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NOTES ON THE STANDARD ATMOSPHERE.

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Summary.

This report contains the derivation of a series of relations between temperature, pressure, density and altitude in a "standard atmosphere" which assumes a uniform decrease of temperature with altitude. The equations are collected and given with proper constants in both metric and English units for the temperature gradient adopted by the National Advisory Committee for Aeronautics. A table of values of temperature pressure and density at various altitudes in this standard atmosphere is included in the report.

Introduction.

Certain interesting and significant relations exist between pressure, density, temperature, and altitude in a "standard atmosphere" which assumes a linear decrease of temperature with altitude. Such an atmosphere is in extensive use in Europe and ha recently been adopted by the National Advisory Committee for Aero nautics for Official use in the United States. The derivation of the more important of the standard atmosphere relations and their tabulation in form for ready reference seems to be a practical method of indicating the way towards a utilization of the many advantages of a standard atmosphere.

The nomenclature followed throughout this report is standard, i.e.:

p = pressure,

ρ = mass-density,

T = temperature, absolute,

R = gas constant for air,

a = temperature gradient,

y = altitude.

Standard Atmosphere.

A discussion of Standard Atmosphere may be found in National Advisory Committee for Aeronautics Report No. 147, by W. R. Gregg. A few brief remarks will be made for the benefit of those who do do not have a copy of that report available for reference.

"Standard Atmosphere," as the name implies, is an atmosphere in which fixed values of pressure, temperature and density are adopted, more or less arbitrarily, for each altitude. Such an atmosphere is required in comparing the performance of aircraft and aircraft power plants. The most satisfactory of all proposed standards is that based on the so-called "Toussaint's Rule," which assumes the air temperature to decrease uniformly with altitude from 15°C at sea level to -50°C at 10,000 meters, or, expressed in symbols

$$T = T_o - ay \quad . . . . \quad (1)$$

where a has the value of .0065 in the metric system (T in °C, y in meters) and the value .003567 in the English system (T in °F, y in feet); and T_o is the temperature at sea-level, i.e.: 273 + 15 = 288°C (or, 418.6°F).

While it is desirable that a standard atmosphere be a close approximation to actual conditions, this requirement should not be stressed. The chief purpose of a standard atmosphere is to supply a basis for comparison of performance and not to specify the temperature and pressure obtaining at a given absolute altitude at all times. It so happens that "Toussaint's Rule" expresses with considerable accuracy the <u>average annual</u> conditions for latitude 40° in the United States. This is not altogether accidental since the temperature gradient is based on averages, but it is very doubtful if the standard atmosphere ever expresses the true conditions at a given instant. Nevertheless, the "standard atmosphere" when properly understood and used is of great value in aeronautics.

Table 2, taken from National Advisory Committee for Aeronautics Report No. 147, gives values of t, $\left(\frac{T}{T_o}\right)$, $\left(\frac{p}{p_o}\right)$ and $\left(\frac{\rho}{\rho_o}\right)$ for a series of values of y in both the metric and the English units. It should be noted that the standard air density used in the United States and England is based on a temperature of 60°F instead of 59°F. This introduces a slight density difference which may lead to confusion in certain cases unless the proper value be used. The difference is negligible in engineering calculations, however.

Fundamental Relations.

Since for all practical purposes air may be considered as a "perfect gas," the fundamental relations are the assumed

$$T = T_o - ay \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

the perfect gas law

$$p = \rho R T \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

or,

$$\left(\frac{p}{p_o}\right) = \left(\frac{\rho}{\rho_o}\right) \left(\frac{T}{T_o}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2a)$$

and

$$dp = - \rho g \, dy \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

From these three relations there may be derived a series of equations which are of great value to the engineer. The derivation of equations (4) to (8) are original, although equations similar to (4) and (5) were derived by Pistolesi in "Bollettino Tecnicio N.18) of the Italian Air Service, and an equation somewhat similar to (7) appears in R & M 324 of the British Advisory Committee for Aeronautics.

Since the preparation of these notes, articles containing similar equations have appeared in the "Bulletin Technique" (Service Technique de l'Aeronautique) No. 4, March, 1922, and in the Aeronautical Journal for May, 1922.

Derived Relations.

(a) Between temperature and pressure.

Dividing (2) by (3) and substituting $(T_0 - ay)$ for T

$$\frac{dp}{p} = -\frac{g\,dy}{RT} = -\frac{g\,dy}{R(T_0 - ay)}$$

Integrating

$$\frac{aR}{g}\log\left(\frac{p}{p_0}\right) = \log\left(\frac{T}{T_0}\right)$$

and

$$\left(\frac{T}{T_0}\right) = \left(\frac{p}{p_0}\right)^{\frac{aR}{g}} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

The value of the exponent is obviously independent of the system of units. In the English system $a = .003567$ and $R = 53.34g$ or 1716.3. Therefore,

$$\frac{aR}{g} = 0.190$$

and

$$\left(\frac{T}{T_0}\right) = \left(\frac{p}{p_0}\right)^{0.19} \quad \ldots\ldots\ldots\ldots\ldots (4a)$$

$$\left(\frac{p}{p_0}\right) = \left(\frac{T}{T_0}\right)^{5.255} \quad \ldots\ldots\ldots\ldots\ldots (4b)$$

(b) Between density and pressure:

From (2a) and (4a)

$$\left(\frac{\rho}{\rho_0}\right) = \left(\frac{p}{p_0}\right)^{1-\frac{aR}{g}} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

$$\left(\frac{p}{p_0}\right)^{0.81} \quad \ldots\ldots\ldots\ldots\ldots (5a)$$

or

$$\left(\frac{p}{p_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{1.235} \quad \ldots\ldots\ldots\ldots\ldots \quad (5b)$$

(c) Between temperature and density:

From (4) and (5)

$$\left(\frac{T}{T_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{\frac{aR}{g-aR}} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$\left(\frac{T}{T_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{4.255} \quad \ldots\ldots\ldots\ldots\ldots \quad (6a)$$

or

$$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{T}{T_o}\right)^{4.255} \quad \ldots\ldots\ldots\ldots\ldots \quad (6b)$$

(d) Between density and altitude:

From (1) and (6)

$$\left(\frac{\rho}{\rho_o}\right)^{\frac{aR}{g-aR}} = \left(1 - \frac{a}{T_o}\,y\right) \quad \ldots\ldots\ldots\ldots \quad (7)$$

$$\left(\frac{\rho}{\rho_o}\right)^{0.235} = \left(1 - \frac{a}{T_o}\,y\right) \quad \ldots\ldots\ldots\ldots \quad (7a)$$

$$\left(\frac{\rho}{\rho_o}\right) = \left(1 - \frac{a}{T_o}\,y\right)^{4.255} \quad \ldots\ldots\ldots\ldots \quad (7b)$$

(e) Between pressure and altitude:

From (1) and (4)

$$\left(\frac{p}{p_o}\right)^{\frac{aR}{g}} = \left(1 - \frac{a}{T_o}\,y\right) \quad \ldots\ldots\ldots\ldots \quad (8)$$

$$\left(\frac{p}{p_o}\right)^{0.19} = \left(1 - \frac{a}{T_o} y\right) \quad \ldots\ldots\ldots\ldots \quad (8a)$$

or

$$\left(\frac{p}{p_o}\right) = \left(1 - \frac{a}{T_o} y\right)^{5.255} \quad \ldots\ldots\ldots \quad (8b)$$

A modified form of the well-known Halley's equation which is frequently used, is included for convenience:

$$y = 2.3026 \, H \left(\frac{T + T_o}{2 \cdot T_o}\right) \log_{10} \left(\frac{p_o}{p}\right) \quad \ldots\ldots\ldots \quad (9)$$

In this equation H is the height of the "homogeneous atmosphere," i.e., the height to which the atmosphere would have to extend if its density were uniform throughout and of the value required to produce standard pressure at sea-level. This height is 7991 meters, or 26,217 feet.

TABLE 1.

Collected Formulae and Constants.

	General Equations and Constants.	Metric System	English System
	p_o	760 mm (10330 kg/m²)	29.921 in.kg. (2116 lbs/ft²)
	ρ_o	.00125	.00237
	Gravity - g	980 6 cm/sec²	32.172 ft/sec²
	T	280°C	418.6°F
	a	-.0065	-.003567
	R	29.27 g	53.34 g
1.	$T = T_o - ay$	$T = T_o - .0065\,y$	$T = T_o - .003567y$
2.	$p = \rho RT$	$p = \rho RT$	$p = \rho RT$
3.	$dp = - g\rho dy$	$dp = - g\rho\, dy$	$dp = - g\,\rho dy$
4.	$\left(\frac{T}{T_o}\right) = \left(\frac{p}{p_o}\right)^{\frac{aR}{g}}$	$\left(\frac{T}{T_o}\right) = \left(\frac{p}{p_o}\right)^{0.19}$	$\left(\frac{T}{T_o}\right) = \left(\frac{p}{p_o}\right)^{0.19}$
	$\left(\frac{p}{p_o}\right) = \left(\frac{T}{T_o}\right)^{\frac{g}{aR}}$	$\left(\frac{p}{p_o}\right) = \left(\frac{T}{T_o}\right)^{5.255}$	$\left(\frac{p}{p_o}\right) = \left(\frac{T}{T_o}\right)^{5.255}$
5.	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{p}{p_o}\right)^{1-\frac{aR}{g}}$	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{p}{p_o}\right)^{0.81}$	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{p}{p_o}\right)^{0.81}$
	$\left(\frac{p}{p_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{\frac{g}{g-aR}}$	$\left(\frac{p}{p_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{1.235}$	$\left(\frac{p}{p_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{1.235}$
6.	$\left(\frac{T}{T_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{\frac{aR}{g-aR}}$	$\left(\frac{T}{T_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{0.235}$	$\left(\frac{T}{T_o}\right) = \left(\frac{\rho}{\rho_o}\right)^{0.235}$
	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{T}{T_o}\right)^{\frac{g-aR}{aR}}$	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{T}{T_o}\right)^{4.255}$	$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{T}{T_o}\right)^{4.255}$
7.	$\left(\frac{\rho}{\rho_o}\right)^{\frac{aR}{g-aR}} \left(1 - \frac{a}{T_o}\,y\right)$	$\left(\frac{\rho}{\rho_o}\right)^{0.235} = (1-.00002257y)$	$\left(\frac{\rho}{\rho_o}\right)^{0.235} = (1-.000006878y)$
	$\left(\frac{\rho}{\rho_o}\right)\left(1-\frac{a}{T_o}\,y\right)^{\frac{g-aR}{aR}}$	$\left(\frac{\rho}{\rho_o}\right) = (1-.00002257y)^{4.255}$	$\left(\frac{\rho}{\rho_o}\right) = (1-.000006878y)^{4.255}$
8.	$\left(\frac{p}{p_o}\right)^{\frac{aR}{g}} = \left(1 - \frac{a}{T_o}\,y\right)$	$\left(\frac{p}{p_o}\right)^{0.19} = (1-.00002257y)$	$\left(\frac{p}{p_o}\right)^{0.19} = (1-.000006878y)$
	$\left(\frac{p}{p_o}\right) = \left(1 - \frac{a}{T_o}\,y\right)^{\frac{g}{aR}}$	$\left(\frac{p}{p_o}\right) = (1-.00002257y)^{5.255}$	$\left(\frac{p}{p_o}\right) = (1-.000006878y)^{5.255}$
9.	$y = 2.3026\,H\left(\frac{T+T_o}{2T_o}\right) \log_{10}\left(\frac{p_o}{p}\right)$	$y = 18400\left(\frac{T+T_o}{2T_o}\right) \log_{10}\left(\frac{p_o}{p}\right)$	$y = 60370\left(\frac{T+T_o}{2T_o}\right) \log_{10}\left(\frac{p_o}{p}\right)$

TABLE 2.

Standard Atmosphere.

Air Dry — Gravity Constant

Metric					English				
y meters	t °C	$\left(\frac{T}{T_o}\right)$	$\left(\frac{p}{p_o}\right)$	$\left(\frac{\rho}{\rho_o}\right)$	y feet	t °F	$\left(\frac{T}{T_o}\right)$	$\left(\frac{p}{p_o}\right)$	$\left(\frac{\rho}{\rho_o}\right)$
0	15.0	1.000	1.000	1.000	0	59.0	1.000	1.000	1.000
500	11.7	.989	.940	.951	2000	51.9	.986	.928	.943
1000	8.5	.977	.886	.908	4000	44.7	.972	.862	.888
1500	5.2	.966	.834	.865	6000	37.6	.959	.800	.837
2000	2.0	.955	.784	.823	8000	30.5	.945	.741	.786
2500	-1.2	.944	.737	.781	10000	23.3	.931	.686	.738
3000	-4.5	.932	.692	.741	12000	16.2	.917	.635	.693
3500	-7.7	.921	.649	.706	14000	9.1	.904	.587	.649
4000	-11.0	.910	.608	.669	16000	1.9	.890	.542	.608
4500	-14.2	.898	.569	.635	18000	-5.2	.876	.498	.569
5000	-17.5	.887	.533	.600	20000	-12.3	.862	.459	.531
6000	-24.0	.865	.466	.539	22000	-19.5	.849	.421	.496
7000	-30.5	.842	.405	.479	24000	-26.6	.835	.386	.463
8000	-37.0	.819	.351	.429	26000	-33.7	.821	.353	.432
9000	-43.5	.797	.303	.381	28000	-40.9	.807	.324	.402
10000	-50.0	.774	.262	.337	30000	-48.0	.794	.297	.374

Metric:

$\left(\frac{T}{T_o}\right) = (1 - .00002257y)$

$T_o = 288°C$

p_o = 76 cm. Hg. $g\,\rho_o = 1.225$ kg/m^3

English:

$\left(\frac{T}{T_o}\right) = (1 - .000006878y)$

$T_o = 418.6°F$

p_o = 29.921 in. Hg. $g\,\rho_o = .07635$ lbs/f.