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User Guide for Compressible Flow Toolbox Version 2.1 for Use With MATLAB[®] Version 7

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Abstract

This report provides a user guide for the Compressible Flow Toolbox, a collection of algorithms that solve almost 300 linear and nonlinear classical compressible flow relations. The algorithms, implemented in the popular MATLAB[®] programming language, are useful for analysis of one-dimensional steady flow with constant entropy, friction, heat transfer, or shock discontinuities. The solutions do not include any gas dissociative effects. The toolbox also contains functions for comparing and validating the equation-solving algorithms against solutions previously published in the open literature. The classical equations solved by the Compressible Flow Toolbox are: isentropic-flow equations, Fanno flow equations (pertaining to flow of an ideal gas in a pipe with friction), Rayleigh flow equations (pertaining to frictionless flow of an ideal gas, with heat transfer, in a pipe of constant cross section.), normal-shock equations, oblique-shock equations, and Prandtl-Meyer expansion equations. At the time this report was published, the Compressible Flow Toolbox was available without cost from the NASA Software Repository.

1. Introduction

Description

This paper provides a User Guide for the Compressible Flow Toolbox, a collection of algorithms that solve almost 300 linear and nonlinear classical compressible flow relations. The algorithms, implemented in the popular MATLAB[®] programming language, are useful for analysis of one-dimensional steady flow with constant entropy, friction, heat transfer, or shock discontinuities. The solutions do not include any gas dissociative effects. The toolbox also contains functions for comparing and validating the equation-solving algorithms against solutions previously published in the open literature. The classical equations solved by the Compressible Flow Toolbox are:

- The isentropic-flow equations,
- The Fanno flow equations (pertaining to flow of an ideal gas in a pipe with friction),
- The Rayleigh flow equations (pertaining to frictionless flow of an ideal gas, with heat transfer, in a pipe of constant cross section.)
- The normal-shock equations,
- The oblique-shock equations, and
- The Prandtl-Meyer expansion equations.

The user should note that the scope of this guide is limited to documenting the individual functions and providing instruction in using them to solve simple compressible flow examples. Functions in the toolbox can be used together to solve more complex compressible flow problems—that is why they were created. However, instructing the user in the broader context of compressible flow is not the intended purpose of this guide.

Background

Algorithms included in the Compressible Flow Toolbox were originally developed to support controls and dynamics research under the NASA's High Speed Research Program. They were inspired by NACA Report 1135 "Equations Tables, and Charts for Compressible Flow" (ref. 1) which the author studied extensively as part of that research. Early implementations were first published as part of the author's Masters Thesis in 1996. They were subsequently made publicly available via a MATLAB[®] third party software web site hosted by the Mathworks, Inc. After several years, the toolbox was removed from the web site for a variety of reasons, including the need to upgrade the algorithms for compatibility with newer versions of MATLAB[®]. Finally, to appease a number of recent requests for the software, the toolbox has been updated, expanded, and made available to the general public via the NASA Software Repository.

All of the numerical and graphical results shown in this report were generated using functions included in the Compressible Flow Toolbox version 2.1 and running MATLAB[®] version 7.04 on an MS Windows XP, 2.2 GHz Intel Pentium 4 processor-based personal computer. Results may vary slightly based on the precision of the floating point processor used to perform the calculations.

Organization

This User's Guide is organized in five sections. Introduction, Nomenclature, Quick Reference Guide, Function Reference Guide, and References. Section 1. Introduction provides a general description of the User Guide along with historical information on the origin of the toolbox and availability of the software. Section 2. Nomenclature describes the symbols and special formatting conventions used throughout the text. Section 3. Quick Reference Guide provides a comprehensive list of the functions contained in the toolbox and provides a brief description of each function listed. Section 4. Function Reference Guide provides a detailed description of each function in the toolbox including its purpose, syntax, a discussion of how the algorithm works, and examples demonstrating its use. Finally, Section 5. References contains a list of references used in developing and documenting the toolbox.

Availability

At the time this report was published, the Compressible Flow Toolbox was available to the general public without cost through the NASA Software Repository.

https://technology.grc.nasa.gov/software/

2. Nomenclature

Formats and Convensions

Monospace	MATLAB [®] commands, functions names and screen output are displayed in this font. For example: rayleigh.
Italics	Book titles and names of book sections, mathematical symbols and notation, and the introduction of new terms. For example: <i>Introduction</i> .
Bold Initial Caps	Key names, menu names, and items that are selected from menus. For example: the File menu.

Symbols

This document uses the following symbols and notations:

Roman Symbols	Description
A	Cross-sectional area of stream tube or channel
D_H	Hydraulic diameter of the flow cross-sectional area
Ι	Impulse function
M	Mach number, V/a
Р	Static Pressure
P_t	Total Pressure
Т	Static Temperature
T_t	Total Temperature
V	Velocity
$ar{f}$	Average friction factor
q	Dynamic pressure, $\rho V^2/2$

Greek Symbols	Description
β	$\sqrt{ M^2-1 }$
γ	Ratio of specific heats of the working fluid (default = 1.4)
δ	Turning angle (degrees)
θ	Oblique shock angle (degrees)
μ	Mach Angle (degrees)
ν	Prandtl-Meyer angle (degrees)
ρ	Static mass density
ρ	Mass density

Subscripts

Description

*	Critical flow condition (i.e., conditions where the local fluid
	velocity is equal to the local speed of sound)
1	Upstream flow property
2	Downstream flow property

3. Quick Reference Tables

PROPERTIES OF ISENTROPIC FLOW, PRANDTL-MEYER FLOW, AND NORMAL SHOCKS			
ames	Solves the equations for isentropic flow, Prandtl-Meyer flow, and normal shocks to obtain flow properties.		
amesplt	Plots the properties for isentropic flow, Prandtl-Meyer flow, and the normal shocks as a function of Mach number.		
ameserr	Consistency check for function ames. Computes and plots, as a function of Mach number, errors in ames calculations.		
isentbl	Generates text file containing a table of the isentropic flow properties.		
nshktb1	Generates text file containing a table of Prandtl-Meyer flow and normal shock properties.		

PROPERTIES OF OBLIQUE SHOCKS			
oblqshck	Solves the oblique shock equations for both weak and strong shock angles.		
oblqw12	Solves the oblique shock equations to obtain downstream flow properties as a function of upstream flow properties.		
oblqw21	Solves the oblique shock equations to obtain upstream flow properties as a function of downstream flow properties.		
deltason	Computes the theoretical deflection angle that reduces supersonic flow to sonic conditions.		
deltamax	Computes the theoretical maximum angle through which supersonic flow may be deflected or turned without separation.		

PROPERTIES OF FANNO-LINE FLOW			
fanno	Solves the Fanno line equations to obtain properties of flow with friction.		
fannoplt	Plots the Fanno line flow properties as a function of Mach number.		
fannoerr	Consistency check for function fanno. Computes and plots, as a function of Mach number, errors in fanno calculations.		
fannotbl	Generates text file containing a table of the Fanno-line flow properties.		

PROPERTIES OF RAYLEIGH-LINE FLOW			
rayleigh	Solves the Rayleigh-line equations to obtain properties of flow heating or cooling.		
raylplt	Plots the Rayleigh-line flow properties as a function of Mach number.		
raylerr	Consistency check for function rayleigh . Computes and plots, as a function of Mach number, errors in rayleigh calculations.		
rayltbl	Generates text file containing a table of the Rayleigh-line flow properties.		

4. Function Reference Guide

ames

Purpose

Solve the equations for isentropic flow, both subsonic and supersonic, Prandtl-Meyer expansion, and normal shocks.

Synopsis

```
ames
Properties=ames(VarIn,ValuesIn,VarsOut)
Properties=ames(VarIn,ValuesIn,VarsOut,Gamma)
[Properties,PltLbls]=ames(VarIn,ValuesIn,VarsOut,Gamma)
```

Description

ames by itself calls amesplt which plots normalized versions of the isentropic flow, Prandtl-Meyer, and normal shock functions versus Mach number.

Properties=ames(VarIn,ValuesIn,VarsOut), given a number designating one of the flow properties listed in **Table 4.1** and a value or vector of values for that flow property, ames computes corresponding values for isentropic flow, Prandtl-Meyer flow, and normal shock functions. VarIn is a scalar that specifies the property used as the input (independent variable). ValuesIn may be a scalar or vector and contains values of the independent variable for which the other properties will be computed. VarsOut contains a list of Indices corresponding to the flow properties listed in **Table 4.1**. Indices specified in VarsOut may be in any order and may be repeated as desired by the user. Results are returned in the Properties matrix. Columns in this matrix correspond to indices specified in VarsOut. Rows of the Properties matrix contain results corresponding to the elements of ValuesIn.

Note that, when properties 5, 6, or 7 are used as the independent variable, the solution is double-valued. The double-valued solution is provided by making Properties a cell array. Properties{1} contains values of the solution associated with the smaller Mach number, while Properties{2} contains the solution associated with the larger Mach number.

Properties=ames(VarIn, ValuesIn, VarsOut, Gamma) provides a mechanism for specifying values for the ratio of specific heats of the working fluid via Gamma. Gamma is optional. If unspecified, a value of 1.4, the value of the ratio of specific heats of air at standard temperature and pressure, is used. If specified, Gamma may be defined as either a scalar or a vector. If it is a vector, it must have the same length as ValuesIn.

[Properties,PltLbls]=ames(VarIn,ValuesIn,VarsOut,Gamma), in addition to returning the properties of the fluid at user specified conditions, also returns a cell array, PltLbls, containing text strings that may be used when plotting the results.

REF. INDEX	PROPERTY	REF. 1	DESCRIPTION		
ISENTR	ISENTROPIC FLOW PROPERTIES (VALID FOR ALL M):				
1.	$M \text{ or } M_I$		Mach number		
2.	P/P_t	Eq. 44	Ratio of static to total pressure		
3.	ρ / ρ_t	Eq. 45	Ratio of static to total density		
4.	T/T_t	Eq. 43	Ratio of static to total temperature		
5.	β	pg. 1	$\sqrt{M^2 - 1}$		
6.	q/P_t	Eq. 48	Ratio of dynamic to total pressure		
7.	A/A^*	Eq. 80	Ratio of flow area to critical flow area		
8.	V/V^*	Eq. 50	Ratio of flow velocity to critical flow velocity		
PRANDTL-MEYER FLOW (VALID FOR M≥1):					
9.	ν	Eq. 171	Prandtl-Meyer angle (degrees)		
10.	μ	pg. 1	Mach Angle (degrees), $\sin^{-1}(1/M)$		
NORMAL SHOCK PROPERTIES (VALID FOR M≥1):					
11.	M_2	Eq. 96	Mach number downstream of a normal shock		
12.	P_2 / P_1	Eq. 93	Static pressure ratio across a normal shock		
13.	ρ_2 / ρ_1	Eq. 94	Static density ratio across a normal shock		
14.	T_2 / T_1	Eq. 95	Static temperature ratio across a normal shock		
15.	$P_{t,2}/P_{t,1}$	Eq. 99	Total pressure ratio across a normal shock		
16.	$P_{1}/P_{t,2}$	Eq. 100	Ratio of static pressure upstream of a normal shock to total pressure downstream of the same shock		

Table 4.1—Description of Flow Properties Computed by Function ames

Algorithm

ames determines the desired flow properties by first obtaining a Mach number solution for each value, ValuesIn, of the user specified flow property, VarIn. These Mach numbers are then used to compute the other properties, VarSOut, specified by the user. Most of the flow equations may be manipulated analytically to obtain Mach number as a function of the other properties. However, some nonlinear relationships exist which have no simple analytical solution. In these cases, MATLAB's fminbnd function is used determine an approximate solution for Mach number from the nonlinear equations. The search is arbitrarily constrained to Mach numbers less than 100. Solutions associated with Mach numbers larger than 100 are returned as NaN (i.e., not a number).

See Also

ameserr, amesplt, isentbl, and nshcktbl

Example 4.1:

Determine the properties of air at Mach 2.

```
>> ames(1,2,1:16)
ans =
  Columns 1 through 5
    2.0000
              0.1278
                         0.2300
                                   0.5556
                                             1.7321
 Columns 6 through 10
    0.3579
              1.6875
                         1.6330
                                  26.3798
                                            30.0000
  Columns 11 through 15
    0.577
             44.5000
                         2.6667
                                   1.6875
                                             0.7209
 Column 16
    0.1773
```

Example 4.2:

Given a normal shock with downstream Mach number of 0.85, determine the Mach number upstream of the shock.

>> ames(11,0.85,1) ans = 1.1876

Example 4.3:

Determine the properties of air when $A/A_* = 3.007$.

```
>> properties=ames(7,3.007,1:16)
properties =
    [1x16 double]
                     [1x16 double]
>> properties{1}
ans =
 Columns 1 through 5
              0.9733
                         0.9809
                                   0.9923
                                             0.9804
    0.1970
 Columns 6 through 10
    0.0264
              3.0070
                                                NaN
                        0.2149
                                      NaN
 Columns 11 through 15
       NaN
                 NaN
                            NaN
                                      NaN
                                                NaN
  Column 16
       NaN
>> properties{2}
ans =
  Columns 1 through 6
    2.6399
              0.0471
                         0.1128
                                   0.4177
                                             2.4432
 Columns 6 through 10
                        1.8691
                                  42.3049
                                             22.2597
    0.2299
              3.0070
  Columns 11 through 15
    0.5005
              7.9638
                         3.4935
                                   2.2796
                                             0.4453
 Column 16
    0.1058
```

Example 4.4:

Plot the Mach number downstream of a normal shock as a function of the Mach number upstream of the shock.

M1=1:0.1:10; [M2,Lbls]=ames(1,M1,11); plot(M1,M2); xlabel('M_1'); ylabel(Lbls{1});



Figure 4.1.—Result of using function ames to compute Mach number variations across a normal shock.

Purpose

Show the computational errors that result when using function **ames** to solve the equations for isentropic flow, Prandtl-Meyer expansion, and normal shocks.

Synopsis

ameserr
[error,M1]=ameserr

Description

ameserr computes the error between Mach numbers used as inputs to function ames and Mach numbers calculated from the output of function ames. The results are plotted as absolute and percent errors versus Mach number for each of the flow functions shown in **Table 4.1**.

[error,M1]=ameserr returns the computed error in error. If specified, M1 contains the initial vector of Mach numbers.

Algorithm

ameserr first generates a logarithmically spaced vector of 250 Mach numbers from 0.01 to 10. This vector also includes critical Mach number values where numerical stability is important, such as saddle points. ameserr then uses function ames to calculate each of the isentropic flow properties and the normal shock properties corresponding to those Mach numbers. The functions of Mach number, obtained from ames, are then used as input to the ames function in order to obtain a Mach number which corresponds to the function value. Theoretically, the initial and computed Mach numbers should be the same. In general, they are not due to round off, truncation, convergence, and/or optimization errors. The difference in the two Mach numbers is returned as the error in the calculations.

See Also

ames, amesplt, isentbl, and nshcktbl

Example 4.5:

Compute and plot the errors the errors that result from running **ameserr**. Plots are shown in Figure 4.2(a to g).

>> ameserr



(b)

Figure 4.2.—Output of function ameserr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7.





(d)

Figure 4.2.—Output of function ameserr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7 (continued).











Figure 4.2.—Output of function ameserr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7 (continued).

amesplt

Purpose

Plots normalized properties for isentropic flow, Prandtl-Meyer expansion, and normal shocks as a function of Mach number.

Synopsis

amesplt
amesplt(MNmin,MNmax)
amesplt(MNmin,MNmax,Npts)
amesplt(MNmin,MNmax,Npts,Gamma)

Description

amesplt uses function ames to compute and plot the isentropic and normal shock flow properties at 250 points between Mach 0.01 and Mach 10 when the ratio of specific heats of the fluid is 1.4.

amesplt(MNmin,MNmax) plots results for a range of user specified Mach numbers where: MNmin is the minimum Mach number; and MNmax is the maximum Mach number.

amesplt(MNmin,MNmax,Npts) in addition to allowing the user to specify the range of Mach numbers used, this form allows the user to specify the number of data points, Npts, used to plot each curve.

amesplt(MNmin, MNmax, Npts, Gamma) in addition to allowing the user to specify Mach number. and number of points per curve, this form also allows the user to specify a scalar value for the ratio of specific heats, Gamma, of the fluid.

Algorithm

amesplt first generates a logarithmically spaced vector of 250 Mach numbers from 0.01 to 10. This vector also includes critical Mach number values where numerical stability is important, such as solution saddle points. amesplt then uses this vector as inputs to function ames which is used to calculate each of the isentropic flow properties and the normal shock properties corresponding to those Mach numbers. The resulting values are normalized and plotted versus Mach number to provide the user a graphical understanding of the relationship between flow properties and Mach number.

See Also

ames, amesplt, isentbl, and nshcktbl

Example 4.6:

Plot normalized isentropic flow and normal shock properties as a function of Mach number. The resulting plots are shown in Figure 4.3 (a and b).

>> amesplt





Figure 4.3.—Normalized isentropic and normal shock functions as generated by function <code>amesplt</code>.

deltamax

Purpose

For steady state supersonic flow with compressive turning, deltamax computes the maximum flow deflection angle (δ) that can occur without producing separation of the flow from the turning surface. Also, optionally calculates the angle of the oblique shock (θ) that results from turning the flow. Both angles have units of degrees. See Figure 4.4 for a graphical representation of the flow situation.

Synopsis

deltamax
Delta=deltamax(M1)
[Delta,Theta]=deltamax(M1,Gamma)

Description

deltamax by itself, computes and plots the maximum flow deflection and resulting oblique shock angle for a range of Mach numbers from 1.0 to 15.

Delta=deltamax(M1) computes and returns the maximum flow deflection angle, delta, in degrees for user specified Mach numbers, M1. M1 may be a scalar, vector, or matrix.

[Delta, Theta]=deltamax(M1, Gamma) uses optional input Gamma, the ratio of specific heats for the working fluid, to calculate the turning angle Delta and additionally the angle, Theta, of the oblique shock that results from turning the flow. Gamma has a default value of 1.4 and must be a scalar or have dimensions equivalent to M1. The dimensions of Delta and Theta, and the values therein, correspond to the dimensions of M1.

Algorithm

If no input parameters are specified by the user, deltamax first generates a vector of upstream Mach numbers. The function then uses the Mach number(s) to calculate the maximum angle, θ_{max} , of an oblique shock that can occur without separation. The shock angle is then used with the Mach number(s) to calculate the associated flow deflection angle, δ_{max} .



Figure 4.4.—Oblique shock diagram.

The equation used to calculate θ_{max} is:

$$\theta_{\max} = \sin^{-1} \sqrt{\frac{1}{\gamma M_1^2}} \left\{ \frac{(\gamma+1)}{4} M_1^2 + \sqrt{(\gamma+1) \left[1 + \frac{(\gamma-1)}{2} M_1^2 + \frac{(\gamma-1)}{16} M_1^4 \right]} - 1 \right\}$$
(4.1)

The equation used to calculate δ_{max} is:

$$\delta_{\max} = \tan^{-1} \frac{\left(M_1^2 \sin^2 \theta_{\max} - 1\right) \cot \theta_{\max}}{\frac{1}{2} (\gamma + 1) M_1^2 - M_1^2 \sin^2 \theta_{\max} + 1}$$
(4.2)

Similar equations may be found in reference 1, pp. 9 and 12; (ref. 2), p. 586; and (ref. 4), pp. 315 to 316.

See Also

deltason, oblqshck, oblqw12, and oblqw21

Example 4.7:

Calculate and plot the maximum compressive turning angle and oblique shock angle for airflow over a range of Mach numbers from 1 to 15.

>> deltamax



Figure 4.5.—Results of function deltamax showing maximum turning angle and the angle of the resulting oblique shock as a function of upstream Mach number.

Example 4.8:

Calculate the maximum compressive turning angle and oblique shock angle for steam flowing at Mach numbers from 1.5 to 3.0. The ratio of specific heats for steam is 1.327 at standard temperature.

```
>> [Delta,Theta]=deltamax(1.5:0.1:3.0,1.327)
Delta =
  Columns 1 through 5
   12.6726
             15.3598
                       17.8660
                                 20.1780
                                           22.2960
 Columns 6 through 10
   24.2282
             25.9869
                       27.5862
                                 29.0404
                                           30.3634
 Columns 11 through 15
   31.5684
             32.6673
                                           35.4314
                       33.6712
                                 34.5896
  Column 16
   36.2042
Theta =
  Columns 1 through 5
             66.0774
                                 65.3536
                                           65.2072
   66.7820
                       65.6264
 Columns 6 through 10
             65.1587
                                 65.2959
                                           65.4013
   65.1509
                       65.2116
 Columns 11 through 15
   65.5206
             65.6483
                       65.7803
                                 65.9137
                                           66.0466
 Column 16
   66.1772
```

deltason

Purpose

For steady state supersonic flow with compressive turning, deltamax computes the flow deflection angle (δ) that results in sonic flow downstream of the resulting oblique shock (i.e., $M_2 = 1$). Also, optionally calculates the angle of the oblique shock (θ) that produces sonic flow. Both angles have units of degrees. See Figure 4.4 (pp. 19) for a graphical representation of the flow situation.

Synopsis

deltason
Delta=deltason(M1)
[Delta,Theta]=deltason(M1,Gamma)

Description

deltason by itself, computes and plots the sonic flow deflection angle and the resulting oblique shock angle for a range of Mach numbers from 1.0 to 15.

Delta=deltason(M1) computes and returns the flow deflection angle, Delta, that results in sonic flow downstream. Values of Delta are in degrees for user specified Mach numbers, M1. M1 may be a scalar, vector, or matrix.

[Delta, Theta]=deltason(M1, Gamma) uses optional input Gamma, the ratio of specific heats for the working fluid, to calculate the turning angle Delta and additionally the angle, Theta, of the oblique shock that results from turning the flow. Gamma has a default value of 1.4 and must be a scalar or have dimensions equivalent to M1. The dimensions of Delta and Theta, and the values therein, correspond to the dimensions of M1.

Algorithm

If no input parameters are specified by the user, deltason first generates a vector of upstream Mach numbers. The function then uses the Mach number(s) to calculate the angle, θ_* , of an oblique shock that produces sonic flow downstream of the shock. The shock angle is then used with the Mach number(s) to calculate the associated flow deflection angle, δ_* .

The equation used to calculate θ_* is:

$$\theta_* = \sin^{-1} \sqrt{\frac{1}{\gamma M_1^2} \left\{ \frac{(\gamma+1)M_1^2 - (3-\gamma)}{4} + \sqrt{(\gamma+1)\left[\frac{9+\gamma}{16} - \frac{3-\gamma}{8}M_1^2 + \frac{(\gamma+1)}{16}M_1^4\right]} \right\}}$$
(4.1)

The equation used to calculate δ_* is:

$$\delta_* = \tan^{-1} \frac{\left(M_1^2 \sin^2 \theta_* - 1\right) \cot \theta_*}{\frac{1}{2} (\gamma + 1) M_1^2 - M_1^2 \sin^2 \theta_* + 1}$$
(4.2)

Similar equations may be found in reference 1, pp. 9 and 12; (ref. 2), p. 586; and (ref. 4), pp. 315 to 316.

See Also

deltamax, oblqshck, oblqw12, and oblqw21

Example 4.9:

For airflow over a range of Mach numbers from 1 to 15, calculate and plot the compressive turning angle and associated oblique shock angle that results in sonic flow downstream of the shock.

>> deltason



Figure 4.6.—Results of function deltason showing sonic turning angle and the angle of the resulting oblique shock as a function of upstream Mach number.

Example 4.10:

Given a flow of hydrogen gas at a several Mach numbers from 1.0 to 2.0, at each Mach number, calculate the compressive turning angle that produces sonic flow and the associated oblique shock angle. The ratio of specific heats for hydrogen is 1.667 at standard temperature.

```
>> format short e
>> [Delta,Theta]=deltason(1.0:0.1:2.0,1.667)
```

fanno

Purpose

Solve the equations for one-dimensional steady adiabatic flow in a constant area duct with friction.

Synopsis

```
Properties=fanno(VarIn,ValuesIn,VarsOut)
Properties=fanno(VarIn,ValuesIn,VarsOut,Gamma)
[Properties,PltLbls]=fanno(VarIn,ValuesIn,VarsOut,Gamma)
fanno
```

Description

Properties=fanno(VarIn, ValuesIn, VarsOut), given a number designating one of the flow properties listed in **Table 4.2** and a value or vector of values for that flow property, fanno computes corresponding values for adiabatic frictional flow. VarIn is a scalar that specifies the property used as the input (independent variable). ValuesIn may be a scalar or vector and contains values of the independent variable for which the other properties will be computed. VarSOut contains a list of Indices corresponding to the flow properties listed in **Table 4.2**. Indices specified in VarSOut may be in any order and may be repeated as desired by the user. Results are returned in the Properties matrix. Columns in this matrix correspond to indices specified in VarSOut. Rows of the Properties matrix contain results corresponding to the elements of ValuesIn.

Note that, when properties 4, 6, or 7 are used as the independent variable, the solution is double-valued. The double-valued solution is provided by making Properties a cell array. Properties{1} contains values of the solution associated with the smaller Mach number, while Properties{2} contains the solution associated with the larger Mach number.

Properties=fanno(VarIn, ValuesIn, VarsOut, Gamma) provides a mechanism for specifying values for the ratio of specific heats of the working fluid via Gamma. Gamma is optional. If unspecified, a value of 1.4, the value of the ratio of specific heats of air at standard temperature and pressure, is used. If specified, Gamma may be defined as either a scalar or a vector. If it is a vector, it must have the same length as ValuesIn.

[Properties,PltLbls]=fanno(VarIn,ValuesIn,VarSOut,Gamma), in addition to returning the properties of the fluid at user specified conditions, also returns a cell array, PltLbls, containing text strings that may be used when plotting the results.

fanno by itself calls fannoplt which plots the Fanno-line flow properties versus Mach number.

REF. INDEX	PROPERTY	REF. 4	DESCRIPTION	
1.	$M \text{ or } M_l$		Mach number	
2.	T/T_*	Eq. 5.31	Ratio of static temperature at M_1 to static temperature at sonic conditions.	
3.	P/P_*	Eq. 5.30	Ratio of static pressure at M_1 to static pressure at sonic conditions.	
4.	$P_t/P_{t,*}$	Eq. 5.34	Ratio of total pressure at M_1 to total pressure at sonic conditions.	
5.	V/V_*	Eq. 5.29	Ratio of flow velocity at M_I to flow velocity at sonic conditions. Also, ratio of static density at sonic	
	P*/P		conditions to static density at M_1 .	
6.	I/I_*	Eq. 3.42	Ratio of the impulse function at M_I to the impulse function at sonic conditions.	
7.	$4\bar{f}L_*/D_H$	Eq. 5.35	Friction factor	

Table 4.2.—Description of Flow Properties Computed by Function fanno

Algorithm

fanno determines the desired flow properties by first obtaining a Mach number solution for each value, ValuesIn, of the user specified flow property, VarIn. The resulting Mach numbers are then used to compute the other properties, VarsOut. Some of the flow equations may be manipulated analytically to obtain Mach number as a function of the other properties. However, some nonlinear relationships exist which have no simple analytical solution. In these cases, MATLAB's fminbnd function is used determine an approximate solution for Mach number from the nonlinear equations. The search is arbitrarily constrained to Mach numbers less than or equal to 100. Solutions associated with Mach numbers larger than 100 are returned as NaN (i.e., not a number).

See Also

fannoerr, fannoplt, and fannotbl

Example 4.11:

For air flowing at Mach 3.5, determine the Fanno-line flow properties.

```
>> fanno(1,3.5,1:7)
ans =
Columns 1 through 5
3.5000 0.3478 0.1685 6.7896 2.0642
Columns 6 through 7
1.2743 0.5864
```

Example 4.12:

For a range of friction factors from 0.5 to 1.0, determine the static pressure ratio (P/P_*) and upstream Mach number of air flowing adiabatically through a constant area duct.

>> fric=[0.5:0.1:1.0]'; properties=fanno(7,fric,[3,1])

properties =				
[6x2 double]		[6x2 double]		
>> [fric prop	erties{1]	}]		
ans =				
0.5000	1.7706	0.5977		
0.6000	1.8459	0.5748		
0.7000	1.9154	0.5551		
0.8000	1.9804	0.5378		
0.9000	2.0416	0.5225		
1.0000	2.0996	0.5087		
<pre>>> [fric prop</pre>	erties{2	}]		
ans =				
0.5000	0.2359	2.8603		
0.6000	0.1583	3.6302		
0.7000	0.0850	5.1405		
0.8000	0.0148	12.7693		
0.9000	NaN	NaN		
1.0000	NaN	NaN		

Here, properties{1} is the subsonic solution, and properties{2} is the supersonic solution. Also, column 1 of ans contains values for friction factor. Column 2 and 3 contain corresponding solutions for pressure ratio and Mach number, respectively. Note that NaN results imply solution has exceeded internal limit for intermediate Mach number calcualtion (i.e., M > 100).
fannoerr

Purpose

Show the computational errors that result when using function fanno to solve equations for one-dimensional steady adiabatic flow in a constant-area duct with friction.

Synopsis

fannoerr
[error,M1]=fannoerr

Description

fannoerr computes the error between Mach numbers used as inputs to function fanno and Mach numbers calculated from the output of function fanno. The results are plotted as absolute and percent errors versus Mach number for each of the flow functions shown in **Table 4.2**.

[error,M1]=fannoerr returns the computed error in error. If specified, M1 contains the initial vector of Mach numbers.

Algorithm

fannoerr first generates a logarithmically spaced vector of 250 Mach numbers from 0.01 to 10. This vector also includes critical Mach number values where numerical stability is important, such as saddle points. fannoerr then uses function fanno to calculate each of the Fanno-line flow properties corresponding to those Mach numbers. The functions of Mach number, obtained from fanno, are then used as input to the fanno function in order to obtain a Mach number which corresponds to the function value. Theoretically, the initial and computed Mach numbers should be the same. In general, they are not due to round off, truncation, convergence, and/or optimization errors. The difference in the two Mach numbers is returned as the error in the calculations.

See Also

fanno, fannoplt, and fannotbl

Example 4.13:

Compute and plot the errors the errors that result from running fannoerr. Plots are shown in Figure 4.7(a to d)

>> fannoerr



(b)

Figure 4.7.—utput of function fannoerr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7.



Figure 4.7.—utput of function fannoerr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7 (continued).

fannoplt

Purpose

Plot properties for Fanno-line flow, i.e., one-dimensional steady adiabatic flow in a constant-area duct with friction.

Synopsis

fannoplt
fannoplt(MNmin,MNmax)
fannoplt(MNmin,MNmax,Npts)
fannoplt(MNmin,MNmax,Npts,Gamma)

Description

fannoplt uses function fanno to compute and plot the Fanno-line flow properties at 250 points between Mach 0.05 and Mach 2.5 when the ratio of specific heats of the fluid is 1.4. This plot resembles Figure 5.4 in (ref. 4).

fannoplt(MNmin, MNmax) plots results for a range of user specified Mach numbers where: MNmin is the minimum Mach number; and MNmax is the maximum Mach number.

fannoplt(MNmin,MNmax,Npts) in addition to allowing the user to specify the range of Mach numbers used, this form allows the user to specify the number of data points, Npts, used to plot each curve.

fannoplt(MNmin, MNmax, Npts, Gamma) in addition to allowing the user to specify Mach No. and number of points per curve, this form also allows the user to specify a scalar value for the ratio of specific heats, Gamma, of the fluid.

Algorithm

fannoplt first generates a logarithmically spaced vector of 250 Mach numbers from 0.05 to 2.5. This vector also includes critical Mach number values where numerical stability is important, such as solution saddle points. fannoplt then uses this vector as inputs to function fanno which is used to calculate each of the isentropic flow properties and the normal shock properties corresponding to those Mach numbers. The resulting values are plotted versus Mach number to provide the user a graphical understanding of the relationship between flow properties and Mach number.

See Also

fanno, fannoerr, and fannotbl

Example 4.14:

Plot Fanno-line flow properties over a range of Mach numbers from 0.05 to 2.5. The resulting plot is shown in Figure 4.8.

>> fannoplt



Figure 4.8.—Fanno-line flow properties as generated by function fannop1t.

fannotbl

Purpose

Generate a text file containing tables of Fanno-line flow properties. The tables generated by this function may be useful when computational solution of the equations is not practical.

Synopsis

fannotbl
fannotbl(Filename,Mn,Gamma)

Description

fannotbl uses function fanno to generate a table of values for Fanno-line flow properties as a function of Mach numbers from 0.01 to 10. Properties 2 through 7 of **Table 4.2** are written to the text file, fannotbl.txt.

fannotbl(Filename, Mn, Gamma) computes the flow functions and writes the ASCII data to the file specified by the string variable, Filename. Functions are evaluated at Mach numbers specified in Mn. Gamma is an optional scalar variable specifying the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used for Gamma.

See Also

fanno, fannoplt, and fannotbl

Example 4–15:

Create a table containing values for Fanno-line flow functions over a range of Mach numbers from 0.50 to 0.70 in increments of 0.01. Results are shown in **Table 4.3** on the following page.

>> fannotbl('fannotbl.txt.',0.5:0.01:0.7)

Table 4.3.—Output of function fannotb1 for a range of Mach numbers from 0.5 to 0.7

	4fL*/D	1.06906e+000	9.90414e-001	9.17418e-001	8.49624e-001	7.86625e-001	7.28053e-001	6.73571e-001	6.22874e-001	5.75683e-001	5.31743e-001	4.90822e-001	4.52705e-001	4.17197e-001	3.84116e-001	3.53299e-001	3.24591e-001	2.97853e-001	2.72955e-001	2.49775e-001	2.28204e-001	2.08139e-001
operties for Gamma=1.400000	×I/I	1.20268e+000	1.19030e+000	1.17860e+000	1.16753e+000	1.15705e+000	1.14715e+000	1.13777e+000	1.12890e+000	1.12050e+000	1.11256e+000	1.10504e+000	1.09793e+000	1.09120e+000	1.08484e+000	1.07883e+000	1.07314e+000	1.06777e+000	1.06270e+000	1.05792e+000	1.05340e+000	1.04915e+000
Gamma=1.40000	Λ/V^*	5.34522e-001	5.44689e-001	5.54826e-001	5.64934e-001	5.75011e-001	5.85057e-001	5.95072e-001	6.05055e-001	6.15006e-001	6.24925e-001	6.34811e-001	6.44664e-001	6.54483e-001	6.64269e-001	6.74020e-001	6.83737e-001	6.93419e-001	7.03066e-001	7.12677e-001	7.22252e-001	7.31792e-001
roperties for	P0/P0*	1.33984e+000	1.32117e+000	1.30339e+000	1.28645e+000	1.27032e+000	1.25495e+000	1.24029e+000	1.22633e+000	1.21301e+000	1.20031e+000	1.18820e+000	1.17665e+000	1.16565e+000	1.15515e+000	1.14515e+000	1.13562e+000	1.12654e+000	1.11789e+000	1.10965e+000	1.10182e+000	1.09437e+000
no-line Flow Pro	P/P*	2.13809e+000	2.09415e+000	2.05187e+000	2.01116e+000	1.97192e+000	1.93407e+000	1.89755e+000	1.86228e+000	1.82820e+000	1.79525e+000	1.76336e+000	1.73250e+000	1.70261e+000	1.67364e+000	1.64556e+000	1.61831e+000	1.59187e+000	1.56620e+000	1.54126e+000	1.51702e+000	1.49345e+000
Fan	T/T^*	1.14286e+000	1.14066e+000	1.13843e+000	1.13617e+000	1.13387e+000	1.13154e+000	1.12918e+000	1.12678e+000	1.12435e+000	1.12189e+000	1.11940e+000	1.11688e+000	1.11433e+000	1.11175e+000	1.10914e+000	1.10650e+000	1.10383e+000	1.10114e+000	1.09842e+000	1.09567e+000	1.09290e+000
	Μ	5.00000e-001	5.10000e-001	5.20000e-001	5.30000e-001	5.40000e-001	5.50000e-001	5.60000e-001	5.70000e-001	5.80000e-001	5.90000e-001	6.00000e-001	6.10000e-001	6.20000e-001	6.30000e-001	6.40000e-001	6.50000e-001	6.60000e-001	6.70000e-001	6.80000e-001	6.90000e-001	7.00000e-001

isentbl

Purpose

Generate a text file containing tables of isentropic flow properties. The tables generated by this function may be useful when computational solution of the equations is not practical.

Synopsis

isentbl
isentbl(Filename,Mn,Gamma)

Description

isentbl uses function ames to generate a table of values for isentropic flow properties as a function of Mach numbers from 0.01 to 10. Properties 2 through 8 of Table 4.1 are written to the text file, isentbl.txt.

isentbl(Filename, Mn, Gamma) computes the flow functions and writes the ASCII data to the file specified by the string variable, Filename. Functions are evaluated at Mach numbers specified in Mn. Gamma is an optional scalar variable specifying the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used for Gamma.

See Also

ames, fannotbl, nshcktbl, and rayltbl

Example 4–16:

Create a table containing isentropic functions for a range of Mach numbers from 0.9 to 1.1 in increments of 0.01. Results are shown in **Table 4.4** on the following page.

```
>> isentbl('isentbl.txt.',0.9:0.01:1.1);
```

Table 4.4.—Output of function isentb1 for a range of Mach numbers from 0.9 to 1.1

..00831e+000 .01658e+000 l.02481e+000 .03300e+000 l.04114e+000 .05731e+000 l.06533e+000 l.07331e+000 .00000e+000 .04925e+000 1.08124e+000 9.91646e-001 9.23323e-001 9.32007e-001 9.40650e-001 9.49253e-001 9.57814e-001 9.74813e-001 9.83250e-001 9.14598e-001 9.66334e-001 $^{\Lambda/\Lambda}$ 1.00311e+000 1.00560e+000 1.00215e+000 1.00136e+000 1.00076e+000 1.00034e+000 1.00008e+000 1.00000e+000 1.00008e+000 1.00033e+000 1.00074e+000 ..00131e+000 1.00203e+000 .00291e+000 1.00394e+000 L.00645e+000 1.00793e+000 1.00886e+000 1.00713e+000 1.00426e+000 L.00512e+000 A/A* 3.42735e-001 3.39025e-001 3.53439e-001 3.60208e-001 3.63481e-001 3.72840e-001 3.84233e-001 3.91949e-001 3.35244e-001 3.46375e-001 3.49943e-001 3.56861e-001 3.66677e-001 3.69797e-001 3.75806e-001 3.78693e-001 3.81502e-001 3.86884e-001 3.89456e-001 3.94362e-001 3.96696e-001 g/Pt Isentropic Flow Table for Gamma=1.400000 4.35890e-001 4.14608e-001 3.67560e-001 3.12250e-001 2.80000e-001 2.43105e-001 1.41067e-001 0.00000e+000 1.41774e-001 2.85657e-001 3.91918e-001 3.41174e-001 1.98997e-001 2.00998e-001 2.46779e-001 3.20156e-001 3.51568e-001 3.80657e-001 4.07922e-001 4.33705e-001 4.58258e-001 Beta 8.60585e-001 8.57913e-001 8.55227e-001 8.52529e-001 8.47099e-001 8.36106e-001 8.19336e-001 8.49820e-001 8.44366e-001 8.41623e-001 8.38870e-001 8.33333e-001 8.27760e-001 8.24960e-001 8.22152e-001 8.16513e-001 8.13683e-001 8.10846e-001 8.08002e-001 8.05153e-001 8.30551e-001 T/Tt 6.87044e-001 6.81722e-001 6.76400e-001 6.71079e-001 6.39225e-001 5.81696e-001 6.65759e-001 6.60443e-001 6.55130e-001 6.49822e-001 6.44520e-001 6.33938e-001 6.28660e-001 6.23391e-001 6.18133e-001 6.12887e-001 6.07653e-001 6.02432e-001 5.97225e-001 5.92033e-001 5.86856e-001 p/pt 5.91260e-001 5.84858e-001 5.78476e-001 5.72114e-001 5.65775e-001 5.59460e-001 5.53170e-001 5.46905e-001 5.40669e-001 5.34460e-001 5.28282e-001 5.22134e-001 5.16018e-001 5.09935e-001 4.97872e-001 4.91894e-001 4.85952e-001 4.68354e-001 5.03886e-001 4.80047e-001 4.74181e-001 P/Pt .09000e+000 1.00000e+000 1.01000e+000 .02000e+000 .03000e+000 .04000e+000 .05000e+000 .06000e+000 .07000e+000 .08000e+000 1.10000e+000 9.30000e-001 9.50000e-001 9.00000e-001 9.10000e-001 9.20000e-001 9.40000e-001 9.60000e-001 9.70000e-001 9.80000e-001 9.90000e-001 M or M1

nshktbl

Purpose

Generate a text file containing tables of supersonic flow and normal shock properties. The tables generated by this function may be useful when computational solution of the equations is not practical.

Synopsis

nshktbl
nshktbl(Filename,Mn,Gamma)

Description

nshktbl uses function ames to generate a table of values for supersonic flow and normal shock properties as a function of Mach numbers from 1 to 10. Properties 9 through 16 of **Table 4.1** are written to the text file, nshktbl.txt.

nshktbl(Filename,Mn,Gamma) computes the flow functions and writes the ASCII data to the file specified by the string variable, Filename. Functions are evaluated at Mach numbers specified in Mn. Gamma is an optional scalar variable specifying the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used for Gamma.

See Also

ames, isentbl, fannotbl, and rayltbl

Example 4–17:

Create a table containing supersonic flow and normal shock functions for a range of Mach numbers from 1.0 to 2.5 in increments of 0.1. Results are shown in **Table 4.5** on the following page.

```
>> nshktbl('nshktbl.txt.',1.0:0.1:2.5);
```

Table 4.5.—Output of function nshktb1 for a range of Mach numbers from 1.0 to 2.5

5.28282e-001 4.68857e-001 4.15368e-001 3.68515e-001 2.92974e-001 2.62814e-001 2.36752e-001 2.14155e-001 1.94485e-001 .77291e-001 1.62196e-001 .48888e-001 1.37105e-001 1.26632e-001 .17286e-001 1.08917e-001 .01395e-001 9.46129e-002 8.84780e-002 8.29121e-002 3.27951e-001 P1/PT2 1.00000e+000 8.12684e-001 7.20874e-001 9.92798e-001 9.79374e-001 9.58194e-001 8.95200e-001 8.55721e-001 7.67357e-001 6.74203e-001 6.28136e-001 5.83295e-001 5.40144e-001 4.99015e-001 4.60123e-001 4.23590e-001 3.89464e-001 3.28344e-001 9.98928e-001 9.29787e-001 3.57733e-001 PT2/PT1 1.00000e+000 1.06494e+000 1.12799e+000 1.19087e+000 1.25469e+000 1.32022e+000 1.38797e+000 1.45833e+000 1.53158e+000 1.60792e+000 1.68750e+000 1.77045e+000 1.85686e+000 1.94680e+000 2.04033e+000 2.13750e+000 2.23834e+000 2.34289e+000 2.45117e+000 2.56321e+000 2.67901e+000 T2/T1 Gamma=1.400000 2.35922e+000 3.33333e+000 1.00000e+000 1.16908e+000 1.34161e+000 1.51570e+000 1.68966e+000 1.86207e+000 2.03175e+000 .19772e+000 2.51568e+000 2.66667e+000 2.81190e+000 2.95122e+000 3.08455e+000 3.21190e+000 3.44898e+000 .55899e+000 3.66355e+000 3.76286e+000 3.85714e+000 p2/p1 Supersonic Flow & Normal Shock Properties for 1.00000e+000 1.24500e+000 1.51333e+000 1.80500e+000 2.12000e+000 2.45833e+000 2.82000e+000 3.20500e+000 3.61333e+000 4.04500e+000 4.50000e+000 4.97833e+000 5.48000e+000 6.00500e+000 6.55333e+000 7.12500e+000 7.72000e+000 8.33833e+000 8.98000e+000 9.64500e+000 1.03333e+001 P2/P1 1.00000e+000 9.11770e-001 8.42170e-001 7.85957e-001 7.39709e-001 7.01089e-001 6.68437e-001 6.16501e-001 5.95616e-001 5.77350e-001 5.61277e-001 5.47056e-001 5.34411e-001 5.23118e-001 5.03871e-001 4.95634e-001 4.75191e-001 6.40544e-001 5.12989e-001 4.88167e-001 4.81380e-001 MZ 9.00000e+001 5.02849e+001 4.55847e+001 3.00000e+001 2.84369e+001 2.46243e+001 2.17385e+001 6.53800e+001 5.64427e+001 4.18103e+001 3.86822e+001 3.60319e+001 3.37490e+001 3.17569e+001 2.70357e+001 2.57715e+001 2.35782e+001 2.26199e+001 2.09248e+001 2.01713e+001 1.94712e+001 ШM 0.00000e+000 8.98702e+000 1.33620e+000 3.55823e+000 6.17029e+000 1.19052e+001 1.48604e+001 1.78099e+001 .07251e+001 2.35861e+001 2.63798e+001 2.90971e+001 3.17325e+001 3.42828e+001 3.67465e+001 3.91236e+001 4.14147e+001 4.36215e+001 4.57459e+001 4.77903e+001 4.97573e+001 ΠN 1.30000e+000 L.40000e+000 L.50000e+000 2.70000e+000 2.80000e+000 2.90000e+000 3.00000e+000 L.00000e+000 1.10000e+000 1.20000e+000 L.60000e+000 1.70000e+000 1.80000e+000 1.90000e+000 2.00000e+000 2.10000e+000 2.20000e+000 2.30000e+000 2.40000e+000 2.50000e+000 2.60000e+000 Σ

oblqshck

Purpose

For steady state supersonic flow, oblqshck computes the angle of the oblique shock that results from compressively turning the flow through angle (δ). One of two solutions will occur for each Mach number specified, a weak shock solution, or a strong shock solution. Angles have units of degrees. The flow situation is similar to that depicted in Figure 4.4 (pp. 19).

Synopsis

```
oblqshck
ThetaW=oblqshck(M1,delta)
[ThetaW,ThetaS]=oblqshck(M1,delta,gamma)
```

Description

oblqshck by itself generates a plot showing shock angle vs. deflection angle for lines of constant Mach from 1 to 20. This plot is a representation of Chart 2 in (ref. 1).

[ThetaW, ThetaS]=oblqshck(M1, delta) computes both the weak oblique shock angle, ThetaW, and the strong oblique shock angle, ThetaS, that are the result of compressively turning supersonic flow at Mach number, M1, through angle delta. M1 and delta may be either scalars or vectors. If both are vectors, they must have identical dimensions.

[ThetaW, ThetaS, DELmax, DELson]=oblqshck(M1, delta, gamma) uses optional scalar input Gamma to specify the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used for Gamma. In addition to returning the oblique shock angles, this form also returns the maximum flow deflection angle, DELmax, and the flow deflection angle that results in sonic flow downstream of the oblique shock, DELson.

Algorithm

oblqshck uses the upstream Mach number, the flow deflection angle, and the ratio of specific heats to calculate the solution of the cubic equation for both weak and strong oblique shock angles using the method given in (ref. 5). If flow deflection angles are specified as outputs, oblqshck uses functions deltamax and deltason to compute values for those parameters at Mach number(s), M1.

See Also

deltason, deltamax, oblqw12, and oblqw21

Example 4–18:

Replicate the results in Chart 2 of (ref. 1).

>> oblqshck



Figure 4.9.—Oblique shock angle versus deflection angle for lines of constant Mach number.

Example 4-19:

Given freestream airflow at Mach 2.2, calculate the shock angle that results from turning the flow through a range of deflection angles from zero to the maximum deflection possible without separating the flow.

```
>> M1=2.2, DELmax=deltamax(M)
M1 =
    2.2000
DELmax =
   26.1028
>> delta=DELmax*[0:10]'/10; M1=M1*ones(size(delta));
>> ThetaW=oblgshck(M1,delta); [delta ThetaW]
ans =
         0
             27.0357
    2.6103
             29.0843
             31.2899
    5.2206
    7.8308
             33.6666
   10.4411
             36.2338
   13.0514
             39.0200
   15.6617
             42.0707
   18.2719
             45.4660
   20.8822
             49.3676
   23.4925
             54.2056
   26.1028
             64.6203
```

oblqw12

Purpose

For steady state supersonic flow, oblqw12 uses the upstream fluid properties to compute properties of the flow downstream of a weak oblique shock. The flow situation is similar to that depicted in Figure 4.4 (pp. 19).

Synopsis

```
oblqw12
M2=oblqw12(M1,delta)
[M2,Theta,PTratio]=oblqw12(M1,delta,gamma)
```

Description

oblqw12 by itself generates a series of plots showing properties of oblique shocks for lines of constant Mach from 1 to 20. Figure 4.9(a and b) replicates the weak shock portions of Charts 2 and 4 in (ref. 1). Figure 4.9(c) shows variations in total pressure across and oblique shock as a function shock angle. In these figures, the solid lines represent lines of constant Mach number.

M2=oblqw12(M1, delta) computes the Mach number, M2, downstream of an oblique shock that results from compressively turning supersonic flow at Mach number, M1, through angle delta. M1 and delta may be either scalars or vectors. If both are vectors, they must have identical dimensions.

[M2, Theta, PTratio]=oblqw12(M1, delta, gamma) uses optional scalar input Gamma to specify the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used. In addition to returning the downstream Mach number, this form also returns the resulting oblique shock angle as Theta, and the total pressure ratio across the shock, $P_{t,2}/P_{t,1}$, as PTratio.

Algorithm

Oblqw12 uses the upstream Mach number, the flow deflection angle, and the ratio of specific heats to calculate the solution of the cubic equation for both weak and strong oblique shock angles using the method given in reference 5. It then uses equation 131 and 142 from reference 1—alternatively, equation 7.31 and 7.25 from reference 4—to calculate the downstream Mach number and total pressure ratio across the oblique shock.

See Also

deltason, deltamax, oblqshck, and oblqw21

Example 4–20:

Replicate the results in Chart 2 and 4 of reference 1. Results are shown in Figure 4.10.

>> oblqshck



Figure 4.10.—Plots generated by function oblqw12.

Example 4–21:

Given flow deflection of freestream airflow at Mach 2.2, calculate the shock angle that results from turning the flow through a range of deflection angles from zero to the maximum deflection possible without separating the flow.

```
>> M1=2.2, DELmax=deltamax(M)
M1 =
    2.2000
DELmax =
   26.1028
>> delta=DELmax*[0:10]'/10; M1=M1*ones(size(delta));
>> ThetaW=oblqshck(M1,delta); [delta ThetaW]
ans =
         0
             27.0357
             29.0843
    2.6103
   5.2206
             31.2899
    7.8308
             33.6666
             36.2338
   10.4411
   13.0514
             39.0200
   15.6617
             42.0707
   18.2719
             45.4660
   20.8822
             49.3676
   23.4925
             54.2056
   26.1028
             64.6203
```

Purpose

For steady state supersonic flow, oblqw21 uses the downstream fluid properties to compute properties of the flow upstream of a weak oblique shock. The flow situation is similar to that depicted in Figure 4.4 (pp. 19).

Synopsis

oblqw21 M1=oblqw21(M2,delta) [M1,Theta,PTratio]=oblqw21(M2,delta,gamma)

Description

oblqw21 generates a consistency check for the oblique shock equations used by functions oblqshck, oblqw12, and oblqw21. The absolute error is plotted as a function of flow deflection angle for lines of constant Mach number.

M1=oblqw21(M2,delta) computes the Mach number, M1, upstream of an oblique shock that is required to achieve downstream Mach number, M2, after compressively turning the flow through angle delta. M2 and delta may be either scalars or vectors. If both are vectors, they must have identical dimensions.

[M1, Theta, PTratio]=oblqw21(M2, delta, gamma) uses optional scalar input Gamma to specify the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used. In addition to returning the downstream Mach number, this form also returns the resulting oblique shock angle as Theta, and the total pressure ratio across the shock, $P_{\rm t,2}/P_{\rm t,1}$, as PTratio.

Algorithm

oblqw21 uses MATLAB's fminbnd function to find a solution to the nonlinear equation for downstream Mach number as a function of upstream Mach number, the flow deflection angle, and the ratio of specific heats. fminbnd solves the nonlinear equation by employing an inline function to compute the error between the user specified Mach number, M_2 , and a downstream Mach number, $M_{2,Guess}$, that is calculated using function oblqw12 and a guess for the upstream Mach number, $M_{1,Guess}$. The search is arbitrarily constrained to Mach numbers less than or equal to 100. Solutions associated with Mach numbers larger than 100 are returned as NaN (i.e., not a number).

See Also

deltason, deltamax, oblqshck, and oblqw12

Example 4–22:

Generate the consistency check for the oblique shock functions.

>> oblw21



Figure 4.11.—Consistency check for oblique shock functions oblqshck, oblqw12, and oblqw21.

Example 4-23:

Given the values for downstream Mach number, M2, shown below, calculate the upstream Mach number required to produce M2 when the flow deflection angle is 0.5 degrees. Also calculate the oblique shock angle and the total pressure ratio across the oblique shock.

```
>> M2=[ 0.9000 1.0000 1.9819 2.9742 3.9624 ...
61.9798 69.5625 76.8279 83.7768 90.4113];
>> delta=0.5;
>> [M1,Theta,PTratio]=oblqw21(M2,delta);
Warning: FANNO: Some Mach number solutions exceed internal limit (M
> 100). Solutions set to NaN
> In oblqw21 at 119
```

>>	[M2(:) M	1(:) Theta	n(:) PTrat ⁻	io(:)]						
ans =										
	0.9000	NaN	NaN	NaN						
	1.0000	1.0490	77.7993	1.0000						
	1.9819	2.0000	30.4029	1.0000						
	2.9742	3.0000	19.8116	1.0000						
	3.9624	4.0000	14.8009	1.0000						
	61.9798	70.0000	1.1719	0.9500						
	69.5625	80.0001	1.0766	0.9288						
	76.8279	90.0000	1.0038	0.9037						
	83.7768	99.9999	0.9468	0.8750						
	90.4113	NaN	NaN	NaN						

Note that a weak oblique shock solution does not exist for the case where M2 = 0.9. Therefore, the associated solutions are defined as NaN. Also, because the search space is limited to Mach numbers less than or equal to 100, a solution for M2 = 90.4113, M1 = 110, is not computed by oblqw21.

rayleigh

Purpose

Solve the equations for one-dimensional steady flow in a constant area duct with heat transfer.

Synopsis

```
Properties=rayleigh(VarIn,ValuesIn,VarsOut)
Properties=rayleigh(VarIn,ValuesIn,VarsOut,Gamma)
[Properties,PltLbls]=rayleigh(VarIn,ValuesIn,VarsOut,Gamma)
rayleigh
```

Description

Properties=rayleigh(VarIn, ValuesIn, VarsOut), given a number designating one of the flow properties listed in **Table 4.6** and a value or vector of values for that flow property, rayleigh computes corresponding values for flow though a constant area duct with heat transfer. VarIn is a scalar that specifies the property used as the input (independent variable). ValuesIn may be a scalar or vector and contains values of the independent variable for which the other properties will be computed. VarsOut contains a list of indices corresponding to the flow properties listed in **Table 4.6**. Indices specified in VarsOut may be in any order and may be repeated as desired by the user. Results are returned in the Properties matrix. Columns in this matrix correspond to indices specified in VarsOut. Rows of the Properties matrix contain results corresponding to the elements of ValuesIn.

Note that, when properties 2, 3, or 5 are used as the independent variable, the solution is double-valued. The double-valued solution is provided by making Properties a cell array. Properties{1} contains values of the solution associated with the smaller Mach number, while Properties{2} contains the solution associated with the larger Mach number.

Properties=rayleigh(VarIn, ValuesIn, VarsOut, Gamma) provides a mechanism for specifying values for the ratio of specific heats of the working fluid via Gamma. Gamma is optional. If unspecified, a value of 1.4, the value of the ratio of specific heats of air at standard temperature and pressure, is used. If specified, Gamma may be defined as either a scalar or a vector. If it is a vector, it must have the same length as ValuesIn.

[Properties,PltLbls]=rayleigh(VarIn,ValuesIn,VarSOut,Gamma), in addition to returning the properties of the fluid at user specified conditions, also returns a cell array, PltLbls, containing text strings that may be used when plotting the results.

rayleigh by itself calls raylplt which plots the Rayleigh-line flow properties versus Mach number. The resulting plot is similar to figure 6.5 in reference 4.

REF. INDEX	PROPERTY	REF. 4 [*] EQN.	DESCRIPTION
1.	$M \text{ or } M_l$		Mach number
2.	$T_t/T_{t,*}$	Eq. 6.22	Ratio of total temperature at M_1 to total temperature at sonic conditions.
3.	T/T_*	Eq. 6.19	Ratio of static temperature at M_1 to static temperature at sonic conditions.
4.	P/P_*	Eq. 6.20	Ratio of static pressure at M_1 to static pressure at sonic conditions.
5.	$P_t/P_{t,*}$	Eq. 6.23	Ratio of total pressure at M_1 to total pressure at sonic conditions.
6.	V/V_*	Eq. 6.21	Ratio of flow velocity at M_1 to flow velocity at sonic conditions.

Table 4.6.—Description of Flow Properties Computed by Function rayleigh

* Similar equations may also be found in reference 2 on p. 196.

Algorithm

rayleigh determines the desired flow properties by first obtaining a Mach number solution for each value, ValuesIn, of the user specified flow property, VarIn. The resulting Mach numbers are then used to compute the other properties, VarSOut. Most of the flow equations may be manipulated analytically to obtain Mach number as a function of the other properties. However, in the case of total pressure ratio, a nonlinear relationships exists which has no simple analytical solution. In this case, MATLAB's fminbnd function is used determine an approximate solution for Mach number from the nonlinear equation. The search is arbitrarily constrained to Mach numbers less than or equal to 100. Solutions associated with Mach numbers larger than 100 are returned as NaN (i.e., not a number).

See Also

raylerr, raylplt, and rayltbl

Example 4.24:

For air flowing at Mach 0.72 and 2.85, determine the Rayleigh-line flow properties.

>> rayleigh(1,[0.72 2.85],2:6)											
ans =	ans =										
0.9221	1.0026	1.3907	1.0376	0.7209							
0.6685	0.3057	0.1940	3.0014	1.5757							

Example 4.25:

Given sonic conditions at a point in a constant area duct with total temperature of 1000 K and total pressure of 300 kPa, find the Mach number and total pressure as the total temperature decreases to 800, 600, and 400 K.

First, divide duct total temperature by sonic total temperature to obtain the ratio of total temperatures.

>> TTratio = [900 750 600]/1000
TTratio =
0.9000 0.7500 0.6000

Next, use function rayleigh to compute the Mach number (ref. index 1) and total pressure ratio (ref. index 5) for the speficied total temperature ratios.

```
>> Properties=rayleigh(2,TTratio,[1 5])
Properties =
    [3x2 double] [3x2 double]
```

The solution is double-valued. The subsonic solution is returned as Properties{1}, and the supersonic solution is returned as Properties{2}. Mach number values are returned in column one of each of the solutions. The subsonic and supersonic solutions for Mach number are shown below in columns one and two, respectively. Here, the rows correspond to the elements of TTratio, with the first row corresponding to the element one.

Values for total pressure ratio are returned in column two of each of the cells in **Properties**. The subsonic and supersonic solutions are shown below in columns one and two, respectively. Here, the rows correspond to the elements of **TTratio**, with the first row corresponding to the element one. The total pressure ratios are multiplied by the pressure ratio at sonic conditions (i.e., 300 kPa) to obtain total pressure at the specified total temperature conditions.

```
>> PT=[Properties{2}(:,2); Properties{2}(:,2)]*300
PT =
    1.0e+003 *
    0.3139    0.3421
    0.3291    0.5379
    0.3416    2.0328
```

Note that NaN results would imply that the intermediate solution for Mach number exceeded the internal limit (i.e., M > 100).

Purpose

Show the computational errors that result when using function rayleigh to solve equations for one-dimensional steady adiabatic flow in a constant-area duct with friction.

Synopsis

raylerr [error,M1]=raylerr

Description

raylerr computes the error between Mach numbers used as inputs to function rayleigh and Mach numbers calculated from the output of function rayleigh. The results are plotted as absolute and percent errors versus Mach number for each of the flow functions shown in **Table 4.6**.

[error,M1]=raylerr returns the computed error in error. If specified, M1 contains the initial vector of Mach numbers.

Algorithm

raylerr first generates a logarithmically spaced vector of 250 Mach numbers from 0.01 to 10. This vector also includes critical Mach number values where numerical stability is important, such as saddle points. raylerr then uses function rayleigh to calculate each of the Rayleigh-line flow properties corresponding to those Mach numbers. The functions of Mach number, obtained from rayleigh, are then used as input to the rayleigh function in order to obtain a Mach number which corresponds to the function value. Theoretically, the initial and computed Mach numbers should be the same. In general, they are not due to round off, truncation, convergence, and/or optimization errors. The difference in the two Mach numbers is returned as the error in the calculations.

See Also

rayleigh, raylplt, and rayltbl

Example 4.26:

Perform a consistency check on the calculations in function rayleigh. Plots are shown in Figure 4.12(a to c).

>> raylerr



Figure 4.12.—Output of function raylerr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7.



Figure 4.12.—Output of function raylerr as computed on an Intel Pentium4 processor-based computer running MATLAB[®] 7 (continued).

raylplt

Purpose

Plot properties for Rayleigh-line flow, i.e., one-dimensional steady adiabatic flow in a constant-area duct with friction.

Synopsis

raylplt
raylplt(MNmin,MNmax)
raylplt(MNmin,MNmax,Npts)
raylplt(MNmin,MNmax,Npts,Gamma)

Description

raylplt uses function rayleigh to compute and plot the Rayleigh-line flow properties at 250 points between Mach 0.05 and Mach 2.5 when the ratio of specific heats of the fluid is 1.4. This plot resembles figure 6.5 in reference 4.

raylplt(MNmin,MNmax) plots results for a range of user specified Mach numbers where: MNmin is the minimum Mach number; and MNmax is the maximum Mach number.

raylplt(MNmin,MNmax,Npts) in addition to allowing the user to specify the range of Mach numbers used, this form allows the user to specify the number of data points, Npts, used to plot each curve.

raylplt(MNmin, MNmax, Npts, Gamma) in addition to allowing the user to specify Mach No. and number of points per curve, this form also allows the user to specify a scalar value for the ratio of specific heats, Gamma, of the fluid.

Algorithm

raylplt first generates a logarithmically spaced vector of 250 Mach numbers from 0.05 to 2.5. This vector also includes critical Mach number values where numerical stability is important, such as solution saddle points. raylplt then uses this vector as inputs to function rayleigh which is used to calculate each of the isentropic flow properties and the normal shock properties corresponding to those Mach numbers. The resulting values are plotted versus Mach number to provide the user a graphical understanding of the relationship between flow properties and Mach number.

See Also

rayleigh, raylerr, and rayltbl

Example 4.27:

Plot Rayleigh-line flow properties over a range of Mach numbers from 0.5 to 2.5. The resulting plot is shown in Figure 4.13.

>> raylplt



Figure 4.13.—Rayleigh-line flow properties as generated by function raylplt.

Purpose

Generate a text file containing tables of Rayleigh-line flow properties. The tables generated by this function may be useful when computational solution of the equations is not practical.

Synopsis

rayltbl
rayltbl(Filename,Mn,Gamma)

Description

rayltbl uses function rayleigh to generate a table of values for Rayleigh-line flow properties as a function of Mach numbers from 0.01 to 10. Properties 2 through 6 of **Table 4.2** are written to the text file, rayltbl.txt.

rayltbl(Filename,Mn,Gamma) computes the flow functions and writes the ASCII data to the file specified by the string variable, Filename. Functions are evaluated at Mach numbers specified in Mn. Gamma is an optional scalar variable specifying the ratio of specific heats of the working fluid. If unspecified, a value of 1.4 is used for Gamma.

See Also

rayleigh, raylplt, and rayltbl

Example 4–28:

Create a table containing values for Rayleigh-line flow functions over a range of Mach numbers from 0.55 to 059 in increments of 0.01, and from 2.0 to 2.4 in increments of 0.1. Results are shown in **Table 4.7** on the following page.

>> M1=[0.55:0.01:0.59 2.0:0.1:2.4];

>> rayltbl('rayltbl.txt.',M1)

Table 4.7.—Output of function ray1tb1 for a range of Mach numbers specified in Example 4.28

5. References

- 1. Ames Research Staff: "Equations Tables and Charts for Compressible Flow," NACA Report 1135, 1953.
- Melcher, Kevin J.: "A Method for Perturbing the Flow Field in Supersonic Wind Tunnels for Dynamic Analysis of High Speed Inlets," Masters Thesis, Cleveland State University, June 1996.
- 3. Shapiro, A.: *The Dynamics and Thermodynamics of Compressible Fluid Flow*, John Wiley & Sons, Inc., 1953.
- 4. Saad, M.: Compressible Fluid Flow, Prentice-Hall., 1993.
- 5. Hartley, T.T., et al.: "Exact and Approximate Solutions to the Oblique Shock Equations for Real-Time Applications," NASA CR-187173, August 1991.

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This report provides a user	guide for the Compressible Flow	v Toolbox a collection of al	porithms that solve almost						
300 linear and nonlinear cl	assical compressible flow relation	ns The algorithms impleme	ented in the popular MATLAB [®]						
programming language, and	e useful for analysis of one-dime	nsional steady flow with con	stant entropy, friction, heat						
transfer. or shock discontir	nuities. The solutions do not inclu	ide any gas dissociative effect	cts. The toolbox also contains						
functions for comparing ar	nd validating the equation-solving	g algorithms against solution	s previously published in the						
open literature. The classic	al equations solved by the Comp	ressible Flow Toolbox are: is	sentropic-flow equations, Fanno						
flow equations (pertaining	to flow of an ideal gas in a pipe	with friction), Rayleigh flow	equations (pertaining to friction-						
less flow of an ideal gas, w	with heat transfer, in a pipe of con	stant cross section.), normal	-shock equations, oblique-shock						
equations, and Prandtl-Meyer expansion equations. At the time this report was published, the Compressible Flow									
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