NATIONAL BUREAU OF STANDARDS REPORT

3795

PROJECTS and PUBLICATIONS

of the

APPLIED MATHEMATICS DIVISION

A Quarterly Report

July through September 1954

<NBS>

U. S. DEPARTMENT OF COMMERCE

NATIONAL BUREAU OF STANDARDS

THE NATIONAL BUREAU OF STANDARDS

The scope of activities of the National Bureau of Standards is suggested in the following listing of the divisions and sections engaged in technical work. In general, each section is engaged in specialized research, development, and engineering in the field indicated by its title. A brief description of the activities, and of the resultant reports and publications, appears on the inside of the back cover of this report.

Electricity. Resistance and Reactance Measurements. Electrical Instruments. Magnetic Measurements. Electrochemistry.

Optics and Metrology. Photometry and Colorimetry. Optical Instruments. Photographic Technology. Length. Engineering Metrology.

Heat and Power. Temperature Measurements. Thermodynamics. Cryogenic Physics. Engines and Lubrication. Engine Fuels. Cryogenic Engineering.

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Electronics. Engineering Electronics. Electron Tubes. Electronic Computers. Electronic Instrumentation. Process Technology.

Radio Propagation. Upper Atmosphere Research. Ionospheric Research. Regular Propagation Services. Frequency Utilization Research. Tropospheric Propagation Research. High Frequency Standards. Microwave Standards.

● Office of Basic Instrumentation   ● Office of Weights and Measures.

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NBS PROJECT

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APPLIED MATHEMATICS DIVISION

July through September 1954

U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

APPLIED MATHEMATICS DIVISION
July 1 through September 30, 1954

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CONTENTS

Status of Projects

September 30, 1954

I. NUMERICAL ANALYSIS SECTION
(Section 11.1)

RESEARCH IN NUMERICAL ANALYSIS AND RELATED FIELDS
Task 1101-10-1104/55-55

Origin and Sponsor: NBS   Authorized 8/13/54
Managers: J. Todd, P. Davis   Revised   8/29/54

Objective: To develop new numerical techniques for the solution of problems in mathematics, physics, chemistry, and engineering, and to conduct research in special topics of mathematics of immediate applicability to the techniques of computation and to the construction of mathematical tables.

Background: The use of high speed automatic digital computing machines as tools for research, development, production, and management is increasing rapidly. In order fully to exploit their capabilities and to insure the effective use of this expensive equipment it is essential to carry out research in all branches of numerical analysis.

Comments: Work on the following former tasks will be continued as part of this task:

1102-10-1104/50-1   Research in classical numerical analysis
1102-10-1104/51-6   Solution of Laplace equation by Monte Carlo method
1102-10-1104/53-15   Differential equation for nerve fiber reaction
1102-10-5116/52-33   Tables of integrals involving higher transcendental
                          functions
1102-10-5116/53-54   Water waves

Also that portion of task 1102-10-1104/50-5, Bibliography of Coding Procedures, Mathematical Tables, and Numerical Analysis, that pertains to numerical analysis, is continued under this task.

Status: NEW. H. Antosiewicz, in a new manuscript, "Stable systems of nonlinear differential equations," studies the differential equation $dx/dt=p(x,t)$ under conditions which make $x\equiv0$ a stable solution in the sense of Liapunov. He derives sufficient conditions on a perturbation $q(x,t)$ which is integrable in t over the positive half-line so that every solution of $dx/dt=p(x,t)+q(x,t)$ is stable.

In their paper, "A representation for solutions of analytic systems of differential equations," H. Antosiewicz and M. Abramowitz show that the general solution of the n-dimension vector differential equation $dx/dt=A(t,z)x$, where $A(t,z)$ satisfies certain simple conditions,

may be represented as $X(t,z)Y(t)c$, where $Y(t)c$ is the general solution of some equation $dy/dt=B(t)y$. The usefulness of the theorem is illustrated by obtaining convergent series representations for Coulomb wave functions and parabolic cylinder functions.

The study of differential equations for nerve fiber reaction (previously reported under task 1102-10-1104/53-15) by H. Antosiewicz and P. Rabinowitz is continuing. The code is being rewritten to study a second order system of nonlinear differential equations. A report on the mathematical aspects of the work performed so far has been prepared and will be incorporated in a joint report with the sponsor.

M. Abramowitz and P. Henrici have begun work on an expository account of the practical summation of series. P. Henrici has completed a paper on the "Application of two methods of numerical analysis to the computation of the reflected radiation of a point source." The paper discusses the effect of applying Aitken's δ^2-method to the summation of the series for the hypergeometric function $F(3/4,5/4;1;z)$, and then applies the method of Davis and Rabinowitz (see publications (7) and (8) below) to the study of the error due to numerical quadrature of an integral involving this function.

J. Todd completed a note on the work of L. F. Richardson. J. Cameron, M. Newman, O. Taussky-Todd, and J. Todd completed the manuscript on "Generating of random numbers on SEAC."

Publications: (1) "Coulomb wave functions along the transition line," by M. Abramowitz and P. Rabinowitz; to appear in the Physical Review. (2) "Coulomb wave functions in the transition region," by M. Abramowitz and H. Antosiewicz; to appear in the Physical Review. (3) "A representation for solutions of analytic systems of differential equations," by H. Antosiewicz and M. Abramowitz; to appear in the Journal of the Washington Academy of Sciences. (4) "Some implications of Liapunov's conditions for stability," by H. Antosiewicz and P. Davis; Journal of Rational Mechanics and Analysis 3, 447-457 (July 1954). (5) "Stable systems of nonlinear differential equations with integrable forcing term," by H. Antosiewicz; submitted to a technical journal. (6) "Computation of vibration modes and frequencies on SEAC," by W. Cahill and S. Levy (NBS 6.4); submitted to a technical journal. (7) "On a problem in the theory of mechanical quadratures," by P. Davis; to appear in Pacific Journal of Mathematics. (8) "On the estimations of quadrature errors for analytic functions," by P. Davis and P. Rabinowitz; to appear in Mathematical Tables and Other Aids to Computation. (9) "A multi-purpose orthonormalizing code and its uses," by P. Davis and P. Rabinowitz; to appear in the Journal of the Association for Computing Machinery. (10) "Application of two methods of numerical analysis to the computation of the reflected radiation of a point source," by P. Henrici; IN MANUSCRIPT. (11) "On the Lerch zeta function," by F. Oberhettinger; submitted to a technical journal. (12) "Generation and testing of random numbers on SEAC," by O. Taussky-Todd, J. Todd, M. Newman, and J. Cameron; IN MANUSCRIPT. (13) "Motivation for working in numerical analysis," by J. Todd; to appear in the Transactions of a Symposium in Applied Mathematics, held by the American Mathematical Society and the Office of Ordnance Research in Chicago, Ill., April 1954.

TURBULENT ATMOSPHERIC CONTAMINATION
Task 1101-10-1104/55-59

Origin: Applied Physics Laboratory, Johns Hopkins   Authorized 7/22/54
          University, and National Bureau of Standards
Sponsor: "   "   "
Managers: E. Marden, G. Hawkins

Objective: To devise computational methods for determining the
transport of suspended particles in the atmosphere as a result of
turbulent air flow. In particular, to evaluate the total contamination
which accumulates from a number of points and finite area sources where
the amount of contamination from a single point source can be expressed as

$$\bar{S} = \frac{Q}{(2\pi\bar{x}^2)^{3/2}} \exp \frac{-(x^2+y^2+z^2)}{2\bar{x}^2} \;,$$

and where the standard deviation in the x-direction, $\bar{x}^2$, is given by

$$\bar{x}^2 = 2\bar{u}^2 \int_0^t (t-\alpha)R(\alpha)d\alpha,$$

Q = source strength

$\bar{u}$ = intensity of fluctuating velocity
R = Lagrangian correlation coefficient
t = dispersion time.

Background: The programming of this problem for solution on SEAC
arises from a proposal made by F.N.Frenkiel, of the Applied Physics
Laboratory, to calculate the concentration of contaminants contained in
smog by application of the statistical theory of turbulence to existing
or predicted meteorological conditions and to a known or assumed distribution
of sources. Data for the Los Angeles area are to be used for a numerical
example.

Status: NEW. A code for SEAC has been prepared, and computations
have been completed for several test cases using single point sources
of unit strength and mean velocities simulating prevailing winds in the
Los Angeles area.

BASIC RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/50-2

Origin: Office of Scientific Research, ARDC, USAF,   Authorized 3/31/50
        and Office of the Air Comptroller, USAF
Sponsor: "   "   "
Managers: J. Todd, A. Hoffman
Full task description: Jan-Mar 1950 issue, p. 24

Status: CONTINUED. K. Fan, O. Taussky-Todd, and J. Todd have
revised their manuscript, "Discrete analogs of inequalities of Wirtinger."
In collaboration with I. Glicksberg, K. Fan has introduced the concept
of a "fully convex" normed linear space as a generalization of the notion
of uniformly convex space. The principal results of the manuscript they
are preparing are as follows: (1) Every fully convex Banach space is
reflexive: (2) a characterization of reflexive spaces in terms of "weak
full convexity;" (3) A mean-value theorem in fully convex Banach spaces

that generalizes G. Birkhoff's mean ergodic theorem.

K. Fan and G. Pall have determined all possible sets of n-1 complex numbers $(\beta_1,\ldots,\beta_{n-1})$ that may be the eigenvalues of a principal submatrix of a normal matrix with prescribed eigenvalues $(\alpha_1,\ldots,\alpha_n)$.

This generalizes an earlier result of H. Wielandt for the Hermitian case. A. Hoffman has pointed out that Wielandt's theorem yields a simple proof of Horn's result characterizing the set of diagonal elements of a Hermitian matrix with prescribed eigenvalues.

A first draft of a monograph "On systems of linear inequalities," has been prepared by K. Fan. Chapter I deals with purely algebraic aspects of finite systems of linear inequalities in a linear space that is not necessarily normed, topological, or finite-dimensional. Most of the results in this chapter are not new, but have usually appeared in the literature in a form which had meaning only in the finite-dimensional case. Chapter II is devoted to results which require that the space be finite-dimensional, and Chapter III is a study of linear inequalities from the viewpoint of functional analysis, requiring either that the space be normed or topological. The results obtained are related to or generalize work of Hahn-Banach, Rosenbloom, Hoffman, Schoenberg, Nikando, and Hestenes-McShane.

Publications: (1) "A theorem on alternatives for pairs of matrices," by H. A. Antosiewicz; to appear in Pacific Journal of Mathematics. (2) "Lower bounds for the rank and location of the eigenvalues of a matrix," by K. Fan and A. J. Hoffman; included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39, pp. 117-130. (3) "Some metric inequalities in the space of matrices," by K. Fan and A. J. Hoffman; to appear in Proceedings of the American Mathematical Society. (4) "Discrete analogs of inequalities of Wirtinger," by K. Fan, O. Taussky, and J. Todd; submitted to a technical journal. (5) "A determinantal inequality," by K. Fan and J. Todd; to appear in the Journal of the London Mathematical Society. (6) "On the relevance of LeChatelier's principle to linear programming," by A. J. Hoffman; IN MANUSCRIPT. (7) "An extremum property of sums of eigenvalues," by H. Wielandt; to appear in the Proceedings of the American Mathematical Society.

SUPPORTING RESEARCH IN LINEAR PROGRAMMING
Task 1102-10-5116/54-9

Origin: Office of the Air Comptroller, USAF   Authorized 9/25/53
Sponsor: Office of Scientific Research, Air Research and
         Development Command, USAF
Managers: J. Todd, A. J. Hoffman
Full task description: July-Sept 1953 issue, p. 35

Status: CONTINUED. In collaboration with the U. S. Air Force, the National Bureau of Standards will conduct a symposium on linear programming to be held in Washington on January 27, 28, and 29, 1955. Sessions are planned on computation procedures, economic theory, relations with other branches of mathematics, and practical applications. Each session will include a one-hour address as well as shorter papers. Acceptances have been obtained for over half the program.

The code for solving linear programs by the so-called modified simplex method (i.e., systematic use of the inverse of the basis leaving the initial tableau unaltered) has been written by L. Joel and will be tested in comparison with the original simplex method. In this connection it has been observed that, for sparse matrices containing many unit vectors

a good proportion of which are expected to belong to any basis, a variation of the modified simplex method will save arithmetic operations. The observation is based on the fact that only the inverse of the submatrix complimentary to the identity submatrix contained in the basis need be carried.

In studying the paper of Charnes and Lemke which developed a technique for minimizing on a polyhedron a convex function that can be written as the sum of convex functions of one variable, it has been observed that the sum condition is not an intrinsic feature of their approach. By altering their procedure slightly, any convex function may be treated by linear programming techniques. It is necessary,however, that the function whose minimum is to be located must be convex; i.e., if one formulates the Charnes-Lemke procedure in an abstract way, the function which it may handle is required to be convex. The setting in which this result is proved seems so general as to discourage hopes for a linear programming approach to the location of minima of non-convex functions.

The following problem (arising in an economic context) has been studied: Let (1) $Ax+By \geqslant c$ be a polyhedron in (x,y) space, and let y vary in a convex set K, such that for any $y \in K$, there exists an x satisfying (1). Let d be a given vector in the x-space. The problem is to find y yielding

$$\min_{y \in K} \ \max_{(x,y) \in L} \ (d,x) \ .$$

For the particular problem under consideration, the problem is computationally feasible; but, more generally it leads (via the dual) to the problem: minimize (z,w), where $z \in Z$, $w \in W$, Z, W given polyhedra. The problem of devising an algorithm to solve this problem is under study. So far, procedures for finding z_0,w_0 such that $(z_0,w_0) \leq (z_0,w)$, $(z_0,w_0) \leq (z,w_0)$ have been worked out.

Publications: (1) "On 'overshoot' in the 'furthest hyperplane' method," by R. Bryce; IN MANUSCRIPT. (2) "Linear programming in bid evaluation," by L. Gainen, D. J. Honig, and E. D. Stanley; Logistics Research Quar. 1, 48-54 (Mar. 1954). (3) "On the solution of the caterer problem," by J. W. Gaddum, A. J. Hoffman, and D. Sokolowsky; to appear in Logistics Research Quarterly. (4) "An algorithm for solving the transportation problem," by A. Gleyzal; to appear in Journal of Research of the NBS. (5) "Smooth patterns of production," by A.J.Hoffman and W. Jacobs; to appear in the Transactions of the Institute of Management Science. (6) "On the optimal ordering of items for a two-stage process," by A. J. Hoffman; IN MANUSCRIPT. (7) "On block relaxation," by L.S. Joel; IN MANUSCRIPT. (8) "A remark on the smoothing problem," by H.A.Antosiewicz and A. J. Hoffman; to appear in the Transactions of the Institute of Management Science.

RESEARCH IN MATHEMATICAL TOPICS APPLICABLE TO NUMERICAL ANALYSIS
Task 1101-10-5116/55-56

Origin: Office of Naval Research   Authorized 8/13/54
Sponsor: "   "   Revised   8/29/54
Managers: O. Taussky-Todd, M. Newman

Objective: To conduct basic and supporting research in various branches of mathematics of particular interest to agencies of the Federal

government, especially in fields related to numerical analysis.

Background: Various agencies in the Federal government have problems
in mathematics, and in particular in several aspects of numerical analysis,
for solution of which the staff and facilities of the National Bureau of
Standards are uniquely fitted.

Comments: Work on the following former tasks will be continued as
part of this task:
1102-10-1104/50-4   Miscellaneous studies in pure mathematics
1102-10-1104/50-5a   Number-theoretical test problems for SEAC
1102-10-1104/52-34   Special problems in finite matrix theory

Status: NEW. M. Newman has completed a manuscript entitled
"The coefficients of certain modular forms," in which are studied the
coefficients Pr(u) arising in the formal expansion

$$\prod_{n=1}^{\infty} (1-x^n)^r = \sum_{n=0} P_r(n)x^n$$

The principal result is the following: If p is a prime greater than
3, $r(p+1)\equiv 0 \mod 24$, $\Delta = r(p^2-1)/24$, then $P_r(np+\Delta)=(-p)^{(r-2)/2}P_r(n/p)$

holds for r=2,4,6,8,10,14,26 and for no other values of r. Many of the
computations involved in showing the exhaustiveness of the set of r's
were carried out and checked on SEAC. Without the aid of the computer,
the labor would have been excessive. This is one of the few instances
in number theory where a theoretical question was definitively settled
with the aid of a high-speed computer.
    In connection with this work, many values of Pr(n) have been
computed on SEAC, and it is anticipated that these will be made available
in a report soon. It was found that $P_{15}(482)=0$, whereas $P_{15}(n)\neq 0$ for
all n<482 except the well-known case n=53.
    M. Newman has submitted to a technical journal a manuscript
entitled "$ax^m-by^n=c$." It is shown that this diophantine equation has
only a finite number of solutions for nonzero a,b,c and all positive
integral m, n except m=n=2, m=1 or n=1.
    In collaboration with J. D. Hoffman of NBS Division 7.6,
K. Goldberg is studying the eigenvalues of certain matrices arising in
the theory of dielectric relaxation. The mathematical problem is to
show that, apart from exceptional cases which are physically trivial,
certain non-symmetric matrices have distinct, real eigenvalues. So far,
the reality has been proved in general, and the distinctness only for
those matrices of order 3.
    O. Taussky-Todd has continued her work on the uniqueness of the
normal base in cyclic fields, and has now shown that it is not unique in
fields of degree 8. She further studied the possibility of finding a
base for cyclic fields that, together with its conjugates, generates
a normal matrix (not merely a permutation of a normal matrix). She has
shown, with the assistance of K. Goldberg, that this is not possible for
fields of degree 2,3,4,5,6.
    As an aid to a statistical problem, A. Hoffman has shown how
to construct, for every n, a latin square of order n with distinct diagonal
elements. He has further constructed, for every even n, a latin square
such that any set of n elements, no two in the same row or column, cannot
be distinct.
    J. Todd has begun a study of the condition of the systems of
equations approximating to integral equations.

Publications: (1) "Linear functional equations and interpolation
series," by P. Davis; to appear in Pacific Journal of Mathematics.

(2) "Some L^2 aspects of Faber polynomials," by P. Davis and H. Pollack (Bell Telephone Laboratories). (3) "Continuity in terms of connectedness," by K. Fan and R. A. Stuble; to appear in Proc. Nederl.Akad., Wetensch. Ind. Math. (4) "Compactification of completely regular spaces," by K. Fan and F. Wagner; IN MANUSCRIPT. (5) "Inequalities for eigenvalues of Hermitian matrices," by K. Fan; included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39, pp. 131-139. (6) "Some inequalities concerning positive definite Hermitian matrices," by K. Fan; submitted to a technical journal. (7) "A comparison theorem for eigenvalues of normal operators," by K. Fan; submitted to a technical journal. (8) "Power series for log $(e^x.e^y)$", by K. Goldberg; IN MANUSCRIPT. (9) "On certain series expansions involving Whittaker functions and Jacobi polynomials," by P. Henrici; to appear in Pacific Journal of Mathematics. (10) "On generating functions of the Jacobi polynomials," by P. Henrici; to appear in Pacific Journal of Mathematics. (11) "The number of absolute points of a correlation," by A. Hoffman, M. Newman, E. Straus, and O. Taussky; submitted to a technical journal. (12) "The diophantine equation $ax^m - by^n = c$," by M. Newman; submitted to a technical journal. (13) "Note on a certain determinant," by M. Newman; IN MANUSCRIPT. (14) "Structure theorems for modular subgroups," by M. Newman; to appear in Duke Mathematical Journal. (15) "The coefficients of certain modular forms," by M. Newman; IN MANUSCRIPT. (16) "Some computational problems in algebraic number theory," by O. Taussky; to appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica, California, August 1953. (17) "Generalized commutators of matrices and permutations of factors in a product of three matrices," by O. Taussky; to appear in the von Mises anniversary volume. (18) "The condition of the finite segments of the Hilbert matrix," by J. Todd; included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39, pp. 109-116. (19) "Error bounds for eigenvalues of symmetric integral equations," by H. Wielandt; to appear in the Proceedings of the American Mathematical Society's Sixth Symposium on Applied Mathematics, held at Santa Monica, August, 1953. (20) "On eigenvalues of sums of normal matrices," by H. Wielandt; submitted to a technical journal.

II. COMPUTATION LABORATORY

(Section 11.2)

1. Mathematical Tables

1102-40-1110/43-3   TABLES OF $E_1(z)$, $(z=x+iy)$
Origin: Canadian National Research Council
Full task description: Apr-June 1949 issue, p. 41
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 28.

1102-40-1110/46-1   TABLE OF THE GAMMA FUNCTION FOR COMPLEX ARGUMENTS
Origin: NBS
Full task description: Apr-June 1949 issue, p. 43
Status: Completed.
Publication: "Table of the gamma function for complex arguments,"
  NBS Applied Mathematics Series 34; available from U. S. Government
  Printing Office, Washington 25, D. C., $2.00.

1102-40-1110/47-2   TABLES OF COULOMB WAVE FUNCTIONS
Origin: NBS
Full task description: Apr-June 1949 issue, p. 45
Status: Continued. Computations of F_0, F_0^1, G_0, G_0^1 are in progress for
  L=0, $\rho=\eta/3, \eta/2, \eta$, 2η, 3η, 4η for ρ varying from 0 to 50. The results
  are to be included in the second volume of Coulomb wave functions.

1102-40-1110/50-7   WAVE FUNCTION FOR LITHIUM
Origin: NBS
Sponsor: Bureau of Ordnance, USN
Full task description: Apr-June 1950 issue, p. 36
Status: Continued. The iterations for the last approximation to the
  wave function have been started and have been run as machine time was
  available. A code to compute the new approximation to the eigenvalue
  has been checked out.

1102-40-1110/51-4   COLLECTED SHORT MATHEMATICAL TABLES OF THE
                     COMPUTATION LABORATORY
Origin: NBS
Full task description: July-Sept 1950 issue, p. 43
Status: Continued. The first volume is in press.
Publication: "Tables of functions and of zeros of functions: Collected
  short tables of the Computation Laboratory," NBS Applied Mathematics
  Series 37; in press, U. S. Government Printing Office.

1102-40-1110/51-8   TABLES OF POWER POINTS OF ANALYSIS OF VARIANCE TESTS
<u>Origin</u>: Section 11.3, NBS
<u>Full task description</u>: Apr-June 1951 issue, p. 49
<u>Status</u>: Continued. This project has been inactive since March 1953 and
   is now being resumed. Following is a summary of the status to date.
     The original task was deemed too large for available funds and was,
   for the present, reduced. In the notation of the full task description,
   it was decided to compute nineteen (α, β) tables based on all pairs α, β
   derivable from $\alpha = .01, .02, .05, .20, \beta = .01, .05, .10, .50, .90$ with
   $\beta < 1 - \alpha$.
     Preliminary to the main computation it is necessary to have available
   inverse tables of the incomplete beta function ratio,

$$I_x(\tfrac{1}{2}f_1, \tfrac{1}{2}f_2) = B_x(\tfrac{1}{2}f_1, \tfrac{1}{2}f_2)/B(\tfrac{1}{2}f_1, \tfrac{1}{2}f_2) = \alpha.$$

For $\alpha = .01, .05$, these had been calculated previously (C.M. Thompson,
Biometrika, vol. 32, p. 168). For $\alpha = .02, .20$, they were computed by
a code written by L. Joel. The code performed inverse interpolation
on the function $I_x(p,q)$, which was calculated using a well-known double
recurrence formula that allows easy estimates of the rounding error.
   The main computation is being performed generally along the lines
suggested by Emma Lehmer in <u>Annals of Mathematical Statistics</u>, December
1944. When f_2 is an even integer, the infinite series can be replaced
by a finite series, and this is worth while for $f_2 \leq 60$. For these values
of f_2 and all finite f_1, the corresponding entries in the nineteen

(α, β) tables have been computed using a code written by S. Tsingou.
The remaining finite values of f_2 require computation of the infinite

series. A code which sums this series and performs the inverse interpolation
has been written by Miss Tsingou and is being checked. The terms of
the series are computed using the recurrence

$$I_x(p+1,q) = I_x(p,q) - \frac{(1-x)^q x^p}{p B(p,q)}$$

   Whether the method is feasible for f_2 small and odd remains to be

seen, but no difficulty is anticipated for f_2 large. When this computation
is completed, all that will remain to be done are the entries corresponding
to the infinite values of f_1 and f_2.

1102-40-1110/52-7   REVISION OF MATHEMATICAL TABLES
<u>Origin</u>: NBS
<u>Full task description</u>: July-Sept 1951 issue, p. 41
<u>Status</u>: Continued. Following is the status of those mathematical tables
   the sales stock of which has been exhausted and for which reissue is
   planned:
     "Tables of sine and cosine integrals for arguments from 10 to 100,"
   NBS Applied Mathematics Series 32; reissued July 1954 and available
   from the U. S. Government Printing Office, Washington 25, D. C., $2.25.
     "Tables of the error function and its derivative," NBS Applied
   Mathematics Series 41; in press, U. S. Government Printing Office.
   This was originally Mathematical Table MT8, "Tables of probability
   functions," vol. I (1941).
     "Tables of sines and cosines for radian arguments," NBS Applied
   Mathematics Series 43; in press, U. S. Government Printing Office.
   This table was formerly designated as Mathematical Table MT4.
     "Tables of natural logarithms," vol. IV, Mathematical Table MT12
   (1941); revision in progress for reissue in the Applied Mathematics
   Series.

1102-40-1110/52-14   TABLE OF ARCSIN FOR COMPLEX ARGUMENTS
Origin: NBS
Full task description: July-Sept 1951 issue, p. 41
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 31.

1102-40-1110/52-18   EXTENSION OF THE TABLE OF HYPERBOLIC SINES AND COSINES
Origin: NBS
Full task description: July-Sept 1951 issue, p. 41
Status: Continued. Typing of the manuscript has been completed, and
   checking is in progress.

1102-40-1110/52-23   TABLE OF THE MODIFIED AIRY INTEGRAL
Origin: NBS
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 31

1102-40-1110/52-25   TABLE OF ERROR FUNCTION FOR COMPLEX ARGUMENTS
Origin: NBS
Full task description: July-Sept 1951 issue, p. 42
Status: Inactive. The 20-place table for x,y=0(.1)2 is available on
   punched cards.

1102-40-1110/52-31   EXTENSION OF TABLES OF THE EXPONENTIAL FUNCTION FOR
                      NEGATIVE ARGUMENTS
Origin: NBS
Full task description: July-Sept 1951 issue, p. 43
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 32.

1102-40-1110/52-37   SPHEROIDAL WAVE FUNCTIONS
Origin: NBS
Full task description: Oct-Dec 1951 issue, p. 38
Status: Inactive. For status to date, see Jan-Mar 1954 issue, p. 31.

1102-40-1110/52-49   RADIAL MATHIEU FUNCTIONS
Origin: NBS
Full task description: Jan-Mar 1952 issue, p. 45
Status: Continued. Editing and differencing of the results is in progress.

1102-40-1110/52-57   SIEVERT'S INTEGRAL
Origin: NBS
Full task description: Jan-Mar 1952 issue, p. 46
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 33.

1102-40-1110/53-35   HYPERGEOMETRIC FUNCTIONS
Origin: NBS
Full task description: Jan-Mar 1953 issue, p. 36
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 27.

1102-40-1110/53-52   L-SHELL CONVERSION COEFFICIENTS
Origin: Oak Ridge National Laboratory
Full task description: Apr-June 1953 issue, p. 45
Status: Continued. Part III of the problem (see Apr-June 1954 issue,
   pp. 27-28) involves the integration of a differential equation with

radial distance r as independent variable. A singularity occurring at r=0 is overcome by using a series expansion up to a point $r=r_s$ and numerical integration beyond that point. A number of computations were carried out in order to explore the effect of varying r_s.

3711-60-0009/55-65   RESEARCH AND DEVELOPMENT IN AUTOMATIC CODING
Origin: NBS
Objective: To investigate methods of improving programming techniques for high speed computers and make the results generally available. The immediate objectives are: (1) to improve the general purpose interpretive floating decimal routine now in use; (2) to develop new service routines and incorporate them in the system now in use, which facilitates rapid access to the computer; (3) to complete the improved version of the input conversion routine.
    The long-term objectives are: (1) to study current and completed programs in order to develop special techniques for general use; (2) to develop a general purpose pseudo-code.
Background: The present library of subroutines and service routines has been produced largely as by-products of other projects. Their usefulness has indicated the need for separate study of automatic programming techniques. The experience of the staff in operating other high speed automatic digital machines will be applied. While the object basically is the exploitation of SEAC, nevertheless attempts will be made to keep the work on as general a footing as possible.
Comments: That portion of the former task 1102-10-1104/50-5, Bibliography of Coding Procedures, Mathematical Tables, and Numerical Analysis, that pertains to coding procedures, is continued under this task.
Status: New. (1) An input conversion routine called Sebbe 3 which composes codes, converts floating decimal numbers to floating binary numbers, converts decimal numbers to hexadecimal numbers, and supplies each of the above groups with appropriate read-in orders, was put into service. (2) A course is being conducted for NBS personnel in which programming for SEAC is being taught. (3) The sine, cosine and arctangent subroutines are in process of being incorporated in the interpretive floating-point routine (base 00).

2. Mathematical Services

Note: The tasks under Mathematical Services are arranged serially according to the digits following the slant lines in the task number. The first two digits following the slant line designate the fiscal year in which the task was authorized.

1102-40-5126/50-13   RAY TRACING
Origin and Sponsor: NBS, Section 2.2
Full task description: Jan-Mar 1950 issue, p. 33
Status: Continued. Several aerial camera lens systems are being analyzed.

1102-40-5126/51-37   MOLECULAR STRUCTURE, III
Origin: Naval Research Laboratory, USN
Full task description: July-Sept 1951 issue, p. 50
Status: Continued. Computations were performed as requested.

1102-40-5126/52-20   SPHERICAL BLAST
Origin and Sponsor: Naval Ordnance Laboratory
Full task description: July-Sept 1951 issue, p. 56
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 30

1102-40-5126/52-44   CALCULATIONS FOR d SPACINGS
Origin and Sponsor: NBS, Division 9
Full task description: Oct-Dec 1951 issue, p. 47
Status: Continued. Programming, coding, and checking on SEAC have been
   completed. About 20 problems have been run during the past quarter.

1102-40-5126/53-4   NEUTRON DIFFUSION, II
Origin and Sponsor: Atomic Energy Commission, New York Office (NDA)
Full task description: July-Sept 1952 issue, p. 60
Status: Terminated. The objectives of this project have been changed,and
   future work growing out of this task will be performed under task
   1102-40-5126/55-78, to be established in the next quarter.

1102-40-5126/53-11   STRENGTH OF WING COMPONENTS
Origin and Sponsor: National Advisory Committee for Aeronautics,
                     Langley Field, Va.
Full task description: Oct-Dec 1952 issue, p. 54
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 31.

1102-40-5126/53-20   RADIANT HEATING OF SOLIDS
Origin and Sponsor: NBS, Section 10.2
Full task description: Oct-Dec 1952 issue, p. 55
Status: Terminated. The objectives of this project have been changed,and
   current work is being performed under task 1102-40-5126/54-45, p.16.

1102-40-5126/53-27   COMPUTATION OF THERMODYNAMIC FUNCTIONS
Origin and Sponsor: NBS, Division 5
Full task description: Jan-Mar 1953 issue, p. 57
Status: Continued. The SEAC code was revised to include temperatures up
   to 15,000°K, and thermodynamic functions were calculated for approximately
   30 molecules that included these higher temperature ranges.

1102-40-5126/53-29   DYNAMIC BEHAVIOR OF AIRCRAFT STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Full task description: Jan-Mar 1953 issue, p. 58
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 32.

1102-40-5126/53-39   ACOUSTICAL IMPEDANCES
Origin and Sponsor: NBS, Section 6.1
Full task description: Apr-June 1953 issue, p. 55
Status: Inactive. For status to date, see Apr-June 1953 issue, p. 55.

1102-40-5126/53-41   LORAN UNIVAC CODE
Origin and Sponsor: Hydrographic Office, U. S. Navy
Full task description: Apr-June 1953 issue, p. 56
Status: Continued. Code checking has been started on the Baseline
   Extension and on the Skywave Correction and Trainer Codes.

1102-40-5126/53-51   RADIATION DIFFUSION
Origin: NBS, Section 4.8
Sponsor: Armed Forces Special Weapons Project
Full task description: Apr-June 1953 issue, p. 57 (Neutron Diffusion III)
Status: Continued. Codes have been written and checked (a) for extension
  of reflection calculations to cover the energy range from 50 to 1000
  Kev., (b) for the evaluation of the energy ratios received behind a
  slab and behind an equal thickness of an infinite medium, and (c) for
  energy and angular spectra of transmitted and reflected radiation.
     A code has also been written for the determination of the spectrum of
  pulses produced by gamma rays in a sodium iodide crystal, a widely-used
  type of radiation detector.
     Information on the above problem is difficult to obtain experimentally
  and is important for the interpretations of measurements made with
  scintillation counters.

1102-40-5126/54-4   HIGH TEMPERATURE PROPERTIES OF WATER
Origin: NBS, Division 3.2
Sponsor: Bureau of Ordnance, USN
Full task description: Oct-Dec 1953 issue, p. 41
Status: Continued. A code has been written to compute the thermal
  functions for the one-ionic species O_{VIII} using the formulation of
  H. W. Wooley mentioned in the previous quarterly report. Most of
  these functions have been computed for the temperature range between
  10^3 and 10^8 degrees Kelvin at 50 intervals of temperature logarithmically
  spaced. The same code will be used to compute thermal functions for
  the singly ionized species H and these functions, along with the rest
  of the O_{VIII} for the same range of temperature.

3711-60-0009/54-7   DISTRIBUTION OF NORMAL MODES OF VIBRATION OF CUBIC
                        LATTICES, II
Origin and Sponsor: NBS, Division 30
Full task description: Oct-Dec 1953 issue, p. 42
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 33.

3711-60-0009/54-8   NORMAL VIBRATIONS IN MOLECULES
Origin and Sponsor: NBS, Section 3.2
Full task description: Oct-Dec 1953 issue, p. 42
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 34.

3711-60-0009/54-11   COMPUTATION OF VIBRATION MODES AND FREQUENCIES
Origin and Sponsor: NBS, Section 6.4
Full task description: Oct-Dec 1953 issue, p. 42
Status: Continued. The first five symmetric modes for three aircraft
  structures (using 31 mass points) were computed.
Publications: "Computation of vibration modes and frequencies on SEAC,"
  by W. F. Cahill and S. Levy (NBS Section 6.4); submitted to a technical
  journal.

1102-40-5126/54-13   AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: New York Quartermaster Procurement Agency
Full task description: Oct-Dec 1953 issue, p. 43
Status: Continued. Contract awards for the Quartermaster Corps are being
  evaluated on SEAC at the average rate of two per week. Similar UNIVAC
  routines have been written and checked out in cooperation with personnel
  of the Office of the Air Comptroller, USAF.

1102-40-5126/54-14   FLOW IN SUPERSONIC NOZZLE
Origin and Sponsor: National Advisory Committee for Aeronautics,
                        Langley Field, Va.
Full task description: Oct-Dec 1953 issue, p. 44
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 35.

3711-60-0009/54-17   DEPOLYMERIZATION
Origin and Sponsor: NBS, Section 7.6
Full task description: Oct-Dec 1953 issue, p. 44
Status: Inactive. For status to date, see Oct-Dec 1953 issue, p. 44.

3711-60-0009/54-19   ENERGY LEVELS OF COMPLEX ATOMS
Origin and Sponsor: NBS, Section 4.1
Full task description: Jan-Mar 1954 issue, p. 41
Status: Continued. During this quarter the first nine characteristic
  roots of a 25x25 symmetric matrix were computed.

1102-40-5126/54-20   LOW TEMPERATURE PROPERTIES OF BORON COMPOUNDS
Origin and Sponsor: NBS, Section 3.2
Full task description: Jan-Mar 1954 issue, p. 41
Status: Inactive. For status to date, see Jan-Mar 1954 issue, p. 41.

1102-40-5126/54-21   EXPERIMENTAL PROGRAM FOR MULTIPLE INPUT AND OUTPUT
Origin and Sponsor: NBS, Section 12.3
Full task description: Jan-Mar 1954 issue, p. 42
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 36.

3711-60-0009/54-22   ENERGY DISTRIBUTIONS ON OPTICAL IMAGE
Origin and Sponsor: NBS, Section 2.2
Full task description: Jan-Mar 1954, p. 43
Status: Continued. Computations have been completed for $W=9x(x^2+y^2)$
  for ξ and η ranging from 1 to 16. Computations for $W=100(x^2+y^2)+200(x^2+y^2)^2$
  $-300(x^2+y^2)^3$ are being carried out for various values of ξ and η.

1102-40-5126/54-24   SOLUTION OF THE DIFFUSION EQUATION FOR ELECTRONS
Origin and Sponsor: NBS, Division 4
Full task description: Jan-Mar 1954 issue, p. 43
Status: Inactive. For status to date, see Jan-Mar 1954 issue, p. 43.

1102-40-5126/54-25   DEFLECTED RADIATION FROM AN INFINITE LAMBERT PLANE
Origin and Sponsor: Armed Forces Special Weapons Group
Full task description: Jan-Mar 1954 issue, p. 44
Status: Continued. Values of $\psi(u,v)$ for u,v = .25(.25)16.0 have been
  computed and have been submitted to the sponsor.

3711-60-0009/54-27   EMF OF CELLS AT HIGH TEMPERATURE
Origin and Sponsor: NBS, Section 1.8
Full task description: Jan-Mar 1954 issue, p. 45
Status: Continued. Electromotive forces for all fluorides and iodides
  have been computed.

3711-60-0009/54-28   LOVIBOND NETWORK FOR CIE SOURCE A
Origin and Sponsor: NBS, Section 2.1
Full task description: Jan-Mar 1954 issue, p. 46
Status: Inactive. For status to date, see Apr-June issue, p. 37.

3711-60-0009/54-30   SPECTRUM ANALYSIS
Origin and Sponsor: NBS, Division 4
Full task description: Jan-Mar 1954 issue, p. 46
Status: Continued. Searching of differences between all known and
  predicted levels for pairs of lines which confirm known levels and predict
  new levels was performed for approximately 20 differences for Tc I.
  The wave numbers (σ) for over 6,000 lines were computed for ruthenium.

1102-40-5126/54-33   AIR CONDITIONING IN UNDERGROUND STRUCTURES
Origin and Sponsor: NBS, Section 10.3
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. A supplementary code has been written to evaluate
  the integral

$$\frac{4Gua}{\pi^2}\int_0^\infty \frac{e^{-\frac{ty^2}{a^2}}}{[yJ_1(y)+aGJ_0(y)]^2 + [yY_1(y) + aGY_0(y)]^2}\frac{dy}{y} \; .$$

  Results for this integral and the original integral have been obtained
  as they were requested by the originator.

1102-40-5126/54-34   EQUILIBRIUM CALCULATIONS FOR WATER
Origin and Sponsor: Naval Ordnance Laboratory
Full task description: Apr-June 1954 issue, p. 38
Status: Continued. A set of 46 simultaneous nonlinear equations has been
  solved for numerous parameters submitted by the sponsor. For some values
  of the parameters the solution method used in the original program
  fails to converge and consequently a technique was devised to force
  convergence. The problem is being recoded to include this new technique.

1102-40-5126/54-35   THERMAL STRESSES IN STRUCTURES
Origin and Sponsor: NBS, Section 6.4
Full task description: Apr-June 1954 issue, p. 38
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 38.

3711-60-0009/54-36   VELOCITY OF LIGHT
Origin and Sponsor: NBS, Section 11.3
Full task description: Jan-Mar 1954 issue, p. 47
Status: Continued. Ten third degree and ten fourth degree polynomials
  were obtained from data submitted by the sponsor using the orthonormalization
  code previously programmed.

3711-60-0009/54-38   COMPRESSIBILITY FACTORS OF DRY AIR
Origin and Sponsor: NBS, Section 3.2
Full task description: Jan-Mar 1954 issue, p. 48
Status: Continued. The code has been completed and is being checked.

1102-40-5126/55-39   MOLECULAR VIBRATIONS
Origin and Sponsor: NBS, Section 3.2
Objective: To develop four routines for solving matrix problems with
  floating decimal point operations: (1) multiplication of matrices;
  (2) extraction of eigenvalues; (3) extracting of eigenvectors;
  (4) inversion. Routines (2), (3), and (4) are to be limited to
  symmetric matrices.
Background: A problem arising from molecular vibration theory is the
  prediction of frequencies of vibrations given the force constants of
  the molecules, and conversely. This can be translated into an eigenvalue
  problem in which the frequencies can be determined from the eigenvalues
  of the matrix of force constants or the matrix of force constants can be
  approximated from the eigenvalues. In the latter case the force constants
  obtained are used to predict the force constants of other molecules of
  a related structure.
Status: New. A routine for the evaluation of eigenvalues of symmetric
  matrices using floating decimal point operation has been written and
  checked out, and it will be put into use for production in the future.

1102-40-5126/54-41   FREQUENCY CORRELATION
Origin and Sponsor: NBS, Division 6
Full task description: Apr-June 1954 issue, p. 39
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 39.

1102-40-5126/54-42   RESOLUTION CORRECTION FOR SCINTILLATION SPECTROMETER
Origin and Sponsor: NBS, Section 4.13
Full task description: Apr-June 1954 issue, p. 40
Status: Continued. Twenty systems of simultaneous equations of order 9
  and 20 systems of order 10 have been solved.

1102-40-5126/54-43   CHARACTERISTICS OF CONDUCTING RESISTORS
Origin and Sponsor: NBS, Section 12.1
Full task description: Apr-June 1954 issue, p. 40
Status: Continued. The code has been completed and checked, and the
  computations are being processed with data submitted by the sponsor.

1102-40-5126/54-45   COMPUTATION OF TRANSIENT HEAT FLOW PROBLEMS
Origin and Sponsor: NBS, Section 10.2
Full task description: Apr-June 1954 issue, p. 41
Status: Continued. The problem has been run for various diffusivities
  and heat transfer coefficients, and the results are being transmitted
  to the sponsor as they are obtained.

1102-40-5126/54-46   STUDY IN OPTIMIZATION OF POWER OUTPUT
Origin: Raytheon Manufacturing Company
Sponsor: U. S. Corps of Engineers
Full task description: Apr-June 1954 issue, p. 41
Status: Inactive. For status to date, see Apr-June 1954 issue, p. 41.

1102-40-5126/55-49   WIECHERT DISTRIBUTION FUNCTION
Origin and Sponsor: NBS, Section 7.1
Objective: To evaluate integrals of the following type,

$$L_k(n) = \int_{-\infty}^{\infty} e^{(n-z)} \, e^{-e^{(n-z)}} \, L_\ell(z) \, dz.$$

Background: The integrals arise in the study of approximation methods in linear viscoelasticity theory.
Status: New. Programming and coding are in progress.

1102-10-5126/55-50 AWARD OF PROCUREMENT CONTRACTS BY LINEAR PROGRAMMING
Origin and Sponsor: Navy Purchasing Office
Objective: To solve problems, arising in the sponsor's work, of awarding purchase contracts at minimum cost to the Government.
Background: The calculations to be performed are similar to those now being conducted for the Philadelphia Quartermaster Depot (see task 1102-40-5126/54-13, p.13).
Status: New.

1102-40-5126/55-52 ATMOSPHERIC WAVES
Origin and Sponsor: Weather Bureau
Objective: To solve by finite differences the system of non-linear differential equations

$$\frac{\partial u}{\partial t} = -F_0 u \frac{\partial u}{\partial x} -F_0 c \frac{\partial c}{\partial x} + v, \quad \frac{\partial c}{\partial t} = -\frac{F_0}{2}c \frac{\partial u}{\partial x} -F_0 u \frac{\partial c}{\partial x}, \quad \frac{\partial v}{\partial t} = -F_0 u \frac{\partial v}{\partial x} - u$$

in the region $0 \leqslant t \leqslant 2.4$, $-4.2 \leqslant x \leqslant 4.2$, where F_0 is a constant and $u(x,0)$, $c(x,0)$ and $v(x,0)$ are given.
Background: This problem arises in the study of non-linear atmospheric gravity waves. The equations determine the velocity components u (east) and v (north) and the function c, representing height, in their dependence on time, t, and east-west distance, x. Similar computations had previously been performed at the Weather Bureau on a small scale, using a characteristic grid. The purpose of the present task is to extend that work and to investigate the effect of using various fixed grids.
Status: Completed (New). The results have been transmitted to the sponsor.

1102-10-5126/55-62 INTEGRALS INVOLVED IN SUPERSONIC FLUTTER
Origin and Sponsor: National Advisory Committee for Aeronautics
Objective: To compute tables of the integrals

$$F_\lambda(M,\varpi) = \frac{1}{\sigma} \int_0^\sigma \left(\frac{x}{\sigma}\right)^\lambda e^{-iMx} J_0(x)dx$$

where $\lambda = 0(1)11$, M ranges from 1.2 to 5, and $\sigma = 2KM/M^2-1$ where $K=0(.005).15(.01).20(.05)1(.1)2$. Results are to be tabulated to seven decimal places.
Background: The integrals in question occur in the Possio theory of non-stationary flow for small disturbances in a two-dimensional supersonic flow and the application of the theory to the determination of aerodynamic forces on an oscillating airfoil.
Status: New. The code has been written, and checking is in progress.

1102-40-5126/55-63 VIRIAL INTEGRAL INVOLVING A MORSE POTENTIAL FUNCTION
Origin and Sponsor: NBS, Section 3.2
Objective: To evaluate the integrals

$$Mn = \int_0^\infty [1-e^{-x(e^{-2y}-2e^{-y})}]y^n dy - \int_{-y_0}^0 e^{-x(e^{-2y}-2e^{-y})}y^n dy$$

and their first and second derivatives with respect to x for n=0,1,2, with x ranging from .2 to 20, and y_0 ranging from .9 to 5. To compute

specified combinations of these functions.
Background: Previously the SEAC has been programmed to calculate ideal
   gas thermal properties of molecular species up to 15,000°K, employing
   the usual partition function expression. At more elevated temperatures
   a different procedure is employed. Since the thermal properties at
   these temperatures are sensitive to the upper regions of the potential
   energy curve, and the usual partition function sum tends to diverge
   when accurately summed over all the necessary energy levels, an expression
   was derived for the internal partition function by considering the
   equilibria between the molecular species and its dissociation products.
   The equilibrium constant can be related to a virial coefficient. Thus
   the internal partition function can be directly related to a virial
   type integral, which explicitly includes the dependence on the potential
   function.
Status: New. The program employing the 16-point Gaussian quadrature
   formula for evaluating the integrals on SEAC has been written and code
   checking is in progress.

1102-40-5126/55-64   HIGH-TEMPERATURE THERMODYNAMIC TABLES (FMT)
Origin and Sponsor: NBS, Section 3.2
Objective: To prepare a table of ten generalized thermodynamic properties
   (i.e. enthalpy, Helmholtz free energy, etc.) for the elements at high
   temperatures. Reduced temperature and atomic volume are to be used as
   arguments so that the properties are independent of atomic number.
Background: The method to be followed is that of Feynman, Metropolis,
   and Teller, Phys. Rev. 75, 1561 (1949) using their exact treatment
   of high temperature effects. The solutions of the temperature dependent
   Fermi-Thomas equation are to be extended beyond those obtained in the
   FMT paper in the course of these new calculations.
Status: New.

3711-60-0009/55-66   RECONSTITUTION OF MONOCHROMATIC LIGHT INTENSITIES
Origin and Sponsor: NBS, Division 30
Objective: To solve integral equations of the type

(1) $$E(x) = I \int P(y) \, K(x,y) dy \equiv I\hat{P},$$

where the domain of integration is a suitably defined neighborhood
of x. Specifically, to investigate the convergence of iterative methods
of the form

(2) $$P_{i+1} = P_i \, (\hat{P}/\hat{P}_i)$$

or

(3) $$P_{i+1} = P_i \, (\hat{P}/\hat{P}_i)^{\alpha}$$

Background: Integral equations of the form (1) arise in the problem of
   deblurring of photographic negatives. E(x) is the light incident on the
   point x of a photographic negative, P(x) the light emanating from the
   original. The kernel K(x,y) represents the effect of light from the
   point y on the original on the point x of the negative; in other words,
   it represents the degree of imperfection of the photographic system.
   If the iterations (2) or (3) converged, then it would be possible to
   design automatic equipment for carrying them out. Such equipment could
   carry out the operation $\hat{P}_i$ by means of the original photographic system,
   and therefore would not require explicit knowledge of the kernel K(x,y).
Comment: This problem was proposed specifically by C. H. Page (NBS, Div.30).
Status: New. First trials have been made on the SEAC at the numerical

solution using an assumed rectangular kernel of the following form:

$$
\begin{array}{ccc}
1 & 4 & 1 \\
4 & 16 & 1 \\
1 & 4 & 1
\end{array}
\qquad
\begin{array}{l}
\text{(each element normalized by the sum} \\
\text{of all elements of the kernel)}
\end{array}
$$

For the runs that have been completed, $\alpha=1$ and $\alpha=2$ have been used.
Behavior of the iterated values shows marked instability for slight
scaling variations of the original kernel and investigation will
continue.

1102-40-5126/55-67 DOSAGE INTEGRAL
Origin and Sponsor: U. S. Weather Bureau
Objective: To tabulate D(d) for various values of the parameters, where

$$
D(d) = \frac{BQ_0}{4\pi^{5/2}\,C^3} \cdot
$$

$$
\cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{\mathrm{Exp}\left\{-(ut)^{n-2}\left[\dfrac{x^2+y^2+z^2}{C^2}\right] - \gamma[(d-ut-x)^2+y^2+z^2]^{\frac{1}{2}}\right\}}{t^a(ut)^{3/2(2-n)}[(d-ut-x)^2 + y^2+z^2]}\,dx\,dy\,dz\,dt
$$

The parameters are: the initial source strength (Q_0), the Sutton virtual
diffusion coefficient (C), the average wind speed (u), the Sutton
stability index (n), the gamma absorption coefficient in air (γ), the
decay parameter (a), the conversion factor to roentgen units (B), and
the distance of receptor at ground downwind of point of release (d).
Background: The problem arises in calculating the potential hazard to
surrounding communities in the event of an accidental release of
radioactive fission products from a nuclear reactor. The problem is
complicated by the fact that during their travel in the atmosphere
these products would be subject to radioactive decay as well as
atmospheric dilution through the action of turbulent eddies. In
addition the exposure received by a person on the ground is further
affected by the absorption and scattering of the rays on route from
various parts of the cloud.
Status: New. The quadruple integral has been reduced to a double
integral and D(d) has been computed on the SEAC for one set of
parameters and two values of d.

3711-60-0009/55-69 THEORY OF DIELECTRIC RELAXATION
Origin and Sponsor: NBS, Section 7.6
Objective: To investigate the properties of the characteristic roots of
certain matrices arising in dielectric relaxation theory.
Background: A problem in dielectric relaxation theory is to determine the
possible relaxation times of the dipoles in a crystal when an electric
field which has been applied to the crystal is removed. These relaxation
times can be determined as the eigenvalues of a matrix of transition
probabilities depending on the crystal model.
Status: New. It has been proved that all of the matrices arising in
this connection have real eigenvalues. It remains to be shown that the
roots are distinct in non-trivial cases.

1102-40-5126/55-72   MARYLAND INTER-INDUSTRY STUDY
Origin and Sponsor: Office of Scientific Research, ARDC, USAF
Objective: To undertake linear algebraic computation for a State of
  Maryland inter-industry study.
Background: This analysis of the economy of the State of Maryland is
  based on the reduction of all Maryland economic activity to 24 basic
  industrial categories and the study of the interrelationships among
  these classifications in terms of input and output of the industries
  in dollars.
Status: New. The Leontieff matrix $|I-A|$ where A is an input-output
  matrix of order 24 was inverted; also $|I-A|$ X=Y was solved for six
  vectors Y.

III. STATISTICAL ENGINEERING LABORATORY
(Section 11.3)

1. Fundamental Research in Mathematical Statistics

BIBLIOGRAPHY AND GUIDE TO STATISTICAL LITERATURE
Task 1103-10-1107/49-1a

Origin: NBS   Authorized 1/9/49
Manager: L. S. Deming
Full task description: Apr-June 1949 issue, p. 75

     Status: CONTINUED. For a description of the continuing activity
on this task, see the Jan-Mar 1954 issue, p. 49.

MANUAL ON FITTING STRAIGHT LINES
Task 1103-10-1107/50-2

Origin: NBS   Authorized 3/1/50
Manager: F. S. Acton
Full task description: Jan-Mar 1950 issue, p. 42

     Status: CONTINUED. F. S. Acton has very nearly completed the
remaining sections of the first draft. About half of the manuscript is
currently in the hands of technical advisors for comment.

TABLES TO FACILITATE DRAWING RANDOM SAMPLES
Task 1103-10-1107/51-1

Origin: NBS   Authorized 7/1/50
Managers: C. Eisenhart, L. S. Deming
Full task description: July-Sept 1950 issue, p. 57

     Status: INACTIVE. For status to date see July-Sept 1952 issue,
p. 64.

MISCELLANEOUS STUDIES IN PROBABILITY AND STATISTICS
Task 1103-10-1107/51-2

Origin: NBS   Authorized 7/1/50
Manager: C. Eisenhart
Full task description: July-Sept 1950 issue, p. 58

     Status: CONTINUED. Mary G. Natrella prepared a composite table of

the 5, 2, and 1 percent significance levels of the smaller rank sum for the Wilcoxon test, by compilation from various published tables, with correction of all known errata in the originals. A copy of the table has been deposited in the Unpublished Mathematical Tables file maintained by the editor of Mathematical Tables and Other Aids to Computation.

Publications: (1) "Optimum grouping in one-criterion variance components analysis," by E. P. King; J. Am. Stat. Assoc. $\underline{49}$, No. 267, 637-639 (Sept. 1954). (2) "Time-discrete stochastic processes in arbitrary sets, with applications to processes with absorbing regions and to the problem of loops in Markoff chains," by D. van Dantzig; accepted for publication (in French) in Annales de l'Institut Henri Poincaré (Paris). (3) "Two early papers on the relation between extreme values and tensile strength," (Formerly "A historical note on the application of the 'weakest link' idea to tensile strengths,") by J. Lieblein; accepted for publication in Biometrika. (4) "Tables of the inverses of finite segments of the Hilbert matrix," by I. R. Savage and E. Lukacs; included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39, pp. 105-108. (5) "Bounds on a distribution function which are functions of moments to order four," by M. Zelen; accepted for publication in the Journal of Research of the NBS. (6) "On the variances and covariances of order statistics from the Weibull distribution," by J. Lieblein; submitted to a technical journal.

LAW OF PROPAGATION OF ERROR
Task 1103-10-1107/52-1

Origin: NBS   Authorized 6/23/51
Managers: C. Eisenhart, I. R. Savage
Full task description: July-Sept 1951 issue, p. 65

Status: TERMINATED. For the last report on this task see July-Sept 1951 issue, p. 65.

PROCEDURES OF NON-PARAMETRIC STATISTICS
Task 1103-10-1107/52-2

Origin: NBS   Authorized 9/17/51
Manager: I. R. Savage
Full task description: July-Sept 1951 issue, p. 66

Status: CONTINUED. I. R. Savage completed an extensive manuscript entitled, "Contributions to the theory of rank order statistics," which provides a unified treatment of his research under this project to date.
The problem of the construction of optimum nonparametric tests and the evaluation of their power functions is considered. All of the situations treated involve mutually independent random variables $X_1,\ldots,X_m$, $Y_1,\ldots,Y_n$ where the X's have a continuous cumulative distribution function $F(x)$ and the Y's have a continuous cumulative distribution function $G(x)$. When the observed values of the random variables are arranged in an ascending sequence the rank order is defined as $z=(z_1,\ldots,z_i,\ldots,z_{m+n})$ where $z_i=0(1)$ if the i-th observation in the ascending sequence is from the F(G) distribution. A rank order test is formed by assigning

a probability a(z) to each rank order and if a rank order z occurs H_0:
F = G is rejected with probability a(z).
   If for the rank order z and z' we have P(Z=z) > P(Z=z') for
all alternatives under consideration then the criterion of admissibility
requires that a = 1 whenever a' > 0. When enough relationships of the
above type fail to exist the construction of best tests becomes extremely
complicated. It is shown that for such alternatives as H_S: $F(x) \geq G(x)$,
where the inequality holds for some x, there does not exist an adequate
number of the desired relationships. For the alternative H_{KD}: F and G
have density functions $f(x) = A(x)B(\theta_1) \exp[C(x)D(\theta_1)]$ and g(x) =
$A(x)B(\theta_2) \exp[C(x)D(\theta_2)]$ where C(x) and D(θ) are increasing functions,
it is shown that if two rank orders z and z' are identical except for the
i and j (i<j) elements which are (0,1) for z and (1,0) for z' then
P(Z=z) > P(Z=z'). It is then shown that the Wilcoxon statistic and the
c_1 statistic of Terry satisfy the above criterion of admissibility but
that the one-sided Smirnov test is inadmissible.
   Next the hypothesis H_L: $G(x) = [F(x)]^\delta$ where $\delta \geq 1$ is considered.
H_L is a special case of H_{KD} when F(x) has a density function, but even
if there is no density function the previously mentioned result regarding
the ordering of the probabilities of the rank orders is applicable.
However, in this case more can be done towards the construction of
optimum tests. It is shown that

$$P(Z=z) = m! \; n! \; \delta^n \Big/ \prod_{i=1}^{m+n} \Big[\sum_{j=1}^{i} (1-z_j+\delta z_j) \Big] .$$

Thus for a particular value of δ under H_L it is possible to compute the
probabilities of the rank orders and to construct the most powerful
rank order test. For small sample sizes it is possible to examine the
expressions given for the probabilities of the rank orders and to find
other pairs z and z' than those given above such that P(Z=z) > P(Z=z').
For very small sample sizes enough ordering relationships are obtained
for constructing uniformly most powerful rank order tests. For slightly
larger sample sizes where, for some levels of significance, uniformly
most powerful rank order tests fail to exist, it is possible using the
above expression for the probability of a rank order to construct most
stringent rank order tests. Under H_L it is shown that tests based on
small values of

$$T = \sum_{i=1}^{m+n} z_i D_{Ni}, \quad \text{where} \quad D_{Ni} = \sum_{j=1}^{m+n} j^{-1}$$

are locally most powerful and that T is approximately normally distributed
for large samples. It is indicated that decisions resulting from the T
and the Wilcoxon tests will frequently be the same; but the decisions of
the Wilcoxon and Terry tests will be in agreement even more often.
   Various tables are given of probabilities of the rank orders
under H_L, of the distribution of T, and of the power functions of
proposed tests. These tables enable one to conduct tests of H_L against
H_0 and to estimate their power for all combinations of sample sizes. An
important empirical conclusion which can be drawn from these tables is
that for all practical purposes the test based on T will give results
almost as powerful as the best test even for very small sample sizes.

   Publications: (1) "Easily used simultaneous confidence limits
for a line," by W. S. Connor; submitted to a technical journal. (2)
"Contributions to the theory of rank order statistics," by I. R. Savage;
submitted to a technical journal.

STUDIES IN THE MATHEMATICS OF EXPERIMENT DESIGN
Task 1103-10-1107/53-1

Origin: NBS   Authorized 10/15/52
Manager: W. S. Connor
Full task description: Oct-Dec 1952 issue, p. 60

Status: CONTINUED. 1. An alternate procedure which results in a more simplified computational technique has been worked out for analyzing chain block designs.

2. Let an observation y_{ij} have the mathematical model

$y_{ij} = \mu + t_i + \delta_j + \epsilon_{ij}$, $i=1,\ldots,v$; $j=1,\ldots,b$ where μ and t_i are fixed constants and ϵ_{ij} and δ_j are mutually independent random variables such that $E \epsilon_{ij} = 0$, $E \delta_j = 0$

$$\text{var } \epsilon_{ij} = \sigma^2, \quad \text{cov}(\epsilon_{ij}, \epsilon_{i'j'}) = 0 \quad \text{for } i \neq i' \text{ or } j \neq j',$$

$$\text{var } \delta_j = \sigma_b^2, \quad \text{cov}(\delta_j, \delta_{j'}) = 0 \quad \text{for } j \neq j' .$$

If the observations are taken using an incomplete block design satisfying the restriction $b > v$, then methods have been worked out for placing confidence limits on the ratio

$$p = \frac{\sigma_b^2}{\sigma^2} .$$

3. The present theory of factorial arrangements is useful only where the levels of the different factors are all at the same prime number (i.e., p^n series) or are all powers of the same prime number. Also, the present theory only allows the construction of designs having block sizes which are multiples of the level of the factors. Methods have recently been found for adapting incomplete block designs to mixed factorials where the restriction on the size of the block is less severe.

Publications: (1) "Analysis for some partially balanced incomplete block designs having a missing block," by M. Zelen; Biometrics 10, No. 2, 273-281 (June 1954). (2) "A note on partially balanced designs," by M. Zelen; Ann. Math. Stat. 25, No. 3, 599-602 (Sept. 1954). (3) "Partially balanced incomplete block designs with two associate classes and two treatments per block," by W. H. Clatworthy; accepted for publication in the Journal of Research of the NBS. (4) "New experimental designs for paired observations," by W. S. Connor and W. J. Youden; accepted for publication in the Journal of Research of the NBS. (5) "On the enumeration of partially balanced designs with two associate classes," by W.H.Clatworthy; submitted to a technical journal.

2. Applied Research in Mathematical Statistics

COLLABORATION ON STATISTICAL ASPECTS OF NBS
RESEARCH AND TESTING
Task 3737-60-0002/51-1

Origin: NBS   Authorized 7/1/50
Managers: W. J. Youden, J. Cameron
Full task description: July-Sept 1950 issue, p. 60

      Status: CONTINUED. Personnel of the Section are collaborating
with G. Conrad (Section 1.6, Engineering Electronics) on the statistical
aspects of his studies of resistor noise. Resistor noise varies from
resistor to resistor (and even for repeat measurements on the same
resistor). It is necessary to characterize the behavior of resistors
by statistical techniques which describe the behavior of groups of
resistors. The statistical aspects of this investigation have been
concerned with:
            (a) the underlying distribution of resistor noise
            (b) the statistical design of experiments to determine the
behavior of noise with respect to changes in frequency, current, and
resistance
            (c) the behavior of individual resistors over a long period
of time
            (d) comparison of noise characteristics of resistors from
different manufacturers.

      Publications: (1) "Instrumental drift," by W. J. Youden;
accepted for publication in Science. (2) "A comparison of four national
radium standards. Part II: Statistical procedures and survey," by
W. J. Youden and W. S. Connor; J. Res. NBS 53, 191-196 (Sept. 1954).

STATISTICAL ASPECTS OF NBS ADMINISTRATIVE OPERATIONS
Task 3737-60-0002/52-1

Origin: NBS   Authorized 10/1/51
Manager: I. R. Savage
Full task description: Oct-Dec 1951 issue, p. 56

      Status: INACTIVE. For status to date see July-Sept 1952 issue,
p. 68.

STATISTICAL SERVICES FOR COMMITTEE ON SHIP STEEL, NRC
Task 1103-40-5105/52-1

Origin: Ship Structure Committee, NRC   Authorized 12/1/51
Sponsor: "   "   "
Manager: W. J. Youden
Full task description: Oct-Dec 1951 issue, p. 58

      Status: CONTINUED. Work was begun on the construction of a program
of sampling of production of ship steel that would give information on
sources of variability in manufacturing.

RESEARCH IN APPLICATIONS OF MATHEMATICAL STATISTICS TO
PROBLEMS OF THE CHEMICAL CORPS
Task 1103-40-5118/52-1

Origin: Biological Laboratories, Chemical Corps   Authorized 10/1/51
          Dept. of the Army
Sponsor: "   "   "
Manager: C. Eisenhart
Full task description: Oct-Dec 1951 issue, p. 57

    Status: CONTINUED. A report giving a tabulation of fractional replications arranged in blocks of the 2^n series of factorial designs was submitted. Some work was done on methods of programming analysis of variance computations for large scale computers.

STATISTICAL ANALYSIS AND DESIGN OF EXPERIMENTS
FOR THE U. S. GEOLOGICAL SURVEY
Task 1103-40-5140/54-1

Origin: U. S. Geological Survey, Department of   Authorized 10/9/53
        Interior
Sponsor: "   "   "
Managers: C. Eisenhart, W. J. Youden
Full task description: Oct-Dec 1953 issue, p. 50

    Status: CONTINUED. Studies are being made of the variation in lithology of collections of pebbles taken at different sites, and the variations of samples taken at the same site.

STATISTICAL ANALYSIS OF BALL BEARING FATIGUE DATA
Task 1103-40-5145/54-1

Origin: American Standards Association   Authorized 1/13/54
Sponsor: "   "
Manager: J. Lieblein
Full task description: Jan-Mar 1954 issue, p. 54

    Status: CONTINUED. The method aimed at obtaining improved life estimates referred to in the previous report has been worked out. This method is in process of being applied to the test groups of ball bearing data. Programming of the necessary computations for the electronic computer (SEAC) has been completed and is undergoing a trial run. The revised estimates of L_{10} and L_{50} will form the basis of a regression study of the various parameters in the life formula including the exponent p.

IV. MATHEMATICAL PHYSICS SECTION
(Section 11.4)

RESEARCH IN MATHEMATICAL PHYSICS AND RELATED FIELDS
Task 1104-10-1115/55-57

Origin and Sponsor: NBS   Authorized 9/1/54
Manager: E. W. Cannon

  Objective: To perform research in various fields of applied
mathematics important in theoretical physics and engineering science,
with special emphasis on problems connected with the program of other
divisions of the Bureau.

  Background: During the fiscal year 1954 this program was carried
out under project 1102-10-1104. This project covers the basic formulation
of mathematical theories in new and currently developing fields, the
derivation of solutions to specific problems either in analytic form or
by reformulation in a manner suitable for computational procedures, and
the study of mathematical disciplines underlying these problems, such as
differential equations, special functions, stochastic processes, etc.

  Status: NEW.

RESEARCH IN MECHANICS OF CONTINUA
Task 1104-10-5160/54-23

Origin: NBS   Authorized 12/29/53
Sponsor: Office of Naval Research, USN
Manager: R. F. Dressler
Full task description: Oct-Dec 1953 issue, p. 27

  Status: CONTINUED. A talk on "Entropy changes in the equations
for rarefaction waves" was presented by R. F. Dressler at the International
Congress of Mathematicians, held in Amsterdam, in September. The manuscript
incorporating these results is now being brought into final form for
publication. Another talk on the "Comparison of theories and experiments
for the hydraulic dam-break wave" was given by Dr. Dressler at the meeting
of the International Union of Geodesy and Geophysics, Rome, Italy, in
September, and the manuscript presenting this material has been accepted
for publication in the Proceedings of the I.U.G.G., Hydrology Section.
  Results on an aerodynamic model for unsteady flow which considers
frictional forces and varying shock tube cross-section, but neglects heat
generation, was completed; the paper describing these results appears in the
Journal of Research of the NBS, October 1954 (RP2541).
  P. Henrici, in collaboration with P. Chaix, completed a paper
concerning the design of two-dimensional supersonic nozzles. The authors
use the Mach line originating from the throat (the S-line) as carrier
of the initial data. The geometrical data of the S-line are expressed
in terms of three geometrical parameters of the nozzle. Velocity
distribution on the center line is not assumed to be linear. A study
of the transonic region in nozzles using T. M. Cherry's hodograph method

has also been started.

   M. Abramowitz has resumed a study of heat transfer in laminar cylindrical flow. In this investigation the axial temperature gradients are not suppressed in comparison with radial gradiants.   '

   Publications: (1) "Turbulent flow in shock tubes of varying cross-section," by R. F. Dressler; accepted for publication in the Journal of Research of the NBS. (2) "Comparison of theories and experiments for the hydraulic dam-break wave," by R. F. Dressler; to appear in Proceedings of the International Union of Geodesy and Geophysics, Rome, Italy, 1954. (3) "Entropy changes in rarefaction waves," by R. F. Dressler; IN MANUSCRIPT. (4) "On helical elastic springs of finite cross-section," by P. Henrici; to appear in the Quarterly of Applied Mathematics. (5) "On rotational viscous flow through a tube" by P. Henrici; to appear in Zeitschrift für angewandte Mathematik und Physik. (6) "On the design of two-dimensional supersonic nozzles" by B. Chaix and P. Henrici; IN MANUSCRIPT.

RESEARCH IN ELECTROMAGNETIC THEORY
Task 1104-10-5160/54-47

Origin and Sponsor: Diamond Ordnance Fuze Laboratory, Authorized 6/29/54
        Department of the Army   Revised 9/29/54
Manager: F. Oberhettinger

   Objective: (1) To investigate the asymptotic expansion of certain integrals. (2) To investigate the convergence of various Green's function representations for the electromagnetic field in homogeneous and inhomogeneous enclosed media. (3) To investigate the problem of the diffraction of electric and acoustic waves by certain obstacles which consist of an array of half planes.

   Background: (1) The solution of a number of diffraction problems can be reduced to integral representations which are of the form of a Kontorovich transform. The behavior of such an integral transform for large values of the variable is of importance for the knowledge of the behavior of certain asymptotic properties of the diffracted field. (2) Green's function of the wave equation for the three-dimensional case of a rectangular cross section (rectangular wave guide) can be represented in the form of an infinite double series (representation by modes). This representation becomes useless when the cross section which contains the source point is approached. Alternate expressions which do not show this behavior are therefore desired. (3) This research is directed towards an extension and generalization of previously obtained results (task 1102-10-5160/54-51, p. 29).

   Status: CONTINUED. F. Oberhettinger did preliminary work on the problem of diffraction of a wave field generated by a dipole with arbitrary orientation on an ideally conducting half-plane.

DIFFRACTION OF WAVES
Task 1104-10-5160/54-51

Origin and Sponsor: NBS, Division 14   Authorized 6/29/54
Manager: F. Oberhettinger
Full task description: Apr-June 1954 issue, p. 22

        Status: COMPLETED.

        Publications: (1) "Diffraction of waves by a wedge," by F. Oberhettinger;
Communications on Pure and Applied Mathematics VII, 551-563 (Aug. 1954).
(2) "On asymptotic series for functions occurring in the theory of
diffraction of waves by wedges," by F. Oberhettinger; submitted to a
technical journal.

NATIONAL BUREAU OF STANDARDS EASTERN AUTOMATIC COMPUTER (SEAC)

As a result of recent reorganizations it has become necessary to move SEAC to another building on the Bureau grounds. During this quarter the new site has been prepared. It is anticipated that the installation will be shut down on November 15 for the move and will be out of operation for an estimated period of six to eight weeks. In preparation for the move a new chassis layout for the computer proper has been made, and a signal lead harness identifying every lead has been designed and is being installed. A new frame and base have been erected in the new location, and a contractor is now in process of installing a redesigned primary power distribution system. Also, a new clock distribution system has been installed so that as soon as SEAC is shut down each chassis can be taken from the old to the new location and immediately plugged in. This advance engineering has been done in the hope that it will minimize the length of time required for putting SEAC back into operation.

During this quarter (the third quarter of its operation as a general Bureau facility), the SEAC has been operated with an over-all efficiency of 89°/o during scheduled operation. This compares quite favorably with its average of 75°/o for its first four years of operation (May 1950 to May 1954). Scheduled computation time was distributed as follows:

```
Total scheduled time available for computation . . 843 hours
Good operating time  . . . . . . . . . . . . . . . 742   "
Idle-in-order time . . . . . . . . . . . . . . . .   9   "
Time lost during or following malfunction  . . . .  92   "
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In addition, the SEAC was utilized for 363 hours of nonscheduled computation, i.e., operation without a duty engineer in attendance for machine maintenance, and 379 hours for engineering and preventive maintenance.

The record of SEAC operations from July 1 through September 30 is as follows:

Task No.	Title	Hours Used: Code Checking	Productive Operation
NBS:			
1104/55-55	Research in numerical analysis	61	25
5116/54-9	Supporting research in linear programming	5	10
1110/47-2	Tables of Coulomb wave function		13
1110/50-7	Wave function for lithium	6	50
1110/51-8	Power points of analysis of variance tests	1	
1110/52-49	Mathieu functions		7
1110/53-52	Internal conversion coefficients		23
1110/55-65	Automatic coding	3	3
5126/50-13	Ray tracing	9	33
0009/52-44	Calculations for d spacings	6	8
5126/53-27	Thermodynamic functions		4
5126/53-29	Dynamic behavior of aircraft structures	1	43
5126/53-51	Radiation diffusion	24	143
5126/54-4	High temperature properties of water	27	8
0009/54-8	Normal vibrations in molecules		2
0009/54-15	Matrix reduction	10	12
0009/54-19	Energy levels of complex atoms		2

30

Task No.	Title	Hours Used:	
		Code Checking	Productive Operation
NBS:			
0009/54-22	Energy distribution on optical image	3	24
0009/54-27	EMF of cells at high temperature		2
5126/54-33	Air conditioning in underground structures	3	6
0009/54-36	Velocity of light		3
5126/54-43	Characteristics of conducting resistors	11	1
5126/54-45	Computation of transient heat flow		34
5126/54-49	Wiechert distribution function	20	
5126/55-39	Molecular vibrations	12	1
5126/55-63	Morse integrals	9	45
5126/55-64	High-temperature thermodynamic tables	1	
0009/55-66	Reconstitution of light intensities	12	2
5126/55-67	Dosage integrals	1	1
5126/55-70	Ball bearing fatigue data	3	
	Totals:	228	505
Other:			
1104/55-59	Turbulent atmospheric contamination	7	23
5126/53-4	Neutron diffusion	2	30
5126/53-45	Theory of games	17	2
5126/53-53	Computing services to Chemical Corps	2	132
5126/54-13	Award of procurement contracts by linear programming	4	47
5126/54-25	Reflected radiation from an infinite Lambert plane	2	6
5126/54-34	Equilibrium calculations for water	2	2
5126/54-40	Flight simulator		7
5126/54-44	Flight performance	2	54
5126/55-52	Atmospheric waves	1	15
5126/55-60	Detonator voltage and current on oscillograms	9	
5126/55-61	Elastic neutron scattering	6	
5126/55-62	Integrals involved in supersonic flutter		6
5126/55-71	Loran tables		19
5126/55-72	Maryland inter-industry study		3
	Totals:	54	346

Lectures and Symposia

Note: In general, copies of papers or talks listed in this section are not available from the National Bureau of Standards. If and when a paper is to be published, it will be listed in the section of this report on Publication Activities.

<u>Papers and Invited Talks</u>
<u>Presented by Members of the Staff</u>
<u>at Meetings of Outside Organizations</u>

Papers presented at the International Congress of Mathematicians, Amsterdam, Netherlands, September 2-9:

ALT, F. L. Recent results on the equation of burning.

DRESSLER, R. F. Entropy changes in the equation for rarefaction waves.

TAUSSKY-TODD, O. Normal matrices in some problems in algebraic number theory.

TODD, J. The condition of matrices.

Papers presented at the meeting of the American Mathematical Society, Laramie, Wyoming, August 31 - September 3:

ANTOSIEWICZ, H. A. Stable systems of differential equations with integrable forcing term.

DAVIS, P., and H. O. POLLACK (Bell Telephone Laboratories). Complex biorthogonality for certain sets of polynomials.

Papers presented at the annual meeting of the American Statistical Association, Montreal, Canada, September 10-13:

EISENHART, C. The joint effects of reading errors and grouping on standard methods of statistical inference. Presented at the session sponsored by the Committee on Statistics in the Physical Sciences.

ZELEN, M. (1) The use of incomplete block designs for factorial experiments. Presented at a session sponsored by the Committee on Statistics in the Physical Sciences. (2) Exact tests of significance for combining inter- and intra-block information in incomplete block designs.

Papers presented at other meetings:

ABRAMOWITZ, M.   Program for mathematical tables at the National Bureau of
                Standards. Presented at the Symposium on Mathematical
                Tables held at the Massachusetts Institute of Technology,
                Cambridge, Mass., September 15-16.

CONNOR, W. S.   Statistical procedures. Presented at the AAAS Gordon
                Research Conference on Coal, held in New Hampton, N. H.,
                August 25.

DRESSLER, R. F.   Comparison of theories and experiments for the hydraulic
                dam-break wave. Presented at the International Geophysics
                Meeting, Rome, Italy, September 1954.

TAUSSKY-TODD, O.   Bounds for characteristic roots of finite matrices,
                and their application in numerical analysis. (1) Presented
                at the Institute of Mathematics, Mechanics, and Theoretical
                Physics, University of Lund, Sweden, September 23. (2)
                Presented at the Mathematical Institute, University of
                Stockholm, Sweden, September 27.

TODD, J.   (1) Relative difficulty of the solution of differential equations
                of different orders and dimensions. Presented at the
                Institute of Mathematics, Mechanics, and Theoretical
                Physics, University of Lund, Sweden, September 23.
                (2) The solution of large systems of linear equations --
                theory and practice. Presented at the Mathematical
                Institute, University of Stockholm, Sweden, September 24.

WEGSTEIN, J. H.   (1) Automatic programming and a general purpose pseudo-
                code. Presented at the David Taylor Model Basin,
                Washington, D. C., August 23. (2) Automatic coding.
                Presented to the Logistics Research Project, George
                Washington University, Washington, D. C., September 30.

YOUDEN, W. J.   (1) Control of experimental error by statistical design,
                and (2) New statistical designs useful in the physical
                sciences. Presented at the Shell Oil Company Seminar on
                Statistical Methods, Houston, Texas, July 1-2. (3) The
                statistical part of the planning of chemical experiments.
                Presented at the American Association for the Advancement
                of Science, Gordon Research Conference on Statistics in
                Chemistry, New Hampton, N. H., July 26. (4) Statistical
                design as an aid to improving the precision of instruments.
                Presented at the American Association for the Advancement
                of Science, Gordon Conference on Instrumentation, New
                London, N. H., July 30. (5) Interpretation of chemical
                data. Presented to the Lehigh Valley Section of the
                American Chemical Society, Palmerton, Pa., September 17.
                (6) Marbles, measurements, and mathematics. Presented
                to the Washington Junior Academy of Science, Washington, D.C.,
                September 18. (7) The gun problem. Presented to the
                staff of Logistics Research Project, George Washington
                University, Washington, D. C., September 23.

Publication Activities

1. PUBLICATIONS WHICH APPEARED DURING THE QUARTER

1.1 Mathematical Tables

(1) Table of sine and cosine integrals for arguments from 10 to 100. NBS Applied Mathematics Series 32. (A reissue of NBS Mathematical Table MT13). Available from the U. S. Government Printing Office, Washington 25, D. C., $2.25.

(2) Table of the Gamma function for complex arguments. NBS Applied Mathematics Series 34. Available from the U. S. Government Printing Office, Washington 25, D. C., $2.00.

(3) Tables of the inverses of finite segments of the Hilbert matrix. I. R. Savage and E. Lukacs. Included in Contributions to the solution of systems of linear equations and the determination of eigenvalues, NBS Applied Mathematics Series 39, pp. 105-108; available from the U. S. Government Printing Office, Washington, 25, D. C., $2.00.

1.3 Technical Papers

The first six papers listed below are included in "Contributions to the Solution of Linear Equations and the Determination of Eigenvalues," NBS Applied Mathematics Series 39, available from the U. S. Government Printing Office, Washington 25, D. C., $2.00.

(1) Inequalities for eigenvalues of Hermitian matrices. K. Fan. Pp. 131-139.

(2) Lower bounds for the rank and location of the eigenvalues of a matrix. K. Fan and A. J. Hoffman. Pp. 117-130.

(3) Punched-card experiments with accelerated gradient methods for linear equations. A. I. Forsythe and G. E. Forsythe. Pp. 55-69.

(4) Practical solution of linear equations and inversion of matrices. L. Fox. Pp. 1-54.

(5) Iterative methods of solving linear problems on Hilbert space. R. M. Hayes. Pp. 71-103.

(6) The condition of the finite segment of the Hilbert matrix. J. Todd. Pp. 109-116.

. . . .

(7) The relaxation method for linear inequalities. S. Agmon. Can. J. Math. 6, 382-392 (1954).

(8) Some implications of Liapunov's conditions for stability. H. A. Antosiewicz and P. Davis. Journal of Rational Mechanics and Analysis 3, 447-457 (July 1954).

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(9) Asymptotic lower bounds for the frequencies of polygonal membranes. G. E. Forsythe. Pac. J. Math. $\underline{4}$, 467-480 (Sept. 1954).

(10) Linear programming in bid evaluation. L. Gainen, D. J. Honig, and E. D. Stanley. Logistics Research Quar. $\underline{1}$, 48-54 (Mar. 1954).

(11) Variational principles for the acoustic field. E. Gerjuoy and D. S. Saxon. The Physical Review $\underline{94}$, 1445 (June 15, 1954).

(12) Optimum grouping in one-criterion variance components analysis. E. P. King. J. Am. Stat. Assoc. $\underline{49}$, 637-639 (Sept. 1954).

(13) On the relaxation method for linear inequalities. T. S. Motzkin and I. J. Schoenberg. Can. J. Math. $\underline{6}$, 393-404 (1954).

(14) Diffraction of waves by a wedge. F. Oberhettinger. Communications on Pure and Applied Mathematics $\underline{7}$, 551-563 (Aug. 1954).

(15) The evaluation of the exponential integral for large complex arguments. J. Todd. J. Res. NBS $\underline{52}$, 313-318 (June 1954).

(16) Statistics and planning tests at elevated temperatures. W.J.Youden. Proceedings of the Society for Experimental Stress Analysis $\underline{XI}$, No. 2, 219-222 (Nov. 1950).

(17) A comparison of four national radium standards. Part II: Statistical procedures and survey. W. J. Youden and W. S. Connor. J. Res. NBS $\underline{53}$, 191-196 (Sept. 1954).

(18) A note on partially balanced designs. M. Zelen. Ann. Math. Stat. $\underline{25}$, 599-602 (Sept. 1954).

(19) Analysis for some partially balanced incomplete block designs having a missing block. M. Zelen. Biometrics $\underline{10}$, 273-281 (June 1954).

1.5 Miscellaneous Publications

(1) Contributions to the solution of systems of linear equations and the determination of eigenvalues. NBS Applied Mathematics Series 39. Available from U. S. Government Printing Office, Washington 25, D. C., $2.00.

2. MANUSCRIPTS IN THE PROCESS OF PUBLICATION SEPTEMBER 30, 1954

2.1 Mathematical Tables

(1) Tables of functions and of zeros of functions. Collected short tables of the Computation Laboratory. NBS Applied Mathematics Series 37. In press, Government Printing Office.

(2) Tables of the error function and its derivative. NBS Applied Mathematics Series 41. (A reissue of NBS Mathematical Table 8, Tables of probability functions, vol. I.) In press, Government Printing Office.

(3) Tables of sines and cosines for radian arguments. NBS Applied Mathematics Series 43. (A reissue of NBS Mathematical Table 4.) In press, Government Printing Office.

(4) Table of salvo kill probabilities for square targets. NBS Applied Mathematics Series 43. In press, Government Printing Office.

2.3 Technical Papers

(1) Coulomb wave functions in the transition region. M. Abramowitz
    and H. Antosiewicz. Accepted for publication in The Physical Review.

(2) Approximate method for rapid Loran computation. M. Abramowitz,
    D. H. Call, and J. C. Mathews. Submitted to a technical journal.

(3) Coulomb wave functions along the transition line. M. Abramowitz
    and P. Rabinowitz. Accepted for publication in The Physical Review.

(4) A theorem on alternatives for pairs of matrices. H. Antosiewicz.
    Accepted for publication in the Pacific Journal of Mathematics.

(5) Asymptotic solution of linear differential equations with a
    parameter. H. Antosiewicz. Submitted to a technical journal.

(6) On a certain integral involving Bessel functions. H. Antosiewicz.
    Submitted to a technical journal.

(7) On nonlinear differential equations of the second order with
    integrable forcing term. H. Antosiewicz. Accepted for publication
    in the Journal of the London Mathematical Society.

(8) On the differential equation $\ddot{x} + k(f(x) + g(x)\dot{x})\dot{x} = ke(t)$.
    H. A. Antosiewicz. Submitted to a technical journal.

(9) Stable systems of differential equations with integrable forcing
    term. H. Antosiewicz. Submitted to a technical journal.

(10) A representation for solutions of analytic systems of linear
     differential equations. H. A. Antosiewicz and M. Abramowitz.
     Accepted for publication in the Journal of the Washington Academy
     of Sciences.

(11) A remark on the smoothing problem. H. Antosiewicz and A. J. Hoffman.
     Accepted for publication in the Transactions of the Institute of
     Management Science.

(12) Computation of vibration modes and frequencies on SEAC.
     W. F. Cahill and S. Levy (NBS 6.4). Submitted to a technical
     journal.

(13) On the enumeration of partially balanced designs with two associate
     classes. W. H. Clatworthy. Submitted to a technical journal.

(14) Partially balanced incomplete block designs with two associate
     classes and two treatments per block. W. H. Clatworthy. Accepted
     for publication in the Journal of Research of the NBS.

(15) Easily used simultaneous confidence limits for a line. W.S.Connor.
     Submitted to a technical journal.

(16) Experimental designs for paired observations. W. S. Connor and
     W. J. Youden. Accepted for publication in the Journal of Research
     of the NBS.

(17) Time-discrete stochastic processes in arbitrary sets, with
     applications to processes with absorbing regions and to the problem
     of loops in Markoff chains. D. van Dantzig. Accepted for
     publication (in French) in Annales de l'Institut Henri Poincaré
     (Paris).

(18) Linear functional equations and interpolation series. P. Davis.
Accepted for publication in the Pacific Journal of Mathematics.

(19) On a problem in the theory of mechanical quadratures. P. Davis.
Accepted for publication in the Pacific Journal of Mathematics.

(20) Some L^2 aspects of Faber polynomials. P. Davis and H. Pollack
(Bell Telephone Laboratories). Accepted for publication in the
Duke Mathematical Journal.

(21) A multi-purpose orthonormalizing code and its uses. P. Davis
and P. Rabinowitz. Accepted for publication in the Journal of
the Association for Computing Machinery.

(22) On the estimation of quadrature errors for analytic functions.
P. Davis and P. Rabinowitz. Accepted for publication in
Mathematical Tables and Other Aids to Computation.

(23) Some SEAC computations of subsonic fluid flows by Bergman's
method of integral operators. P. Davis and P. Rabinowitz.
To be an appendix in the volume, "Bergman's Operator Method,"
by M.Z. v.Krzywoblocki (University of Illinois).

(24) Some sampling results on the power of nonparametric tests against
normal alternatives. W. J. Dixon (University of Oregon) and
D. Teichroew. Submitted to a technical journal.

(25) Turbulent flow in shock tubes of varying cross-section. R.F.Dressler.
Accepted by the Journal of Research of the NBS.

(26) On the optimal character of the (s,S) policy in inventory theory.
A. Dvoretzky, J. Kiefer, and J. Wolfowitz. Submitted to a
technical journal.

(27) A comparison theorem for eigenvalues of normal matrices. K. Fan.
Submitted to a technical journal.

(28) Some inequalities concerning positive-definite Hermitian matrices.
K. Fan. Submitted to a technical journal.

(29) Some metric inequalities in the space of matrices. K. Fan and
A. J. Hoffman. Accepted for publication in the Proceedings of
the American Mathematical Society.

(30) Discrete analogs of inequalities of Wirtinger. K. Fan, O.Taussky,
and J. Todd. Submitted to a technical journal.

(31) A determinantal inequality. K. Fan and J. Todd. Accepted for
publication in the Journal of the London Mathematical Society.

(32) Asymptotic lower bounds for the fundamental frequency of convex
membranes. G. E. Forsythe. Accepted for publication in the
Pacific Journal of Mathematics.

(33) On the solution of the caterer problem. J. W. Gaddum, A.J. Hoffman,
and D. Sokolowsky. To appear in the Logistics Research Quarterly.

(34) An algorithm for solving the transportation problem. A. Gleyzal.
Accepted for publication in the Journal of Research of the NBS.

(35) Equations of physics in general Newtonian space-time. A.N.Gleyzal.
Submitted to a technical journal.

(36) The use of extreme-value methods in engineering problems.
     E. J. Gumbel and J. Lieblein. Accepted for publication in the
     American Statistician.

(37) On certain series expansions involving Whittaker functions and
     Jacobi polynomials. P. Henrici. Accepted for publication in
     the Pacific Journal of Mathematics.

(38) On generating functions of the Jacobi polynomials. P. Henrici.
     Accepted for publication in the Pacific Journal of Mathematics.

(39) On helical springs of finite thickness. P. Henrici. Accepted
     for publication in the Quarterly of Applied Mathematics.

(40) On rotational viscous flow through a tube. P. Henrici. Accepted
     for publication in ZAMP.

(41) Smooth patterns of production. A. J. Hoffman and W. Jacobs.
     Accepted for publication in the Transactions of the Institute
     of Management Science.

(42) The number of absolute points of a correlation. A. J. Hoffman,
     M. Newman, E. G. Straus, and O. Taussky. Submitted to a technical
     journal.

(43) Contractibility and convexity. H. W. Kuhn (Bryn Mawr College).
     Accepted for publication in the Proceedings of the American
     Mathematical Society.

(44) On certain character matrices. D. H. Lehmer. Submitted to a
     technical journal.

(45) Indentation pressure of a smooth punch. E. Levin. Accepted for
     publication in the Quarterly of Applied Mathematics.

(46) Acoustic radiation pressure on a circular disk. H. Levine.
     To appear in the Proceedings of the Fifth Symposium on Applied
     Mathematics of the American Mathematical Society, held in
     Pittsburgh, Pa., June 1952.

(47) On the variances and covariances of order statistics from the
     Weibull distribution. J. Lieblein. Submitted to a technical
     journal.

(48) A proof of Hilbert's Nullstellensatz. T. S. Motzkin. Submitted
     to a technical journal.

(49) The assignment problem. M. Motzkin. To appear in the Proceedings
     of the American Mathematical Society's Sixth Symposium on Applied
     Mathematics, held at Santa Monica City College, August 1953.

(50) On Fejér sets in linear and spherical spaces. T. S. Motzkin and
     I. J. Schoenberg. Accepted for publication in Annals of
     Mathematics.

(51) Pairs of matrices with property L, II. T. S. Motzkin and O. Taussky.
     Submitted to a technical journal.

(52) Least p-th power polynomials on a real finite point set. T.S. Motzkin
     and J. L. Walsh. Submitted to a technical journal.

(53) The diophantine equation $ax^m - by^n = c$. M. Newman. Submitted to a
     technical journal.

(54) Structure theorems for modular subgroups. M. Newman. Accepted for publication in the Duke Mathematical Journal.

(55) On asymptotic series for functions occurring in the theory of diffraction of waves by a wedge. F. Oberhettinger. Submitted to a technical journal.

(56) On the Lerch zeta function. F. Oberhettinger. Submitted to a technical journal.

(57) On two problems in abstract algebra connected with Horner's rule. A. M. Ostrowski. To appear in the von Mises Anniversary Volume.

(58) Determinanten mit ueberwiegender Hauptdiagonale und die absolute Konvergenz von linearen Iterationsprozessen. A. M. Ostrowski. Submitted to a technical journal.

(59) On the convergence of Gauss' alternating procedure in the method of the least squares, I. A. M. Ostrowski. Submitted to a technical journal.

(60) On the linear iteration procedures for symmetric matrices. A. M. Ostrowski. Accepted for publication in Rendiconti di Matematico, Rome.

(61) On the spectrum of a one parametric family of matrices. A. M. Ostrowski. Submitted to a technical journal.

(62) On Gauss' speeding up device in the theory of single step iteration. A. M. Ostrowski. Submitted to a technical journal.

(63) On absolute convergence of linear iteration processes. A. M. Ostrowski. Submitted to a technical journal.

(64) On spectra of second-order differential operators. D. Ray. Submitted to a technical journal.

(65) On the convergence of asymptotic solutions of linear differential equations. R. M. Redheffer (U.C.L.A.) and W. Wasow. Submitted to a technical journal.

(66) Numerical computation of low moments of order statistics from a normal population. J. B. Rosser. Submitted to a technical journal.

(67) Osculatory interpolation in the complex plane. H. E. Salzer (Department of the Army). Accepted for publication in the Journal of Research of the NBS.

(68) Contributions to the theory of rank order statistics. I.R.Savage. Submitted to a technical journal.

(69) An isoperimetric inequality for closed curves convex in even-dimensional Euclidean space. I. J. Schoenberg. Accepted for publication in Acta Mathematica.

(70) Generalized commutators of matrices and permutations of factors in a product of three matrices. O. Taussky. For publication in the von Mises Anniversary volume.

(71) Some computational problems in algebraic number theory. O.Taussky. To appear in the Proceedings of the American Mathematical Society Sixth Symposium on Applied Mathematics, held at Santa Monica City College, August 26-28, 1953.

(72) Generation and testing of pseudo-random numbers. O.Taussky
and J. Todd. Submitted to a technical journal.

(73) An improved cathode ray tube storage system. R. Thorensen. To
appear in the Proceedings of the Western Computer Conference of
the AIEE-IRE-ACM held in Los Angeles, Calif., February 4,5,6,1953.

(74) Experiments in the solution of differential equations by Monte Carlo
methods. J. Todd. Submitted to a technical journal.

(75) Motivation for working in numerical analysis. J. Todd. To appear
in the Transactions of a Symposium on Applied Mathematics sponsored
by the Office of Ordnance Research and held in Chicago, Ill.,
April 29, 1954.

(76) The condition of certain matrices, II. J. Todd. To appear in
Archiv der Mathematik.

(77) An extension of a theorem of Dantzig's. C. Tompkins and I.Heller
(George Washington University). Submitted to a technical journal.

(78) A method for the numerical integration of differential equations
of second order without explicit first derivatives. R. de Vogelaere.
Accepted for the Journal of Research of the NBS.

(79) Discrete approximations to elliptic differential equations.
W. Wasow. Accepted for publication in Zeitschrift fur angewandte
Mathematik und Physik.

(80) On the asymptotic transformation of certain distributions into
the normal distribution. W. Wasow. To appear in the Proceedings
of the American Mathematical Society Sixth Symposium on Applied
Mathematics, held at Santa Monica City College, August 26-28,1953.

(81) An extremum property of sums of eigenvalues. H. Wielandt.
Accepted for publication in the Proceedings of the American
Mathematical Society.

(82) Error bounds for the eigenvalues of symmetric integral equations.
H. Wielandt. To appear in the Proceedings of the American
Mathematical Society Sixth Symposium on Applied Mathematics, held
at Santa Monica City College, August 26-28, 1953.

(83) On eigenvalues of sums of normal matrices. H. Wielandt. Submitted
to a technical journal.

(84) Instrumental drift. W. J. Youden. To appear in Science.

(85) Bounds on a distribution function which are functions of moments
to order four. M. Zelen. Accepted by the Journal of Research
of the NBS.

2.4 Reviews, Notes

(1) Note on the circle theorem of hydrodynamics. E. Levin. Accepted
for publication by the Quarterly of Applied Mathematics.

(2) Two early papers on the relation between extreme values and tensile
strength. (Formerly "A historical note on the application of the
'weakest link' idea to tensile strengths.") J. Lieblein.
To appear in Biometrika.

2.5 Miscellaneous Publications

    (1) Experiments in the computation of conformal maps. NBS Applied
        Mathematics Series 42. In press, Government Printing Office.

THE NATIONAL BUREAU OF STANDARDS

Functions and Activities

The functions of the National Bureau of Standards are set forth in the Act of Congress, March 3, 1901, as amended by Congress in Public Law 619, 1950. These include the development and maintenance of the national standards of measurement and the provision of means and methods for making measurements consistent with these standards; the determination of physical constants and properties of materials; the development of methods and instruments for testing materials, devices, and structures; advisory services to Government Agencies on scientific and technical problems; invention and development of devices to serve special needs of the Government; and the development of standard practices, codes, and specifications. The work includes basic and applied research, development, engineering, instrumentation, testing, evaluation, calibration services, and various consultation and information services. A major portion of the Bureau's work is performed for other Government Agencies, particularly the Department of Defense and the Atomic Energy Commission. The scope of activities is suggested by the listing of divisions and sections on the inside of the front cover.

Reports and Publications

The results of the Bureau's work take the form of either actual equipment and devices or published papers and reports. Reports are issued to the sponsoring agency of a particular project or program. Published papers appear either in the Bureau's own series of publications or in the journals of professional and scientific societies. The Bureau itself publishes three monthly periodicals, available from the Government Printing Office: The Journal of Research, which presents complete papers reporting technical investigations; the Technical News Bulletin, which presents summary and preliminary reports on work in progress; and Basic Radio Propagation Predictions, which provides data for determining the best frequencies to use for radio communications throughout the world. There are also five series of nonperiodical publications: The Applied Mathematics Series, Circulars, Handbooks, Building Materials and Structures Reports, and Miscellaneous Publications.

Information on the Bureau's publications can be found in NBS Circular 460, Publications of the National Bureau of Standards ($1.00). Information on calibration services and fees can be found in NBS Circular 483, Testing by the National Bureau of Standards (25 cents). Both are available from the Government Printing Office. Inquiries regarding the Bureau's reports and publications should be addressed to the Office of Scientific Publications, National Bureau of Standards, Washington 25, D. C.

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