A Rainfall-Runoff Simulation Model for Estimation of Flood Peaks for Small Drainage Basins

GEOLOGICAL SURVEY PROFESSIONAL PAPER 506-B

A Rainfall-Runoff Simulation Model for Estimation of Flood Peaks for Small Drainage Basins

By DAVID R. DAWDY, ROBERT W. LICHTY, *and* JAMES M. BERGMANN

SYNTHESIS IN HYDROLOGY

GEOLOGICAL SURVEY PROFESSIONAL PAPER 506-B

A parametric rainfall-runoff simulation model is used with rainfall data and daily potential evaporation data to predict flood volume and peak rates of runoff for small drainage areas

---

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1972

UNITED STATES DEPARTMENT OF THE INTERIOR

ROGERS C. B. MORTON, *Secretary*

GEOLOGICAL SURVEY

V. E. McKelvey, *Director*

Library of Congress catalog-c. rd No. 78-185756

First printing 1972
Second printing 1972 (with minor revisions)

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price 40 cents (paper cover)

Stock Number 2401-1226

CONTENTS

ILLUSTRATIONS

TABLES

A RAINFALL-RUNOFF SIMULATION MODEL FOR ESTIMATION OF FLOOD PEAKS FOR SMALL DRAINAGE BASINS

By DAVID R. DAWDY, ROBERT W. LICHTY, and JAMES M. BERGMANN

ABSTRACT

A parametric rainfall-runoff simulation model is used with data from a point rainfall gage and data on daily potential evapotranspiration to predict flood volume and peak rates of runoff for small drainage areas. The model is based on bulk-parameter approximations to the physical laws governing infiltration, soil-moisture accretion and depletion, and surface streamflow. Three case studies are presented in which an objective fitting method is used for determining optimal best-fit sets of parameter values for the data available for use in predicting flood peaks. Errors of prediction result both from errors in rainfall input and from lack of model equivalence to the physical prototype. These two sources of error seem to be of the same order of magnitude for a model of the level of simplicity of that presented. Major gains in accuracy of simulation will require improvements in both data and model. The limit of accuracy of prediction of flood peaks by simulation with a bulk-parameter model using data obtained from a single rain gage seems to be on the order of 25 percent.

INTRODUCTION

The development of the digital computer has added a new dimension to hydrology. Solutions to problems took hours with pen and pencil but now they take seconds with the computer. In addition, much more complex methods of analysis are feasible because of the speed of solution by the computer. The impact of the computer has been particularly great in the area of rainfall-runoff modeling. Historically, surface-water hydrology has been concerned with modeling, for flood routing and unit-hydrograph analysis are mathematical modeling. Complete rainfall-runoff simulation models date back at least to the 1920's. However, the present burst of activity in hydrologic simulation is a direct result of widespread availability of the computer.

Computers have made rainfall-runoff simulation on a large scale economically feasible. Practicality, however, depends upon applicability and accuracy of the simulation results. Simulation may be practical if one of the following applications is realized.

1. A rainfall record can be used to supplement a streamflow record having a shorter period of record than the rainfall record.
2. Model parameters for ungaged sites can be estimated on the basis of the parameters derived for gaged sites, and information can be gained at the ungaged sites through the use of recorded or simulated rainfall data and the use of estimated parameters at the ungaged sites.
3. The effect of man-made changes on a basin can be related to changes in model parameters, so that measured "before" conditions can be compared with simulated "after" conditions of sufficient accuracy for planning purposes.

Predictions obtained from rainfall-runoff simulation models are successfully applied in any of the above determinations only where the level of accuracy of the predictions is known. Measures of accuracy must be presented to the user in understandable terms. Accuracy should be measured in terms of prediction, rather than in terms of fitting. Accuracy of fitting indicates only how well the model can reproduce a set of data from adjusted model parameters. Accuracy of prediction indicates how well the model can reproduce a set of data that was not used to derive the parameter values. Therefore, prediction involves an independent test of accuracy of the model.

The U.S. Geological Survey research program is developing rainfall-runoff simulation models. Research emphasis has been on the utility of the models for practical field application to current projects and has centered upon both development of models and testing of their accuracy of prediction. This report is a statement of progress to date (1971) on model development, including illustrative examples of the results of prediction for several basins in various hydrologic settings.

Hydrologic models have been developed in response to hydrologic needs. The use of computers has led to the development of more sophisticated hydrologic models. The more sophisticated models should be more accurate to justify their existence, and their accuracy must be measured in terms of their ultimate use.

The models discussed in this report are parametric models, or models that try to simulate physical conditions by a deterministic mathematical description, which includes, as much as possible, approximations to the physical laws governing surface-water hydrology. Wherever possible, a physical interpretation is placed upon the parameters used in the models. A separate field of modeling not included in this study is that of stochastic simulation. These models describe the hydrologic record in statistical terms and use that statistical description to generate synthetic "equally likely" records. Each type of model has its advantages and disadvantages for application to meet a particular need.

The derivation of a set of optimum parameters representing the hydrology of a basin must be based on data. A parametric model requires both streamflow and rainfall data and, perhaps, other hydrologic data. Certain data other than that on streamflow also contain streamflow information, and use of this additional information should reduce the time required to collect streamflow data necessary to achieve a given level of accuracy of prediction.

Most studies involving rainfall-runoff models include the assumption of a stationary time series, at least during a period of calibration. Thus, the model parameter values can not change with time. Often, an assumption is made that, if parameters do change, any such changes can be related to physical changes on the drainage basin, particularly to man-made changes.

HISTORICAL DEVELOPMENT OF PARAMETRIC RAINFALL-RUNOFF MODELS

Parametric hydrology is that field of mathematical hydrology which attempts to synthesize a model of the land phase of the hydrologic cycle, by approximating the physical laws governing the various components of the rainfall-runoff system. Infiltration, soil-moisture storage, percolation to ground water, evapotranspiration, and surface- and subsurface-flow routing are modeled by sets of equations that, hopefully, give a response equivalent to the response of the component modeled. The components and all necessary interrelations among components are described by means of parameters, some of which are empirical, and some of which have a physical interpretation.

One of the earliest overall models of the hydrologic cycle was developed by Folse (1929). Development of that model was begun in 1916 and continued through the 1920's. During the 1930's, advances were made in the description of all components of the hydrologic cycle. Sherman (1932) introduced the theory of the unit hydrograph, which led to a flurry of developments, culminating with Dooge's general linear theory of flood-flow routing (1959). The Horton (1939) infiltration equation was an empirical attempt to describe unsaturated flow. Philip (1954) extended this by deriving an approximation based upon the Darcy equation for infiltration at a point. Theis (1935) showed the analogy of Darcy's equation for flow through saturated porous media to the heat-flow equation. Many simplifications for specified boundary conditions were subsequently developed and became the basis for routing of ground-water discharges, such as the equation of Kraijenhoff (1958) for instantaneous recharge in two-dimensional flow.

The digital computer made it possible to combine these many approximations into one overall approximation describing the operation of the land phase of the hydrologic cycle. Linsley was the first to take advantage of this possibility, and his efforts led to the development of the Stanford Watershed Model (Crawford and Linsley, 1966). Similar models have been developed at many universities and in government agencies, both in the United States and abroad.

The many models currently available or being developed must meet certain criteria to be useful in practical application. They must:

1. Require only input data that are generally available.
2. Be simple enough for the user to operate and to understand.
3. Provide the output desired at an acceptable level of accuracy for the application for which it is used.

The U.S. Geological Survey flood-hydrograph simulation model follows directly from the historical developments previously described, and is designed to meet the criteria outlined above.

TRANSFERABILITY OF RESULTS OF MODELING

For modeling results to be transferable, the parameters derived from simulation studies at measured sites must be constant or must possess invariant relations with physical variables which can be measured in other basins. Time invariance is required, or else any changes in time must be the result of measureable physical changes within the basin. Certain types of information may be transferable without the use of simulation. For instance, Benson (1962, 1964) showed that raw-data analysis can lead to regionalization of flood-frequency characteristics for a region. Simulation might aid such a study by extending the data base available for analysis. In addition, simulation is necessary if the time sequence of flows, rather than just their frequency of occurrence, is needed.

Parametric simulation is so structured as to include those parameters related to physical measures of the basin. Therefore, transferability is implicit in parametric simulation. To date (1970), however, no studies have presented results leading to regionalization of the parameters of either stochastic or parametric simulation, although some thought has been given to the problem (Benson and Matalas, 1967; Matalas and Gilroy, 1968). Feasibility of any such regionalization of parameters can be tested by comparing the sensitivity of results of simulation with the accuracy of the parameter estimates. That a statistical or physical parameter can be related to some characteristic of a basin is of no value if the standard error of estimate of the resulting relation is such that the simulation would be grossly in error. Therefore, the transferability of parameters is limited by the sensitivity of the modeling results to the magnitude of the errors in parameter values.

ADVANTAGES AND DISADVANTAGES OF PARAMETRIC SIMULATION

Rainfall-runoff models, in general, are lumped-parameter models, although often the surface-streamflow routing is accomplished by the use of a finite-difference approximation to the drainage system. A lumped-parameter model attempts to use a single parameter value to represent a physical measure that also has spatial variability. The models are, therefore, at least one step removed from simulating the actual flow mechanics at each point in the watershed. Derived parameter values are, at best, average values for the basin, and are an index to, rather than a measure of, the underlying physical system. This approximation introduces a major source of error in a lumped-parameter model and limits the accuracy of prediction obtained by the use of the model.

The parameters in parametric-simulation models should require a shorter period of record in order to be as well defined as those for either deterministic or stochastic black-box models. This has advantages when data must be collected and the analysis postponed until sufficient data are available. Transferability should be easier for parametric models, although this is yet to be demonstrated. Parametric models require more types of data for each event modeled, both for system identification (fitting of parameters) and for simulation of synthetic records.

The emphasis of the models presented in this study is on flood-hydrograph simulation for small drainage areas. Generally, there is little or no data on small drainage areas; therefore, results must be obtained on the basis of short records. In addition, only a small percentage of smaller basins can be gaged. Consequently, results must have transferability if the smaller, ungaged basins are to be simulated. Concentration on the development of a parametric model, thus, seemed to be warranted.

ACKNOWLEDGMENTS

The present report represents several years' research by the authors. The project was begun as the result of the efforts of R. W. Carter, U.S. Geological Survey, and his helpful encouragement throughout was important. Terence O'Donnell, Imperial College, London, joined the Survey's research group during his year's sabbatical leave from Imperial College, and his continued interest has contributed to the development of ideas, particularly on the use of objective fitting methods. Jaime Amorocho, University of California at Davis, Chester Kisiel, University of Arizona, and Jacob Rubin, U.S. Geological Survey, Menlo Park, Calif., all helped encourage the authors to "say what they mean, and mean what they say."

STRUCTURE OF THE MODEL

GENERAL STRUCTURE

The rainfall-runoff model described in this report deals with three components of the hydrologic cycle—antecedent moisture, infiltration, and surface runoff. Structure of the model is shown in figure 1.

Particular effort was made to design a model with a degree of equivalence to the physical system. Therefore, this model should be very similar in structure to any other bulk-parameter model for rainfall-runoff simulation. The antecedent-moisture-

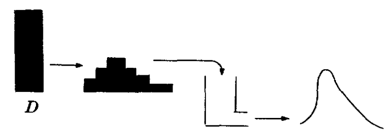
ANTECEDENT-MOISTURE ACCOUNTING COMPONENT	INFILTRATION COMPONENT	ROUTING COMPONENT
Saturated-unsaturated soil moisture regimes	Philip infiltration equation	Clark instantaneous unit hydrograph
	$\frac{di}{dt} = K\left(1 + \frac{P(m-m_o)}{i}\right)$	
Parameter: *EVC*, *RR*, *BMSM*, *DRN* Variable: *BMS*, *SMS*	Parameter: *SWF*, *KSAT*, *RGF* Variable: *BMS*, *SMS*	Parameter: *KSW* Variable: *SW* Time-area curve
INPUT DATA		
Daily rainfall Daily pan evaporation Initial condition	Unit rainfall "*BMS*" "*SMS*"	Rainfall excess
OUTPUT DATA		
BMS *SMS*	Rainfall excess	Discharge

FIGURE 1.—Schematic outline of the model, showing components, parameters, and variables.

accounting component is a more sophisticated version of the antecedent-precipitation index (*API*), which is designed to determine the initial infiltration rate for a storm. The infiltration component uses the Philip equation, which is believed to be a somewhat better approximation to the differential equation for unsaturated flow than the classical Horton exponential-decay-infiltration equation. Surface routing is based on a linear approximation developed more than 20 years ago (Clark, 1945).

The operation of the antecedent moisture accounting component is designed to simulate moisture redistribution in the soil column and evapotranspiration from the soil. It contains four parameters: *EVC,* a pan coefficient converting measured pan evaporation to potential evapotranspiration; *RR,* a coefficient that determines the relative amounts of infiltration and surface runoff for periods with daily rainfall input; *BMSM,* a maximum effective amount of base moisture storage, and *DRN,* a coefficient controlling the rate of drainage of the infiltrated soil moisture. The input to this latter component is daily rainfall and daily pan evaporation. The output is the amount of base-moisture storage (*BMS*) and of infiltrated surface-moisture storage (*SMS*). *BMS* represents a uniform antecedent moisture content of the active soil column, and its range of values should simulate the moisture range from wilting-point conditions to field capacity. *SMS* represents the moisture content of the surface layer that forms during infiltration.

The infiltration component is based on an approximation to the differential equation for unsaturated flow (Philip, 1954). The equation is based on a two-part accounting of the soil moisture, with a wetting layer overlying a layer of uniform moisture content (determined by antecedent events). The infiltration component contains three parameters: *SWF*, which represents the combined effects of moisture content and capillary potential at the wetting front for field-capacity conditions; *RGF*, a parameter that varies the effective value of *SWF* as a function of *BMS*; and *KSAT*, the hydraulic conductivity of the transmission zone. Inputs to this component are unit rainfall data and the values of *BMS* and *SMS* derived from previous times. The output is rainfall excess—that is, the remaining rainfall after abstractions by infiltration.

Surface-runoff routing is based on the Clark form of the instantaneous unit hydrograph. The single parameter is a linear reservoir routing coefficient (*KSW*). In addition, a time-area curve is derived which distributes the excess rainfall into a translation hydrograph. The input to this component is the rainfall-excess output computed from the infiltration component, and the output is the storm-runoff hydrograph. Table 1 summarizes the eight model parameters.

TABLE 1.—*The eight model parameters and their application in the modeling process*

Parameter identifier code[1]	Units	Application
SWF	Inches	Effect of moisture content and capillary potential at the wetting front for field-capacity conditions.
RGF		Varies the effective value of *SWF* as a function of *BMS*.
KSAT	Inches per hour.	The minimum (saturated) value of hydraulic conductivity used to determine infiltration soil rates.
BMSM	Inches	Soil moisture-storage volume at field capacity.
EVC		Coefficient to convert pan evaporation to potential evapotranspiration values.
DRN	Inches per hour.	A constant drainage rate for redistribution of soil moisture.
RR		Proportion of daily rainfall that infiltrates the soil.
KSW	Hours	Time characteristic for linear reservoir routing.

[1] For explanation of the parameter identifier codes, see preceding text.

The output from one component is the input to the next. Even a model as simple as this one has many interactions among the parameters. This is particularly true of the antecedent-moisture-accounting and infiltration components. Often, adjustments of a parameter in one component can be compensated for by an adjustment in a different component in another parameter. Over some error range, many sets of parameter values may fit a given set of data equally well. Even though the parameters of the model are chosen so as to be analogous to physical parameters in a basin, the degree of similarity in the optimum set of derived parameter values may mask the relation of the values to their supposed physical prototype. Thus, the conceptual physical equivalence of the model may be lost in the fitting process. (This point is discussed more fully in the section entitled "Fitting Errors.")

INFILTRATION COMPONENT

"Infiltration" is the term used to describe the entry into the soil of water available at the soil surface. When rain falls on a soil it either infiltrates, goes into detention storage, or becomes surface runoff. The rate of infiltration into the soil is, of course, limited by the supply rate of rainfall. Darcy's law describes flow of a liquid in a homogeneous porous medium and is the basic mathematical description of the infiltration process.

Many empirical equations have been used to approximate the infiltration process. One of the more physically meaningful equations is that of Philip (1954; Green and Ampt, 1911), which has been used as the basis for the infiltration component in the flood hydrograph synthesis program. The Philip equation assumes a two-part soil moisture distribution, as shown in figure 2.

A soil column of initial moisture content, m_o, is infiltrated by water which wets a thickness of soil, x, to a uniform liquid content, m. Both m and m_0 are relative moisture contents of their respective soil columns, with m representing moisture content near saturation.

The wetting front is at the depth, x, below the soil surface. The equation assumes that both the velocity of flow throughout the wetted column and the soil suction at the wetting front are constant. The capillary potential of an unsaturated soil acts to move moisture from wetter to drier portions of a soil column.

With these assumptions, Darcy's law reduces to

$$\frac{Vx}{k_h} = P + x + H,$$

or

$$V = k_h \left[1 + \frac{P + H}{x} \right], \qquad (1)$$

where V is the downward velocity of flow in the infiltrating column (units of L/T); k_h is the capillary conductivity (units of L/T) at soil moisture

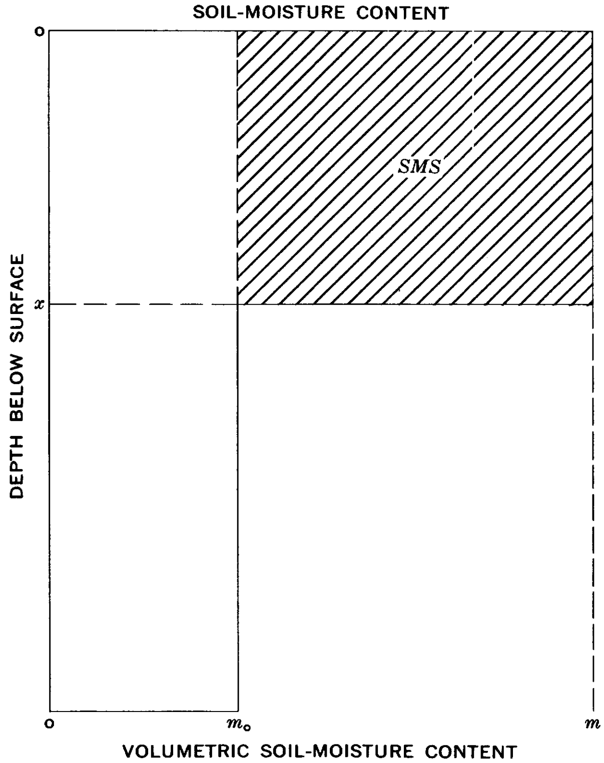


FIGURE 2.—Schematic diagram of the two-layered soil-moisture profile used with the infiltration equation in the model. The initial uniform soil-moisture content, stated as a proportion of total volume, is m_0, and the depth of the wetted layer is x. The amount of infiltrated moisture (SMS) is $(m-m_0) \bullet x$, and is shown as the hachured area. The amount of antecedent moisture (BMS) is shown as the unhachured area. That portion of antecedent moisture contained in the wetted layer is $m_0 \bullet x$, and is shown as the unhachured area above the depth x in the profile.

m; P is capillary potential at the wetting front (units of L); and H is the depth of ponded water at the surface (units of L). The capillary potential generally is several orders of magnitude larger than the depth of ponded water, so that the H term may be ignored. Because

$$V = di/dt, \tag{2}$$

and

$$x = i/(m-m_0), \tag{3}$$

equation 1 becomes

$$\frac{di}{dt} = k_h \left[1 + \frac{P(m-m_0)}{i} \right], \tag{4}$$

where i is the accumulated infiltration in the wetting column (denoted by patterned area in fig. 2). The mnemonic identifiers used to designate equation 4 in the computer program and in this paper are

$$FR = KSAT \left[1 + \frac{PS}{SMS} \right], \tag{5}$$

where

$FR = di/dt$ (units of L/T),

$KSAT = k_h$ (units of L/T),

$PS = P(m-m_0)$ | effective (units of L), and

$SMS = i$ (units of L).

The capillary potential at the wetting front is not a constant, but varies according to the initial soil-moisture condition. Colman and Bodman (1944) stated (in a paper used by Philip for some of the justification for his equation) that "of the changed conditions brought about by using moist rather than air-dry soils, the observed results indicate the particular importance of the lowered potential gradient at the wet front." However, neither Philip nor Colman and Bodman gave a method for determining the variation of the potential. The flood-hydrograph simulation program determines the effective value of PS as varying linearly between a value at plant wilting point and a value at field capacity. This requires two parameters. The first is the effective value of the product $P(m-m_0)$ at field capacity (SWF). The other is the ratio (RGF) of the product at wilting point to that at field capacity. The effective value of the product of capillary potential and soil-moisture deficit is described by a linear relation to soil moisture deficit and is computed as

$$PS = SWF \left[RGF - (RGF-1)\frac{BMS}{BMSM} \right], \tag{6}$$

where BMS is the beginning soil-moisture storage in the soil column, and $BMSM$ is the maximum moisture storage in the soil column at field capacity. This relationship is shown in figure 3.

Equations 5 and 6 represent the approximation used for infiltration at a point. Equation 5 is a differential equation with a variable coefficient because the soil-suction coefficient, PS, is a function of soil moisture, as shown in equation 6. Infiltration occurs throughout a basin at varying rates; however, the flood-hydrograph synthesis program uses a scheme

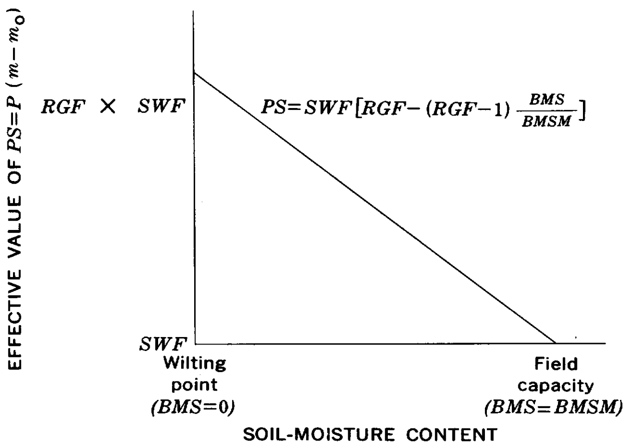


FIGURE 3.—The relation which determines the effective value of moisture content and capillary potential (PS) for use in the infiltration equation.

first presented by Crawford and Linsley (1966, p. 210) to convert point potential infiltration to net infiltration over a basin. Letting SR represent the supply rate of rainfall for infiltration, and QR represent the rate of generation of excess precipitation that does not infiltrate, the equations are

$$QR = SR^2/2FR \qquad SR < FR \qquad (7a)$$

$$QR = SR-(FR/2) \qquad SR > FR \qquad (7b)$$

The schematic representation of the relations is shown in figure 4. The relation may be interpreted as describing the probability distribution of potential infiltration throughout the basin by a straight line, with net infiltration being the average throughout the basin. However, no claim is made that equation 7 actually is a representation of the probability distribution of potential infiltration. Certainly, such a distribution would not be uniform, as implied by the equation, nor would its shape be similar in time. Rather, equation 7 is an empirical tool which eliminates the absolute threshold value for infiltration. Thus, there is some runoff from any volume of rainfall, although for low-intensity rains where soil conditions are dry, the runoff is very small. The major justification for equation 7 is that it aids the modeling of the runoff volumes for the smaller, low-intensity storms.

Equations 5, 6, and 7 together describe the infiltration component. The flow chart for the infiltration component is shown in figure 5.

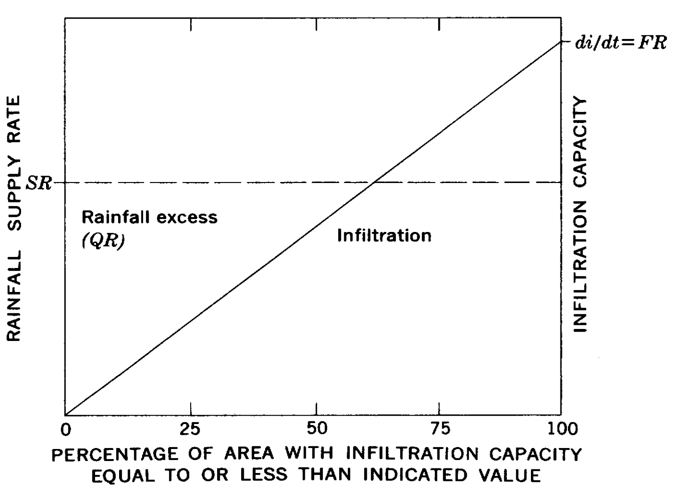


FIGURE 4.—The relation which determines rainfall excess (QR) as a function of maximum-infiltration capacity (FR) and supply rate of rainfall (SR).

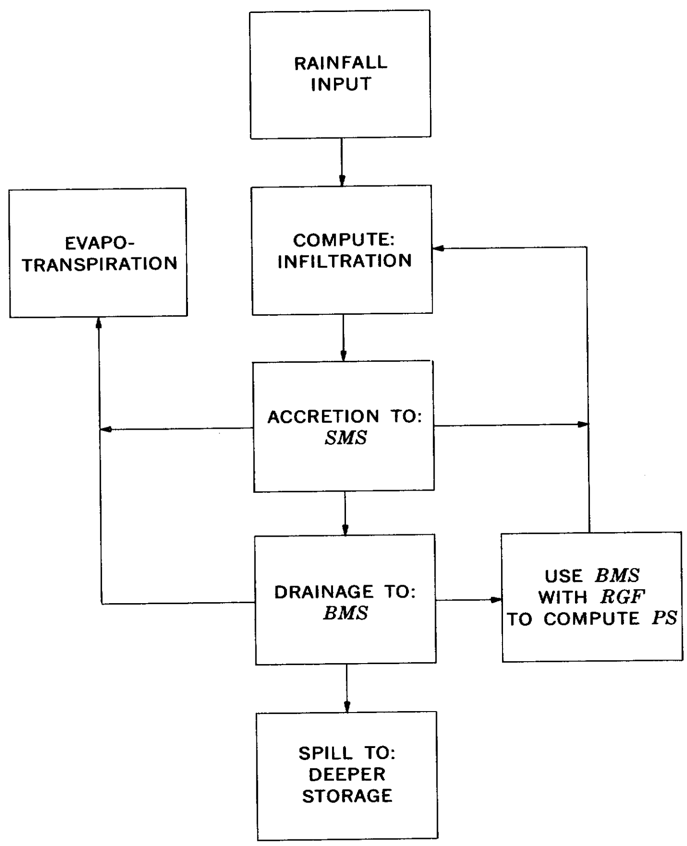


FIGURE 5.—Schematic flow chart of the flood-hydrograph simulation program.

SOIL-MOISTURE-ACCOUNTING COMPONENT

The soil-moisture-accounting component in a rainfall-runoff simulation model determines the effect of antecedent conditions on the infiltration component. Although the moisture accounting system in this model was designed to represent the physical process to a large degree, the lack of full physical equivalence, in application, may result in a curve-fitting process, so that the fitted parameter values have more apparent than real physical meaning. In addition, there is a necessary constraint that the soil-moisture accounting component must be compatible with the infiltration component, if a water budget is to be maintained throughout the system. These two facts should be kept in mind throughout the description of the soil-moisture accounting component.

The soil moisture component in the flood hydrograph simulation program is based upon the Philip scheme described for the infiltration component. The total moisture in storage in the soil column is divided into two parts. The first is contained in a base moisture storage (*BMS*) at a soil moisture which can vary from field capacity to wilting-point conditions. The second is a surface moisture storage (*SMS*) near saturation. Thus, the total infiltrated column is assumed to be near saturation capacity. This assumption is based upon the results shown by Colman and Bodman (1944). A schematic diagram of the soil moisture accounting is shown in figure 2.

SMS depicts accumulated infiltration, and all infiltration during storm periods is added to *SMS*. *BMS*, on the other hand, is used to compute the relative soil moisture deficit. The unpatterned area in figure 2 represents *BMS*. *BMS* and the ratio *RGF* are together used to compute *PS*, the effective value of the product of the capillary potential and the soil moisture deficit, also a part of the infiltration equation.

Evapotranspiration losses are assumed to occur at the potential rate. All evapotranspiration demand is met from *SMS*, if possible. When storage in *SMS* is zero, the evapotranspiration demand is met from *BMS*.

Drainage occurs from *SMS* to *BMS* at a constant rate as long as storage exists in *SMS*. Storage in *BMS* has a maximum value (*BMSM*) equivalent to the field-capacity moisture storage of an active soil zone. Zero storage in *BMS* is assumed to correspond to wilting-point conditions in the active soil zone. When storage in *BMS* exceeds *BMSM*, the excess is spilled to deeper storage. The spills could be the basis for routing interflow and base-flow components, if desired. However, these components of streamflow are not modeled in the flood-hydrograph simulation program. If other components of flow make up a significant part of the flood peak, a routing of these spills would be necessary.

SURFACE ROUTING COMPONENT

The excess precipitation generated in the flood-hydrograph simulation program must be converted into a flood hydrograph by a routing method. The Clark flood-routing method (1945) is used to develop the unit hydrograph for the basin. The Clark method has two parts. First, the excess precipitation is converted into a translation hydrograph representing the effect of varying traveltimes in the basin. The translation hydrograph for the basin is represented by a time-discharge histogram. The time-discharge histogram is developed from the distance-area histogram for the basin. In essence, the derivation assumes that distance and traveltime are directly proportional. Because of variation both in resistance to flow and in channel slope throughout the basin, the assumption of proportionality of distance and traveltime does not necessarily hold. Therefore, a comparison of the shapes of simulated and observed hydrographs for several flood events can be used to revise the time-area histogram to a more appropriate shape for a study basin.

The translation hydrograph must be routed through some element representing storage in the basin. For an instantaneously developed excess precipitation of 1 inch, this results in the instantaneous unit hydrograph. The Clark method assumes a linear time-invariant storage. Dooge (1959) presented an excellent discussion on unit hydrograph methods and the place of the Clark method in the general theory. Figure 6 illustrates the operation of the Clark method.

SYSTEM IDENTIFICATION

The method of determining optimum parameter values is based on an optimization technique devised by Rosenbrock (1960). Wilde (1964) referred to the method as "the method of rotating coordinates." It is a hill-climbing procedure that does not require evaluation of partial derivatives of the objective function with respect to the parameters. All parameters must be bounded for the method to be used. Thus, parameter values may be constrained to a range of "reasonable" values if desired. The utility of the procedure, as related to system identification in hydrologic modeling, was discussed by Dawdy and O'Donnell (1965).

The method revises the parameter values and recomputes the objective function, using the revised set of values. If the result is an improvement, the revised set is accepted; if not, the method returns to the previous best set of parameters. The objec-

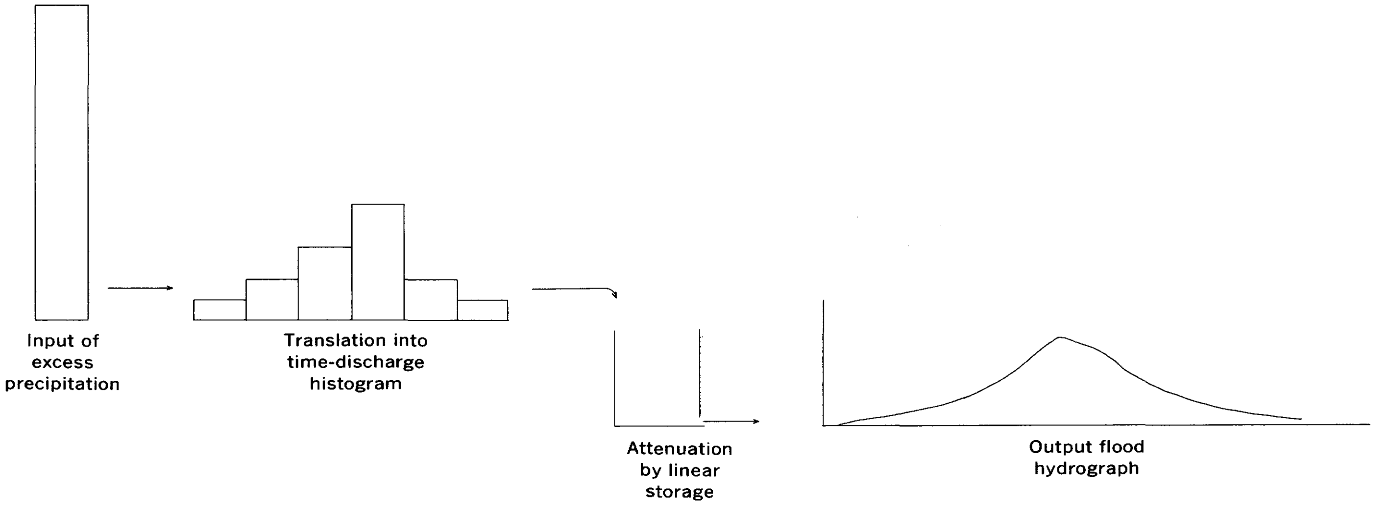


FIGURE 6.—Schematic drawing of the Clark unit hydrography used in the surface-runoff routing component.

tive function, or *U*-function, throughout this study is based upon the sum of the squared deviations of the logarithms of peak flows, storm volumes, or some combination of both. Thus, the fitting procedure develops a nonlinear least-squares solution.

The logarithms of flows are used because streamflow errors are generally more nearly equal in percentage than they are in absolute terms. Thus, if a peak of 1,000 cfs (cubic feet per second) is estimated in error by an average of 100 cfs (10 percent), a peak of 5,000 cfs will have a greater probability of an average error of 500 cfs (10 percent) than of 100 cfs (2 percent). The logarithmic transformation is meant to make the error of estimation more commensurable for the large and the small peaks. The sum of the squared errors is used as an objective function because of the mathematical property that it is a convex function, and because of its direct analogy to least-squares fitting in standard linear statistical theory. More concerning this point will be discussed in the section entitled "Response of the Model."

Rosenbrock's method of optimization proceeds by stages. During the first stage, each parameter represents one axis until arbitrary end-of-stage criteria are satisfied. At the end of each stage, a new set of orthogonal directions is computed, based on the experience of parameter movement during the preceding stage. The major feature of this procedure is that, after the first stage, one axis is alined in a direction reflecting the net parameter movement experienced during the previous stage.

To start the fitting process, the hydrologic model is assigned an initial set of parameter values, and the resulting simulated flood-hydrograph response is computed. The objective function is calculated and then stored in the computer memory bank as a reference value; later, this reference value is used to evaluate the results of subsequent trials. A step of arbitrary length is attempted in the first-search direction. If the resulting value of the objective function is less than or equal to the reference value, the trial is registered as a success, and the appropriate step-size, e, is multiplited by $\partial>1$. If a failure results, the step is not allowed and e is multiplied by $-\beta$, where $0<\beta<1$. An attempt is made in the next search direction, and the process continues until the end-of-stage criteria are met. At this point, a new search pattern is determined, and another stage of optimization undertaken. Only a limited amount of information is output during optimization. The *U*-function value and associated parameter values are printed for each successful trial. Also, a listing by flood event of the simulated hydrologic response and of observed data are output at the start of each stage.

Note particularly that the concept of automatically determining optimum model parameters requires the objective function to be compatible with the intended use for which the fitting is undertaken. In order to give weight to both the volume and the shape characteristics of the flood hydrograph, a weighted objective function ($U3$)—including both peak and volume error—was used. One component of the objective function used in optimization is the sum of squared log deviations between recorded and simulated flood peaks ($U1$). Another component,

($U2$), is the sum of squared log deviations between estimated and simulated surface runoff for each storm period.

Estimated surface runoff is calculated by a crude hydrograph-separation technique that integrates the volume of runoff under the flood hydrograph, from the start of the storm period through the period of rise, and for a duration of recession after the peak. The contribution from base flow is deducted and is assumed to be equal to the volume derived by projecting the level of discharge from the start of the rise through the period of integration. Recorded flood peaks are similarly reduced by the antecedent discharge level to account for the contribution from base flow.

RESPONSE OF THE MODEL

The "game" of hydrologic simulation is based upon engineering approximations. Approximations introduce errors into simulation results. To properly utilize a model, therefore, there must be some understanding of the magnitude of errors produced by use of the model.

Errors in data are reflected in errors in the fitted parameters in a simulation model. If perfect input data are routed through a perfect model, the output produced would agree perfectly with an error-free output record. If errors are introduced into the input or output record, or both, the output results will not be exactly reproduced even from a perfect model. If a fitting process is used, the parameters will deviate from their true values in order to minimize the deviations between the simulated and recorded traces, as specified in the objective function. The "optimal" set of parameters will now be in error, and the value of the objective function after fitting will be less than its "true" value. This is so because the value has been derived by a method used to find the minimum value for the objective function.

The fitting process is analogous to a statistical least-squares analysis. The fitted parameters deviate from their population values because of random errors in the data. The standard error of estimate is a measure of error in the data. The standard error of prediction, however, is somewhat greater than the standard error of estimate, for it includes both the measure of lack of fit of the data used to calibrate the model and the measure of error in the fitted parameters. These relationships are given in table 2.

TABLE 2.—*Errors involved in hydrologic modeling qualitatively compared with analogous errors resulting from standard linear statistical analysis*

Source of error	Qualitative size of error variance	Statistical analog
Measured data	a	Measurement and sampling error variance.
Differences between measured and simulated flows *during* the calibration period.	$a-b$	Square of standard error of estimate.
Differences between measured and simulated flows *outside* the calibration period.	$a+c$	Square of standard error of prediction.

If the assumptions of regression theory were valid, for a linear model with normally distributed and homoscedastic errors of the dependent variable (that is, the variance about the regression is independent of the independent variables), the standard error of prediction could be computed from the standard error of estimate, the deviations of the independent variables from their mean, and the error in the coefficients for the independent variables. These assumptions seldom hold, however, so that competent statisticians often resort to split-sample testing. The assumptions also fail for hydrologic simulation, and the models are nonlinear, as well; hence, there is no theory by which to compute the error of prediction. Therefore, split-sample testing must be used in hydrologic simulation modeling whenever possible.

At present, nonlinearity of the hydrologic process precludes any theoretical description of the mechanism by which errors in data are transferred to model parameters and, in turn, are combined with input errors in the test period to produce errors in the simulated streamflow. An empirical study for the response of the model to input and output errors is shown in table 3.

A recorded rainfall trace was assumed to be error free and was routed through an optimized set of parameters for the Little Beaver Creek basin near Rolla, Mo. The optimized parameter values were assumed to be the correct values to obtain a "true" streamflow trace. Then, a random error with mean zero and standard deviation of 10 percent was applied to all rainfall values. These "erroneous" rainfall values were then routed through the model with the "true" parameter values, and the resulting value of the objective function for the simulated streamflow trace computed. Next, an optimization run was made which adjusted the parameters to minimize that value. The "optimized" set of parameters is

TABLE 3.—*Results of an empirical study of the response of the model to input and output errors*

Parameter identifier[1]	Assumed true values	Values optimized to rainfall errors of: 10 percent	Values optimized to rainfall errors of: 20 percent	Values optimized to streamflow errors of: 5 percent	Values optimized to streamflow errors of: 10 percent
SWF (in.)	3.6	3.6	3.8	3.7	3.7
KSAT (in. per hr)	.063	.063	.06	.063	.061
KSW (hr)	1.0	1.04	1.06	.98	.98
EVC	.56	.57	.58	.559	.56
BMSM (in.)	4.0	4.02	3.98	4.04	4.04
RGF	12.0	11.9	11.94	12.12	12.21
RR	.8	.796	.8	.796	.796
DRN (in. per hr)	.020	.018	.017	.020	.019
U:					
Pd[2]	------	[3] .0150 (12)	.0538 (23)	.00233 (4.8)	.00915 (9.6)
pd	------	.0097 (9.9)	.0493 (22)	.00170 (4.1)	.00708 (8.3)
pD	------	.0039 (6.3)	.0152 (12)	----------	----------
Test[4]	------	.0196 (14)	.0890 (30)	----------	----------

[1] For explanation of parameter identifier codes, see p. B3–B5.

[2] *P*, true parameters; *p*, optimized parameters; *D*, correct data; *d*, erroneous data.

[3] First value, shown without parentheses, is the average of two-thirds of the squares of natural logarithms of the sample peaks plus one-third of the squares for the sample storm volumes. The second value, shown in parentheses, converts the first value to an equivalent "percent standard error" by *SE*=antilog *U*, and averaging plus and minus percentages.

[4] Average of nine separate test runs.

shown, along with the resulting value. The "true" rainfall trace was then routed through the new optimized parameters and the objective function evaluated. Assuming independence of the two sources of error—one in the input data, and the other in the model parameters—the error of prediction should be approximately equal to the square root of the sum of the squares of the two separate estimates of error. To test this relation, nine independent sets of random errors were applied to the rainfall values and routed through the model using the optimized parameter values. The average U value for the nine test runs is also given in table 3.

For the case of 10-percent rainfall errors, the error introduced by data errors (Pd) is 0.0150, while that for parameter errors (pD) is 0.0039. The sum of these is 0.0189 which is to be compared with 0.0196 (U test). The comparison of the error of prediction, based upon the variance given above, is 13.8 percent ($Pd + pD$), as compared with 14.3 (U test).

Similar results are shown for input rainfall errors with a 20 percent standard error. As also true of the 10-percent errors, the error in simulated output was magnified so that it is about 20 percent greater than the rainfall error (Pd is 23 percent as compared with the previous value of 12 percent). Once again $Pd + pD$ should combine to produce a value comparable to that for the test results, and $0.0538 + 0.0152 = 0.0690$ is to be compared with 0.0890. The respective percentages are 26 and 30 for estimates of the error of prediction.

Errors in streamflow measurement are transferred to model parameters in the fitting process. An example of this is shown in table 3. Errors of 5 and 10 percent were introduced into runoff estimates, and a set of best-fit parameters derived. The rainfall and runoff errors are independent in this study, so that the square of the error of prediction for 10-percent runoff errors and 20-percent rainfall errors would be on the order of the sum of the two variance terms, 0.0890 for rainfall errors and 0.00915 for runoff errors, or a 32-percent error for the two combined, as compared with 30 percent for rainfall alone.

Two points are particularly noteworthy in the above results. First, rainfall errors have a magnified effect on the simulated streamflow for basins similar to the one chosen for the study. This probably is true for most basins with drainage area less than 10 square miles. Therefore, rainfall errors probably are the controlling factor determining accuracy of streamflow simulation. Second, the response of this wholly nonlinear hydrologic model is approximately linear for errors in rainfall on the order of magnitude investigated, which probably are on the order of magnitude generally found in a field study. One would expect that errors in rainfall input would result in greater proportional errors in predicted streamflow, because streamflow is a residual after abstractions. However, amounts of excess rainfall for different time periods from different parts of the basin are combined in the translation routing, and the storage routing attenuates errors by averaging by means of the storage process. Therefore, the relative size of rainfall errors and of errors of estimated streamflow depends upon the extent to which the model of the routing process attenuates the magnification of errors produced in model estimates of excess rainfall.

That the errors of streamflow estimates are approximately linearly related to the errors of rainfall input data is particularly important. The linearity of errors indicates that there may be some hope for the derivation of a theory of errors for streamflow simulation. In addition, the linearity gives some post hoc justification for the nonlinear least square fitting technique used in the fitting process.

SIMULATION MODEL STUDIES

The proof of the pudding is in the eating thereof. The empirical study described in the preceding section does give insight into the modeling process and, in particular, into the operation of the model. To further illustrate the utility of the model in field application, three basin studies are presented. They represent a range in location and hydrology. The basins are Santa Anita Creek near Pasadena, Calif., a semiarid basin; Beetree Creek near Swannanoa, N.C., a humid basin; and Little Beaver Creek near Rolla, Mo., a basin in which hydrology is typical of the interior United States. All three basins have pronounced relief.

The data available varied from basin to basin. Also, the relative stage of development of the model led to the emphasis of different research goals during the analysis of the different areas. Sufficient rainfall data were available for the Santa Anita Creek basin for a study to be made of the effect of bias of rainfall measurements and of the effect of time and space variability of rainfall on modeling results. Beetree Creek basin was used to study the effects of split-sample testing to study the methodology of the use and limitations of the objective curve-fitting method. Each basin will be discussed separately, and a discussion of the overall results and of the problems encountered will be presented.

SANTA ANITA BASIN

GENERAL PHYSIOGRAPHY

The Santa Anita Creek drainage basin is a 9.7 square mile (25 sq km) area of the San Gabriel Mountains in southern California. The rugged topography ranges in elevation from 1,500 to 5,700 feet (460 to 1,700 m above sea level with the mean elevation about 3,600 feet (1,100 m). Thin porous soils covering a highly fractured bedrock combine to give the basin high moisture-retention and absorption properties. The southerly facing basin receives about three-fourths of its rainfall during the cool winters. The climate and soils support a thin to dense growth of chaparral native to the area.

PRECIPITATION

The precipitation-measuring network on the Santa Anita Creek basin consists of 6 stations for a 14-year period ending with the 1962 water year. The six sites provide good areal (fig. 7) and elevation (table 4) coverage of the annual precipitation

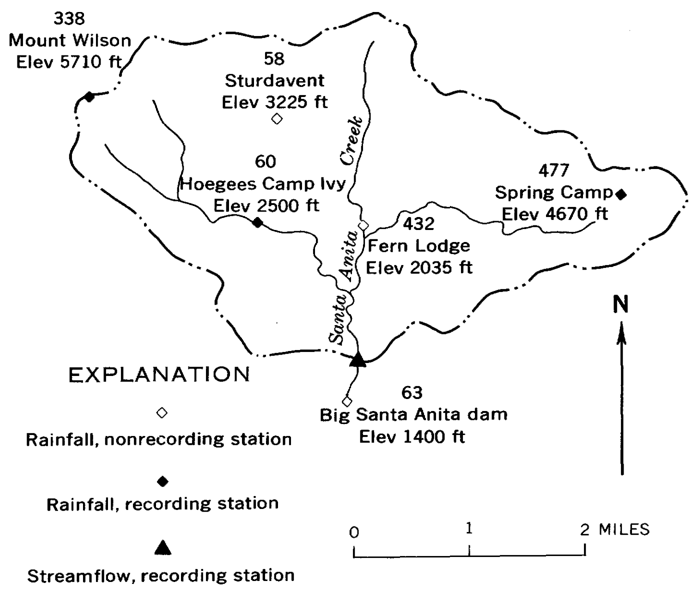


FIGURE 7.—Santa Anita Creek basin, above the stream-gaging station near Pasadena, Calif.

TABLE 4.—*Mean annual rainfall, Santa Anita Creek basin, California, 1949–62*

Station No.	Station name	Elevation (ft)	Mean annual rainfall (in.)	Deviation from basin mean [1] (percent)
58	Sturdavent	3,225	30.57	-----
60	Hoegees Camp Ivy	2,500	30.44	+3.2
63	Big Santa Anita dam	1,400	19.47	-----
338	Mount Wilson	5,710	25.21	−13.5
432	Fern Lodge	2,035	26.30	-----
477	Spring Camp	4,670	[2] 28.89	+0.6
	Basin mean			
	Thiessen method:			
	All stations	----	28.3	-----
	Three stations [3]	----	29.5	-----
	Elevation-area method	----	29.5	-----

[1] Basin mean used was 29.5 inches.
[2] Adjusted on the basis of double-mass analysis.
[3] Stations 60, 338, and 477.

in the basin. A double-mass analysis of the 14-year annual precipitation values show fair measurement consistency among the 6 stations. Three sites with continuous recorders (stas. 477, 60, 338) provided the rainfall data required by the simulation model.

Two methods were used to determine the basin mean annual rainfall during the 14-year period (table 4). First, the standard Thiessen method gave

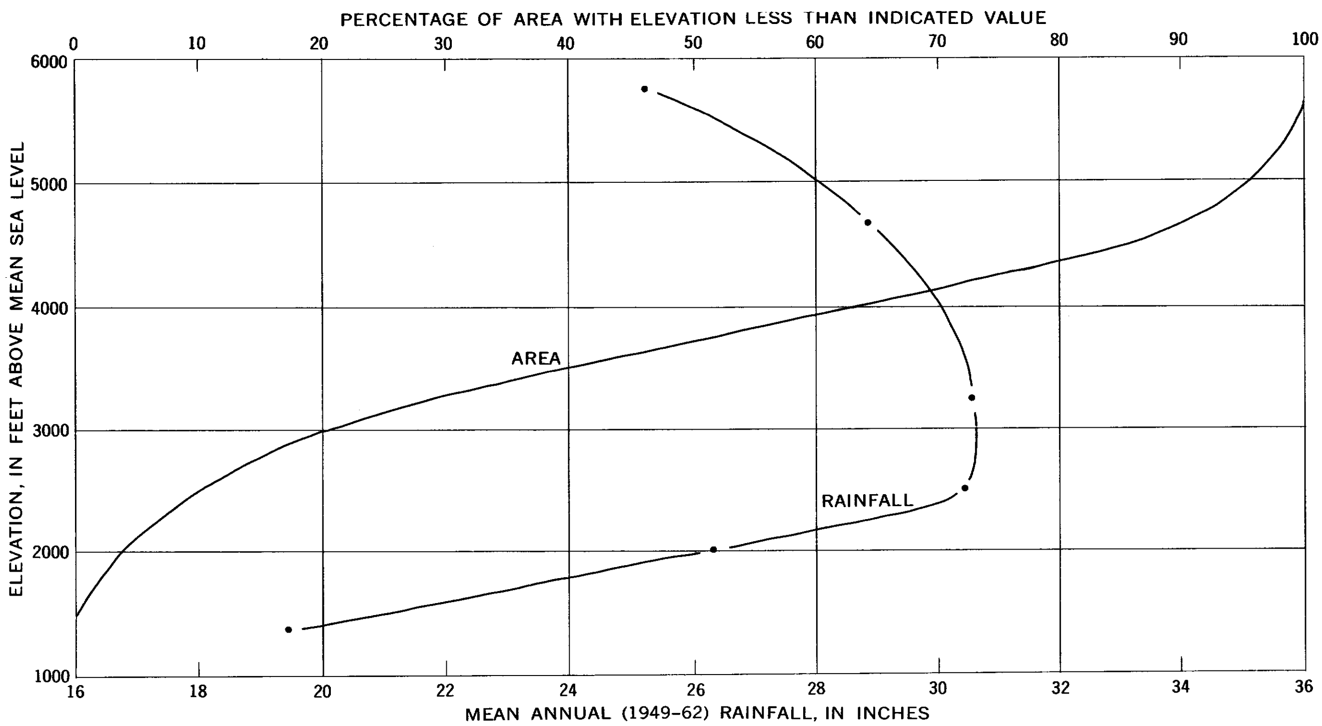


FIGURE 8.—The relation of mean annual rainfall and of area in the basin to elevation for the Santa Anita Creek basin above the stream-gaging station near Pasadena, Calif.

a mean of 28.3 inches (71.9 cm) when all six stations were considered and a mean of 29.5 inches when only the three recording sites were used. The second method, a numerical integration of the relationships for elevation-percent area obtained from topographic maps and elevation-annual rainfall defined by the six gage records (fig. 8), gave an annual mean of 29.5 inches. A value of 29.5 inches was chosen as the estimate of the mean annual rainfall and was used to evaluate the relation of individual-station rainfall to basin-mean rainfall.

The 24 storm periods selected for simulation had both complete records of the rainfall occurring at the three recording stations and a significant rise in stream discharge. The records for the storm periods were converted to discharge volumes for 15-minute time intervals. Daily rainfall records were used between storms. The storm data were compiled from gage charts provided by the Los Angeles County Flood Control District.

STREAMFLOW

The streamflow data used for fitting the model to the Santa Anita Creek basin were those for the U.S. Geological Survey gaging station near Pasadena, Calif. The site has been gaged since 1916. The mean flow has been 5.5 cfs (0.16 m³/sec) or 7.7 inches (0.2 m) (meters) per year. The maximum flow of about 5,200 cfs (147 m³/sec) occurred in March 1938. The peak discharges during the storm periods selected for study ranged from 17 to 2,530 cfs (0.5 to 71.6 m³/sec). (See table 5.)

TABLE 5.—*Simulated peak discharges, using fitted parameters*

[Observed and simulated discharges, reported in cubic feet per second (cfs), do not include base flow]

Storm No.	Observed discharge (cfs)	Simulated discharge (cfs) with adjusted data (*C*)[1]			
		Station			Mean discharge[2] (cfs)
		477	60	338	
1	2,529	2,506	2,318	2,722	2,515
2[3]	1,472	2,846	1,667	1,198	1,904
3[3]	338	917	296	346	520
4	194	224	221	225	223
5	342	532	405	454	464
6	45.8	44.4	53.4	63.8	53.9
7	30.6	37.2	38.5	35.2	37.0
8	108	184	146	148	159
9[3]	34.1	35.9	21.5	30.3	29.2
10[3]	50.1	51.9	14.5	27.3	31.2
11	660	601	618	472	547
12	150	121	149	130	133
13	156	166	190	129	162
14	111	113	99.6	96.6	103
15	361	283	417	318	339
16	332	238	228	253	240
17	837	779	696	512	662
18[3]	709	372	97.9	42.3	171
19	243	190	196	161	182
20	15.4	31.6	28.9	35.4	32.0
21[3]	58.4	18.3	36.2	14.3	22.9
22	55.0	28.1	29.7	21.9	26.6
23	91.2	54.1	45.9	69.3	56.4
24	1,235	1,238	1,303	1,362	1,301
U1[4]		2.07	1.76	2.73	1.92

[1] Set C is data for each storm event adjusted to average basin volume.
[2] Average of the simulated peaks for the three stations.
[3] Not a component of the objective function used in optimization.
[4] *U1* is the sum of squared log deviations between recorded and simulated flood peaks.

EVAPORATION

Daily values of pan evaporation at Tanbark Flat were obtained from the Pacific Southwest Forest and Range Experiment Station, U.S. Department of Agriculture. The Tanbark Flat climatic station is located in the San Dimas Experimental Forest about 10 miles (16 km) east of the Santa Anita Creek basin and is at an elevation of 2,800 feet (850 m). The mean annual evaporation from a standard [U.S.] Weather Bureau pan is in excess of 60 inches (1.5 m).

DATA SCREENING

The amount of rainfall data available for the Santa Anita Creek basin was sufficient to investigate the effects of variability of measured rainfall upon simulation results. The record of several storm events indicated a large spatial variation in total storm rainfall over the basin, as indicated by the deviation in percent of measured storm volume at each site from the weighted mean for each storm. Several storms also appeared to have a large spatial variation in total storm rainfall over the basin, as indicated by the deviation in percent of measured storm volume at each site from the weighted mean for each storm. Several storms also appeared to have a large spatial variation in rainfall intensities over the basin. On the basis of preliminary screening, six storms of the 24 available for analysis were not used in fitting the parameters. However, these peaks were simulated, and results are shown in the scatter diagrams. Only one of the excluded storms might have significantly changed the results. The records for that storm show extremely high intensities for very short periods of time; the 15-minute time interval used to define the rainfall records appears to be inadequate for an accurate simulation for that storm. The purpose of screening is to eliminate storms with extreme errors in data input, so as to minimize the effect of data errors on the fitting process.

PARAMETER DEFINITION

Nine model parameter determinations were made as a series of three fittings for each of the three rainfall stations (stations 60, 338, 477). The first in the series of three fittings was made by using the data as recorded at the stations (set A). These results are analogous to those for simulation studies for which a single recording rain gage is available in a basin, and for which there is no basis for adjusting the record to obtain a better estimate of the mean basin rainfall.

The second series of parameter determinations was made for each station by adjusting the recorded storm volumes by a constant station factor (set B). These factors were computed in order to adjust the mean annual depth at the station site to 29.5 inches (computed for basin mean annual rainfall, as explained earlier). These results are analogous to those for simulation studies for which a recording gage is available in a basin, and supplementary data are available to determine an average annual rainfall on the basin and at the gaged site.

The third fit was made to the data with the storm volumes adjusted to a three-station Thiessen weighted mean for each event (set C)—that is, the mean basin volume was distributed in time in accordance with the rainfall-intensity pattern for each individual station. These results are analogous to those for simulation studies for which a recording gage plus several nonrecording rain gages are available in a basin. Thus, a weighted mean basin rainfall for each storm can be derived.

To summarize, the various rainfall intensities are adjusted as follows:

Set	Adjustment
A	$\widehat{R}_{ij} = R_{ij}$
B	$\widehat{R}_{ij} = a_{\cdot j} \cdot R_{ij}$
C	$\widehat{R}_{ij} = a_{ij} \cdot R_{ij}$

where R_{ij} is the measured intensity for period i at station j; $\widehat{R}_{ij}$ is the adjusted intensity used in the given simulation set; $a_{\cdot j}$ is an average adjustment, which is the ratio of mean annual rainfall over the basin to the mean annual rainfall measured at station j; and a_{ij} is the ratio of average rainfall over the basin for storm i to that volume measured for storm i at station j.

The results of the nine fittings are given in table 6. In addition to parameter values the average squares of deviations between logarithms of simulated and observed peaks, $U1$, is given for each set of parameters. Table 5 shows the values of the simulated peaks for data-set C. Figure 9 shows typical scatter diagrams for data-set A.

PARAMETER SENSITIVITY

The sensitivity of the goodness of fit criterion to changes in parameter values is helpful in discussing parameter importance and simulation results. An expression of sensitivity of the error criterion to given parameters can be obtained by performing repeated simulations while incrementing the parameters, holding all other parameters to their fitted value, and observing the change in value of the fit-

TABLE 6.—*Fitted parameter values*

[Input series: *A*, rainfall data as recorded; *B*, rainfall volumes adjusted by mean annual factors; *C*, rainfall volumes adjusted by mean storm factors. Figures shown in parentheses are average errors, in percent]

Parameter identifier [1]	Input series	Station 477	Station 60	Station 338
SWF	*A*	21	22	11
	B	20	20	16
	C	20	18	17
RGF	*A*	7.5	6.1	4.4
	B	6.7	5.9	5.5
	C	5.6	6.5	6.1
KSAT	*A*	.32	.32	.31
	B	.31	.32	.25
	C	.32	.32	.31
BMSM	*A*	4.1	3.4	2.1
	B	4.0	3.4	3.6
	C	3.5	3.5	3.5
EVC	*A*	.52	.73	.52
	B	.59	.71	.80
	C	.74	.72	.74
DRN	*A*	.049	.058	.030
	B	.045	.057	.043
	C	.056	.059	.057
RR	*A*	1.14	.98	1.03
	B	1.08	1.00	.90
	C	.96	1.01	.95
KSW	*A*	2.4	2.8	2.2
	B	2.5	2.7	2.3
	C	2.6	2.8	2.3
*U*1, fit criteria [2]	*A*	.097 (32)	.123 (35)	.440 (__)
	B	.100 (32)	.122 (35)	.438 (__)
	C	.115 (35)	.098 (32)	.153 (40)

[1] For explanation of parameter identifier codes, see p. B3–B5.
[2] A component of the objective function used in optimization. *U*1 is the average of squared log deviations between recorded and simulated flood peaks.

ting criterion. This gives no measure of interaction of the parameters but is a simple measure of how critically the simulation results are dependent upon the individual parameters. The results of this procedure as applied to data set *C* for station 60 (table 6) are shown graphically in figure 10. The figure is a plot of criterion value versus the percent change in parameter values. Applying this procedure to the other data sets produced similar relationships.

ANALYSIS OF RESULTS

The results of the nine separate optimization runs —three for each of the three stations—are summarized in table 6. Shown are fitted parameter values and the resulting goodness of fit, *U*1, which is the peak-simulation part of the total fitting criterion used. The representativeness of the rainfall data is least for inputs *A*, and most for inputs *C*. As the data for the three records became more nearly similar to each other, the fitted parameter values would be expected to converge to common values for the three stations. As the data become more representative, the accuracy of fitting should increase, and the *U*1 values should decrease. The effects of the various components of error can be seen by comparing the variability of parameter values and the goodness of fit between stations for a given input set.

PARAMETER VALUES

Prediction depends upon the fitted parameter values for the model, as well as upon the data used for the prediction period. The more stable the estimated parameter values, the better the possibility of relating the fitted values to measures of the basin. Thus, variability of fitted parameters for the nine optimization runs may give insight into the degree to which model parameters are influenced by data errors.

A wide range in fitted parameter values resulted when the data were used as recorded (input series *A*) at the three stations. None of the three sets of parameters can be considered unlikely when viewed individually; however, together the sets of values illustrate a possible range, depending on the data representativeness. In practical application, the available data may consist of only one record, which must be used without knowledge of its degree of representativeness. The variability of fitted parameter values, such as those for series *A*, will affect the feasibility and accuracy of any regionalization of parameter values.

Input series *B* contains both time-distribution and total-volume errors but has been adjusted to reduce the gaging bias resulting from errors in the estimated mean annual precipitation over the basin. The reduced range in parameter values, except for *KSAT* and *RR*, indicates a better estimation than was obtained in series A. The relative insensitivity shown for *KSAT* in figure 10 is for the independent effect of *KSAT* in the first term of the Philip infiltration equation. Accurate determination is not possible and may not be important. The range of *RR* values between series *A* and *B* are about equal. Values of *RR* greater than 1.0 reflect curve fitting in the model, and result, in part, from the differences between rainfall measured at a given point and average rainfall over the basin. No constraint was placed on the value of *RR* for these optimization runs.

Input series *C* has the same estimate for storm volume at all stations for each storm. The only variability is that introduced by the different time distribution during a given storm, as recorded at the three stations. All parameters have relatively stable fitted values. The variation has been reduced to within ±10 percent for all parameters, with only the infiltration parameters (*SWF* and *RGF*) and the routing parameter (*KSW*) varying by more than 5 percent. The overall correspondence of station 477 to station 60 in series *C* is very close, espe-

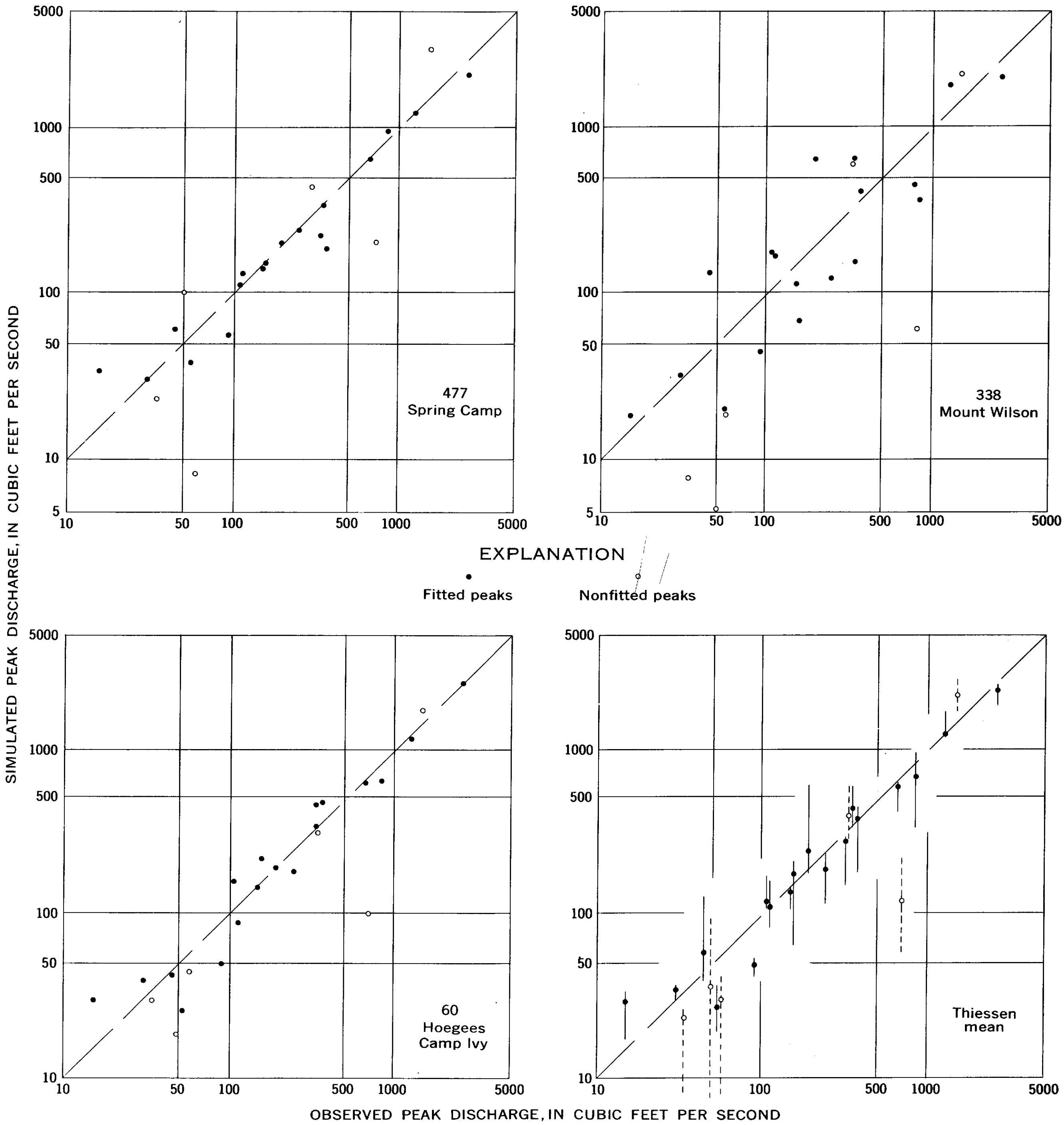


FIGURE 9.—Typical scatter diagrams for simulation results in the Santa Anita Creek basin. Results shown are for optimum fits, using rainfall data without adjustment.

cially when the direct interaction of *SWF* and *RGF* is considered.

FITTING ERRORS

The measure of goodness of fit, *U*1 (table 6), is the average of the squared deviations between logarithms of computed and simulated peaks and is analogous to a variance or the square of a standard error.

Several components of error occur in the simulation results. On the assumption that the errors are independent, the results of a simulation run, stated in terms of components of variance, can be represented as:

$$Q+M+R+V+T-C=U, \quad (8)$$

where Q is the variance of error in the computation of discharge that results from measurement error, from error in rating analysis, and from undefined rating changes. M is the variance resulting from the approximations used in the model that results from the fact the physical laws are not exactly known; where known, these laws may be approximated for convenience or speed in computation. Both Q and M remain the same for all three sets of data. R is the bias error resulting from the use of incorrect mean annual rainfall values for the basin. The purpose of the adjustments for data set B was to minimize this bias as much as possible for the given amount of data. This was accomplished, as stated, by using all the data to estimate mean basin rainfall and then adjusting each measured station mean to the estimate of the basin mean. V is error introduced owing to the fact that a point measurement of volume for a given storm differs from the mean basin volume for that storm. Adjustments made to obtain input set C were intended to minimize this error component. This was accomplished by using all data available to estimate mean storm volume for each storm. T is error introduced by the fact that point measurements of time variability of intensity during a storm differ, and any point measurement differs from an "effective time distribution" which best represents average conditions over the basin for simulation purposes. Probably, the only way to minimize the component V would be to use an input that varies over the basin. C is the curve-fitting error introduced into the model parameters by a fitting process. The parameter values are perturbed from a global "best" set of values in order to minimize the fitting criterion, U, so that C is negative in sign. For use of the model in prediction, the curve fitting adds to the error. (See table 2.)

The fitted-error criteria of set A for all three stations are closely similar to those for set B, although rainfall values for set A are not adjusted to mean basin conditions. The bias in the recorded rainfall at each station was compensated for by the curve-fitting ability of the model to adjust parameter values. On the basis of these data, bias in amount of recorded rainfall affects the resulting fitted-parameter values, rather than the accuracy of fit. Although the change in value of the fit criterion was less than 1 percent, the parameter values for station 338 changed so much that the parameter values for set B have a maximum of 1.36 for the

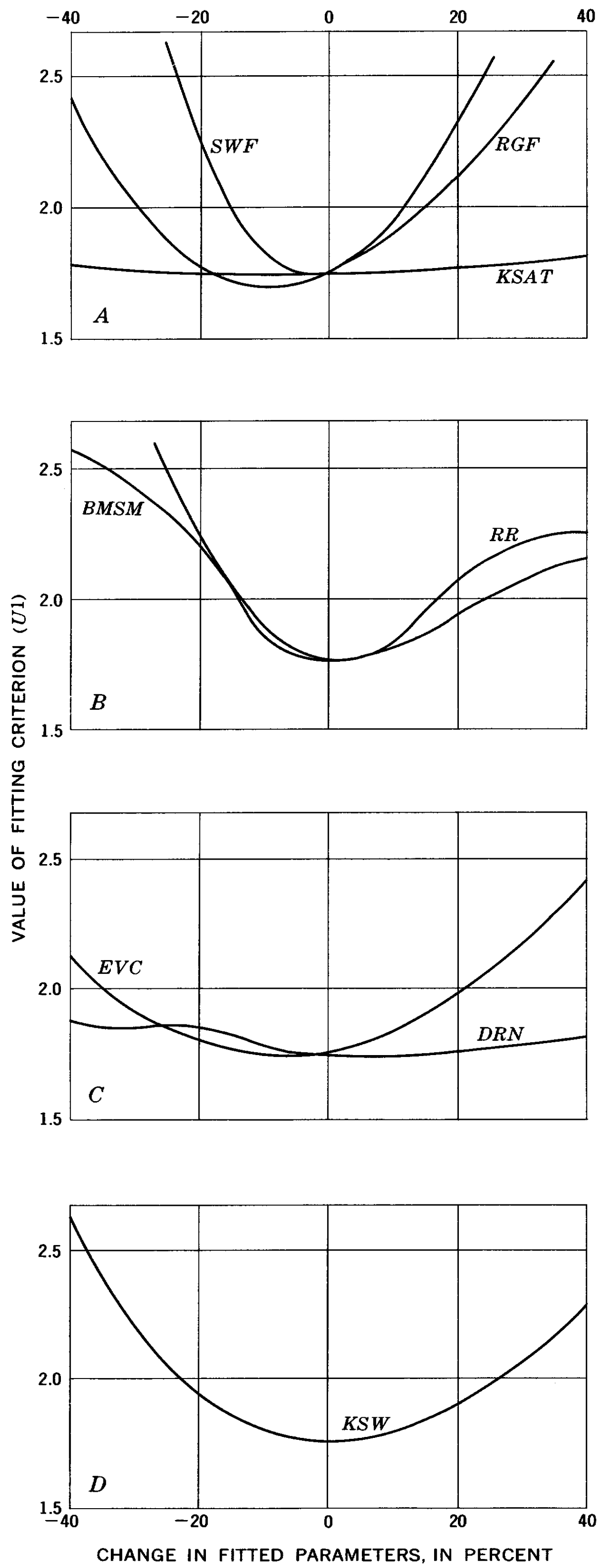


FIGURE 10.—Typical response curves, showing sensitivity of fitting criterion to percentage changes in parameter values. Results are for station 60 time distribution applied to Thiessen weighted storm-rainfall volumes.

ratio of highest to lowest value, the ratio for parameter *EVC*. For set *A*, five parameters (*SWF, RGF, BMSM, EVC,* and *DRN*) had ratios greater than 1.36. The fitted-parameter value for station 338 is one of the extreme values for each of those five parameters in both sets *A* and *B*. Thus, the errors seem to be transferred from the data to the parameters, as is particularly evident for station 338.

Input set *C* contains variability among the three inputs only in the time distribution of rainfall. The goodness of fit for this set ranged from 0.098 for station 60 to 0.153 for station 338. Converting the range value of 0.055 to an average percentage error for the peak discharges yields an estimated 23-percent error in peak-discharge reproduction, introduced by time variability alone. The fitted parameter for a basin having this degree of variation in rainfall patterns also reflects the relative smoothing action introduced by the model and, hopefully, by the hydrology; nonetheless, an average error of as much as 20 percent for simulated flood peaks can be introduced by the time-distribution error alone. Considering only the two better, or seemingly more representative gages, the difference in fitted *U*1 values is 0.017, which gives an average percentage error of 13 percent, introduced by time-distribution error in a good record.

In set *C*, the most representative gage, in terms of goodness of fit, was that closest to the center of the basins; the least representative was that on the perimeter and at the highest elevation of the basin. Therefore, relative representativeness was found to be about as expected.

Input set *B* contains both time-distribution errors within a given storm and storm-volume errors. The records were adjusted to minimize only the station bias in relation to basin mean annual rainfall. The results of input set *B* runs indicate that station 477 probably is the most representative station for predicting storm volumes, just as results of input set *C* runs indicate that station 60 probably is the most representative for time distribution of rainfall during a storm.

An estimate of the volume-error component for station 60 should be, approximately, the sum of the differences between the values of the objective functions for the runs of input sets *B* and *C* for the two stations. This follows from the fact that the *B* runs contain both volume and time errors. Therefore, other errors being constant,

$$V_{60} - \mathrm{V}_{477} = U_{60} - U_{477} + T_{477} - T_{60} = 0.022 + 0.017 = 0.039$$

yields an estimate for the volume-error component. Thus, volume errors can introduce as much as 0.04 to *U*1, which is on the order of 20 percent errors. The compounding of the time-distribution errors of station 477 and the storm-volume errors of station 60 would give a *U*1 value of 0.057, which leads to a possible combined rainfall-data-error component on the order of a 24-percent standard error.

EFFECT OF SCREENED DATA

All data used in fitting was screened for gross flyers, or outliers. The fitted parameters will predict within the indicated range of accuracy for other data containing the same range of errors as in the screened data. The screened data used for fitting contain the usual range of errors normally introduced. However, grossly inadequate or unrepresentative data will produce outliers well beyond the errors of the indicated prediction. If data are grossly in error, modeling results using such data should also be expected to be in error.

ACCURACY OF SIMULATION FOR SANTA ANITA BASIN

In general, accuracy of simulation of flood peaks for the 18 peaks used in the analysis was on the order of a standard error of 32 to 35 percent. Errors introduced by rainfall variability over the basin were on the order of 24 percent. Assuming that data errors and model errors were independent, other sources of error are believed to have contributed about the same amount to the total error. This follows from the fact that, with independence of errors, variances should be additive. Therefore, the variance contributed by data error ($24^2 = 576$) plus that contributed by model approximations (*M*) is equal to the total variance ($34^2 = 1{,}156$). To reduce errors of simulation on this basin, the rainfall input must be refined by the use of information from more than one gage or by some means of using estimates of areal variability in the model other than by the assumption of uniform-rainfall distribution, as is assumed in the model.

BEETREE BASIN

GENERAL PHYSIOGRAPHY

Beetree Creek drains an area of 5.41 square miles (14 sq km) of rough terrain near Swannanoa, N.C., on the western slope of the Great Craggy Mountains in the Blue Ridge province of the Appalachian Highlands (Fenneman, 1938). Land and channel slopes are steep, with elevations ranging from 2,700 feet (820 m) at the stream-gaging station to 5,600 feet (1,700 m) at the headwater-drainage divide. The basin is approximately rectangular, having a main channel length of about 3.2 miles (5.1 km) and an average width of about 1.5 miles (2.4 km). The index of channel slope, given by the ratio of fall over the reach of channel from 0.1 to 0.85 of main channel length, is 490 feet per mile (0.00928 ft per ft). The predominant soil is mapped as "stony rough land of Porters soil material" and described as a gray-brown podzolic type derived from granite, gneiss, and schist (Goldstone and others, 1954). Practically all the land supports native forest, with small areas of pasture at lower elevations.

PRECIPITATION

The Tennessee Valley Authority has operated a recording rain gage since 1935 at the Beetree Dam, 4,000 feet (1,200 m) downstream from the stream-gaging station. For the period 1935–59 the mean annual precipitation was measured as 46.4 inches (1.18 m) (Tennessee Valley Authority, 1961). In 1948 new equipment was installed for the recording gage, and problems of calibration caused the installation of a nonrecording rain gage beside the recording gage. In addition, a recording gage has been maintained at various points in the upper area of the basin, as indicated in figure 11.

Data for 40 flood events that occurred during the period from April 1936, through October 1964 were assembled by personnel of the U.S. Geological Survey from published records and copies of original recording charts. Storm-period rainfall data were compiled on the basis of 15-minute time intervals. An analysis of annual rainfall data indicated that an inconsistency occurred in the Beetree Creek dam record in 1949. A review of the history of the rain gage showed that a change in instrumentation was made in July 1948, when the originally installed Ferguson recording gage was replaced by a Universal recording gage. On the basis of this information, 16 flood events prior to July 1948 were selected for detailed study.

STREAMFLOW

The streamflow data used for fitting the model to the Beetree Creek basin were those for the U.S. Geological Survey gaging station near Swannanoa, N.C. The site has been gaged since 1926. The mean discharge during the period 1926–60 was 10.4 cfs (0.29 m^3 per sec) or 25 inches over the basin (0.64 m). The maximum flow of 1,370 cfs (39 m^3 per sec) occurred August 13, 1940. The peak discharges during the periods selected for study ranged from 82 to 1,370 cfs (2.3 to 39 m^3 per sec), as shown in table 7.

TABLE 7.—*Storm-period data*

Storm No.	Date	Storm rainfall (in.)	Peak discharge (cfs)	Surface runoff (in.)
	Sample A			
1	Apr. 4, 5, 1936	2.08	220	0.66
3	Nov. 14, 15, 1938	2.29	82	12
5	Aug. 17, 18, 1939	2.49	236	.50
7	Aug. 29, 30, 1940	7.36	1,180	4.28
9	Aug. 24, 25, 1941	1.22	94	.15
11	Mar. 8, 9, 1942	1.27	151	.43
13	Sept. 20, 21, 1944	1.42	115	.09
15	Oct. 5, 6, 1945	2.22	117	.39
	Sample B			
2	Oct. 15, 16, 1936	3.08	218	0.62
4	Jan. 29, 30, 1938	1.74	167	.43
6	Aug. 11, 13, 1940	10.33	1,370	4.42
8	Dec. 27, 28, 1940	2.59	263	.59
10	Feb. 16, 17, 1942	1.72	107	.26
12	Dec. 29, 30, 1942	2.06	208	.74
14	Mar. 26, 27, 1945	1.88	100	.28
16	Feb. 10, 11, 1946	1.82	141	.41

EVAPORATION

Daily values of pan evaporation were obtained from the Tennessee Valley Authority, which maintains a climatic station 4,000 feet (1,200 m) downstream from the gaging station, at an elevation of 2,540 feet (770 m). The evaporation record has been collected since 1935, and during the period 1935–59, the average annual pan evaporation was 39.9 inches (1.01 m).

PARAMETER DEFINITION

To facilitate a split-sample comparison of the results of simulation, the screened test sample of 16 storms was divided into two sets of eight storm events each. To achieve an approximate balance in the range in magnitude of peak-discharge rates represented in each sample, the odd-numbered events were selected to make up sample A, and the even-numbered events were assigned to sample B. A summary of the storm-period data appears in table 7.

Three types of optimization were performed on the pre-1948 flood events. First, sample A was used for fitting, and optimum model parameters were derived to predict the events of sample B. In the second, sample B was used for fitting to produce a set of optimum parameters used to predict sample A. In the third, all 16 events were used to de-

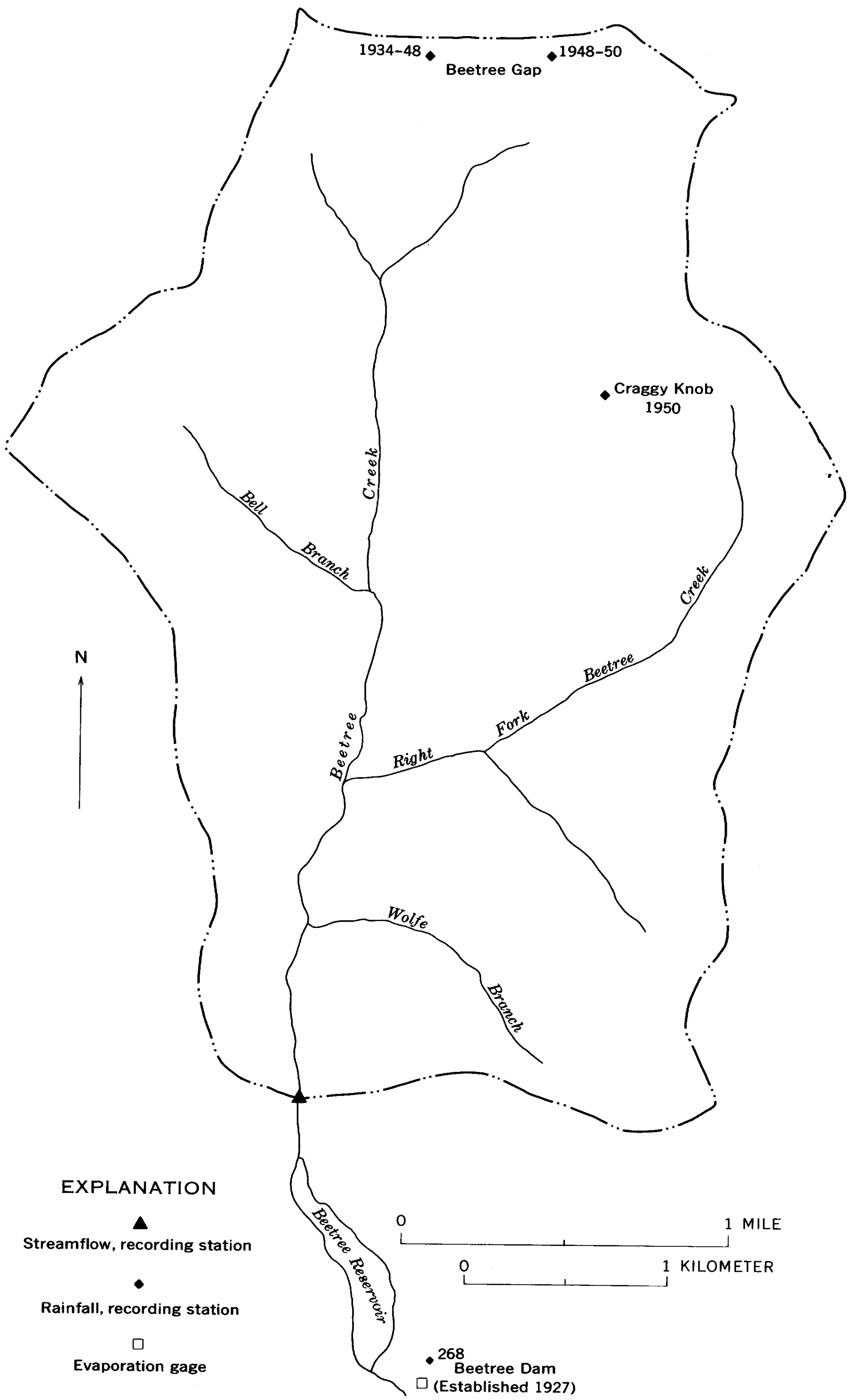


FIGURE 11.—Beetree Creek basin, above the stream-gaging station near Swannanoa, N.C.

termine the best-fit parameters for the pre-1948 record.

In each optimization run, a 5-week period of daily rainfall and pan evaporation was monitored, prior to the first storm event, to reduce the effect of arbitrarily initializing storage values for *SMS* and *BMS* (0 and *BMSM*, respectively). A similar lead-in period was used for all basins and for all results shown in this paper. In addition, initial optimization runs for all three types were started with the same set of initial-parameter values. These were assigned on the basis of (1) assumptions about average soil characteristics, (2) an estimate of the ratio of potential evapotranspiration to pan evaporation, and (3) the recession and timing characteristics of observed flood hydrographs.

Results for the three optimization runs are given in table 8. Both the optimum fitted-parameter values

TABLE 8.—*Results of fitting of model parameters to data and of split-sample testing for Beetree Creek near Swannanoa, N.C.*

[Figures in parentheses are root mean square error presented as average percentage]

Parameter identifier [1]	Optimum fitted-parameter value		
	Sample A	Sample B	All storms
SWF ________(in.)_	3.36	4.26	3.62
*KSAT*_(in. per hr)_	0.101	0.097	0.095
KSW ________(hr)_	4.97	6.24	5.67
EVC ______________	0.597	0.541	0.58
BMSM ______(in.)_	1.60	1.67	1.87
RGF ______________	14.0	8.15	14.0
RR _______________	0.78	0.81	0.75
*DRN*__(in. per hr)_	0.0050	0.0051	0.0048
U3:			
A ____________	0.069(27)	[2]0.191	0.074(27)
B _____________	[2]0.132	0.099(32)	0.107(33)
All ___________	0.101(32)	[2]0.145	0.090(30)
Test adjusted [3] _	0.079(28)	0.098(32)	_________

[1] For explanation of parameter identifier codes, see p. B5; for explanation of *U3*, see p. B9.
[2] Average error not computed.
[3] Peak most in error is removed from the predicted set.

and the fitted-objective-function values are shown. In addition, for each set of eight peaks used for fitting, the remaining set of eight peaks is used as a test sample, and the accuracy of prediction is shown. An adjusted accuracy of prediction is also given, in which the peak value most in error is removed from the predicted set, to give some indication of the effect of extreme errors on the fitting criterion.

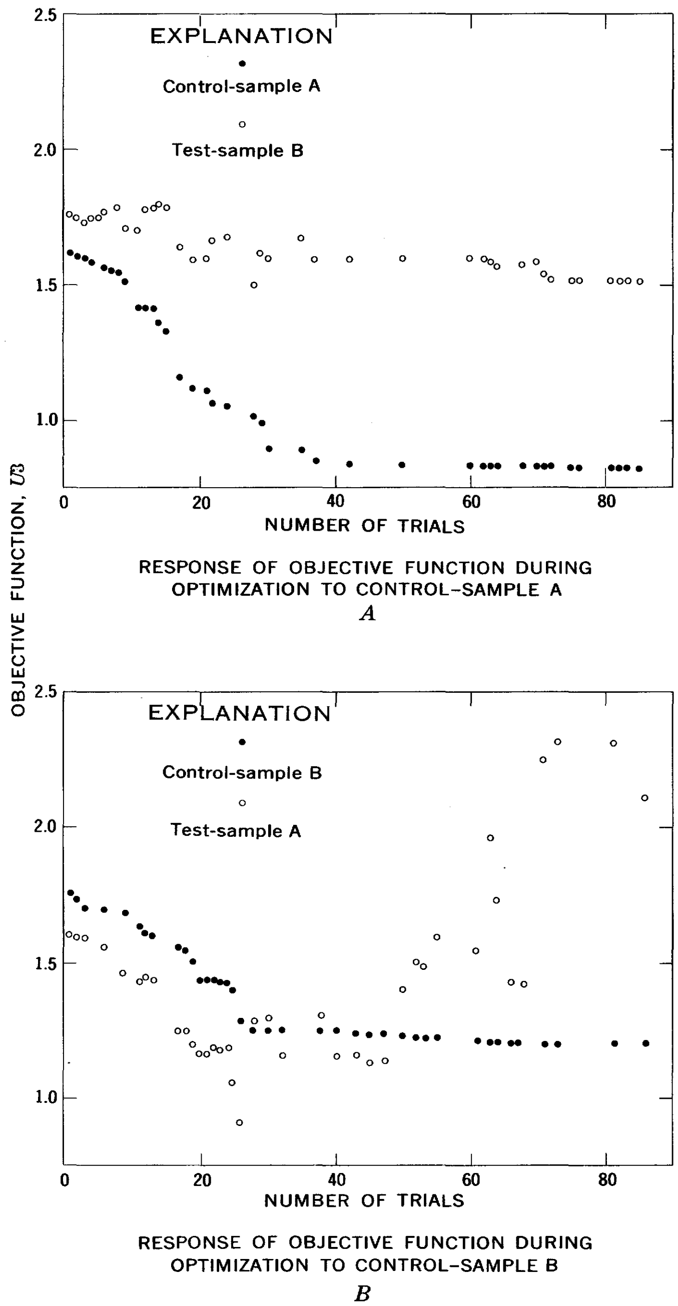


FIGURE 12.—Response of objective function during optimization with the split sample for Beetree Creek basin. The control sample in each response is included in the optimization procedure, and the concurrent value of the objective function for the test sample is shown for comparison.

RESULTS AND CONCLUSION

The response of the objective function during two optimization runs is shown in figure 12. Figure 12*A* shows the response with sample A as the control used for estimation of parameters, and the corresponding response for the test-sample B used for independent prediction of flood peaks. Similarly, figure 12*B* illustrates the results of optimization with sample B used as the control. In both responses, the rate of improvement of the objective function for the control samples decreased markedly, with little progress achieved after about 30 trials, when a plateau of best fit was noted. Rapid improvement of the objective function during the early stage of fitting, followed by an extended period of decreasing improvement, is a characteristic of the optimization procedure. Figure 12*A* shows that test-sample B is virtually unaffected by, and independent of, parameter adjustments made to improve the goodness-of-fit measured over control-sample A. However, figure 12*B* shows that the response of the error criterion for test-sample A is strongly related to that of control-sample B during the early stage of optimization. Eventually, the response diverges, becoming progressively worse after a near-optimum solution has been achieved for the control sample.

The degradation of the error criterion measured over test-sample A (fig. 12*B*) can be attributed to the influence of episodes of low magnitude that produced highly variable simulated-flood runoff in response to small changes in the parameters associated with antecedent-moisture accounting. However, the variable response of these events does not appear to bias the parameters generated from a control sample in which they are included. For example, the results of simulation for test-sample B, using parameter values derived for control-sample A, compare favorably with the results based on optimization. Furthermore, the results of simulation for test-sample A are similar to those based on optimization, when the influence of those events is discounted. With the exclusion of event 9, for instance, the objective function for test-sample A would be reduced by about 50 percent and would compare favorably with a best-fit results of 0.069, illustrating the fact that an understanding of the distribution of error is important in evaluating the results of optimization.

The simulated response from the split-sample fitting and testing procedure is shown in figure 13. Figure 13*A* is a scatter diagram of observed versus simulated flood peaks based on optimization to sample A. Similarly, figure 13*B* shows the observed versus simulated peaks based on optimization to sample B. Figure 13*C* shows the scatter of fit, using all 16 events in the optimization. The distribution of errors is related both to the approximations and simplifications inherent in the hydrologic model and to the errors in storm rainfall, known to vary considerably throughout the area.

The analysis of objective-function response to change in optimum-parameter values offers a means of evaluating the significance of the optimum solution and illustrates interaction between individual

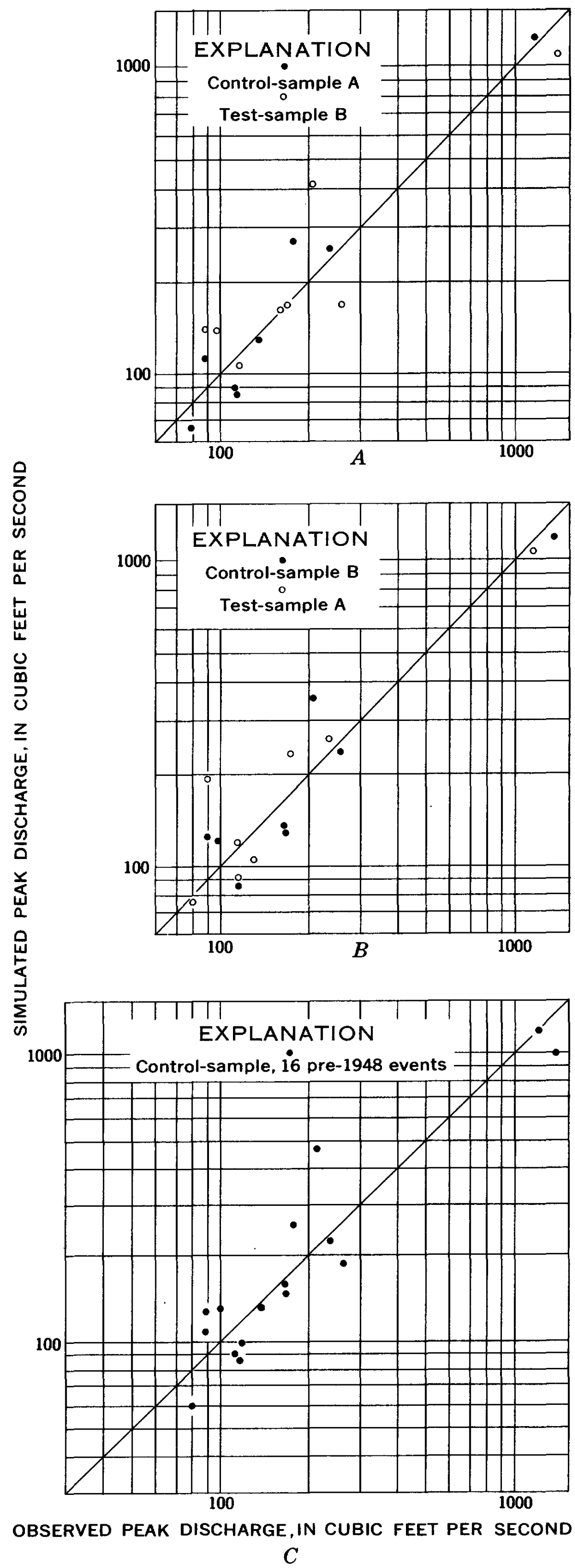


FIGURE 13.—Scatter diagrams for simulation results in the Beetree Creek basin. In *A* and *B*,

the results are shown for the split-sample optimization, in which the control sample used for the optimization and the test sample are plotted together for comparison. In *C* the results of optimization, using all 16 events with no split-sample testing, are shown.

parameters and groups of parameters. However, the objective function will be importantly influenced by the nature of the events over which it is computed and may not reflect the overall significance of model parameters. For example, figure 14 shows the response of the objective function, at 5-percent increments from the optimum value of the parameter *RR*, for both control-samples A and B (*RR* = 0.78 and 0.81, respectively). The plots indicate that optimization provided best-fit solutions for both samples, in that the objective function would be degraded by either positive or negative incrementations. However, the objective function computed for sample B is much less sensitive to the parameter *RR* than is that for sample A. The sensitivity of

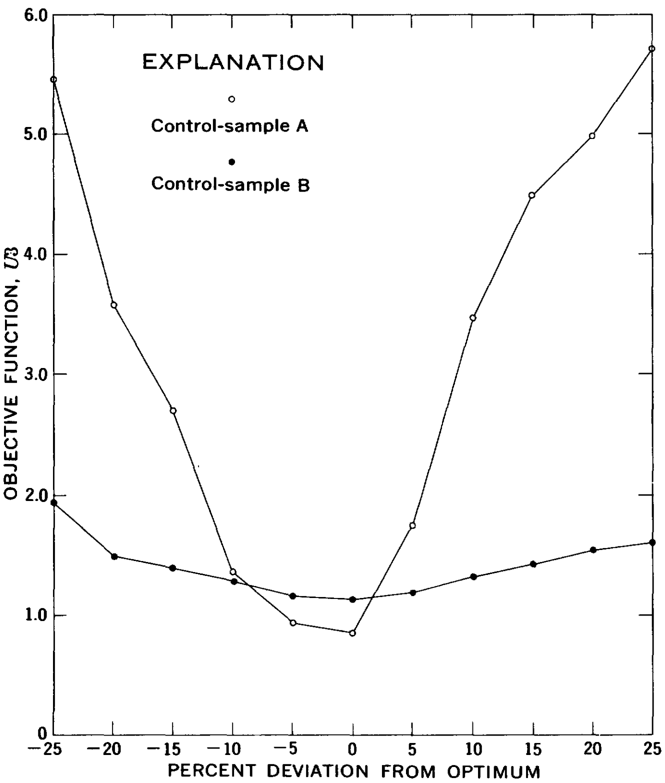


FIGURE 14.—Response of the objective function to changes from the optimum value of parameter *RR*. Sample A is much more sensitive to changes in *RR*, indicating that antecedent-soil-moisture conditions are more important in the determination of flood volumes and peaks for that sample.

RR for control-sample A results from the critical nature of antecedent-soil-moisture conditions in determining the peak of several of the smaller storms. The sensitivity for sample A is highly related to one event. Deletion of event No. 9 has little effect for drier conditions (*RR* small), but brings control-samples A and B into relative agreement for wetter conditions (*RR* large). Apparently too high a value of *RR* causes event 9 to be overestimated, and the optimum value (0.780) is a result of reducing this value sufficiently to estimate event 9 without reducing the accuracy of estimation of other events. Note that without event 9, a value of *RR* of 0.819 yields a lower error for the remaining eight events than does the overall optimum value of 0.78.

The final optimization to determine best-fit parameters for the pre-1948 flood events produced an objective function of 0.090. Results of the optimization procedure are given in table 8 for several different test runs. With the sample of 16 events, the model produces a fit very similar to that achieved for the smaller control samples. For example, the magnitude of errors in the optimum solution for all storm events was only 8 percent greater than the average of the objective functions for the control-samples A and B.

Inspection of objective-function sensitivity for each of the three control samples indicated a consistent hierarchy of parameter influence. The parameters associated with the method of antecedent-moisture accounting (*RR, EVC, DRN*) grossly controlled the objective function. The Philip infiltration parameters (*SWF, KSAT*) and the routing coefficient (*KSW*) were intermediate in importance. The range factor (*RGF*) and field-capacity-moisture storage (*BMSM*) had little influence on the objective function for the various control samples and may be poorly identified.

A sufficient number of events is not the only requirement to obtain a meaningful identification of model parameters. Equally important is the need for a wide range in both antecedent and storm-period conditions. For example, if all the flood events included in a control sample were associated with similar antecedent conditions, then one or more of the parameters may exert little influence on the results of simulation and be poorly identified, and others may be "overdetermined." In addition, an interpretation of the hierarchy of parameter sensitivity must be tempered by not only an understanding of the limitations of the model and its lack of equivalence to the physical system, but also by careful evaluation of the characteristics of the criterion used to express the sensitivity. The response of low-magnitude events to small changes in some parameter values prevents a straightforward assessment of model sensitivity and demonstrates the need for development of alternative measures of sensitivity.

The authors cannot overstress that in the split-sample testing for this station, eight events were used to determine eight model parameters. This clearly places this study in the area of small-sample theory. The relative consistency of results, both in accuracy and in derived parameter values, is therefore very encouraging. The various results of split-sample testing indicate that the root-mean-square error of prediction is about 30 percent for these data, with, apparently, about one small storm being grossly in error for each test.

LITTLE BEAVER BASIN

GENERAL PHYSIOGRAPHY

The Little Beaver Creek drainage basin is a 6.41-square-mile (16.6 sq km) area of the Gasconade Hills in the Ozark Mountains, just west of Rolla, Mo. The range in elevation is from 790 feet (240 m) at the U.S. Geological Survey gaging station to 1,180 feet (360 m). The gently rolling hills are covered with a stony porous soil. Rainfall in the southerly facing basin is fairly evenly distributed throughout the year, although the amounts are somewhat greater in the summer than in the winter.

PRECIPITATION

The U.S. Geological Survey maintains a recording rain gage—the Rolla 3–W gage—near the center of the basin. (See fig. 15.) The record obtained from that gage was used for simulation of rainfall for the entire 1948–64 period of record. In addition, a rain gage is maintained at the Missouri School of Mines and Metallurgy, about 1 mile east of the east boundary of the basin. The average annual rainfall during the period 1948–64 was 36.7 inches (0.93 m).

Data for 29 flood events during the period 1948–64 were reduced to rainfall intensities for 15-minute intervals. These storms were split into a control sample of 14 events during the period 1948–53 and a test period of 15 events during the period 1954–64.

STREAMFLOW

The streamflow data used for fitting the model to the Little Beaver Creek basin were those recorded at the U.S. Geological Survey stream-gaging station near Rolla, Mo. The site has been gaged since 1948. The mean discharge for the period of record 1948–64 was 3.77 cfs (0.11 m^3 per sec) or 11.1 inches (0.28 m) throughout the basin. The maximum flow of 7,420 cfs (210 m^3 per sec) occurred July 17, 1958. The annual peak discharges during the period of study varied from 524 cfs (15 m^3 per sec) to 7,420 cfs (210 m^3 per sec). However, individual peaks selected for the present study were as low as 200 cfs (5.8 m^3 per sec).

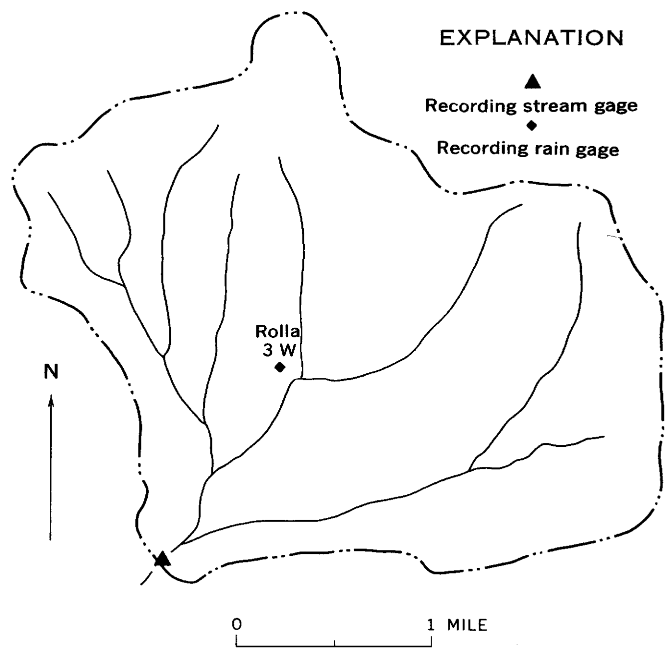


FIGURE 15.—Little Beaver Creek basin, above the stream-gaging station near Rolla, Mo.

EVAPORATION

Daily values of pan evaporation, obtained from the U.S. Weather Bureau, were recorded at the pan evaporation station at Lakeside, Mo., located about 45 miles west of the Little Beaver Creek basin and at an elevation of 595 feet (181 m). The average pan evaporation during the period 1948–64 was 53 inches (1.35 m).

PARAMETER DEFINITION

Three sets of model-parameter determinations were made, using the control period 1948–53. The results of these fittings plus two sets of starting parameters are given in table 9. The first derivation was of set 2 from the starting set 1. The accuracy of fit of 0.065 gives a standard error of fit of about 25 percent. The value of *RR* of 0.98 seemed to be

TABLE 9.—*Results of fitting of model parameters to data for Little Beaver Creek near Rolla, Mo., using the Rolla 3–W rain gage*

Parameter indicator [1]	Start: Set 1	Optimum: Set 2	Optimum: Set 3	Start: Set 4	Optimum: Set 5	Test (Set 5 parameters)
SWF (in.)	2.0	2.5	10.1	4.0	4.1	4.1
KSAT (in. per hr)	0.1	0.08	0.07	0.05	0.047	0.047
KSW (hr)	1.0	[2] 1.0	[2] 1.0	0.85	0.84	0.84
EVC	0.7	0.56	[2] 0.56	0.55	0.52	0.52
BMSM (in.)	2.0	2.8	2.3	3.0	2.4	2.4
RGF	10.0	9.4	9.3	10.0	11.7	11.7
RR	0.8	0.98	[2] 0.8	0.85	[2] 0.85	0.85
DRN (in. per hr)	0.1	0.02	0.02	0.025	0.022	0.022
*U*1	---	0.065	0.075	0.061	0.055	0.073
Standard error (percent)	---	25	27	25	23	27

[1] For explanation of parameter identifier codes, see p. B3–B5; for *U*1, see p. B9.
[2] Parameter values held constant for the run indicated.

high and was believed to be too much of a curve-fitting parameter. Therefore, set 3 was derived by fixing the evaporation pan coefficient (*EVC*) at its optimum value and the daily-rainfall-infiltration coefficient (*RR*) at 0.8. A lower limit for *RR* should be 0.7, because the mean annual flow is about 30 percent of the mean annual rainfall. Therefore, 0.8 to 0.85 is a reasonable value. The accuracy of fit for the parameters for set 3 is 0.075, or about 27 percent.

On the basis of hydrograph plots for the results of set 3, the routing component was recomputed. Both the time-area histrogram and the surface routing coefficient (*KSW*) were revised, and *KSW* was included in the next optimization run. *RR* was held fixed at 0.85. The fit of set 5 is 0.055, which yields about a 23-percent accuracy. The test group of 15 floods during the period 1954–64 were then simulated with set 5 parameter values. The accuracy of fit for the test-set was 0.073, which yields an estimate of 27 percent for a standard error of prediction.

A separate fitting for the Little Beaver Creek basin was made to the Missouri School of Mines and Metallurgy rain gage, which lies outside the basin. The results of the fitting are given in table 10. A comparison of rainfall volumes for the two gages and of the simulated volumes and peaks is shown in table 11.

TABLE 10.—*Results of fitting of model parameters to data for Little Beaver Creek near Rolla, Mo., using the Missouri School of Mines and Metallurgy rain gage*

Parameter indicator	Start 1	Optimum 2
SWF (in.)	4.0	1.75
KSAT (in. per hr)	.05	.063
KSW (hr)	.85	.97
EVC	.55	.39
BMSM (in.)	3.0	2.2
RGF	10.0	8.0
RR	.85	[1] .85
DRN (in. per hr)	.025	.038
*U*1 (13 events)	0.21	0.19
Standard error (percent)	46	44
*U*1 (9 events)	0.121	0.099
Standard error (percent)	35	31

[1] Parameter value held fixed for the run.

TABLE 11.—*Comparison of estimates for flood volumes and peaks for Little Beaver Creek by the use of the two rain gages*

Date	Measured: Runoff (in.)	Measured: Peak [1] (cfs)	Rolla 3–W rain gage: Measured *RF* (in.)	Rolla 3–W: Simulated Runoff (in.)	Rolla 3–W: Simulated Peak [1] (cfs)	Missouri School of Mines and Metallurgy rain gage: Measured rainfall (in.)	MSM&M: Simulated Runoff (in.)	MSM&M: Simulated Peak [1] (cfs)
6–17–48	0.12	376	1.17	0.13	351	0.76	0.21	545
6–2–49 [2]	1.05	1,228	2.59	.82	1,328	.89	---	----
7–22–49	.33	1,199	1.21	.47	1,253	.85	.47	1,247
10–11–49	2.76	3,121	4.26	2.90	2,848	[3] 6.05	4.64	3,589
10–20–49	.55	1,142	1.33	.79	1,639	.95	.54	1,321
1–13–50	.64	1,348	1.03	.38	990	1.10	.49	1,124
4–10–50	.24	811	.85	.37	1,053	.88	.52	1,392
5–19–50	1.06	1,575	1.85	.94	1,446	1.77	.92	1,145
5–26–50	.25	742	1.34	.43	1,167	[3] .48	.15	406
6–9–50	1.78	4,177	3.36	1.73	3,683	[3] 2.01	1.33	2,461
6–22–51	.31	848	1.16	.38	979	1.08	.58	1,338
6–30–51	1.64	2,079	2.40	1.35	1,514	2.67	1.69	2,218
4–23–53	.73	2,054	1.56	.74	1,829	[3] 2.66	2.19	5,380
5–17–53	.15	416	.56	.10	301	.38	.11	308

[1] Peak rates are surface-runoff rates only; base flow has been subtracted from the measured rate.
[2] Not included in the Missouri School of Mines and Metallurgy optimization because the measured storm runoff exceeded the measured rainfall.
[3] Missouri School of Mines and Metallurgy gage storm rainfall apparently grossly in error.

Two conclusions can be drawn from this second fitting. First, the School of Mines gage is not an adequately representative measure of rainfall on the basin, even though it is just outside the basin. The growth of trees in the vicinity of the gage created an increasing amount of interference effects. In 1956 the gage was moved 100 feet to the south to correct the problems of measurement caused by the trees. The accuracy of fit is 44 percent. Five measured storm volumes are grossly different from those measured for Rolla 3–W. One of the five was excluded from the fitting, but the other four influenced the fitting and probably caused the higher value of *KSW* and reduced volumes of infiltration. However, the School of Mines gage does given some indication of the effect of variability of storm volume over the basin. For nine of the 14 storms, the Rolla 3–W gage simulation overestimated peaks when its measured storm volume exceeded that at the School of Mines. This held true for seven of the nine peaks above 1,000 cfs, and for nine of the 12 above 500 cfs. Therefore, although the School of Mines gage alone gives much less accurate results than those for Rolla 3–W, the two sets of results used together could give a better estimate for flood-peak simulation.

COMPARISON OF DERIVED PARAMETER VALUES

The model is based, at least in part, upon a simulation of the physical processes operating upon the basin modeled. The parameter values derived should therefore be related to the physical parameters involved. However, the model is a bulk-parameter model—that is, it models all the infiltration in the basin as if it were uniform over the basin. The parameter values derived are in some way optimal average values and can be, at best, indices to the "true" parameters or to their distribution over the basin.

If the model is to be used in regional studies, it can serve either of two purposes: First, it can be used to extend a record in time. For that use, the most important consideration is the error of prediction. For the three basins for which results are presented in this study, a standard error of prediction of about 30 to 35 percent was achieved. This was found to be largely dependent upon the accuracy of rainfall measurement. In particular, the use of a single rain gage to estimate rainfall variability over the basin seems to introduce an error of about 20 to 25 percent into the simulation. A decision must be made as to whether the point rainfall data that produce errors of this magnitude add information to the record. Second, the model can be used in regional studies by relating the derived parameter values to physical characteristics measurable in the basins which are simulated. The derived relations could then be used to estimate parameter values for ungaged sites. The accuracy of prediction in this use would be a function both of the errors in rainfall input and of the errors in predicted values for the model parameters. This accuracy of prediction would be compared to the accuracy of flood-frequency methods presently in use.

The derived parameter values for the three basins used in this developmental study are shown in table 12. All are reasonable values. However, there are too few results to draw any general conclusions at this time. Each parameter will be discussed as to its relation among stations and the reasons for variability. *RR* is a measure of percentage of infiltration for daily rainfall amounts for periods not simulated in detail, either because rainfall amounts are too small or because records are not accurate enough to use for detailed simulation. Also shown in table 12 are values of 1 minus the ratio of measured runoff to measured rainfall for each basin during the study period. This sets a lower limit on *RR*, and for each basin the fitted value exceeds this lower limit. Actually, the lower limit should be somewhat higher, because all base flow should be subtracted from the runoff to derive the limiting value. Beetree Creek basin has the highest base flow; thus, the fitted value exceeding the limiting value by a relatively large amount is consistent.

TABLE 12.—*Summary of results of optimization for the three study basins*

Parameter indicator	Basin		
	Santa Anita Creek	Beetree Creek	Little Beaver Creek
SWF (in.)	20	3.6	4.1
KSAT (in. per hr)	.32	.1	.05
KSW (hr)	2.7	5.7	.84
EVC	.73	.58	.52
BMSM (in.)	3.5	1.9	2.4
RGF	6	14	12
RR	1.0	.75	.85
DRN (in. per hr)	.058	.005	.022
L (mi)	4.7	3.2	3.25
S (ft per ft)	.12	.00928	.0124
$L\sqrt{S}$	13.5	33.5	29.3
$1-(RO/RF)$	.74	.46	.7

KSAT, *SWF*, and *RGF* determine the infiltration equation during detailed storm simulation and, therefore, should be discussed together. *SWF* determines the soil-suction characteristics for wet conditions, *SWF* multiplied by *RGF* determines them for dry conditions, and *KSAT* represents the soil's saturated permeability, or minimum infiltration rate. The range of soil suction is from 4 to 50 inches (10 to 125 cm) for both Beetree Creek and Little Beaver Creek basins, and is from 20 to 120 inches (50 to 300 cm) for Santa Anita Creek basin. Comparable experimental ranges for a sandy loam are about 30 to 130 centimeters for Yolo sandy loam, and 30 to 200 centimeters for Yolo silt loam (Colman and Bodman, 1944). Seemingly, the minimum infiltration rates are anomalous for the measured basins, in that 0.3 inches per hour seems to correspond to a sandy loam rate, whereas 0.05 to 0.10 inches per hour seems to correspond to a rate for a silt loam (Musgrave, 1055). Some attempts should be made to relate the fitted values to ring infiltrometer or other data collected for study basins.

BMSM represents an effective maximum soil-moisture retention, and the low values indicate shallow soils. Of the three study basins, Beetree Creek basin appears to have the thinnest effective soil mantle, and Santa Anita Creek basin, the least shallow. This agrees qualitatively with descriptions of the geology and soils. *DRN* represents the drainage rate from the saturated layer to the unsaturated layer. This parameter is critical for determining the antecedent conditions for some storms, but has no effect on most storms. Therefore, it is probably poorly defined for all basins. The derived values are considerably less than *KSAT* in each basin (which is as expected), but nothing can be said as to the reasonableness of the values otherwise.

EVC should represent an effective average pan coefficient for the basin. However, this meaning is compounded by the fact that for each basin a correction also must be made to adjust the pan evaporation to average basin conditions. For Little Beaver Creek basin, the nearest pan evaporation record was 45 miles away; for Santa Anita Creek basin it was 10 miles away; for Beetree Creek basin the evaporation record was nearby, but at a lower elevation. All records are for U.S. Weather Bureau Class A pans, for which the pan coefficient should range from 0.6 to 0.8. *EVC* should be somewhat lower than these values, if an altitude correction is involved. Little or no altitude correction should be necessary for Santa Anita Creek basin, as the pan is at an elevation well above the lowest point in the basin. Both of the other records are for sites at elevations below the lowest point in the basin, and for Little Beaver Creek basin, considerably lower. Therefore, the derived values seem to be of the right order of magnitude.

Neither the hydrograph recession rate (*KSW*) nor the translation hydrograph ordinates enter directly into the fitting process, as both are derived from the measured hydrograph shapes. The Little Beaver basin has an unusually rapid recession. Values of $L/\sqrt{S}$ are shown, where L is the length of the main channel, in miles, and S is the slope of the basin, in feet per foot, for the reach from 10 percent to 85 percent of the distance from the discharge gaging station to the point on the ridge that represents the extension of the main channel (Benson, 1962). Although $L/\sqrt{S}$ values for Beetree Creek and Little Beaver Creek basins are very similar, the values of *KSW* differ by a ratio of 7. Santa Anita Creek basin is consistent with Beetree Creek basin in this regard, in that both $L/\sqrt{S}$ and *KSW* are about half the values for Beetree. The reason for the anomalous value for Little Beaver Creek basin is unknown, but it may be related to the drainage pattern. Both Santa Anita Creek and Beetree Creek basins are dendritic, whereas Little Beaver Creek basin seems to be more palmate.

SOURCES OF ERROR AND THEIR IMPACT

The accuracy of fit for the three basins studied was similar. An accuracy of about a 30-percent standard error is obtainable. The detailed study for Santa Anita Creek basin indicated that about a 20-percent standard error was attributable to rainfall sampling alone. If the rainfall errors are independent of other modeling errors, then

$$RE^2 + ME^2 = TE^2,$$

where *RE* is the modeling error resulting from rainfall-input error, *ME* is other modeling error, and *TE* is the total error of simulation. For the Santa Anita Creek basin,

$$20^2 + ME^2 = 30^2; ME^2 = 500.$$

According to Eagleson (1967), if one rain gage gives an error of 20 percent, then two properly placed rain gages would give an error of about 15 percent. The use of the information from two gages with the present model structure should thus result in an error of

$$TE^2 = 15^2 + 500 = 725,$$

or a standard error of 27 percent, rather than 30 percent.

The improvement of the structure of the model can also lead to more accurate prediction. If the model error were cut in half, the resulting standard error would be

$$TE^2 = 20^2 + 250 = 650,$$

or a standard error of 25 percent. Thus, to achieve any major improvement in the accuracy of simulation, the improvement in both the model and the accuracy of rainfall input must be simultaneous. Model improvements alone will increase the accuracy of prediction, but there will be a limiting accuracy which must be accepted if the constraint of a single rain gage is to be maintained.

The marginal gains in accuracy which should be expected from model improvement influence the strategy for judging model improvements. Changes should be accepted as improvements if they (1) add to the simplicity of the model, (2) aid in the regionalization of the parameter values, or (3) gain accuracy. The search will continue for a better model, but, to date, an imperfect model must be accepted.

CONCLUSIONS

The development of the model demonstrates the feasibility of rainfall-runoff simulation. Such simulation is not new, so that such a demonstration of feasibility is not unexpected. However, the constraints placed upon the model developed were that a single rain gage be used for simulation on a basin. This led to the development of a bulk-parameter model. Thus, model parameter values are indices of average conditions on the basin that only approximate real parameter values. Both the errors of rainfall input and the lack of model equivalence to the physical prototype limit the prediction ability of simulation. These two sources of error are of similar order of magnitude for the basins studied; hence, major gains in accuracy will depend upon simultaneous improvement in both. The limit of accuracy of prediction of flood peaks by simulation with a single rain gage seems to be on the order of about 25 percent, and this level of accuracy should be understood to have resulted from the imposed constraint.

REFERENCES CITED

Benson, M. A., 1962, Factors influencing the occurrence of floods in a humid region of diverse terrain: U.S. Geol. Survey Water-Supply Paper 1580–B, 64 p.

———1964, Factors affecting the occurrence of floods in the Southwest: U.S. Geol. Survey Water-Supply Paper 1580–D, 72 p.

Benson, M. A., and Matalas, N. C., 1967, Synthetic hydrology based on regional statistical parameters: Water Resources Research, v. 3, no. 4, p. 931–945.

Clark, C. O., 1945, Storage and the unit hydrograph: Am. Soc. Civil Engineers Trans., v. 110, p. 1419–1488.

Colman, E. A., and Bodman, G. B., 1944, Moisture and energy conditions during downward entry of water into moist and layered soils: Soil Sci. Soc. America Proc., v. 9, p. 3–11.

Crawford, N. H., and Linsley, R. K., 1966, Digital simulation in hydrology: Stanford, Calif., Stanford Watershed Model IV, Tech. Rept. 39, Civil Eng. Dept., Stanford Univ., 210 p.

Dawdy, D. R., and O'Donnell, Terence, 1965, Mathematical models of catchment behavior: Am. Soc. Civil Engineers Proc., v. 91, no. HY 4, Paper 4410, p. 123–137.

Dooge, J. C. I., 1959, A general theory of the unit hydrograph: Jour. Geophys. Research, v. 64, no. 2, p. 241–256.

Eagleson, P. S., 1967, Optimum density of rainfall networks: Water Resources Research, v. 3, no. 4, p. 1021–1033.

Fenneman, N. M., 1938, Physiography of Eastern United States: New York, McGraw-Hill Book Co., 714 p.

Folse, J. A., 1929, A new method of estimating streamflow: Carnegie Inst. Washington Rept. 400, 237 p.

Goldstone, E. F., and others, 1954, Soil survey of Buncombe County, North Carolina: U.S. Dept. Agriculture, Soil Survey Ser. 1942, no. 6, 122 p.

Green, W. H., and Ampt, G. A., 1911, Studies on soil physics; I, Flow of air and water through soils: Jour. Agr. Research, v. 4, p. 1–24.

Horton, R. E., 1939, Approach toward a physical interpretation of infiltration capacity: Soil Sci. Soc. Am. Proc., v. 5, p. 399–417.

Kraijenhoff Van De Leur, D. A., 1958, A study of nonsteady groundwater flow with special reference to a reservoir-coefficient: Ingenieur, v. 70, no. 19, p. B87–B94.

Matalas, N. C., and Gilroy, E. J., 1968, Some comments on regionalization in hydrologic studies: Water Resources Research, v. 4, no. 6, p. 1361–1369.

Musgrave, G. W., 1955, How much of the rain enters the soil?, *in* Water: U.S. Dept. Agriculture Handb., p. 151–159.

Philip, J. R., 1954, An infiltration equation with physical significance: Soil Sci. Soc. Am. Proc., v. 77, p. 153–157.

Rosenbrock, H. H., 1960, An automatic method of finding the greatest or least value of a function: Computer Jour., v. 3, p. 175–184.

Sherman, L. K., 1932, Streamflow from rainfall by the unit hydrograph method: Eng. News Record, v. 108, p. 501–505.

Tennessee Valley Authority, 1961, Evaporation in Tennessee River Basin: Div. Water Control Planning, Hydraulic Data Branch, Rept. 0–243–310–A, 49 p. [Supp. to "Precipitation in Tennessee River Basin."]

Theis, C. V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., pt. 2, p. 519–524.

Wilde, D. J., 1964, Optimum-seeking methods: Englewood Cliffs, N.J., Prentice-Hall, Inc.

U. S. GOVERNMENT PRINTING OFFICE : 1972 O - 489-368