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By Dr. Adolf Rohrbach.

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1. Basis of Problem.- Structural safety is now generally expressed by a number, termed the factor of safety, which indicates how many times larger the breaking strength of a structural part is than the force exerted on said part in rectilinear unaccelerated flight. This conception of structural safety has two fundamental, oft experienced defects, the lesser of which is due to the following cause.

It is known that a structural part can permanently support a given load, only when the stress exerted on it does not exceed the proportionality limit. For different materials and different structural parts, the breaking stress and the claim on the proportionality limit stand by no means in the same relation to each other. Only for the same materials and similar shapes can we therefore utilize directly the breaking strength, instead of the elasticity limit, for the determination of the factor of safety. The conception of the proportionality limit would not have been easy to determine during the last war for the main structural parts of airplanes were made for the most part out of greatly differing materials. Its experimental determination by testing samples would have resulted in a great loss of time. The practical determination of the proportionality limit of wings, etc., could not have brought improvements commensurate with the expense, since the military airplanes were then very similar, both in materials and method of construction. Hence the breaking test was a sufficiently accurate and a very economical method for determining the factor of safety. The destruction of a few in breaking tests guaranteed the strength of very large numbers of airplanes, and the value of the broken ones constituted only a small premium for the safety of the many other airplanes of similar construction. Since, at the present time, only small numbers, often only a single airplane, of a given type are constructed, they must be calculated and constructed with especial care, in order to save the present extremely high cost of breaking tests with whole airplanes. By taking into consideration the data obtained during the war and supplementary data concerning individual parts and their combinations, it is probable that the actual factor of safety of such carefully calculated structures will have very nearly the desired value.

Very much less satisfactorily, however, could we hitherto answer the question, as to what factor of safety should be given new airplanes, either for war or peace. The previous work is intended to help in answering this question and therewith remove the present far more serious defect of the conception of structual safety mentioned at the beginning of this article.

It is inherent in the nature of an airplane that its factor of safety, like that of any other vehicle, is less needed during rectilinear unaccelerated motion (on which, however, it is based), than during those portions of the flight when the motion of the airplane is accelerated. The value of the still existing factor of safety under certain flight conditions of an aircraft (to which we shall now confine ourselves) will accordingly be given by the greatest possible acceleration of the aircraft under said conditions. Hence it is customary, for the strength of the wings, to distinguish several cases of loading characterized by the angles of attack. It is known that, at a large angle of attack, considerably greater accelerations occur than at a small angle of attack. Also an easily managed combat airplane can be considerably more overloaded than a sluggish bombing airplane. Both these considerations were taken into account during the war by requiring the highest factors of safety for large angles of attack and light combat airplanes and smaller factors of safety for smaller angles of attack and heavier airplanes.

The factors of safety during the war rested, on the one hand, on the practical experience that wings of a certain strength held, while weaker ones broke, in flight, and, on the other hand, on a few acceleration tests made on flying airplanes, in taking off, etc as well as ultimately on pure mathematical calculations.

The factors of safety first required by the B.L.V. (military specifications) in 1916 (Table 1) were modified in 1918 (Table 2) on the basis of the enormously increased experience. The safety factors for the most maneuverable airplanes were raised considerably, while they were somewhat lowered for giant airplanes. It is probably true that the safety factors of the 1918 B.L.V. military airplanes were just large enough to guarantee the wings against breaking.

It will be a long time before new data on the requisite strength of the wings can be obtained in such abundance as given in Table 2. Hence the factors of safety of the 1918 B.L.V. must be used for the near future, as the basis for the determination of the factors of safety of new airplane types. Nevertheless, we cannot set down offhand for the safety factors of new airplane types, differing greatly from those of the last war, any safety factors whatsoever of the 1918 B.L.V. and then wait to see whether the absence or occurrence of wing breaks speaks for or against the use of the old factors of safety for other airplane types. Too small factors of safety would weaken confidence in aviation. Unnecessarily high factors of safety would hinder the development of airplanes, on account of their excessive weight and lack of economy. Hence the question still remains to be answered, as to what factors of safety, in accord with military experience with the 1918 B.L.V., are the right ones for airplanes differing greatly from those of that time, as, for example, the 1000 HP all-metal monoplane of the Zeppelin works at Staaken. The following considerations are intended to aid in the solution of this problem.

First let us consider the safety factor of a wing at large angles of attack (Case A). Slow flight at a large angle of attack, gliding and stalled flight do not overload the wings and are therefore of no interest in this connection. Large angles of attack are however combined with such high speeds, in taking off and during curved flight, that the force acting on the wings exceeds the weight of the airplane and causes a bending of the flight path toward the upper side of the wings. It may be regarded as undecided whether the wings are taxed more in taking off or in sharp curves. In any event, the difference between the maximum values of the wing stresses generated in these two cases cannot be great, else experience would have shown one of them to be more dangerous than the other. As a matter of fact, wing breaks have resulted about as often from one cause as the other.

Numerous attempts have already been made to determine the relations between the maximum load according to mathematical calculations and the factor of safety found necessary in the light of experience. Any factor of safety thus determined, from the flight path of the airplane in taking off, would also answer for curved flight, since both forms of flight seem to stress the wings about equally. Of course the reverse must also be true, that a safety factor obtained by overloading the wings during curved flight answers likewise for taking off. As a matter of fact, only the latter path leads to the goal, since, as far as such calculations have to do with over-stressing the wings in taking off, they must remain worthless for the present, because there are not even approximately accurate experimental values to serve as a basis for reproducing this motion with sufficient accuracy, i.e., with an approximation of 10 to 20%. The speed at the beginning of the take-off is only approximately known. Its changes might be approximately calculated, but we do not know with what speed the elevator is moved and therefore cannot calculate the turning speeds of the airplane and its angles of attack. Aside from this, such calculations would make a great deal of work, on account of the necessary graphic integrations. Much more favorable conditions are presented by curved flight in a horizontal plane, with engine running. Here we can, on the basis of comparatively accurate experimental data, calculate, in a simple manner, a condition of permanence with much greater accuracy. Hence curving flight has already been theoretically treated with considerably more thoroughness than the phenomena of taking-off .* These calculations give, first of all, the diameters of the smallest circles for different flight altitudes, as likewise the times required for traversing them. What is more, even the excess loads of the wings belonging to various radii are calculated for a D and a C airplane. Thereby, contrary to the results of this work, the greatest wing stress is found at the angle of attack at the height limit, hence for the sharpest curve. Starting with these earlier works on curved flight, on the one hand, and with the safety factors of war time, on the other, a simple relation must now be established between the maximum loading of the wings during curved flight and the requisite safety factor for Case A.

2. <u>Accomplishment of Task</u>.- A general relation between the maximum wing loading and the safety factor of the airplane can only be found, when both are known for several different airplane types. For this purpose, the maximum wing loading must first be found for several airplanes for a series of radii. This calculation needs to be made only for the air density at sea-level, since the wings are stressed most at that level.

The formulas in the works mentioned are little suited to such frequent repetition of the calculation. Therefore a "nomogram" was worked out for the calculation of curved flights, by adding another bunch of lines to a long since verified diagram for the calculation of flight performances.** After this completion of the diagram, the requisite gliding capacity can be represented on the same sheet in a simple manner for the curving flight of an airplane of given lift and drag coefficients, for any desired radii and wing loading. In order to be able to find conveniently the maximum propeller efficiency on the basis of the new American propeller experiments,*** a new nomogram was worked out, by means of which the propeller efficiency can be calculated for different propeller shapes, diameters and flight speeds, which are available with the employment of a given engine, for which we need to know the relation between the r.p.m. and the torque. The description and application of this nomogram will be given in another place, since it can interest only a portion of our readers.

The calculation of the curving flights was made for the following airplanes: SSW.D IV; DFW.C.V; Alb.D.III; Staak.R.XIV. As the

* Kann, "Der wagerechte Kurvenflug des Flugzeuges," T.B.III,p.26.
Salkowsky, "Der Kurvenflug eines Flugzeuges," T.B.III, p.267.
** Rohrbach and Lupberger, "Zeichnerisches Verfahren zur Berechnung der Geschwindigkeit und des Steigvermögens der Flugzeuge," T.B.III, p.218.
***Durand and Ashley, "Experimental Research on Air Propellers," Reports Nos. 14, 30, 64, 109 of National Advisory Committee for Aeronautics, U.S.A.

results of a nomogram calculation of this kind, there are drawn in Fig. 1 the requisite gliding capacities for the curving flight of the DFW.C.V in relation to the flight speed for various curve radii There is also introduced on the same scale the attainable propeller efficiency with a 220 HP Benz engine. This presentation refers only to flight near the ground. A horizontal curve is flown at such speed that the line of propeller efficiency cuts the line of gliding capacity for the given radius. If the airplane flies faster (slower), it falls (climbs) with a speed proportional to the difference between gliding and propeller efficiency, i.e., to the corresponding perpendicular distance between the two efficiency curves. From the radius of the circle and the corresponding flight speed we can now calculate the mean force resulting from the combination of weight and propeller thrust, which must be borne by the wings. Thus we obtain the curve excess loading α according to the formula

$$\alpha = \sqrt{1 + \frac{v^4}{g^2 r^2}}, \text{ in which} \qquad (1)$$

$$\alpha = \frac{\text{load borne by wings}}{\text{weight of airplane}}$$

v = flight speed in m/sec.
r = radius of circle in m.
g = acceleration due to gravity in m/sec²

For the purposes of the present investigation, the repeated use of this difficult equation was however avoided, since the curve excess loading of the wings was very easily obtained by means of the nomogram. The four airplanes mentioned are characterized by the data given in Table 3. The excess loads of their wings occurring at various radii are represented in Fig. 2. It is clearly shown how the wings of the maneuverable airplanes of less weight per horsepower are much more stressed than those of the heavy airplanes with small reserve power. It is known that the maximum loading does not occur in the sharpest curve, for this is flown over at such a relatively low speed (on account of the small available propeller efficiency at low speeds) that only small flight forces are generated.

In order to see how the excess loading in the curve is related to the drag coefficients, the wing loading, the weight per horse power, etc. (Equation 1) with the utilization of the reasoning methods of earlier works, is transformed to

$$\alpha = \sqrt{\frac{\frac{C_\alpha^3}{C_D^2} \; 358 \; \eta^2}{\left(\frac{W}{N}\right)^2 \left(\frac{W}{S}\right)}} \qquad (2)$$

and hence: $\alpha = \frac{\gamma}{\gamma_g}$ (3)

in which, W = weight of airplane in kg, N = HP of engine at sea-level, S = area of wings in m^2, η = propeller efficiency, γ = air density in kg/m^3 at sea-level, and γ_g = air density at "ceiling." Equations 2 and 3 naturally give correct results, only in so far as the assumptions made in deducing them agree with the facts. These assumptions, aside from the customary ones, are, in particular, reduction of engine power (HP) in proportion to the air density and propeller efficiency independent of the flight speed. For accurate calculations, the employment of the nomogram and formula 1 is therefore preferable. However, we must also know the aerodynamic coefficients of the airplane, as well as its propeller efficiency and the other quantities in the formula. When this is not the case, the simple Equation 3 will often give more reliable results, than the attempt to reach the goal, by means of any assumptions for the unknown quantities, with the nomogram or with Equation 2.

Equation 2 shows plainly to what extent the excess wing loading increases during horizontal curving flight, when the airplane is aerodynamically improved, or when the propeller efficiency is raised, or the load per HP and wing loading are diminished.

3. Result.- The following numerical example, which was solved by means of the nomogram, gives, for curved flights with different radii, the excess wing loading of two airplanes, Nos. 1 and 2, of like load per HP and wing loading, 4.75 kg/HP and 45 kg/m^2, as also the same propeller efficiency, 0.78, which are only distinguished from each other by their different aerodynamic quality (Figs. 3-4). The maximum speed of airplane No. 1 is found to be 225 km/hr, while that of No. 2 is only 175 km/hr. The maximum climbing speed of No. 1 is 7 m/sec, and of No. 2 is 6.5 m/sec. We see, from Fig. 4, that the wings of the aerodynamically better airplane must withstand, in 44% curves, greater stresses than the wings of the poorer airplane, No. 2. Hence although a safety factor of 5.5 suffices for No. 2, No. 1 must have a safety factor of 8, in order to furnish the same protection against failure in circling flight. In this example, the better airplane, No. 1, was provided with the propeller which gives great speed. For this reason, the climbing speeds of the two airplanes did not differ much, in spite of the great difference in their head resistance (Fig. 5). If we should give airplane No. 1 (with the relinquishment of a small part of the maximum speed) a propeller as well suited for climbing as the propeller of No. 2, the climbing speed of No. 1 would then be considerably greater than that of No. 2 and the values of α would differ far more than 44%. It would, in any case, be exceedingly dangerous to give the same factor of safety to the wings of such an aerodynamically better and consequently more efficient airplane

(on the assumption that we have to do with an experimental value), which factor of safety had always been found sufficient for a poorer airplane.

Since some of the present-day commercial airplanes are considerably better, in their aerodynamic properties, than the airplanes at the end of the war, for which alone there was sufficient knowledge regarding structural safety, and since we may count on still further aerodynamic improvements, warning must be given, with reference to the stress during curved flight, against the direct application of the safety factors of war-time stipulations to very good new airplanes. We often hear it said that maneuverability is less important for traffic airplanes. Doubtless, they do not need to be built expressly for maximum maneuverability, but, apart from the fact that, in exhibitions or other circumstances, many pilots, without being obliged to do so, make sharp curves even with commercial airplanes, there are many conceivable circumstances in which sharp curves cannot be avoided, as, for example, in evading a suddenly appearing obstacle or quickly reaching a small field in the event of a forced landing. Therefore the wings of every commercial airplane must be able to withstand the stress of any curve or take-off. The experimental values for these cases are given by the comparison of the maximum loads during curved flight, shown on Fig. 2, with the corresponding safety factors of Case A, as contained in the military specifications. We designate the factor, by which the maximum load during curved flight is to be multiplied, as the safety factor for curved flight, σ_A. The values of α and σ_A are given together in Table 4 for the four known military types.

In addition to horizontal curving flight with engine running, there are also other possible circumstances in which the wings must withstand a greater load than in unaccelerated rectilinear flight, since the wings are similarly stressed, when curved flight becomes spiral flight, in climbing or descending. With respect to the air density, which varies with the altitude, the curvature in such spiral flights must constantly grow sharper toward the ground, if the loading of the wings is to remain constant. From the very great change in the perpendicular component of the flight speed for short radii (Fig. 5), it follows that only in very steep and narrow spiral flights the wings can be much more heavily loaded than during horizontal curving flight. We may therefore assume that the values of α, corresponding to the latter, will seldom, if ever, be exceeded, whereby the magnitude of such a possible excess would be given by the same factors, which hold good in Equations 1 to 3 for horizontal curving flight. The same probably holds good for the most-to-be-feared forces, to which the wings are subjected in a combination of curving flight and taking off. All these considerations indicate that σ_A represents a value which (although it does not give directly the factor of safety present in the most unfavorable cases) is still proportional to this minimum factor of safety.

In fact, the very good agreement of the values of σ_A, for entirely different airplane types, shows that a very reliable empirical factor is contained in σ_A. We may therefore employ the same value of σ_A and find the right wing-load factor according to Equations 1 to 3, as:

$$\text{Load factor of Case A} = \sigma_A \sqrt{1 + \frac{v^4}{g^2 r^2}} \qquad (1a)$$

$$= \sigma_A \sqrt[3]{\frac{\frac{C_L^3}{C_D^2} \; 358\,\eta^2}{\left(\frac{W}{N}\right)^2 \left(\frac{W}{S}\right)}} \qquad (2a)$$

$$= \sigma_A' \frac{\gamma}{\gamma_g} \qquad (3a)$$

In this connection the value of σ_A, according to the empirical values of Table 4, cannot be much smaller than 2.5, if the wing is to be strong enough.

After such an unobjectionable method has been found for determining the wing-load factors for large angles of attack, the questions arise as to what will accordingly hold good for the wing strength for other angles of attack and what are the consequences for the strength of the tail unit and fuselage, in so far as the latter serves for the transfer of the air forces from the tail unit.

For small positive angles of attack, hence in Case B, the wing strength will be utilized especially in steep spiral glides and in coming out of a dive. The latter is much more susceptible than the spiral glide to mathematical calculation, for the reasons already mentioned in connection with the take-off. The fitness of an airplane for spiral and horizontal curving flight is dependent on the same factors. Hence, it follows that the wing-load factors for Case B can also be determined in the manner worked out for Case A, according to formulas 1a to 3a, by means of a similar empirical value σ_B, instead of σ_A. In Table 4 there are accordingly found also the load factors required from the military specifications for Case B, as likewise the values of σ_B found by dividing the same by α. These also agree very well with each other.

The figures of the military specifications for diving flight, Case C, should, in all important cases, be replaced by the results of special aerodynamic calculations of the forces and moments, to which the wings are subjected in a dive.

As yet we have no basis for determining the safety factor of inverted flight, Case D, which is, however, of the least importance.

It would be well also to raise the load factors of Case D correspondingly with reference to the military specifications, if we find higher load factors in Cases A and B than customary at the end of the war.

The air forces on the tail unit produce equilibrium between the air forces on the wings and on the remaining parts, on the one hand, and between the propeller thrust and momentum, on the other. From this it follows directly that the strength requirements of the tail unit depend on those for the wings. Hence, we must first determine the forces, for the angles of attack corresponding to the Cases A, B, C, and D, which must act on the elevator in order to produce equilibrium with the other forces. The same load factors must be provided for the reception of these elevator forces, as for the wings under corresponding flight conditions. The rudder will be strong enough, if calculated on the basis of the most unfavorable case for the elevator. Of how much greater significance this individual determination of the safety factors of the tail unit can be, is shown by the construction of the Staaken 1000 HP monoplane. As a result of the high wing loading, there are so great pressures on the tail unit, that the breaking loads (given in the military specifications simply in kg/m^2) for tail planes would have given only about half as strong elevators as are, in fact, necessary, according to aerodynamic calculations.

The further construction of airplanes is effecting a constant diminution of their head resistance. As was shown, the structural safety must, at the same time, be continuously increased. The constructor may regret that the hitherto requisite excess weights somewhat diminish the gain from the lessening of the head resistance. On the other hand, the traffic manager, who produces and uses the airplanes, will rejoice that the development of airplanes must bring him not only more efficient, but also (on account of their high factor of safety) stronger and more stable airplanes.

Table 1.

Safety factors of wings according to the 1916 B.L.V.

Airplane type	D	C & G	R
Taking off	5.0	4.5	4.0
Gliding	3.5	3.0	2.5
Diving	2.5	2.0	1.5
Inverted	3.0	2.5	2.0

Table 2.

Safety factors of wings according to the 1918 B.L.V.

Total weight of airplane, kg.	Over 5000	2500-5000	2500-4000	1200-2500	Less than 1200
Carrying capacity, kg.		1000-2000	800-1500	400-800	Less than 400
Taking off	3.5	4.8	5.5	5.8	6.5
Gliding	2.5	2.6	3.2	3.3	4.0
Diving	1.2	1.5	1.75	2.0	2.0
Inverted	-	-	2.8	2.8	3.5

Table 3.

Airplanes of the illustrative calculations.

Airplane	Weight in kg.	Brake HP of engine	Area of wings, m^2	Coefficient of air resistance
SSW. D. IV	700	200	15.2	T B III, p.262
Alb. D III	910	178	22.0	Like DFW C.V.
DFW. C. V.	1540	235	42.2	T B III, p.270
Staak.R.XIV	14000	1225	332.0	From a few experiments.

Table 4.

Curved-flight loads and factors of safety.

Airplane	Wing load factors Case A	Wing load factors Case B	Maximum wing load Curve radius	Value of α	σ_A	σ_B
SSW. D. IV	6.5	4.0	82 m	2.74	2.38	1.46
Alb. D. III	5.0	3.5	86	2.12	2.36	1.65
DFW. C. V.	4.5	3.0	72	1.93	2.34	1.55
Staak.R.XIV	3.5	2.25	105	1.43	2.45	1.57

APPENDIX.

I. Graphic Determination of Airplane Speeds during Rectilinear and Curved Flight.

1. The diagram.- In order to save time and to see at a glance the effect of structural changes on flight efficiency, a diagram was made years ago for the graphic determination of flight speed and climbing ability in straight ahead flight. The addition of another set of curves in each nomogram now renders its application possible to the determination of speed, climbing ability and excess wing loading during curved flight: Although the diagram, in so far as it concerns rectilinear flight, has already been described in "Technische Berichte,"* we will here give enough of it for the explanation of the newly added curving flight calculation.

By means of the nomogram (Figs. 6-7), we can represent, for airplanes of given drag coefficients for any desirable wing loads and flight altitudes, the values of N_g/W (line sg) and N_k/W (line sk) as ordinates over the corresponding flight speeds, which serve as abscissas. Here W represents the weight of the airplane in kilograms; N_g the HP in straight ahead flight; N_k the HP during curved flight, just sufficient for overcoming the head resistance. Hence N_g/W and N_k/W receive the dimension of a speed. This speed is equal to the falling speed at which the airplane must drop, in order to be able (when there is no available engine power) to glide with the speed $v_g (v_k)$ corresponding to $N_g (N_k)$.

If we first consider only the calculation of rectilinear flight, then the basis for the nomogram is

$$\frac{N_g}{W} = \frac{C_D}{C_L} v_g \qquad (1)$$

in which

$$v_g = \sqrt{\frac{W}{S} \frac{}{C_L \, g}} \qquad (2)$$

Here S = wing area in m^2 and g = pressure in kg/m^2.

The diagram consists of the upper main part, on which are drawn the air resistance values C_L and C_D of the whole airplane

* Rohrbach and Lupberger, "Zeichnerisches Verfahren zur. Berechnung der Geschwindigkeit und des Steigvermögens der Flugzeuge," T.B.III, p.218.

in the form of the Lilienthal polars p, as also the computed dat sg corresponding to Equation 1, and also of the lower auxiliary portion, in which, by means of two sets of lines f and h, at a certain flight altitude and a given wing loading, the desired lift coefficients of coordinated flight speeds are obtained according to Equation 2.

The right-hand lower corner of the upper main part of the diagram serves as the origin of the coordinates for the air resistance coefficients and the results of the computations. The values of C_D are indicated on the ordinates, on the scale of 5 mm equals unity, and the values of C_L as abscissas, with 1 mm = unity. The scale for N_g/W and N_k/W is so chosen that, corresponding to Equation 1, the right triangle with the sides C_D and C_L adjacent to the right angle is similar to the resulting triangle with the sides v_g and N_g/W adjacent to the right angle, so that thus the coordinated points of the polars and of the N_g/W or sg lines lie on one and the same line i passing through the origin.

The lower auxiliary portion of the diagram contains, on the right, the bundle of hyperbolas of like wing loading. In order to obtain this set, the corresponding values of C_L and v_g must first be determined according to Equation 2 for certain values of W/S (here 30, 40, 50, etc. kg/m²). The values of v_g are now entered below as ordinates with the corresponding abscissas C_L. Then the hyperbolas f can be drawn. The origin of these lines of like wing loading is 40 mm above the line of separation between the upper and lower portion of the diagram. The scale for v_g values may be arbitrary. Here 2 mm = 1 m/sec. The inclination of the straight lines for flight at sea level must be so adapted to the scale chosen for v_g, that the scale already used for the values of C_L can be used, i.e., 1 mm = 1 km/hr. The remaining altitude lines, marked for altitudes from 1000 to 9000 m, also pass through the origin of the coordinates of the hyperbolas of like wing loading. Their inclination is such that the distances, from the right side of the diagram to their points of intersection with any desired horizontal line, are inversely proportional to the square roots of the corresponding air densities.

During curved flight, the wings are subjected to the resultant force of the weight and the propeller thrust. Therefore the speed of every airplane in curving flight is greater than in rectilinear flight at the same altitude with the same angle of attack and the same load. The relation between these speeds is given by

$$K = \frac{v_k}{v_g} \sqrt[4]{\frac{1}{1 - \left(\frac{v_g^2}{g\,r}\right)^2}} \tag{3}$$

in which r = radius of curve in meters and g = acceleration due to gravity in m/sec². The set of altitude lines on the left side of the diagram corresponds to this equation, as follows: The straight lines for $r = \infty$, drawn in any desired direction, and a set of curves r give intersection points with every horizontal line through the set of curves. The abscissas of these intersection points now stand in the relation given by Equation 3, if we indicate by v_g the abscissas of the intersection of any horizontal line with the straight line for $r = \infty$, as likewise the abscissas of the intersection of the same horizontal line with one of the curves for a finite value of r with v_k. Every line, which we imagine drawn through the intersection of the straight lines for $r = \infty$ with the right side of the diagram so that it cuts the set of r curves, corresponds, therefore, to a certain value of k. By themselves, the values of k are considerably less important than the excess wing loading α, which occurs during curved flight and which is connected with k in the expression $k^2 = \alpha$. We therefore obtain the straight lines of like α by drawing from the values of α, given in the left lower corner of the nomogram, the connecting lines to the intersection of the straight lines for $r = \infty$ with the right side of the nomogram. The intersection points of the lines of like curve radii, with such a α = straight lines, then show the speed at which a curve with the given radius must be flown, if the wings are to be more heavily loaded, by the value α, than in rectilinear flight. Corresponding to Equation 1, we may write

$$\frac{N_k}{W\alpha} = \frac{C_D}{C_L} v_k.$$

From this equation the value of N_k/W can be obtained, either by simple computation or by a combination of drawing and computation and be set over the corresponding v_k. Thereby a value of N_k/W corresponding to a climbing speed of 1 m/sec, as also for N_g/W, is represented by a length of 18 mm.

2. <u>The calculation</u>.- We first draw the polar diagram p at 90° from its usual position. The hyperbola f, corresponding to the wing area and load in question according to Equation 2, is drawn, preferably in color. Likewise, the straight lines h, corresponding to the given altitude, and the curves r, coordinated to the assumed curve radius, are drawn. Starting from any point A of the polars p, the line g is broken at right angles on the hyperbola f and on the straight line h. The intersection of g with the line i, from the origin of the coordinates through the exit point A gives the point B, whose abscissa gives the flight speed corresponding to the remaining values for rectilinear flight and whose ordinate gives the coordinated value of N_k/W. If we now go from the intersection point of g and the altitude line h to

the line $r = \infty$ and thence, making a right-angle turn, to the curve r of the chosen radius, then the abscissa of the point of intersection C with the latter gives the speed during curved flight corresponding to the other values and to the point A of the polars. The line i intersects the perpendicular through C at the point D. According to Equation 4, the ordinate of D in the ratio $1 : \alpha$ must be extended to E, if we wish to obtain N_k/W. Thus a point B of the curve sg and a point E of the line sk are coordinated. If we introduce, on the same scale, the line ss of the values of N_s/W, given by the available propeller efficiency N_s, we thus obtain the values corresponding to any desired flight speed of the (positive or negative) climbing speeds, since the latter are proportional to the differences of the ordinates of ss, on the one hand, and of sg or sk, on the other.

In order to determine the relation of the maximum speed or of the climbing speed to the flight altitude or the radius of the curve, the computation must be repeated for a series of flight altitudes or curve radii, by drawing further pairs of lines sg, sk, and ss.

3. Proof.- By means of the flight lines g, we can determine the apex H of the triangle HFG, which, however, was not drawn during the computation, but was completed in Fig. 7, for the sake of clearness. In this connection, the distance FG is proportional, according to the diagram, to the flight speed v computed by Equation 2, which belongs to the hyperbola f and the lift coefficient of the point A. The line HG represents the same speed v_g on such a scale that the C_L distribution can be simultaneously employed for reading the speed in km/hr, in which connection the air density will also be taken into account through the different inclination of the h line. According to this construction, the line KM in the triangle KMB is proportional to the speed v_g, while the line MB, on account of the similarity of the triangle LAK and MBK, according to Equation 1, is proportional to N_g/W. By drawing the line HIC, the side KN of the triangle NDK, likewise similar to both the above-mentioned triangles, is proportional to v_k, while its other side ND is proportional, as before, to v_k C_L/C_D. The extension of ND by thus gives in NE a distance proportional to N_k/W.

II. Graphic Determination of Propeller Efficiency According to Experiments with Models.

1. The task.- The best and most comprehensive experiments with propeller models are those published by Durand and Lesley.* For the

* Durand and Lesley, "Experimental Research on Air Propellers," Reports Nos. 14, 30, 64, and 109 of the National Advisory Committee for Aeronautics, U.S.A.

utilization of the experimental results, there are appended to these reports special diagrams, on which the computation can be carried out with a logarithmic scale according to the method of the comparison of variously directed lines. This method of computation is so lacking in clearness and the lines drawn overlap one another so much, that errors easily occur. In order to eliminate this disadvantage and to obtain the values of N_s/W directly, as a curve dependent on the flight speed, in the same scale in which the just described nomogram gives N_g/W and N_k/W, a new diagram (Figs. 8 and 9) was worked out for the graphic computation of the propeller.

The basis of this nomogram is the equation for the energy absorbed by the propeller:

$$\frac{N_m}{W} = \frac{Q_c \; \gamma \; v^2 \; n \; D^3}{W \; 716000} \qquad (1)$$

in which
- D = diameter of propeller in meters,
- W = weight of airplane in kilograms,
- N_m = energy in HP absorbed by propeller,
- N_s = energy in HP given out by propeller,
- M = torque-moment in kg-m absorbed by propeller,
- v = flight speed in meters per second,
- n = r.p.m.,
- γ = air density in kg/m³,
- Q_c = coefficient of torque-moment of the American reports, which is characterized by the dimension

$\left(\frac{m}{sec^2}\right)^{-1}$ corresponding to the expression

$$Q_c = \frac{1000 \; M}{\gamma \; v^2 \; D^3} \qquad (2)$$

We find, in the above-mentioned reports, the values of Q_c, as well as the values of the propeller efficiency η, plotted against $60 \frac{v}{Dn}$. The values D and W, contained in Equation (1), and the number 716000 are combined in

$$c = \frac{D^3}{W \; 716000} \qquad (3)$$

so that

$$\frac{N_m}{W} = Q_c \; \gamma \; v^2 \; n \; c \qquad (1a)$$

After we have plotted the values of N_m/W against the flight speed by means of the nomogram for various revolution speeds (in accordance with Equation 1a), we have no difficulty in determining under what conditions the available engine power (for the same revolution speed of the propeller) is exactly equal to N_m. By multiplication with the propeller efficiencies corresponding to like running conditions, we obtain the values N_s/W, corresponding to the energy N_s given out by the propeller, plotted against the flight speed.

In order to be able to transfer the values of N_s/W, found in this nomogram, in a simple manner for computing the flight performances, to the first diagram, we must have the quantities N_m/W, corresponding to the left side of Equation 1a, in the same scale in which N_s/W and N_k/W are there drawn. The remaining scales must accordingly be so chosen that we will have, for the left side of Equation 1a,

$$\frac{1}{75} \times \frac{HP}{kg} = 1 \text{ m/sec} = 18 \text{ mm.}$$

2. The diagram.- In the finished diagram (Fig. 8), we have, on the same sheet, the initial values, namely, the results of the experiments with models, the lines Q_c and η (Fig. 9), and the final values of the computation, hence the values N_s/W, line s in Fig. 9. The lower left-hand corner of the diagram is the origin for presenting the results of the experiments with models. The ordinates are measured by the values of $60\frac{V}{Dn}$, inscribed on the left margin, in which the value 1 of this product is equal to a distance of 180 mm. The experimental values of the coefficient of torque-moment Q_c, corresponding to different values of $60\frac{V}{Dn}$, and of the propeller efficiency η are recorded as abscissas. Here one Q_c unit equals 40 mm, while a distance of 400 mm corresponds to the unit of propeller efficiency $\eta = 1$.

For the transition from this presentation of experimental results to the real computation, use is made of a set of lines a, in Fig. 9, from which, in Fig. 8, are drawn the straight lines corresponding to 10 different values of Dn progressing in uniform steps. The origin of the line a, conjointly with the origin of the coordinates for representing the result of the computation, lies on the extension of the bottom line of the drawing, 400 mm from its left end.

For the computation results of the diagram, corresponding to this origin of the coordinates, the reciprocal values of N/W (load per HP) are measured as ordinates by a scale of 1 m/sec = 18 mm. On the axis of the abscissas, we find how the flight speeds are given, with the accompanying flight performance nomogram, where 1 km/hr = 1 mm. For carrying out the computation, we need also a set of parabolas b (values of c), both sets of lines c (values of n) and d (values of the flight altitude A), as also the "proportionality-axis" e. The origin of the set of parabolas coincides with the origin of the set of lines a, the origin of the coordinates. For 20 different values of c, progressing in uniform steps, we find in Fig. 8 the corresponding parabolas, whose abscissas equal v and whose ordinates equal $c v^2$. Here the set c shows the lines for 13 uniformly distributed values of n between 800 and 2000. In the same manner, the set d gives 10 lines from sea-level to 9000 meters flight altitude. The proportionality axis e is 59.2 mm from the left margin of the drawing perpendicular to the axis of the abscissas.

3. The computation.- We first represent the experimental results for the chosen propeller shape in the form of Q_c and η lines in the diagram. In the sets of lines, a, b, c, d, we draw lines, preferably colored, corresponding to the given or assumed values of Dn, n, c, and H.

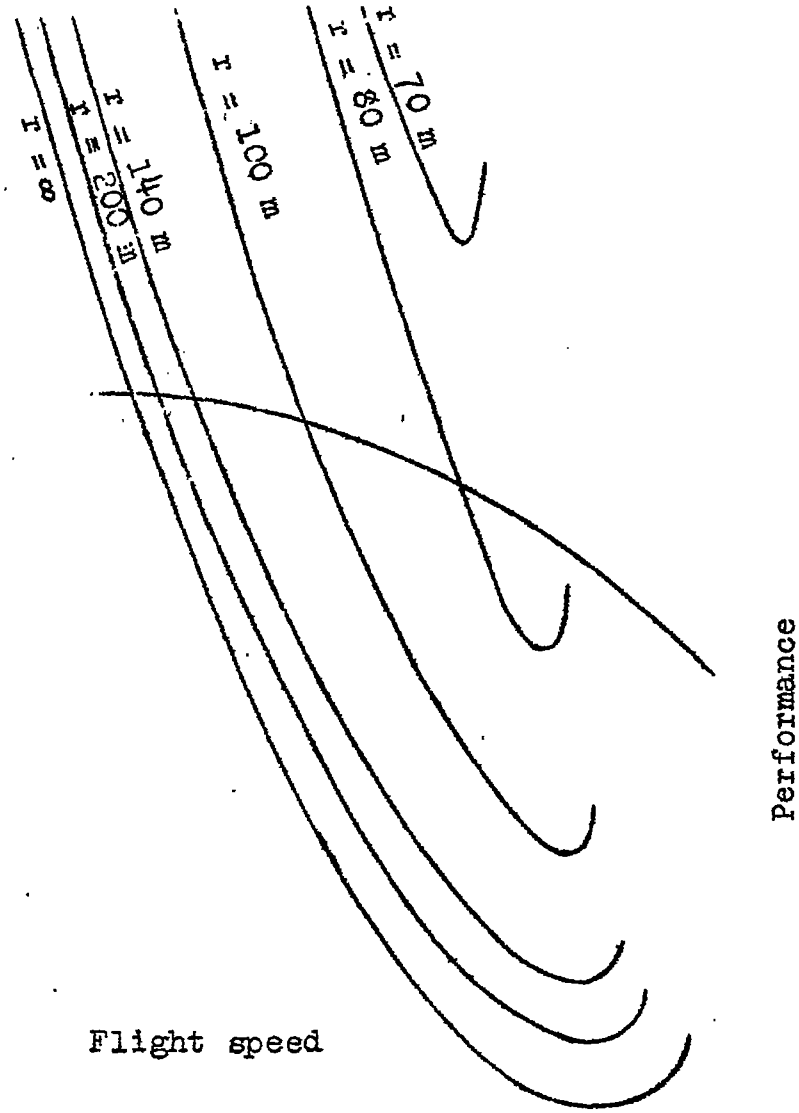
We first determine the energy absorbed by the propeller at the revolution speed n. For this purpose, we draw the ordinate f corresponding to any flight speed. Through its point of intersection with the parabola b we pass along the line g, which is broken in several places by the lines c, d, and "sea-level", to the proportionality axis e. We connect the point of intersection H with this axis, by means of the line i, to the left lower corner k of the diagram. We then draw, from the point of intersection of the ordinate f with the line a, the line h (broken by the Q_c curve) to its point of intersection with i. The distance IF, between the latter intersection point of i with h and the axis of the abscissas is the desired quantity N/W. It will be indicated on the ordinate f by the mark l. In like manner, the value of N/W, which is proportional to the energy absorbed by the propeller, is determined for other flight speeds. Thus we obtain the n = constant curve, as the representation of all the quotients N/W.

The determination of the energy N_s given out by the propeller is now simple. For the brake horsepower N_m at the revolution speed n, corresponding to the n-constant curve, the length of the ordinate, corresponding to the value N_m/W in the scale of the nomogram, is calculated. The point A of the n-constant curve, whose ordinate p is equal to the one thus computed, gives the flight speed at which the propeller exactly absorbs the energy imparted by the engine. We now extend the ordinate p through A and thus obtain the propeller efficiency corresponding to A, by bending at right angles at the line a at the point of intersection with the η curve. The propeller efficiency shows in what ratio the ordinate of A must be diminished, so that the ordinate of B will represent the value of N_s/W corresponding to the energy given off by the propeller. The repetition of this computation for other values of n gives further points A and B and, as their total, finally the line m coordinated with the energy given out by the engine, as also the curve s corresponding to the propeller output. The latter must then be transferred to the previously described nomogram for the computation of the flight efficiency. The work may be greatly simplified by again utilizing most of the original lines f, g, h, etc. in repeating the computation for other revolution speeds and flight altitudes.

4. Proof.- The distance CD represents the value of $60\frac{v}{Dn}$, which corresponds to the Dn of the straight lines a and to the flight speed of the point D. According to this, however, the distance KF, the abscissa of $E_{,v}$ is equal to Q_c, which is coordinated with the value of $60\frac{v}{Dn}$ represented by DC. It is obvious that the distance HG, found by means of the line g out of the distance DL which represents the product $c\ v^2$, must be proportional to the product $\gamma\ v^2 n\ c$ (Equation 1a). From the similarity of the triangles HGK and IFK, whose side KF is equal to Q_c, the distance IF is proportional to the total value of left side of Equation 1a. The scales of the drawing and the distance KG are so chosen that the distance IF represents the value of N_m/W and, corresponding to the desired agreement of the scales in both nomograms,

$$\frac{1}{75} \times \frac{HP}{kg} = 1 \text{ m/sec} = 18 \text{ mm.}$$

Translated from the German by the National Advisory Committee for Aeronautics.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r = 80 m
r = 100 m
r = 140 m
r = 200 m
r = ∞
Performance
Flight speed

Fig. 1.

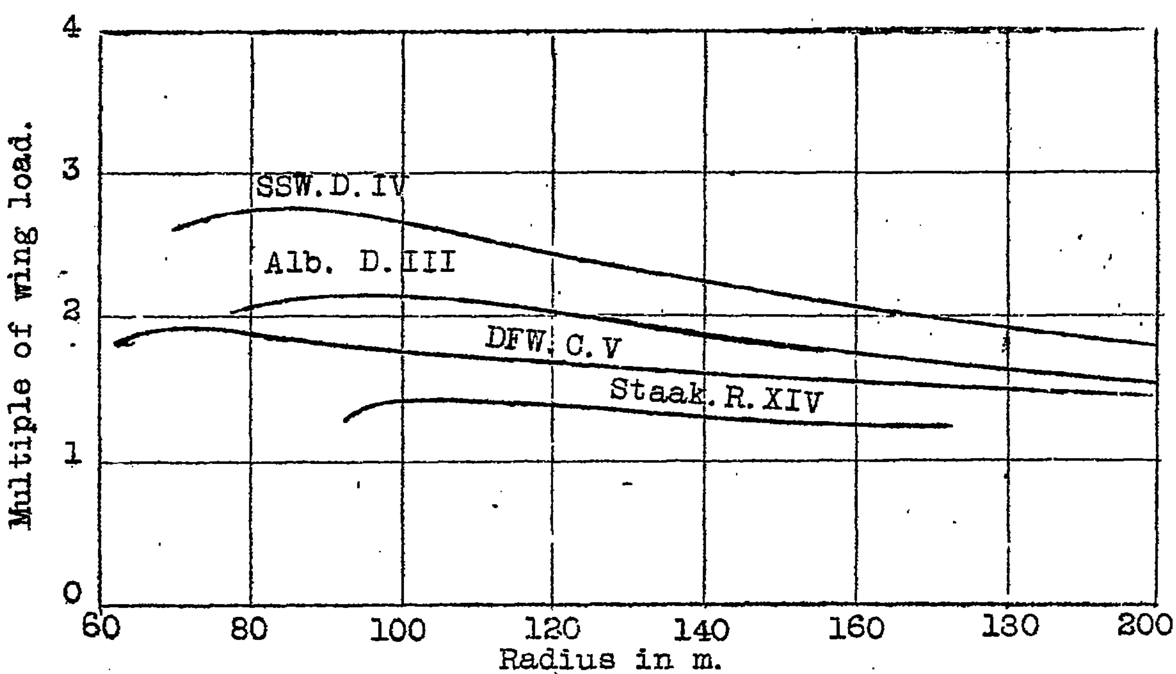

Multiple of wing load.
4
3
2
1
0
SSW.D.IV
Alb. D.III
DFW. C.V
Staak.R.XIV
60
80
100
120
140
160
130
200
Radius in m.

Fig. 2

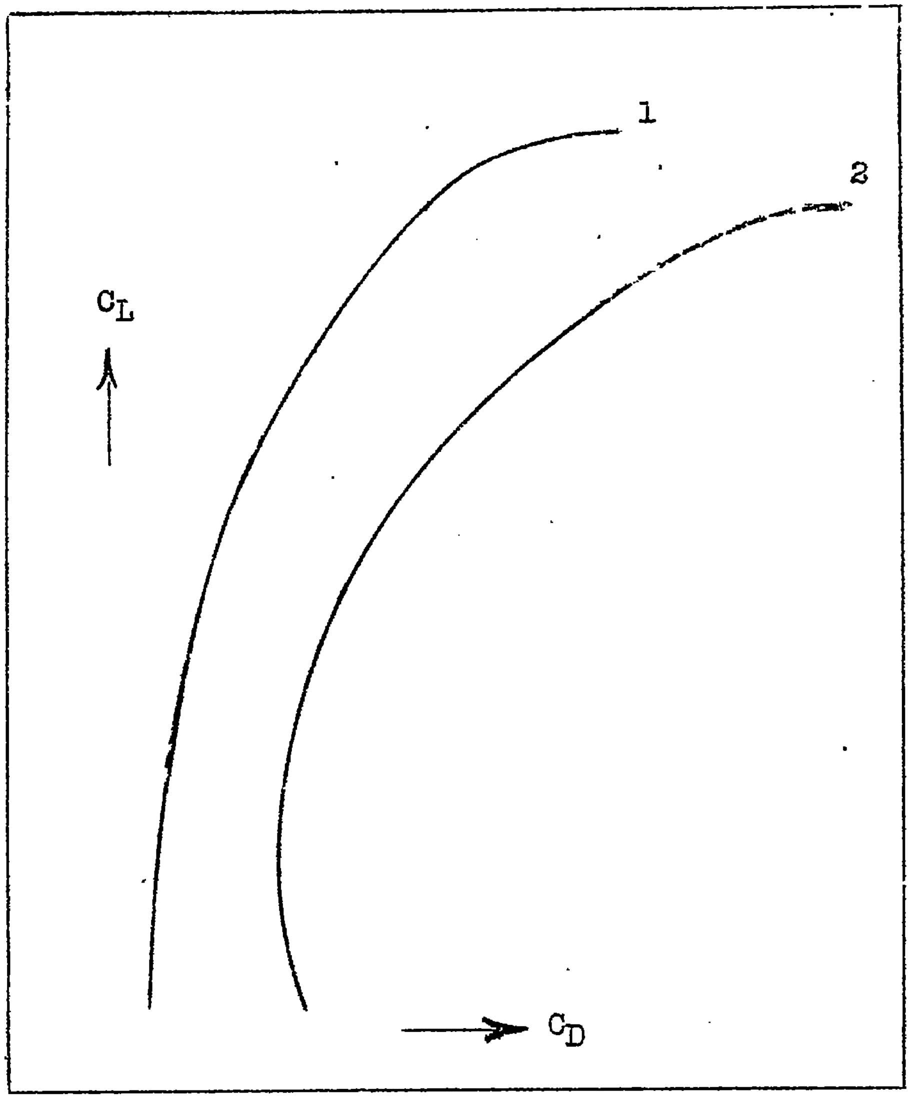

1
2
C_L
C_D

Fig. 3

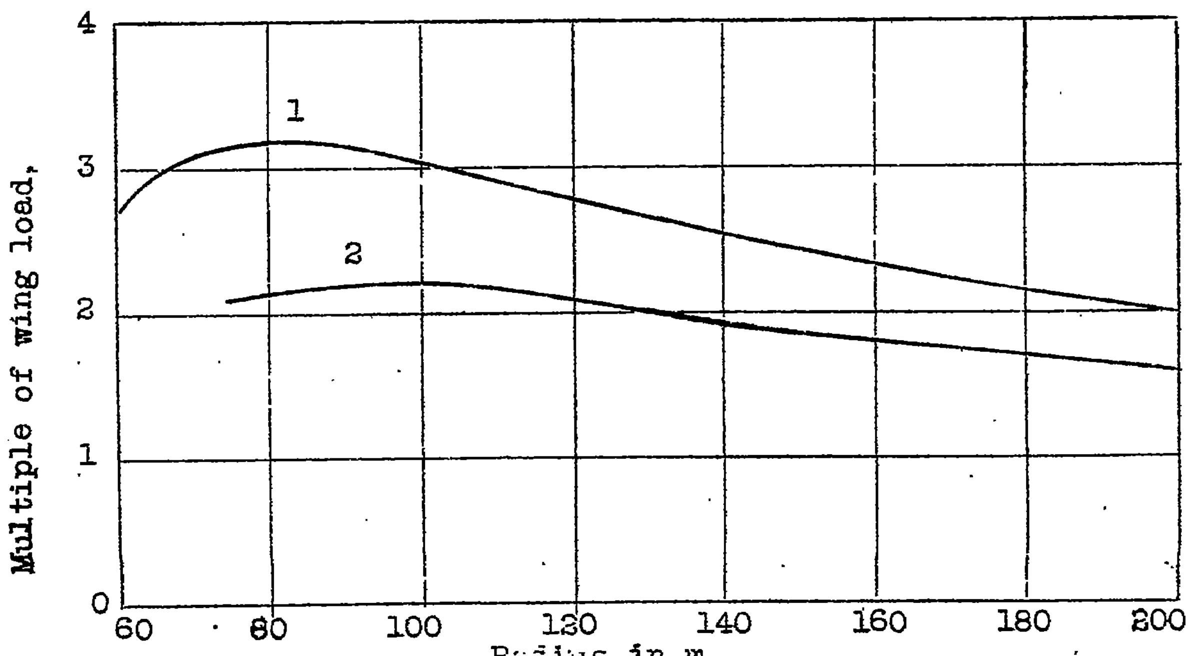

Multiple of wing load.
4
3
2
1
0
1
2
60
80
100
120
140
160
180
200
Radius in m.

Fig. 4

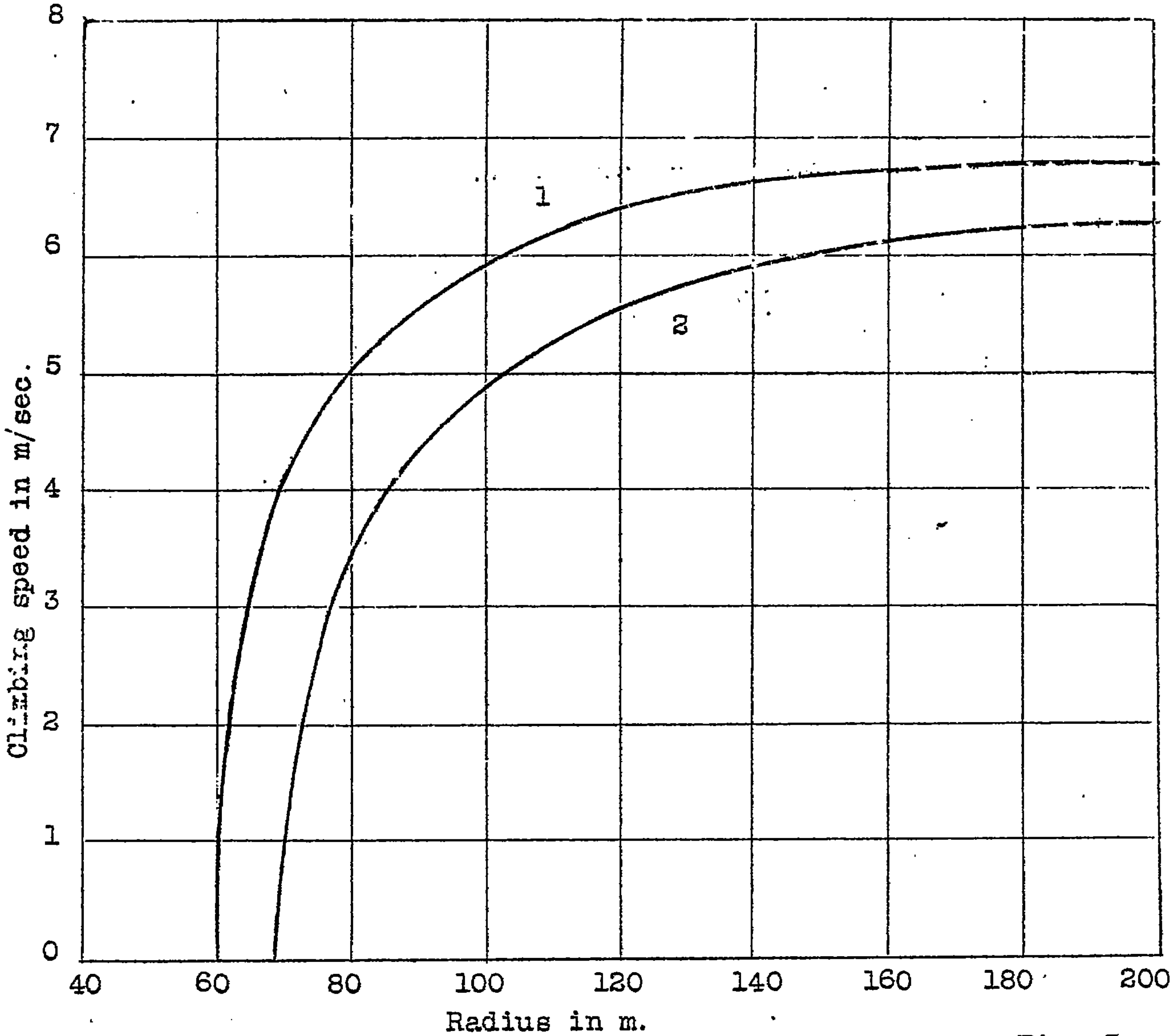
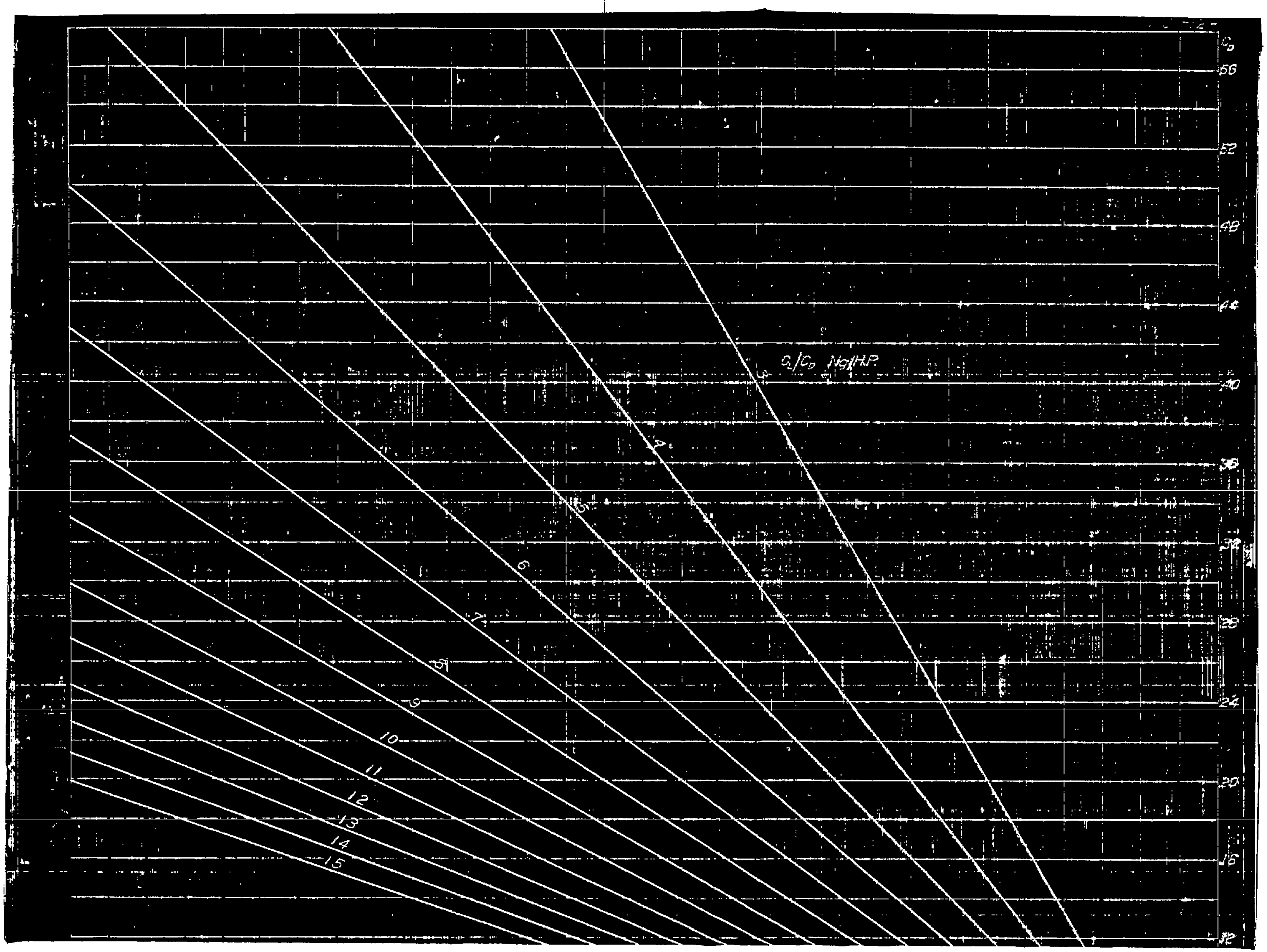
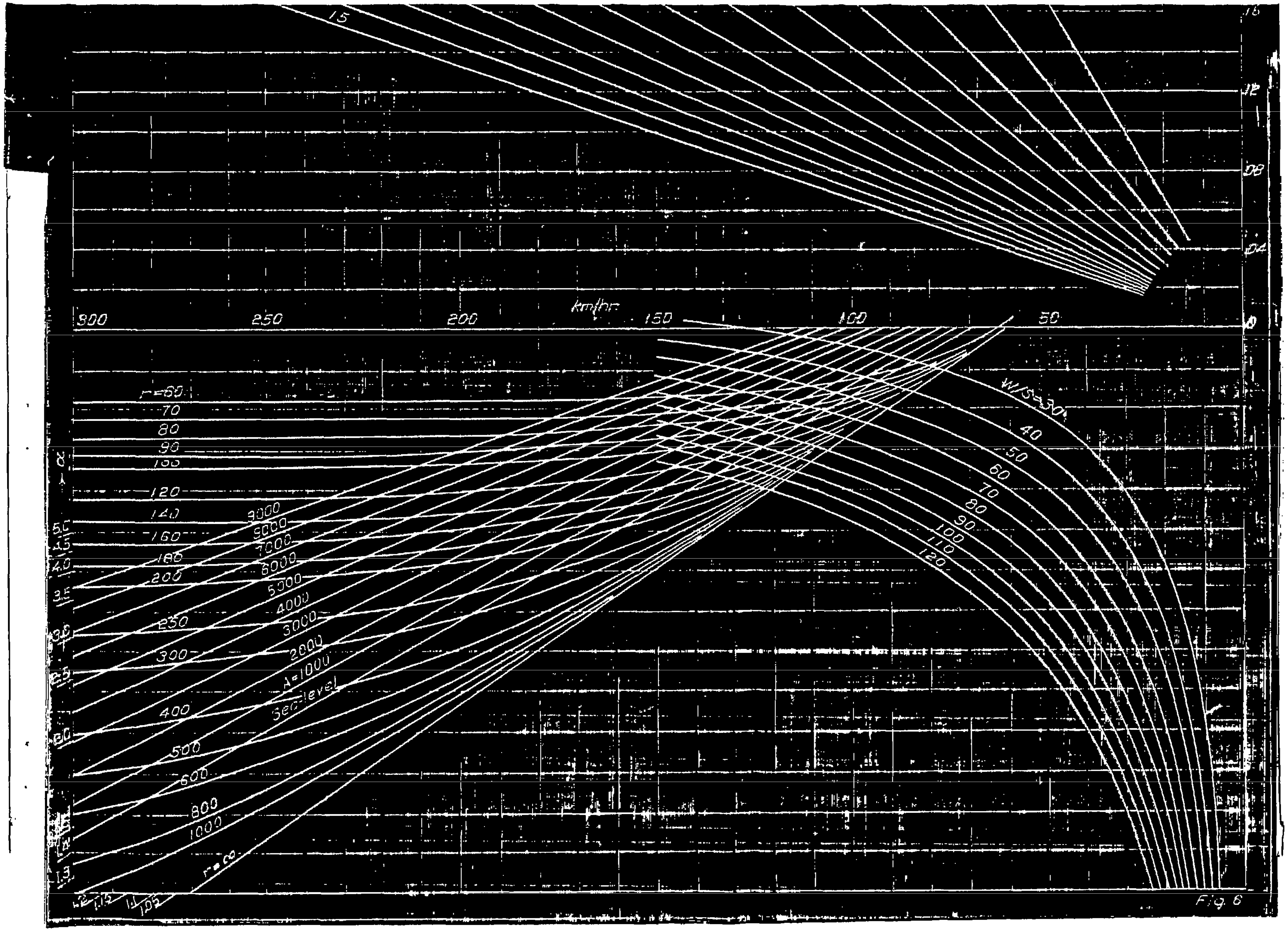

Climbing speed in m/sec.
8
7
6
5
4
3
2
1
0
1
2
40
60
80
100
120
140
160
180
200
Radius in m.

Fig. 5



Cp
56
52
48
44
40
36
32
28
24
20
16
12
Q/Cp Ng/H.P.
3
4
5
6
7
8
9
10
11
12
13
14
15



km/hr
300
250
200
150
100
50
W/S=30
40
50
60
70
80
90
100
110
120
r=60
70
80
90
120
140
160
180
200
250
300
400
500
600
800
1000
r=∞
9000
8000
7000
6000
5000
4000
3000
2000
h=1000
Sea-level
15

Fig. 6

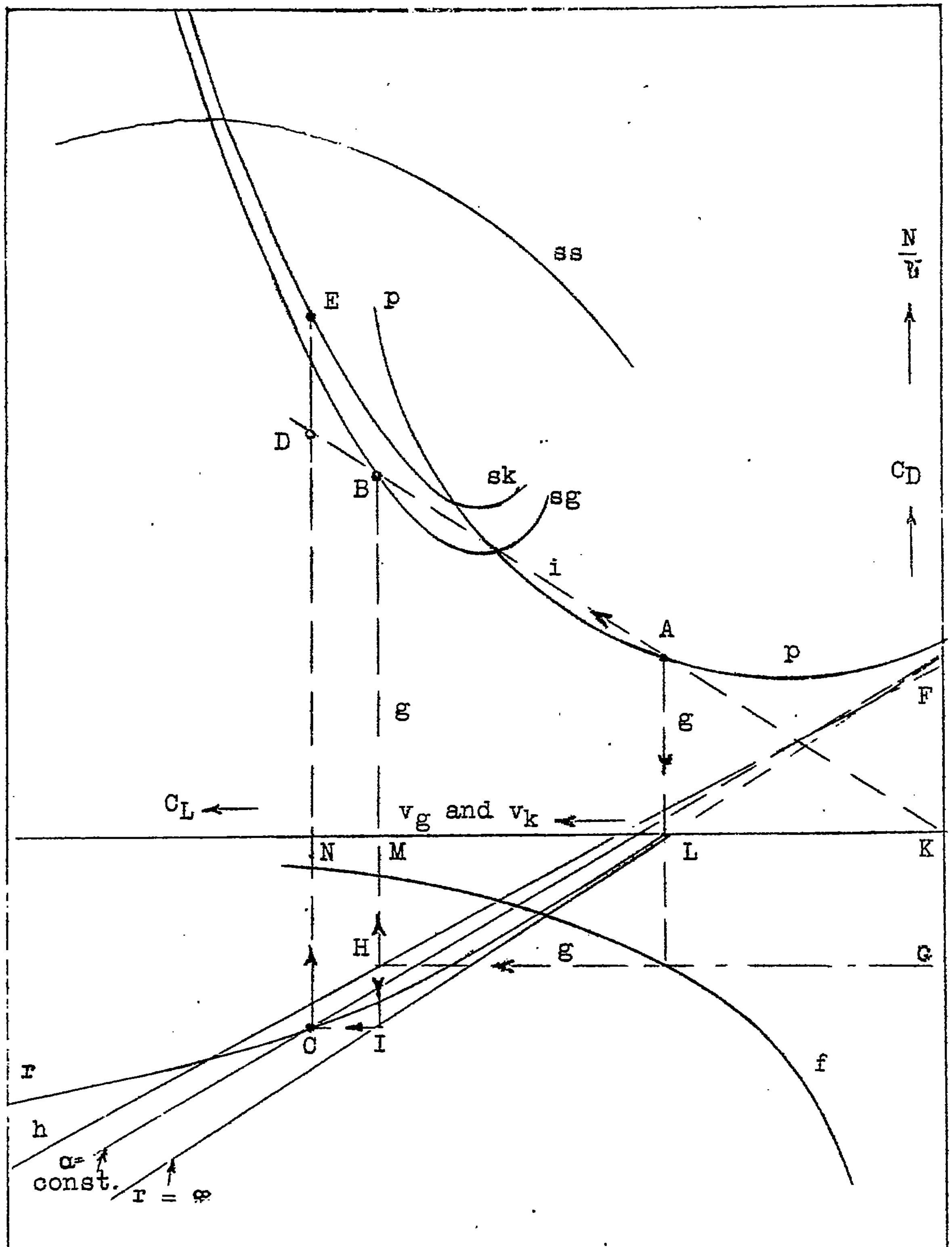
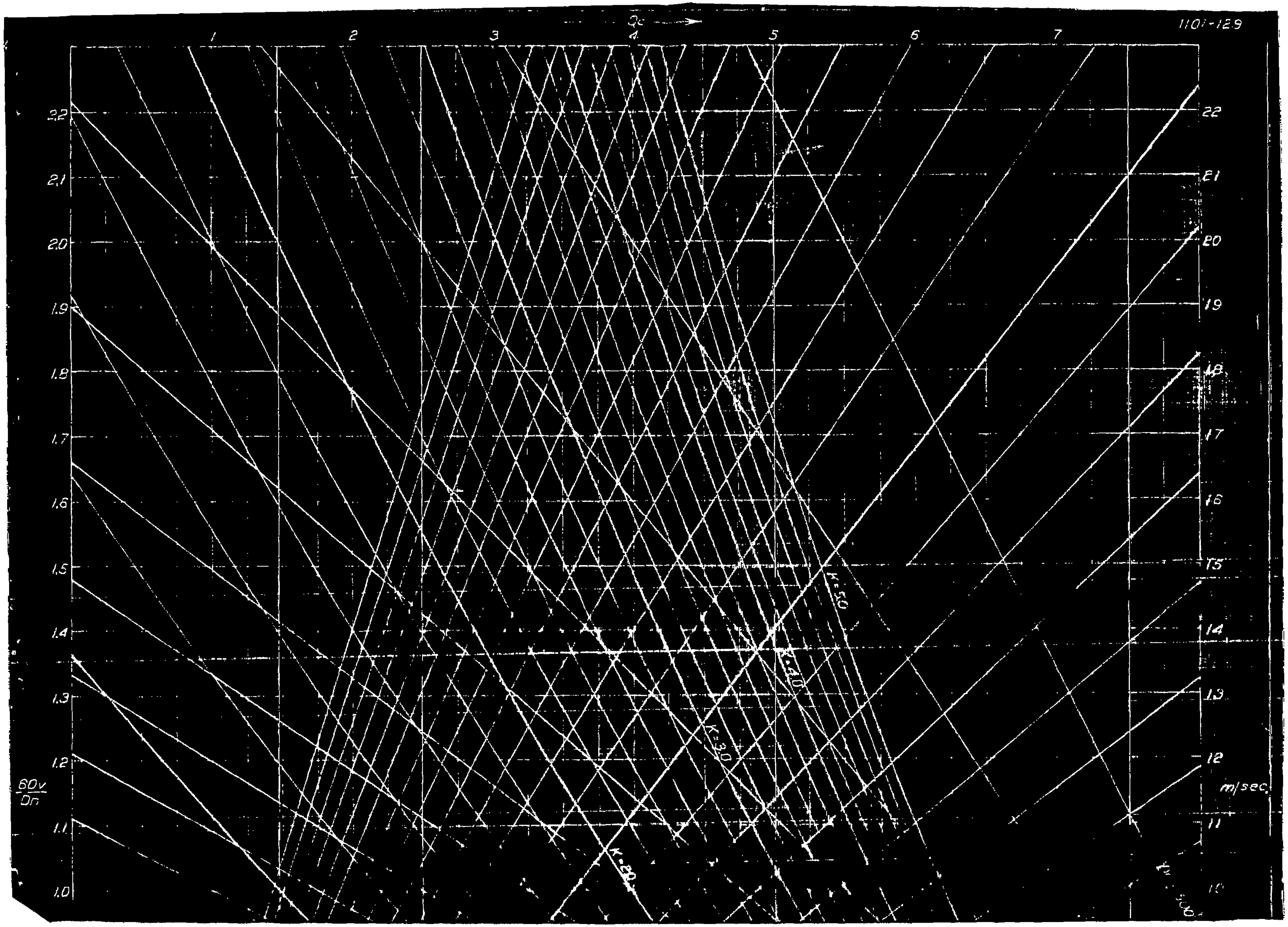
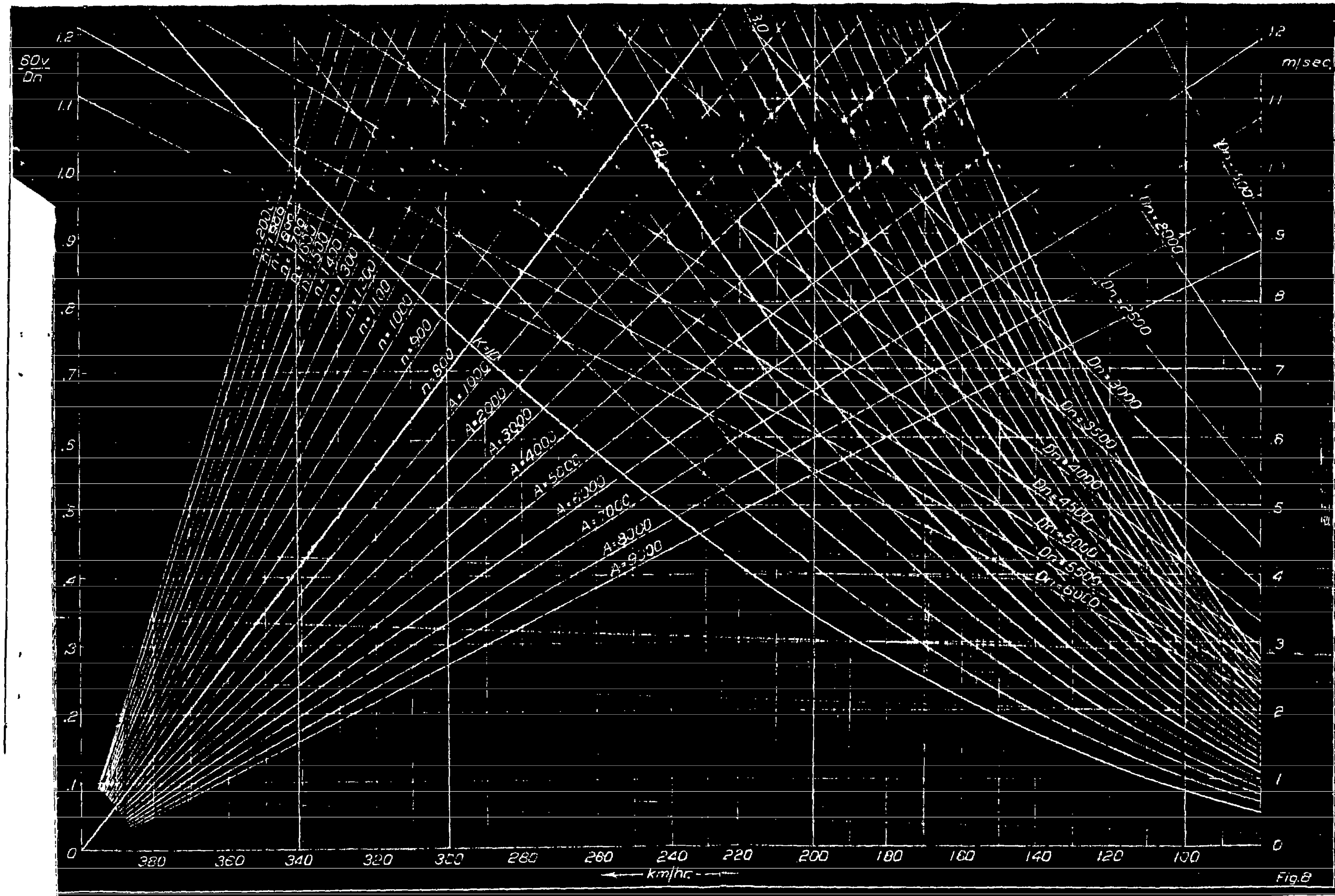


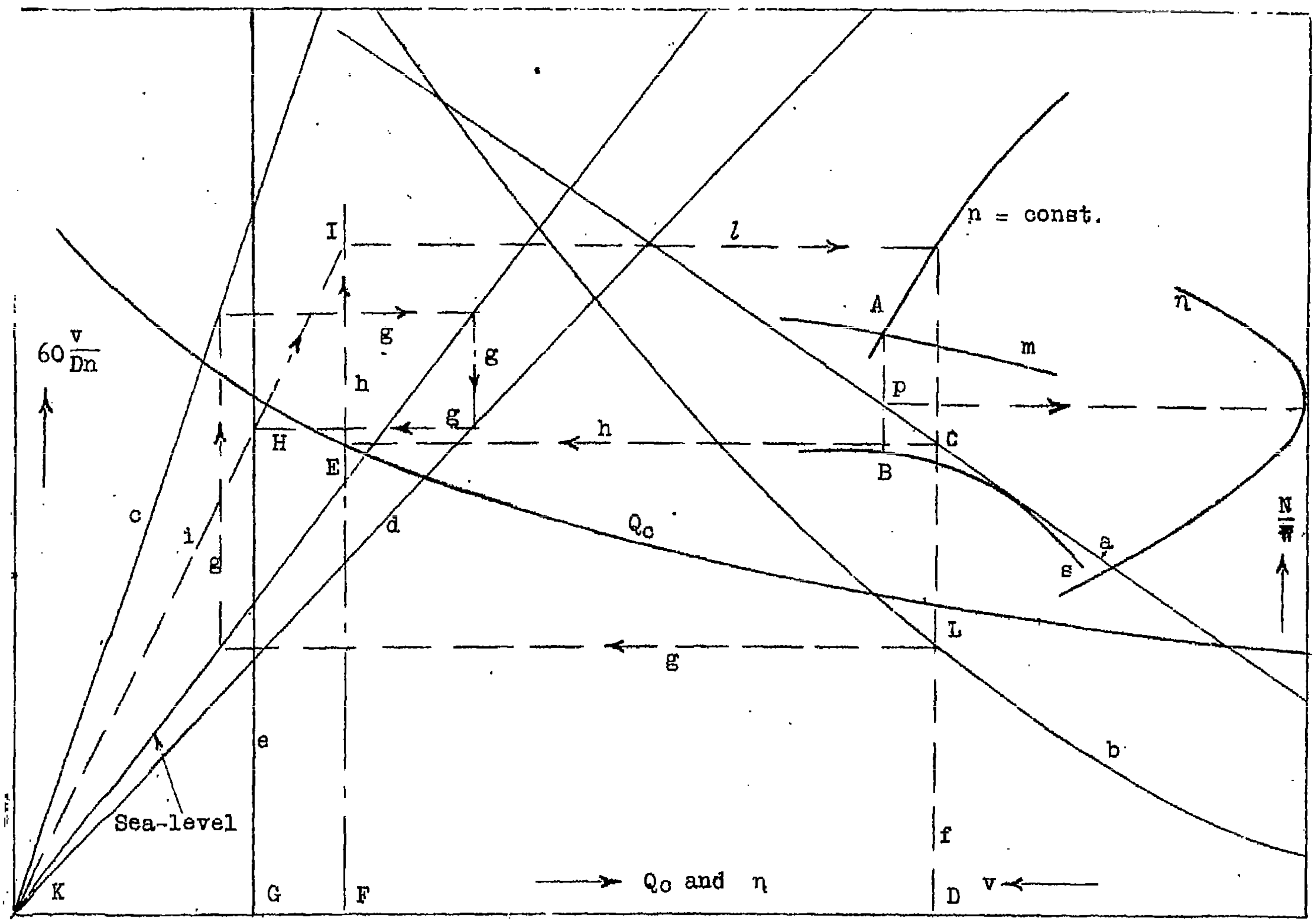
Fig. 7.



Qc
1
2
3
4
5
6
7
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
60v/Dn
22
21
20
19
18
17
16
15
14
13
12
11
m/sec
K=5.0
K=4.0
K=3.0
K=2.0



60v / Dn
m/sec
km/hr
0
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
Fig.8



n = const.
l
I
A
m
η
g
g
g
h
h
p
$60\frac{v}{Dn}$
H
E
C
B
c
i
g
d
Q_c
$\frac{N}{W}$
a
s
L
g
e
b
Sea-level
f
K
G
F
Q_c and η
D
v

Fig. 9.