMISSION AMOUNT BY NO.
            C       VEHICLES IN GROUP
            C
ISN 0062            VN=NVIG
ISN 0063            DO 5030 I=1,3
ISN 0064       5030 C(I)=C(I)*VN
            C       WRITE OUT EMISSION RESULTS....
ISN 0065            WRITE(6,500)(C(L),L=1,3)
ISN 0066        500 FORMAT(1H ,'HC=',F10.2,2X,'CO=',F10.2,2X,'NOX=',F10.2)
ISN 0067            STOP
ISN 0068            END
```

```
        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
ISN 0002                   SUBROUTINE SETUP(VTM,AA,AS)
                 C         ******************************************************************
                 C
                 C         SUBROUTINE SETUP COMPUTES THE BASIS FUNCTION FACTOR ARRAYS FOR
                 C         ACCEL/DECEL AND STEADY STATE EMISSION RATE FUNCTION DETERMINATIONS
                 C         ,ARRAYS 'AA' AND 'AS' RESPECTIVELY, GIVEN THE VELOCITY VS. TIME
                 C         HISTORY OF THE SURVEILLANCE DRIVING SEQUENCE(ARRAY VTM).
                 C
                 C         VTM(I)=>VELOCITY VS. TIME(IN ONE SEC. INTERVALS) OF THE SURVEILL-
                 C         ANCE DRIVING SEQUENCE. VTM(I)=VELOCITY AT TIME I.(REAL*4)
                 C
                 C         MVT(I)=TIME I'TH ACCEL/DECEL MODE STARTS IN THE SURVEILLANCE
                 C         DRIVING SEQUENCE.(REAL*4)
                 C
                 C         TM(I)=TIME(SEC) IN I'TH  ACCEL/DECEL MODE.(REAL*4)
                 C
                 C         AA=> BASIS FUNCTION FACTOR ARRAY FOR ACCEL/DECEL.(REAL*8)
                 C
                 C         AS=>BASIS FUNCTION FACTOR ARRAY FOR STEADY STATE.(REAL*8)
                 C
                 C         ******************************************************************
ISN 0003                   DIMENSION VTM(1055),MVT(32),TM(32)
ISN 0004                   REAL*8 X(32,9),SV,SA,TMD,C(9,9),AA(9,32),SUM,AS(3,5)
ISN 0005                   DATA MVT/11.,38.,64.,87.,113.,141.,167.,199.,227.,257.,302.,343.,
                          C374.,421.,459.,483.,501.,538.,569.,602.,631.,671.,709.,739.,780.,
                          C814.,834.,887.,932.,965.,1001.,1030./
ISN 0006                   DATA TM/12.,16.,8.,11.,13.,12.,17.,12.,14.,30.,26.,21.,32.,23.,9.,
                          C8.,22.,16.,18.,19.,25.,28.,15.,25.,18.,10.,38.,35.,18.,21.,14.,13.
                          C/
ISN 0007                   NOBSA=32
ISN 0008                   NOBSS=5
ISN 0009                   NBFA=9
ISN 0010                   NBFS=3
                 C
                 C         ****** CALCULATE AA  *************
                 C
                 C         CALCULATE BASIS FUNCTION ARRAY X
                 C
ISN 0011                   DO 1000 IM=1,NOBSA
ISN 0012                   NTS=MVT(IM)
ISN 0013                   NTM=TM(IM)
ISN 0014                   NTF=NTS+NTM-1
ISN 0015                   DO 999 IK=2,NBFA
ISN 0016               999 X(IM,IK)=0.0D0
ISN 0017                   X(IM,1)=1.0D0
ISN 0018                   DO 1001 IT=NTS,NTF
ISN 0019                   KT=IT+1
ISN 0020                   SV=(VTM(IT)+VTM(KT))/2.0
ISN 0021                   SA=VTM(KT)-VTM(IT)
ISN 0022                   X(IM,2)=X(IM,2)+SV
ISN 0023                   X(IM,3)=X(IM,3)+SA
ISN 0024                   X(IM,4)=X(IM,4)+(SV*SA)
ISN 0025                   X(IM,5)=X(IM,5)+(SV**2)
ISN 0026                   X(IM,6)=X(IM,6)+(SA**2)
ISN 0027                   X(IM,7)=X(IM,7)+((SV**2)*SA)
ISN 0028                   X(IM,8)=X(IM,8)+((SA**2)*SV)
```

```
ISN 0029            X(IM,9)=X(IM,9)+((SV**2)*(SA**2))
ISN 0030       1001 CONTINUE
ISN 0031            DO 1002 IK=2,NBFA
ISN 0032            TMD=TM(IM)
ISN 0033            X(IM,IK)=X(IM,IK)/TMD
ISN 0034       1002 CONTINUE
ISN 0035       1000 CONTINUE
              C
              C     SET UP C ARRAY, C=(X'X)-'
              C
ISN 0036            DO 1003 I=1,NBFA
ISN 0037            DO 1004 J=1,NBFA
ISN 0038            SUM=0.0D0
ISN 0039            DO 1005 K=1,NOBSA
ISN 0040            SUM=SUM+(X(K,I)*X(K,J))
ISN 0041       1005 CONTINUE
ISN 0042            C(I,J)=SUM
ISN 0043       1004 CONTINUE
ISN 0044       1003 CONTINUE
              C
              C
ISN 0045            CALL INVERS(C,NBFA,NBFA)
              C
              C
              C      SET UP AA ARRAY
              C
ISN 0046             DO 1006 I=1,NBFA
ISN 0047            DO 1007 J=1,NOBSA
ISN 0048            SUM=0.0D0
ISN 0049            DO 1008 K=1,NBFA
ISN 0050            SUM=SUM+(C(I,K)*X(J,K))
ISN 0051       1008 CONTINUE
ISN 0052            AA(I,J)=SUM
ISN 0053       1007 CONTINUE
ISN 0054       1006 CONTINUE
              C
              C     ****** CALCULATE AS ARRAY *********
              C
              C     CALCULATE BASIS FUNCTIONS
              C
ISN 0055            DO 2000 I=1,NOBSS
ISN 0056            XI=I-1
ISN 0057            V=XI*15.0
ISN 0058            X(I,1)=1.0D0
ISN 0059            X(I,2)=V
ISN 0060            X(I,3)=V**2
ISN 0061       2000 CONTINUE
              C
              C     SET UP C ARRAY,C=(X'X)-'
              C
ISN 0062            DO 2001 I=1,NBFS
ISN 0063            DO 2002 J=1,NBFS
ISN 0064            SUM=0.0D0
ISN 0065            DO 2003 K=1,NOBSS
ISN 0066            SUM=SUM+(X(K,I)*X(K,J))
ISN 0067       2003 CONTINUE
ISN 0068            C(I,J)=SUM
ISN 0069       2002 CONTINUE
```

```
ISN 0070        2001 CONTINUE
                C
                C
ISN 0071             CALL INVERS(C,3,9)
                C
                C
ISN 0072             DO 2004 I=1,NBFS
ISN 0073             DO 2005 J=1,NOBSS
ISN 0074             SUM=0.0D0
ISN 0075             DO 2006 K=1,NBFS
ISN 0076             SUM=SUM+(C(I,K)*X(J,K))
ISN 0077        2006 CONTINUE
ISN 0078             AS(I,J)=SUM
ISN 0079        2005 CONTINUE
ISN 0080        2004 CONTINUE
ISN 0081             RETURN
ISN 0082             END
```

```
        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
ISN 0002                   SUBROUTINE EDOT(AMTC,AA,AS,BAD)
                     C     ******************************************************************
                     C      SUBROUTINE EDOT COMPUTES THE COEFFICIENTS THAT SPECIFY AN AUTO'S
                     C     INSTANTANEOUS EMISSION RATE FUNCTIONS FOR HC,CO,NOX(ARRAY 'BAD'),
                     C     GIVEN THE AMOUNT OF EACH EMITTANT GIVEN OFF BY THE AUTO IN 32 A/D
                     C     MODES AND 5 STEADY STATE MODES(ARRAY 'AMTC'),AND THE BASIS
                     C     FUNCTION FACTOR ARRAYS(AA,AS).
                     C
                     C      AMTC(I,J)=AMOUNT(GMS) OF THE I'TH EMITTANT GIVEN OFF BY THIS AUTO
                     C     IN THE J'TH MODE.I=1=>HC,I=2=>CO,I=3=>NOX,J=1,37(32 A/D MODES
                     C     5 STEADY STATE MODES).(REAL*4)
                     C
                     C     BAD(I,J)=J'TH COEFFICIENT OF THIS AUTO'S INSTANTANEOUS EMISSION
                     C     RATE FUNCTION FOR THE I(TH KIND OF EMITTANT.I=1=>HC,I=2=>CO,
                     C     I=3=>NOX.(REAL*8)
                     C
                     C     AA=>BASIS FI
                     C     AA=BASIS FUNCTION FACTOR ARRAY FOR ACEL/DECEL(CALCULATED BY SUBROU
                     C    -TINE SETUP).
                     C
                     C     AS=BASIS FUNCTION FACTOR ARRAY FOR STEADY STATE(CALCULATED BY
                     C     SUBROUTINE SETUP).
                     C
                     C     TM(I)=TIME(SEC) IN I'TH MODE.(REAL*4)
                     C
                     C     ******************************************************************
ISN 0003                   DIMENSION TM(37),AMTC(3,37)
ISN 0004                   REAL*8 AA(9,32),AS(3,5),BAD(3,12),SUM,YA(32),YS(5),B(3),XO,X1,X2
                          C,A1,A2
ISN 0005                   DATA TM/12.,16.,8.,11.,13.,12.,17.,12.,14.,30.,26.,21.,32.,23.,9.,
                          C8.,22.,16.,18.,19.,25.,28.,15.,25.,18.,10.,38.,35.,18.,21.,14.,13.
                          C,60.,60.,60.,60.,60./
ISN 0006                   NOBSA=32
ISN 0007                   NOBSS=5
ISN 0008                   NBFA=9
ISN 0009                   NBFS=3
                     C
ISN 0010                   DO 1000 IC=1,3
                     C
                     C      IC=1=>HC,IC=2=>CO,I=3=>NOX
                     C
                     C     CALCULATE OBSERVED AVERAGE EMISSION RATES OVER 32 A/D MODES
                     C
ISN 0011                   DO 1100 I=1,32
ISN 0012                   A1=AMTC(IC,I)
ISN 0013                   A2=TM(I)
ISN 0014                   YA(I)=A1/A2
ISN 0015              1100 CONTINUE
                     C
                     C      CALCULATE COEFFICIENTS THAT SPECIFY A/D EMISSION RATE FUNCTIONS
                     C
ISN 0016                   DO 1200 I=1,NBFA
ISN 0017                   SUM=0.0D0
ISN 0018                   DO 1250 J=1,NOBSA
ISN 0019                   SUM=SUM+(AA(I,J)*YA(J))
ISN 0020              1250 CONTINUE
```

```
ISN 0021            BAD(IC,I)=SUM
ISN 0022       1200 CONTINUE
              C
              C     CALCULATE OBSERVED AVERAGE EMISSION RATES OVER 5 SS MODES
              C
ISN 0023            DO 2000 I=33,37
ISN 0024            IP=I-32
ISN 0025            A1=AMTC(IC,I)
ISN 0026            A2=TM(I)
ISN 0027            YS(IP)=A1/A2
ISN 0028       2000 CONTINUE
              C
              C     CALCULATE COEFFICIENTS THAT SPECIFY SS EMISSION RATE FUNCTIONS
              C
ISN 0029            DO 2001 I=1,NBFS
ISN 0030            SUM=0.0D0
ISN 0031            DO 2100 J=1,NOBSS
ISN 0032            SUM=SUM+(AS(I,J)*YS(J))
ISN 0033       2100 CONTINUE
ISN 0034            B(I)=SUM
ISN 0035       2001 CONTINUE
              C
              C     CHECK ON EXISTANCE OF NEGATIVE EMISSION RATES
              C
ISN 0036            LOOP=0
ISN 0037            IF(B(3).EQ.0.0D0)GOTO2151
ISN 0039            XO=(B(2)**2)-(4.0D0*B(3)*B(1))
ISN 0040            IF(XO.LT.0.0D0)GOTO2153
ISN 0042            XO=DSQRT((B(2)**2)-(4.0D0*B(3)*B(1)))
ISN 0043            X1=(-B(2)+XO)/(2.0D0*B(3))
ISN 0044            X2=(-B(2)-XO)/(2.0D0*B(3))
ISN 0045            IF((X1.GT.0.0D0.AND.X1.LT.60.0D0).OR.(X2.GT.0.0D0.AND.X2.LT.60.0D0
                   C))LOOP=1
ISN 0047            GOTO2153
ISN 0048       2151 XO=-B(1)/B(2)
ISN 0049            IF(XO.GT.0.0D0.AND.XO.LT.60.0D0)LOOP=2
ISN 0051       2153 IF(LOOP.EQ.0)GOTO2154
              C
              C      IF LOOP=0=>NO NEGATIVE EMISSIONS FOR VELOCITYS BETWEEN 0,60
              C
              C     IF LOOP=1 OR 2=> NEGATIVE EMISSION RATES BETWEEN 0,60MPH.
              C
              C     CALL SUBROUTINE PAD TO FIND COEFFICIENTS WHICH DO NOT PRODUCE
              C     NEGATIVE EMISSION RATES.
              C
ISN 0053            CALL PAD(YS,B)
              C
ISN 0054       2154 BAD(IC,10)=B(1)
ISN 0055            BAD(IC,11)=B(2)
ISN 0056            BAD(IC,12)=B(3)
ISN 0057       1000 CONTINUE
ISN 0058            RETURN
ISN 0059            END
```

```
          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002                  SUBROUTINE PAD(Z,BT)
                   C        ******************************************************************
                   C
                   C        SUBROUTINE PAD COMPUTES A SET OF COEFFICIENTS THAT SPECIFYS AN
                   C        EMISSION RATE FUNCTION FOR STEADY STATE CONDITIONS THAT IS
                   C        NON NEGATIVE BETWEEN VELOCTY 0 AND 60 MPH.
                   C
                   C        ******************************************************************
                   C
  ISN 0003                  REAL*8 Z(5),ZP(5),BT(3),Z1,Z2,A,B,SUM1,SUM2,V,C1
  ISN 0004                  Z1=Z(1)
  ISN 0005                  Z2=Z(2)
  ISN 0006                  I1=1
  ISN 0007                  I2=2
  ISN 0008                  IF(Z1.LT.Z2)GOTO1
  ISN 0010                  Z1=Z(2)
  ISN 0011                  Z2=Z(1)
  ISN 0012                  I1=2
  ISN 0013                  I2=1
  ISN 0014                1 DO 2 I=3,5
  ISN 0015                  IF(Z(I).GT.Z2)GOTO2
  ISN 0017                  Z2=Z(I)
  ISN 0018                  I2=I
  ISN 0019                  IF(Z1.LT.Z2)GOTO2
  ISN 0021                  C1=Z1
  ISN 0022                  Z1=Z2
  ISN 0023                  Z2=C1
  ISN 0024                  IX=I1
  ISN 0025                  I1=I2
  ISN 0026                  I2=IX
  ISN 0027                2 CONTINUE
  ISN 0028                  B=(Z1+Z2)/2.0D0
  ISN 0029                  V1=I1
  ISN 0030                  V2=I2
  ISN 0031                  V1=(V1-1.0)*15.0
  ISN 0032                  V2=(V2-1.0)*15.0
  ISN 0033                  A=(V1+V2)/2.0
  ISN 0034                  DO 4 I=1,5
  ISN 0035                4 ZP(I)=Z(I)-B
  ISN 0036                  SUM1=0.0D0
  ISN 0037                  SUM2=0.0D0
  ISN 0038                  DO 5 I=1,5
  ISN 0039                  V1=I
  ISN 0040                  V1=(V1-1.0)*15.0
  ISN 0041                  V=V1
  ISN 0042                  X=(V**2)+(-2.0D0*A*V)+(A**2)
  ISN 0043                  SUM1=SUM1+(ZP(I)*X)
  ISN 0044                  SUM2=SUM2+(X**2)
  ISN 0045                5 CONTINUE
  ISN 0046                  BT(3)=SUM1/SUM2
  ISN 0047                  BT(1)=B+(BT(3)*(A**2))
  ISN 0048                  BT(2)=-2.0D0*BT(3)*A
  ISN 0049                  RETURN
  ISN 0050                  END
```

```
         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002              SUBROUTINE ESUM(VVT,NT,BAD,AHC,ACO,ANOX,DIST)
                  C     ************************************************************************
                  C
                  C     SUBROUTINE ESUM CALCULATES THE AMOUNT OF HC, CO, NOX EMITTED
                  C     AND THE DISTANCE TRAVELED BY THE AUTO THAT HAS THE INSTANTANEOUS
                  C     EMISSION RATE FUNCTION SPECIFIED BY THE ARRAY 'BAD', AND A DRIVING
                  C     CYCLE SPECIFIED BY THE ARRAY 'VVT'.
                  C
                  C     VVT(I)=>VELOCITY VS. TIME HISTORY(DRIVING CYCLE) IN ONE SECOND
                  C             INTERVALS.VVT(I)=VELOCITY(MPH) AT THE I'TH SECOND.REAL*4
                  C
                  C     NT=>MAXIMUM NUMBER SECONDS IN DRIVING CYCLE+1 SECOND
                  C
                  C     BAD(I,J)=>J'TH COEFFICIENT OF THIS AUTO'S INSTANTANEOUS EMMISSION
                  C               RATE FUNCTION FOR THE I'TH KIND OF EMITTANT,I=1=>HC,
                  C               I=2=>CO,I=3=>NOX.(REAL*8)
                  C
                  C     AHC=AMT(GMS) HC GIVEN OFF BY THIS AUTO IN GOING THRU DRIVING CYCLE
                  C         REAL*4
                  C
                  C     ACO=AMT(GMS) CO GIVEN OFF BY THIS AUTO IN GOING THRU DRIVING CYCLE
                  C         REAL*4
                  C
                  C     ANOX=AMT(GMS) NOX GIVEN OFF BY THIS AUTO IN GOING THRU DRIVE CYCLE
                  C          REAL*4
                  C
                  C     DIST=DISTANCE(MILES)IN SPECIFIED DRIVING CYCLE,REAL*4
                  C
                  C     ************************************************************************
  ISN 0003              DIMENSION VVT(NT)
  ISN 0004              REAL*8 BAD(3,12),AMT(3),X(12),DIS,AMIN,AMAX,A1,A2,HOA,SOA
  ISN 0005              AMAX=1.0D0
  ISN 0006              AMIN=-1.20D0
  ISN 0007              A1=-1.0D0/AMIN
  ISN 0008              A2=-1.0D0/AMAX
                  C
                  C     CLEAR AMT ARRAY
                  C
  ISN 0009              DO 1000 I=1,3
  ISN 0010         1000 AMT(I)=0.0D0
                  C
                  C     INTEGRATE AUTO'S EMISSION RATE FUNCTION OVER DRIVING CYCLE
                  C
  ISN 0011              DIS=0.0D0
  ISN 0012              NTT=NT-1
  ISN 0013              DO 3000 IT=1,NTT
  ISN 0014              KT=IT+1
  ISN 0015              X(1)=1.0D0
  ISN 0016              X(2)=DBLE((VVT(IT)+VVT(KT))/2.0)
  ISN 0017              X(3)=DBLE(VVT(KT)-VVT(IT))
  ISN 0018              X(4)=X(2)*X(3)
  ISN 0019              X(5)=X(2)**2
  ISN 0020              X(6)=X(3)**2
  ISN 0021              X(7)=(X(2)**2)*X(3)
  ISN 0022              X(8)=(X(3)**2)*X(2)
  ISN 0023              X(9)=(X(2)**2)*(X(3)**2)
```

```
ISN 0024          X(10)=X(1)
ISN 0025          X(11)=X(2)
ISN 0026          X(12)=X(5)
ISN 0027          IF(X(3).GE.AMAX)HOA=0.0D0
ISN 0029          IF(X(3).LE.AMIN)HOA=0.0D0
ISN 0031          IF(X(3).GE.0.0D0.AND.X(3).LT.AMAX)HOA=(A2*X(3))+1.0D0
ISN 0033          IF(X(3).LE.0.0D0.AND.X(3).GT.AMIN)HOA=(A1*X(3))+1.0D0
ISN 0035          DO 2999IC=1,3
ISN 0036          DO 2998 IE=1,12
ISN 0037          SOA=1.0D0-HOA
ISN 0038          IF(IE.GT.9)SOA=HOA
ISN 0040          AMT(IC)=AMT(IC)+(X(IE)*SOA*BAD(IC,IE))
ISN 0041     2998 CONTINUE
ISN 0042     2999 CONTINUE
ISN 0043          DIS=DIS+X(2)
ISN 0044     3000 CONTINUE
ISN 0045          AHC=AMT(1)
ISN 0046          ACO=AMT(2)
ISN 0047          ANOX=AMT(3)
ISN 0048          DIS=DIS/3600.0D0
ISN 0049          DIST=DIS
ISN 0050          DO 4000 ICK=1,3
ISN 0051     4000 IF(AMT(ICK).LT.0.0D0)GOTO4001
ISN 0053          GOTO4444
ISN 0054     4001 WRITE(6,4002)
ISN 0055     4002 FORMAT(1H ,'MODEL IS UNABLE TO PREDICT THIS VEHICLES EMISSIONS')
ISN 0056     4444 RETURN
ISN 0057          END
```

```
      COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                         SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
 ISN 0002                SUBROUTINE EDGRP(BAD,INT)
               C         ******************************************************************
               C
               C          SUBROUTINE EDGRP COMPUTES THE COEFFICIENTS THAT SPECIFY THE
               C         INSTANTANEOUS EMISSION RATE FUNCTIONS OF THE 'AVERAGE' AUTO OF
               C          SOME GROUP OF AUTOS. EDGRP IS CALLED ONCE FOR EACH AUTO IN THE
               C         GROUP, SPECIFYING EACH TIME THE INSTANTANEOUS EMISSION RATE
               C         FUNCTIONS COEFFICIENT ARRAY 'BAD' FOR THE INDIVIDUAL AUTO. AFTER
               C         EDGRP HAS BEEN CALLED FOR ALL AUTOS WITHIN THE GROUP, THE 'AVERAGE
               C         AUTO'S INSTANTANEOUS EMISSION RATE FUNCTIONS ARRAY IS GIVEN BY
               C         ARRAY 'BAD'.
               C
               C         BAD=>INSTANTANEOUS EMISSION RATE FUNCTIONS ARRAY.(REAL*8)
               C
               C         SET INT=1 FOR THE FIRST AUTO IN THE GROUP
               C
               C         SET INT=2 FOR THE REST OF THE AUTOS IN THE GROUP
               C
               C         SET INT=3 AFTER ALL THE AUTOS IN THE GROUP HAVE BEEN CONSIDERED
               C
               C         ******************************************************************
 ISN 0003                REAL*8 BAD(3,12),B(3,12),CTN
               C
               C
 ISN 0004                IF(INT.GT.1)GOTO2000
 ISN 0006                DO 1000 I=1,3
 ISN 0007                DO 1001 J=1,12
 ISN 0008                B(I,J)=0.0D0
 ISN 0009           1001 CONTINUE
 ISN 0010           1000 CONTINUE
 ISN 0011                CTN=0.0D0
 ISN 0012           2000 IF(INT.EQ.3)GOTO4133
 ISN 0014                CTN=CTN+1.0D0
 ISN 0015                DO 2001 I=1,3
 ISN 0016                DO 2002 J=1,12
 ISN 0017                B(I,J)=B(I,J)+BAD(I,J)
 ISN 0018           2002 CONTINUE
 ISN 0019           2001 CONTINUE
 ISN 0020                IF(INT.NE.3)GOTO4000
 ISN 0022           4133 DO 3000 I=1,3
 ISN 0023                DO 3001 J=1,12
 ISN 0024                BAD(I,J)=B(I,J)/CTN
 ISN 0025           3001 CONTINUE
 ISN 0026           3000 CONTINUE
 ISN 0027           4000 RETURN
 ISN 0028                END
```

```
        COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                           SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
  ISN 0002            SUBROUTINE INVERS(A,N,N1)
              C*                                                                     *
              C********************************  INVERSE  ***********************************
              C*                                                                     *
              C*                                                                     *
              C*  REMARKS : THE ROUTINE COMPUTES THE INVERSE OF MATRIXA; DIMENSION <20,  *
              C*                WHERE N IS THE DIMENSION OF A.                       *
              C*                                                                     *
              C*                THE INVERSE IS RETURNED IN MATRIX A.                 *
              C*                                                                     *
              C*                N IS RETURNED  AS -1 WHEN A IS SINGULAR              *
              C*                                                                     *
              C**************************************************************************
              C
  ISN 0003            DIMENSION A(N1,N1),S(10,20)
  ISN 0004            DOUBLE PRECISION S,BUFF,DVH,FPY,A
  ISN 0005            K= N+N
              C
              C                                        INITIALIZATION  :     S = (A ,I)
  ISN 0006            DO 501 I=1,10
  ISN 0007            DO 502 J=1,20
  ISN 0008            S(I,J)=0.D0
  ISN 0009        502 CONTINUE
  ISN 0010        501 CONTINUE
  ISN 0011            DO  12 I = 1,N
  ISN 0012            DO  10 J = 1,N
  ISN 0013            S(I,J) = A(I,J)
  ISN 0014         10 CONTINUE
  ISN 0015            IN = I + N
  ISN 0016            S(I,IN) = 1.D0
  ISN 0017         12 CONTINUE
              C
              C                                   COMPUTE INVERSE OF A MATRIX S
              C
              C
              C                                        LOWER TRIANGULAZATION OF S
  ISN 0018         15 DO 150  I=1,N
  ISN 0019            L = I
  ISN 0020            M = I+1
  ISN 0021            JIN = I
  ISN 0022         20 IF(S(I,I).NE.0.D0)GOTO45
  ISN 0024         25 LE = I +1
  ISN 0025         26 IF(S(LE,I).NE.0.D0)GOTO800
  ISN 0027         27 LE = LE + 1
  ISN 0028         28 IF (LE-N) 26,26,900
  ISN 0029        800 DO 35  J=1,K
  ISN 0030            BUFF = S(I,J)
  ISN 0031            S(I,J) = S(LE,J)
  ISN 0032            S(LE,J) = BUFF
  ISN 0033         35 CONTINUE
  ISN 0034         41 GO TO 20
  ISN 0035         45 DVH = S(I,I)
  ISN 0036            DO 46  J=1,K
  ISN 0037         46 S(I,J) = S(I,J)/DVH
  ISN 0038            S(I,I)    =     1.0D0
  ISN 0039         48 IF(I .GE. N  )  GO  TO  149
```

```
ISN 0041            49 FPY = S(M,L)
ISN 0042               IF ( FPY   . EQ.   0.D0       )   GO   TO  75
ISN 0044            50 DO 70   J=1,K
ISN 0045               BUFF = FPY * S(I,J)
ISN 0046               S(M,J) = S(M,J) - BUFF
ISN 0047            70 CONTINUE
ISN 0048            75 JIN = M+1
ISN 0049               IF ( JIN  . GT.N ) GO TO  149
ISN 0051           100 M = M+1
ISN 0052           120 GO TO 49
ISN 0053           149 CONTINUE
ISN 0054           150 CONTINUE
               C                                                  DIAGONALIZATION OF S
ISN 0055               DO 385   I=2,N
ISN 0056               L = I
ISN 0057               M = I-1
ISN 0058           350 FPY = S(M,L)
ISN 0059               IF ( FPY   . EQ.   0.D0       )   GO   TO 375
ISN 0061           351 DO 370   J=1,K
ISN 0062               BUFF = FPY * S(I,J)
ISN 0063               S(M,J) = S(M,J) - BUFF
ISN 0064           370 CONTINUE
ISN 0065           375 IF ( M  .LE.  1 )   GO TO    384
ISN 0067           380 M = M-1
ISN 0068               GO TO 350
ISN 0069           384 CONTINUE
ISN 0070           385 CONTINUE
               C                                              STORE INVERSE IN A
ISN 0071           390 DO 402   I1=1,N
ISN 0072               LL=I1
ISN 0073           395 DO 400   J1=1,N
ISN 0074               KK = N + J1
ISN 0075           396 A(I1,J1) = S(LL,KK)
ISN 0076           400 CONTINUE
ISN 0077           402 CONTINUE
ISN 0078               RETURN
               C                                                        NO INVERSE
ISN 0079           900 WRITE(6,7000)
ISN 0080          7000 FORMAT(1H0,'NO INVERSE')
ISN 0081               N=-1
ISN 0082               RETURN
ISN 0083               END
```

APPENDIX V

VEHICLE CLASSIFICATION BY DISCRIMINANT FUNCTION ANALYSIS

The implementation of the emission-rate function model is assessing the impact of a collection of vehicles on the environment requires that the input data be appropriately chosen according to the problem under consideration. A matter of central concern in this regard is whether available data can be employed in a setting different from that in which it originated. In particular, such questions as geographic location and time frame are of interest. For example, can surveillance data obtained in one location be employed as a basis for modeling emissions in a different location, and can data collected in the past be used as a basis for modeling present or future problems?

In a previous report, it was noted that a distinction exists between the emissions of vehicles in Denver as compared with five other cities of lower altitudes. Also, as would be expected, the implementation of emission controls clearly affected the emissions of vehicles in both geographic categories. The concept of discriminant function analysis was employed to crystallize and quantify these differences.

The distinctions alluded to above were based on an analysis of total emissions of HC, CO and NO_x over the surveillance driving sequence. Though it is presumed that these differences among categories of vehicles would persist in other driving sequences, it was considered of interest to investigate this assumption in a systematic way. Toward this end a discriminant function approach was undertaken.

The logic of this approach is as follows. Each driving cycle consists of a series of accel/decel and steady-state modes. Conceivably some of these modes might be more sensititve to geographic or time differences than others. This would result in a situation where driving sequences consisting of combinations of these modes could themselves differ in this regard. A linear discriminant function is a weighted combination of the modes constructed in such a way that differences between vehicle categories, if they exist at all, will be most clearly delineated.

Suppose that only Denver versus non-Denver vehicles are considered and the emissions for a particular pollutant for the various modes are denoted X_1, X_2, ..., X_{37}. Then the linear discriminant function is defined as

$$F = a_1x_1 + a_2x_2 + \ldots\ldots + a_{37}x_{37}$$

where a_1, a_2, ..., a_{37} are coefficients selected according to an optimization logic. For each vehicle, one can compute a value of F. If one groups together all F values for Denver vehicles and all F values for non-Denver vehicles, one can compute a mean value of F for each group and a variance or standard deviation for each group. The two groups will separate clearly, with minimal overlap of the two distributions of F values, if the difference between the mean values for the groups is large and the dispersion or scatter of F values within each group is small. The coefficients a_1, a_2, ..., a_{37} are chosen so as to make as large as possible the ratio of the dispersion of the group means to the pooled dispersion of the individual vehicles within groups.

If only two groups are involved, it is possible to compute only one discriminant function for each vehicle. However, if there are G groups, one can compute G-1 such functions, provided G-1 does not exceed the number of modes.

For example, consider the following groups of vehicles

GROUP 1 - Non-Denver, pre-emission control
GROUP 2 - Denver, pre-emission control
GROUP 3 - Non-Denver, emission control
GROUP 4 - Denver, emission control

In this case three discriminant functions can be computed as follows:

$$F_1 = a_1x_1 + a_2x_2 + \text{----} \quad a_{37}\,x_{37}$$

$$F_2 = b_1x_1 + b_2x_2 + \text{----} \quad b_{37}\,x_{37}$$

$$F_3 = c_1x_1 + c_2x_2 + \text{----} \quad c_{37}\,x_{37}$$

These functions employ three distinct sets of weighting coefficients $\{a_i\}$, $\{b_i\}$, $\{c_i\}$ the efficacy of which can be appreciated best by a geometric argument.

For each vehicle, one can compute F_1, F_2, and F_3. Consider these three values as the coordinates of a point in three-dimensional space. If each vehicle is plotted as such a point in 3-space, then there will result a collection of points which, hopefully, will tend to cluster into four distinct "clouds", these clouds being associated with the four groups of vehicles under study. If the separation of these groups is complete, one can partition the space into disjoint compartments and there will be no overlap or spillover from one compartment to the other. In reality, this will not be the case, and there will be a certain amount of "mixing" or "confusion" if one attempts to assign a particular vehicle to its correct group merely by looking at the F-value coordinates of that vehicle. Quantification of the degree of correct and incorrect classification is therefore required in order to assess how "good" such a classification matrix is.

To appreciate the nature of this quantification, consider first the case in which there are only two categories--say, Denver and non-Denver--and only a single discriminant function F. Divide all vehicles into two sets on the basis of their known membership--that is, consider all Denver vehicles as a group and all non-Denver vehicles as a group. There are 169 vehicles in the Denver group, 851 in the non-Denver group. For the Denver group, compute F for each of the 169 vehicles and display these results as a histogram. Similarly, compute F for each of the 859 vehicles in the non-Denver group and display these results as a histogram. The results are shown in Figures 5-1, 5-2, and 5-3 for HC, CO and NO_x respectively. Quite clearly, a certain amount of overlap is evident, so that if one attempts to assign class membership on the basis of the F value only, a certain number of both the Denver and non-Denver vehicles will be misclassified.

A technique which can be used to determine the quantitative separation of groups is the construction of a "classification matrix" for the groups. The concept, as illustrated below, represents classifying automobiles as coming from City A or City B on the basis of a particular test value.

	CITY A	CITY B
CITY A	83	17
CITY B	42	58

CLASSIFICATION MATRIX FOR TWO CITIES

The classification matrix should be interpreted in the following way. Assume a technique which takes a test value for an automobile from City A and, on the basis of probabilities for this test value, classifies this automobile as coming from either City A or City B, whichever has the highest probability for the given test value. If all the automobiles from City A are subjected to this probability test, one will find that some are correctly classified as coming from City A while others are incorrectly classified as coming from City B. The same test can be performed on the automobiles from City B. Since the number of automobiles from each city is known, one can convert the number of automobiles correctly or incorrectly classified into percent of automobiles from each city. The diagonal elements of the matrix represent the percent of automobiles coming from City A and City B which were correctly classified as coming from City A and City B. The off-diagonal elements give the percent of automobiles incorrectly classified. Therefore, 83 percent of the automobiles from City A were correctly classified as coming from City A, while 17 percent were incorrectly classified as coming from City B. Likewise, 58 percent of the automobiles from City B were classified as coming from City B while 42 percent were classified as coming from City A. In this case, the probability for classifying automobiles was more heavily weighted in favor of City A.

Classification matrices for Denver versus non-Denver vehicles are shown in Table 5-1 and the corresponding histograms on which they are based are shown in Figures 5-1, 5-2 and 5-3. It can be seen that the separation of the two groups is somewhat better for CO and NO_x than for HC. Also, in the case of HC, it is more likely that Denver vehicles will be misclassified as non-Denver vehicles than that non-Denver vehicles will be classified as Denver vehicles. Thus once the probabilities for a set of groups have been determined for values of a given test the classification matrix technique can be used to give an easy-to-interpret quantitative measure of the separation between groups.

If the number of groups used in the classification is increased, the size of the classification matrix increases accordingly, with the diagonal elements indicating the percent correct classification. In this more elaborate situation, a vehicle can be misclassified in more than one way, and the nature of the misclassification can provide insight into which categories of vehicles are most "alike".

Just as the number of groups (cities) in the classification matrix can be increased, the number of discriminant functions to be used as the basis of classification can also be increased. In this case one calculates, for a given automobile, the probability of occurrence of each of the discriminant-function values associated with the vehicle. For reasons discussed later, it can be assumed that the several discriminant functions are statistically independent. Therefore, these probabilities can be multiplied together and it can be assumed that the product is the probability of the joint occurrence of the particular discriminant function values encountered for the vehicle in question. Then compare the probability products calculated for all the groups and assign the automobile to the group with the highest probability product.

In the case of Denver and non-Denver vehicles before and after the advent of emission controls, there are four categories and three F values. By virtue of the theory on which discriminant function analysis rests, the three F values are statistically uncorrelated. Also, by virtue of the fact that each of the F values is a linear combination of the outcomes of 37 random variables, it can be presumed that the Central Limit Theorem of statistics will cause the distribution of the F values to tend toward a Gaussian distribut on. Under these conditions, the assumption of independence is believed justified.

The classification matrices for HC, CO and NO_x according to Denver and non-Denver, pre-control and control eras are presented in Table 5-2. The type of inferences which can be drawn from these tables is exemplified by noting the classification matrix for NO_x. For example, note that non-Denver, pre-emission controls (category 1) is misclassified more frequently as non-Denver emission controls (category 3) than as either of the Denver categories (categories 2 and 4). Similarly, the Denver pre-emission control is confused more with Denver post-emission controls than with the other two categories, and the Denver emission

TABLE 5-1
CLASSIFICATION MATRICES

GROUP 1 = NON-DENVER

GROUP 2 = DENVER

HC

	1	2
1)	82	17
2)	42	57

CO

	1	2
1)	84	15
2)	11	88

NO_x

	1	2
1)	88	11
2)	11	88

TABLE 5-2

CLASSIFICATION MATRICES

GROUP 1 = NON-DENVER PRE-EMISSION CONTROL

GROUP 2 = DENVER PRE-EMISSION CONTROL

GROUP 3 = NON-DENVER EMISSION CONTROL

GROUP 4 = DENVER EMISSION CONTROL

HC

	1	2	3	4
1)	47	13	23	14
2)	19	48	3	28
3)	16	3	58	21
4)	5	15	11	68

CO

	1	2	3	4
1)	56	8	25	9
2)	12	72	0	15
3)	15	1	79	3
4)	12	11	4	72

NO_x

	1	2	3	4
1)	58	10	20	9
2)	7	73	2	17
3)	27	2	66	3
4)	5	23	2	68

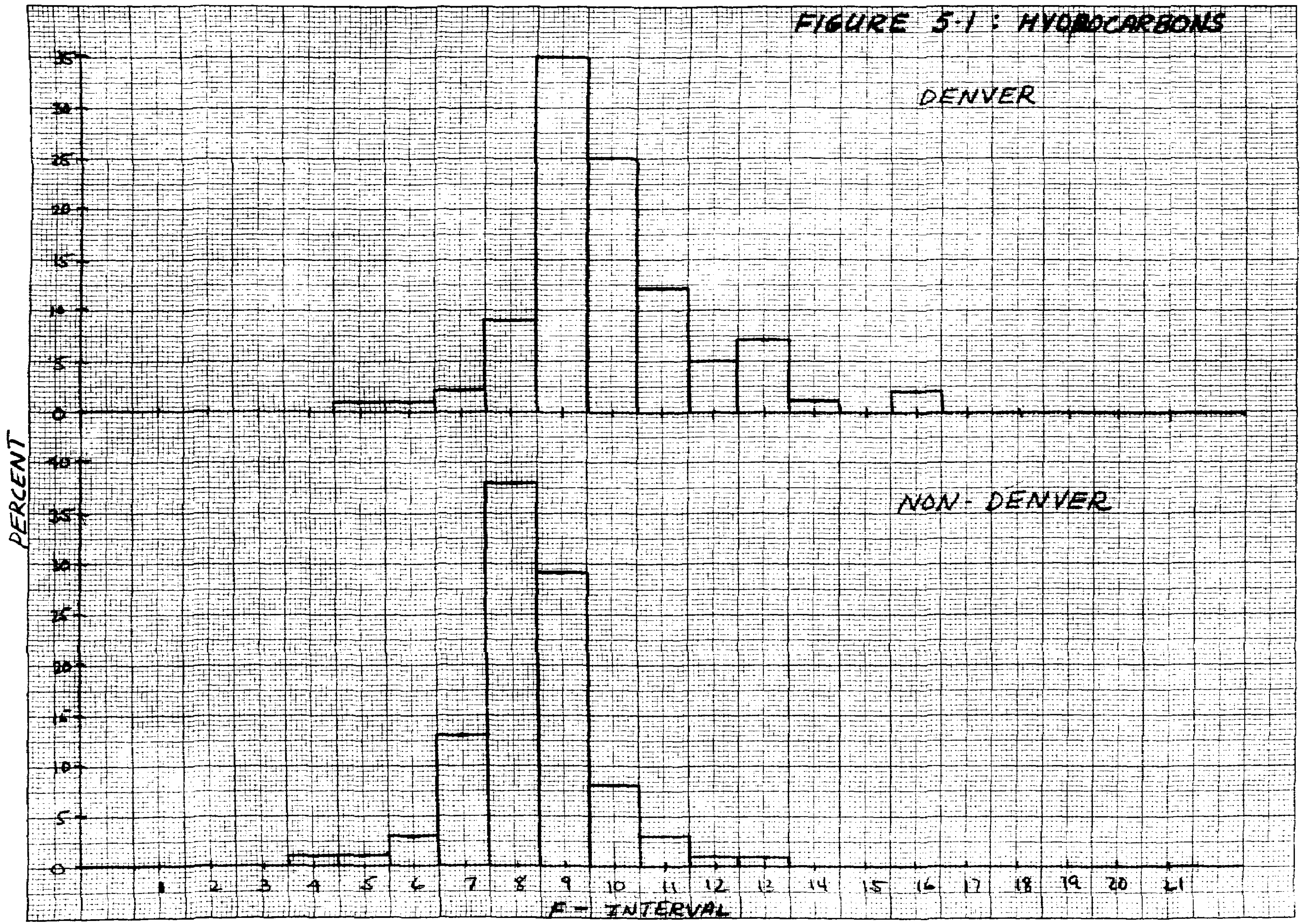
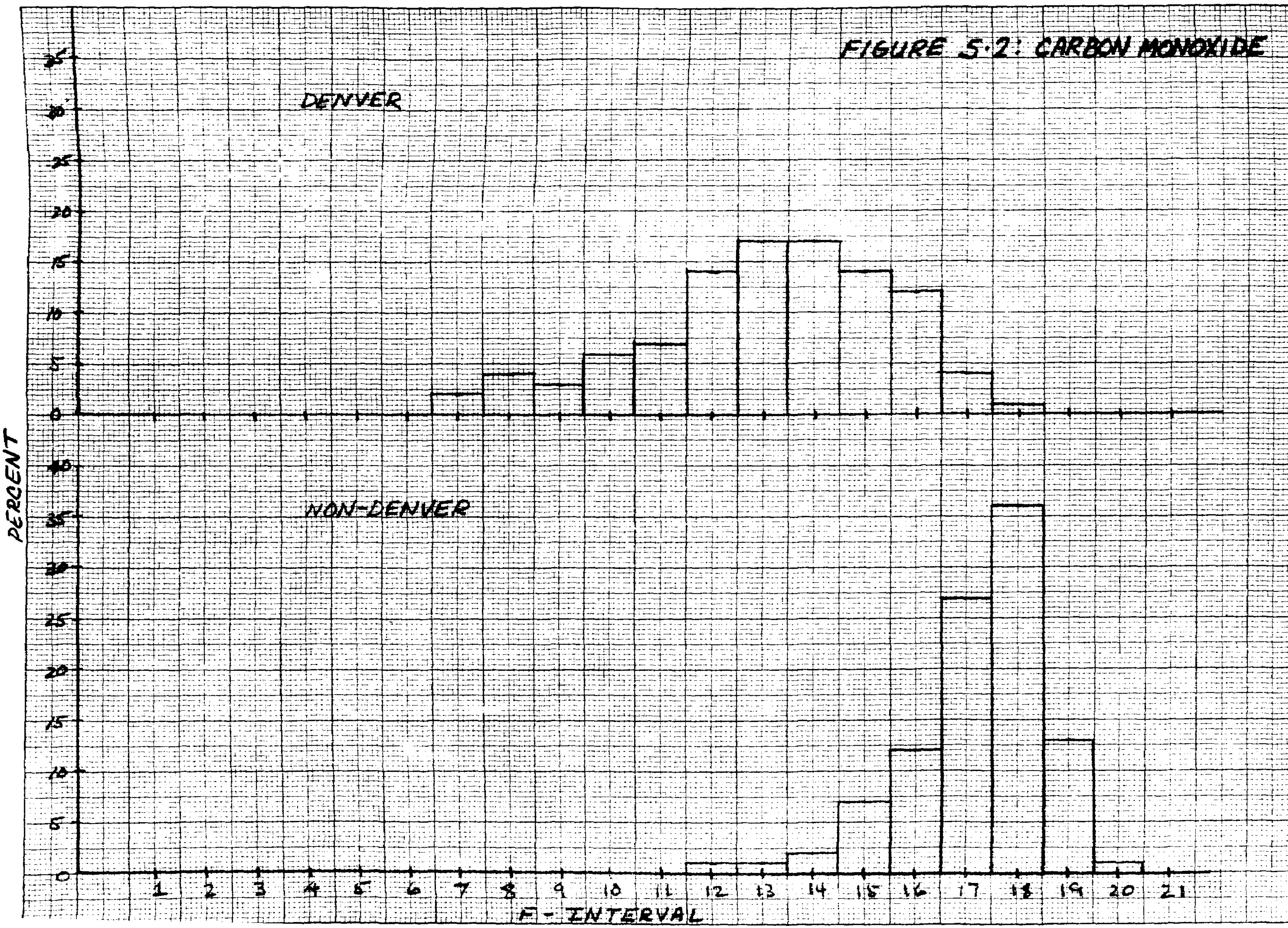


FIGURE 5-1 : HYDROCARBONS
DENVER
NON- DENVER
PERCENT
F - INTERVAL
0
5
10
15
20
25
30
35
40
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21

K+E 10 X 10 TO THE CENTIMETER 18 X 25 CM. 46 1512 MADE IN U.S.A. KEUFFEL & ESSER CO.



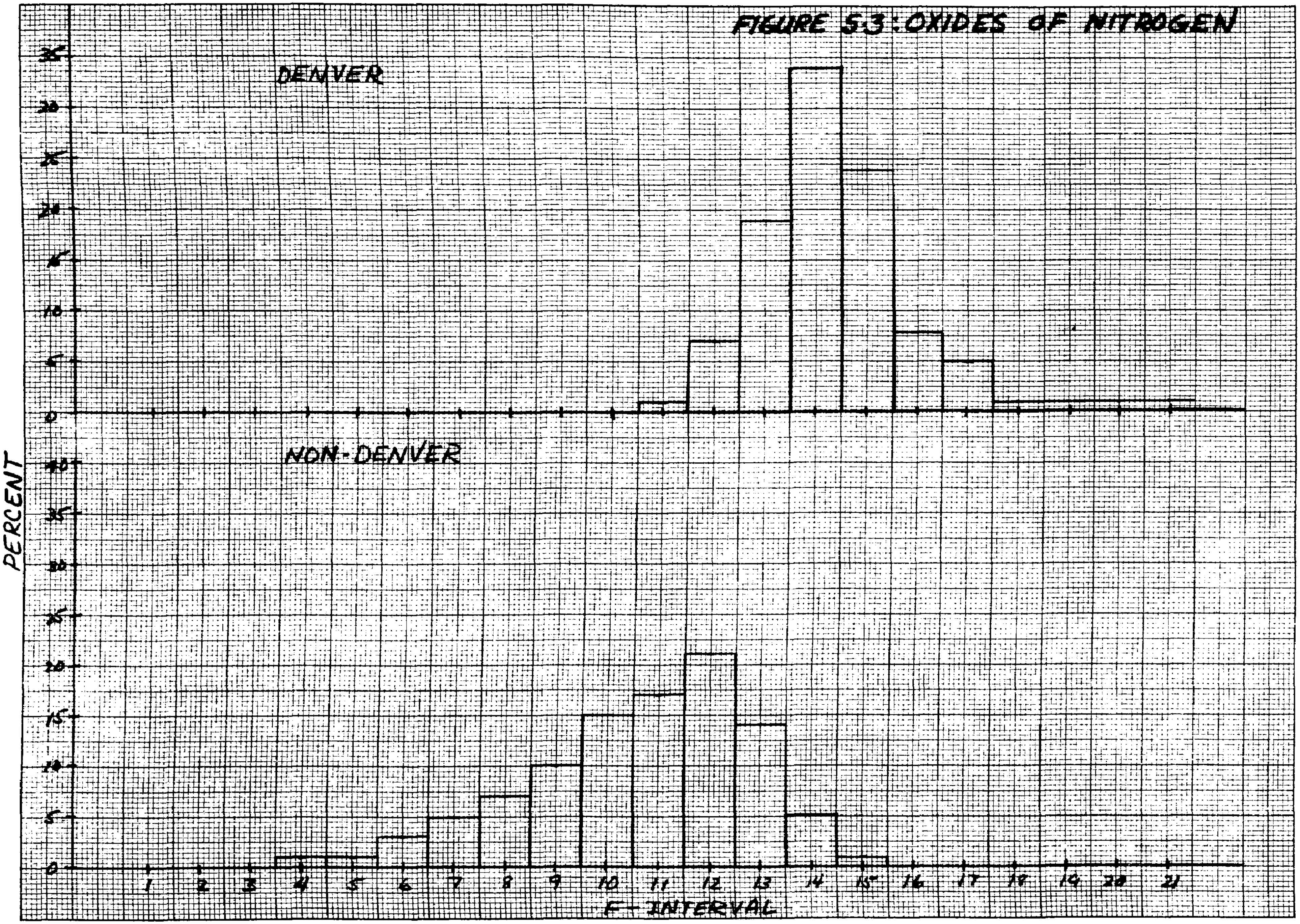


FIGURE 53: OXIDES OF NITROGEN
DENVER
NON-DENVER
PERCENT
F-INTERVAL
0
5
10
15
20
25
30
35
40
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21