TEMPERATURE CORRECTION FORMULAS FOR ADJUSTING ESTIMATES OF AUTOMOBILE FUEL CONSUMPTION

by

Norman Morse

Report 3520-1/BUF-35 May 1980

Whittaker
FALCON RESEARCH AND DEVELOPMENT

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TEMPERATURE CORRECTION FORMULAS FOR ADJUSTING ESTIMATES OF AUTOMOBILE FUEL CONSUMPTION

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Ann Arbor, MI 48105

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1.0 INTRODUCTION

This report describes an analysis of test data leading to formulas reflecting temperature effects on automobile fuel consumption. The analysis was conducted by Falcon Research and Development Company as a task under Contract No. 68-03-2835 for the Environmental Protection Agency. The purpose of the task was to provide factors which, when used to multiply fuel consumption estimates for vehicle operation at standard FTP temperatures, would yield "corrected" estimates of fuel consumption for operation outside the FTP ambient temperature range.

The report is divided into three additional sections, supported by an appendix. Section 2 summarizes the work and the outputs of the study. Section 3 presents the derived correction formulas in both graphical and tabular forms. In the graphs, the plotted formulas are superimposed on scatter diagrams of the input data. The output tables are series of temperature correction factors calculated at intervals of 5°F. In the final section some observations are offered on the results of the study and on the analytical method employed.

2.0 DESCRIPTION OF THE STUDY

This study was performed according to a task order generated at EPA in which the analysis method was prescribed. The method itself is one which had previously been used by Farrell[1] in deriving temperature correction formulas for automobile emissions. The important features of the study are as follows.

2.1 Data Base

The data base for the study was the same emissions vs. temperature data base[2] used by Farrell in deriving the temperature correction formulas for emissions. Data from several test series are contained in this data base:

Bureau of Mines, 1974 (BM1)
Bureau of Mines, 1975 (BM2)
Bureau of Mines, 1977 (BM3)
Canadian Data, 1975 (CA1)
Canadian Data, 1978 (CA2)
Gulf Data, 1977-79 (GU1)
Chicago Cold Test, 1978 (CH1)

The references contain full descriptions of these test series. In total, after appropriate editing, the input contained data from 143 vehicles. There were 854 individual test points pertinent to this analysis.

Each test point consisted of a run of a test vehicle through the FTP cycle at a given ambient temperature. The standard FTP terminology, in which "bag number" refers to one of the three major regimes within the driving cycle, is used herein. Of the total of 854 tests, 499 were at temperatures below the FTP range, 315 were within the FTP range, and 90 were at higher temperatures. Each test yielded CO, HC, and CO_2 emissions for each bag number. This allowed composite fuel consumption to be calculated by the carbon balance method, and by weighting Bag 1, Bag 2, and Bag 3 fuel consumptions in the ratio 21:52:27. In some of the older test series, the CO_2 values were not available, but composite fuel economy was provided as an explicit input datum.

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[1] Robert L. Farrell, "Temperature Correction Formulae for Adjusting Estimates of Emissions from Automobiles," Vector Research, Inc., Report No. VRI-EPA-6 (Draft), September 1979

[2] G. Miller, and K. Wilkinson, "Data Base for the Development of Improved Temperature Correction Factors for Emissions," Vector Research, Inc., Report No. VRI-EPA-5, July 27, 1979

Considerable effort was expended checking the validity of individual data points before performing the analysis. Because the input data varied with respect to availability of fuel economy and/or CO_2 data, the checking process differed from test series to test series. Where fuel economy (mpg) data were provided as inputs, these were converted to fuel consumption (gpm) data before analysis began.

2.2 Model-Year/Standard Groups of Vehicles

Fifteen groups or subpopulations of automobiles had been defined for the previous work on temperature correction factors for emissions, and the task work statement prescribed use of these same groups. These were termed model-year/standard (MYST) groups, which were as follows:

MYST =	1	67 FED
	2	69 FED
	3	70 FED
	4	71 FED
	5	72 FED
	6	73 FED
	7	74 FED
	8	75 CAL
	9	75 FED
	10	76 FED
	11	77 CAL
	12	77 FED
	13	78 CAL
	14	78 FED
	15	80 FED

The work statement required that individual correction formulas and sets of correction factors be derived for each MYST group.

2.3 Analytical Method

Each temperature datum was classified as being in the COLD range ($T \leq 67^oF$), in the HOT range ($T \geq 87^oF$), or in the FTP range ($68^oF \leq T \leq 86^oF$). Each vehicle was tested at one or more COLD or HOT temperatures and at one or more FTP temperatures. This allowed each fuel consumption (FC) figure obtained in the COLD or HOT ranges to be expressed as a ratio relative to FC at FTP temperatures for the same vehicle.

Explicitly, let C be an individual FC value for a vehicle, obtained at temperature T not in the FTP range. Let $\bar{C}_{FTP}$ be the geometric mean of all the FC's obtained for the same vehicle at FTP temperatures. Define

$$U = C/\bar{C}_{FTP}.$$

U will then provide an estimate of the ratio by which FTP FC estimates must be multiplied in order to produce an FC estimate for temperature T.

Suppose T is in the HOT range. It is conjectured that, except for random errors, the correction required is one at the boundary between the FTP and HOT regions, and changes exponentially with distance from that boundary. The input temperature data had been rounded to the nearest integer. For analysis purposes, the boundary is given the idealized location $T = 86.5^{o}$. Thus the correction formula is of the form

$$U = \exp \left[b(T - 86.5) \right] \quad (T \geqq 86.5^{o})$$

where b is appropriately chosen. The constant b is estimated by providing that value which "best fits" the HOT temperature data from all vehicles in the given MYST group. To determine that value, let $Y = \ln U$. Then

$$Y = b\ (T - 86.5) \qquad (T \geqq 86.5^{o})$$

and note that $Y = \ln 1 = 0$ when $T = 86.5^{o}$. If there are n HOT data points in the given MYST group, then each is represented by a pair of values (T_i, Y_i). The constant b can then be obtained by the method of least squares.

The most common form of linear regression, which allows for a Y-intercept to be estimated, is not appropriate in the present case. Here, since the fitting equation is constrained to go through the point $(T,Y) = (86.5,0)$, only the slope b has to be estimated. Equations suitable for this constrained regression analysis are given in the Appendix.

For COLD temperatures, the same basic approach is followed, which results only in the change of one or two details. The boundary between COLD and FTP temperature ranges is idealized as $T = 67.5^{o}$ for analytical purposes. The model becomes

$$U = \exp \left[b\ (67.5 - T) \right] \quad (T \leqq 67.5^{o})$$

which goes into the form

$$Y = b\ (67.5 - T) \qquad (T \leq 67.5)$$

after taking natural logarithms. The fitting equation is then constrained to go through the point (T,Y) = (67.5,0).

Where sufficient data were available, two fitting equations were obtained for each MYST group: one for COLD and one for HOT temperatures. In one case (75 CAL) insufficient data were available to allow the analysis for HOT to be performed.

2.4 Temperature Correction Coefficients Obtained

Table 2.1 contains the constants b obtained for the COLD and HOT ranges for the various MYST groups. These constants may be used to "correct" FC estimates based on FTP temperatures for temperature effects outside the normal range. Specifically, if b is the coefficient obtained from the table, then

$$\frac{\text{FC at temp. T}}{\text{FC at FTP}} = \exp\left[b\ (67.5 - T)\right]$$

for COLD temperatures, and

$$\frac{\text{FC at temp. T}}{\text{FC at FTP}} = \exp\left[b\ (T - 86.5)\right]$$

for HOT temperatures.

Table 2.1

COMPOSITE FUEL CONSUMPTION TEMPERATURE EFFECTS COEFFICIENTS

Model Year/Std.	Low Temperatures	High Temperatures
67 FED	.002037	.000161
69 FED	.002682	-.000048
70 FED	.001697	-.002261
71 FED	.002261	-.000933
72 FED	.002555	-.000733
73 FED	.001775	-.000305
74 FED	.003021	-.000627
75 CAL	.003203	.000000
75 FED	.002941	-.002192
76 FED	.002310	.000000
77 CAL	.001521	.000304
77 FED	.002608	-.000593
78 CAL	.002600	-.000483
78 FED	.002982	.002810
80 FED	.002958	-.002456

3.0 GRAPHICAL AND TABULAR RESULTS

On the following pages, Tables 3.1 through 3.15 and Figures 3.1 through 3.15 give the results of the study in graphical and tabular form. Each figure contains the input data, displayed by means of a scatter diagram, for the COLD and HOT temperature correction formulas for the given MYST group. The corresponding correction equations are also depicted. Note that the vertical, or U axis, has a logarithmic scale, so that the fitting equations appear as straight lines.

Each figure also conveys percentages labelled STD ERROR. One is given for each of the b coefficients, COLD and HOT. The figure is the estimate of the standard error of b, s_b, expressed as a percent of b itself, i.e.,

$$\text{STD ERROR} = 100 \times (s_b/b)\ \%$$

The formula for s_b is given in the Appendix. Note that large percentages are cases with large variability in the estimated values of b, and vice versa. Thus only where small percentages appear could the estimated b be significantly different from zero. Explicit significance tests were not performed because of concerns with the form of the distribution of deviations from the regression lines.

Each of the Tables 3.1 through 3.15 contains the correction factors obtained by substituting various values of T into the fitted equations.

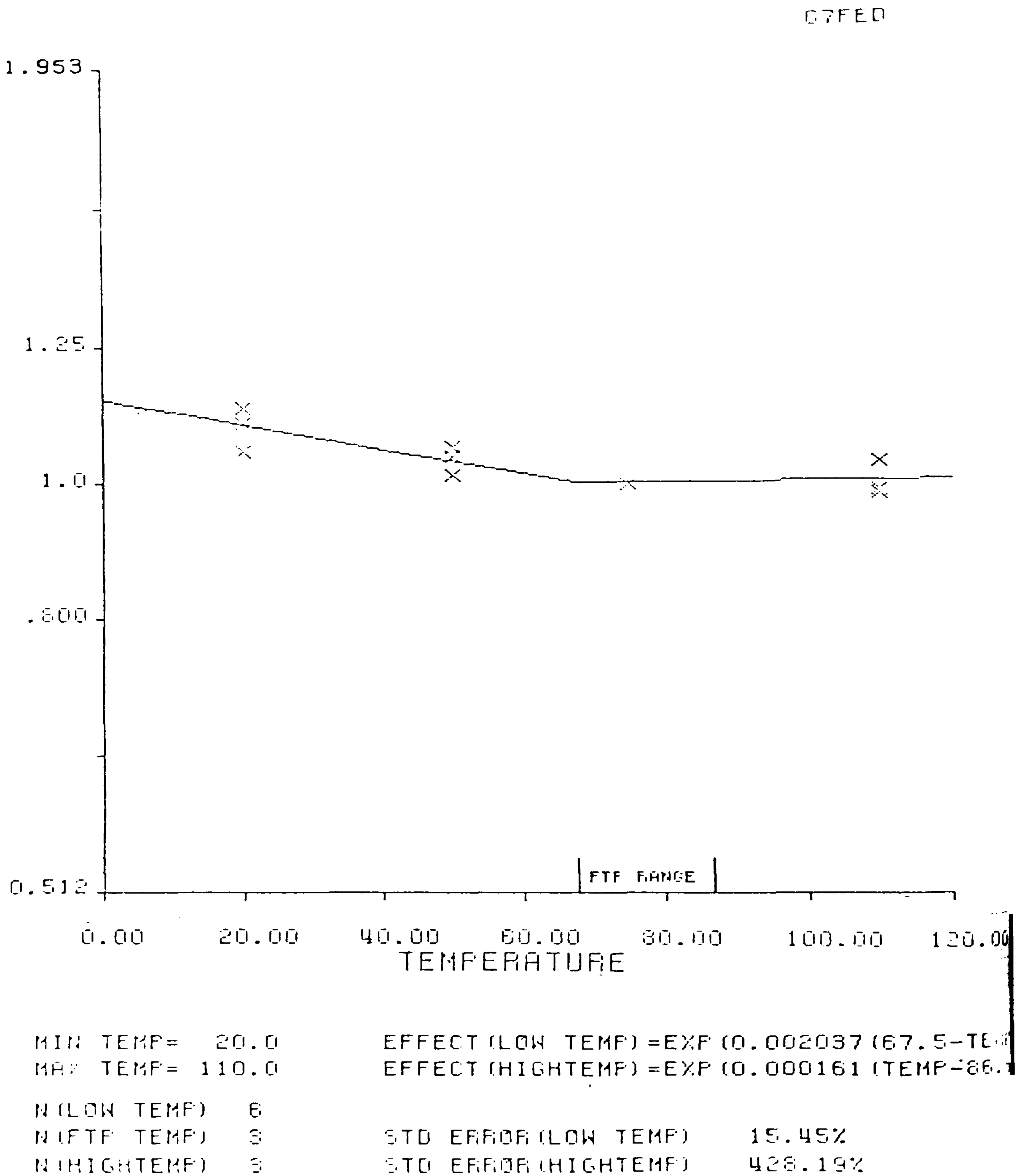


FIGURE 3.1

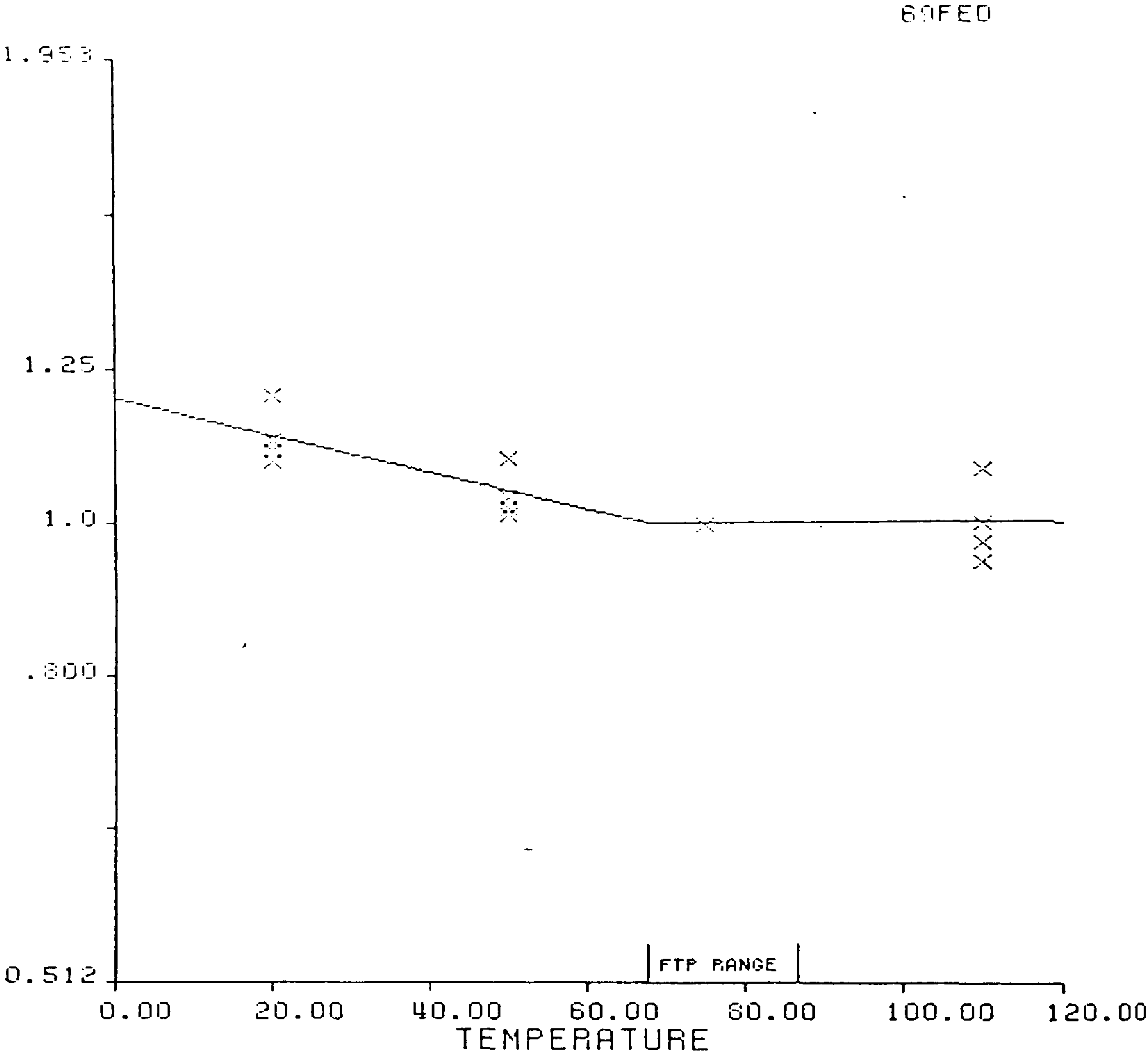
Temperature Effects on Fuel Consumption, Model-Year/Standard = 67 FED

Table 3.1

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 67 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.1474
5.0	1.1358
10.0	1.1243
15.0	1.1129
20.0	1.1016
25.0	1.0904
30.0	1.0794
35.0	1.0684
40.0	1.0576
45.0	1.0469
50.0	1.0363
55.0	1.0258
60.0	1.0154
65.0	1.0051
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	1.0006
95.0	1.0014
100.0	1.0022
105.0	1.0030
110.0	1.0038



MIN TEMP=	20.0	EFFECT (LOW TEMP) =EXP (0.002682 (67.5-TEMP))	
MAX TEMP=	110.0	EFFECT (HIGHTEMP) =EXP (-0.000048 (TEMP-86.5)	
N (LOW TEMP)	8		
N (FTP TEMP)	4	STD ERROR (LOW TEMP)	13.41%
N (HIGHTEMP)	4	STD ERROR (HIGHTEMP)	2511.17%

FIGURE 3.2

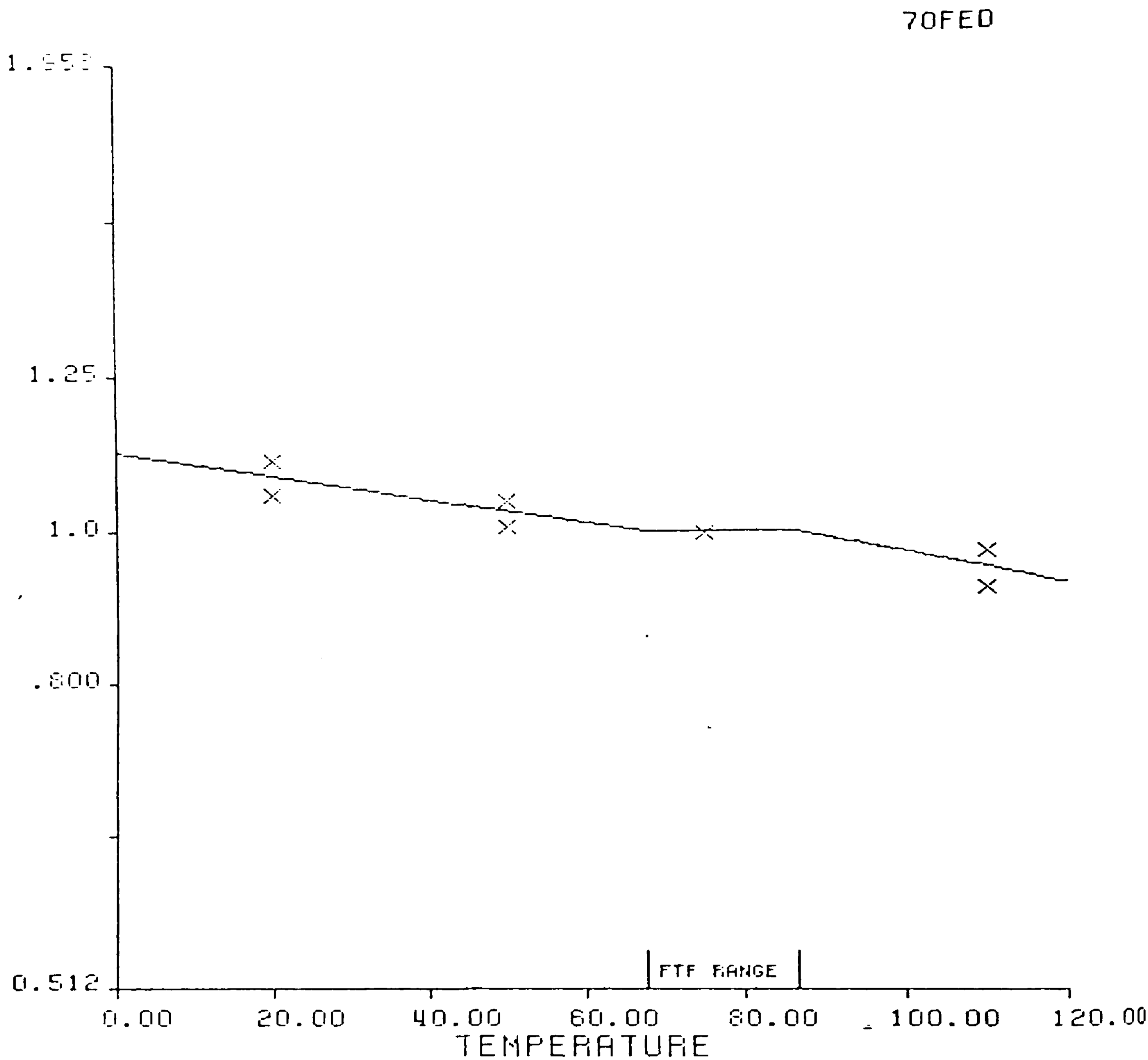
Temperature Effects on Fuel Consumption, Model-Year/Standard = 69 FED

Table 3.2

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 69 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.1985
5.0	1.1825
10.0	1.1667
15.0	1.1512
20.0	1.1359
25.0	1.1207
30.0	1.1058
35.0	.1.0911
40.0	1.0765
45.0	1.0622
50.0	1.0481
55.0	1.0341
60.0	1.0203
65.0	1.0067
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9998
95.0	0.9996
100.0	0.9994
105.0	0.9991
110.0	0.9989



MIN TEMP= 20.0
MAX TEMP= 110.0

N(LOW TEMP) 4
N(FTF TEMP) 2
N(HIGHTEMP) 2

EFFECT(LOW TEMP)=EXP(0.001697(67.5-TEMP)
EFFECT(HIGHTEMP)=EXP(-0.002261(TEMP-86.5

STD ERROR(LOW TEMP) 21.18%
STD ERROR(HIGHTEMP) 49.34%

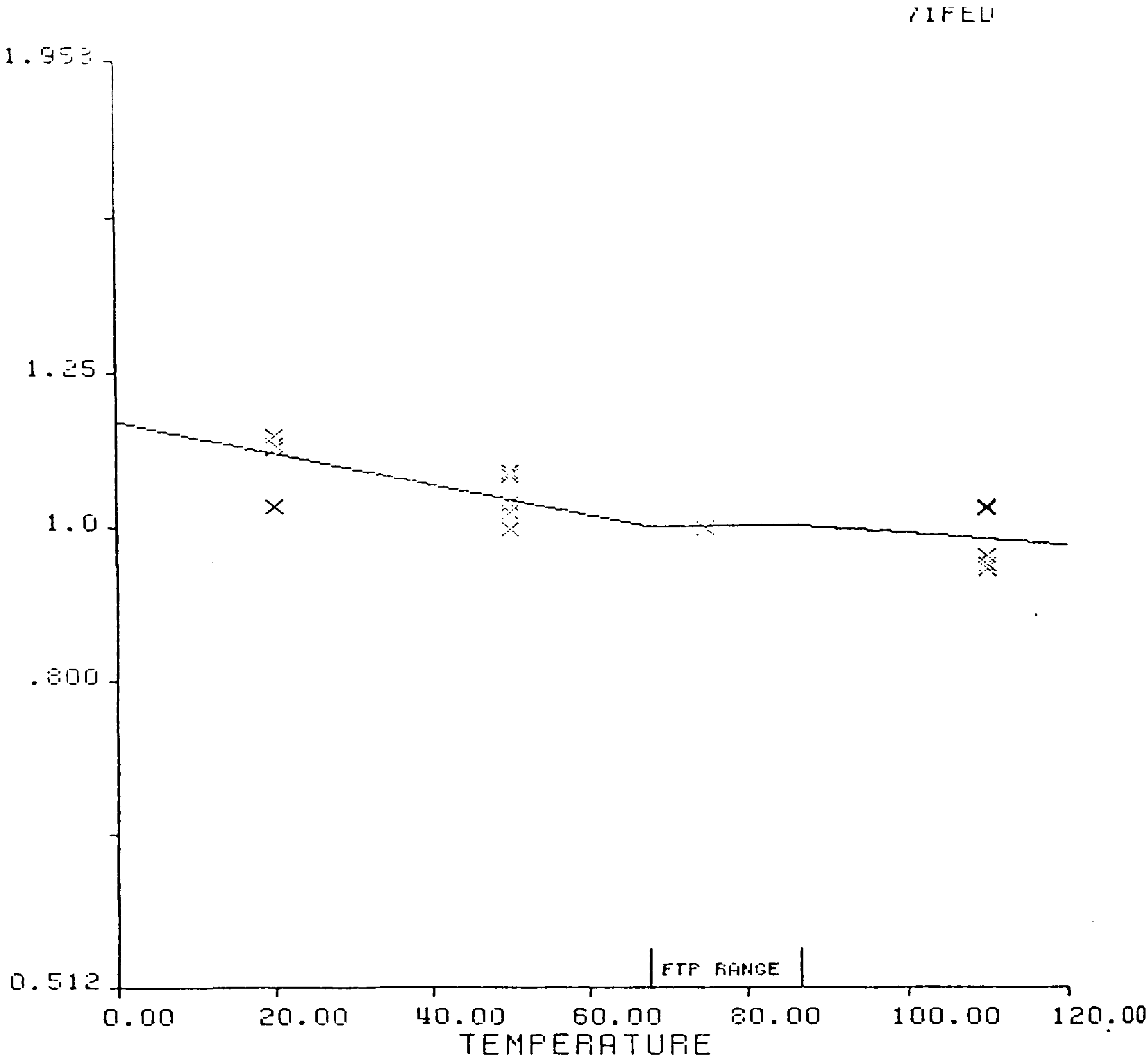
FIGURE 3.3

Temperature Effects on Fuel Consumption, Model-Year/Standard = 70 FED

Table 3.3

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION	70 FED
TEMP (F)	CORRECTION FACTOR
0.0	1.1214
5.0	1.1119
10.0	1.1025
15.0	1.0932
20.0	1.0839
25.0	1.0748
30.0	1.0657
35.0	1.0567
40.0	1.0478
45.0	1.0389
50.0	1.0301
55.0	1.0214
60.0	1.0128
65.0	1.0043
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9921
95.0	0.9810
100.0	0.9699
105.0	0.9590
110.0	0.9483



MIN TEMP=	20.0	EFFECT(LOW TEMP)=EXP(0.002261(67.5-TEMP))	
MAX TEMP=	110.0	EFFECT(HIGHTEMP)=EXP(-0.000933(TEMP-86.5	
N(LOW TEMP)	10		
N(FTP TEMP)	5	STD ERROR(LOW TEMP)	14.15%
N(HIGHTEMP)	5	STD ERROR(HIGHTEMP)	89.87%

FIGURE 3.4

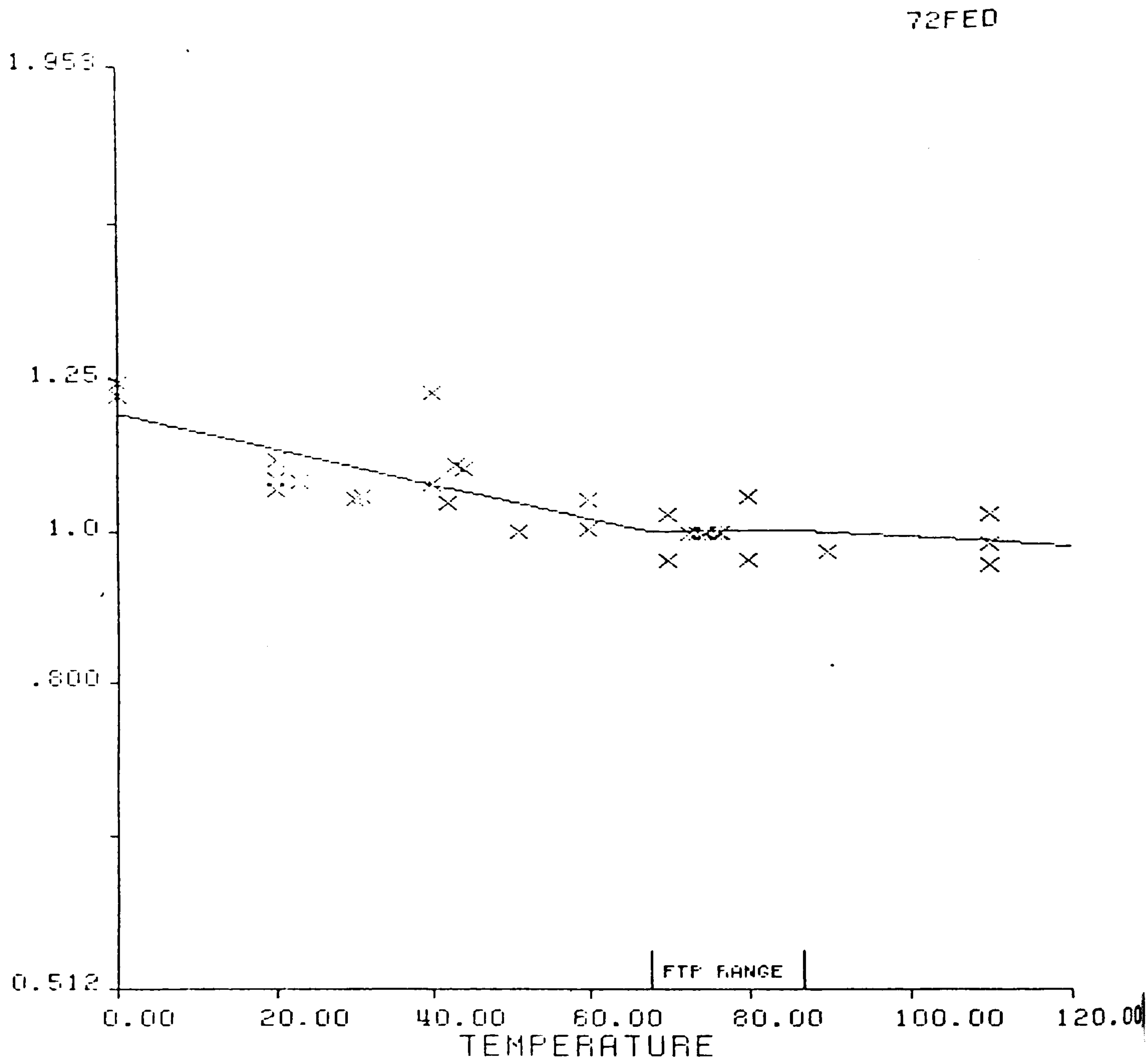
Temperature Effects on Fuel Consumption, Model-Year/Standard = 71 FED

Table 3.4

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 71 FED

TEMP. (F)	CORRECTION FACTOR
0.0	1.1649
5.0	1.1518
10.0	1.1388
15.0	1.1260
20.0	1.1134
25.0	1.1009
30.0	1.0885
35.0	1.0762
40.0	1.0642
45.0	1.0522
50.0	1.0404
55.0	1.0287
60.0	1.0171
65.0	1.0057
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9967
95.0	0.9921
100.0	0.9875
105.0	0.9829
110.0	0.9783



MIN TEMP= 0.0
MAX TEMP= 110.0

N(LOW TEMP) 16
N(FTP TEMP) 12
N(HIGHTEMP) 5

EFFECT(LOW TEMP)=EXP(0.002555(67.5-TEMP
EFFECT(HIGHTEMP)=EXP(-0.000733(TEMP-86.

STD ERROR(LOW TEMP) 12.52%
STD ERROR(HIGHTEMP) 105.12%

FIGURE 3.5

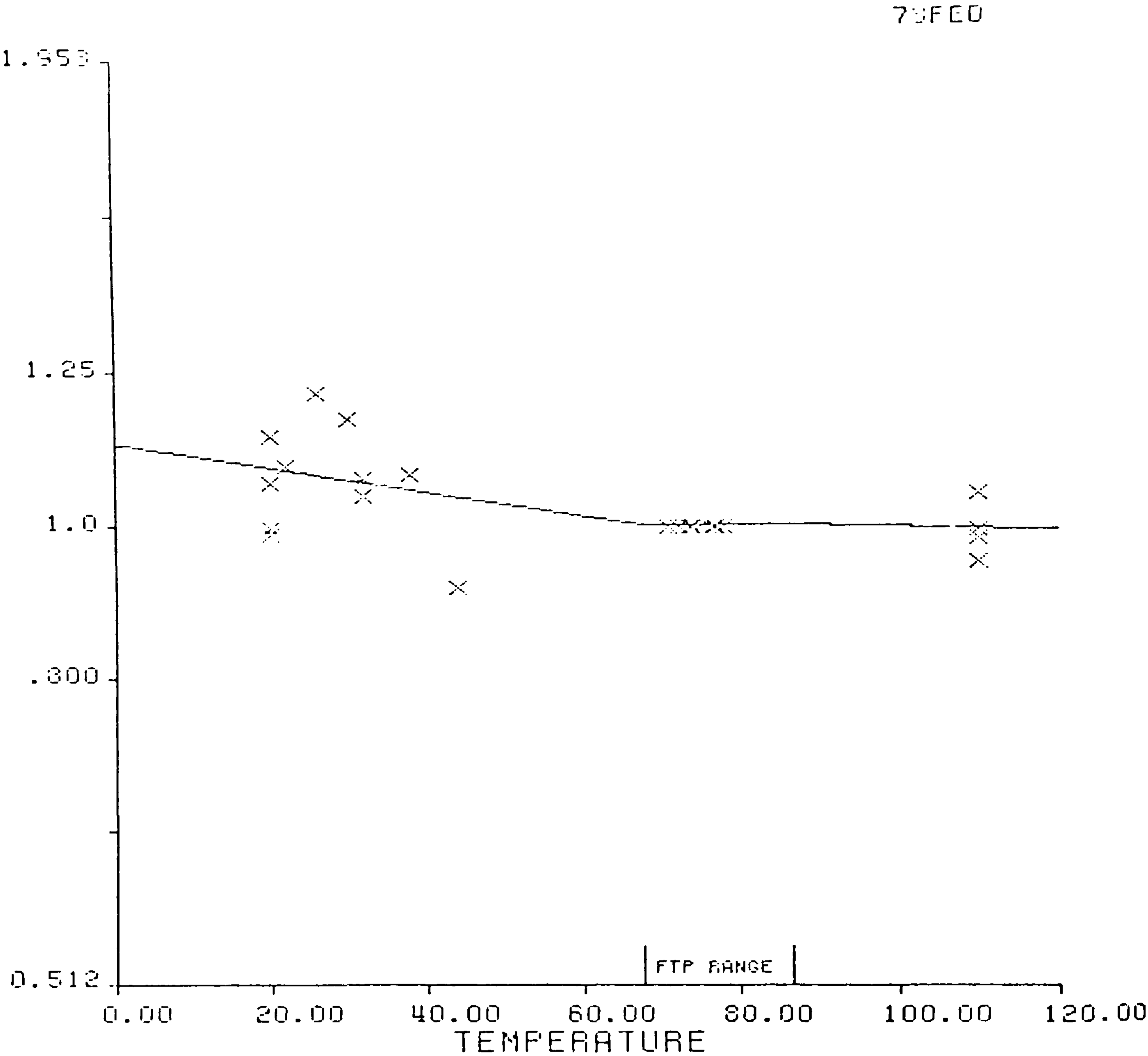
Temperature Effects on Fuel Consumption, Model-Year/Standard = 72 FED

Table 3.5

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 72 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.1882
5.0	1.1731
10.0	1.1583
15.0	1.1436
20.0	1.1290
25.0	1.1147
30.0	1.1006
35.0	1.0866
40.0	1.0728
45.0	1.0592
50.0	1.0457
55.0	1.0325
60.0	1.0193
65.0	1.0064
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9974
95.0	0.9938
100.0	0.9902
105.0	0.9865
110.0	0.9829



MIN TEMP= 20.0	EFFECT(LOW TEMP)=EXP(0.001775(67.5-TEMP))
MAX TEMP= 110.0	EFFECT(HIGHTEMP)=EXP(-0.000305(TEMP-86.5))
N(LOW TEMP) 12	
N(FTP TEMP) 12	STD ERROR(LOW TEMP) 30.30%
N(HIGHTEMP) 4	STD ERROR(HIGHTEMP) 281.43%

FIGURE 3.6

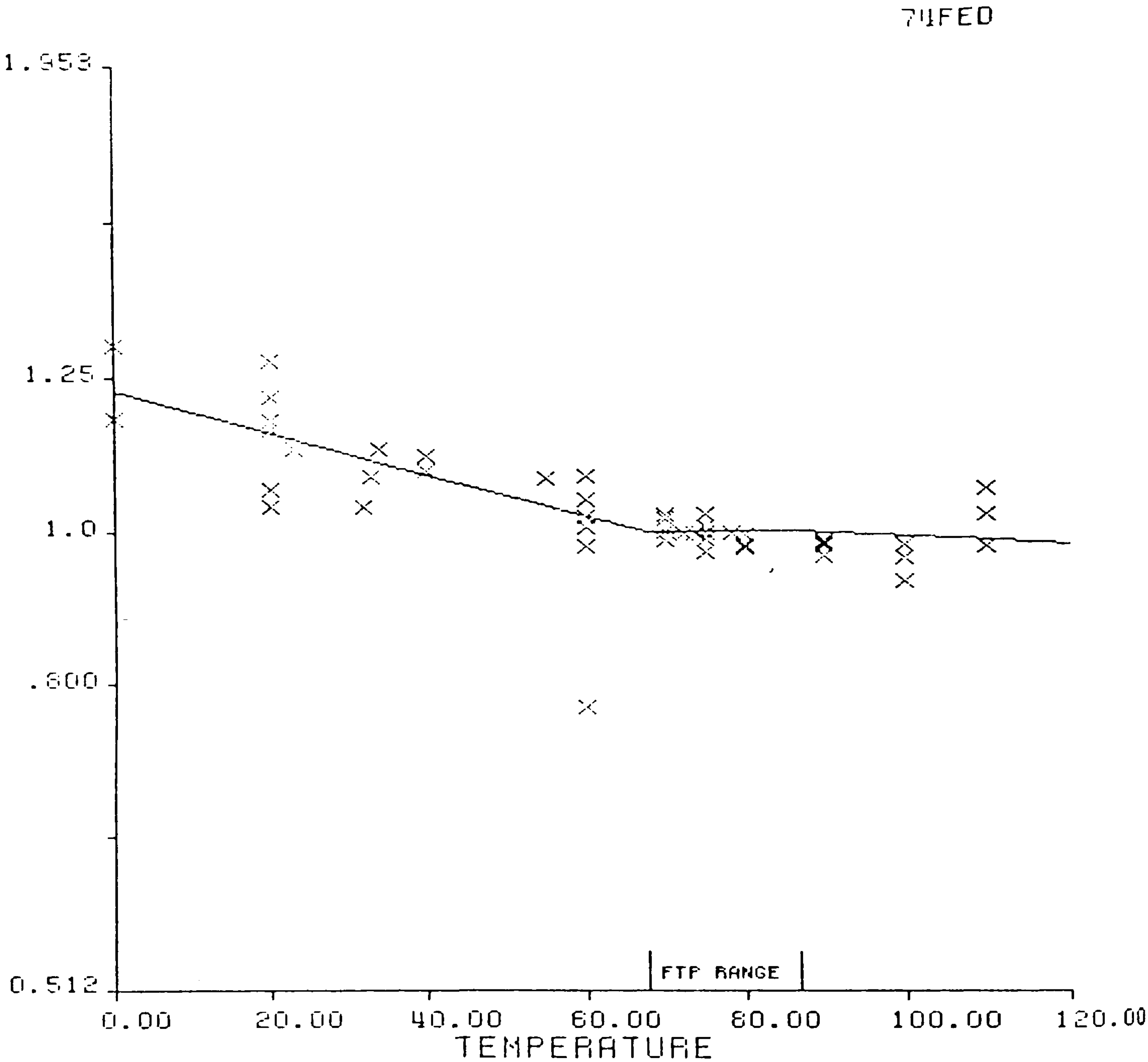
Temperature Effects on Fuel Consumption, Model-Year/Standard = 73 FED

Table 3.6

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 73 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.1273
5.0	1.1173
10.0	1.1075
15.0	1.0977
20.0	1.0880
25.0	1.0784
30.0	1.0688
35.0	1.0594
40.0	1.0500
45.0	1.0407
50.0	1.0315
55.0	1.0224
60.0	1.0134
65.0	1.0044
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9989
95.0	0.9974
100.0	0.9959
105.0	0.9944
110.0	0.9929



MIN TEMP= 0.0
MAX TEMP= 110.0

N(LOW TEMP) 22
N(FTP TEMP) 20
N(HIGHTEMP) 12

EFFECT(LOW TEMP)=EXP(0.003021(67.5-TEMP)
EFFECT(HIGHTEMP)=EXP(-0.000627(TEMP-86.5

STD ERROR(LOW TEMP) 14.44%
STD ERROR(HIGHTEMP) 133.78%

FIGURE 3.7

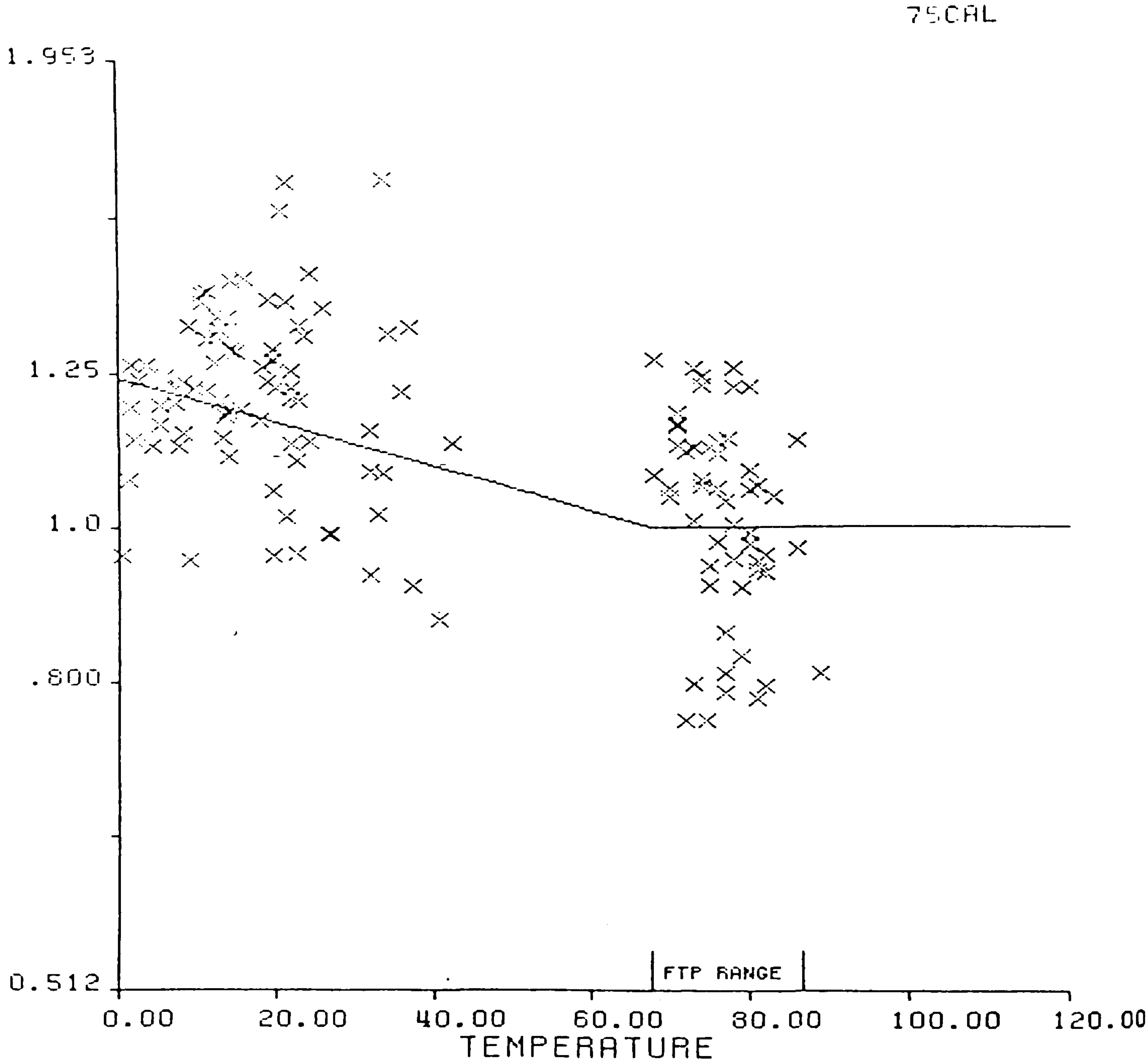
Temperature Effects on Fuel Consumption, Model-Year/Standard = 74 FED

Table 3.7

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 74 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.2262
5.0	1.2078
10.0	1.1897
15.0	1.1719
20.0	1.1543
25.0	1.1370
30.0	1.1200
35.0	1.1032
40.0	1.0866
45.0	1.0703
50.0	1.0543
55.0	1.0385
60.0	1.0229
65.0	1.0076
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9978
95.0	0.9947
100.0	0.9916
105.0	0.9885
110.0	0.9854



MIN TEMP= -22.0* EFFECT(LOW TEMP)=EXP(0.003203(67.5-TEMP))
MAX TEMP= 89.0

N(LOW TEMP) 93
N(FTP TEMP) 53 STD ERROR(LOW TEMP) 7.91%
N(HIGHTEMP) 1

---

* The data set on which the fitting equation was based contained an additional 15 points at temperatures below 0^{o}F, and one point in the FTP range which fell below the lower boundary of the graph.

FIGURE 3.8

Temperature Effects on Fuel Consumption, Model-Year/Standard = 75 CAL

Table 3.8

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 75 CAL

TEMP (F)	CORRECTION FACTOR
0.0	1.2414
5.0	1.2216
10.0	1.2022
15.0	1.1831
20.0	1.1643
25.0	1.1458
30.0	1.1276
35.0	1.1097
40.0	1.0921
45.0	1.0747
50.0	1.0577
55.0	1.0408
60.0	1.0243
65.0	1.0080
70.0	1.0000
75.0	1.0000
80.0	1.0000
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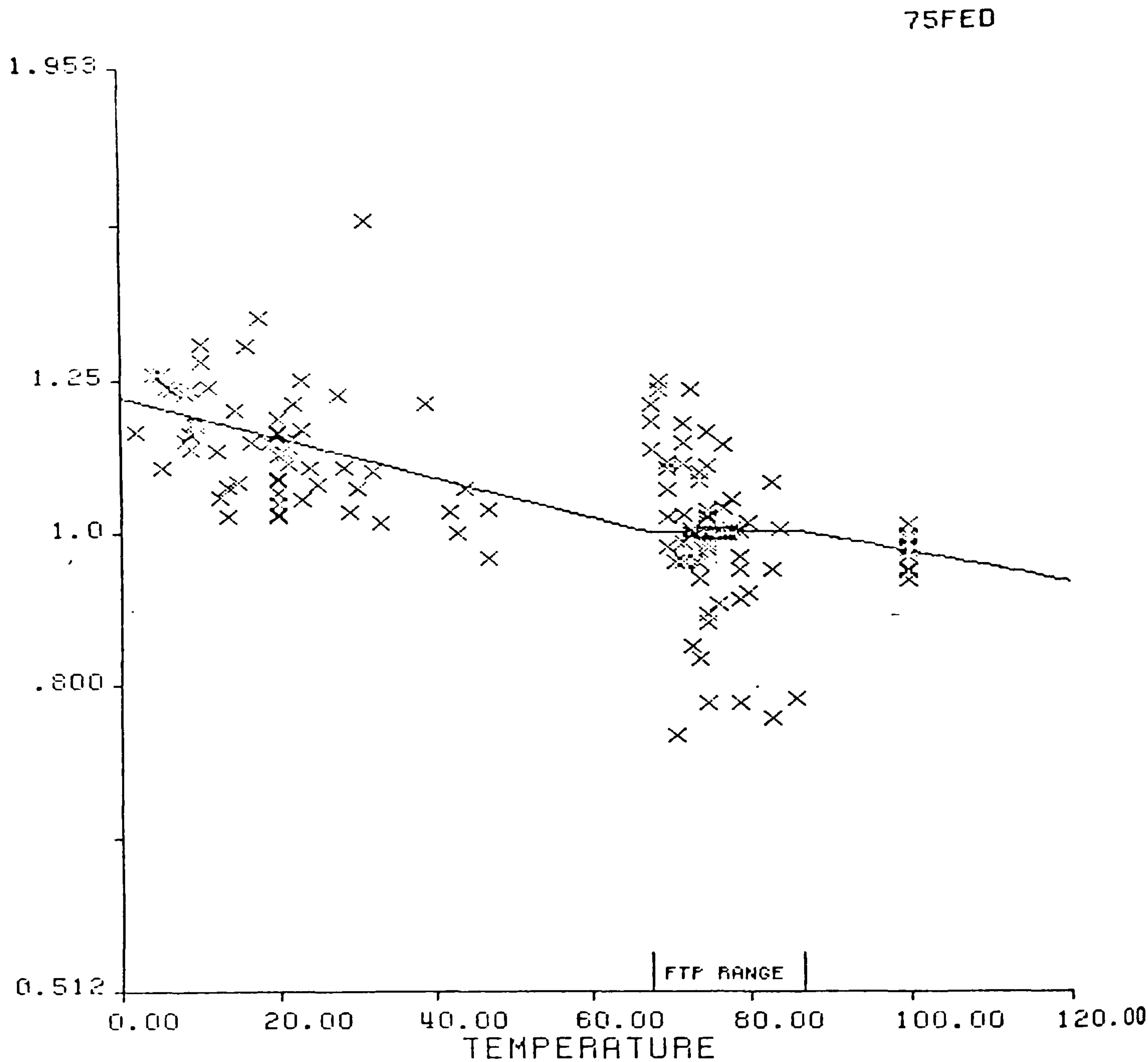
90.0	1.0000
95.0	1.0000
100.0	1.0000
105.0	1.0000
110.0	1.0000



MIN TEMP= -8.5* EFFECT(LOW TEMP)=EXP(0.002941(67.5-TEMP)

MAX TEMP= 100.0 EFFECT(HIGHTEMP)=EXP(-0.002192(TEMP-86.5

N(LOW TEMP)	71		
N(FTP TEMP)	81	STD ERROR(LOW TEMP)	6.63%
N(HIGHTEMP)	11	STD ERROR(HIGHTEMP)	25.77%

---

* The data set on which the fitting equation was based contained an additional 10 points at temperatures below $0^{O}F$.

FIGURE 3.9

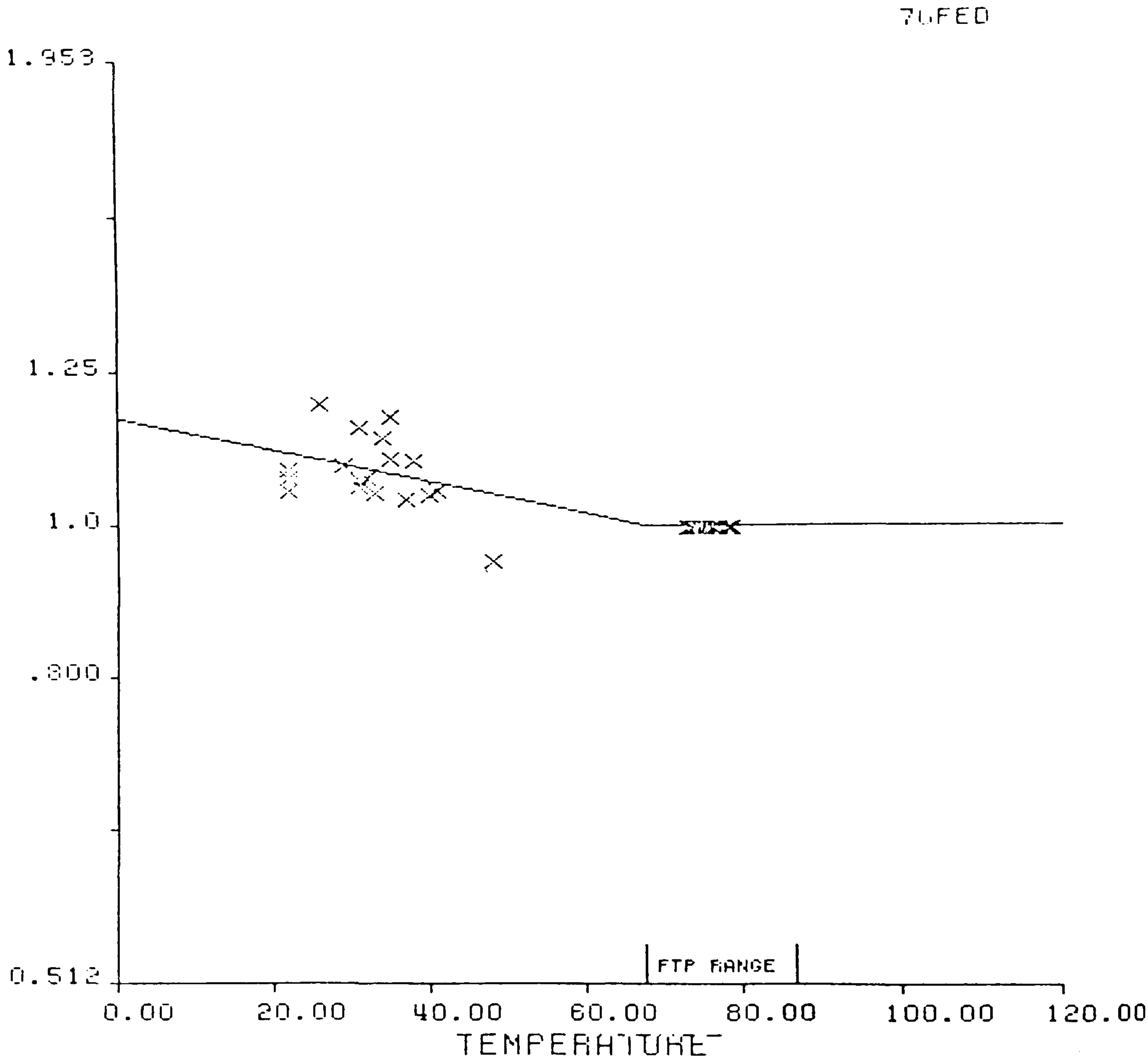
Temperature Effects on Fuel Consumption, Model-Year/Standard = 75 FED

Table 3.9

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 75 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.2196
5.0	1.2018
10.0	1.1842
15.0	1.1670
20.0	1.1499
25.0	1.1331
30.0	1.1166
35.0	1.1003
40.0	1.0842
45.0	1.0684
50.0	1.0528
55.0	1.0374
60.0	1.0223
65.0	1.0074
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9924
95.0	0.9815
100.0	0.9708
105.0	0.9603
110.0	0.9498



MIN TEMP= 22.0
MAX TEMP= 78.7

N (LOW TEMP) 19
N (FTP TEMP) 19
N (HIGHTEMP) 0

EFFECT (LOW TEMP) =EXP (0.002310 (67.5-TEMP))
EFFECT (HIGHTEMP) =EXP (0.000000 (TEMP-86.5))

STD ERROR (LOW TEMP) 13.02%

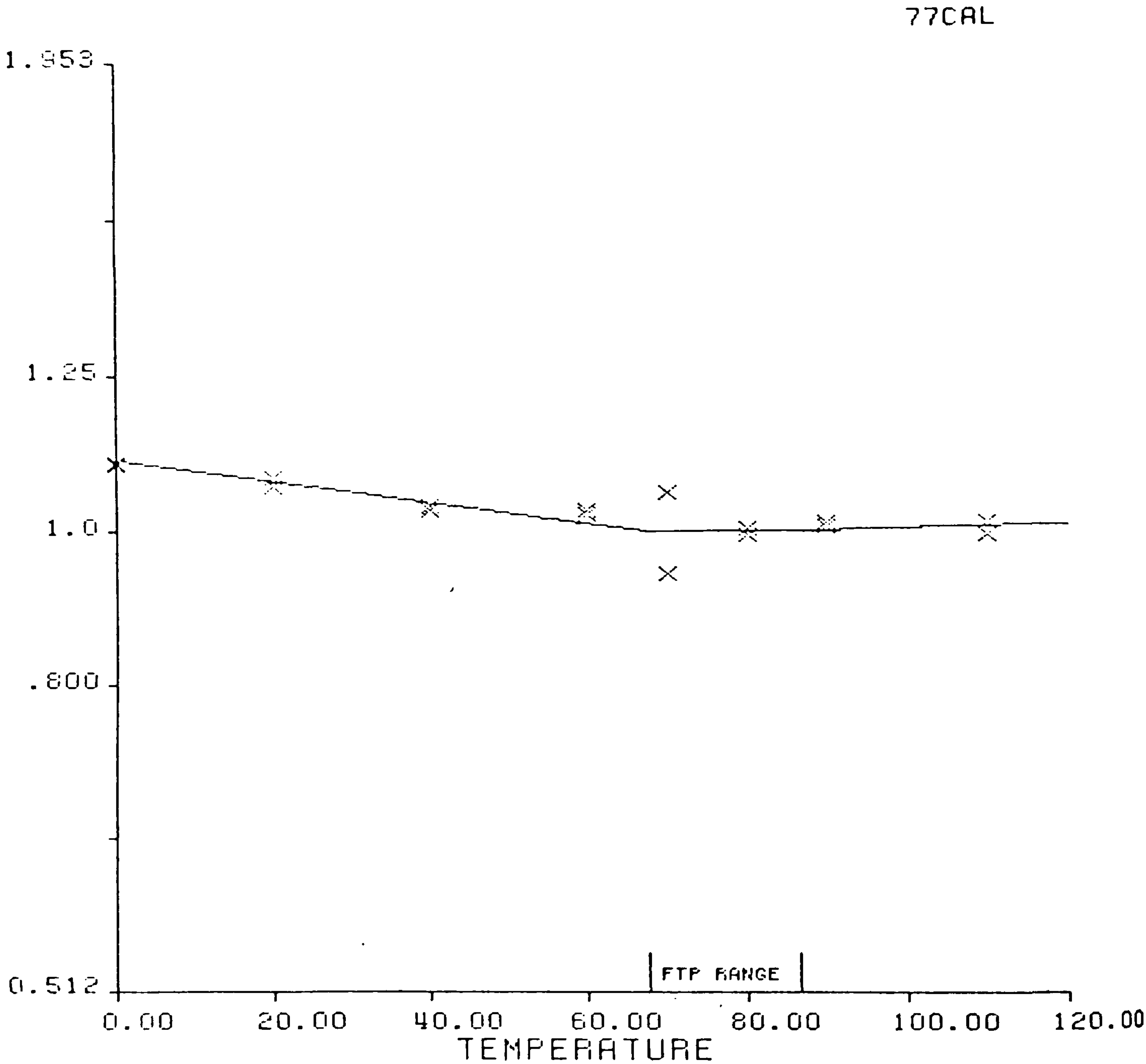
FIGURE 3.10
Temperature Effects on Fuel Consumption, Model-Year/Standard = 76 FED

Table 3.10

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 76 FED

TEMP. (F)	CORRECTION FACTOR
0.0	1.1687
5.0	1.1553
10.0	1.1421
15.0	1.1289
20.0	1.1160
25.0	1.1032
30.0	1.0905
35.0	1.0780
40.0	1.0656
45.0	1.0533
50.0	1.0413
55.0	1.0293
60.0	1.0175
65.0	1.0058
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	1.0000
95.0	1.0000
100.0	1.0000
105.0	1.0000
110.0	1.0000



MIN TEMP= 0.0
MAX TEMP= 110.0

EFFECT(LOW TEMP)=EXP(0.001521(67.5-TEMP)
EFFECT(HIGHTEMP)=EXP(0.000304(TEMP-86.5)

N(LOW TEMP) 8
N(FTP TEMP) 4
N(HIGHTEMP) 4

STD ERROR(LOW TEMP) 8.58%
STD ERROR(HIGHTEMP) 120.15%

FIGURE 3.11

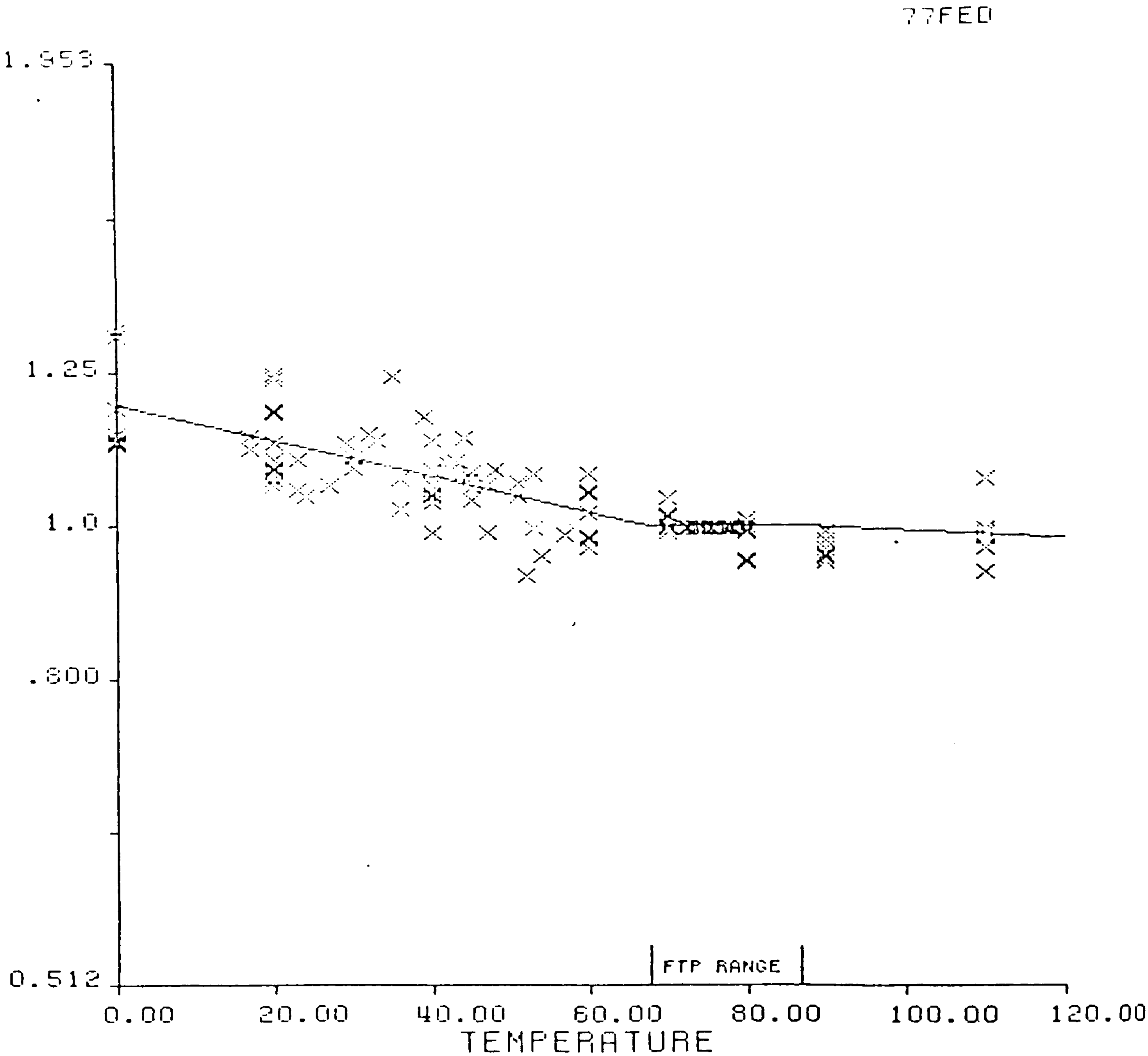
Temperature Effects on Fuel Consumption, Model-Year/Standard = 77 CAL

Table 3.11

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 77 CAL

TEMP. (F)	CORRECTION FACTOR
0.0	1.1081
5.0	1.0997
10.0	1.0914
15.0	1.0831
20.0	1.0749
25.0	1.0668
30.0	1.0587
35.0	1.0507
40.0	1.0427
45.0	1.0348
50.0	1.0270
55.0	1.0192
60.0	1.0115
65.0	1.0038
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	1.0011
95.0	1.0026
100.0	1.0041
105.0	1.0056
110.0	1.0072



MIN TEMP= 0.0 EFFECT (LOW TEMP) =EXP (0.002608 (67.5-TEMP))
MAX TEMP= 110.0 EFFECT (HIGHTEMP) =EXP (-0.000593 (TEMP-86.5)

N (LOW TEMP) 65
N (FTP TEMP) 48 STD ERROR (LOW TEMP) 8.42%
N (HIGHTEMP) 12 STD ERROR (HIGHTEMP) 110.57%

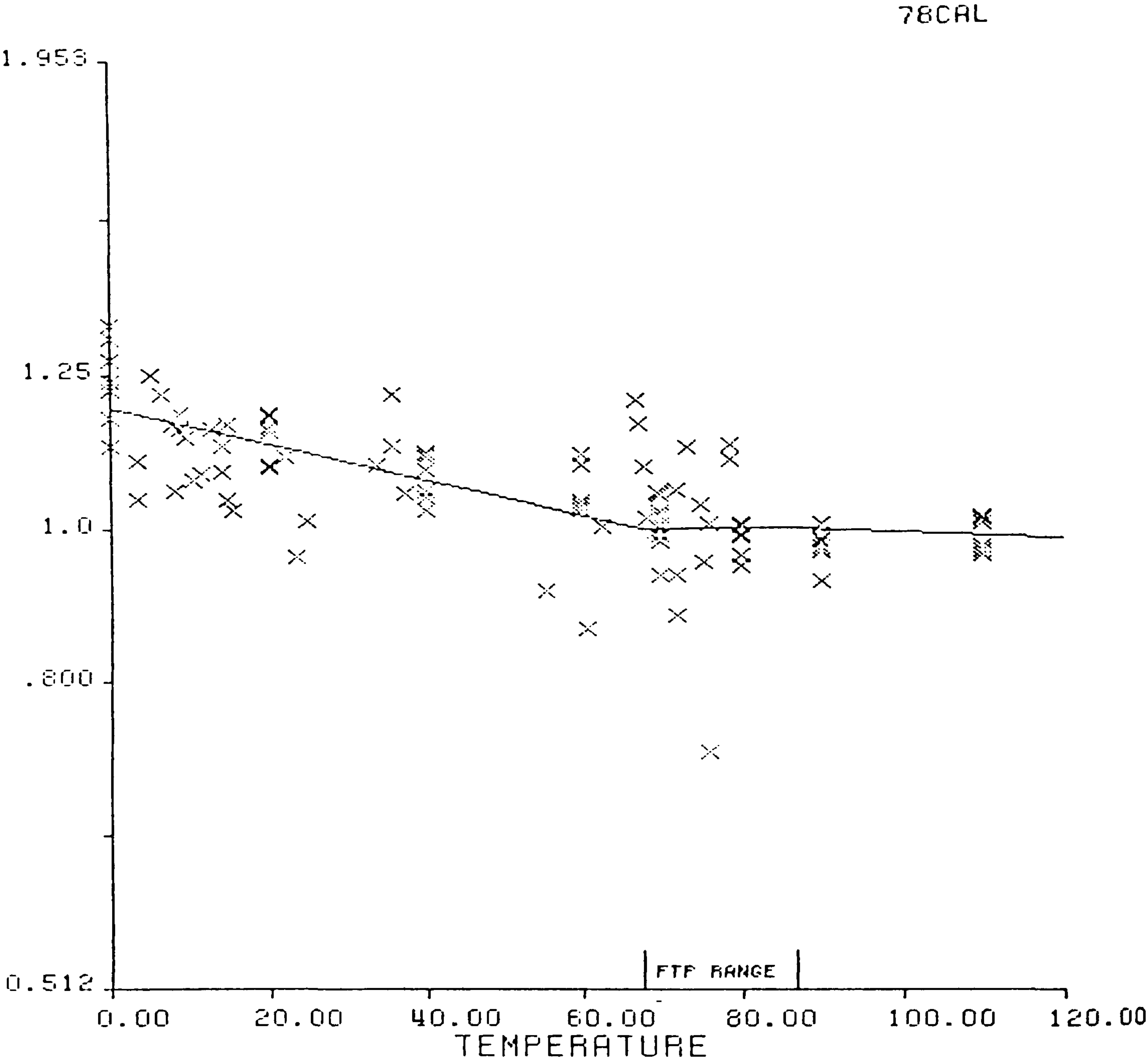
FIGURE 3.12

Temperature Effects on Fuel Consumption, Model-Year/Standard = 77 FED

Table 3.12

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION TEMP. (F)	77 FED CORRECTION FACTOR
0.0	1.1925
5.0	1.1770
10.0	1.1618
15.0	1.1467
20.0	1.1319
25.0	1.1172
30.0	1.1027
35.0	1.0885
40.0	1.0744
45.0	1.0604
50.0	1.0467
55.0	1.0331
60.0	1.0198
65.0	1.0065
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9979
95.0	0.9950
100.0	0.9920
105.0	0.9891
110.0	0.9862




```
MIN TEMP=   -3.8*        EFFECT(LOW TEMP)=EXP(0.002600(67.5-TEMP))
MAX TEMP= 110.0          EFFECT(HIGHTEMP)=EXP(-0.000483(TEMP-86.5)

N(LOW TEMP)    57
N(FTP TEMP)    29        STD ERROR(LOW TEMP)     8.24%
N(HIGHTEMP)    13        STD ERROR(HIGHTEMP)     104.95%
```

---

* The data set on which the fitting equation was based contained an additional 2 points at temperatures below 0°F.

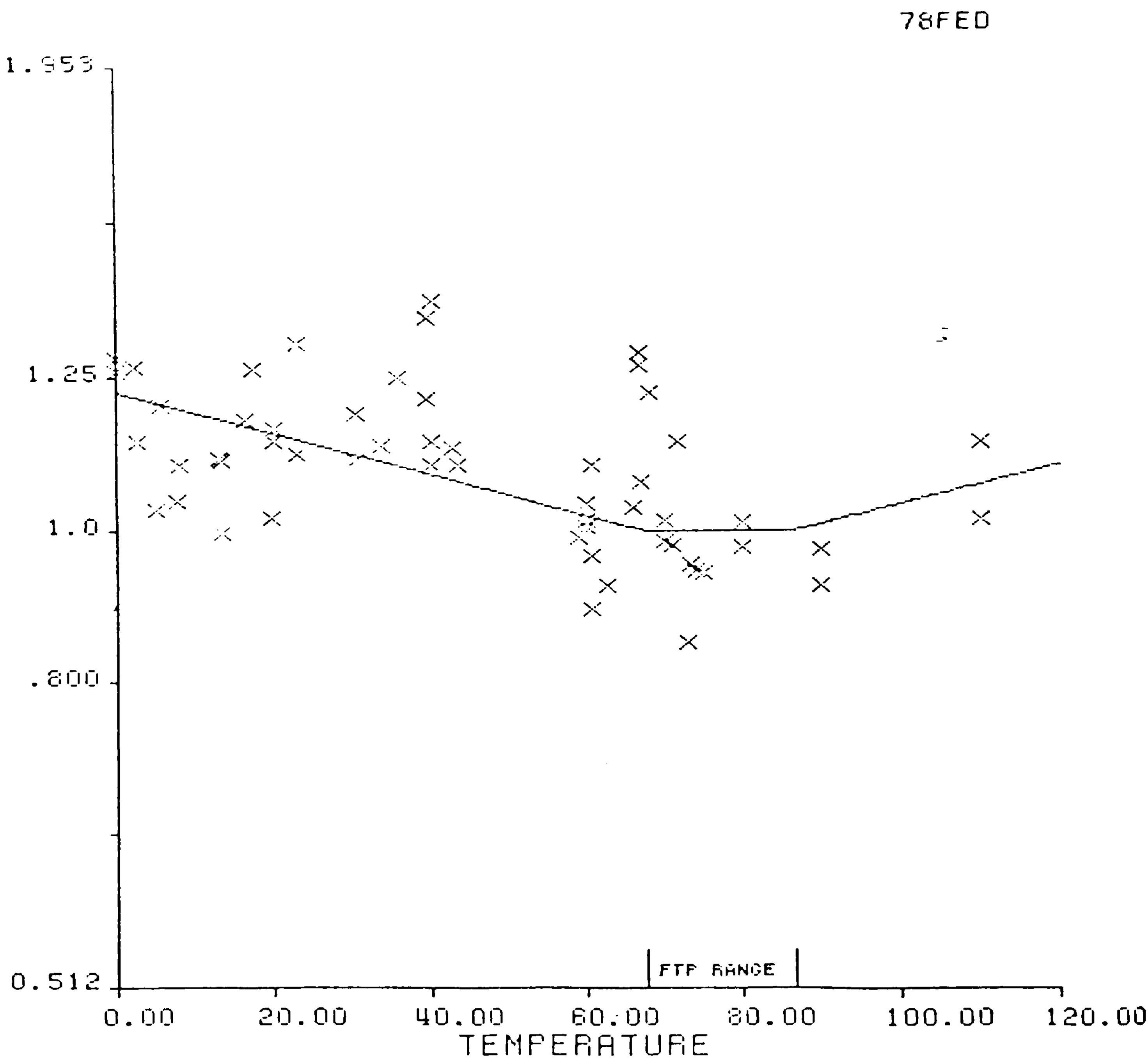
FIGURE 3.13

Temperature Effects on Fuel Consumption, Model-Year/Standard = 78 CAL

Table 3.13

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION TEMP (F)	78 CAL CORRECTION FACTOR
0.0	1.1918
5.0	1.1764
10.0	1.1613
15.0	1.1463
20.0	1.1314
25.0	1.1168
30.0	1.1024
35.0	1.0882
40.0	1.0741
45.0	1.0602
50.0	1.0466
55.0	1.0330
60.0	1.0197
65.0	1.0065
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9983
95.0	0.9959
100.0	0.9935
105.0	0.9911
110.0	0.9887



MIN TEMP= 0.0
MAX TEMP= 110.0

EFFECT(LOW TEMP)=EXP(0.002982(67.5-TEMP)
EFFECT(HIGHTEMP)=EXP(0.002810(TEMP-86.5)

N(LOW TEMP) 41
N(FTP TEMP) 11
N(HIGHTEMP) 4

STD ERROR(LOW TEMP) 13.83%
STD ERROR(HIGHTEMP) 75.55%

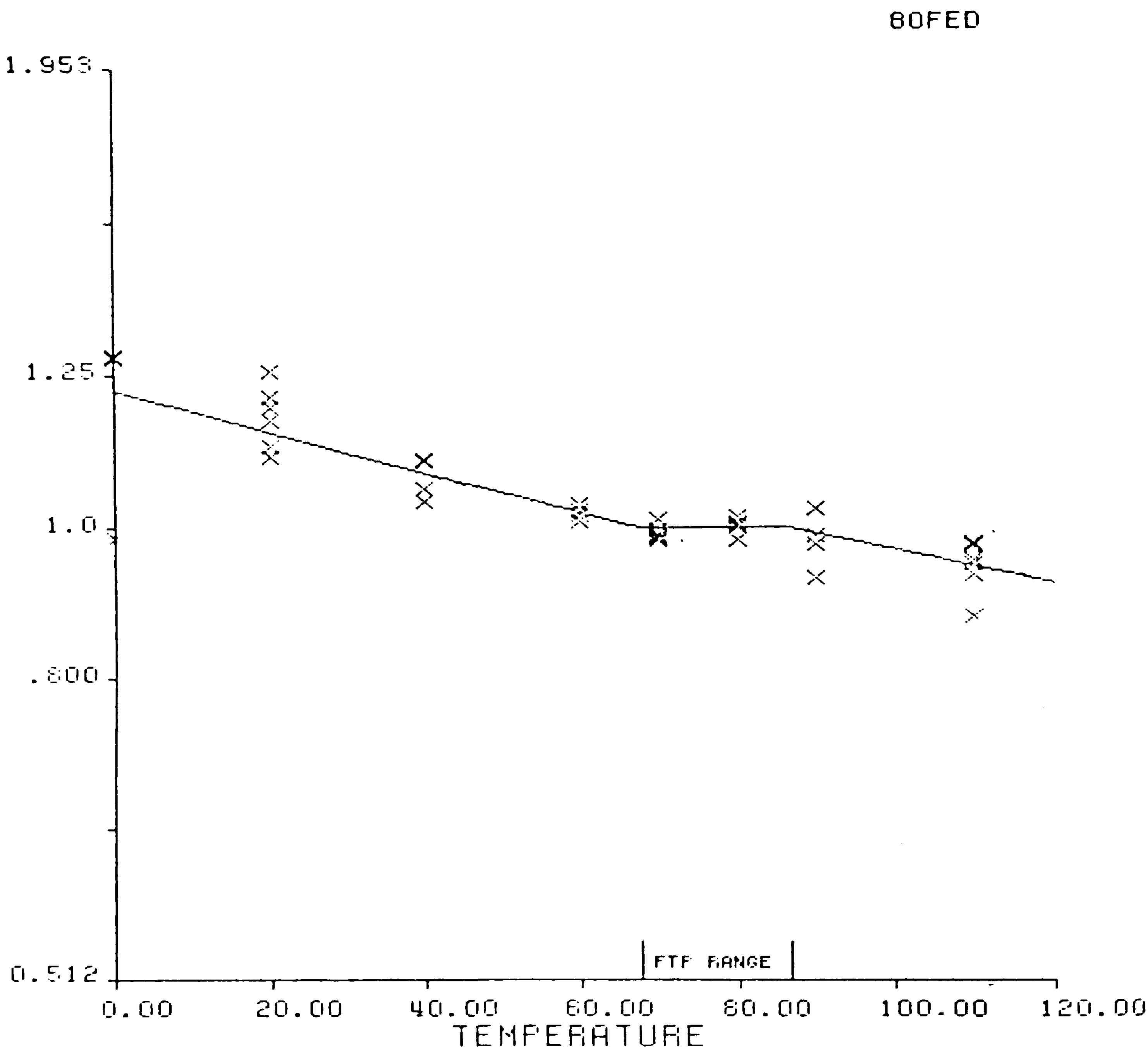
FIGURE 3.14

Temperature Effects on Fuel Consumption, Model-Year/Standard = 78 FED

Table 3.14

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION	78 FED
TEMP. (F)	CORRECTION FACTOR
0.0	1.2230
5.0	1.2049
10.0	1.1870
15.0	1.1695
20.0	1.1522
25.0	1.1351
30.0	1.1183
35.0	1.1018
40.0	1.0855
45.0	1.0694
50.0	1.0536
55.0	1.0380
60.0	1.0226
65.0	1.0075
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	1.0099
95.0	1.0242
100.0	1.0387
105.0	1.0534
110.0	1.0683




```
MIN TEMP=   0.0          EFFECT(LOW TEMP)=EXP(0.002958(67.5-TEMP))
MAX TEMP= 110.0          EFFECT(HIGHTEMP)=EXP(-0.002456(TEMP-86.5))

N(LOW TEMP)   17
N(FTP TEMP)   12         STD ERROR(LOW TEMP)     12.61%
N(HIGHTEMP)   10         STD ERROR(HIGHTEMP)     27.11%
```

FIGURE 3.15

Temperature Effects on Fuel Consumption, Model-Year/Standard = 80 FED

Table 3.15

TABLE OF ESTIMATED TEMPERATURE EFFECTS

FUEL CONSUMPTION 80 FED

TEMP (F)	CORRECTION FACTOR
0.0	1.2210
5.0	1.2031
10.0	1.1854
15.0	1.1680
20.0	1.1509
25.0	1.1340
30.0	1.1173
35.0	1.1009
40.0	1.0847
45.0	1.0688
50.0	1.0531
55.0	1.0377
60.0	1.0224
65.0	1.0074
70.0	1.0000
75.0	1.0000
80.0	1.0000
85.0	1.0000
90.0	0.9914
95.0	0.9793
100.0	0.9674
105.0	0.9556
110.0	0.9439

4.0 LIMITATIONS OF PRESENT ANALYSIS AND RECOMMENDATIONS FOR FURTHER STUDY

The results of this report were produced by a "first-step" analysis which is subject to a number of refinements. It is appropriate to comment on the limitations of the analysis in its current form, and on related questions that are subject to further investigation.

Users of the correction factors should be aware that the derived equations are only appropriate for vehicle operations reproducing the FTP cycle, and not other mixes of operating regimes. It might be useful to derive sets of correction factors for the individual bag numbers, which could then be combined appropriately for given operating cycles. Using the existing data base and little additional effort, temperature correction factors for fuel consumption could be produced for individual bag numbers.

The assumptions of additive errors whose variance is constant with respect to the independent variable (temperature) are implicit in applying linear regression methods to the transformed variables. With respect to the untransformed data (the U's) these assumptions imply a model with multiplicative errors with constant variance. There is implied a tendency for sampling errors to be proportional to U, and thus to increase with distance from the FTP range. This is an aspect of adoption of the exponential model used on this study which should be considered in judging its validity.

Users should also be aware that the temperature "effect" implied by the derived fitting equation might, in some cases, be estimated with wide variability due to large sampling errors. Some of the estimates have been based on extremely small samples. The given standard error values should be used as a guide to detect cases where the fitted equation should be used with caution.

Furthermore, note that the constrained regression method can yield an estimate of the slope even where only one value of the independent variable is represented. This is the case for two of the HOT temperature analyses: only one temperature was represented. The result of any such analysis depends even more heavily than usual on the linearity assumption, since the sample itself provides no information with which to check the form of the assumed relationship. Depending on the number of observations concentrated at that single value, it may even happen that the standard error is small in such cases.

Examination of the coefficients and correction factors could lead one to question whether there is a HOT temperature effect at all, or whether the coefficients obtained for the different MYST groups are in fact randomly distributed estimates of the same zero coefficients. Similarly, although there appear to be significant COLD temperature effects, it is questionable that there are fifteen individual effects rather than some smaller set, or indeed a single one. These questions suggest areas for further investigation. In a further study, it would be useful to consider whether or not there are, in fact, significant HOT temperature effects, and whether or not there is a smaller set of COLD temperature correction equations applicable over broader classes of vehicles.

Finally, some users may object to the fact that the correction relationship for a given MYST group, viewed as a function over the entire temperature range, has discontinuous slopes at $T = 67.5$ and $T = 86.5$. It would be easy to "smooth" the function in the neighborhoods of these values by appropriate weighting of the adjacent relationships. This might also be considered in further investigations.

APPENDIX

APPLYING CONSTRAINED LEAST SQUARES TO THE Y-vs-T DATA

For HOT temperature cases, the fitting equation is of the form

$$Y = b(T - 86.5)$$

Let $X = T - 86.5$. Then $Y = bX$. Thus the relationship is constrained to go through the origin, just as the Y-vs-T relationship was constrained to go through (86.5, 0). For COLD temperature cases, the fitting equation is of the form

$$Y = b(67.5 - T)$$

Letting $X = 67.5 - T$, one again obtains the constrained relationship $Y = bX$.

Except for the constraint, the assumptions and the approach are as with ordinary (i.e., including a Y-intercept) linear regression. The model is

$$Y_i = bX_i + e_i \qquad (i = 1, \ldots, n)$$

where $E(e_i) = 0$, $Var(e_i) = \sigma^2$ for all i, and where the e_i's are independent of one another. There is only one "normal equation" in this case, namely

$$\Sigma X_i Y_i = b \Sigma X_i^2 .$$

Thus the fitting equation is $Y = \hat{b}X$, where

$$\hat{b} = \Sigma X_i Y_i / \Sigma X_i^2$$

An estimate of σ^2 is given by

$$\hat{\sigma}^2 = \frac{1}{n-1} \Sigma (Y_i - \hat{b}X_i)^2$$

$$= \frac{1}{n-1} \left[\Sigma Y_i^2 - \hat{b}^2 \Sigma X_i^2 \right]$$

It can be shown that $E(\hat{\sigma}^2) = \sigma^2$. It can also be shown that

$$\sigma_b^2 = \mathrm{Var}(\hat{b}) = \sigma^2/\Sigma X_i^2$$

$$\sigma_b = \sigma/\sqrt{\Sigma X_i^2}$$

Thus an estimate of σ_b is provided by

$$s_b = \hat{\sigma}/\sqrt{\Sigma X_i^2}$$

It is clear from the above that $E(s_b^2) = \sigma_b^2$.