NASA Technical Memorandum 87125

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(NASA-TM-87125) A NUMERICAL METHOD FOR APPROXIMATING ANTENNA SURFACES DEFINED BY DISCRETE SURFACE POINTS (NASA) 10 p HC A02/MF A01 CSCL 20N

N86-10381

Unclas G3/32 27537

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Prepared for the
1985 Antenna Applications Symposium
cosponsored by the University of Illinois
and the Rome Air Development Center
Monticello, Illinois, September 18–20, 1985

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A NUMERICAL METHOD FOR APPROXIMATING ANTENNA SURFACES DEFINED BY DISCRETE SURFACE POINTS

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E-2738

1. INTRODUCTION

Reflector antennas on earth orbiting spacecrafts generally cannot be described analytically. The reflector surface is subjected to large temperature fluctuation and gradients, and is thus warped from its true geometrical shape. Aside from distortion by thermal stresses, reflector surfaces are often purposely shaped to minimize phase aberations and scanning losses. To analyze distorted reflector antennas defined by discrete surface points, a numerical technique must be applied to compute an interpolatory surface passing through a grid of discrete points. Although numerical techniques for analyzing reflector surface distortions and doubly curved reflector surfaces have been reported in open literatures,[1-3] all these techniques are rather complicated and involve lengthy computations. In this paper, a simple numerical technique based on Taylor's expansion and finite differences approximation is presented. By applying the numerical technique to approximate the surface normals of a distorted reflector surface, the aperture field over a near-field plane is obtained using geometric optics technique. The aperture integration method is then applied to find the radiation patterns.[4]

2. FORMULATION

The proposed numerical technique is a quadratic approximation of a distorted surface defined by a grid of nine points. Let (x,y,z) be the coordinate of any point, P, on the surface which is described by

$$z = z(x,y)$$

The position vector of P is given by

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

To obtain an approximation for the surface $z(x,y)$, the position vector is replaced by its Taylor's expansion around $\vec{r}_o(x_o, y_o, z_o)$, the closest point to the point of incidence. That is

$$\vec{r}(x,y,z) = \vec{r}_o(x_o,y_o,z_o) + (x - x_o)\frac{\partial\vec{r}}{\partial x} + (y - y_o)\frac{\partial\vec{r}}{\partial y}$$
$$+ \frac{1}{2}\left[(x - x_o)^2\frac{\partial^2\vec{r}}{\partial x^2} + (y - y_o)^2\frac{\partial^2\vec{r}}{\partial y^2}\right.$$
$$\left. + 2(x - x_o)(y - y_o)\frac{\partial^2\vec{r}}{\partial x\partial y}\right] + O(x^3,y^3)$$

where the partial derivatives of $\vec{r}$ are evaluated at $\vec{r}_o(x_o, y_o, z_o)$. All terms higher than the quadratics of x and y have been neglected as indicated by $O(x^3, y^3)$. The surface may then be approximated by

$$z(x,y) = \hat{z} \cdot \vec{r} = z_o + (x - x_o)\frac{\partial z}{\partial x} + (y - y_o)\frac{\partial z}{\partial y}$$
$$+ \frac{1}{2}\left[(x - x_o)^2 \frac{\partial^2 z}{\partial x^2} + 2(x - x_o)(y - y_o)\frac{\partial^2 z}{\partial x \partial y}\right.$$
$$\left. + (y - y_o)^2 \frac{\partial^2 z}{\partial yz}\right] + O(x^3, y^3)$$

All the partial derivatives of z are evaluated at $\vec{r}_o(x_o, y_o, z_o)$. The normals to the interpolatory surface can be found by taking the gradient of $z(x,y)$ as follows:

$$\hat{n} = \frac{\partial z}{\partial x}\hat{x} + \frac{\partial z}{\partial y}\hat{y} + \hat{z}$$

$$n_x = \frac{\partial z}{\partial x} + (x - x_o)\frac{\partial^2 z}{\partial x^2} + (y - y_o)\frac{\partial^2 z}{\partial x \partial y} + O(x^2, y^2)$$

$$n_y = \frac{\partial z}{\partial y} + (x - x_o)\frac{\partial^2 z}{\partial x \partial y} + (y - y_o)\frac{\partial^2 z}{\partial y^2} + O(x^2, y^2)$$

$$n_z = -1$$

where (n_x, n_y, n_z) are the x, y, and z components of $\hat{n}$ with the normal pointing toward the source. Finite differences can be applied to approximate the partial derivatives. Since the partial derivatives are of second order, a grid of nine points is required.

The partial derivatives may be expressed as

$$\frac{\partial^2 z}{\partial x^2} \sim \frac{\dfrac{(z_4 - z_0)}{(x_4 - x_0)} - \dfrac{(z_0 - z_5)}{(x_0 - x_5)}}{\dfrac{(x_4 - x_5)}{2}}$$

$$\frac{\partial z}{\partial x} \sim \frac{z_4 - z_5}{x_4 - x_5}$$

$$\frac{\partial^2 z}{\partial y^2} \sim \frac{\dfrac{(z_2 - z_0)}{(y_2 - y_0)} - \dfrac{(z_0 - z_7)}{(y_0 - y_7)}}{\dfrac{(y_2 - y_7)}{2}}$$

$$\frac{\partial z}{\partial y} \sim \frac{z_2 - z_7}{y_2 - y_7}$$

$$\frac{\partial^2 z}{\partial x \partial y} \sim \frac{\dfrac{(z_1 - z_3)}{(x_1 - x_3)} - \dfrac{(z_6 - z_8)}{(x_6 - x_8)}}{y_2 - y_7}$$

where (x_1, y_1, z_1), (x_2, y_2, z_2) ... $(x_8, y_8 z_8)$ are the coordinates of the points around the closest point (x_o, y_o, z_o).

3. DISCUSSION AND RESULT

In applying the numerical technique to compute the surface normals, the nine closest points to the point of incidence are located by searching an array of discrete surface points defining the distorted antenna surface. Ray-tracing is then used to compute the reflected field from the approximated normals. The secondary far field patterns are obtained from the Fourier transformation of the aperture field distribution.

To illustrate the numerical technique, the numerical algorithm has been programmed and applied to the antenna configuration shown in figure 1. The surface grid points for an artificially "enlarged" parabolic surface are generated. Thus, the boundary points along the rim of the actual reflector surface can be included in the computation. The computed secondary far field patterns for the symmetric and the offset cases are shown in figures 2 and 3 respectively. In the figures, the E-plane radiation patterns computed from an analytic expression of a parabolic surface are drawn with solid lines, while that computed numerically are drawn with dots and broken lines. The E-plane radiation patterns computed numerically from a grid of 50x50 surface points of ~0.7 λ spacing was found to compare exactly to that computed from an analytic expression. For the offset case, the aperture surface is described by fewer points; therefore, a higher density of target points is required to produce the same accuracy. With a scan step of $\lambda/3$ for the symmetric case and $\lambda/6$ for the offset case, the computer time required to compute the secondary radiation patterns numerically is 25 sec for the symmetric case and 17 sec for the offset case, respectively. For comparison purpose, the radiation pattern for the offset case has been computed using spline function approximation.[5] The computed E-plane pattern is shown in figure 3.

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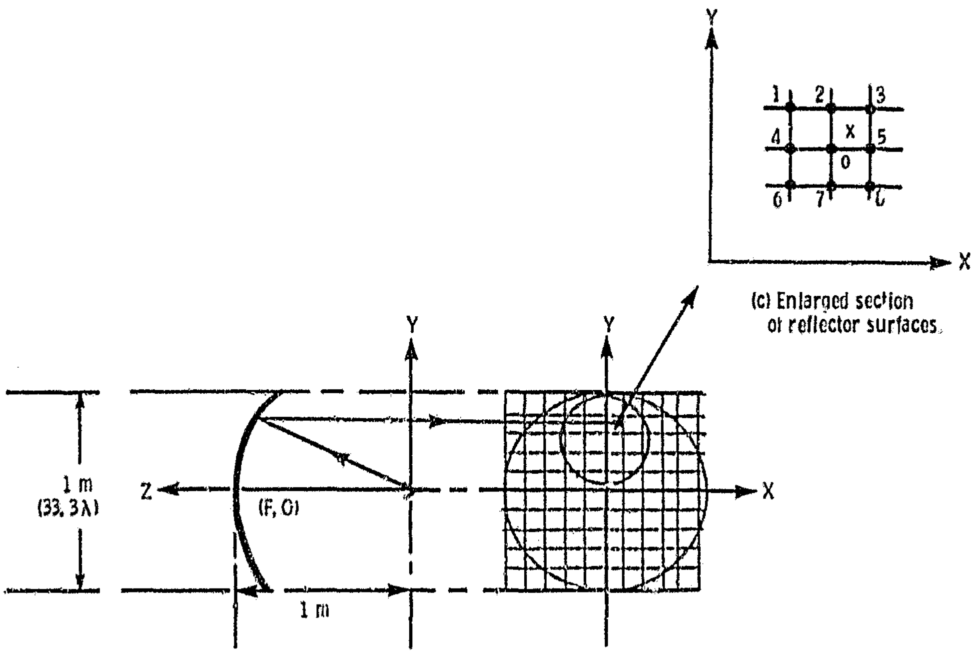

(a) Antenna configuration.

(b) Grid of surface points describing the reflector surface.

Figure 1. - Antenna configuration used in the study.

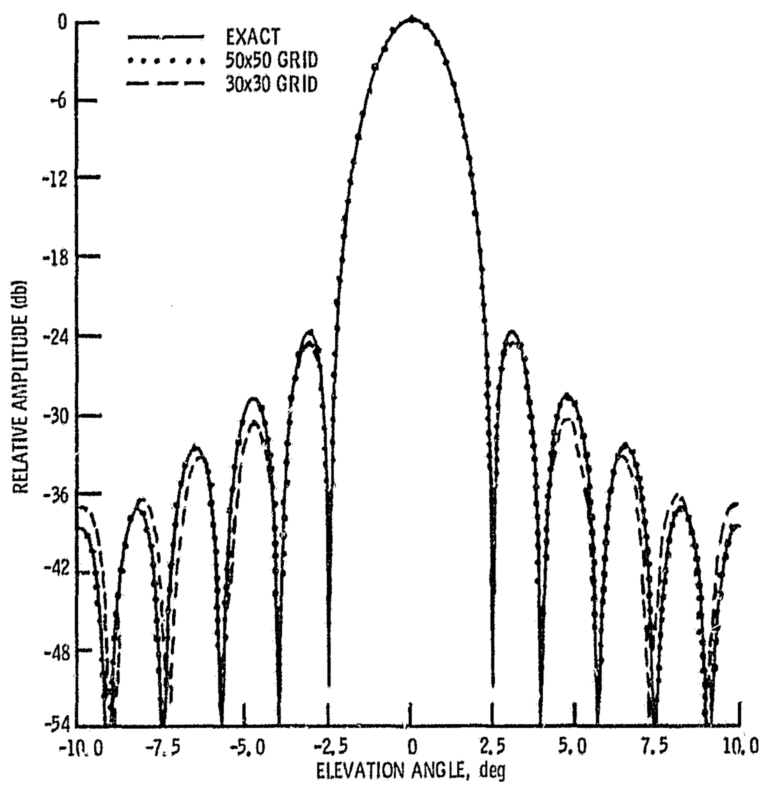

Figure 2. - E - plane pattern for the symmetric case.

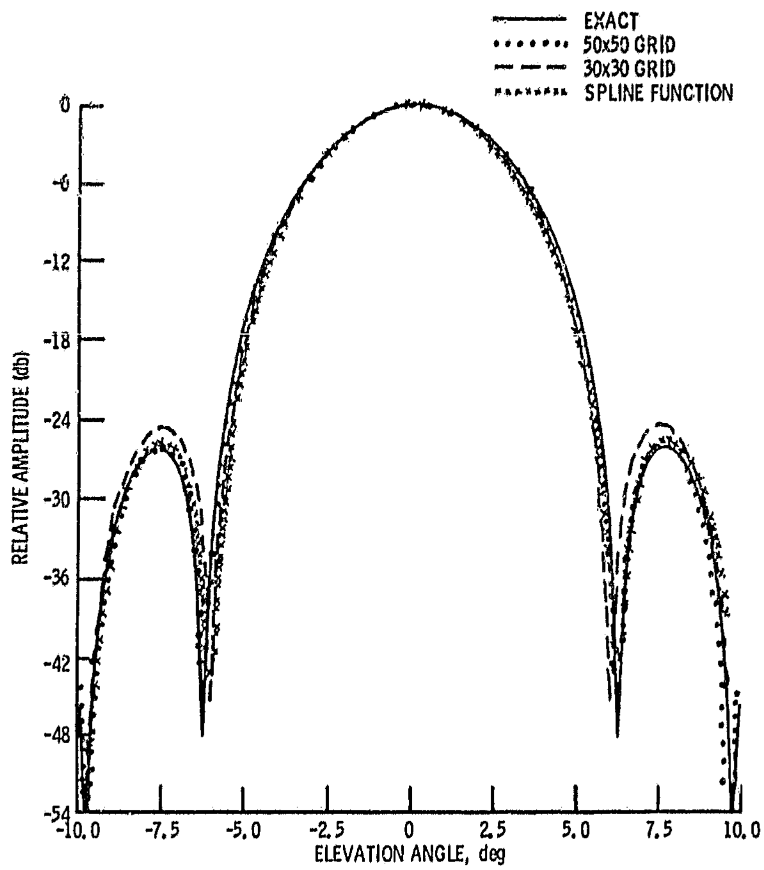

Figure 3. - E - plane pattern for the offset case.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-87125		

4. Title and Subtitle	5. Report Date
A Numerical Method for Approximating Antenna Surfaces Defined by Discrete Surface Points	
	6. Performing Organization Code: 506-58-22

7. Author(s)	8. Performing Organization Report No.
Richard Q. Lee and Roberto Acosta	E-2738
	10. Work Unit No.

9. Performing Organization Name and Address	11. Contract or Grant No.
National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	
	13. Type of Report and Period Covered: Technical Memorandum

12. Sponsoring Agency Name and Address	14. Sponsoring Agency Code
National Aeronautics and Space Administration Washington, D.C. 20546	

15. Supplementary Notes

Prepared for the 1985 Antenna Applications Symposium, cosponsored by the University of Illinois and the Rome Air Development Center, Monticello, Illinois, September 18-20, 1985.

16. Abstract

A simple numerical method for the quadratic approximation of a discretely defined reflector surface is described. The numerical method was applied to interpolate the surface normal of a parabolic reflector surface from a grid of nine closest surface points to the point of incidence. After computing the surface normals, the geometrical optics and the aperture integration method using the discrete Fast Fourier Transform (FFT) were applied to compute the radiaton patterns for a symmetric and an offset antenna configuratons. The computed patterns are compared to that of the analytic case and to the patterns generated from another numerical technique using the spline function approximation. In the paper, examples of computations are given. The accuracy of the numerical method is discussed.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Numerical method Reflector antenna	Unclassified - unlimited STAR Category 32

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		

*For sale by the National Technical Information Service, Springfield, Virginia 22161