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NASA TECHNICAL MEMORANDUM NASA TM-75667

STUDY OF PERIODIC MOTIONS OF A SATELLITE WITH A MAGNETIC DAMPER

by

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(NASA-TM-75667) STUDY OF PERIODIC MOTIONS OF A SATELLITE WITH A MAGNETIC DAMPER (National Aeronautics and Space Administration) 29 p HC A03/MF A01 CSCL 22A N80-10273
Unclas
G3/15 45939

Translation of: Issledovaniye Periodicheskikh Dvizheniy Sputnika s Magnitnym Dempferom, Academy of Sciences USSR, Moscow, Institute of Applied Mathematics imeni M. V. Keldysh, Preprint No. 35, 1979, pp 1-32.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. AUGUST 1979

STANDARD TITLE PAGE

1. Report No. NASA TM-75667	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle STUDY OF PERIODIC MOTIONS OF A SATELLITE WITH A MAGNETIC DAMPER		5. Report Date August 1979
		6. Performing Organization Code
7. Author(s) Yu. A. Sadov and A. D. Teterin		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Leo Kanner Associates Redwood City, California 94063		11. Contract or Grant No. NASW-3199
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration, Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: Issledovaniye Periodicheskikh Dvizheniy Sputnika s Magnitnym Dempferom, Academy of Sciences USSR, Moscow, Institute of Applied Mathematics imeni M. V. Keldysh, Preprint No. 35, 1979, pp 1-32.

16. Abstract Periodic motion of a satellite with a magnetic damper in the plane of a circular polar orbit is studied. The asymptotics of periodic solutions are constructed for a satellite close to axisymmetric and the radius of convergence is evaluated for the power series obtained. In a broad range of values of parameters, a periodic solution is obtained by numerical integration of equations of motion of the satellite. The asymptotics of a bifurcated curve obtained (the curve on which origin of a pair of periodic solutions occurs) in the space of the parameters agrees well with the results of numerical computation with all physical values of these parameters. A breakdown is made of the space of the initial data of phase variables in the field of effect of different types of periodic motion.

17. Key Words (Selected by Author(s))		18. Distribution Statement Unclassified-Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 23	22. Price

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INTRODUCTION

The final phase of motion of a satellite equipped with a system of gravitational stabilization with a magnetic damper is studied in this work; after completion of the transition process it goes into a stationary state. Then, interaction of dissipative and excitation factors are balanced in such a way that further evolution does not occur and a stationary type of motion is retained for an indefinitely long time period (for the assumptions made in this work).

A system of gravitational stabilization with a magnetic damper includes a gravitational rod at whose end the magnetic damper is fastened. At a certain stage in motion of the satellite, the gravitational rod is opened to form the required ellipsoid of inertia.

One of the possible designs for the damper for a gravitational stabilized satellite is described in reference [1]. It includes a spherical permanent magnet (a damper float) placed inside a spherical cavity in the damper housing. The gap between the float and the housing is filled with a viscous fluid in which dissipation of energy occurs during relative motion of these bodies. Another dissipation mechanism is the Foucault currents which occur in the metal housing with a relative shift of the float and damper.

In reference [2] possible periodic motions of this system are studied in a plane of a circular polar orbit assuming that the coefficient of damping is fairly small and the permanent magnet of the damper precisely follows the vector of directivity of the geomagnetic field. It is pointed out that with these conditions, periodic oscillations and periodic rotation with frequency exist commensurate with the orbital. In reference [2] asymptotic series which describes periodic oscillations produced by stationary solutions of an unperturbed system are constructed which converge with fairly small values of the coefficient of damping. On the other hand, it is clear that when considering the problem posed, when the floats of the damper track the direction of the force lines of a geomagnetic field, with an increase in the coefficient of damping, the presence of a strong connection between the satellite and the damper causes rotation along with the magnetic field of the entire satellite. With infinitely large value, the coefficient of damping remains only one type of periodic motion, periodic rotation of the satellite together with the vector of directivity of the magnetic field. Consequently, all of the other types of periodic motion disappear with certain finite values of the coefficient of damping. Computation of this value results actually in finding the radius of convergence of the appropriate power series.

In this work, an estimate is made of this value for periodic oscillations of a satellite close to axisymmetric produced by a stable stationary point of an unperturbed system and an asymptotic formula is obtained which describes this motion up to the maximum value. The asymptotics obtained of a bifurcated curve (the curve on which the origin of pairs of periodic solutions occurs) agrees well with all allowable values of the parameters and the results of numerical computation.

In all fields of change of the parameters by a numerical method, initial data of periodic solutions were obtained both for an oscillation and for a rotating type. For large values of the coefficient of damping, where numerical location of initial data of periodic solutions is difficult, asymptotic formulas are constructed.

As is indicated in [2], at certain values introduced into the equation of motion of parameters, it has fairly asymptotically stable periodic solutions which correspond to different stable motions of the satellite with a magnetic damper. Which of these motions is realized depends on the initial conditions. Therefore, the entire plane of initial values of phase variables can break down into those regions so that motion beginning inside one of these regions occurs in a uniform type of periodic motion. Each of these regions will be called a field of effect of a corresponding periodic motion.

In this work, the problem of breaking down the space of initial data into fields of effect of different types of periodic motion will be considered. By numerical integration of equations of motion of a satellite for certain values the parameters introduced into it, the fields of effect of different types of periodic motions will be constructed.

STUDY OF PERIODIC MOTIONS OF A SATELLITE WITH A MAGNETIC DAMPER

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Yu. A. Sadov and A. D. Teterin

1. Equation of Motion of a Satellite /8*

In this work, a study is continued begun in the article [2]. As in that article here we propose that a satellite moves in a circular polar orbit, the magnetic field of Earth is modeled by a straight central dipole, and the initial conditions are selected in such a way that motion occurs in the plane of the orbit. Also, it is assumed that the permanent magnet of the damper follows precisely the direction of the magnetic force line. This assumption is achieved with existing hypotheses as to the values of parameters contained in the equation of motion of the float of a magnetic damper, namely, when assuming the smallness of the ratio of moments of inertia of the float and the satellite and the smallness of the moment of viscous forces acting on the float in comparison with the magnetic moment.

With these hypotheses, the equation of motion of a satellite relative to the center of mass will have the following form

$$\frac{d^2\vartheta}{d\nu^2} + \alpha^2 \sin\vartheta\cos\vartheta = 2\varepsilon\left(\frac{2}{1+3\sin^2\nu} - \frac{d\vartheta}{d\nu}\right), \qquad (1)$$

where ν - is the independent variable of width,

ϕ - angle of inclination of the largest axis of the ellipsoid of inertia from the local vertical,

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*Numbers in the margin indicate pagination in the foreign text

α^2 - inertia parameter,

2ε - dimensionless coefficient of damping,

$$\alpha^2 = 3\frac{B-A}{C}, \qquad 2\varepsilon = \frac{K_g}{C\omega_0}. \tag{2}$$

Here A, B, C - are the main central moments of inertia of a satellite located in increasing magnitude; Kg - is the coefficient of damping; ω_0 - is the orbital angular velocity.

We note that the right part of equation (1) was apparent due to the presence of the damper and introduces excitation into the orientation of the satellite. Operation of the system of stabilization can be considered as satisfactory only in a case where these perturbations are small. Therefore, the dimensionless coefficient of damping ε must be fairly small. /9

For convenience we make a change in equation (1) of the variables

$$\theta = 2\vartheta, \qquad \tau = 2\nu. \tag{3}$$

Then, for describing plane motion of a satellite with a magnetic damper, we find the equation

$$\ddot{\theta} + \frac{\alpha^2}{4}\sin\theta = \varepsilon\left(\frac{4}{5-3\cos\tau} - \dot{\theta}\right), \tag{4}$$

where differentiation is precisely designated according to T. Equation (4) will be the main object of our study.

As was noted above, parameter ε, introduced into equation (4), is fairly small. This makes it possible to use, in this or another form, a method of a small parameter. For this adaptation it is necessary to know the solution of the unperturbed equation which is obtained from the initial when $\varepsilon=0$. In this case, the unperturbed equation is presented as the well known equation of a mathematical pendulum

$$\ddot{\theta} + \frac{\alpha^2}{4}\sin\theta = 0 . \tag{5}$$

Equation (5) has the integral of energy which can be written in the form

$$\nu^2 = \frac{\dot{\theta}^2}{\alpha^2} + \sin^2\frac{\theta}{2} = const \qquad (\alpha^2 \neq 0). \tag{6}$$

Using this integral, it is possible to integrate equation (5) to its end. The general solution of the equation is expressed by elliptical Jacobi functions [3].

Where $0<\nu<1$, equation (5) describes oscillation motion /10
with the period

$$T_1 = \frac{8K(k)}{\alpha}, \qquad k = \nu , \tag{7}$$

where $K(k)$ - is the full elliptical integral of the first order.

When $\nu>1$, equation (5) has a periodic solution which describes rotation with the period

$$T_2 = \frac{4kK(k)}{\alpha}, \qquad k = \frac{1}{\nu} . \tag{8}$$

The case $\nu=1$ corresponds to motion along the separatrix separated from the phase plane $(\theta, \dot{\theta})$, the field of oscillations from the field of rotation.

Besides these solutions, equation (5) has two stationary solutions

$$\theta = 0, \quad \dot{\theta} = 0 \qquad (\nu = 0) , \tag{9}$$

$$\theta = \pi, \quad \dot{\theta} = 0 \qquad (\nu = 1) . \tag{10}$$

Equalities (10) define the trough special point of unperturbed motion. The second special point is the center which is described by equation (9).

Formulas (6)-(8) lose their meaning when $\alpha=0$ (an axisymmetric satellite). In this case, equation (5) describes a uniform rotation

$$\theta = \dot{\theta}_0 (\tau + \tau_0) . \tag{11}$$

The picture of the trajectory, as is seen from equation (5), is periodic according to θ, and therefore the phase plane $(\theta, \dot{\theta})$ can reduce to a cylinder $(-\pi<\theta\leq\pi;\ -\infty<\theta<+\infty)$. All of the phase trajectories on this cylinder except for the separatrix are closed and periodic according to T.

2. Classes of Solutions Generated /11

Periodic solutions are of particular interest for studying equation (4). Equation (4) is periodic according to the variables T and θ, that is, its form does not change with substitutions

$$T \to T+2\pi,$$
$$\theta \to \theta+2\pi.$$

Therefore, periodic solutions of this equation must satisfy the condition

$$\theta(\tau + 2\pi m) = \theta(\tau) + 2\pi p \quad (m = 1, 2, \ldots;\ p = 0, \pm 1, \pm 2, \ldots) \tag{12}$$

The trajectory which satisfies this condition for m periods according to T circles the phase cylinder $(\theta, \dot{\theta})$ p times and is closed going to the initial point. When $\varepsilon \to 0$, such a trajectory changes to a certain integral curve of unperturbed motion which is called generated for the solution considered. Due to the continuous dependence of the solution on the parameter ε, the generated solution also satisfies condition (12). This makes it possible to separate, among the solutions of an unperturbed system, those which can be generated for periodic solutions of a perturbed system.

When $\alpha=0$ from equations (11) and (12) we find that on the generated trajectories one must fulfill the condition

$$\dot{\theta}_0 = \frac{p}{m} . \tag{13}$$

If $\alpha\neq 0$, then among the generated solutions one finds stationary solutions (9), (10) and also oscillatory (7) and rotational (8).

Let us consider oscillation solutions which are defined by formula (7). For these, the relationship following is /12
fulfilled

$$\theta(t+T_1) = \theta(t)$$

and, consequently,

$$\theta(\tau+nT_1) = \theta(\tau) , \quad n=1,2,3,\ldots \tag{14}$$

Comparing (14) and (12) we find that on oscillation trajectories, (12) is fulfilled when $p=0$, if only

$$T_1 = 2\pi \frac{m}{n} . \tag{15}$$

For rotational solutions from formula (8) we have

$$\theta(\tau+T_2) = \theta(\tau) \pm 2\pi ,$$

or

$$\theta(\tau+nT_2) = \theta(\tau) \pm 2\pi n , \quad n=1,2,3,\ldots \tag{16}$$

In this way, condition (12) is fulfilled, if

$$T_2 = 2\pi \frac{m}{n} . \tag{17}$$

Then, in formula (12) $p=\pm n$, where the sign for n is selected in accordance with the direction of rotation of unperturbed motion.

In reference [2] it is pointed out that periodic solutions do not exist for equation (4), generated by oscillation solutions (7) of an unperturbed system. Also periodic solutions do not exist generated by rotations (8) toward the side opposite the direction of rotation of the magnetic field. Rotation (8) in a straight direction can generate slow (with a frequency no larger than the orbital, $m\geq n$) nondamped rotations of the satellite. With this rotation at an orbital frequency ($m=n$) it is possible with all physically allowable values of the parameters, ($0<\alpha^2\leq 3$; $0\leq\varepsilon<\infty$). Periodic rotations of other types have a very narrow field of existence in the space of parameters of α, ε and, accordingly, a low probability of falling into this condition. Stationary solutions of an unperturbed system are generated with stable (9) and unstable (10) oscillations. When $\alpha=0$ (an axisymmetric satellite) for any $\varepsilon>0$, a single parameter family of stable periodic trajectories exists, corresponding to $m=p=1$: /13

$$\theta=\tau+2\sum_{k=1}^{\infty}\frac{\varepsilon}{k3^k}\cdot\frac{\varepsilon\sin k\tau-k\cos k\tau}{\varepsilon^2+k^2}+C. \qquad (17')$$

3. Construction of Periodic Solutions of the Equation of Motion of a Satellite Close to Axisymmetric, Generated by Stationary Solutions of an Unperturbed Equation and an Evaluation of the Field of their Existence in the Space of Parameters (ε, α).

Having used the smallest values of parameter ε, one can construct the asymptotic formulas which describe periodic motion of a satellite, generated by stable (9) and unstable

(10) stationary points of unperturbed equation (5). As is pointed out in reference [2], they have the following form

$$\theta = 2\varepsilon\left[\frac{2}{\alpha^2} - \sum_{j=1}^{\infty} \frac{\cos j\tau}{j^2\left(j^2 - \frac{\alpha^2}{4}\right)}\right] + 2\varepsilon^2 \sum_{j=1}^{\infty} \frac{j \sin j\tau}{j^2\left(j^2 - \frac{\alpha^2}{4}\right)^2} + \dots, \tag{18}$$

$$\theta = \pi - 2\varepsilon\left[\frac{2}{\alpha^2} + \sum_{j=1}^{\infty} \frac{\cos j\tau}{j^2\left(j^2 + \frac{\alpha^2}{4}\right)}\right] + 2\varepsilon^2 \sum_{j=1}^{\infty} \frac{j \sin j\tau}{j^2\left(j^2 + \frac{\alpha^2}{4}\right)^2} + \dots, \tag{19}$$

for all physically allowable values of the inertia parameter $\alpha (0 < \alpha \leq \sqrt{3})$. /14

With the proposed posing of the problem (when the float of the damper follows the direction of the force lines of the magnetic field) 2π - periodic oscillations (18) and (19) will exist not for all values of a dimensionless coefficient of damping ε, but only for $0 \leq \varepsilon < \varepsilon_*$, where the value of ε_* depends on α. Actually, for fairly large values of ε, the presence of a strong connection between the satellite and the damper float causes rotation along with the magnetic field of the whole satellite. Rotation of $m:n=1:1$ is a unique type of periodic motion in this case.

Numerical values of ε_* result actually in finding the radius of convergence of series (18) and (19). This problem, as a rule, is extremely cumbersome, that is, it requires computation of the common member of the series (18) and (19) which in the case of nonlinear equations presents considerable difficulty. Therefore, we will attempt to attain certain estimates of the value of ε_*. For this, actually we will construct a periodic solution generated by the stable position of equilibrium (9) in a case when the inertia parameter α is small. Then one can expect that the critical value of ε_* also will be small and one can use asymptotic methods right up to this value. Here, however, it is impossible to use asymptotics of (18), because the amplitude of oscillation when ε is close to ε_* is large. On the other hand, angular velocity $\dot{\theta}$, as one sees from the integral of energy in unperturbed motion (6) remains a small value on the order of α.

Therefore we assume $\dot{\theta} = \alpha \cdot p$.

$$\tag{20}$$

Then /15

$$\dot{p} + \frac{\alpha}{4}\sin\theta = \frac{\varepsilon}{\alpha}\left(\frac{4}{5-3\cos\tau} - \alpha p\right). \tag{21}$$

We assume

$$\alpha = \kappa\sqrt{\varepsilon} \tag{22}$$

and rewrite system (20), (21) in the form

$$\begin{cases} \dot{\theta} = \sqrt{\varepsilon}\,\kappa p, \\ \dot{p} = \sqrt{\varepsilon}\left(\frac{1}{\kappa}\frac{4}{5-3\cos\tau} - \frac{\kappa}{4}\sin\theta\right) - \varepsilon p. \end{cases} \tag{23}$$

Finding the solution of this sytem in the form of the series according to the power $\sqrt{\varepsilon}$

$$\begin{cases} \theta = \theta_0 + \sqrt{\varepsilon}\,\theta_1 + \varepsilon\theta_2 + \varepsilon^{3/2}\theta_3 + \cdots, \\ p = p_0 + \sqrt{\varepsilon}\,p_1 + \varepsilon p_2 + \varepsilon^{3/2} p_3 + \cdots, \end{cases} \tag{24}$$

we find the following equations for sequential approximations

$$\begin{cases} \dot{\theta}_0 = 0, \quad \dot{p}_0 = 0; \\ \dot{\theta}_1 = \kappa p_0, \quad \dot{p}_1 = \frac{1}{\kappa}\frac{4}{5-3\cos\tau} - \frac{\kappa}{4}\sin\theta_0; \\ \dot{\theta}_2 = \kappa p_1, \quad \dot{p}_2 = -\frac{\kappa}{4}\theta_1\cos\theta_0 - p_0; \\ \dot{\theta}_3 = \kappa p_2, \quad \dot{p}_3 = -\frac{\kappa}{4}\theta_2\cos\theta_0 + \frac{\kappa}{8}\theta_1^2\sin\theta_0 - p_1; \ \cdots \end{cases} \tag{25}$$

Using the solution of the equations of a zero approximation

$$\theta_0 = C_0 = const, \quad p_0 = D_0 = const, \tag{26}$$

we find in the first approximation

$$\theta_1 = \kappa D_0 \tau + C_1, \quad \rho_1 = \frac{2}{\kappa} \operatorname{Arctg}\left(2 \operatorname{tg}\frac{\tau}{2}\right) - \frac{\kappa}{4}\tau \sin C_0 + D_1, \tag{27}$$

where /16

$$\operatorname{Arctg}\left(2\operatorname{tg}\frac{\tau}{2}\right) = \int_0^\tau \frac{2dx}{5-3\cos x} = \frac{\tau}{2} + \sum_{j=1}^{\infty} \frac{\sin j\tau}{j \cdot 3^j}. \tag{28}$$

For this, so that solution (27) would correspond to periodic oscillation, it is necessary and adequate that

$$D_0 = 0, \quad \sin C_0 = \frac{4}{\kappa^2}. \tag{29}$$

The latter condition can be fulfilled only when $k \geq 2$. For the calculation of (22) this result can be formulated as: when

$$\varepsilon > \varepsilon' = \frac{\alpha^2}{4} \tag{30}$$

does not exist of periodic solutions of equation (4), generated by a stable stationary point. The value of ε', defined by formula (30) gives an estimate above the boundary of ε_* of the field of existence of periodic solutions of the type considered ($\varepsilon_* < \varepsilon'$).

The constants C_1 and D_1 entering into (27) are determined from the condition of periodicity of the solution of equations of the second approximation

$$\begin{cases} \theta_2 = 2\int_0^\tau \operatorname{Arctg}\left(2\operatorname{tg}\frac{x}{2}\right)dx - \frac{1}{2}\tau^2 + \kappa D_1 \tau + C_2 = \\ \quad = -2\sum_{j=1}^{\infty} \frac{\cos j\tau}{j^2 \cdot 3^j} + 2\sum_{j=1}^{\infty} \frac{1}{j^2 \cdot 3^j} + \kappa D_1 \tau + C_2, \\ \rho_2 = -\frac{\kappa\tau}{4} C_1 \cos C_0 + D_2. \end{cases} \tag{31}$$

From this we find

$$C_1 = 0, \quad D_1 = 0. \tag{32}$$

In a completely analogous way, the conditions of periodicity for all sequential approximations give an equation for determining the additive constants of the preceding approximation. These equations are linear and solved when $\cos\theta_0 \neq 0$; therefore, construction of the series (24) is possible for all ε, which satisfy the inequality /17

$$\varepsilon < \varepsilon' . \tag{33}$$

The dependence of ε' on α is graphically shown in Figure 1. Here, along the axis of the abscissa, the parameter $a = \alpha^2/4$ is applied and $\varepsilon'(a)$ is expressed by a dashed straight line. The solid line indicates the boundary of the field of existence of the solutions considered, obtained using numerical calculations described in the following paragraph.

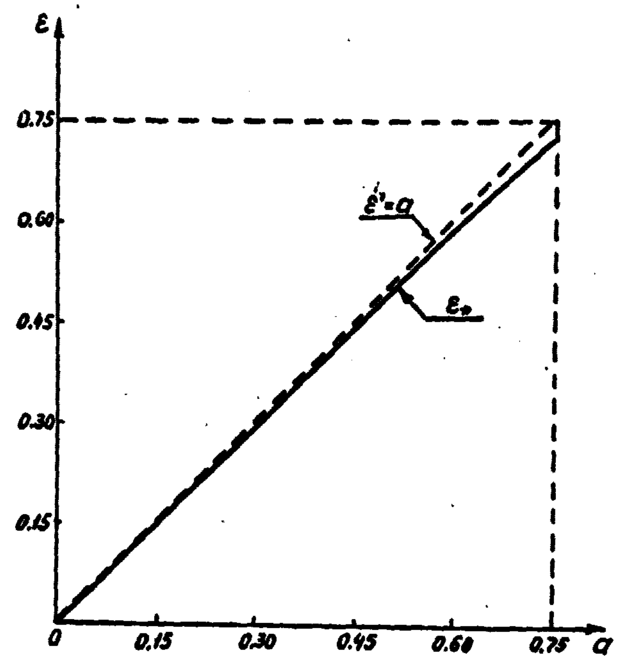

Figure 1

A total of the constants C_2 and D_2 in formula (31) gives us

$$C_2 = -2\sum_{j=1}^{\infty} \frac{1}{j^2 \cdot 3^j} \approx -0,7325 , \quad D_2 = 0 . \tag{34}$$

Consequently, with small values of α, the periodic solutions sought can be written in the form of a series

$$\theta = \arcsin\frac{4\varepsilon}{\alpha^2} - \varepsilon \cdot 2\sum_{j=1}^{\infty} \frac{\cos j\tau}{j^2 \cdot 3^j} + O(\varepsilon^2) . \tag{35}$$

4. Numerical Finding of Periodic Solutions

In a broad range of values of the parameters a and ε, periodic solutions are found by numerical integration of equation (4). The problem amounted to a selection of the initial conditions θ, $\dot{\theta}$ when T=0 so that at the moment $T=2\pi$ the following relationship is fulfilled

$$\theta(2\pi)=\theta(0), \quad \dot{\theta}(2\pi)=\dot{\theta}(0). \qquad (36)$$ /18

In this way, periodic oscillations were calculated, generated by stable and unstable stationary points and also periodic rotations of the type $m:n=1:1$. Computation encompassed a range of values $a=\alpha^2/4$ from 0 to 0.750. The latter number corresponds to the boundary of physically allowable values of the inertia parameter.

The values of the initial conditions of integration obtained from computation $\theta^{(0)}\equiv\theta(0)$ and $\dot{\theta}^{(0)}\equiv\dot{\theta}(0)$ depend on the two parameters a and ε

$$\theta^{(0)}=\theta^{(0)}(a,\varepsilon), \quad \dot{\theta}^{(0)}=\dot{\theta}^{(0)}(a,\varepsilon) \qquad (37)$$

and are shown in Figure 2. In this drawing, three series of curves are visible. Each of the curves is the geometrical location of the initial conditions of periodic solutions with a fixed value of the parameter a and a change in ε from 0 to ε_*. In each series, it is expressed on the three curves corresponding to a=0.3; 0.6 and 0.75.

Curves in the upper part of the drawing when $\dot{\theta}^{(0)}>1$ correspond to rotation, then when $\theta^{(0)}>0$, stable rotations are obtained; when $\theta^{(0)}<0$ - they are unstable. Such motions exist with all values of ε and therefore the curves of each series begins when $\varepsilon=0$ and are completed in general for each of the series at a point corresponding to $\varepsilon\to\infty$. In this maximum case, the satellite like the float of the damper rotates

/31

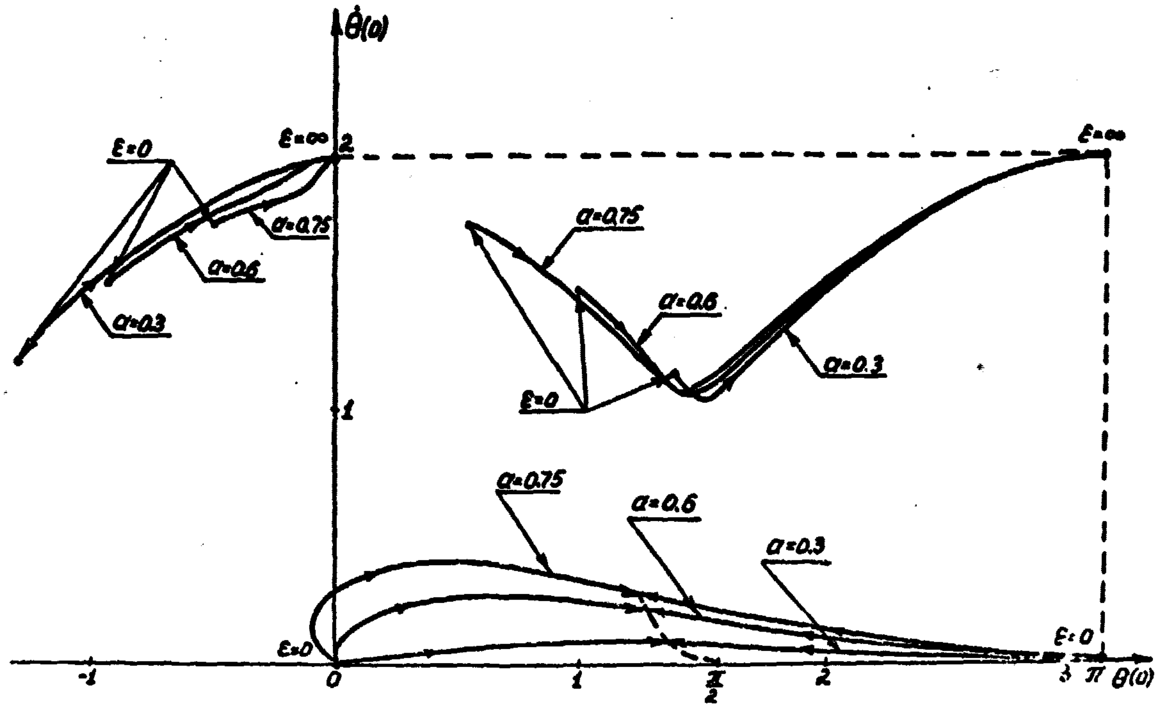

Figure 2

along with the force lines of the magnetic field and such motion does not depend on gravitational moments. The numerical /19 location of the solutions with large values of ε are difficult but in this case one can construct asymptotic formulas:

$$\begin{cases} \theta^{(0)} = -\dfrac{2-a\rho}{\varepsilon} + O\left(\dfrac{1}{\varepsilon^3}\right), \\ \dot{\theta}^{(0)} = 2 - \dfrac{3-4a+a^2\rho}{\varepsilon^2} + O\left(\dfrac{1}{\varepsilon^3}\right); \end{cases} \quad \text{- for unstable rotation,} \quad (38)$$

$$\begin{cases} \theta^{(0)} = \pi - \dfrac{2+a\rho}{\varepsilon} + O\left(\dfrac{1}{\varepsilon^3}\right), \\ \dot{\theta}^{(0)} = 2 - \dfrac{3+4a+a^2\rho}{\varepsilon^2} + O\left(\dfrac{1}{\varepsilon^3}\right). \end{cases} \quad \text{- for stable rotation} \quad (39)$$

In these formulas ρ is constant

$$\rho = \frac{8}{3}(9\ln 3 - 13\ln 2) \approx 2{,}3389. \qquad (40)$$

For obtaining asymptotic formulas (38), (39) we rewrite equation (4) in the form

$$\frac{4}{5-3\cos\tau}-\dot{\theta}=\frac{1}{\varepsilon}(\ddot{\theta}+a\sin\theta). \qquad (41)$$

Solution of equation (41) will be found in the form of a series:

$$\theta(\tau)=\theta_0(\tau)+\frac{1}{\varepsilon}\theta_1(\tau)+\frac{1}{\varepsilon^2}\theta_2(\tau)+\frac{1}{\varepsilon^3}\theta_3(\tau)+\ldots, \qquad (42)$$

By substituting (42) in equation (41) and equalizing the coefficients of uniform powers of the parameter ε, we find that the function $\theta_i(T)$, $i=0,1,2,\ldots$ must satisfy the following equations:

$$\dot{\theta}_0=\frac{4}{5-3\cos\tau}, \qquad (43)$$

$$\dot{\theta}_1=-\ddot{\theta}_0-a\sin\theta_0, \qquad (44)$$

$$\dot{\theta}_2=-\ddot{\theta}_1-a\theta_1\cos\theta_0, \qquad (45)$$ /29

$$\dot{\theta}_3=-\ddot{\theta}_2-a\left(\theta_2\cos\theta_0-\frac{1}{2}\theta_1^2\sin\theta_0\right), \ldots, \qquad (46)$$

Then for any value of T one must satisfy the condition

$$\theta_0(\tau+2\pi n)=\theta_0(\tau)+2\pi n, \quad \theta_i(\tau+2\pi n)=\theta_i(\tau), \quad i=1,2,\ldots;\ n=\pm1,\pm2,\ldots \qquad (47)$$

Integrating equation (43), we have in a zero approximation

$$\theta_0(\tau)=2\,\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right)+C_0=\tau+2\sum_{j=1}^{\infty}\frac{\sin j\tau}{j\cdot 3^j}+C_0. \qquad (48)$$

The zero approximation describes the motion of the satellite when $\varepsilon=\infty$, that is, when it rotates along with the force lines of the magnetic field.

From equation (44) we find the first approximation

$$\theta_1(\tau) = -\frac{4}{5-3\cos\tau} - \frac{4}{3}a\cos C_0 \ln\frac{5-3\cos\tau}{2} + \\ + a\sin C_0\left[\frac{5}{3}\tau - \frac{8}{3}\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right)\right] + C_1 = \\ = -\frac{4}{5-3\cos\tau} - \frac{4}{3}a\cos C_0 \ln\frac{5-3\cos\tau}{2} + \\ + a\sin C_0\left[\frac{\tau}{3} - \frac{8}{3}\sum_{j=1}^{\infty}\frac{\sin j\tau}{j\cdot 3^j}\right] + C_1. \tag{49}$$

From the condition of periodicity (47) it follows that

$$\sin C_0 = 0,$$

that is,

$$C_0^{(1)} = 0, \qquad C_0^{(2)} = \pi. \tag{50}$$

The first value of the constant C_0 corresponds to unstable rotation, and the second to stable.

The second approximation we find by integrating equation /21 (45):

$$\theta_2(\tau) = -\frac{12\sin\tau}{(5-3\cos\tau)^2} + 8a\cos C_0\frac{\sin\tau}{5-3\cos\tau} + \\ + aC_1\cos C_0\left[\frac{5}{3}\tau - \frac{8}{3}\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right)\right] + \\ + \frac{4}{3}a^2\int_0^{\tau}\frac{3-5\cos x}{5-3\cos x}\ln\frac{2}{5-3\cos x}\,dx + C_2. \tag{51}$$

The condition of periodicity (47) for $\Theta_2(T)$ takes on the following form

$$\frac{2\pi}{3}a\cdot C_1\cos C_0 + \frac{4}{3}a^2\int_0^{2\pi}\frac{3-5\cos x}{5-3\cos x}\ln\frac{2}{5-3\cos x}\,dx = 0. \tag{52}$$

Having used formulas 4.413 (3.4) [5], we find

$$C_1 = a\rho\cos C_0, \tag{53}$$

where ρ is determined by formula (40).

The constant C_2 in (51) we will define from the condition of periodicity for the third approximation. It appears equal to zero:

$$C_2 = 0 \ . \tag{54}$$

Thus, for adequately large values of ε, equation (4) has two 2π-periodic solutions of a roatating type (unstable and stable, respectively) which according to formulas (42)-(54) can be presented in the form

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$$\begin{cases} \theta(\tau) = 2\,\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right) + \frac{1}{\varepsilon}\left[-\frac{4}{5-3\cos\tau} + \alpha\left(\rho - \frac{4}{3}\ln\frac{5-3\cos\tau}{2}\right)\right] + \\ \quad + \frac{1}{\varepsilon^2}\left\{-\frac{12\sin\tau}{(5-3\cos\tau)^2} + \frac{8\alpha\sin\tau}{5-3\cos\tau} + \right. \\ \quad + \alpha^2\left[\rho\left(\frac{5}{3}\tau - \frac{8}{3}\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right)\right) + \right. \\ \quad \left.\left. + \frac{4}{3}\int_0^\tau \frac{5\cos x - 3}{5-3\cos x}\ln\frac{5-3\cos x}{2}\,dx\right]\right\} + O\left(\frac{1}{\varepsilon^3}\right), \\ \dot\theta(\tau) = \frac{4}{5-3\cos\tau} + \frac{1}{\varepsilon}\,\frac{4\sin\tau}{5-3\cos\tau}\left(\frac{3}{5-3\cos\tau} - \alpha\right) + \\ \quad + \frac{1}{\varepsilon^2}\left[\frac{12(6-5\cos\tau-3\cos^2\tau)}{(5-3\cos\tau)^3} + \frac{8\alpha(5\cos\tau-3)}{(5-3\cos\tau)^2} - \right. \\ \quad \left. - \alpha^2\frac{5\cos\tau-3}{5-3\cos\tau}\left(\rho - \frac{4}{3}\ln\frac{5-3\cos\tau}{2}\right)\right] + O\left(\frac{1}{\varepsilon^3}\right); \end{cases}$$

$$\begin{cases} \theta(\tau) = \pi + 2\,\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right) + \frac{1}{\varepsilon}\left[-\frac{4}{5-3\cos\tau} - \alpha\left(\rho - \frac{4}{3}\ln\frac{5-3\cos\tau}{2}\right)\right] + \\ \quad + \frac{1}{\varepsilon^2}\left\{-\frac{12\sin\tau}{(5-3\cos\tau)^2} - \frac{8\alpha\sin\tau}{5-3\cos\tau} + \right. \\ \quad + \alpha^2\left[\rho\left(\frac{5}{3}\tau - \frac{8}{3}\mathrm{Arctg}\left(2\,\mathrm{tg}\frac{\tau}{2}\right)\right) + \right. \\ \quad \left.\left. + \frac{4}{3}\int_0^\tau \frac{5\cos x - 3}{5-3\cos x}\ln\frac{5-3\cos x}{2}\,dx\right]\right\} + O\left(\frac{1}{\varepsilon^3}\right), \\ \dot\theta(\tau) = \frac{4}{5-3\cos\tau} + \frac{1}{\varepsilon}\,\frac{4\sin\tau}{5-3\cos\tau}\left(\frac{3}{5-3\cos\tau} + \alpha\right) + \\ \quad + \frac{1}{\varepsilon^2}\left[\frac{12(6-5\cos\tau-3\cos^2\tau)}{(5-3\cos\tau)^3} - \frac{8\alpha(5\cos\tau-3)}{(5-3\cos\tau)^2} - \right. \\ \quad \left. - \alpha^2\frac{5\cos\tau-3}{5-3\cos\tau}\left(\rho - \frac{4}{3}\ln\frac{5-3\cos\tau}{2}\right)\right] + O\left(\frac{1}{\varepsilon^3}\right). \end{cases}$$

When $T=0$, from (55) and (56) we find asymptotic formulas (38) and (39).

In the lower part of Figure 2 when $\dot{\theta}^{(0)}<0.4$, there is a series of curves corresponding to oscillation motion of the satellite. Each curve consists of two branches -- stable and unstable. These branches occur when $\varepsilon=0$ respectively from the stable and unstable stationary points of an unperturbed system. With an increase in ε, the points shown for stable and unstable periodic solutions move along their branches and when $\varepsilon=\varepsilon_*(a)$ they merge at the branching point. With large values of ε, 2π-periodic oscillation solutions do not exist. The bifurcated curve on which merging of two periodic solutions occurs is shown in Figure 2 by the dashed line. The critical values of ε_* depending on a are shown by the solid line in Figure 1. This curve, for all physically possible values of the parameter a coincide fairly well with straight line /23

$$\varepsilon=a, \tag{57}$$

defined by asymptotics (27).

The fields of existence of two types of periodic solutions according to the parameter ε are considerably narrower. For example, when $\alpha=\frac{\pi}{2}$ ($a=0.616$) 4π-periodic rotations corresponding to $m:n=2:1$, it can be successfully obtained numerically only when $\varepsilon<0.0035$.

5. Fields of Effect of Different Types of Periodic Motion

As one sees from reference [2], besides the periodic solutions studied above, with certain values of the parameters α and ε, equation (4) has other types of asymptotically stable periodic solutions which correspond to different

established motion of a satellite with a magnetic damper. /24
Which of these motions is realized depends on the initial conditions. Therefore, the entire plane of the initial values of θ, $\dot{\theta}$ can be divided in these fields so that motion, beginning inside one of the fields results in a uniform type of periodic motion.

Let us determine the transformation of the phase plane of θ, $\dot{\theta}$ for a period like this transformation, in which a given point θ, $\dot{\theta}$ is the point $(\theta(2\pi), \dot{\theta}(2\pi))$ of the trajectory of equation (4) which occurs at the moment $T=0$ through $(\theta(0), \dot{\theta}(0))$. By virtue of the 2π-periodicity of equation (4) according to the T properties of the trajectory, corresponding to point $(\theta(0), \dot{\theta}(0))$, like the initial, are invariant with such a transformation. The invariant, in particular, is broken down in the field of effect. The 2π-periodic solution of equation (4) corresponds to immovable points of the transformation considered. It is clear that asymptotically stable stationary points belong to the field of effect on the periodic solution determined by them. Invariant curves which pass through unstable stationary points are the separatrixes of a given transformation; they divide the field with qualitatively different behavior of the invariant curves. In particular, the sum of the separatrixes divide the field of effect. Therefore, the problem of finding the fields of effect leads to calculation of the separatrix of transformation of the phase space caused by a shift in the period along the trajectory of differential equation (4).

As one sees from what has been presented above, such a transformation for any fixed values of the parameters $a=\alpha^2/4$ and ε $(0<a\leq 0.75;\ 0<\varepsilon<\varepsilon_*(a))$ has four stationary points (Figure 2): two stable and two unstable. Computation of the separatrix began in the environment of unstable stationary points. Close to such a point, the transformation of the phase plane is close to linear. The intrinsic vectors of this linear transformation indicate the direction of the branches of the separatrix, which pass through a given stationary point. The intrinsic vector which has an intrinsic value in absolute size larger than one corresponds to outgoing branches of the separatrix and the intrinsic vector with intrinsic value according to the model smaller than one corresponds to the incoming branch. For constructing the separatrix, the initial conditions are selected lying in the environment of unstable stationary points in a direction determined by the intrinsic vector and numerical integration is made of the system with these initial conditions. The points of the trajectory corresponding to the moment of time $T=2k\pi$, $k=1,2,\ldots$, belongs to the appropriate separatrix. For this, in order to obtain in this way the incoming branches, integration must be done in a direction of decrease of the independent variable ($T=2k\pi$, $k=1,-2,\ldots$). Obtaining the sequence of points when moving away from the initial stationary point becomes very rare and does not give the correct concept of the behavior of the separatrix. For obtaining a more detailed picture, one must follow the indicated procedure more than once, selecting as the initial difference of the point one or another branch in the environment of the unstable stationary point. /25

Figure 3 shows the breakdown of the phase plane obtained in this way corresponding to $a=0.750$, $\varepsilon=0.1$. In this case, as has already been indicated, we have four periodic solutions: two stable and two unstable. In Figure 3, the four stationary points correspond to it. At each of the unstable stationary points, the separatrixes converge along four branches: two incoming and two outgoing. These curves break down the phase cylinder in four parts. One of these parts shaded in Figure 3 forms the field of effect of the rotating periodic motion, the three others belong to the field of effect of oscillation motion. One of the latter three sections corresponds to the trajectory beginning in the field of reverse motion with negative values of $\dot{\theta}$. The trajectories from the remaining two sections begin in the field of forward rotation and end with periodic oscillations; they differ from each other in that the trajectory from one section (in Figure 3 it corresponds to the broader nonshaded band in the upper part of the drawing) separate only rotation around the phase cylinder in comparison with the other trajectories. /26

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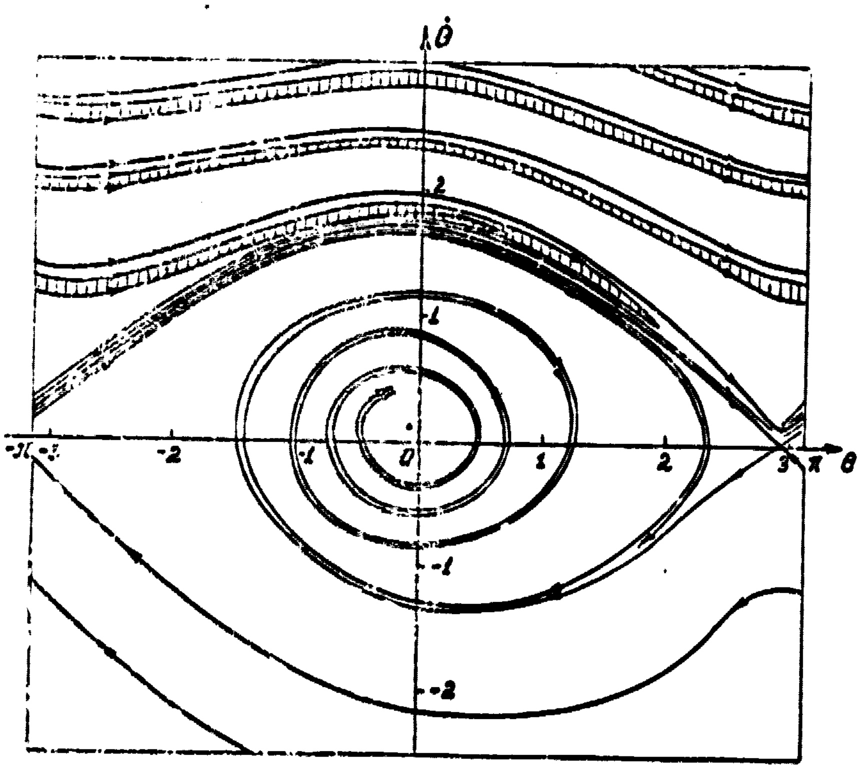

Figure 3. $(a=0.75;\ \varepsilon=0.1)$

From Figure 3 it is apparent that the field of effect of a rotational periodic condition is fairly narrow and is /26 located as a whole in the field of forward rotation when $\dot{\theta}>1$. Therefore, one of the methods to avoid capture of the satellite in rotation consists of imparting to it a certain initial twist in rotation opposite the direction of orbital motion.

When increasing the coefficient of damping ε, the field of effect of the rotational condition is expanded and at certain values of ε the separatrix going away from the point corresponding to unstable oscillation falls at a point corresponding to unstable rotation. After this, the picture of breakdown of phase space in the field of effect changes qualitatively. In the field of effect of the stationary rotational condition there is a band passing into the field of reverse rotation. Thus, for example, when $a=\pi/2$ and $\varepsilon=0.4$, one can find the initial conditions in the field of reverse rotation leading to stationary rotation of the satellite. A further increase in ε results in a larger contraction of the field of effect of the oscillation condition which disappears when $\varepsilon=\varepsilon_{*}(a)$ and rotation along with the magnetic field remains the single type of established motion.

Conclusions /27

In this work, periodic motion of a gravitationally stabilized satellite with a magnetic damper in the plane of a circular polar orbit is studied. In the system considered, besides the small established rotations of the satellite relative to the oriented position, also small (with frequency, not large orbital) undamped rotations are also possible. Then, rotation with the orbital frequency is possible for all physical values of the inertia parameter and for all values of the coefficient of damping. The periodic rotations of other types have a very narrow field of effect and, correspondingly, a low probability of falling into this condition.

With large values of the coefficient of damping due to the strong connection between the satellite and the float of the magnetic damper, a constant magnet which precisely tracks the force lines of the magnetic field, the satellite also begins to rotate along with the float. Therefore, periodic oscillations of the satellite around the oriented position exist only for values of the damping coefficient which do not exceed a certain maximum value depending on the value of the inertia parameter.

In the work, an evaluation is obtained higher than the maximum values of the coefficient of damping for a satellite close to axisymmetric (at low values of the inertia parameter). The results of numerical calculation showed that the value obtained corresponds well to the actuality for all physically possible values of the inertia parameter. From the evaluation it follows that for coefficients of damping $\varepsilon > 0.75$, periodic oscillations of the satellite do not exist nor at such physical values of the inertia parameter as $a (0 \leq a \leq 0.75)$.

At certain values of the parameters a and ε, the equation of motion of a satellite has several asymptotically stable periodic solutions which correspond to different stabilized motions of the satellite with a magnetic damper. Which of these motions is realized depends on the initial conditions. Therefore, the entire plane of initial values of phase variables can break down in the field of effect of different types of periodic motions, that is, into those fields where motion beginning inside one or another field results in a uniform type of periodic motion. In the work, a breakdown of the phase plane in fields of effect for values of the parameter $a = 0.750$, $\varepsilon = 0.1$ is obtained. In this case, the equation of motion has two asymptotically stable 2π-periodic solutions; one is oscillatory and the other is /28 rotational. As is seen from the breakdown, the field of effect of the rotational periodic condition is fairly narrow and is located as a whole within the field of forward rotations. Therefore, one of the methods to avoid capture of the satellite in rotation is to impart to it a certain initial twist in a direction opposite the direction of orbital motion. With an increase in the coefficient of damping ε, the field of effect of the rotational periodic conditions is increased and the oscillatory is decreased and when $\varepsilon=\varepsilon_*(a)$, the field of effect of the oscillation periodic condition disappears.

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